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Credit Risk Fundamentals

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Section 1

Credit risk and its types

Default Risk

Default risk is that a counterparty does not honour their obligations.
We call the risk of a payment default **credit risk**.

- Default Risk is ubiquitous as any transaction carries a risk that one of two parties involved does not deliver.

- Examples of obligors:
  - A company has borrowed money from a bank
  - A company has issued bonds.
  - A household borrowed money from a bank, to buy a house (a mortgage).
  - A bank has entered into a bilateral financial contract (*e.g* an IR swap) with another bank.

- Examples of defaults:
  - A company goes bankrupt.
  - As company fails to pay a coupon for some of its issued bonds on agreed time.
  - A household fails to pay amortisation or IR on their mortgage.

Credit risk types

There are at least four aspects of the credit risk.

- **arrival risk**, the risk connected to whether or not a default will happen in a given time-period, for a obligor
- **timing risk**, the risk connected to the uncertainness of the exact time-point of the arrival risk
- **recovery risk**, the rist connected to the size of the actual loss, should a default occur.
- **dependency risk**, the risk that several obligors jointly defaults during some specific time period.

  This is one of the most important risk factors that has to be considered within the credit portfolio framework.

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Credit Loss

Credit loss, loss due to default of obligor = EAD × LGD,

Expected credit loss ECL = PD × EAD × LGD

- The credit risk may be described by the following processes:
  - $\mathbf{1}_\tau(t)$, the **default indicator** process
    equals to 1 if there was a default by the time *t*, and 0 if the obligor is still alive;
  - **E**(*t*), the **exposure at default** process (EAD),
    the total amount of payment obligations of obligor at time *t;*
  - **L**(*t*), the **loss given default** process,
    a proportion of the exposure to be recovered in bankruptcy proceeding.

- The expected credit loss, assuming <u>mutual independence</u> of the above processes:
  - ECL = PD × EAD × LGD.

- Modelling PD and LGD is one of the main tasks of Credit Risk Quants

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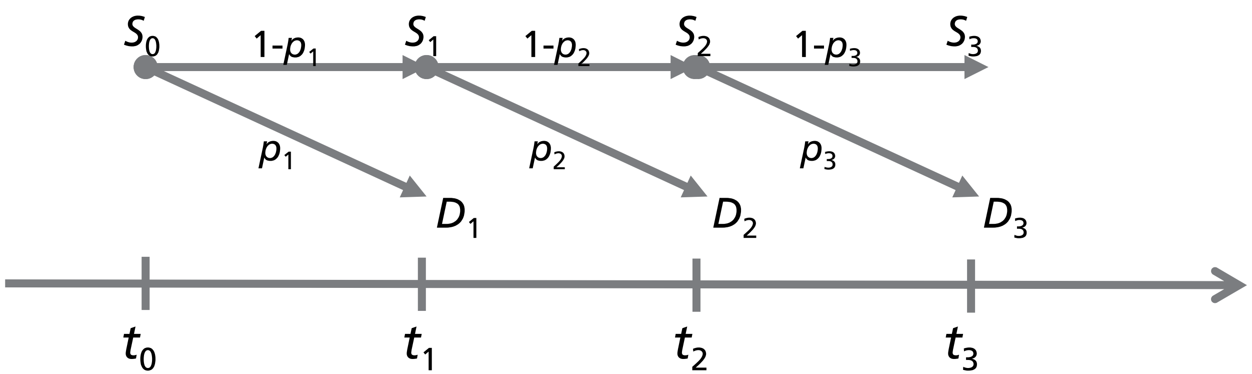
More details on default probabilities

PD is usually **time dependent.** It depends on specific features of the borrower and the loan itself, however very often it is impacted by the economical situation.

- PD language:
  - Cumulative PD : $F(t) = P(\ \tau \le t\ ) = E(\mathbf{1}_{\tau}(t))$
  - Marginal PD: $F(t_i, t_{i+1}) = P(t_i < \tau \le t_{i+1}) = F(t_{i+1}) - F(t_i)$
  - Forward PD, point in time: $p(t_i, t_{i+1}) = P(t_i < \tau \le t_{i+1} \mid t_i < \tau\ )$
- Multi-step default tree, $p_i = p(t_{i-1}, t_i)$:



- $F(t_i) = 1-(1-p_1)\,(1-p_2)\ldots\,(1-p_i) = 1-s_1 s_2 \ldots\, s_i$, where survival probability (pit) $s_i = 1-p_i$

Section 2

Estimation of PD

Credit Ratings – Historical DRs

Credit Rating is the assessment of the credit risk of the obligor. Agency rating it is one of the tools to assess likelihood that an obligor defaults based on historical default rates.

Table 33

Average Multiyear Global Corporate Transition Matrix (1981-2015) (%)

	--One-year transition rates--								
From/to	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC/C**	**D**	**NR**
AAA	87.08	9.00	0.53	0.05	0.08	0.03	0.05	0.00	3.18
	(7.14)	(7.16)	(0.83)	(0.25)	(0.25)	(0.17)	(0.35)	(0.00)	(2.44)
AA	0.53	86.69	8.06	0.53	0.06	0.07	0.02	0.02	4.02
	(0.52)	(5.32)	(4.28)	(0.69)	(0.20)	(0.21)	(0.07)	(0.08)	(1.94)
A	0.03	1.81	87.65	5.39	0.33	0.13	0.02	0.06	4.58
	(0.09)	(1.02)	(3.55)	(2.14)	(0.39)	(0.27)	(0.07)	(0.11)	(1.81)
BBB	0.01	0.11	3.55	85.43	3.82	0.52	0.12	0.19	6.24
	(0.04)	(0.16)	(1.68)	(3.80)	(1.56)	(0.72)	(0.23)	(0.26)	(1.63)
BB	0.01	0.03	0.13	5.08	76.78	6.96	0.64	0.73	9.63
	(0.06)	(0.09)	(0.27)	(1.89)	(4.47)	(3.21)	(0.77)	(0.87)	(2.46)
B	0.00	0.03	0.09	0.21	5.25	74.27	4.39	3.77	11.99
	(0.00)	(0.09)	(0.22)	(0.22)	(2.07)	(4.37)	(2.25)	(3.37)	(2.25)
CCC/C									

Note: Numbers in par
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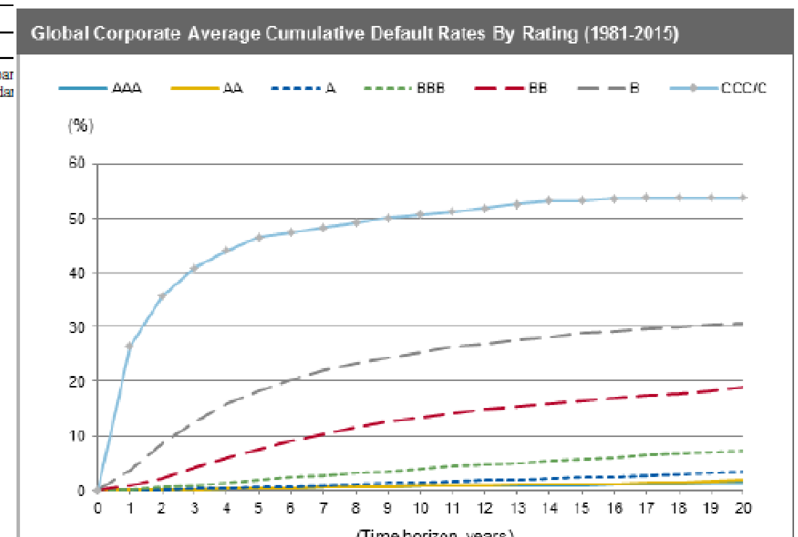


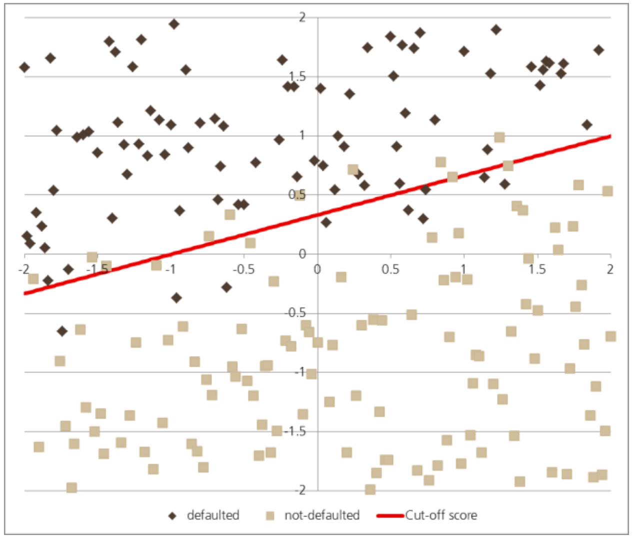
https://www.investing.com/

Vazza D., Kraemer N. K; *2015 Annual Global Corporate Default Study And Rating Transitions*; www.standardandpoors.com/ratingsdirect

Credit Scoring

The ratings from agencies are published only for relatively large corporates. Therefore banks need to prepare internal ratings based (IRB) approach for determining PDs for small and medium enterprises as well as individual clients.

- Altman's *Z*-Score
  - the first credit scoring models; relies on accounting ratios:
    - *A* = Working Capital / Total Assets
    - *B* = Retained Earnings / Total Assets
    - *C* = Earnings before Interest and Taxes / Total Assets
    - *D* = Market Capitalisation / Debt
    - *E* = Sales / Total Assets.
  - $Z = 1.2A + 1.4B + 3.3C + 0.6D + E.$



- Credit score is used also to rank obligors: **higher** score implies **better** credit quality.
- Accounting ratios are used to measure: indebtedness, cash flow available for debt service, profitability.

Market capitalisation refers to the total market value of company's outstanding shares. Commonly referred to as "market cap"; calculated by multiplying company's shares outstanding by the current market price of one share.

Zones of discrimination:

$Z > 2.99$ – “Safe” Zone
$1.81 < Z < 2.99$ – “Grey” Zone
$Z < 1.81$ – “Distress” Zone

If $Z<1.81$ the obligor is likely to default and a loan should be refused.

Estimation of PD

Another way of measuring credit risk is estimation of the direct default probability via an assumed functional relationship: $p = f\,(\beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n)$

- Probit model:
  - $f(x) = \Phi(x)$,
    i.e., cdf of the standard normal distribution.
- Logit model:
  - $f(x) = 1 / (1 + \exp x)$

In general, one can use $f(x)$ as a mapping from something like Z-score into the default probability space.

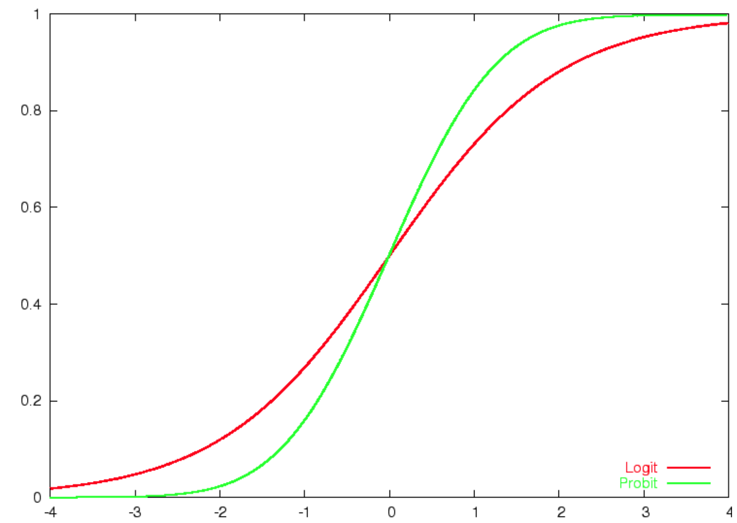
- Let ε be a standard normally distributed noise component. Then we define the credit index:
  $-(\beta_0 + \beta_1 X_1 + \ldots \beta_n X_n) + \varepsilon.$

Fact. Assuming that the obligor defaults when his **credit index drops below zero**, the default probability of the obligor is exactly equal to the p given by the probit model.

Can you prove it?

Logit and probit models

The logit and probit models regress a function of the probability that a case falls into a certain category of the dependant variable



- In the logit model it is assumed that

  $P(Y = 1 \mid x_1, \ldots, x_k) = 1 / (1 + \exp(b_0 + b_1 x_1 + \ldots + b_n x_n))$.

Binary logistic regression major assumptions:

- The dependent variable should be dichotomous
  (*e.g.*, presence vs. absent, default vs. alive).
- There *should be no outliers* in the data.
  This can be assessed by converting the continuous predictors to standardised scores, and removing values below -3.29 or greater than 3.29 [why 3.29?].
- There should be no *high* correlations (*multicollinearity*) among the predictors.
  This can be assessed by a correlation matrix among the predictors. It is suggested that as long correlation coefficients among variables are **less than 0.90** the assumption is met.

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Logit and probit models

The logit and probit models regress a function of the probability that a case falls into a certain category of the dependant variable

- In the probit model it is assumed that
  - $P(Y = 1 \mid x_1, \ldots, x_k) = \Phi(b_0 + b_1 x_1 + \ldots + b_n x_n)$,
    the cdf of the standard normal distribution.
- The reqression coefficients of the probit model are effects on Φ of the probabilities that $Y = 1$, that is, that the obligor defaults.
- Logistic curve has *slightly* flatter tails. *i.e.*, probit curve approaches the axes more quickly than the curve.
- Is logit better than probit, or *vice versa*?

Both methods will yield similar (though not identical) inferences. Logit seems to be more popular partly because coefficients can be interpreted in terms of odds ratios. Probit models can be generalised to account for non-constant error variances in more advanced econometric settings (*heteroskedastic probit models*).

If these more advanced applications are not of relevance, than it **does not really matter** which method you choose to go with.

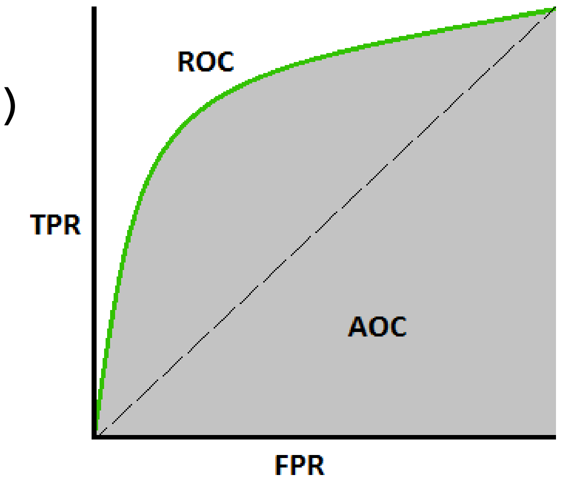
Receiver operating characteristic (ROC)

A graphical plot that illustrates the diagnostic ability of a binary classifier system as its discrimination threshold is varied

- The ROC curve says *how much the model is capable of distinguishing* between classes. Higher the AUC (**area under the curve = integral**), better the model is at predicting 0s as 0s and 1s as 1s.

  By example: higher the AUC, better the model is at distinguishing between clients who'd default from those who will survive.

- The ROC curve is plotted with TPR against the FPR where TPR is on y-axis and FPR is on the x-axis.
  - **TPR (True Positive Rate, sensitivity)** = TP / (TP + FN)
  - **Specificity** = TN / (TN + FP)
  - **FPR** = 1 – Specificity = FP / (TN + FP)



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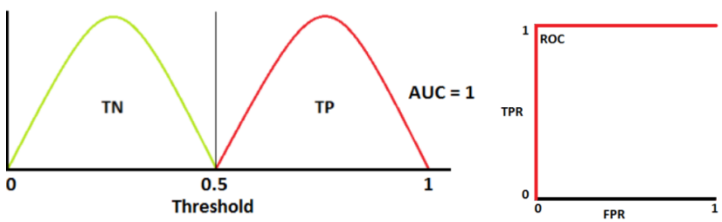
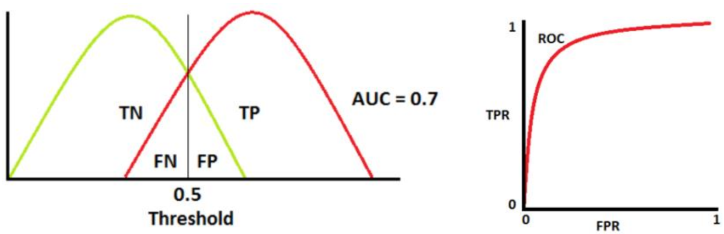
Receiver operating characteristic (ROC)

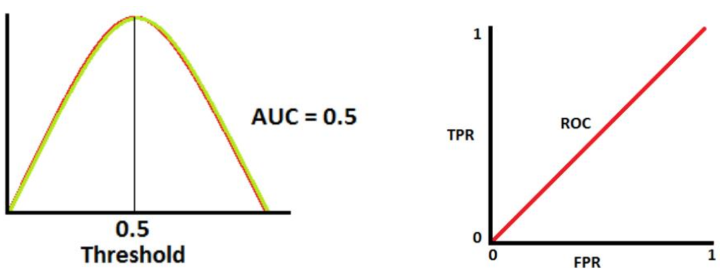
The higher AUC the better



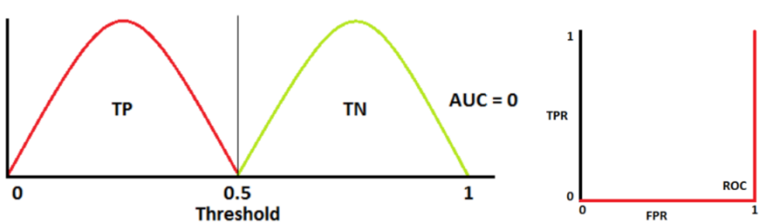
An ideal situation. When two curves do not overlap at all means model has an ideal measure of separability. It is perfectly distinguishes between 1s and 0s.



When two distributions overlap, we introduce type 1 and type 2 error. Depending upon the threshold, we can minimise or maximise them. AUC = 0.7 means there is a 70% chance that model will be able to distinguish between 1s and 0s.



This is the worst situation. When AUC is approximately 0.5, model has no discrimination capacity to distinguish between 1s and 0s.



What does AUC = 0 mean?

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https://towardsdatascience.com/understanding-auc-roc-curve-68b2303cc9c5

The Merton model

a simple model of the firm that provides a way of relating credit risk to the capital structure of the firm

Assumptions:

- the firm has issued two classes of securities: equity and debt;
- the debt is a pure discount bond where the payment of $\overline{D}$ is promised at maturity T;
- the value of the firm is assumed to be a tradable asset that obeys a log-normal diffusion process with **constant volatility** and **interest rate** (equity shareholders receive no dividend). Under the risk neutral measure:

$$dA(t) = r\, A(t)dt + \sigma_A\, A(t)dW(t)$$

The limited liability feature of equity means that the equity holders have the right, but not the obligation, to pay off the debt holders and take over the remaining assets of the firm. [Sounds familiar?]

The Merton model

a simple model of the firm that provides a way of relating credit risk to the capital structure of the firm

At time T two situations may occur:

- if the firm's asset value exceeds the promised payment $\overline{D}$, the lenders are paid the promised amount and the shareholders receive the residual asset value;
- if the **asset value is less than the promised payment** $\overline{D}$, the firms **defaults**, the lenders receive a payment equal to the asset value and the shareholders get nothing.

Key idea: Equity is the same as a call option on the firm's assets with a strike price equal to the book value of the firm's liabilities.

The Merton model

a simple model of the firm that provides a way of relating credit risk to the capital structure of the firm

We set

E = value of company's equity (E_T at time T and E_0 at t = 0);

A = value of company's assets (A_T at time T and A_0 at t = 0).

Observation: since **equity** is a **call option** on the **assets** of the firm with **strike** equal to the promised **debt** payment, the payment to the shareholders at time T is given by:

$$E_T = \max[A_T - \bar{D}, 0]$$

where

$$d_1 = \frac{\ln(A_0 e^{rT}/\bar{D})}{\sigma_A \sqrt{T}} + \frac{1}{2}\sigma_A \sqrt{T} \qquad d_2 = d_1 - \sigma_A \sqrt{T}$$

and r is the risk-free interest rate.

The Merton model

PD = $N(-d_2)$

The end-game: The risk neutral **probability that the company defaults** by time T is the probability that **shareholders will not exercise their call option** to buy the assets of the company for $\overline{D}$ at T:

$$\begin{aligned}
\mathbb{Q}(A_T < \bar{D}) &= \mathbb{Q}(\ln(A_T) < \ln(\bar{D})) \\
&= \mathbb{Q}\left(\frac{W_T}{\sqrt{T}} < \frac{\ln \frac{\bar{D}}{A_0} - \left(r - \frac{\sigma_A^2}{2}\right) T}{\sigma_A \sqrt{T}}\right) \\
&= N\left(-\frac{\ln \frac{A_0}{\bar{D}} + \left(r - \frac{\sigma_A^2}{2}\right) T}{\sigma_A \sqrt{T}}\right) = N(-d_2)
\end{aligned}$$

The usual problem is how to estimate the volatility.

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Section 3

Summary

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Summary

Credit Quant role in the bank is to help credit officers decide if a loan should be granted. Then, after positive decision, credit quants prepares a methodology for risk monitoring and estimation of credit losses, what is needed for capital requirements forecasting

- One of the essential tasks for credit quants is to prepare a methodology for estimation of the probability of a default.
- The latter is used for credit losses estimation via formula:
  - $ECL = PD \times EAD \times LGD$.
- PD can be estimated using external ratings or via internal models.
- In general, internal models aim at finding the best set of parameters (loan and economical situation related) that fit the historical observations.
- The job itself is fun!

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