

Unit 1: Functions and Transformations

Important Vocabulary

Function - A relation such that each input has one output

Relation - A set of ordered pairs (relationship between x and y)

One-to-one - Each output has exactly one input

Transformations - Changes to a function such as translations, reflections, stretches/compressions

Continuous - No breaks in a graph (covered more in depth in calculus)

Zeros/ x -intercepts - x -values when $y=0$ (when it hits the x -axis)

End behavior - what y approaches as x approaches ∞ or $-\infty$

Domain - all x -values valid in a function

Range - all y -values valid in a function

Asymptote - a line that a graph gets arbitrarily close to (vertical, horizontal, slant)

Basic Parent Functions

quadratic ($f(x) = x^2$)

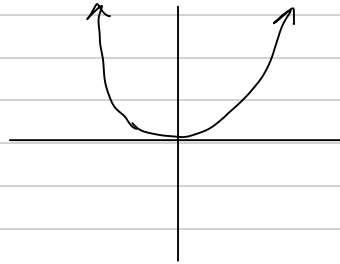
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

asymptotes: none

continuous / non-continuous

even / odd / neither



cubic ($f(x) = x^3$)

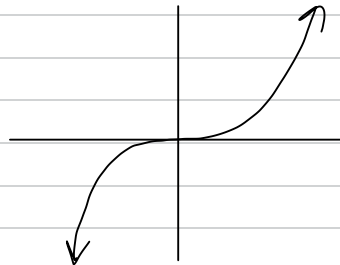
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

asymptotes: none

continuous / non-continuous

even / odd / neither



absolute value ($f(x) = |x|$)

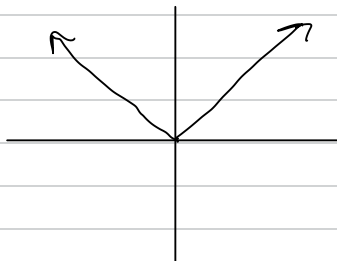
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

asymptotes: none

continuous / non-continuous

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radical ($f(x) = \sqrt{x}$)

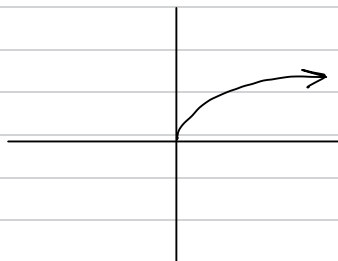
Domain: $[0, \infty)$

Range: $[0, \infty)$

asymptotes: none

continuous / non-continuous

even / odd / neither



exponential ($f(x) = a^x$)

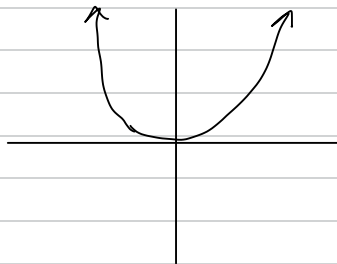
Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

asymptotes: $y = 0$

continuous / non-continuous

even / odd / neither



logarithmic ($f(x) = x^2$)

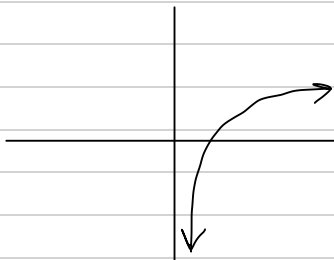
Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

asymptotes: $x = 0$

continuous / non-continuous

even / odd / neither



trig ($f(x) = \sin x$ or $\cos x$)

Domain: $(-\infty, \infty)$

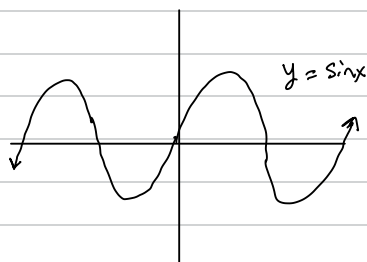
Range: $[-1, 1]$

asymptotes: none

Continuous / non-continuous

even / odd / neither for $\sin x$

even / odd / neither for $\cos x$



reciprocal ($f(x) = \frac{1}{x}$)

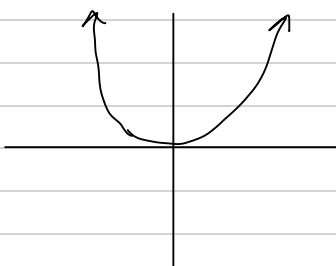
Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

asymptotes: $x=0$, $y=0$

Continuous / non-continuous

even / odd / neither



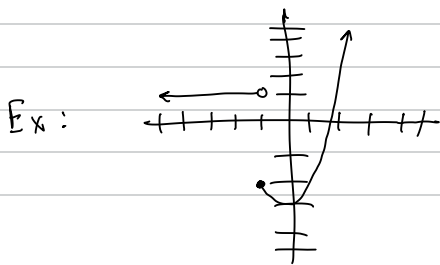
Piecewise Functions

Piecewise Functions are functions that contain multiple other functions for specific domains

Ex: $|x|$ can be written as a piecewise function

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Some piecewise functions are non continuous, filled in circles indicate the x value is included, hollow circles indicate the x -value is not included



$$f(x) = \begin{cases} 1, & x \in (-\infty, -1) \\ x^2 - 3, & x \in [-1, \infty) \end{cases}$$

Inverse functions

Inverse functions are obtained by switching the input and output

Ex: $f(x) = 2x + 3$

$$x = 2y + 3$$

$$x - 3 = 2y$$

$$y = \frac{x-3}{2}$$

$$\rightarrow f^{-1}(x) = \frac{x-3}{2}$$

the domain and range are switched for inverse functions

Ex: $f(x)$

Domain: $[2, 4]$

Range: $(-\infty, \infty)$

$f^{-1}(x)$

Domain: $(-\infty, \infty)$

Range: $[2, 4]$

Composite functions

Composite functions use another function as the input of the function

Ex: $f(x) = 2x$ $h(x) = x^3 + 3$

$$f(h(x)) = 2(x^3 + 3) \quad f(f(x)) = 2(2x)$$

function transformations come in the form

$$a \cdot f(b(x+c)) + d$$

Changing the letter will

a - stretches/compresses vertically by a

b - stretches/compresses horizontally by

c - translates $f(x)$ horizontally by $-c$

d - translates $f(x)$ vertically by $+d$

to think about why ... think about how changing each constant is affecting the input or the output of the function