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1. Optimization Model

Parameters

- Let $t=1,...,T$ denote the time horizon for the vaccination program for the first dose. The second dose is therefore administrated over $t=22,...,T'(=T+26)$.
- Let c_t denote the capacity of the vaccination program up till time t.
- Let $\tilde{s_t^u}$ be the supply schedule of the vaccine, and $\tilde{S_t^u}$ denote the cumulative supply of the vaccine up till time t.
- Let $\tilde{s_t^l}$ be the expired amount vaccine at time t, and $\tilde{S_t^l}$ denote the cumulative expired vaccine up till time t.
- Let $\tilde{d}_t$ denote the choice probability at time t.
- Let $\tilde{r}_k$ denote the response rate of invited population of wave k.
- Let R be the number of days of the second-dose reservation required.
- Let N be the total number of population.
- Denote I as invitation interval and K as total number of invitation waves.

Assume demand and supply are independent, i.e., $(\tilde{r_k}, \tilde{d_t})$ and $(\tilde{S_t^u}, \tilde{S_t^l})$ are independent. The ambiguity set of $(\tilde{r_k}, \tilde{d_t})$, denoted as $\mathcal{D}^+$, and that of $(\tilde{S_t^u}, \tilde{S_t^l})$, denoted as $\mathcal{S}^+$, are assumed to be characterized by the first-two moments.

Decision variables

- x_t denotes the first dose booking limit at time t.
- y_t denotes the second dose booking limit at time t.
- $z_{t,t'}$ denotes the number of patients with first dose at t and second dose at t'.
- v_t denotes the buffer for accounting expired vaccines.

- n_k denotes the number of people invited in wave k.

$$
\begin{aligned}
\min \ & \left\{ \max_{(\tilde{r}_t, \tilde{d}_t) \sim \mathcal{D}^+} \mathbb{E}[Z^w(\boldsymbol{n}, \boldsymbol{x})] + \gamma(k) n_k \right\} \\
\text{s.t.} \ & \sum_{k=1}^{K} n_k = N \\
& z_{t,t'} = 0, && \forall t' \neq t+4. \\
& \sum_{t'} z_{t,t'} = x_t, && \forall t = 1, ..., T. \\
& \sum_{t} z_{t,t'} = y_{t'}, && \forall t' = 1, ..., T'. \\
& \sum_{t=1}^{T'} x_t + y_t \leq c_t && \forall t = 1, ..., T'. \\
& \sum_{i: i \leq t} (x_i + y_i + v_i) + \sum_{j=t+1}^{t+R} y_j \leq \tilde{S}^u_t, && \forall t = 1, ..., T'. \\
& \sum_{i: i \leq t} (x_i + y_i + v_i) + \sum_{j=t+1}^{t+R} y_j \geq \tilde{S}^l_t, && \forall t = 1, ..., T'. \\
& x_t, y_t, z_t, v_t, n_k \geq 0, && \forall t = 1, ..., T'
\end{aligned} \tag{1}
$$

where

$$
\begin{aligned}
Z^w(\boldsymbol{n}, \boldsymbol{x}) = \max_{\alpha_t, \beta_t} \ & \sum_{t=1}^{T} \left(\sum_{k=1}^{t/I+1} n_k \tilde{\gamma}_k \tilde{d}_{t-(k-1)I} - x_t \right) \alpha_t \\
\text{s.t.} \ & \alpha_t - \alpha_{t-1} - \beta_t = -1, \forall t = 2, .., T, \\
& -\alpha_T - \beta_{T+1} = -1, \\
& \alpha_t, \beta_t \geq 0, \forall t = 1, ..., T.
\end{aligned} \tag{2}
$$

The first term in the objective function corresponds to the worst-case total backlog, and the second term regulates more people should be invited in earlier waves to maximize the throughput. The weight function $\gamma(k)$ can take a form that gives higher weight to a latter invitation, e.g., $\gamma(k) = (T'k)^2$.

To account for supply and expiry uncertainty, we relax the supply upper and lower bound hard constraints by considering the Lagrangean form and hedge against the worst case.

$$
\begin{aligned}
\min \ & \left\{ \max_{(\tilde{r}_t, \tilde{d}_t) \sim \mathcal{D}^+} \mathbb{E}[Z^w(\boldsymbol{n}, \boldsymbol{x})] + \gamma(k) n_k + \max_{(\tilde{S}^u_t, \tilde{S}^l_t) \sim \mathcal{S}^+} \mathbb{E}[Z^s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{v})] \right\} \\
\text{s.t.} \ & \sum_{k=1}^{K} n_k = N \\
& z_{t,t'} = 0, && \forall t' \neq t+4. \\
& \sum_{t'} z_{t,t'} = x_t, && \forall t = 1, ..., T. \\
& \sum_{t} z_{t,t'} = y_{t'}, && \forall t' = 1, ..., T'. \\
& \sum_{t=1}^{T'} x_t + y_t \leq c_t && \forall t = 1, ..., T'. \\
& x_t, y_t, z_t, v_t, n_k \geq 0, && \forall t = 1, ..., T'
\end{aligned} \tag{3}
$$

where

$$Z^s(\boldsymbol{x},\boldsymbol{y},\boldsymbol{v}) = \max_{\lambda_t,\delta_t\geq 0} \sum_{t=1}^{T'} \left(\left(\sum_{i:i\leq t} (x_i + y_i + v_i) + \sum_{j=t+1}^{t+R} y_j \right) (\lambda_t - \delta_t) + \tilde{S}_t^l \delta_t - \tilde{S}_t^u \lambda_t \right) \tag{4}$$

$$\begin{aligned} Z^w(\boldsymbol{n},\boldsymbol{x}) = \max_{\alpha_t,\beta_t} \; & \sum_{t=1}^{T} \Big(\sum_{k=1}^{t/I+1} n_k \tilde{\gamma}_k \tilde{d}_{t-(k-1)I} - x_t \Big) \alpha_t \\ \text{s.t.} \; & \alpha_t - \alpha_{t-1} - \beta_t = -1, \forall t = 2,..,T, \\ & -\alpha_T - \beta_{T+1} = -1, \\ & \alpha_t, \beta_t \geq 0, \forall t = 1,...,T. \end{aligned} \tag{5}$$

Both $\max\limits_{(\tilde{r_t},\tilde{d_t})\sim\mathcal{D}^+} \mathbb{E}[Z^w(\boldsymbol{q},\boldsymbol{x})]$ and $\max\limits_{(\tilde{S_t^u},\tilde{S_t^l})\sim\mathcal{S}^+} \mathbb{E}[Z^s(\boldsymbol{x},\boldsymbol{y},\boldsymbol{v})]$ can be reformulated as completely positive programs. By considering their the dual program, the optimization problem 3 is equivalent to a copositive program.