Unit 3: Polynomial and Rational Functions

Polynomial Division

There are two different ways to divide polynomials

Long Division ↓ uses terms

Synthetic Division ↓ uses coefficients

however even though it is fast, it has limitations. It can only divide linear factors that have a leading coefficient of 1 effectively

polynomials can be written in the form...

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

or

$$\text{dividend} = (\text{divisor})(\text{quotient}) + \text{remainder}$$

Remainder Theorem

when a polynomial is divided by $x-a$, the remainder will be $f(a)$

It is more useful to use over division when you only require the remainder

Dividing Tips

When a polynomial is given, the factor is also given in the question or from the calculator

Graphing Rational Functions

A rational function could have 3 important qualities

$$r(x) = \frac{p(x)}{q(x)}$$

Zero ↓ $p(x) = 0$ and $q(x) \neq 0$

hole ↓ $q(x) = 0$ but not in simplified function

vertical asymptote ↓ $q(x) = 0$ and also in simplified function

The y-coordinate of holes can be found by subbing into the simplified function

End Behavior can be seen by looking at the degrees of the numerator and denominator

If degree numerator > degree denominator $\lim_{x\to\infty} f(x) = \infty$

If degree numerator < degree denominator $\lim_{x\to\infty} f(x) = 0$

If degree numerator = degree denominator $\lim_{x\to\infty} f(x) = \dfrac{\text{leading coefficient numerator}}{\text{leading coefficient denominator}}$

A function CAN cross its horizontal & slant asymptote but CANNOT cross its vertical asymptote

This is because end behavior does not tell what happens at the middle of the graph while at the vertical asymptote values are undefined

Evaluating Limits

Limits can tell about a graph's behavior (a & b are constants)

$\lim_{x\to\infty} f(x) = a$ horizontal asymptote at $y=a$

opposite behavior

$\lim_{x\to a^-} f(x) = \infty$ $\lim_{x\to a^+} f(x) = -\infty$ VA at $x=a$

(- and + indicate approaching from left and right respectively)

$\lim_{x\to a} f(x) = b$ but $f(a)$ is undefined

there is a hole at (a, b)

Rational Inequalities

When solving Rational Inequalities it is the same process as solving Quadratic or Absolute Value Inequalities

Make it an equation and include test points

However, this time the end points will include solutions along with undefined values of the function in the original function

Transformations

$\frac{1}{f(x)}$ transformation is taking the reciprocal of the y-values

There are some important things to note...

- zeros of $f(x)$ become vertical asymptotes
- y-values of 1 remain the same
- |y-values| less than 1 become bigger
- |y-values| greater than 1 become smaller
- undefined values are still undefined