Unit 5: Sequences, Series, and Methods of Proof

Arithmetic vs Geometric Series

Arithmetic Series have a common difference

The nth term can be expressed by

$$a_n = a_1 + (n-1)d$$

The sum of the series can be expressed by

$$S_n = \frac{n}{2}(a_1 + a_n) \quad \text{or} \quad S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

Geometric Series have a common ratio

The nth term can be expressed by

$$a_n = a_1 r^{n-1}$$

The sum of the series can be expressed by

$$S_n = \frac{a_1(r^n - 1)}{r-1} = \frac{a_1(1-r^n)}{1-r}$$

Infinite Geometric Series

Infinite Geometric Series only converge when $-1 < r < 1$ otherwise it diverges

When the series converges its sum can be expressed as

$$S_\infty = \frac{a_1}{1-r}$$

this is because as as n approaches infinity, r^n will become smaller and smaller until it becomes virtually zero as a result, the numerator will be expressed as $a_1(1-0)$

Tips for Series Questions

Most IB questions involve a system of equations in order to find an unknown such as the common difference or common ratio

Most questions can be done with an understanding of the equations and how to manipulate them properly

Proof by Induction

A proof by induction has three parts

i) Base Case

this is used to prove that the proposition is true for the first instance

typically if $n \in \mathbb{Z}$, $n = 0$ and

$n \in \mathbb{Z}^+$, $n = 1$

ii) Induction Hypothesis

this is used as an assumption that the LHS and RHS holds true for any instance $n = k$

iii) Prove Proposition for $n = k + 1$

Once the proposition is true for $n = k+1$, the original statement will be proven true

This step often uses the assumption from the induction hypothesis

Common uses for a proof by induction are proving that a series converges to a sum or divisibility proofs

Proof by Contradiction

This proof does not need to be understood in as much detail

Assume that statement is false then show how it leads to a contradiction