Ilie Petre-Cristian 23/11/2021

Grupa 233

$i = 29$

Test Laborator

1) $P(\text{la o aruncare, să iasă } (6,6)) = \frac{1}{6^2} = \frac{1}{36}$

$\Rightarrow P(\ldots \text{ să } \underline{\text{nu}} \text{ iasă}) = \frac{35}{36}$

Cel puțin o dată = $P(\text{fix o dată}) + P(\text{de 2 ori}) + \ldots$

$\ldots + P(\text{de 29 ori})$

$P(\text{fix o dată } (6,6)) = \frac{1}{36} \cdot \left(\frac{35}{36}\right)^{28}$

$P(\text{de 2 ori} \ldots) = \left(\frac{1}{36}\right)^2 \cdot \left(\frac{35}{36}\right)^{27}$

$\vdots$

$P(\text{de 29 ori}) = \left(\frac{1}{36}\right)^{29}$

$\Rightarrow P(\text{să iasă cel puțin o dată } (6,6)) = \frac{35^{28}}{36^{29}} + \frac{35^{27}}{36^{29}} + \ldots + \frac{35}{36^{29}} + \frac{1}{36^{29}}$

$= \frac{1 + 35 + 35^2 + \ldots + 35^{28}}{36^{29}} = \frac{\frac{1 - 35^{29}}{1 - 35}}{36^{29}}$ (sir geometric)

1/6

2) $X: \begin{pmatrix} n \\ \frac{1}{29} \cdot q^n \end{pmatrix}$, $n \in \mathbb{N}$, $q \in (0,1)$

a) $q = ?$ a.î. X v.a.

b) $Var(X) = EX^2 - (EX)^2$

a) $\sum_{n \in \mathbb{N}} \frac{q^n}{29} = 1 \Rightarrow \sum_{n \in \mathbb{N}} \frac{q}{\sqrt[n]{29}} = 1 \Rightarrow \sum_{n=\mathbb{N}} \frac{1}{\sqrt[n]{29}} = \frac{1}{q}$

3.

X/Y	-1	0	1	2	
-1	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{4}{24}+\frac{2}{24}+\frac{2}{24}+\frac{1}{24}=\frac{9}{24}$
0	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{24}$	$\rightarrow \frac{1+4+2+1}{24}=\frac{8}{24}$
1	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{1}{24}$	$\rightarrow \frac{3+4}{24}=\frac{7}{24}$

a)

	-1	0	1
	$\frac{9}{24}$	$\frac{8}{24}$	$\frac{7}{24}$

$$X \sim \begin{pmatrix} -1 & 0 & 1 \\ \frac{9}{24} & \frac{8}{24} & \frac{7}{24} \end{pmatrix}$$

$$Y \sim \begin{pmatrix} -1 & 0 & 1 & 2 \\ \frac{6}{24} & \frac{7}{24} & \frac{8}{24} & \frac{3}{24} \end{pmatrix}$$

b) $EX = (-1 \cdot \frac{9}{24}) + 0 \cdot \frac{8}{24} + 1 \cdot \frac{7}{24} = \frac{-9+7}{24} = -\frac{2}{24} =$

$$= -\frac{1}{12}$$

$$EY = (-\frac{6}{24}) + 0 + \frac{8}{24} + \frac{6}{24} = \frac{-6+6+8}{24} = \frac{8}{24} = \frac{1}{3}$$

~~XY | -1 | 0 | 1 | 2~~

~~1 | 1/12 | 2/12 | 6/12 | 2/12 | 1/12~~

~~$\Rightarrow EXY = -\frac{2}{12} + \left(\frac{2}{12}\right) + 0 + \frac{2}{12} + \frac{2}{12} = 0$~~

X și Y independente ? $\Leftrightarrow P(X|Y) = P(X)$

Exemplu: Luăm $X=-1$, $Y=0 \Rightarrow \frac{9}{24} \cdot \frac{7}{24} \neq \frac{9}{24} \Rightarrow$ (F) $\Rightarrow$

$\Rightarrow X, Y$ Nu sunt independente

$$EXY = 1 \cdot \frac{1}{6} + (-1) \cdot \left(\frac{1}{12}\right) + (-2) \cdot \left(\frac{1}{24}\right) + (-1) \cdot \frac{1}{24} + \frac{1}{6} + \frac{2}{24} =$$

$$= \frac{1}{6} - \frac{1}{12} - \frac{1}{12} - \frac{1}{24} + \frac{1}{6} + \frac{1}{12} = \frac{8}{24} - \frac{3}{24} = \frac{5}{24}$$

c) $Var(X) = EX^2 - (EX)^2 = \left(\frac{9}{24} + \frac{7}{24}\right) - \left(-\frac{1}{12}\right)^2 =$

$$= \frac{16}{24} - \frac{1}{144} = \frac{96-1}{144} = \frac{95}{144}$$

$$Var(Y) = EY^2 - (EY)^2 = \left(\frac{6}{24} + \frac{8}{24} + \frac{12}{24}\right) - \left(\frac{1}{3}\right)^2 =$$

$$= \frac{26}{24} - \frac{1}{9} = \frac{78-8}{72} = \frac{70}{72}$$

$Cov(X,Y) = \rho(X,Y)\sqrt{Var X \cdot Var Y} \Rightarrow$

$$\Rightarrow \rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var X \cdot Var Y}}$$

$$Cov(X,Y) = E\left((X-\mu_X)(Y-\mu_Y)\right) =$$

$$= \frac{1}{6}\cdot\left(-1+\frac{1}{12}\right)\left(-1-\frac{1}{3}\right) + \frac{1}{12}\cdot\left(-1+\frac{1}{12}\right)\left(-1-\frac{1}{3}\right) +$$

$$+ \ldots + \frac{1}{24}\left(-1+\frac{1}{12}\right)\left(-1-\frac{1}{3}\right) =$$

$$= \frac{1}{6}\left(-\frac{11}{9}\right) + \frac{1}{12}\left(-\frac{11}{9}\right) + \frac{1}{12}\left(-\frac{11}{9}\right) + \frac{1}{24}\cdot\left(-\frac{11}{9}\right) + \ldots$$

$$\ldots = \frac{1}{6}\cdot 3\cdot\left(-\frac{11}{9}\right) + \frac{1}{12}\cdot 3\left(-\frac{11}{9}\right) + \frac{1}{24}\cdot 6\cdot\left(-\frac{11}{9}\right) =$$

$$= -\frac{11}{18} + \left(-\frac{11}{36}\right) + \left(-\frac{11}{36}\right) = -\frac{22}{36} - \frac{11}{36} - \frac{11}{36} =$$

$$= -\frac{44}{36}$$

$$\Rightarrow \underset{\uparrow}{\rho}(X,Y) = -\frac{44}{36}\cdot\frac{1}{\sqrt{\frac{95}{144}\cdot\frac{20}{22}}} = -\frac{44}{36}\cdot\frac{1}{\sqrt{\frac{95\cdot 20}{144\cdot 22}}}$$

coeficient de corelație

4) $X \sim \mathcal{U}([-1,1])$; $Y = 2X+1$

$\mathcal{U}([-29,29])$

$$E(X) = \int_{-29}^{29} x f(x)\,dx = \int_{-29}^{29} x\,dx = \frac{x^2}{2}\Big|_{-29}^{29} =$$

$$= \frac{29^2 - (-29)^2}{2} = 0$$

~~$X \to 2X+1 \Rightarrow X \sim \mathcal{U}([-29,29])$~~

$2X+1 \sim \mathcal{U}([-57,59])$