A Recipe For Pricing

11 March 2024

How to become a Millionaire in 1 year!!!

Data from `https://finance.yahoo.com` and `https:google.com`, retrieved on 1/3/2024.

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UBS $\rightarrow$ 100CHF
🚀🚀🚀🌕!!1!!!1

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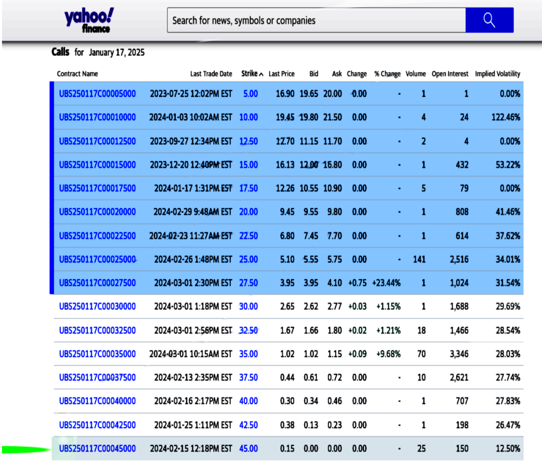

Calls for January 17, 2025

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
UBS250117C00005000	2023-07-25 12:02PM EST	5.00	16.90	19.65	20.00	0.00	-	1	1	0.00%
UBS250117C00010000	2024-01-03 10:02AM EST	10.00	19.45	19.80	21.50	0.00	-	4	24	122.46%
UBS250117C00012500	2023-09-27 12:34PM EST	12.50	12.70	11.15	11.70	0.00	-	2	4	0.00%
UBS250117C00015000	2023-12-20 12:40PM EST	15.00	16.13	12.00	16.80	0.00	-	1	432	53.22%
UBS250117C00017500	2024-01-17 1:31PM EST	17.50	12.26	10.55	10.90	0.00	-	5	79	0.00%
UBS250117C00020000	2024-02-29 9:48AM EST	20.00	9.45	9.55	9.80	0.00	-	1	808	41.46%
UBS250117C00022500	2024-02-23 11:27AM EST	22.50	6.80	7.45	7.70	0.00	-	1	614	37.62%
UBS250117C00025000	2024-02-26 1:48PM EST	25.00	5.10	5.55	5.75	0.00	-	141	2,516	34.01%
UBS250117C00027500	2024-03-01 2:30PM EST	27.50	3.95	3.95	4.10	+0.75	+23.44%	1	1,024	31.54%
UBS250117C00030000	2024-03-01 1:18PM EST	30.00	2.65	2.62	2.77	+0.03	+1.15%	1	1,688	29.69%
UBS250117C00032500	2024-03-01 2:56PM EST	32.50	1.67	1.66	1.80	+0.02	+1.21%	18	1,466	28.54%
UBS250117C00035000	2024-03-01 10:15AM EST	35.00	1.02	1.02	1.15	+0.09	+9.68%	70	3,346	28.03%
UBS250117C00037500	2024-02-13 2:35PM EST	37.50	0.44	0.61	0.72	0.00	-	10	2,621	27.74%
UBS250117C00040000	2024-02-16 2:17PM EST	40.00	0.30	0.34	0.46	0.00	-	1	707	27.83%
UBS250117C00042500	2024-01-25 1:11PM EST	42.50	0.38	0.13	0.23	0.00	-	1	198	26.47%
UBS250117C00045000	2024-02-15 12:18PM EST	45.00	0.15	0.00	0.00	0.00	-	25	150	12.50%

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$$0.15 \cdot \frac{1M}{100-45-0.15} = 2734.73\text{CHF}$$

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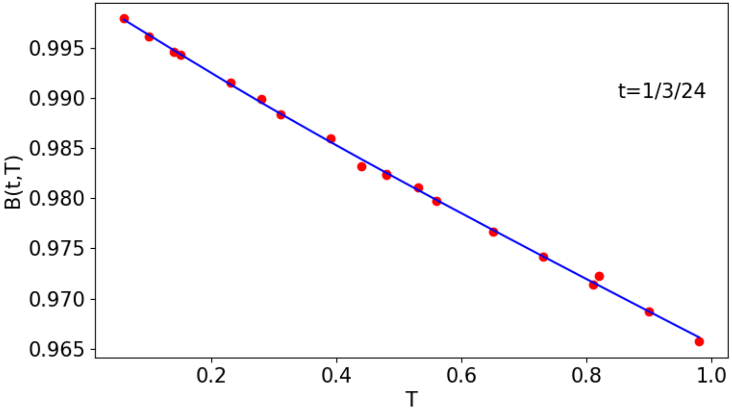
Interest rate

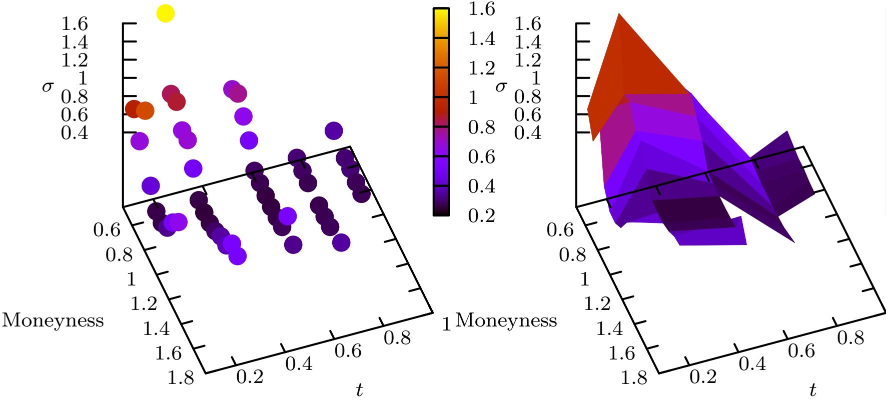
$$dr_t = a(b - r_t)dt + \sigma dW_t$$

$$B(t, T) = E[\mathrm{e}^{-\int_t^T dur_u} | \mathcal{F}_t] = \text{...a lot of maths...} = \mathrm{e}^{\alpha(t,T,a,b,\sigma)-\beta(t,T,a,b,\sigma)r_t}.$$



Data as of 1/03/2024 from `https://www.qualebtp.it`.

Volatility



Data as of 1/03/2024 from `https://finance.yahoo.com`.

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Appendix/Notes

Stochastic Processes

Stochastic processes can be classified as follows:

- **Discrete time**: value of the variable can change only at certain fixed points in time.
- **Continuous time**: changes in value can take place at any time.

Additionally there is a division into:

- **Discrete variable** underlying variable can take only certain discrete values.
- **Continuous variable**: underlying variable can take any value within a certain range.

Important case: Markov processes, i.e. memoryless processes.
In discrete time $\mathbb{E}[X_n|X_{n-1}, X_{n-2}, \cdots, X_1, X_0] = \mathbb{E}[X_n|X_{n-1}]$.

Wiener Process aka Brownian Motion

Foundational object in stochastic processes.

- $W_0 = 0$;
- W has independent increments, it is memoryless;
- has Gaussian increments: $dW_t = W_{t+dt} - W_t \sim \mathcal{N}(0, dt)$.
- W has almost surely continuous paths.[1]

In discrete time $\Delta W_t = \sqrt{\Delta t}\epsilon, \quad \epsilon \sim \mathcal{N}(0, 1)$.

[1] This can actually be proven and not assumed.

Ito Process

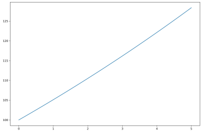
A further type of stochastic process, known as an Ito process, can be defined. This is a generalized Wiener process in which the parameters μ and σ are functions of the value of the underlying variable X and time t.

$$dX + t = \mu\left(X_t, t\right) dt + \sigma\left(X_t, t\right) dW_t.$$

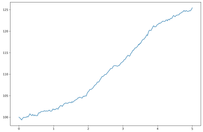
Both the expected drift rate μ and volatility σ of an Ito process are liable to change over time. They are functions of the current value of X and the current time, t.
Special case Geometric Brownian Motion:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

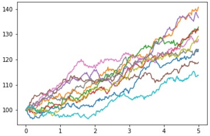
Volatility Impact in Geometric Brownian Motion



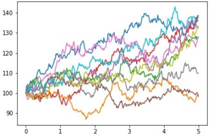
(a) $\sigma = 0$



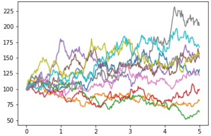
(b) $\sigma = 0.005$



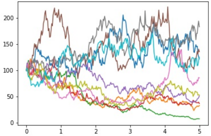
(c) $\sigma = 0.03$



(d) $\sigma = 0.08$



(e) $\sigma = 0.2$



(f) $\sigma = 0.4$

Ito's Lemma

Let X_t be an Ito process and $G(t, X_t)$ a function sufficiently regular, then:

$$\boxed{dG = \left(\frac{\partial G}{\partial t} + u(X_t, t)\frac{\partial G}{\partial X} + \frac{\sigma(X_t, t)^2}{2}\frac{\partial^2 G}{\partial X^2}\right) dt + \sigma(X_t, t)\frac{\partial G}{\partial X} dW_t}$$

Mnemonic rules to perform calculations fast: [2]

$$dW_t dW_t = dt \quad dW_t dt = 0 \quad dt dt = 0$$

[2] They can be proven!!!

Martingales

A fair game according to mathematicians, definition: $\mathbb{E}[M_t|\mathcal{F}_s] = M_s$.
$\mathcal{F}_s$ it the information[3] available at time s.
According to the martingale representation theorem if M_t is a martingale than there exist a process Γ_t such that:

$$M_t = M_0 + \int_0^t \Gamma_u dW_u.$$

Under some additional hypothesis[4] on σ, the converse is also true. If an Ito process is driftless:

$$dX_t = \sigma(t, X_t) dW_t$$

then it is a martingale.

[3] Filtration for friends.
[4] usually true in finance

Girsanov Theorem (oversimplified)

Let W_t be a Brownian motion under the probability measure $\mathbb{P}$, then it is possible to define a martingale $Z_t(\lambda)$ that allows us to perform a change of measure from the old measure $\mathbb{P}$ to a new one $\tilde{\mathbb{P}}$.
Moreover the process

$$\tilde{W}_t = W_t - \int_0^t du\lambda(u)$$

is a Brownian Motion under $\tilde{\mathbb{P}}$.
The theorem tells us how transform a stochastic equation when we change measure.

Black and Scholes Model

The general assumptions of the Black–Scholes model are:

1. The stock price follows the process

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \qquad \sigma > 0, S_0 > 0.$$

   with μ and σ constant.
2. The short selling of securities with full use of proceeds is permitted.
3. There are no transaction costs or taxes. All securities are perfectly divisible.
4. There are no dividends during the life of the derivative.
5. There are no riskless arbitrage opportunities.
6. Security trading is continuous.
7. The risk-free rate of interest, r, is constant and the same for all maturities.

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Call and Put option prices in Black-Scholes model

The value of a call option for a non-dividend-paying underlying stock in terms of the Black–Scholes parameters is:

$$
\begin{aligned}
C(S_t, t) &= N(d_+)S_t - N(d_-)Ke^{-r(T-t)} \\
d_+ &= \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right] \\
d_- &= d_+ - \sigma\sqrt{T-t}
\end{aligned}
$$

The price of a corresponding put option is:

$$
\begin{aligned}
P(S_t, t) &= Ke^{-r(T-t)} - S_t + C(S_t, t) \\
&= N(-d_-)Ke^{-r(T-t)} - N(-d_+)S_t
\end{aligned}
$$

Greeks

Quantities representing the sensitivity of the price of derivatives such as options to a change in underlying parameters on which the value of an instrument or portfolio of financial instruments is dependent. The aim of a trader is to manage the Greeks so that all risks are acceptable.

$$\epsilon = \begin{cases} 1, & \text{for call} \\ -1, & \text{for put} \end{cases}$$

Greek letter	Derivative	Formula
Δ (Delta)	$\frac{\partial}{\partial S}$	$\epsilon N(\epsilon d_+)$
Γ (Gamma)	$\frac{\partial^2}{\partial S^2}$	$\frac{n(d_+)}{S\sigma\sqrt{T-t}}$
$\mathcal{V}$ (Vega)	$\frac{\partial}{\partial \sigma}$	$Sn(d_+)\sqrt{T-t}$
ρ (Rho)	$\frac{\partial}{\partial r}$	$\epsilon K(T-t)e^{-r(T-t)}N(\epsilon d_-)$
Θ (Theta)	$\frac{\partial}{\partial t}$	$-\epsilon rKe^{-r(T-t)}N(\epsilon d_-)$ $-Sn(d_+)\frac{\sigma}{2\sqrt{T-t}}$

Other derivatives

- American option: can be exercised at any time before maturity T.
- Asian Option: payoff is determined by the average underlying price over some period of time: $H_T^{\text{call}} = \left(\frac{1}{T} \int_0^T S(t)\, dt - K \right)^+$
- Equity Swap: a set of future cash flows are agreed to be exchanged between two counterparties at set dates in the future. The two cash flows are usually referred to as "legs" of the swap; one of these "legs" is usually pegged to a floating rate such as LIBOR. This leg is also commonly referred to as the "floating leg". The other leg of the swap is based on the performance of either a share of stock or a stock market index. This leg is commonly referred to as the "equity leg".

Calibration

Process of finding the values of the parameter that make a model fit the data.
It is a hard problem in general, there are several ways of doing it.
Let's supposed to have N observations y_i taken at times t_i. Let's also suppose to have a model f describing the dependence between y_i and t_i, f also depends on $M < N$ parameters β_i.
We define the residues r_i as the difference between the actual observations and the predicted ones:

$$r_i = y_i - f(t_i; \beta_1, \cdots, \beta_M).$$

The family of least squares methods attempts to find the numerical values of the βs that minimize the sum of the squares of the residuals

$$S = \sum_{i=1}^{N} r_i^2.$$