PHY371 Project IV
Integration Methods and Error Assessments *

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1 Introduction

In this project, you will find the integral of $f(x) = \int_0^2 e^{-\frac{1}{2}x} dx$ with three different integration methods, i.e. trapozoidal rule, Simpson's rule and Gauss Quadrature, and conduct careful error assessments on each method.

2 Project

1. Write a program to integrate

$$f(x) = \int_0^2 e^{-\frac{1}{2}x} dx \tag{1}$$

   numercially using the trapezoidal rule, the Simpson's rule and Gauss quadrature.

2. Compute the relative error $\epsilon = |(numerical - exact)/exact|$ in each case. Present your data in the tabular form with spaces or tabs separating the fields. Try N values of 2, 10, 20, 40, 80, 160, ...

N	ϵ_T	ϵ_S	ϵ_G
2	...	...	...
10	...	...	...
...	...	...	...

Table 1: Error assessments of different integration methods

*This problem is developed from section 6.2.5 from *Survey for Computational Physics* by Rubin Landau.

3. Make a lin-lin plot **and** a log-log plot of relative error *versus* N. In the log-log plot, you should observe that

$$\epsilon \simeq CN^{\alpha} \quad \Rightarrow \quad log\epsilon = \alpha logN + constant. \tag{2}$$

   This means that a power-law dependence appears as a straight line on a log-log plot, and that if you use log_{10}, then the ordinate on your log-log plot will be the negative of the number of decimal places of precision in your calculation.

3 Report

Use LaTeX to write a short (around 3 pages) scientific report. Your report should include:

1. Introduction
   In your own words, describe the problem that you are working on.

2. Procedures
   Describe the methods that you use to solve this problem.

3. Results and Discussions

   - Write down the best integral value and its relative error that you have numerically calculated by three integration methods respectively.
   - In a graph with linear $x - y$ scale, plot error *versus* N that are calculated by the Trapozoidal rule, the Simpson's rule and Gaussian quadrature respectively.
   - In another graph, replot the error *versus* N data on a log-log scale. Explain why we should use the log-log scale other than lin-lin scale in this case.
   - Explain why relative errors do not further decrease with N at the large N limit.
   - Recall the error analysis that we have learned in class for Trapozoidal Rule and Simpson's rule: the error of Trapezoidal rule is proportional to $1/N^2$; and Simpson's rule $1/N^3$. Do your error assessments in this project agree with these conclusions?
   - Does the Gaussian Quadrature perform better than Trapezoidal or Simpson's rule? Provide arguments to support your answer.

4. Conclusion
   Summarize your problem, methods and results in a few words.

4 Submission

Please submit your python program (.py file) and your report (.tex and pdf files) to ying.tang at gordon.edu before **11:59pm on Tuesday 10/14**. Your python program should allow me to reproduce data and plots in your report. A late submission will cause a 50% deduction of your grade.