Public

Introduction to Monte Carlo Simulation in Finance

Simulating Interest Rates Derivatives Profiles for Counterparty Credit Risk Management

Renato Barros
UBS

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Section 1

Counterparty Credit Risk

Counterparty Credit Risk

<<Message>>

- UBS, and other IBs, trade derivatives with various counterparties worldwide;
- Typical scenario:
  - Company XYZ issues $1bi of 15y bonds at fixed rate;
  - Company XYZ enters into a swap with UBS to transform its fixed rate liability into a floating one, ie, UBS pays fixed rate to XYZ Co. vs receiving floating for 15y in $1bi;
  - To hedge market risk, UBS enters into offsetting transactions with another Bank.

What can go wrong?

Expected Loss

<<Message>>

- Expected Loss = EAD x PD x LGD
  - EAD: Exposure at Default
  - PD: Probability of Default
  - LGD: Loss Given Default
- Example:

  $100mio 5y loan to a company

  Company has no tangible assets

  Probability of default over next 5y is 5%

  1/ What is the EAD?

  2/ What is the LGD roughly?
- **For a corporate loan, it is easy to determine the EAD (Exposure at Default). But how can we do it for a portfolio of derivatives?**

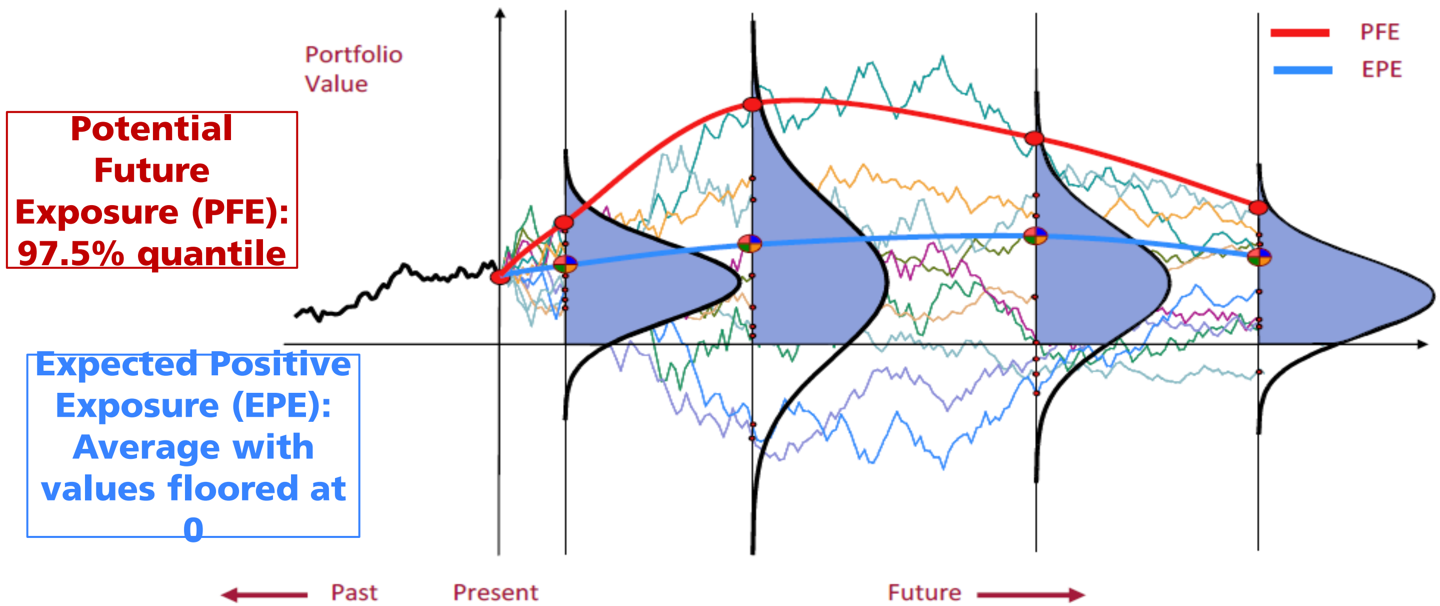
Section 2

Modelling Counterparty Credit Risk

Modelling Counterparty Credit Risk

Expected Loss in the context of derivatives

- We can simulate the evolution of the portfolio value over time



- The equivalent of Expected Loss in derivatives world is called CVA (Credit Valuation Adjustments):

$$\mathrm{CVA}_{0,T} = \overbrace{(1-R_V)}^{\text{LGD}} \int_0^T \mathbb{E}\left[\overbrace{N_u^{-1}\ V_u^{+}}^{\text{EAD part}}\ \overbrace{\mathbb{1}_{\tau=u}}^{\text{PD part}}\, du\right]$$

approx.

Present Value of EPE

$$CVA = \sum_{n=1}^{N} LGD * PD(T_{n-1}, T_n) * EEPV\ (T_{n-1}, T_n)$$

Modelling Counterparty Credit Risk

CCR Modelling Recipe

Goal	**Obtain risk profiles (EPE, PFE, VaR) for a given portfolio;** **In order to compute CVA, RWA and regulatory capital.**
Recipe	1/ Choose a suitable risk factor model 2/ Calibrate the model 3/ Generate simulated paths, ie, simulate a variable of interest, eg.: short rate, stock price, spot fx; 4/ Price the portfolio 5/ Compute profiles 6/ Compute measures like CVA, FVA, RWA, Regulatory Capital
Algorithms Involved	1/ Correlated random number generation (SVD, Cholesky Decomposition) 2/ Calibration – Bootstrapping, Minimum Squared Error, Closed Form Pricing 3/ Pricing 4/ Backward Induction / AMC

Modelling Counterparty Credit Risk

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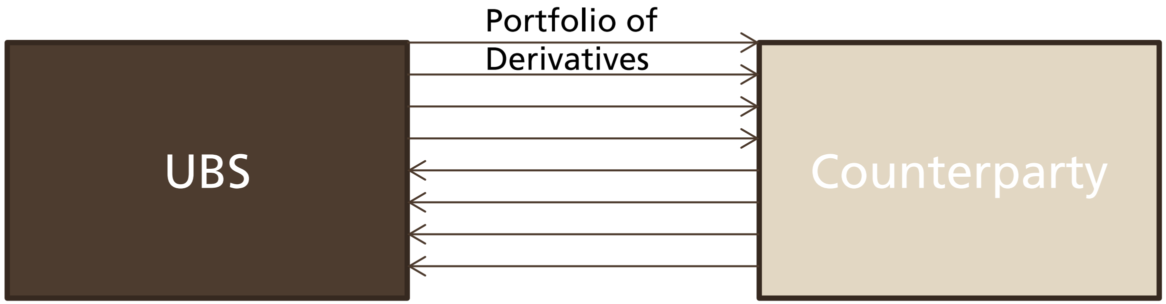
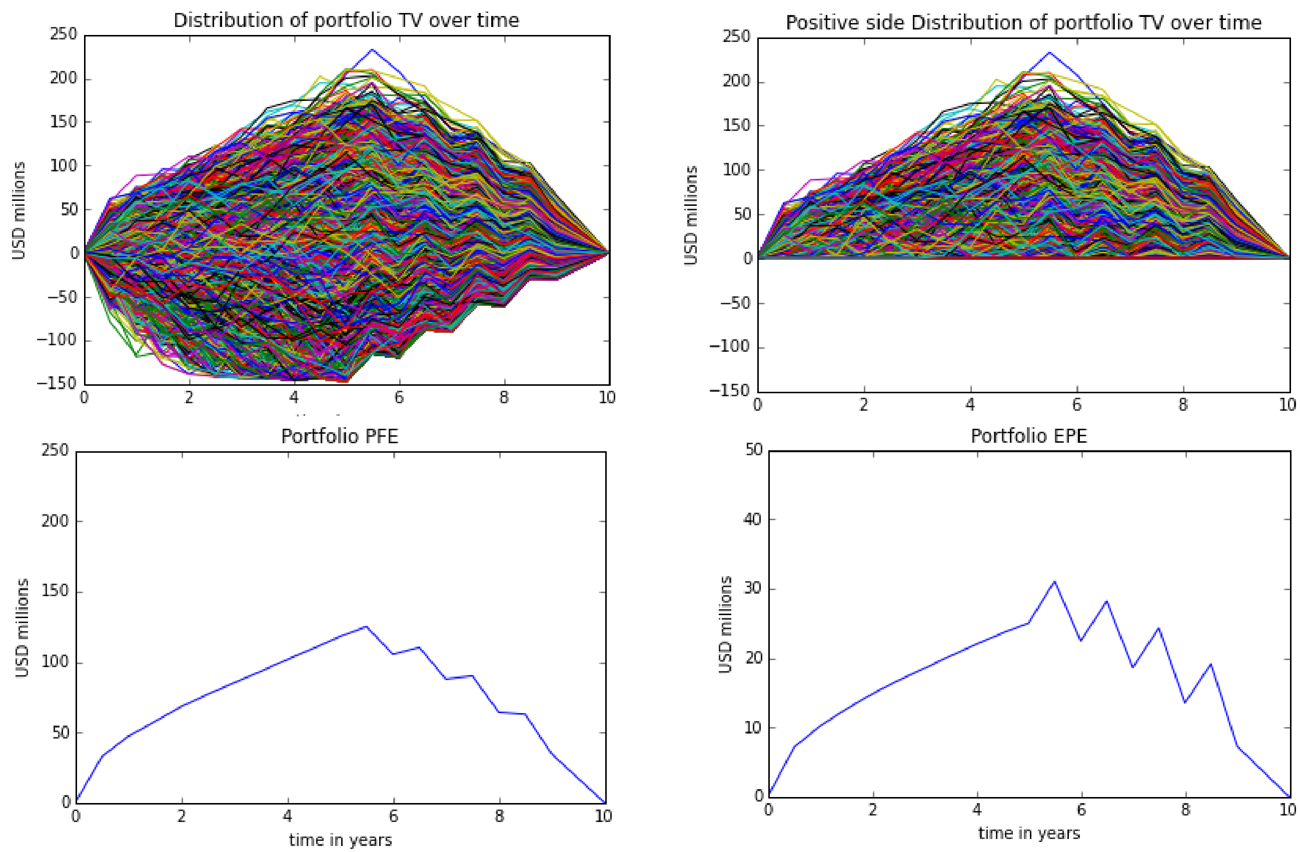
Portfolio Simulation





Modelling Counterparty Credit Risk

Profiles

Profile	Description	How to compute
EPE_t	Expected Postivive Exposure: Expectation of the portolio value floored at zero.	$EPE_t = P(0, t) \text{ x } E[(V_t - C_t)^+ B_t^{-1}]$
PFE_t	Potential Future Exposure: 97.5% quantile of the portfolio value at time t	$PFE_t = \inf\{x: P[(V_t - C_t) \leq x] \geq \alpha\}, \alpha = 97.5$
Reverse EPEt	Equivalent to EPE_t but from the counterparty's point of view	
Total Claim Mean (TCM_t)	For a given close-out period of length δ, also called margin period of risk (MPR), the TCM is given by the formula on the RHS.	$TC_{t,\delta} \sim (V_{t,\delta} - V_t)^+$

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Section 3

Case Overview Revisited

Case Overview Revisited

What is our goal?

*Calculate Risk Weighted Assets for the Bank using the Internal Model approach for three counterparties (*Salzburg Bank, Bank of Cluj, Bank of Mazowsze*) as of 23rd February 2018. Assume the notional for all trades is 1.000.000.000 USD.*

Saltzburg Bank of Tyrol (ID)(netted) :

- 20y Receiver Swap USD6M LIBOR vs fixed 2.94% (paid annually)
- 20y Receiver Swap USD6M LIBOR vs fixed 1.94% (paid annually)
- 20y Receiver Swap USD6M LIBOR vs fixed 3.94% (paid annually)

Draculian Bank of Brasov (ID)(non netted):

- Long 10y Payer Swaption on 10y USD6M LIBOR vs fixed 3.05% (paid annually)
- 30y Receiver Swap USD6M LIBOR vs fixed 2.93% (paid annually)

Bank of Wodzionka (ID)(netted):

- Long 10y (Yearly callable) Payer Bermudan Swaption on USD6M LIBOR vs fixed 3.05% (paid annually)
- Long 10y Payer Swaption on 10y USD6M LIBOR vs fixed 3.05% (paid annually)

Section 4

Modelling CCR for Interest Rates Products

Modelling CCR for Interest Rates Products

t and T	• t: time slice of observation, eg.: prices as of today, t=0 • T: maturity of an instrument	
Bank Account B(t)	• Bank account is a stochastic process • Drift is the short rate r(t) • Zero volatility • B(0) = 1 • B(t) = stochastic as r(t) is stochastic	• Short rate SDE: $dr = \theta\ (r, t)\ dt + \sigma\ (t)\ dZ$ • $B(t) = \int_0^t e^{-r(s)} ds$
Zero Coupon Bond P(t, T)	• Pays one unit at maturity • Before maturity, its price is a stochastic process • P(0, T) can be implied from the curve at t=0 • Can be easily computed for each point in the grid	• P(T, T) = 1 • In a short rate model: $P(t,T) = A(t,T)\ e^{-B(t,T) r(t)}$
Swap s(t, T1, T2) S(t, T1, T2, K)	• Swap rate vs swap value • Swap starts at date T1, matures at date T2 • Swap fixed leg struck at K • Can be easily computed for each point in the grid	• $s(t, T_1, T_2) = f(P(t, T_1), P(t, T_i), \ldots, P(t, T_2))$
Swaption so(t, T1, T2, K)	• Option to enter into a swap • Expires at T1 • Underlying swap starts at T1 and matures at T2 • Easy to price at t=0, trickier for other time slices	• Jeshmidian decomposition • AMC
Bermudan Swaption	• Option to enter into a swap, but with multiple exercise dates. Eg.: 10yNC3y k=2% receiver	• AMC

Modelling CCR for Interest Rates Products

The Bank Account B(t) and the Short Rate r(t)

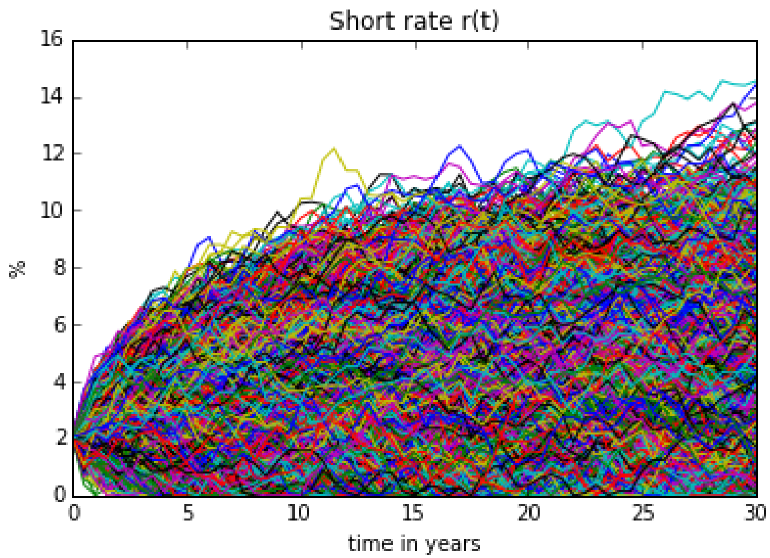
- r is the rate of return on a "presumably" very safe investment, a bank deposit
- The maturity of such deposit is very short, therefore, such investment has no stochastic component, despite r being a stochastic rate
- r can be modelled with a short rate model as

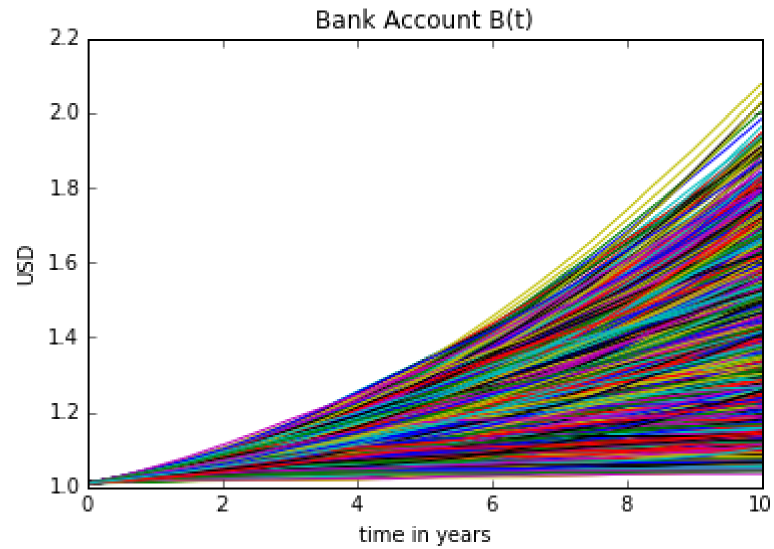
  SDE: $dr = \theta\,(r, t)\,dt + \sigma\,(t)\,dZ$

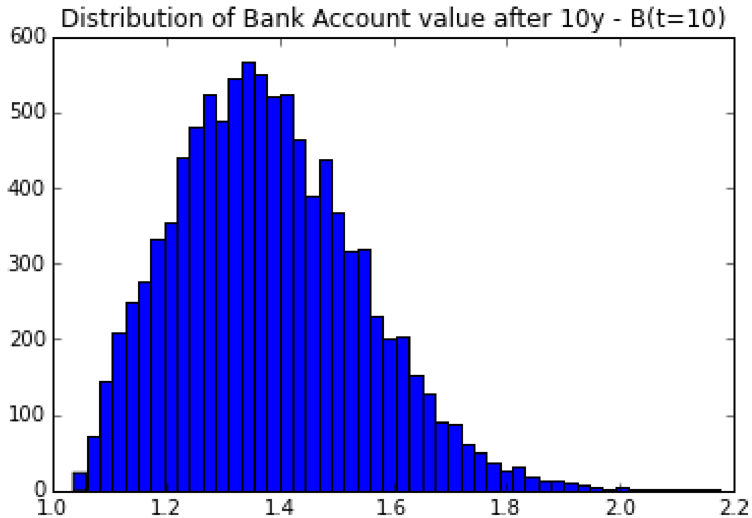
- $B(t) = \int_0^t e^{-r(s)} ds$
- For those familiar with stochastic calculus, B(t) has no stochastic term, hence no volatility. However, its drift is stochastic.

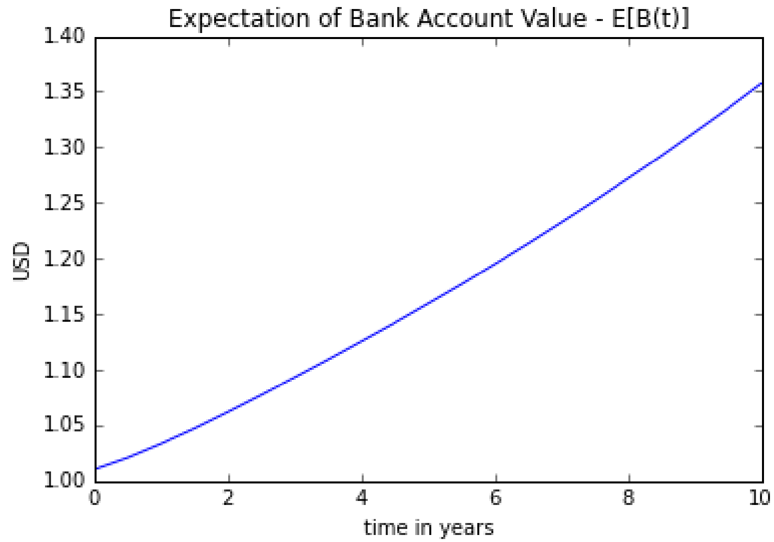
Modelling CCR for Interest Rates Products

The Bank Account B(t) and the Short Rate r(t)





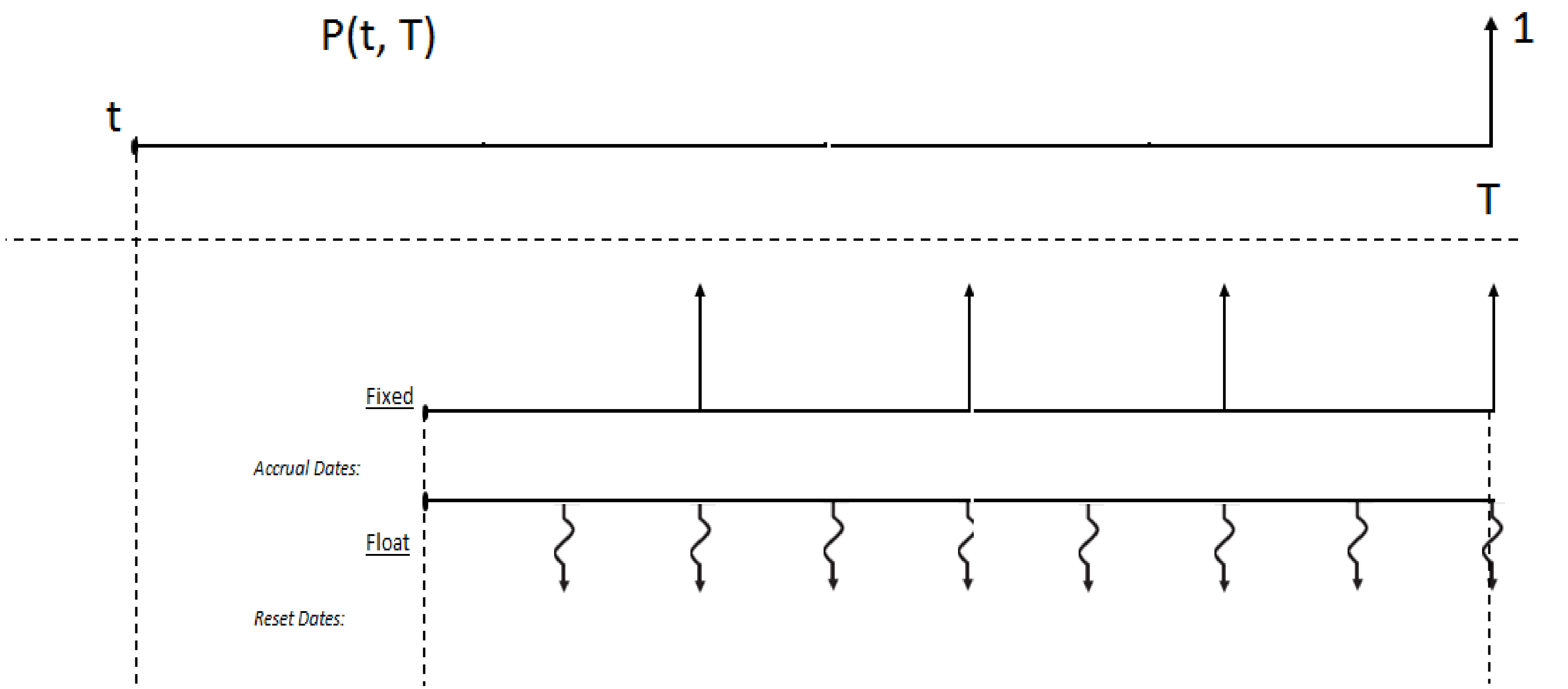




Modelling CCR for Interest Rates Products

Zero Coupon Bonds and Swaps

Zero Coupon Bond (ZCB)

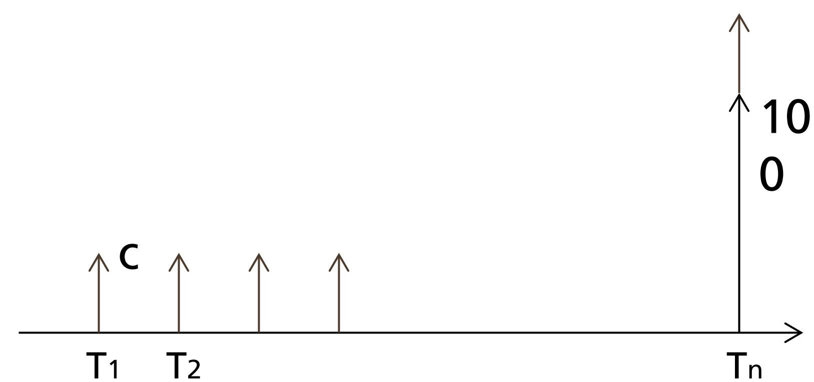


t start start + tenor

Interest Rate Swap (IRS), aka swap

Modelling CCR for Interest Rates Products

Zero to Par Rates Conversion



- Zero rates

  $P(0, T_n) = e^{-R_{ZCB}(T_i)}$

  $DF(T_n) = P(0, T_n)$

  $DF_i = DF(T_i)$

- Par Rates

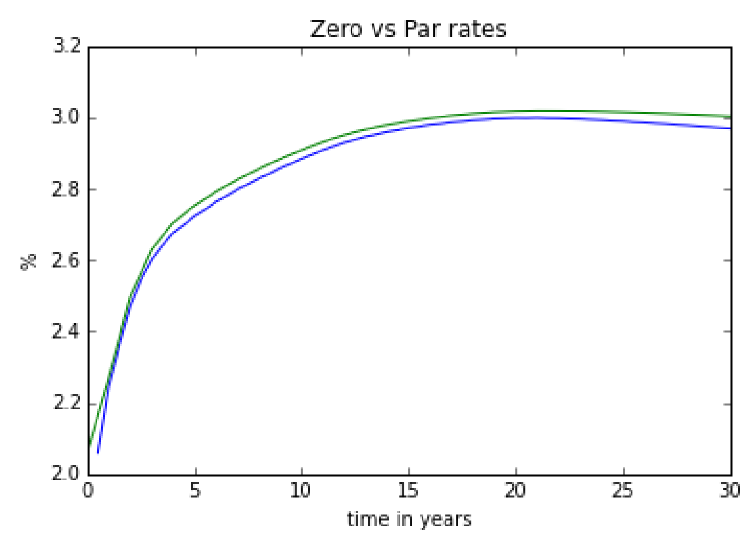
  $100 = c \times DF_1 + c \times DF_2 + \ldots + c \times DF_n + 100 \times DF_n$

  "In Libor flat discounting", it can be shown that, at coupon dates, a bond that pays floating coupons equal to Libor is worth 100

  c is the coupon rate that makes the price of the fixed coupon bond maturing at T_n equal to 100 (par) today*

  c is the rate that makes the price of the swap maturing at T_n equal to zero today*

- Yield Curve
  - For each maturity T, there is a zero coupon rate associated
  - For each maturity Tn, there is a par rate associated
  - We can plot a curve, the famous yield curve



Modelling CCR for Interest Rates Products

Simulation of Zero Coupon Bonds

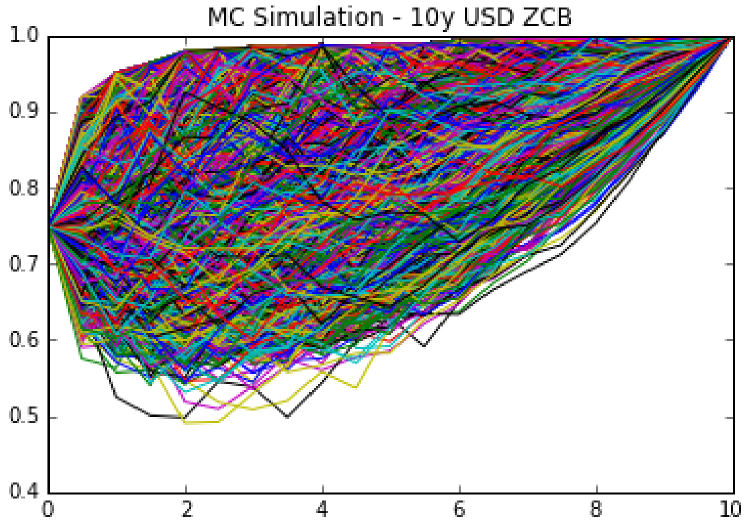
Zero Coupon Bonds:

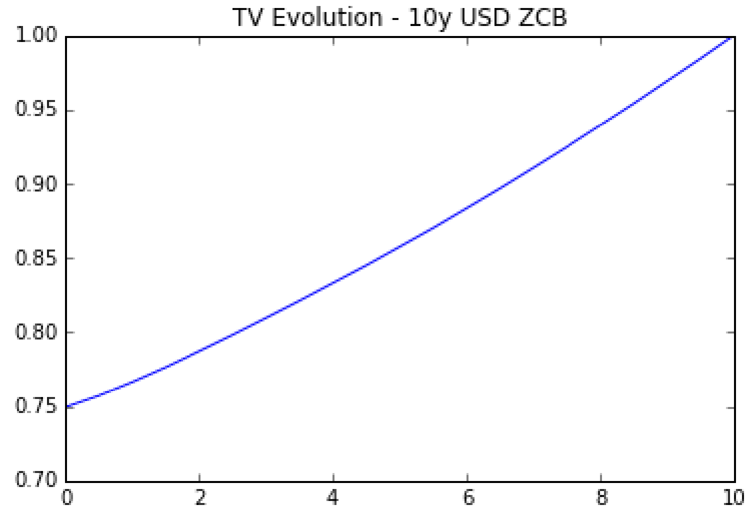
P(0,T) known at t=0

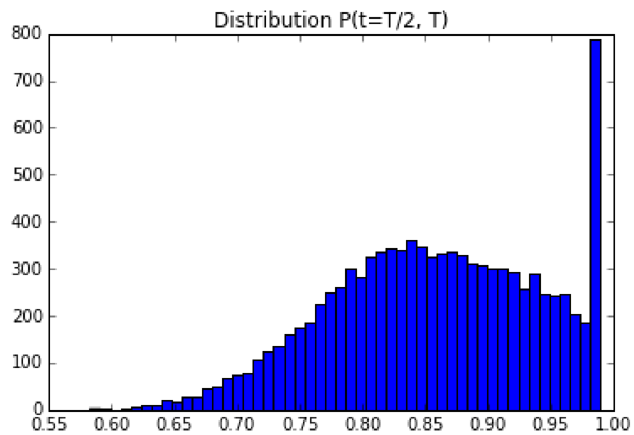
P(T,T) = 1

P(t, T) = exp(-R(t, t, T) x (T-t)) = P(0, T) x exp(-R(t, 0, t) x t)

- Where R(t, T_1, T_2) is the average interest rate between T_1 and T_2 observed at time t;
- If we allow this quantity to be stochastic, then P(t, T) is also stochastic;
- ZCB is a security, its discounted price is a martingale.







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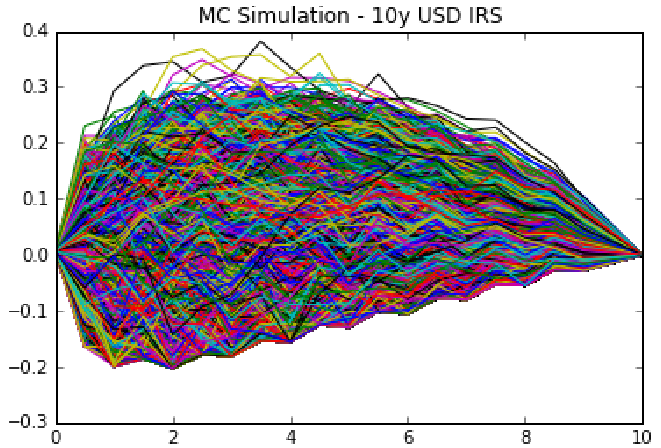
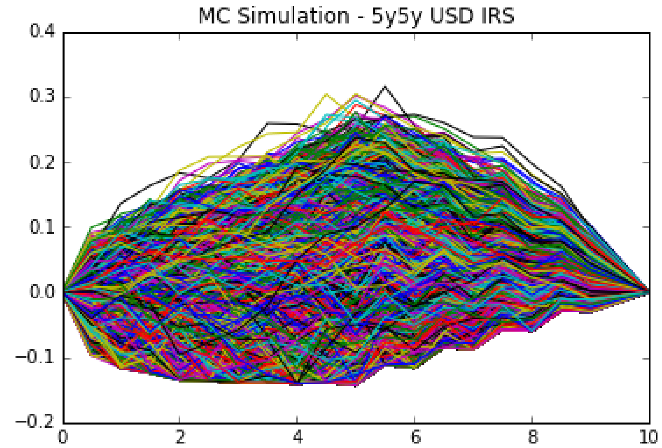
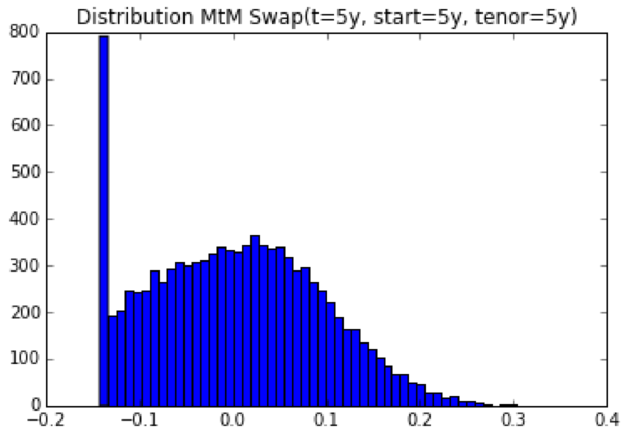
Modelling CCR for Interest Rates Products

Simulation of Swaps

- Spot starting vs forward starting swaps
- At maturity, after all cashflows have been settled, the trade value is obviously equal to zero
- Trade value is zero at inception, zero at maturity but can vary throughout the life of the trade







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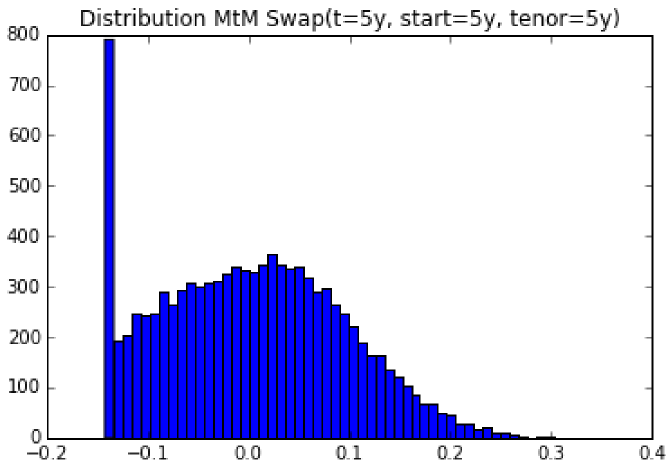
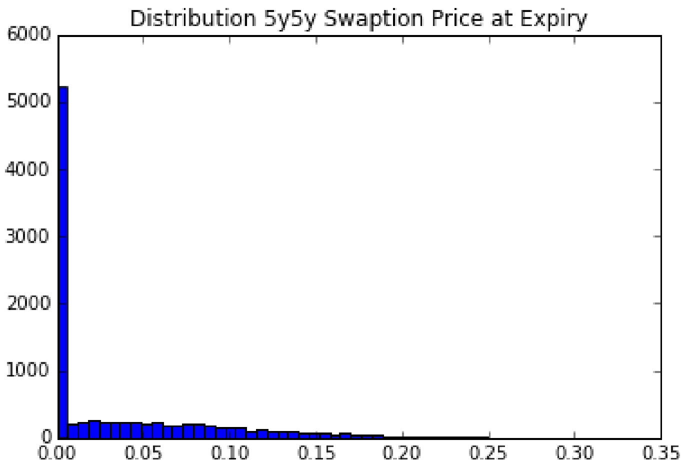
Modelling CCR for Interest Rates Products

Swaption

- European option to enter into a swap that starts immediately after expiry date
- How to price a swaption?
  - Knowing the swap price distribution for t=expiry, it's easy to compute the price of a swaption
  - Jamshidian's decomposition technique => saves computation time
  - Enough to calibrate the model





- Easy to compute the price today, what about price distribution over time?

Modelling CCR for Interest Rates Products

Model Calibration

Iterative Calibration	1. Initialize vol parameters with reasonable values 2. Calibrate drift parameters to match bond prices 3. Calibrate vol parameters to minimize model vs market discrepancy for option prices 4. Go back to step 2 until calibration is satisfactory 5. Remember to re-run step 2	
Bootstrapping	• P(t, T) only depends on r up to time t • r(t), in turn, only depends on θ (r, t) up to time t • Therefore, one can calibrate this parameter for first for t_0 • Then for t_1, for t_2, … t_n	Use this to calibrate the drift parameter of the short rate SDE
Global calibration	• Price a set of instruments • Compute a cost function • Optimize parameters to minimize cost function	• J = $\sum_{i=0}^{n} w_i ({V_i}^{model} - {V_i}^{mkt})^2$ • Use this to calibrate the vol structure

Section 5

Backward Induction and American Monte-Carlo Methods

Backward Induction and American Monte-Carlo Methods

Let's consider a game where we roll a fair die three times. After the first and the second draw we can get a payoff equal to the number drawn or have a chance to roll the die one more time (forgeting about the previous result). If we choose to roll for the third time we get the result of the third rolling.

How do you assess what is the fair price for entering into the game?
At what level drawn the first or second time you should stop rolling and exercise the payoff and at what level you should continue rolling?

Using backward induction:

3rd rolling

we know that in the 3rd rolling if it happens the expected value is:

$$\frac{1}{6}\sum_{i=1}^{6} \mathrm{i} =$$

$$\frac{1}{6}(1+2+3+4+5+6) = 3.5$$

2nd rolling

it means that after the 2nd rolling the player should reject values less than 3.5 ie. 1, 2 and 3.

Then the expected value of the gain from the 2nd and 3rd rolling is:

$$\frac{1}{6}\sum_{i=1}^{6} \max(i, 3.5) =$$

$$\frac{1}{6}(3.5+3.5+3.5+4+5+6) = 4.25$$

1st rolling

hence at the first rolling we should reject values below 4.25 ie. $1, 2, 3, 4$.

Then the expected value of the gain from the whole game (price) is:

$$\frac{1}{6}\sum_{i=1}^{6} \max(i, 4.25) =$$

$$\frac{1}{6}(4.25+4.25+4.25+4.25+5+6) =$$

$$4.66$$

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Example: stock price evolution – 2-step binomial tree

$$\begin{tikzcd} & & S_0u^2 \\ & S_0u \arrow[ur, "p"] \arrow[dr, "1-p"] & \\ S_0 \arrow[ur, "p"] \arrow[dr, "1-p"] & & S_0ud \\ & S_0d \arrow[ur, "p"] \arrow[dr, "1-p"] & \\ & & S_0d^2 \end{tikzcd}$$

$t = t_0 = 0$ $\qquad$ $t = t_1$ $\qquad$ $t = t_2$

European option pricing (call with expiry t_2, strike K)

option price at $t = 0$

price of option at $t = t_1$: $V_t = E\left(e^{-r\Delta t} H(t_2) \middle| F_{t_1}\right)$

price of option at $t = t_2$:

known – $\max(S_{t_2} - K, 0)$

$$\begin{tikzcd} & & V_{t_2}{}^{uu} = \max(S_0u^2 - K, 0) \\ & V_{t_1}{}^{u} = e^{-r\Delta t}[pH_{uu} + (1-p)H_{ud}] \arrow[ur, "p"] \arrow[dr, "1-p"] & \\ V_0 = e^{-r\Delta t}[pV_t{}^u + (1-p)V_t{}^d] \arrow[ur, "p"] \arrow[dr, "1-p"] & & V_{t_2}{}^{ud} = \max(S_0ud - K, 0) \\ & V_{t_1}{}^{d} = e^{-r\Delta t}[pH_{ud} + (1-p)H_{dd}] \arrow[ur, "p"] \arrow[dr, "1-p"] & \\ & & V_{t_2}{}^{dd} = \max(S_0d^2 - K, 0) \end{tikzcd}$$

$t = t_0 = 0$ $\qquad$ $t = t_1$ $\qquad$ $t = t_2$

Backward Induction and American Monte-Carlo Methods

Example: stock price evolution – 2-step binomial tree cd. – american option pricing

American option is an option that can be exercised at any time between the trade start and its contractual expiry.

option price at $t = 0$

price of option at $t = t_1$:

price of option at $t = t_2$:

known – $\max(S_{t_2} - K, 0)$

$$
\begin{tikzcd}[column sep=small]
 & & H_{uu} = \max(S_0 u^2 - K, 0) \\
 & V^u = \max(e^{-r\Delta t}[pH_{uu} + (1-p)H_{ud}]\,, \max(Su - K, 0)) \arrow[ur, "p"] \arrow[dr, "1-p"'] & \\
V_0 = \cdots \arrow[ur, "p"] \arrow[dr, "1-p"'] & & H_{ud} = \max(S_0 ud - K, 0) \\
 & V^d = \max(e^{-r\Delta t}[pH_{ud} + (1-p)H_{dd}]\,, \max(Sd - K, 0)) \arrow[ur, "p"] \arrow[dr, "1-p"'] & \\
 & & H_{dd} = \max(S_0 d^2 - K, 0)
\end{tikzcd}
$$

$t = t_0 = 0$ $\qquad$ $t = t_1$ $\qquad$ $t = t_2$

At $t = t_1$ we can either exercise the option or not.

Whether we do this depends on the information we have up until time $t = t_1$.

If the **continuation value** (price of the option at node $t = t_1$) is higher than what would be our payoff in case of immediate exercise, we hold the option, if not we exercise it.

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Backward Induction and American Monte-Carlo Methods

Bermudan Swaptions

- Option that gives its holder to enter into a swap, that has a certain maturity date, at a few specified exercise dates;
- Upon exercise, the swap cashflows start being exchanged as if it were an usual swap;
- Usually, the exercise dates are yearly after a "non-call" period;
  - Eg.: 10yNC3 bermudan USD receiver struck at 3%
    - Holder has the option to enter into a 7y swap after 3y, rec'ing rates at 3%
    - if after 3y the option has not been exercised, holder still has the option to enter into a 6y swap after 4y
    - …
    - finally, holder has the option to enter into a 1y swap receiving fixed at 3% after 9y. If not exercised after 9y this option expires worthless
- What is this product a natural hedge for?
- How to price such derivative?

Backward Induction and American Monte-Carlo Methods

American Monte Carlo

$V_n = h_n$ payoff at expiry

$V_i = \max\{h_i, {V_i}^P\}$ $i < n$

$h_i = (\ldots)^+$ $i \leq n,$ t_i being an early exercise date,

otherwise, $h_i = 0$

How to compute V_i?

- Backward induction if you have a simulation tree is trivial

- AMC: polynomial linear regression trick

$V_i = (\alpha\, \psi(O_i) + \varepsilon) P(0, t_i)$

where :

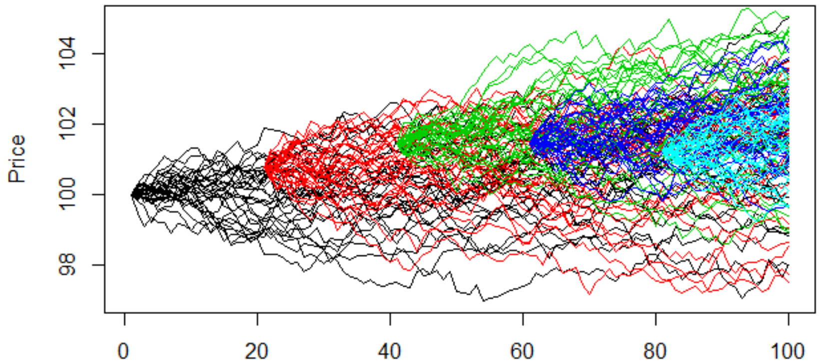
ψ is a polynomial function

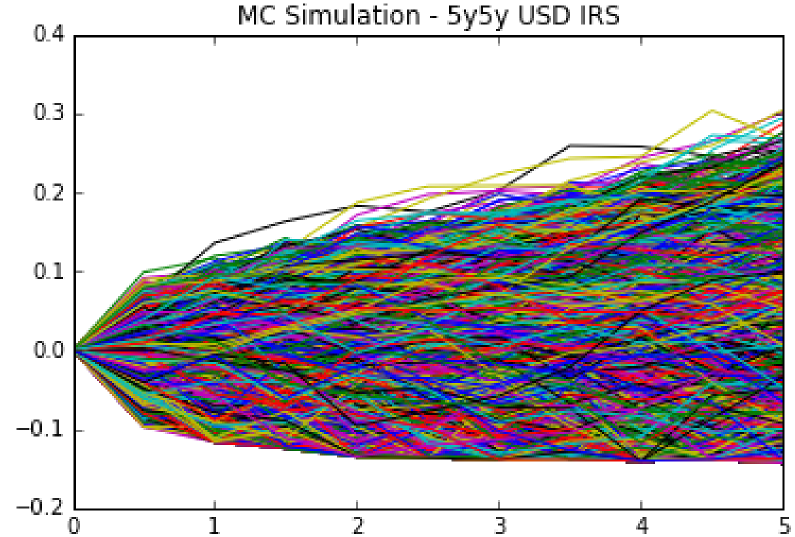
O_i are what we call the set of observables

In addition, let's recall that $B(t_i)V_i$ is a martingale and, therefore:

$$E[B(t_i)V_i | O_i] = E[B(t_{i+1})V_{i+1} | O_i]$$

Nested Monte Carlo





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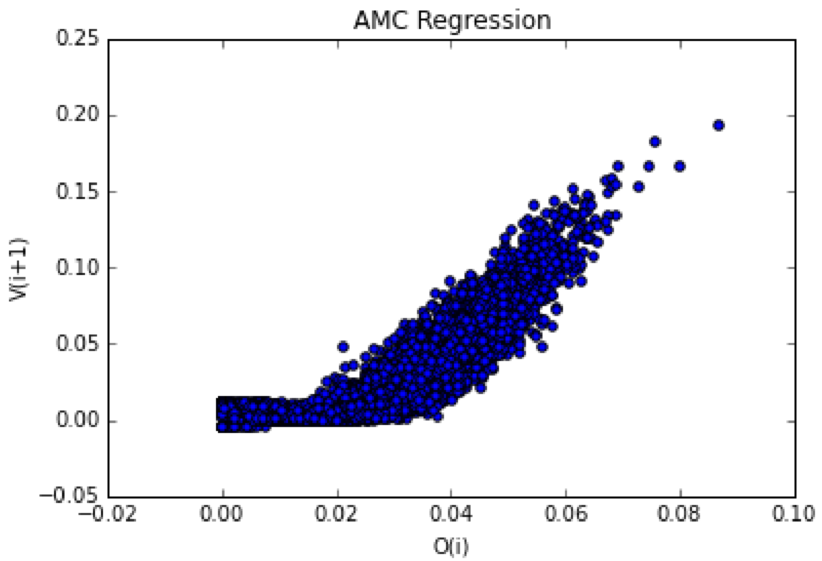
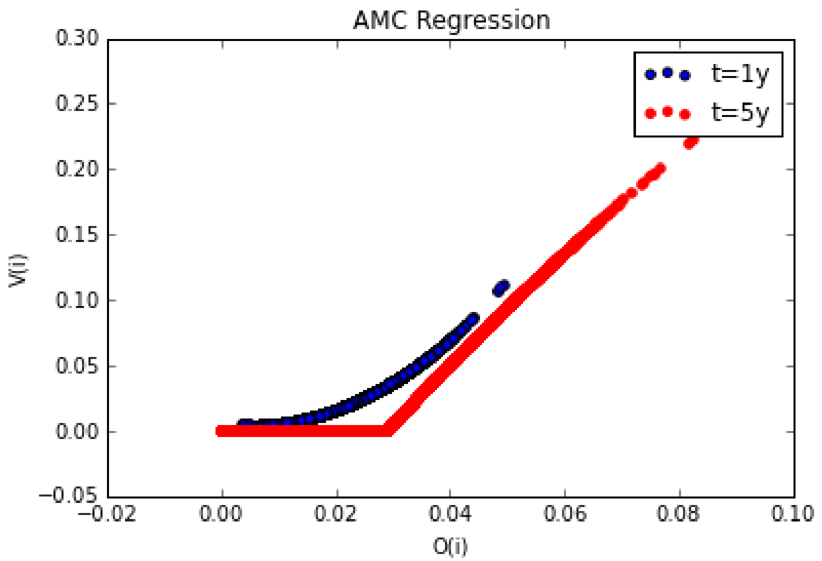
Backward Induction and American Monte-Carlo Methods

AMC Regression Results





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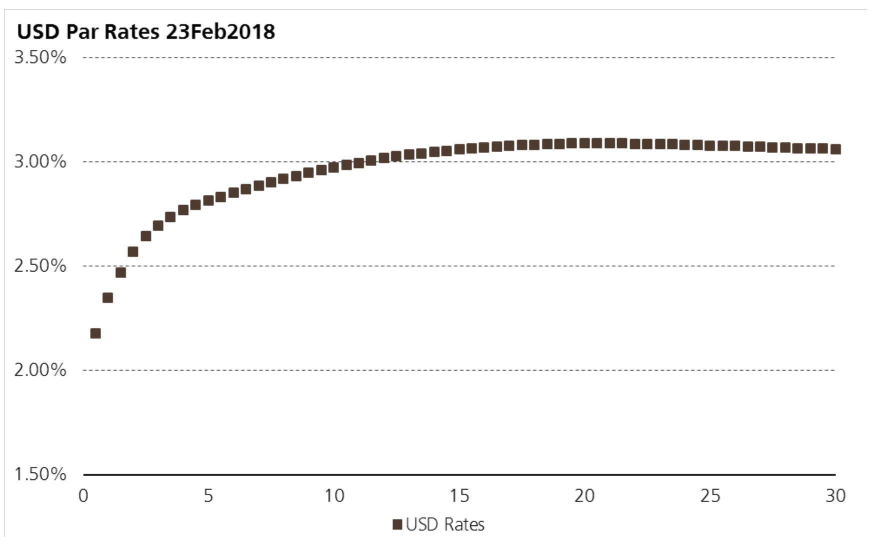
Section 6

Appendix: Market Data

Market Data

Par Rates and Swaption Prices

Par Rates	• Swap rates for tenors between 6m and 30y • Convert to zero rates in order to price zero coupon bonds
Swaptions	• Normal volatilities* for ATM swaptions of various expiries and tenors * To convert to option prices, one needs to multiply by the swap annuity A(t, T_1, T_2) = $\sum_{i=1}^{n}(T_i - T_{i-1})\, P(T_i)$ ATM Swaption price = Normal Vol x A



Swaption Normal Vols				Tenor				
Expiry	**3m**	**6m**	**1y**	**2y**	**5y**	**10y**	**20y**	**30y**
3m	28	25	38	49	63	67	65	65
6m	33	30	41	52	64	67	65	65
1y	42	41	49	58	67	68	66	64
2y	58	57	64	68	71	70	66	65
5y	77	77	76	76	73	71	64	63
10y	71	70	70	69	67	64	57	56
20y	58	58	57	54	53	50	47	46
30y	48	48	48	48	47	47	45	44

Section 7

Appendix: Contact Information

Contact information

What to do in case of problems?

Each team is allowed to ask one Question a day by an email.

- Sent the questions to: piotr-a.morawski@ubs.com
- Be sure to start the subject with phrase: **INTQuant Question – Day X**
- Be mindful to formulate your question properly to pinpoint the core of the problem
  - Questions like**: 'Why it is not working?'** will be answered with **'Because you are doing something wrong.'**
- Make the questions methodology oriented not implementation oriented.

Face-to-face Q&A session - **Friday March 15th at 9:30.**

- The session will be open for all teams
- We will try to split time equally among all the teams
- Try to explore and narrow down the problems before asking about them as we might not have enough time to engage into detailed discussions.

Skype conferences calls

- **Monday March 18th at 9:30 + Tuesday March 19th at 15:00**
- The session will be open for all teams
- We will try to split time equally among all the teams
- Try to explore and narrow down the problems before asking about them as we might not have enough time to engage into detailed discussions

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