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1. Linear Programming Model

Parameters

- Let t = 1, ..., T denote the time horizon for the vaccination program for the first dose. The second dose is therefore administrated over t = 21, ..., T' (= T + 25).
- Let s_t^u be the supply schedule of the vaccine, and S_t^u denote the cumulative supply of the vaccine up till time t.
- Let s_t^l be the expired amount vaccine at time t, and S_t^l denote the cumulative expired vaccine up till time t.
 - Let c_t denote the capacity of the vaccination program up till time t.
- Let d_t denote the number of people in the population who want to be vaccinated at time t. In the LP model, we assume d_t is deterministic and known.
 - Let R be the number of days of the second-dose reservation required.

Decision variables:

- x_t denotes the first dose booking limit at time t.
- y_t denotes the second dose booking limit at time t.
- Let $z_{t,t'}$ denotes the number of patients with first dose at t and second dose T t'.
- v_t denotes the buffer for accounting expired vaccines.

Objective: Design the booking limits constrained by supply and capacity to minimize the total number of backlogs. Let Z(x) denote the number of backlogs.

$$\min_{\substack{x,y,z,v \\ \text{s.t.}}} [Z(x)] \\
\text{s.t.} \quad z_{t,t'} = 0, \qquad \forall t' \neq t + 4. \\
\sum_{\substack{t' \\ t}} z_{t,t'} = x_t, \qquad \forall t = 1, ..., T. \\
\sum_{\substack{t' \\ t}} z_{t,t'} = y_{t'}, \qquad \forall t' = 1, ..., T'. \\
\sum_{\substack{t:i \leq t}} (x_i + y_i + v_i) + \sum_{\substack{j=t+1 \\ t+R}}^{t+R} y_j \leq S_t^u, \forall t = 1, ..., T'. \\
\sum_{\substack{i:i \leq t \\ t+1}} (x_i + y_i + v_i) + \sum_{\substack{j=t+1 \\ t+R}}^{t+R} y_j \geq S_t^l, \forall t = 1, ..., T'. \\
x_t + y_t \leq c_t, \qquad \forall t = 1, ..., T'. \\
x_t, y_t, z_t, v_t \geq 0, \qquad \forall t = 1, ..., T'.$$

It is shown that Z(x) can be obtained by solving the following network flow problem.

$$Z(\mathbf{x}) = \max_{\alpha_{t}, \beta_{t}} \sum_{t=1}^{T} (d_{t} - x_{t}) \alpha_{t}$$
s.t. $\alpha_{t} - \alpha_{t-1} - \beta_{t} = -1, \ \forall t = 2, ..., T,$

$$-\alpha_{T} - \beta_{T+1} = -1,$$

$$\alpha_{t}, \beta_{t} \geq 0, \qquad \forall t = 1, ..., T.$$
(2)

which can be written in a generic form

$$Z(\boldsymbol{x}) = \max_{\alpha_{t}, \beta_{t}} (\boldsymbol{d} - \boldsymbol{x})' \boldsymbol{\alpha}$$
s.t. $\begin{bmatrix} A B \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} = \boldsymbol{b}$

$$\alpha_{t}, \beta_{t} \geq 0, \qquad \forall t = 1, ..., T.$$
(3)

The dual problem is therefore,

$$Z(\boldsymbol{x}) = \min_{\substack{\phi_t \\ \text{s.t.}}} \boldsymbol{d}' \boldsymbol{\phi}$$
s.t. $A' \boldsymbol{\phi} = \boldsymbol{d} - \boldsymbol{x}$

$$B' \boldsymbol{\phi} = 0$$
(4)

Therefore, the overall backlog minimization problem is

$$\min_{x,y,z,v\phi} \mathbf{d}' \phi
\text{s.t.} \quad z_{t,t'} = 0, \qquad \forall t' \neq t + 4.
\sum_{t} z_{t,t'} = x_{t}, \qquad \forall t = 1, ..., T.
\sum_{t} z_{t,t'} = y_{t'}, \qquad \forall t' = 1, ..., T'.
\sum_{i:i \leq t} (x_{i} + y_{i} + v_{i}) + \sum_{j=t+1}^{t+R} y_{j} \leq S_{t}^{u}, \forall t = 1, ..., T'.
\sum_{i:i \leq t} (x_{i} + y_{i} + v_{i}) + \sum_{j=t+1}^{t+R} y_{j} \geq S_{t}^{l}, \forall t = 1, ..., T'.
\sum_{i:i \leq t} (x_{i} + y_{i} + v_{i}) + \sum_{j=t+1}^{t+R} y_{j} \geq S_{t}^{l}, \forall t = 1, ..., T'.
x_{t} + y_{t} \leq c_{t}, \qquad \forall t = 1, ..., T'.
A' \phi = \mathbf{d} - \mathbf{x}
B' \phi = 0
x_{t}, y_{t}, z_{t}, v_{t} \geq 0, \qquad \forall t = 1, ..., T'$$