misconceptions often arise through the use of inconsistent terminology and incomplete definitions of terms. For example, if the term "delay time" is used in a general sense in one case, used to mean carrier delay time in another case, or used in the context of envelope delay time in another instance, misunderstandings are certain to arise. Therefore, a mutual understanding between filter designers and filter users regarding fundamental terms and definitions of terms is highly desirable. Even though a particular delay characteristics may be improperly named, a clear and complete definition of the significance of the term can minimize the confusion of misinterpretation.

Acknowledgment

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A Telemetering System by Code Modulation —Δ-Σ Modulation*

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Summary—**A communication system by code modulation is described which incorporates an integration process in the original delta modulation system and is named delta-sigma modulation after its modulation mechanism. It has an advantage over delta modulation in dc level transmission and stability of performance, although both require essentially an equal bandwidth and complexity of circuitry. An experimental telemetering system employing delta-sigma modulation is also described.**

Introduction

PULSE CODE MODULATION (PCM) and delta modulation (ΔM) have been known as communication systems by code modulation. Although PCM is considered to be the most efficient among the existing communication systems, the circuitry used in modulation and demodulation is quite complicated and expensive. On the other hand, ΔM requires wider bandwidth than PCM, but its circuitry is much simpler.

However, in a ΔM system, pulses which are sent over a transmission line from the sending end carry the information corresponding to the differentiation of the amplitude of the input signal, and in the receiving end these pulses are integrated to obtain the original waveform. Hence transmission disturbances such as noise or hit result in an accumulative error upon the demodulated signal.

This is a drawback of the ΔM system and, in spite of its simplicity of circuitry, has confined its use to the transmission of an audio signal or others which need not transmit the dc level of an input signal.

This drawback of ΔM can be made up for by integrating the input signal before it enters the modulator so as to generate output pulses carrying the information corresponding to the amplitude of the input signal. The delta-sigma modulation (Δ-ΣM) system is a realization of this principle.

The Principle of the Δ-ΣM System

Fig. 1 is the block diagram which shows the principle of this system. It is essentially a feedback system composed of a pulse generator, a pulse modulator, an integrator and a difference circuit. The output pulses $p(t)$ are fed back to the input and subtracted from the input signal $s(t)$ which varies sufficiently slower than the sampling pulses. Then, the difference signal $d(t) = s(t) - p(t)$ is integrated to produce $\epsilon(t) = \int d(t)\,dt$, and enters the pulse modulator. The pulse modulator compares the amplitude of the integrated difference signal $\epsilon(t)$ with a predetermined reference level and opens the gate to pass a pulse from the pulse generator when the polarity of $\epsilon(t)$ is positive, and closes the gate to inhibit the pulse when it is negative. Through this negative feedback procedure, the integrated difference signal is always kept in the vicinity of the reference level of the pulse modulator, provided that the input signal is not too large. Hence, if the amplitude of the input signal becomes large, the output pulses appear more frequently. In other words, the output pulses

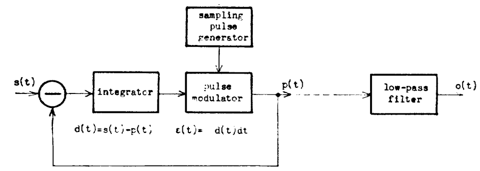

Fig. 1—The block diagram of Δ-Σ modulation system.

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carry the information corresponding to the input signal amplitude.

Demodulation in the receiving end is carried out by reshaping the received pulses and passing them through a low-pass filter and no integration procedure is involved. Therefore, no accumulative error due to transmission disturbances results in the demodulated signal.

An Experimental Telemetering Equipment Using Δ-Σ Modulation

An experimental telemetering equipment is constructed and tested in order to demonstrate the realizability of the communication system based on the above-mentioned principle and to investigate its features.

The equipment is designed for the input signal with the frequency range of dc to 50 cps, and the repetition frequency is chosen as 1 kc.

Fig. 2 and Fig. 3 show, respectively, the block diagram and waveforms of the equipment.

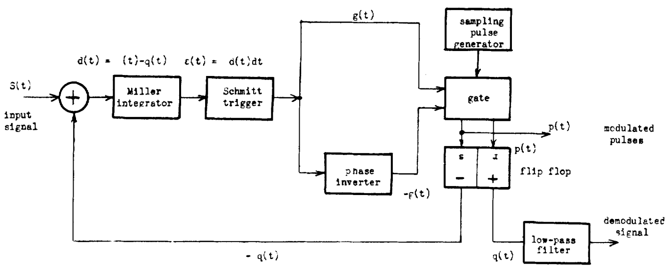

Fig. 2—The block diagram of the telemetering system.

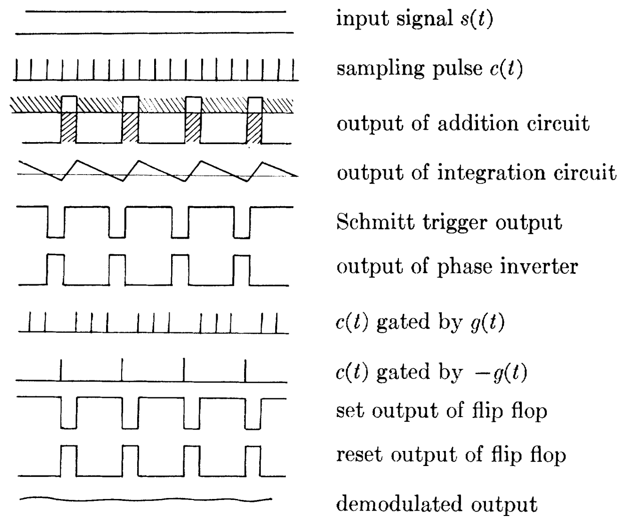

Fig. 3—Waveforms of the telemetering system.

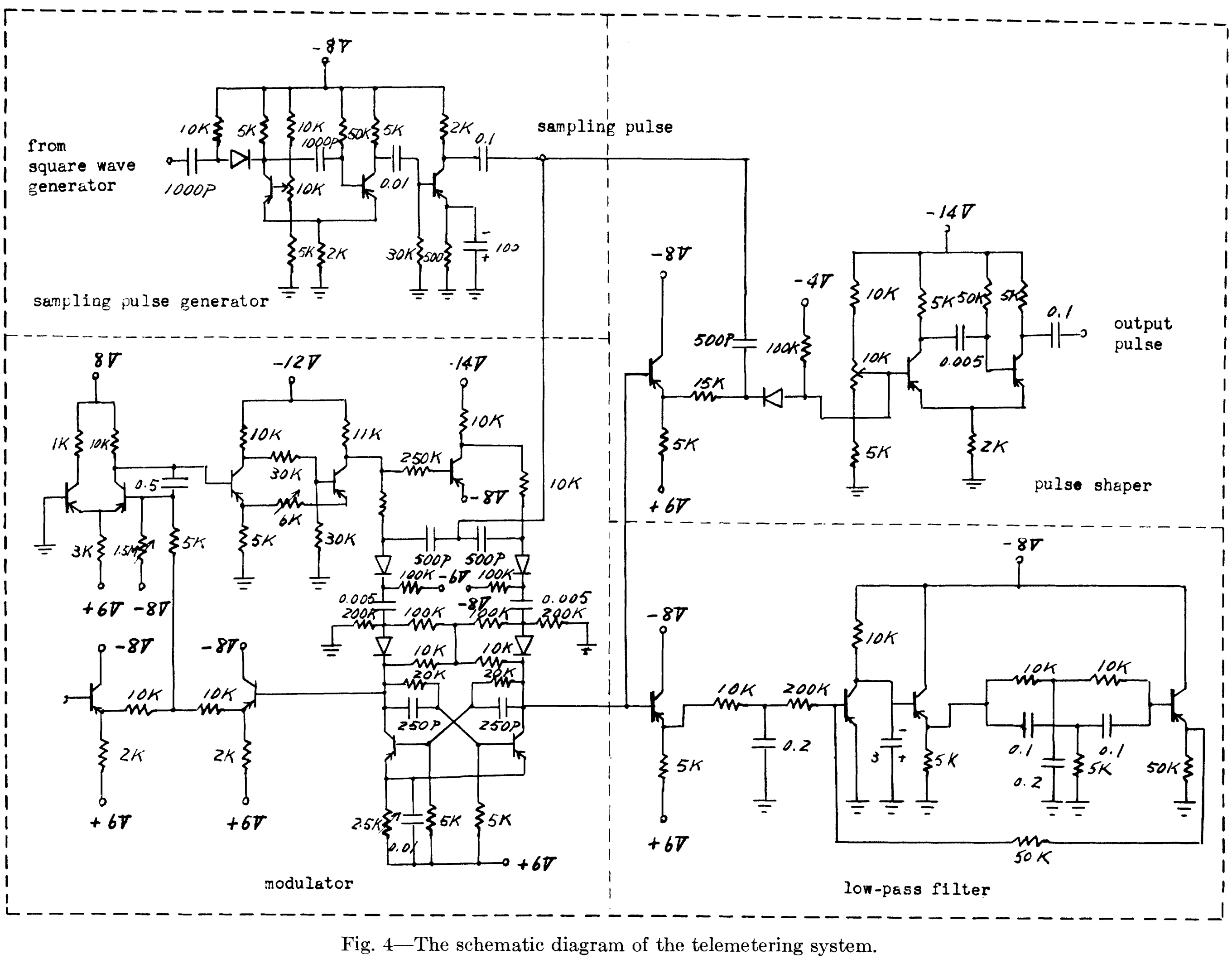

Fig. 4—The schematic diagram of the telemetering system.

The sampling pulse generator generates a series of pulses $c(t)$ with the repetition frequency of 1 kc, the amplitude of 6 v and the pulse width of 5 μsec.

The series of pulses $c(t)$ is gated by $g(t)$ and $-g(t)$, which are the Schmitt trigger outputs of opposite polarity, and becomes, respectively, the set input $p(t)$ and the reset input $\bar{p}(t)$ of the flip flop.

The flip flop reshapes and widens the gated pulse to have the amplitude of ± 3.5 v and to have the width of one sampling period. The reset output $-q(t)$ of the flip flop is fed back to the input, added to the input signal $s(t)$ in the addition circuit composed of resistances and enters the Miller integrator. The output of the Miller integrator is directly coupled to the following Schmitt trigger. When the output of the integrated difference signal $\epsilon(t)$ is larger than the trigger level, the output $g(t)$ of the Schmitt trigger becomes positive and opens the gate and passes a pulse $c(t)$ to the set input of the flip flop.

On the other hand, when it is smaller than the trigger level, $g(t)$ becomes negative and inhibits the pulse $c(t)$. In this case, as the output of the phase inverter $-g(t)$ becomes positive, it passes a sampling pulse to the reset input of the flip flop. Thus the output of the integrator is always kept in the vicinity of the trigger level of the Schmitt trigger. The set input $p(t)$ of the flip flop is fed to the transmitter where it is shaped to match the characteristic of the transmission line.

In the experimental equipment the transmitter is omitted and the set output of the flip flop is directly fed into an emitter-follower and demodulated by means of a low-pass filter with the cutoff frequency of 50 cps.

The schematic diagram of the equipment is shown in Fig. 4. The entire equipment is built with semiconductor components.

Noise in the Δ-Σ Modulation System

The system suffers two sorts of noise, one of which is periodic and the other is random or quantizing noise which generally accompanies in any coding process. With regard to the former, the number of the output pulses in every second, that is, the density f_d is proportional to the amplitudes S of the input signal. If we assume that the amplitude of the flip flop is D and the repetition frequency of the sampling pulses is f_r, the following equation results:

$$f_d = \frac{S}{D} f_r. \tag{1}$$

Then provided that

$$f_d < f_c \tag{2}$$

where f_c is the cutoff frequency of the low-pass filter, the periodic component of the output pulses passes through the low-pass filter, so that this condition should be avoided. Therefore, the dynamic range of the input signal should be given as follows:

$$\frac{f_c}{f_r} D < S < \left(1 - \frac{f_c}{f_r}\right) D. \tag{3}$$

On the other hand, even if $f_d > f_c$, the pattern of the output pulses may contain the frequency component which passes through the low-pass filter. When this is the case, the component adds to the demodulated output as a periodic noise.

Next, as the result of the theoretical calculation described in the Appendix, the signal-to-quantizing noise ratio is given by the following equation:

$$\frac{S}{N} = M \frac{3}{4\pi} \left(\frac{f_r}{f_c}\right)^{3/2} \tag{4}$$

where M is the ratio of the signal amplitude to the maximum amplitude that does not overload the modulator.

In a delta modulation system SNR is given as follows.[1]

$$\frac{S}{N} \sim \frac{f_r^{3/2}}{f_c^{1/2} f_s} \tag{5}$$

where f_s is the signal frequency.

Therefore, both the systems have the same relation as to the pulse repetition frequency, while, in contrast to the delta modulation system, the signal frequency has no relation to the SNR in the Δ-Σ modulation system.

For the purpose of comparison, the SNR of the pulse number modulation is given as follows:

$$\frac{S}{N} \sim \frac{f_r}{f_c^{1/2} f^{1/2}}, \tag{6}$$

where f_r is the pulse repetition frequency and f is the sampling frequency. Reference to (4)–(6) indicates that Δ and Δ-Σ modulation systems have an advantage over the pulse number modulation in the degree of improvement, with regard to the pulse repetition frequency. The reason is that in the former, the pulse repetition frequency is equal to the sampling frequency and as it becomes larger the spectra of the quantizing noise extends to the higher-frequency range, and its component which passes through the low-pass filter is reduced; while in the latter, as the sampling frequency is constant, the frequency composition of the quantizing noise remains unchanged.

The Stability of the Δ-Σ Modulation System

The variation of the reference level of the comparator is generally the most serious problem in an analog-to-digital conversion device when it has to convert the dc level of an input signal exactly into the digital form.

In the Δ-Σ modulation system an input signal is compared with the reference level after it is integrated.

[1] F. de Jager, "Delta modulation—a method of PCM transmission using the one unit code," *Phillips Res. Repts.*, vol. 7, p. 442; 1952.

Therefore, the trigger voltage of the comparator becomes

$$\epsilon(t) = K \int d(t)\,dt = K \int \{s(t) - p(t)\}\,dt. \qquad (7)$$

Now, if we assume that the reference level of the comparator has had a variation of ϵ' volt during the interval sufficiently longer than the time constant of the integrator, the equivalent variation of the input of the integrator becomes d' volt, where

$$\epsilon' = K \int_0^{\tau} d'\,dt = K\tau\, d'$$

or

$$d' = \frac{\epsilon'}{K\tau}. \qquad (8)$$

Then the output pulses appear to compensate for this excursion. As $K \doteqdot 1/RC$ and $\tau \doteqdot ARC$ in the Miller integrator, $K\tau \doteqdot A$. Therefore, provided that the gain is increased, d' can be made negligibly small.

In the experimental telemetering equipment, as the gain A is approximately 100, an error of only 0.01 volt results in the signal, with a variation of the level of 1 volt.

In this way, the variation of the level after integration scarcely effects the system performance. Therefore, only one component of the circuitry which must be well stabilized is the flip flop which feeds the pulse back to the input. The amplitude of the pulses is easily regulated by means of zener diode.

Characteristics of the Experimental Telemetering Equipment

The Input-Output Characteristics

Fig. 5 shows the relation between a dc input level and the output level of the low-pass filter where the repetition frequency $f_r = 3$ kc and the time constant of the integrator $\tau = 10$ msec.

Table I indicates the mean numbers of the output pulses per second which are counted three times for each during 10 seconds.

These results reveal that the linearity of the equipment is satisfactory.

The Frequency Characteristic

Fig. 6 shows the relation between the input signal frequency and the output of the low-pass filter where the amplitude of the input signal is fixed constant. As the characteristic is exactly the same to the frequency characteristic of the low-pass filter, it is known that the modulation system itself has the characteristic which is constant all over the signal frequency range.

The Signal-to-Noise Ratio

The measurement of the SNR is carried out with the method shown in Fig. 7.

Fixing input signal frequency at 30 cps, the relative noise power is obtained from the demodulated signal through a 30 cps rejection amplifier composed of a twin T network. While changing the input signal frequency to pass through the rejection amplifier and fixing its amplitude constant, the component of the signal power is obtained from the demodulated signal.

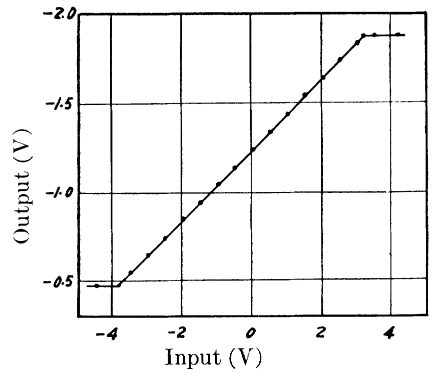

Fig. 5—The input-output characteristics of the telemetering system (dc input).

TABLE I
THE INPUT-OUTPUT CHARACTERISTICS OF THE TELEMETERING MODULATOR (DC INPUT)

Input Level	I	II	III	Average
−4 V	29391	29391	29395	2939/sec
−3	25073	25076	25077	2508
−2	20875	20881	20884	2088
−1	16883	16877	16876	1689
0	12701	12708	12710	1271
1	8593	8593	8590	859
2	4718	4738	4725	473
3	548	547	543	54.6
4	0	0	0	0

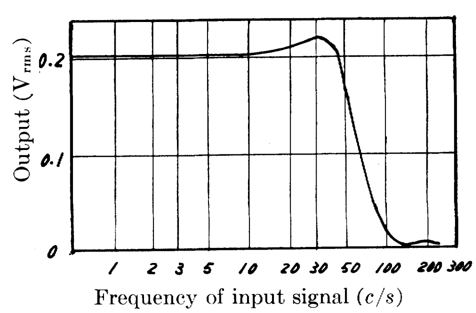

Fig. 6—The frequency characteristics of the telemetering system.

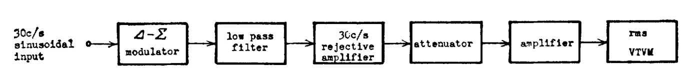

Fig. 7—The method of measuring SNR.

Fig. 8 shows the theoretical and experimental SNR with the input signal amplitude fixed at 2 volts, where f_r is taken as a parameter. Fig. 9 shows the input and output waveforms with various amplitude and frequency ranges which are recorded on a pen recorder.

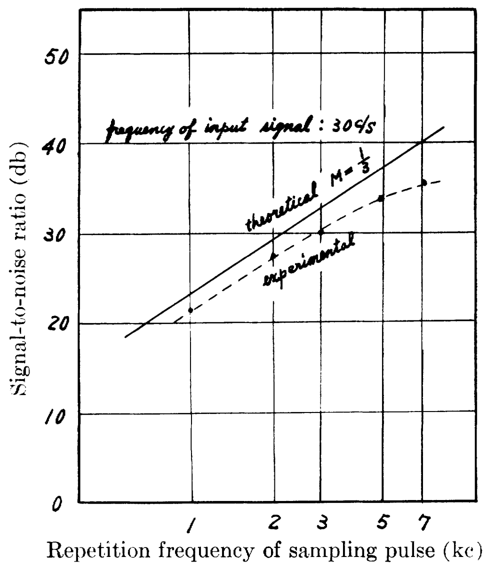

Fig. 8—Measured SNR.

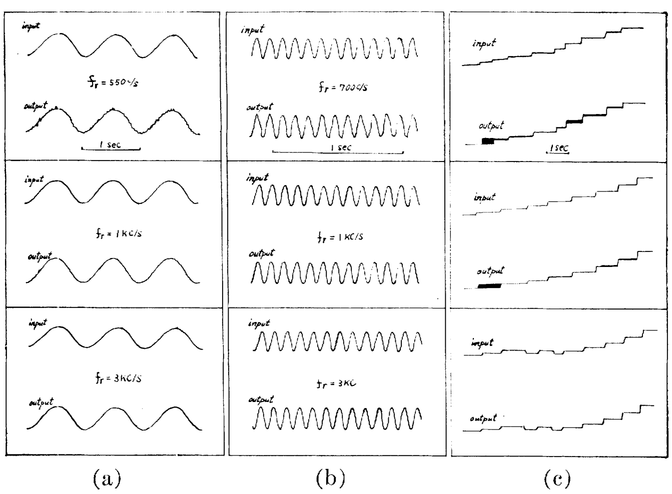

Fig. 9 = Input and output waveforms recorded by a pen recorder. (a) Sinusoidal input (1 cps). (b) Sinusoidal input (10 cps). (c) Staircase input.

Conclusion

Although this system is originally derived from the delta modulation system, it may be considered to be a new communication system by code modulation in view of the characteristics discussed above. It has the following major features:

1) It transmits a dc level of the input signal. This is an advantage over Δ modulation.
2) The stability of performance is satisfactory as compared with other code modulation systems.
3) It requires smaller bandwidth than pulse number modulation.

Appendix
Calculation of SNR

The SNR of the Δ-Σ modulation system can be estimated as follows, where the symbols mean

$s(t)$, $S(\omega)$:	input signal $\mid \omega \mid < \omega_c = 2\pi fc$
$p(t)$, $P(\omega)$:	output pulses
$l(t)$, $L(\omega)$:	difference signal
$\epsilon(t)$, $E(\omega)$:	integrated difference signal
$o(t)$, $O(\omega)$:	demodulated output
$H(\omega)$:	frequency characteristic of the low-pass filter.

$$= \begin{cases} 1 & \mid \omega \mid < \omega_c \\ 0 & \mid \omega \mid > \omega_c \end{cases}$$

$\epsilon(t)$ does not exceed the amplitude Δ of the integrated pulse as long as the input signal is kept in the nonsaturating range.

Therefore, for the sake of convenience, we assume that $\epsilon(t)$ is a sequence of rectangular pulses shown in Fig. 10.

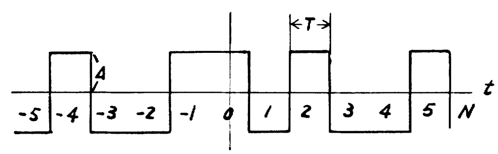

Fig. 10—The waveform of $\epsilon(t)$ assumed for calculation.

Then $\epsilon(t)$ is expressed as follows:

$$\epsilon(t) = \sum_{n=-\infty}^{\infty} \eta_n f(t - nT) \tag{9}$$

where

$$\eta_n = \begin{cases} +1 & \epsilon(t) > 0 \\ -1 & \epsilon(t) < 0. \end{cases}$$

Now, the Fourier transform $F(\omega)$ of $f(t)$ is

$$F(\omega) = \Delta T S\left(\frac{\omega T}{2}\right) \tag{10}$$

where $T = 1/f_r$, the period of the sampling pulses, and $S_a(x) = \sin x/x$.

Hence, the Fourier transform $E_N(\omega)$ of the portion of $\epsilon(t)$ between number $-N$ and N is given as follows:

$$E_N(\omega) = \Delta T \sum_{n=-N}^{N} S_a\left(\frac{\omega T}{2}\right) e^{-in\omega T}. \tag{11}$$

On the other hand

$$\frac{d\epsilon}{dt} = s(t) - p(t) \tag{12}$$

hence,

$$j\omega E(\omega) = S(\omega) - P(\omega). \tag{13}$$

Multiplying $H(\omega)$ to the both sides,

$$j\omega E(\omega)H(\omega) = S(\omega)H(\omega) - P(\omega)H(\omega). \tag{14}$$

Then, by the previous assumption on $H(\omega)$ the following equations result:

$$S(\omega)H(\omega) = S(\omega) \tag{15}$$

$$P(\omega)H(\omega) = O(\omega). \tag{16}$$

Eq. (14) can be rewritten as follows:

$$j\omega E(\omega)H(\omega) = S(\omega) - O(\omega). \tag{17}$$

This is equal to the Fourier transform of the difference signal $L(\omega)$. Then, assuming that the part of the difference signal between number $-N$ and N is $L_N(\omega)$,

$$L_N(\omega) = j\omega E_N(\omega)H(\omega). \tag{18}$$

Therefore, the power spectrum of the difference signal becomes as follows:

$$\begin{aligned} W(\omega) &= \lim_{N\to\infty} \frac{1}{(2N+1)T} L_N(\omega)L_N^*(\omega) \\ &= \omega^2 \lim_{N\to\infty} \frac{\Delta^2 T}{2N+1} S_a^2\left(\frac{\omega T}{2}\right) \\ &\qquad \cdot \sum_n \sum_m \eta_n \eta_m e^{j(n-m)\omega T} H(\omega)H^*(\omega). \end{aligned} \tag{19}$$

Then, defining $K(\omega)$ as

$$K(\omega) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_n \sum_m \eta_n \eta_m e^{j(n-m)\omega T}, \tag{20}$$

the $n \neq m$ terms disappear and $(2N+1)$ $n = m$ terms remain, so that

$$K(\omega) = 1 \tag{21}$$

hence,

$$W(\omega) = \Delta^2 T \omega^2 S_a^2\left(\frac{\omega T}{2}\right) H(\omega)H^*(\omega). \tag{22}$$

The quantizing noise power N which passes through the low-pass filter is given as follows:

$$N^2 = \frac{\Delta^2 T}{2\pi} \int_{-\omega_c}^{\omega_c} \omega^2 S_a^2\left(\frac{\omega T}{2}\right) d\omega \tag{23}$$

where the assumption that $|H(\omega)| = 1$ is used.

In this equation, expanding $S_a^2(\omega T/2)$ in series and leaving its first term

$$N^2 = \frac{8}{3}\pi^2 f_c^3 \Delta^2 T, \tag{24}$$

that is

$$N = 2\sqrt{\frac{2}{3}}\pi f_c^{3/2} \Delta T^{1/2}. \tag{25}$$

On the other hand, when the pulse width of $p(t)$ is equal to the sampling period, the dynamic range of the input signal is approximately as follows:

$$D = 2\frac{\Delta}{T}. \tag{26}$$

Therefore, defining that the ratio of peak-to-peak amplitude of the sinusoidal input signal to the dynamic range is M,

$$S = M \frac{1}{\sqrt{2}} \frac{\Delta}{T}. \tag{27}$$

Hence, the SNR is

$$\frac{S}{N} = M \frac{\sqrt{3}}{4\pi} \left(\frac{f_r}{f_c}\right)^{3/2}. \tag{28}$$

This is obtained from the assumption that $\epsilon(t)$ is composed of rectangular pulses with the constant amplitude Δ. However, in actual cases, the amplitude of $\epsilon(t)$ never exceeds Δ.

Assuming that the amplitude of $\epsilon(t)$ takes the value between zero and Δ with equal probability, the noise power is increased by the following value:

$$\frac{1}{\Delta^2}\int_0^\Delta y^2 \frac{dy}{\Delta} = \frac{1}{3}.$$

Therefore, the SNR can be rewritten as (4).

The SNR of the pulse number modulation can be also calculated in a similar procedure.

Acknowledgment

The authors are deeply indebted to Prof. T. Sakamoto for his suggestions and encouragements.