Homework

Pricing path dependent products

15.04.2020

Calculate the **Exposure Profiles** (i.e. EE/EPE, MLE) for **Interest rate swap** assumption that the interest rate follows a **Black-Sholes** model. For trades profiling please use the **Longstaff-Schwartz** algorithm.

As for the default parameters use the following:

Spot rate: 1%,

Annual Drift: $r = 10\%$ (relative),

Annual Vol: $\sigma = 10\%$ (relative),

Fixed Rate: 1.33%,

Notional: 100000 USD,

Direction: Payers,

Payment Frequency: 3 months,

Maturity: 5y,

Number of simulation paths: 1000.

All other parameters should be chosen ad-hoc as needed.

As a good practice: make sure you leave all the parameters as inputs to the functions so you can modify them easily.

Make sure to illustrate all the points marked as *take notice* in the final project with the appropriate plots. Understand the final result.

Step by step guide:

1. Use the function you created at previous classes to generate Wiener process's increments as need. You would need a number of Wiener process realizations to cover 1000 paths over the entire period of trade at a desired.

2. Use the increments and Black-Sholes formula to calculate the new spot rates at a given time and a given simulation path. **Take notice:** how paths are evolving in time.

3. Calculate the cash-flow at every payment date along each path.

$$\text{CashFlow}(t) = \text{Notional} \times (\text{Rate}(t) - \text{FixedRate}).$$

4. For each date at each path calculate realized future value of the trade (sum of future cash flows along the path)

   Take notice of the following:

   - Towards the end of the trade the distribution of the values (across paths) should converge to zero, because the number of cash flows left is being smaller and smaller.
   - At the beginning of the trade the distribution is the widest.

5. For each paths and each dates calculate the expected future value of the trades in line with Longstaff-Schwartz algorithm:

   (a) linearly regress the realized future values against values of current spot rate for all the paths;

   (b) the expected value of the trade at time t for path i would be the value of the regressed function at a given spot rate (at path i and time t.

   Take notice of the following:

   - Towards the beginning of the trades the Expected values should converge to one number, because the initial rates (at each paths) are close to each other.
   - The distribution of the values (across the paths) grow (initially) wider with time because the spot rates are more and more apart.
   - Towards the end of the trade the distribution of the values (across the paths) converge to zero because less and less cash-flows remain.

6. Calculate and plot the Exposure profiles - Expected Exposure (aka Expected Positive Exposure) and Maximum Likely Exposure. In light of all the previous steps try to understand the final result.

In case of questions do not hesitate to write me an email.