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Multiple Choice Knapsack Problem: Example of planning choice in transportation

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ABSTRACT

Transportation programming, a process of selecting projects for funding given budget and other constraints, is becoming more complex as a result of new federal laws, local planning regulations, and increased public involvement. This article describes the use of an integer programming tool, Multiple Choice Knapsack Problem (MCKP), to provide optimal solutions to transportation programming problems in cases where alternative versions of projects are under consideration. In this paper, optimization methods for use in the transportation programming process are compared and then the process of building and solving the optimization problems is discussed. The concepts about the use of MCKP are presented and a real-world transportation programming example at various budget levels is provided. This article illustrates how the use of MCKP addresses the modern complexities and provides timely solutions in transportation programming practice. While the article uses transportation programming as a case study, MCKP can be useful in other fields where a similar decision among a subset of the alternatives is required.

1. Optimization methods in transportation programming

Transportation programming is the process of selecting a final set of projects to be funded by a transportation agency (Niemeier, Zabinsky, Zeng, & Rutherford, 1995). Two common difficulties associated with transportation programming include a competition for funding among transportation modes that may have differing objectives and an often lack of clear relationships between project implementation and policy goals (Humphrey, 1974). This process is increasingly complicated by requirements in Federal transportation legislation starting with the Intermodal Surface Transportation Efficiency Act of 1991 (ISTEA) and continuing through the current legislation, the Safe, Accountable, Flexible, Efficient Transportation Equity Act—A Legacy for Users (SAFETEA-LU) adopted in 2005. Other legislation including the Federal Clean Air Act and state and local regulations has also added to the complexity. In general, the trend in transportation programming is towards consideration of more factors in the planning process (Gage & McDowell, 1995; Greenstone, 2002; United States Congress, 2005; US DOT & FHWA, 1992). At the same time, increased public involvement is being expected and often required.

The transportation programming process can be completed with or without complex modeling tools. Programming without modeling tools is typically based on rules of thumb, heuristic methods, or decision makers' experience (Turshen & Wester, 1986). These methods are common in current practice because they are easy to apply and do not require quantitative measurements. However, the increasing number of factors to be considered and an increased emphasis on system performance make it difficult to make efficient decisions using these simple methods. For this reason, transportation programming should incorporate advanced modeling tools to help decision makers arrive at final decisions using optimization methods.

Decision making with modeling tools can be considered a decision support system (DSS). Based on the research by Keen and Scott Morton (1978), DSS was developed from the Carnegie Institute of Technology's research on the theoretical studies of organizational decision making during the late 1950s and 1960s and the Massachusetts Institute of Technology's technical work on interactive computer systems in the 1960s. DSS has been applied in such diverse areas as environmental preservation, agriculture, and medical services (Eom, Lee, Kim, & Somarajan, 1998; Hunt, Haynes, & Smith, 1998; Jintrawet, 1995; McCall & Minang, 2005). In the transportation field, DSS has been used for airline flight scheduling and project location choice problems (Jankowski & Nyerges, 2003; Jarrah, Yu, Rishnamurthy, & Rakshit, 1993). DSS has not commonly been applied to transportation programming but the complexity of the process and need for public input make it valuable tool.

Different applications define DSS differently. Finlay (1994) defines it as a computer-based system that aids the process of decision making. Turban (1995) defines it as an interactive, flexible, and adaptable computer-based information system developed for a non-structured management problem that utilizes data, provides an easy-to-use interface, and allows for the decision maker's own insights. Little's (1970) definition, which may by most applicable to the transportation programming problem, is a model-based set of

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procedures for processing data and judgments to assist a manager in his decision-making.

For a model-based system, the choice of model for use in a DSS is very important. Because many factors will be considered during the transportation programming process, Multicriteria Decision Making (MCDM) is a natural choice for this decision situation. Note that Multicriteria Decision Making is sometimes called Multicriteria Decision Analysis (MCDA). MCDM models can be divided into two types: Multiobjective Decision Making (MODM) models and Multiattribute Decision Making (MADM) models.

In transportation programming, decision criteria are measured by project attributes such as travel time savings and construction costs, the projects and their attributes are defined explicitly and, the number of projects under consideration is limited. Because of these traits, transportation programming should use MADM as the MCDM model.

There are many options to consider in building the MADM model. Niemeier et al. (1995) categorizes three types of transportation programming methods:

- Cost-effectiveness evaluation
- Point score ranking
- Cost–benefit analysis

Each of these three models has advantages and disadvantages. Evaluations using cost-effectiveness are simple, but it is difficult to rank projects in terms of desirability (Niemeier et al., 1995). Point score ranking, though simple to perform, usually results in tight clustering of projects because of the subjective category method labels projects as high, medium, or low categories. Cost–benefit analysis is the recommended method of ranking projects (McFarland & Memmott, 1987). However, most of its applications only include those benefits measures that have been used extensively, such as travel time savings. Some measurements (e.g., environmental impacts) or objectives (e.g., increased travel options, economic development potential) that cannot be easily quantified are often not considered. This limitation is unacceptable when regulations require these factors be taken into account.

Hwang and Yoon (1981) provided a method to overcome this issue. All quantitative and qualitative factors with their associated weights are used to produce a composite index (or ranking index). The composite index can then be used in traditional Linear Programming (LP) and Goal Programming (GP) methods. Linear Programming, defined as a method to maximize or minimize a linear objective function of the decision variables (Vanderbei, 2001), has been widely used in the transportation field since the 1970's for such tasks as planning, construction scheduling, and network analysis (Aneja & Nair, 1979; Ben-Ayed, Boyce, & Blair, 1988; Moreb, 1996). Goal Programming, a multiobjective extension of Linear Programming (Ignizio, 1978), has been used in project selection for highway construction and preservation (Muthusubramanyam & Sinha, 1982). Niemeier et al. (1995) points out that one of GP's problems is that its solution cannot meet strict cost constraints. Given recent reductions in federal funding (Turshen & Wester, 1986), funding increases to accommodate GP solutions are not likely. On the other hand, LP solutions can produce an optimized project list while meeting strict cost constraints. In the past, the problem with LP was a lack of computation power necessary to solve the problems quickly. Computation technology has developed so that this is no longer a problem in traditional transportation programming processes.

In addition to LP and GP, the transportation programming problem can also be considered an Integer Programming (IP) problem. The Knapsack Problem (KP) has been well studied as a specific form of IP. In fact, Mathews (1897) performed the early research on this topic in the late 1890's. The Multiple Choice Knapsack Problem (MCKP) is a variant of KP, first used in the late 1970's (Sinha & Zoltners, 1979), that adds additional constraints that prohibit the inclusion of an object in the solution set if another object is selected. For the transportation programming problem, this allows for projects that are solutions to the same transportation issue to be considered for funding. For example, the project list could include a transit and a non-transit alternative to the same congestion problem. The additional MCKP constraints are necessary to ensure that both projects are not funded since they inherently serve the same function.

In the traditional transportation planning process, the decision on which competing project alternative was preferred was made earlier in the planning process, prior to the programming step so all projects under consideration for funding were considered independent choices. As the programming and planning processes become more complex, it has become necessary to advance competing project alternatives into the programming decision process. Increased public involvement in the programming process has also necessitated this change since the choice of a particular project alternatives is often viewed by the public as just as important as the choice to spend funding resources to solve the particular transportation issue.

Like other KP variants, MCKP can be solved using two approaches. One approach is the use of the branch-bound method and the other is dynamic programming. The branch-bound method is an enumeration approach, which reduces its search space by excluding impossible solutions. Several branch-bound algorithms have been developed (Dyer, Kayal, & Walker, 1984; Nauss, 1978; Sinha & Zoltners, 1979). Among them the DyerKayalWalker algorithm is particularly well designed and easily understood. Dudziniski and Walukiewicz (1987) and Pisinger (1995) provided a dynamic programming algorithm approach. To improve on solution efficiency, Dyer, Riha, and Walker (1995) and Pisinger (1995) presented hybrid algorithms that combine dynamic programming and upper boundary tests. Though the hybrid algorithms perform better in computation tests, its integer requirement (either the project cost or project profit must be an integer) limits its application in transportation programming (Kellerer, Pferschy, & Pisinger, 2004). For this reason, the branch-bound method is a good choice for solving the MCKP in transportation programming, though there is a need for improved solution performance for larger scale problems. It should be noted that in practice the integer requirement in dynamic programming can be overcome by scaling the decision and constraint variable by a large constant value (e.g., 1000) to obtain integer values with enough practical precision.

2. Building and solving the optimization problem

Transportation programming aims to find an optimal package of projects for funding, given a budget level, funding priorities (expressed as criteria weights), and other agency constraints such as requirements for regional distribution of projects, that can be translated into an optimization problem. Atallah (1999) defines an optimization problem as "a computational problem in which the object is to find the best of all possible solutions." More formally, to find a solution within the feasible region that has the minimum (or maximum) value of the objective function. The feasible region for the transportation programming problem is a pool of candidate projects funded that is bounded by the decision constraints, such as the total available funds. The objective function is to maximize the sum of the weighted project scores (assuming all project attributes are expressed so that larger scores represent greater desirability). Building the optimization problem requires collecting information about the decision situation that defines the feasible region and the objective function. The general process for building the transportation programming optimization problem consists of the six steps described in the following paragraphs.

First, the project list under consideration for funding must be compiled. Typical sources of these projects include planning documents compiled by state Department of Transportations (DOTs), Metropolitan Planning Organizations (MPOs), and other planning agencies charged with transportation planning. Because of the quantitative requirements for the programming step, considerable effort must be invested in a project prior to being considered for funding in the programming process. As mentioned earlier, traditionally only one alternative for a given transportation issue was advanced for funding consideration and the planning process was charged with selecting the preferred alternative. In modern programming, while the planning process is charged with reducing the number of alternatives to a shortened list, it is possible that competing alternatives are under consideration in the programming step. Recent reductions in available funding have also led to a large backlog of projects being considered for funding, increasing the competition among projects.

The second step is to estimate the project impacts, where the project impacts are directly linked to project criteria. The criteria to be considered in the transportation programming process determine which project impacts will be included in the analysis. Selection of the criteria to be used in the process should reflect the policy priorities of the agency and the public. While it is preferable that all the project impacts are quantitative in nature, sometimes this is not realistic, so qualitative (i.e. categorical) results can be used.

The third step is to collect the weights for the criteria. These weights will be used to create the weighted project scores. Many methods can be used to collect weights, such as assignment of weights directly by decision makers or through more complex method such as an Analytic Hierarchy Process. Ideally this process has a public involvement component since the determination of the criteria weights has considerable impact on the decision process. The criteria weights reflect the relative importance of one criterion compared to others.

The fourth step is to create project scores based on the project impact estimates from the second step. This step is essentially a normalization process to put the different project impacts into a common scale. The results of normalizing project impacts are called project scores in this article. The scores are in the real number range from 0 to 100 (or some similar scale), although technically negative scores are possible. Higher values represent more positive outcomes. Normalization is required because the units of the different project impacts are not consistent. For example, the unit for travel time saving benefits could be US dollars, while qualitative estimates could be unitless categorical values. There are two frequently used normalization methods. The percentile method converts the values of project impacts from real numbers to percentages by dividing the value of each project impact item by the largest value of this item and then multiplying by 100. This method is shown in Eq. (1).

Percentile Normalization

$$s_{ij} = \frac{p_{ij}}{\max_i(p_{ij})} \times 100 \tag{1}$$

where $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$; p_{ij} is the value of the jth project impact for the ith project; s_{ij} is the normalized value of the jth project impact for the ith project.

The value of the deviation variables found using Eq. (1) does not represent an actual geometric distance, which may lead to bias in the problem solution (Young, Barnes, & Rutherford, 2002a). For this reason, the second method, Euclidean normalization, is used in this research. The difference between the percentile normalization and Euclidean normalization is that the Euclidean distance (or Euclidean norm) is used as the deviation in Euclidean normalization. The Euclidean distance is the square root of the sum of the square of the values of project impacts. Eq. (2) shows the Euclidean normalization method.

Euclidean Normalization

$$s_{ij} = \frac{p_{ij}}{\left(\sum_{i=1}^{n} | p_{ij}^2\right)^{1/2}} \times 100 \tag{2}$$

where $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$; p_{ij} is the value of the jth project impact for the ith project; s_{ij} is the normalized value of the jth project impact for the ith project.

The fifth step is to develop the sum of weighted project scores for every project. Linear Weighted Combination and Ordered Weighted Averaging (OWA) (Rinner & Malczewski, 2002) are two methods for performing this step.

The final step in building the transportation programming optimization problem is to construct the objective function and define the constraints. The objective function maximizes (or minimizes) a value that incorporates decision variables. For this problem, the decision variable is a 0–1 binary value with each project having an associated decision variable. If this variable is set to 1, then that project will be included in the final funding package. Otherwise, the value is set to 0 and this project is not included in the funding solution. In this research, the maximization of the sum of weighted project scores is the objective function since this represents the funding solution that provides the greatest transportation impact within the available funding given the project criteria categories and criteria weighting.

After the objective function is built, the next step is to define the constraints. The optimization problem (MCKP) discussed in this article has three constraints: investment constraint, alternative constraint, and 0–1 integer constraint. The investment constraint limits the sum of the funded project costs to be less than the total defined budget. The option constraint requires that, for projects that are alternative solutions to the same problem, at most one alternative can be funded. An alternative constraint is required for every subset of project alternatives (or alternative group). It does not require that an alternative be selected from each subset but limits the maximum number selected to be no greater than one. The 0–1 integer constraint specifies that the decision values for each project can only be 0 or 1.

Eq. (3) defines the objective function and constraints for the MCKP transportation programming optimization problem. The first part of the equation is the objective function. The equations following "subject to" are the three constraints. In this definition, a project with more than one alternative is considered an alternative group. A project without any competing alternatives is considered a group with only one alternative.

MCKP Definition

$$\begin{aligned}
&\text{maximize} \sum_{i=1}^{m} \sum_{j \in N_i} s_{ij} x_{ij} \\
&\text{subject to} \sum_{i=1}^{m} \sum_{j \in N_i} w_{ij} x_{ij} \le c (\text{investment constraint}) \\
&\sum_{j \in N_i} x_{ij} \le 1, \quad i = 1, 2, \ldots, m \ (\text{option constraint}) \\
&x_{ij} \in \{0, 1\}, \quad i = 1, 2, \ldots, m,\ j \in N_i \ (\text{0-1 integer constraint})
\end{aligned} \tag{3}$$

where c is the total investment; w_{ij} is the cost for the alternative j of project I; s_{ij} is the weighted score for the alternative j of project i; x_{ij} is the decision variable for alternative j of project i (1 if funded, 0 if not funded); N_i is the alternative group of project i.

3. Multiple Choice Knapsack Problem solution method

As mentioned earlier, the Multiple Choice Knapsack Problem (MCKP) is an extension of the classic Knapsack Problem (KP) (Kellerer et al., 2004). This section describes the process for solving the MCKP problem. To start the discussion, this section first begins with the solution techniques for the classic Knapsack problem. Eq. (4) shows the definition of the classic KP.

Knapsack Problem Definition

$$\begin{aligned}&\text{maximize} \sum_{j=1}^{n} p_j x_j\\&\text{subject to} \sum_{j=1}^{n} w_j x_j \le c\\&x_j \in \{0,1\}, \quad i = 1, 2, \ldots, n\end{aligned} \tag{4}$$

where p_j is the profit/benefit of jth item; w_j is the weight (i.e. cost) of jth item; c is the capacity (i.e. budget) of a knapsack; x_j is the decision variable for jth item.

The KP is an integer programming problem that has many applications beyond the simple knapsack decision. In transportation programming, the items under consideration are the projects, the benefit of an item (the project score) is also called the profit (p_j), the weight (w_j) of the project is its cost, and the capacity (c) of the knapsack is the available budget. This article investigates the optimization of transportation programming, a variant of KP called MCKP. The KP problem varies when new constraints are added or existing constraints are changed. In addition to MCKP, common variants and extensions of KP include the multi-dimension knapsack problem and multiple knapsack problems (MKP).

MCKP is used when only one of a subset of items can be selected. In the knapsack scenario, there may be two tents among the items to be packed. Because the tents serve the same function, only one should be selected. In this article, the MCKP comes from multiple projects in the project list that serve similar functions and are considered alternatives to the same problem.

Traditionally, the transportation planning process resulted in project lists with the preferred alternative identified prior to the programming step (i.e. no projects were considered as competing alternatives). However, modern planning processes are more complicated and often multiple alternatives are being advanced to the programming step. This creates a scenario that calls for MCKP. Since the MCKP results in optimal funding solutions, the use of MCKP allows the most efficient alternative to be selected given the optimization function and constraints. This may differ from the alternative that might otherwise have been selected in the traditional planning process.

The selection of a particular alternative is usually based on project criteria. When different criteria are emphasized, different alternatives will be chosen. Sometimes it is not possible to select the appropriate project alternative prior to the programming step without "embedding" the importance of criteria into that selection. For example, a congested freeway corridor is to be upgraded. One alternative would be to add a general purpose lane in each direction and another alternative is to add a parallel high capacity transit system. Selecting the second alternative may favor criteria areas, such as air quality and alternative transportation modes, while the first alternative may have more significant impacts on travel time savings. In situations like this, it may be difficult to reach an agreement on the alternative to be selected and both projects must be advanced to the programming stage. Also, when the public is involved in the programming step they may feel that too many decisions have been made outside the process if only one alternative is available. It should be noted that due to increased costs to the agencies in determining the impacts of each projects, situations involving multiple alternatives should be kept as few as possible and should be reserved for only critical, high interest projects.

Of course, compared with traditional alternative selection method, this method has some issues. For example, project alternatives to be evaluated in transportation programming need more data and more detailed construction plans than they do in the earlier transportation planning steps and this will increase cost, require more personnel and time. For this reason, the programming method mentioned in this article to select project alternatives is not a replacement for the traditional planning method. This method is recommended to handle only select projects that have significant impacts on the public or are considered particularly controversial.

4. Process of solving MCKP

In this article, the branch and bound method is used to solve MCKP. Before the branch and bound method is discussed, the linear relaxation problem is introduced.

4.1. Linear relaxation of MCKP

Every KP or its variants have a corresponding linear relaxation problem. Linear relaxation means removing the integer requirement for the decision variable by "relaxing" the 0–1 integer constraint to 0-1 constraint. For example, the linear relaxation of MCKP, denoted as C(MCKP), has the constraint $x_{ij} \in [0, 1]$, $i = 1, 2, \ldots, m$, $j \in N_i$ (0–1 constraint). This means that in C(MCKP) the decision variable can be any real number between 0 and 1. Because of this relaxation, the result of the linear relaxation problem (i.e. the value of the objective function) is always greater than or equal to the result of the original problem (Kellerer et al., 2004). For this reason, linear relaxation KP is the upper bound of the corresponding KP. Solving linear relaxation problem is computationally easier and faster than solving the original KP problem so linear relaxation problem provides a suitable upper bound for situations where the upper bound will be calculated repeatedly.

"MCKP-Greedy" algorithm is used to solve C(MCKP) (Kellerer et al., 2004). There are at most two non-integer decision variables in the solution to C(MCKP). The first non-integer decision variable is called the splitting item. The splitting item will be used in the following explanation of the branch and bound method. The details about this method can be found in the Kellerer et al. (2004).

4.2. Solving the MCKP

The method used to solve the MCKP for this research was created by Dyer et al. (1984). It is a branch-bound method with a pre-processing step.

For the pre-processing step, all dominated and LP-dominated items (as defined below) are removed to reduce the complexity of the MCKP. Dominated items are defined in Definition 1, which states that if the cost of one project (w_{il}) is less than or equal to another project (w_{im}) while the benefit of the first project (p_{il}) is greater than that of the second project (p_{im}) than the second project would never be chosen because the first project would always be preferable because of this dominance. LP-dominated items, which are considered in the linear relaxation problem, are defined by Definition 2. LP-domination describes the relative change between the project benefit and cost of one project versus another. If a project can be funded that provides more benefit than another project with no sacrifice in the overall budget allocation than it should always be chosen over the other project.

Definition 1 (Dominated Items).
If two items l and m are in the same class, N_i, and satisfy the following equation:

$w_{il} \leq w_{im}$ and $p_{il} \geq p_{im}$

then the item m is dominated by item l.

Definition 2 (LP-Dominated Items).
If three items l, m, and n are in the same class, N_i, with $w_{il} < w_{im} < w_{in}$ and $p_{il} < p_{im} < p_{in}$ and satisfy the following equation:

$$\frac{p_{in} - p_{im}}{w_{in} - w_{im}} > \frac{p_{im} - p_{il}}{w_{im} - w_{il}}$$

then the item m is LP-dominated by items l and n.

Fig. 1 shows the dominated and LP-dominated items. Each point in this figure stands for an alternative of a project i. Point 5 is dominated by point 2 and point 3. Point 4 is LP-dominated by point 2 and point 3. The remaining points are undominated points, which comprises the upper left convex limit of project i (all the alternatives outside this convex limit will not be in the optimal solution).

The concept of dominance plays an important role in solving MCKP. Based on a proposition of Kellerer's book (Kellerer et al., 2004), no dominated or LP-dominated item can be in the optimal solution to a MCKP. In the optimal solution, the decision variables associated with these alternatives are equal to 0 (i.e. not funded). For this reason, if all the dominated and LP-dominated alternatives are removed, the complexity of solving MCKP will be reduced.

The second step after removing all dominated items is to solve the optimal problem using the branch and bound method. The branch and bound method divides a complex problem into many small but simple problems (branches) based on the decision variable while removing those branches that cannot have any impact on the final solution (bounding).

Using the branch and bound method, many calculations are required to solve a MCKP problem. The MCKP is NP hard and cannot be solved in polynomial time (Kellerer et al., 2004). Increases in the number of projects and the number of project alternatives will cause the solution run time to increase rapidly. Spreadsheet tools do not have enough power to solve the problem. However, there are two ways to implement the MCKP algorithm. One is to write solution codes in a programming language and the other is to use available commercial optimization software. Though the first method can give researchers more freedom to control the algorithm, the time and cost spent on developing the code is typically more than the cost of buying an existing commercial software package. In addition, the commercial software can use stricter bounds, which will speed up the branch and bound process. For these reasons, this article used LINGO, although other commercial solvers such as CPLEX and Xpress could have been utilized. When compared with other more advance solvers, LINGO is inexpensive and easy to use and provides enough functionality to solve the MCKP discussed in this article. There are some MCKP solvers available online but input requirements for their use are relatively high making them cumbersome to use. The details of how LINGO is used in building and solving MCKP are discussed in the following section.

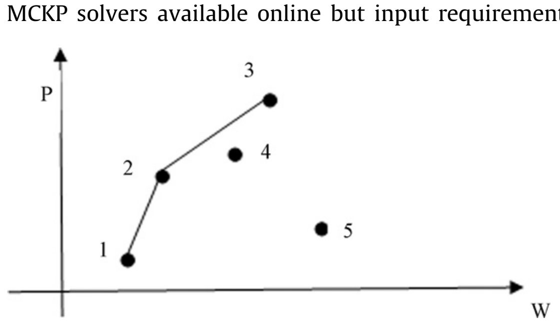

Fig. 1. Dominated and LP-dominated items.

5. An example of building and solving the MCKP

This section explains the process of building and solving the MCKP using a transportation programming example from the Puget Sound Region in the Seattle metropolitan area. This example uses a project list based on a virtual game designed by the *Seattle Times* newspaper called "U Build It". The projects on the "U Build It" list are projects that are regionally significant and included several groups of alternative projects.

5.1. Building MCKP

As mentioned previously, the first step in building the MCKP is determining the project list. The project list used here is a subset of a project list used in U Build It, a virtual transportation programming game created by the *Seattle Times*. Table 1 shows the projects used in this example. Five types of projects were analyzed: General Purpose Lane addition, High Occupancy Vehicle (HOV) Lane addition, New Interchange, Enhanced Transit, and Safety Improvement. The Alternative Group column in Table 1 indicates which alternative group each project belongs to. The Alternative column indicates which alternative each project is within the alternative group. Three alternative groups contain more than one alternative. For example, the Alaskan Way Viaduct (Group 4) has three alternatives. Based on the alternative constraint described previously, the transportation programming solution can incorporate at most one alternative from the Alaskan Way Viaduct option group in the final funding package.

The second step is to calculate the project impacts. Table 2 shows the five impact categories estimated in this example. The five impact areas used are travel time savings cost, operation cost savings, safety cost savings, increased travel options, and support economic growth. The first three criteria are quantitative estimates while the last two are qualitative estimates. The impact estimates shown in Table 2 for all the criteria were calculated using a software program for evaluating transportation projects called Multimodal Investment Choice Analysis (Young et al. 2002a; Young, Barnes, & Rutherford, 2002b, 2002c). The input data for the Multimodal Investment Choice Analysis program were compiled from planning documents and data files obtained from the transportation agencies proposing the projects and were part of a larger research effort into the transportation programming process. The planning year for the calculations and economic assumptions was 2004. While solution sensitivity and data validity are always an issue with any type of analysis, this is considered beyond the scope of this paper. Note that although five criteria are used in this example, the methodology supports more and/or different criteria.

As mentioned before, the project impacts have different units, so these impacts must be normalized. The Euclidean Normalization process described earlier was used to transform all the impacts shown in Table 2 to unitless values between 0 and 100. Table 3 shows the results of normalization. In this example, all impact values are in a scale where larger values represent more positive outcomes. If this were not the case for a particular criterion, the normalization process would have to account for this as well. Every cell of Table 3 is calculated by dividing the cell in Table 2 by the corresponding Euclidean Norm shown in the second row of Table 3 and then multiplying by 100.

After all the project impacts are normalized, the weighted project scores are calculated. Every project score shown in Table 3 was multiplied by the weight of the corresponding criteria (the first row of Table 4). Table 4 shows the weighted project scores.

Table 1
Project list.

Project ID	Project name	Project type	Alternative group	Alternative	Project cost (NPV)
1	I-5 HOV Lane	HOV Lane	1	1	$36,538,462
2	I-5 Industrial Busway	HOV Lane	2	1	$83,849,038
3	Bus Rapid Transit	HOV Lane	3	1	$4,615,385
4	Alaskan Way Viaduct—Alternative 1	GP Lane	4	1	$2,788,461,538
5	Alaskan Way Viaduct—Alternative 2	GP Lane	4	2	$2,692,307,692
6	Alaskan Way Viaduct—Alternative 3	GP Lane	4	3	$5,480,769,231
7	Highway 520—Alternative 1	GP Lane	5	1	$1,634,615,385
8	Highway 520—Alternative 2	GP Lane	5	2	$2,981,730,768
9	Highway 520—Alternative 3	GP Lane	5	3	$7,211,538,462
10	Highway 520—Alternative 4	GP Lane	5	4	$9,038,461,538
11	I-405—Alternative 1	GP Lane	6	1	$649,038,462
12	I-405—Alternative 2	GP Lane	6	2	$650,000,000
13	I-405—Alternative 3	GP Lane	6	3	$216,346,154
14	I-405—Alternative 4	GP Lane	6	4	$212,500,000
15	I-405—Alternative 5	GP Lane	6	5	$3,076,923,077
16	I-405—Alternative 6	GP Lane	6	6	$3,942,307,692
17	I-405—Alternative 7	GP Lane	6	7	$1,144,230,769
18	Highway-509	GP Lane	7	1	$675,721,154
19	Highway 167	GP Lane	8	1	$1,442,307,692
20	I-5/Highway 18/Highway 161 triangle	Interchange	9	1	$99,711,538
21	Highway 509/Highway 518	GP Lane	10	1	$4,807,692
22	I-90 HOV Lanes (Alternative A: add 2 lanes)	HOV Lane	11	1	$123,076,923
23	Highway 18	GP Lane	12	1	$138,461,538
24	Mercer Corridor	GP Lane	13	1	$86,538,462
25	Highway 169–(Jones Road to Highway 516)	GP Lane	14	1	$108,653,846
26	Highway 520/Highway 202 Interchange	GP Lane	15	1	$75,092,404
27	SR 522 (I-5 to SR 405: Multimodal Project)	Safety Improvement	16	1	$6,413,462
28	King Street Station	Transit	17	1	$147,932,692

The next step is to create a synthesized score for each project. In this example, OWA method was utilized. This step is optional and was utilized by this research effort because it allows adjustment to the score based on the user's confidence in the scores. This process is described only briefly in this paper since the focus of the paper is on the optimization process. OWA requires that the scores for a project (a row of Table 4) be in descending order according to the values of the scores. Table 5 shows the reordered results of Table 4, the second row of Table 5 shows the ordered weights used in the analysis. Multiplying the project score by its corresponding ordered weight produces the final score. For example, the final score of the first project is $(3.64*0.3) + (2.97*0.2) + (1.8*0.2) + (0*0.2) + (0*0.1) = 2.046$. If this optional step was not performed the weighted project scores in Table 4 would simply be summed for each row.

5.2. Solving MCKP

To solve the MCKP project, the project list (Table 1) is needed as well as the alternative groups (Table 1). The capacity of the knapsack, the total available funding for transportation programming in this

Table 2
Estimates of project impacts.

Project ID	Travel time cost savings	Operation cost savings	Safety cost savings	Increased travel options	Support economic growth
1	$278,321	$772,035	$26,033,086	100	50
2	$53,207,603	$15,088,591	$3,232,278	100	100
3	$668,963	$934,013	$4,537,357	100	100
4	$61,612,888	$227,197,525	$19,709,507	50	100
5	$32,749,981	$120,765,553	$32,302,617	100	50
6	$63,073,566	$232,583,774	$33,787,726	0	0
7	$140,978,391	$520,079,606	$27,316,601	50	100
8	$77,558,728	$286,119,828	$57,755,100	50	50
9	$147,826,151	$545,341,495	$46,828,459	50	100
10	$231,003,936	$851,976,308	$27,316,601	50	100
11	$216,391,180	$637,908,410	$57,465,925	50	100
12	$23,759,477	$486,763,773	$11,920,079	50	100
13	$3,269,610	$75,503,184	$6,164,255	50	100
14	$2,043,531	$47,190,060	$13,921,930	50	100
15	$2,089,795	$48,258,418	$44,329,425	50	100
16	$85,250,180	$1,968,632,282	$80,127,703	50	100
17	$672,085,647	$1,981,268,771	$57,465,925	50	100
18	$73,050,319	$256,342,399	$9,808,399	100	100
19	$777,758,377	$2,871,024,840	$132,916,273	50	50
20	$8,795,920	$32,473,124	$4,399,189	100	100
21	$2,313,683	$8,536,491	$5,327,091	0	100
22	$115,422,702	$313,020,782	$33,458,024	50	100
23	$178,837,122	$660,846,615	$16,114,865	100	100
24	$32,466,648	$92,092,895	$19,334,051	100	100
25	$291,114,581	$872,818,684	$25,365,438	100	100
26	$23,537,992	$86,796,347	$8,593,870	50	100
27	$0	$0	$40,657,537	0	0
28	$0	$20,241,624	$359,558,166	50	50

Table 3
Results of normalizing project impacts.

Project ID	Travel time savings cost	Operation cost savings	Safety savings cost	Increase travel option	Support economic growth
Euclidean norm	1166682437	4424223876	420575003.6	360.5551	471.6990566
1	0	0	6	28	11
2	5	0	1	28	21
3	0	0	1	28	21
4	5	5	5	14	21
5	3	3	8	28	11
6	5	5	8	0	0
7	12	12	6	14	21
8	7	6	14	14	11
9	13	12	11	14	21
10	20	19	6	14	21
11	19	14	14	14	21
12	2	11	3	14	21
13	0	2	1	14	21
14	0	1	3	14	21
15	0	1	11	14	21
16	7	44	19	14	21
17	58	45	14	14	21
18	6	6	2	28	21
19	67	65	32	14	11
20	1	1	1	28	21
21	0	0	1	0	21
22	10	7	8	14	21
23	15	15	4	28	21
24	3	2	5	28	21
25	25	20	6	28	21
26	2	2	2	14	21
27	0	0	10	0	0
28	0	0	85	14	11

case, is another important building block of MCKP. In order to illustrate the effects of different funding levels on the optimal solutions, several budgets ($15, $10 and $5 billion) are analyzed in this example. The project scores (Table 5) and the project costs (Table 1) are also needed.

The LINGO optimization software, developed by LINDO Systems, was used to solve this MCKP. Like other optimization software, LINGO has its own modeling language. LINGO can understand the model and solve the problem when the MCKP and the relevant information are written in its language. The LINGO code used for this research has been included at the end of this article. Table 6 shows the results from LINGO for different the three different funding levels. A project is funded only when the attribute in fund column for the project is 1. For example, I-5 HOV Lane (project 1) is supported in the solution under $15 billion funding, while the first alternative of I-405 (project 11) is not funded.

Table 4
Weighted project score.

Project ID	Travel time savings	Operation cost savings	Safety savings	Increase travel option	Support economic growth
Weights	0.19	0.11	0.3	0.13	0.27
1	0	0	1.8	3.64	2.97
2	0.95	0	0.3	3.64	5.67
3	0	0	0.3	3.64	5.67
4	0.95	0.55	1.5	1.82	5.67
5	0.57	0.33	2.4	3.64	2.97
6	0.95	0.55	2.4	0	0
7	2.28	1.32	1.8	1.82	5.67
8	1.33	0.66	4.2	1.82	2.97
9	2.47	1.32	3.3	1.82	5.67
10	3.8	2.09	1.8	1.82	5.67
11	3.61	1.54	4.2	1.82	5.67
12	0.38	1.21	0.9	1.82	5.67
13	0	0.22	0.3	1.82	5.67
14	0	0.11	0.9	1.82	5.67
15	0	0.11	3.3	1.82	5.67
16	1.33	4.84	5.7	1.82	5.67
17	11.02	4.95	4.2	1.82	5.67
18	1.14	0.66	0.6	3.64	5.67
19	12.73	7.15	9.6	1.82	2.97
20	0.19	0.11	0.3	3.64	5.67
21	0	0	0.3	0	5.67
22	1.9	0.77	2.4	1.82	5.67
23	2.85	1.65	1.2	3.64	5.67
24	0.57	0.22	1.5	3.64	5.67
25	4.75	2.2	1.8	3.64	5.67
26	0.38	0.22	0.6	1.82	5.67
27	0	0	3	0	0
28	0	0	25.5	1.82	2.97

Table 6 shows that different funding levels produce different solutions and one or two projects play key roles in the solutions. For these project lists, those projects with more than one alternative play important roles. The difference between solutions typically affects these projects. For example, the difference between first solution ($15 billion) and the second solution ($10 billion) is that the first solution chooses the third alternative of "Highway 520" project, while the second solution chooses the first alternative of this project. The third alternative (project #9) is more than $5 billion more than the first alternative (project #7) but offers significantly more safety benefits and slightly more travel time benefits. The third solution ($5 billion) does not fund any of the four alternatives to the Highway 520 project. This is characteristic of the MCKP. For the classic KP, the difference between solutions is usually represented by the change of the number of projects in the solutions. This difference between MCKP solutions is usually represented by changes in the selected project alternatives. For example, though there is a $5 billion difference on the total investment, the number of projects in the first solution and the second solution is same with both solutions having 17 projects.

One issue for MCKP solutions is the "unused investment", which is the difference between the budget level and the total investment. This problem is particularly evident in our example because only large scale, regional projects are considered for funding. For example, the first solution has $0.8 billion unused investments. In reality, an agency would be considering a full array of projects, both small and large, so the unused budget would be minimal.

Table 5
Final project scores produced using OWA.

Project ID	1st	2nd	3rd	4th	5th	Final Scores
Weights	0.3	0.2	0.2	0.2	0.1	
1	3.64	2.97	1.80	0	0	2.046
2	5.67	3.64	0.95	0.30	0	2.679
3	5.67	3.64	0.30	0	0	2.489
4	5.67	1.82	1.50	0.95	0.55	2.61
5	3.64	2.97	2.40	0.57	0.33	2.313
6	2.40	0.95	0.55	0	0	1.02
7	5.67	2.28	1.82	1.80	1.32	3.013
8	4.20	2.97	1.82	1.33	0.66	2.55
9	5.67	3.30	2.47	1.82	1.32	3.351
10	5.67	3.80	2.09	1.82	1.80	3.423
11	5.67	4.20	3.61	1.82	1.54	3.781
12	5.67	1.82	1.21	0.90	0.38	2.525
13	5.67	1.82	0.30	0.22	0	2.169
14	5.67	1.82	0.90	0.11	0	2.267
15	5.67	3.30	1.82	0.11	0	2.747
16	5.70	5.67	4.84	1.82	1.33	4.309
17	11.02	5.67	4.95	4.20	1.82	6.452
18	5.67	3.64	1.14	0.66	0.60	2.849
19	12.73	9.60	7.15	2.97	1.82	7.945
20	5.67	3.64	0.30	0.19	0.11	2.538
21	5.67	0.30	0	0	0	1.761
22	5.67	2.40	1.90	1.82	0.77	32
23	5.67	3.64	2.85	1.65	1.20	3.449
24	5.67	3.64	1.50	0.57	0.22	2.865
25	5.67	4.75	3.64	2.20	1.80	3.999
26	5.67	1.82	0.60	0.38	0.22	2.283
27	3	0	0	0	0	0.9
28	25.50	2.97	1.82	0	0	8.608

LINGO produced these solutions in only three seconds. Different cuts options (i.e. different total investment levels) from $10 billion to $5 billion have been tried on this MCKP, but the results showed that cuts option had no effect on the solution performance. The possible reason is that the solution to the linear relaxation of this MCKP is an optimal solution, so no further steps are needed. The run time for MCKPs with other data sets were also tested, in some cases the number of decision variable was 50 (the limit of the integer variable for an academic LINGO license) and LINGO again produced a solution in three seconds. This run time is very important in participatory environments because the public is usually very sensitive to how much time it takes to produce a solution. Shorter run time means a smaller chance that participants will stop participating. Because the parameters of MCKP, such as the weights for criteria, are easily changed, MCKP must be solved repeatedly. The time spent on such repetitive steps is a key factor in total run time, so a short run time for each iteration is essential.

What the above example illustrates is how the transportation agency can use MCKP to compare how various projects perform under different programming assumptions. For example, which project alternative, if any, is funded under various budget levels. Also, the selection of alternatives is highly dependent on the weighting of criteria. The MCKP solution can provide agencies with insight on how different projects or project alternatives compare for funding as the criteria weighting is changed.

6. Summary

This article presents the steps to building the optimization problem for transportation programming, discusses the method and mathematical processes used to solve the MCKP, and demonstrates the application of the method and processes using an example. The optimization problem built using this six-step process has been constructed to meet the needs of a transportation programming situation.

The branch and bound method and LINGO software can provide a timely solution to the MCKP, which is the key to its usefulness,

Table 6
Solution to MCKP.

Project ID	Project name	Alternative group	Project list (1 = Funded, 0 = Not funded)		
			$15 Billion	$10 Billion	$5 Billion
1	I-5 HOV Lane	1	1	1	1
2	I-5 Industrial Busway	2	1	1	1
3	Bus Rapid Transit	3	1	1	1
4	Alaskan Way Viaduct—Alternative 1	4	1	1	0
5	Alaskan Way Viaduct—Alternative 2	4	0	0	0
6	Alaskan Way Viaduct—Alternative 3	4	0	0	0
7	Highway 520—Alternative 1	5	0	1	0
8	Highway 520—Alternative 2	5	0	0	0
9	Highway 520—Alternative 3	5	1	0	0
10	Highway 520—Alternative 4	5	0	0	0
11	I-405—Alternative 1	6	0	0	0
12	I-405—Alternative 2	6	0	0	0
13	I-405—Alternative 3	6	0	0	0
14	I-405—Alternative 4	6	0	0	0
15	I-405—Alternative 5	6	0	0	0
16	I-405—Alternative 6	6	0	0	0
17	I-405—Alternative 7	6	1	1	1
18	Highway-509	7	1	1	1
19	Highway 167	8	1	1	1
20	I-5/Highway 18/Highway 161 triangle	9	1	1	1
21	Highway 509/Highway 518	10	1	1	1
22	I-90 HOV Lanes (Alternative A: add 2 lanes)	11	1	1	1
23	Highway 18	12	1	1	1
24	Mercer Corridor	13	1	1	1
25	Highway 169–(Jones Road to Highway 516)	14	1	1	1
26	Highway 520/Highway 202 Interchange	15	1	1	1
27	SR 522 (I-5 to SR 405: Multimodal Project)	16	1	1	1
28	King Street Station	17	1	1	1
Total investment (in billion dollars)			14.2	8.6	4.2
Total weighted scores			59.83	59.49	53.87

particularly if various funding scenarios are to be explored by the agency. LINGO's interface and database support, which spreadsheet tools do not provide, can effectively produce the solution to MCKP in an online environment.

The process of building and solving the MCKP can be complex for people without backgrounds in the decision science fields, but a custom interface can be easily developed to reduce the complexity and maintain transparency in the decision process. Users would follow the guidance provided by the system's interface to build the MCKP model. Above all, the purpose of all the techniques covered in this article is to help users participate in transportation programming process by reducing the complexity of task and maintaining their interest in the process.

The introduction of the MCKP over the traditional KP problem is also vital to the programming process. In traditional planning, the selection of alternatives for a particular project was done earlier in the process and only one "best" alternative was advanced to the transportation programming step. It is now understood that the selection of this alternative has preferences about the purpose of the transportation system "embedded" within it. For key regional projects, it is now necessary to advance multiple projects to the transportation programming step to allow the selection of criteria and their associated weights to find the "best" alternative through solving the MCKP problem.

While the article uses transportation programming as a case study, MCKP can be useful in other fields where a similar decision among a subset of the alternatives is required.

LINGO Code

```
! input the number of group;
DATA:
NO_GROUP = 17;
ENDDATA

!setup GROUPS set;
SETS:
GROUPS /1..NO_GROUP/;
ENDSETS

!input the number of projects;
DATA:
NO_PRJ =28;
ENDDATA

!set up PROJECT set and specify its attributes
SETS:
PROJECTS/1..NO_PRJ/ :GROUP,INCLUDE,PROFIT,COST;
ENDSETS

DATA:
!read data from an Excel file;
GROUP COST PROFIT = @OLE ('H:\document\projectdata.xls','Group','Cost','Profit');

!export results to an Excel file
@OLE ('H:\document\projectdata.xls', 'Include')= INCLUDE;
CAPACITY =15000000000;
ENDDATA

!maximize the objective function;
MAX=@SUM ( PROJECTS: PROFIT* INCLUDE);

!investment constraint;
@SUM ( PROJECTS: COST* INCLUDE) <= CAPACITY;

!AlternativeConstraint;
@FOR (GROUPS(I):
 @SUM (PROJECTS(J)| GROUP(J) #EQ# I:INCLUDE(J)) <= 1);

!0-1 integer constraint;
@FOR (PROJECTS: @BIN ( INCLUDE));
```

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