Key Message Statement

Practical Information About

- What backtesting is, and how it can be applied in Finance (or other fields)
  – General backtesting
  – Financial models backtesting
  – Counterparty default backtesting
- Broader lesson we can get in risk assessment and statistical analysis

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Definition of backtesting

- **Backtesting** is the general method for seeing how well a model (or an algorithm or a strategy) will perform if applied in future (or performed if used in past).
- Examples:
  - You created a robot for automated trading, but before investing your money you would like to test it's performances
  - Risk monitoring: you have a model predicting potential future losses of your trades, is it a good model?
- Widely used in finance, but also in other fields where you need forecasting (e.g. meteorology, sport betting)

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Introduction

- Some basic examples on how to **represent probabilities**:
  – Tossing a coin

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Introduction

- Some basic examples on how to **represent probabilities**:
  – Tossing a coin

  – Roll a Dice

  – Clearly: constraint of total probability to be equal to one.

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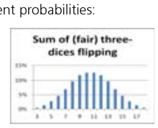
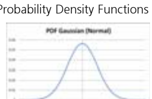
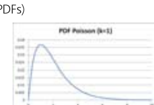
Introduction

- Bit more advanced examples on how to represent probabilities:
  – Rolling mode dices



  – **Continuum limit**: Probability Density Functions (PDFs)





  – Clearly: PDFs are normalized to one. But no constraint to be bounded!

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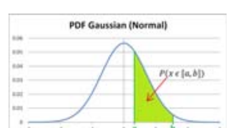
Reminder: PDFs – CDFs

- We consider Gaussian as example, but the definition is general, valid for any PDF
- Given a PDF, the probability to observe an event in a range is given by integration:



$$P(x \in [a,b]) = \int_a^b PDF(x)\,dx$$

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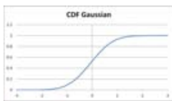
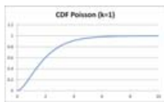
- NB: probability to observe an event in a single point is zero
- Cumulative Density Function (CDF): $\text{CDF}(x) = \int_{-\infty}^{x} PDF(t)\,dt$





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Simple problems

- I give you a coin and a dice. Can you test predicted probabilities?
  - Is the coin fair?
  - Is the dice fair?
  - Are unfair coin/dice as unfair as predicted?

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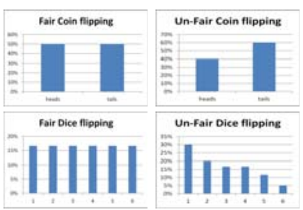

  - Are unfair coin/dice as unfair as predicted?
- Solution: statistics.
  - You just start throwing coin/dice several times, count results and verify if predicted and observed percentages are compatible.

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More complex problem

- I have a pocket of many unfair coins:
  - Each coin has different head/tail probabilities.
  - You are allowed to **<u>flip each coin only once</u>** (or if you are lucky a few times).
  - For each coin, I am predicting the head/tail probability.
  - George is as well providing predictions and he is not in agreement with me.
  - Nicola as well is providing other predictions.
- Can you establish whom of us is providing the right (or most accurate) predictions?
- Can you extend the theory to more complex situations like dices or continuum distributions?

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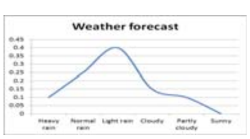
More complex problem: real life example

- Weather forecast:
  - News papers usually state something like: tomorrow at midday will be light raining. This is simplified statement.
  - The meteorologists are collecting data, analysing and performing simulations to provide PDFs of weather conditions:



  - You have a **<u>single occasion to test the predicted result</u>**: tomorrow at midday you check the weather condition.
  - On the other hand, every day we can verify new predictions ;-)
- Can you compare weather forecast from different meteorologists?

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More complex problem: financial example

- Stock market forecast:
  - The value of a financial product is a function of market data.
  - Market data are constantly evolving according to world events.
  - There are several mathematical models that aim to calculate price of a financial product, taking into account possible future evolutions of the value of such a product.
- Example: think you are using your own money! Would you buy a product that my code predicts to be convenient for you? Do you trust me?
- Maybe you trust me (thanks!), or you trust your own model or whatever else, but this is **<u>NOT</u>** the smart way of operating.

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- SMART: first you test the tool and if you get good results you put your money.
- Actually you perform **backtesting**: substantially you test the model using old market data.

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As mathematicians you may also work on some of those models

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This is one our periodic tasks

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Example of Market/Stock forecasting

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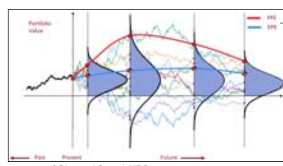
Example of Credit Risk

Credit Risk Pricing



- $\text{CVA} = \underbrace{\text{LGD}}_{\text{Loss Given Default}} \times \int_0^T \overbrace{EAD}^{\text{Exposure At Default}} \times \underbrace{PD}_{\text{Probability of Default}}\, du$
- Pricing the credit risk requires three ingredients:
  - LGD, PD – counterparty dependent
  - EAD – derivative dependent

courtesy of Giovanni Cesari, UBS

- Exposure profilling:
  - Calculating CVA requires knowledge of Trade Value distribution at each point in time
  - MC simulation can provide very general solution to problem of pricing a derivatives regardless of it's complexity
- LGD, PD modelling:
  - For CVA PD/LGD comes from market implied spread
  - For other purposes statistical models based on internal or external historical data

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PIT machinery

- There is the helpful the Probability Integral Transformation (PIT)
- Theorem: Suppose that a random variable X has a continuous PDF and we call F its CDF. Then the random variable Y defined as

$$\text{Y} = F(X)$$

is a **uniform distribution** (i.e. the random variable Y has constant PDF equal to one in the interval [0,1])

- Proof:

$$\begin{aligned} P(Y \le k) &= P(F(X) \le k) \\ &= P(X \le k) \\ &= F\left(F^{-1}(k)\right) \\ &= k \end{aligned}$$

- NB: the hypothesis of continuous PDF can be relaxed (more involved proof).

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PIT machinery in practice

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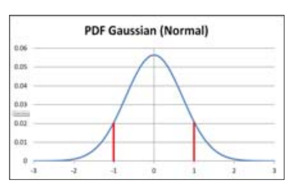

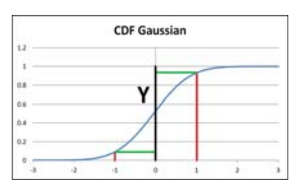

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Backtesting over 1 year historical data

- Consider the following example of Stock Price.
  - For simplicity we consider here a single variable problem, real models takes into account many market data.
  - Suppose you have recorded historical values of last year and you want to use those data to perform backtesting of a model.

- You may sit on day 151 and using available information at that date you run your model to predict day 271 (forecasting horizon 120 days).
  - You have a (post) predicted PDF for day 271 and you know the actual result.
  - You can repeat the exercise by using different dates and different horizons to obtain a large enough statistical set

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Backtesting of a generic model

- **Recap**: usually we have some system governed by a single PDF and several events (or measures, or realizations). To study this test you generally use standard statistical tools (but not always!).
- In case of **backtesting** for each event there can be associated a different PDF.
- Models to be backtested in finance: for a given date and horizon, we can run the simulation for different financial products and trades. Each simulation output is providing different PDF. Playing with different dates, horizons and products the **statistical set can be large enough**!
- **Standard backtesting approach**:
  1. We map all events belonging to different PDF to the same PDF
  2. We can use standard statistical tools to analyse the results
- Comments:
  1. We can choose to map to the easy going uniform distribution (why making our life complicate)
  2. We can also experiment some ad-hoc tools for specific situations

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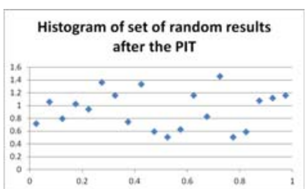
Solution to backtesting problem – part 1

- By using PIT, we can map a set of random events each of them with a different associated PDF, to an uniform distribution.
- As example we can obtain a result looking like:



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Solution to backtesting problem – part 2

- Once the data are mapped to an uniform distribution **the problem is standardized**.
- We can use some **standardized test** to verify whether the distribution is uniform
- **Example 1**: we can define some distance (at PDF level), a quantitative measure to test the result

d= $\int_{-\infty}^{+\infty}(Fn(x)-F(x))\,w(x)\,dx$

where Fn is the empirical CDF and F is the theoretical one. w(x) is a weight function you can choose to emphasize some regions of the unit interval. For example:

- w(x) = 1 Cramer-von-Mises
- w(x) = (1 – F(x))/F(x) Anderson-Darling
- Many more…

- See text books and literature, for example:

Modern Derivatives Pricing and Credit Exposure Analysis – Lichters, Stamm and Gallagher

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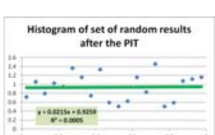
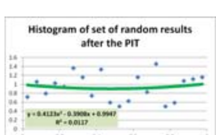
Solution to backtesting problem – part 2

- Once the data are mapped to an uniform distribution **the problem is standardized**.
- We can use some **standardized test** to verify whether the distribution is uniform
- **Example 2**: we can calculate a some fit of the results and verify if the coefficient are compatible with the expected ones
  - $y = ax + b$ where you expect $a = 0, b = 1$:
  - $y = ax^2 + bx + c$ where you expect $a = 0, b = 0, c = 1$:





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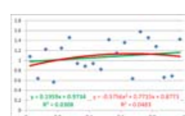
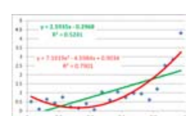
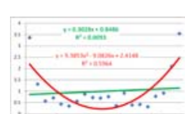
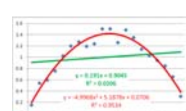
Open problem for you

- You have some realizations of random variables, predicted to be distributed according to Gaussian PDFs. Can you interpret those results? What is going on?









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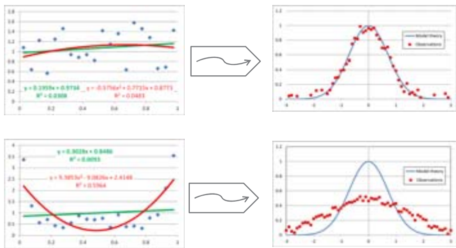
Solution part 1: Open problem for you

- We can guess model quality from results, tr5y to understand what is going on.
- But remember we are plotting together observations related to different predictions (each has a different PDF, not all Gaussian PDFs!).



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Solution part 2: Open problem for you

- We can guess model quality from results, tr5y to understand what is going on.
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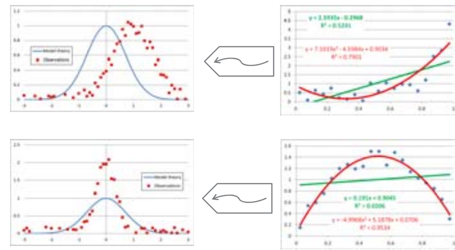

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Another example: default of counterparty (1)

- Investment banks have trades open with many counterparts
  - Some may default and (as consequence) cause losses for the bank.
  - We need some mathematical model to predict possible losses
  - Be need to test model performances: backtesting
- Step one: you rank all counterparties assigning to each of them a probability of default in a given future period (for example)
- After one year you verify how many of them actually default

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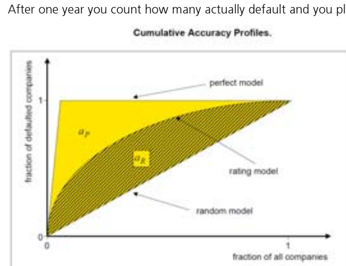
Another example: default of counterparty (2)

- Backtesting strategy: you list all counterparties from left (higher probability of default PD) to right (lower PD)
- After one year you count how many actually default and you plot the result.



The closer the rating model curve is to the perfect model curve, the better you performed

Figure from: Studies on the Validation of Internal Rating Systems - Basel Committee on Banking Supervision https://www.bis.org/publ/bcbs_wp14.htm

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Backtesting vs. Forward Performance Testing

- Forward testing.
  - It is also known as paper trading: may be applied before using an automated trading. Trades are executed in theory, maybe using historical data. After a certain testing period we can assess the quality of trading strategy/algorithm/robot.
  - Pros: we know quality of our trading before risking our money
  - Cons: you are assuming that what appended in the past will be at least statistically equivalent in the future. Namely you assume the world is not changing.
- Backtesting of exposure models.
  - The IB is calculating every day the exposure of all trades, to "predict" potential losses. Alter on we can verify the actual losses
  - Pros: You just test results without historical assumptions
  - Cons: you test model performances only after you used it.

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Regulatory requirements

- Investment banks (IB) are regulated by central banks or prudential authorities (e.g. Fed in US, PRA un UK, KNF Poland).
  - In general IB are required to prove they are reliable, stable and solid. Otherwise they cannot operate!
  - Among several duties (e.g. capital requirements, reserve requirements, corporate governance, financial requirements), they must assess model quality.
- Backtesting is one of the periodic **mandatory** duties for IB.
- Internal rating backtesting (Cumulative Accuracy Profiles) also **mandatory** duties for IB.

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Final general comments about Risk analysis

- In finance and in life in general it is important to take into account the risks.
- Probabilities and statistics are powerful tools to help in risk management.
- Statistical investigation is like police investigation, you must understand about what you are doing and think before act.
- Example: think you are detective investigating on a murder crime scene. You can search for fingerprints.
  - If you find fingerprints of person A, does it implies (s)he is the guilty?
  - If you do not find fingerprints of person B, does it implies (s)he is innocent?
  - Fingerprints is just a test, you must have a larger view of the scene.
- Analogous situation in financial risk management: there is no a single statistical tool telling you: the bank is safe. You need to understand and have an overall view.