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$\boxed{i = 65}$

Examen Probabilități și statistică

1) Mulțimea $(0, 65] \cap \mathbb{N}$

a) Mulțimea are 65 de numere $(1, 2, 3, \dots 64, 65) \Rightarrow$
$\Rightarrow$ 65 de cazuri posibile

Nr. div cu 3 : $3, 6, \dots, 57, 60, 63 \dots \Rightarrow$ 21 cazuri favorabile

$\Rightarrow P(a) = \frac{21}{65} \cong 0,323$

b) Pătrate perfecte : $1, 4, 9, 16, 25, 36, 49, 64 \dots \Rightarrow$ 8 cazuri favorabile

$\Rightarrow P(b) = \frac{8}{65} = 0,123$

c) Nr. prime : $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,$
$43, 47, 53, 59, 61 \dots \Rightarrow$ 18 cazuri favorabile

$\Rightarrow P(c) = \frac{18}{65} = 0,276$

2) $X \sim \begin{pmatrix} -1 & 0 & 1 \\ \frac{65}{1000} & \frac{1}{100} & c \end{pmatrix}$

a) $\frac{65}{1000} + \frac{1}{100} + c = 1 \Rightarrow c = 1 - \left(\frac{65}{1000} + \frac{10}{1000}\right) = 1 - \frac{75}{1000} =$
$= \frac{925}{1000}$

b) $Var(X) = E(X^2) - \{E(X)\}^2$

$E(X) = (-1)\left(\frac{65}{1000}\right) + 0 \cdot \frac{1}{100} + 1 \cdot \frac{925}{1000} =$

$= \frac{925-65}{1000} = \frac{860}{1000} = \frac{86}{100} = \frac{43}{50}$

$X^2 \sim \begin{pmatrix} 0 & 1 \\ \frac{1}{100} & \frac{990}{1000} \end{pmatrix} \Rightarrow E(X^2) = 0 \cdot \frac{1}{100} + \cancel{\frac{1}{1}} + 1 \cdot \frac{990}{1000} = \frac{99}{100}$

$\Rightarrow Var(X) = \frac{99}{100} - \left(\frac{86}{100}\right)^2 = \frac{99}{100} - \frac{7396}{10000} = \frac{9900-7396}{10000} =$

$= \frac{2504}{10000} \cong 0,25$

c) $Var(2X) = E\left((2X)^2\right) - E(2X)^2$

$2X \sim \begin{pmatrix} -2 & 0 & 2 \\ \frac{65}{1000} & \frac{1}{100} & \frac{925}{1000} \end{pmatrix}$

$E(2X) = (-2) \cdot \frac{65}{1000} + 0 + 2 \cdot \frac{925}{1000} = \frac{1850}{1000} - \frac{130}{1000} =$

$= \frac{1720}{1000} \cong 1,72$

$(2X)^2 = 4X^2 \Rightarrow 4X^2 \sim \begin{pmatrix} 0 & 4 \\ \frac{1}{100} & \frac{990}{1000} \end{pmatrix}$

$E\left((2X)^2\right) = 0 + 4 \cdot \frac{990}{1000} = \frac{3960}{1000} = 3,96$

$\Rightarrow Var(2X) = 3,96 - (1,72)^2 = 3,96 - 2,958 = 1,002$

3) X, Y au domeniul [0,65]

$f(x,y) = cxy$

a) $\int_0^{65}\int_0^{65} f(x,y)\, dx\, dy = 1$ (probabilitatea totală)

$$\int_0^{65}\int_0^{65} cxy\, dx\, dy = c\int_0^{65}\int_0^{65} xy\, dx\, dy =$$

$$= c \cdot \frac{x^2}{2}\Big|_{x=0}^{x=65} \cdot \int_0^{65} y\, dy = c \cdot \frac{4225}{2} \cdot \frac{y^2}{2}\Big|_{y=0}^{y=65} =$$

$$= c \cdot \left(\frac{4225}{2}\right)^2 = 1 \Rightarrow c = \frac{1}{\left(\frac{4225}{2}\right)^2} = \left(\frac{2}{4225}\right)^2 =$$

$$= \frac{4}{17.850.625}$$

b) $$f_X(x) = \int_0^{65} cxy\, dy = cx \cdot \frac{y^2}{2}\Big|_0^{65} = cx \cdot \frac{4225}{2} =$$

$$= \left(\frac{2}{4225}\right)^{\cancel{2}} \cdot x \cdot \frac{\cancel{4225}}{\cancel{2}} = \frac{2x}{4225}$$

$$\Rightarrow P(X \leq 3) = \int_0^3 \frac{2x}{4225}\, dx = \frac{2}{4225} \cdot \int_0^3 x\, dx =$$

$$= \frac{2}{4225} \cdot \frac{x^2}{2}\Big|_0^3 = \frac{\cancel{2}}{4225} \cdot \frac{9}{\cancel{2}} = \frac{9}{4225}$$

c) $F(x,y) = \int_0^y \int_0^x cxy \, dx \, dy =$

$$= c \cdot \frac{x^2}{2} \cdot \frac{y^2}{2} = \frac{cx^2y^2}{4} = \frac{x^2y^2}{17850625}$$

$$F_X(x) = F(x,65) = \frac{x^2 \cdot \cancel{4225}}{(4225)^{\cancel{2}}} = \frac{x^2}{4225}$$

$$F_Y(y) = F(65,y) = \frac{\cancel{4225} \cdot y^2}{(4225)^{\cancel{2}}} = \frac{y^2}{4225}$$

$$F_X(x) \cdot F_Y(y) = \frac{x^2}{4225} \cdot \frac{y^2}{4225} = \frac{x^2y^2}{17850625} = F(x,y) \Rightarrow$$

$\Rightarrow$ X și Y sunt independente

4) 200 monede ~~[illegible]~~ $\begin{cases} 65 \text{ de tip A} \; ; P(\text{avers} \mid A) = 0{,}5 \\ 135 \text{ de tip B} \; ; P(\text{avers} \mid B) = 0{,}8 \end{cases}$

$P(A) = \frac{65}{200} = 0{,}325$

$P(B) = \frac{135}{200} = 0{,}675$

$P(\text{avers}) = P(A) \cdot P(\text{avers} \mid A) + P(B) \cdot P(\text{avers} \mid B) =$

$$= \frac{65}{200} \cdot \frac{1}{2} + \frac{135}{200} \cdot \frac{8}{10} =$$

$$= \frac{65}{400} + \frac{1080}{2000} = \frac{1080+325}{2000} = \frac{1405}{2000} \approx 0{,}702$$

$\Rightarrow$ Probabilitatea predictivă a priori $\Rightarrow P(\text{avers}) = 0,702$

$$P(A \mid \text{avers}) \overset{\text{Bayes}}{=} \frac{P(\text{avers} \mid A) \cdot P(A)}{P(\text{avers})} = \frac{\frac{65}{400}}{\frac{1405}{2000}} =$$

$$= \frac{65 \cdot 2000}{400 \cdot 1405} = \frac{130}{562} \simeq 0,231$$

$$P(B \mid \text{avers}) = \frac{P(\text{avers} \mid B) \cdot P(B)}{P(\text{avers})} = 1 - P(A \mid \text{avers}) \simeq 0,769$$

$\Rightarrow$ Probabilitatea predictivă a posteriori $\Rightarrow P(\text{avers}' \mid \text{avers})$

$$= P(\text{avers}' \mid A) \cdot P(A \mid \text{avers}) + P(\text{avers}' \mid B) \cdot P(A \mid \text{avers}) =$$

$$= 0,5 \cdot 0,231 + 0,8 \cdot 0,769 \simeq 0,115 + 0,615 \simeq 0,730$$

5) 110 aruncări $\begin{cases} 65 \text{ aversuri} \\ 45 \text{ reversuri} \end{cases} \Rightarrow$ repartiție binomială$(110, \theta)$

$\Rightarrow$ verosimilitatea este $p(x_1 \mid \theta) = C_{110}^{65} \cdot \theta^{65} (1-\theta)^{45}$

Plecăm cu o a priori plată $\Rightarrow f(\theta) = 1$

$$p(x_1) = \int_0^1 1 \, d\theta = \theta \Big|_0^1 = 1 - 0 = 1$$

$$\Rightarrow f(\theta \mid x_1) \overset{\text{Bayes}}{=} \frac{p(x_1 \mid \theta) \cdot f(\theta)}{p(x_1)} = \frac{p(x_1 \cdot \theta) \cdot 1}{1} =$$

$$= p(x_1|\theta) = C_{110}^{65} \cdot \theta^{65}(1-\theta)^{45}$$