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1. Linear Programming Model

Parameters

- Let $t = 1, ..., T$ denote the time horizon for the vaccination program for the first dose. The second dose is therefore administrated over $t = 21, ..., T'(= T + 25)$.
- Let s_t^u be the supply schedule of the vaccine, and S_t^u denote the cumulative supply of the vaccine up till time t.
- Let s_t^l be the expired amount vaccine at time t, and S_t^l denote the cumulative expired vaccine up till time t.
- Let c_t denote the capacity of the vaccination program up till time t.
- Let d_t denote the number of people in the population who want to be vaccinated at time t. In the LP model, we assume d_t is deterministic and known.
- Let R be the number of days of the second-dose reservation required.

Decision variables:

- x_t denotes the first dose booking limit at time t.
- y_t denotes the second dose booking limit at time t.
- Let $z_{t,t'}$ denotes the number of patients with first dose at t and second dose T t'.
- v_t denotes the buffer for accounting expired vaccines.

Objective: Design the booking limits constrained by supply and capacity to minimize the total number of backlogs. Let $Z(\boldsymbol{x})$ denote the number of backlogs.

$$
\begin{aligned}
\min_{x,y,z,v} \quad & [Z(\boldsymbol{x})] & & \\
\text{s.t.} \quad & z_{t,t'} = 0, & & \forall t' \neq t+4. \\
& \sum_{t'} z_{t,t'} = x_t, & & \forall t = 1,...,T. \\
& \sum_{t} z_{t,t'} = y_{t'}, & & \forall t' = 1,...,T'. \\
& \sum_{i:i \leq t} (x_i + y_i + v_i) + \sum_{j=t+1}^{t+R} y_j \leq S_t^u, & & \forall t = 1,...,T'. \\
& \sum_{i:i \leq t} (x_i + y_i + v_i) + \sum_{j=t+1}^{t+R} y_j \geq S_t^l, & & \forall t = 1,...,T'. \\
& x_t + y_t \leq c_t, & & \forall t = 1,...,T'. \\
& x_t, y_t, z_t, v_t \geq 0, & & \forall t = 1,...,T'
\end{aligned}
\tag{1}
$$

It is shown that $Z(\boldsymbol{x})$ can be obtained by solving the following network flow problem.

$$
\begin{aligned}
Z(\boldsymbol{x}) = \max_{\alpha_t, \beta_t} \quad & \sum_{t=1}^{T} (d_t - x_t)\alpha_t & & \\
\text{s.t.} \quad & \alpha_t - \alpha_{t-1} - \beta_t = -1, & & \forall t = 2,..,T, \\
& -\alpha_T - \beta_{T+1} = -1, & & \\
& \alpha_t, \beta_t \geq 0, & & \forall t = 1,...,T.
\end{aligned}
\tag{2}
$$

which can be written in a generic form

$$
\begin{aligned}
Z(\boldsymbol{x}) = \max_{\alpha_t, \beta_t} \quad & (\boldsymbol{d} - \boldsymbol{x})' \boldsymbol{\alpha} & & \\
\text{s.t.} \quad & \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} = \boldsymbol{b} & & \\
& \alpha_t, \beta_t \geq 0, & & \forall t = 1,...,T.
\end{aligned}
\tag{3}
$$

The dual problem is therefore,

$$
\begin{aligned}
Z(\boldsymbol{x}) = \min_{\phi_t} \quad & \boldsymbol{d}' \boldsymbol{\phi} \\
\text{s.t.} \quad & A' \boldsymbol{\phi} = \boldsymbol{d} - \boldsymbol{x} \\
& B' \boldsymbol{\phi} = 0
\end{aligned}
\tag{4}
$$

Therefore, the overall backlog minimization problem is

$$
\begin{aligned}
\min_{x,y,z,v\phi} \quad & \boldsymbol{d}' \boldsymbol{\phi} & & \\
\text{s.t.} \quad & z_{t,t'} = 0, & & \forall t' \neq t+4. \\
& \sum_{t'} z_{t,t'} = x_t, & & \forall t = 1,...,T. \\
& \sum_{t} z_{t,t'} = y_{t'}, & & \forall t' = 1,...,T'. \\
& \sum_{i:i \leq t} (x_i + y_i + v_i) + \sum_{j=t+1}^{t+R} y_j \leq S_t^u, & & \forall t = 1,...,T'. \\
& \sum_{i:i \leq t} (x_i + y_i + v_i) + \sum_{j=t+1}^{t+R} y_j \geq S_t^l, & & \forall t = 1,...,T'. \\
& x_t + y_t \leq c_t, & & \forall t = 1,...,T'. \\
& A' \boldsymbol{\phi} = \boldsymbol{d} - \boldsymbol{x} & & \\
& B' \boldsymbol{\phi} = 0 & & \\
& x_t, y_t, z_t, v_t \geq 0, & & \forall t = 1,...,T'
\end{aligned}
\tag{5}
$$