Table 2. First 37-term orthonormal Zernike circle polynomials under the Noll indices [25, 36].

j	n	m	$Z_j(\rho, \theta)$	$Z_j(x, y)$	Aberration
1	0	0	1	1	Piston
2	1	1	$2\rho\cos\theta$	$2x$	x-tilt
3		1	$2\rho\sin\theta$	$2y$	y-tilt
4	2	0	$\sqrt{3}(2\rho^2-1)$	$\sqrt{3}[2(x^2+y^2)-1]$	Defocus
5		2	$\sqrt{6}\rho^2\sin 2\theta$	$2\sqrt{6}xy$	45° Primary astigmatism
6		2	$\sqrt{6}\rho^2\cos 2\theta$	$\sqrt{6}(x^2-y^2)$	0° Primary astigmatism
7	3	1	$\sqrt{8}(3\rho^3-2\rho)\sin\theta$	$\sqrt{8}y[3(x^2+y^2)-2]$	Primary y-coma
8		1	$\sqrt{8}(3\rho^3-2\rho)\cos\theta$	$\sqrt{8}x[3(x^2+y^2)-2]$	Primary x-coma
9		3	$\sqrt{8}\rho^3\sin 3\theta$	$\sqrt{8}y(3x^2-y^2)$	
10		3	$\sqrt{8}\rho^3\cos 3\theta$	$\sqrt{8}x(x^2-3y^2)$	
11	4	0	$\sqrt{5}(6\rho^4-6\rho^2+1)$	$\sqrt{5}[6(x^2+y^2)^2-6(x^2+y^2)+1]$	Primary spherical aberration
12		2	$\sqrt{10}(4\rho^4-3\rho^2)\cos 2\theta$	$\sqrt{10}(x^2-y^2)[4(x^2+y^2)-3]$	0° Secondary astigmatism
13		2	$\sqrt{10}(4\rho^4-3\rho^2)\sin 2\theta$	$2\sqrt{10}xy[4(x^2+y^2)-3]$	45° Secondary astigmatism
14		4	$\sqrt{10}\rho^4\cos 4\theta$	$\sqrt{10}[(x^2+y^2)^2-8x^2y^2]$	
15		4	$\sqrt{10}\rho^4\sin 4\theta$	$4\sqrt{10}xy(x^2-y^2)$	
16	5	1	$\sqrt{12}(10\rho^5-12\rho^3+3\rho)\cos\theta$	$\sqrt{12}x[10(x^2+y^2)^2-12(x^2+y^2)+3]$	Secondary x-coma
17		1	$\sqrt{12}(10\rho^5-12\rho^3+3\rho)\sin\theta$	$\sqrt{12}y[10(x^2+y^2)^2-12(x^2+y^2)+3]$	Secondary y-coma
18		3	$\sqrt{12}(5\rho^5-4\rho^3)\cos 3\theta$	$\sqrt{12}x(x^2-3y^2)[5(x^2+y^2)-4]$	
19		3	$\sqrt{12}(5\rho^5-4\rho^3)\sin 3\theta$	$\sqrt{12}y(3x^2-y^2)[5(x^2+y^2)-4]$	
20		5	$\sqrt{12}\rho^5\cos 5\theta$	$\sqrt{12}x[16x^4-20x^2(x^2+y^2)+5(x^2+y^2)^2]$	
21		5	$\sqrt{12}\rho^5\sin 5\theta$	$\sqrt{12}y[16y^4-20y^2(x^2+y^2)+5(x^2+y^2)^2]$	
22	6	0	$\sqrt{7}(20\rho^6-30\rho^4+12\rho^2-1)$	$\sqrt{7}[20(x^2+y^2)^3-30(x^2+y^2)^2+12(x^2+y^2)-1]$	Secondary spherical
23		2	$\sqrt{14}(15\rho^6-20\rho^4+6\rho^2)\sin 2\theta$	$2\sqrt{14}xy[15(x^2+y^2)^2-20(x^2+y^2)+6]$	45° Tertiary astigmatism
24		2	$\sqrt{14}(15\rho^6-20\rho^4+6\rho^2)\cos 2\theta$	$\sqrt{14}(x^2-y^2)[15(x^2+y^2)^2-20(x^2+y^2)+6]$	0° Tertiary astigmatism
25		4	$\sqrt{14}(6\rho^6-5\rho^4)\sin 4\theta$	$4\sqrt{14}xy(x^2-y^2)[6(x^2+y^2)-5]$	
26		4	$\sqrt{14}(6\rho^6-5\rho^4)\cos 4\theta$	$\sqrt{14}[(x^2+y^2)^2-8x^2y^2][6(x^2+y^2)-5]$	
27		6	$\sqrt{14}\rho^6\sin 6\theta$	$\sqrt{14}xy[32x^4-32x^2(x^2+y^2)+6(x^2+y^2)^2]$	
28		6	$\sqrt{14}\rho^6\cos 6\theta$	$\sqrt{14}[32x^6-48x^4(x^2+y^2)+18x^2(x^2+y^2)^2-(x^2+y^2)^3]$	
29	7	1	$4(35\rho^7-60\rho^5+30\rho^3-4\rho)\sin\theta$	$4y[35(x^2+y^2)^3-60(x^2+y^2)^2+30(x^2+y^2)-4]$	Tertiary y-coma
30		1	$4(35\rho^7-60\rho^5+30\rho^3-4\rho)\cos\theta$	$4x[35(x^2+y^2)^3-60(x^2+y^2)^2+30(x^2+y^2)-4]$	Tertiary x-coma
31		3	$4(21\rho^7-30\rho^5+10\rho^3)\sin 3\theta$	$4y(3x^2-y^2)[21(x^2+y^2)^2-30(x^2+y^2)+10]$	
32		3	$4(21\rho^7-30\rho^5+10\rho^3)\cos 3\theta$	$4x(x^2-3y^2)[21(x^2+y^2)^2-30(x^2+y^2)+10]$	
33		5	$4(7\rho^7-6\rho^5)\sin 5\theta$	$4[4x^2y(x^2-y^2)+y(x^2+y^2)^2-8x^2y^3]\times[7(x^2+y^2)-6]$	
34		5	$4(7\rho^7-6\rho^5)\cos 5\theta$	$4[x(x^2+y^2)^2-8x^3y^2-4xy^2(x^2-y^2)]\times[7(x^2+y^2)-6]$	
35		7	$4\rho^7\sin 7\theta$	$8x^2y[3(x^2+y^2)^2-16x^2y^2]+4y(x^2-y^2)[(x^2+y^2)^2-16x^2y^2]$	
36		7	$4\rho^7\cos 7\theta$	$4x(x^2-y^2)[(x^2+y^2)^2-16x^2y^2]-8xy^2[3(x^2+y^2)^2-16x^2y^2]$	
37	8	0	$3(70\rho^8-140\rho^6+90\rho^4-20\rho^2+1)$	$3[70(x^2+y^2)^4-140(x^2+y^2)^3+90(x^2+y^2)^2-20(x^2+y^2)+1]$	Tertiary spherical