Ilie Petre - Cristian

Grupa 233

Temă 2

Ex-1 a) $P(\text{crap la o prindere}) = \frac{3}{10}$

$P(\text{ceilalți 3 pești să nu fie crapi}) = \frac{7}{9} \cdot \frac{6}{8} \cdot \frac{5}{7} = \frac{30}{72} = \frac{15}{36} = \frac{5}{12}$

$\Rightarrow P(\text{ca la 4 aruncări, fix unul să fie crap}) =$

$= P(A) = \frac{3}{10} \cdot \frac{5}{12} = \frac{15}{120} = \frac{5}{40} = \frac{1}{8}$

b) $P(\text{fix 1 crap}) = \frac{1}{8}$

$P(2 \text{ crapi}) = \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{7}{8} \cdot \frac{6}{7} = \frac{36}{720} = \frac{1}{20}$

$P(3 \text{ crapi}) = \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8} \cdot \frac{7}{7} = \frac{6}{720} = \frac{1}{120}$

$P(4 \text{ crapi}) = 0$ (nu sunt decât 3 crapi)

$\Rightarrow P(B) = \frac{1}{8} + \frac{1}{20} + \frac{1}{120} = \frac{15}{120} + \frac{6}{120} + \frac{1}{120} = \frac{22}{120} = \frac{11}{60}$

c) $P(\text{primul pește este crap}) = \frac{3}{10}$ (la început sunt 3 crapi și 10 pești în total)

d) $P(\text{primul crap, al doilea crap}) = \frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90} = \frac{2}{30} = \frac{1}{15}$

$P(\text{primul caras, al doilea crap}) = \frac{7}{10} \cdot \frac{3}{9} = \frac{21}{90} = \frac{7}{30}$

$\Rightarrow P(D) = \frac{1}{15} + \frac{7}{30} = \frac{9}{30} = \frac{3}{10}$

e) $P(E) = \frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90} = \frac{2}{30} = \frac{1}{15}$

f) $P(\text{primul crap}) = \frac{3}{10} \cdot \frac{7}{9} = \frac{21}{90} = \frac{7}{30}$ (al doilea caras)

$P(\text{primul caras, al doilea crap}) = \frac{7}{10} \cdot \frac{3}{9} = \frac{7}{30}$

$P(\text{ambii crapi}) = \frac{3}{10} \cdot \frac{2}{9} = \frac{1}{15}$

$\Rightarrow P(F) = \frac{7}{30} + \frac{7}{30} + \frac{2}{30} = \frac{16}{30} = \frac{8}{15}$

g) Dacă numerotăm peștii de la 1 la 10, după greutate avem cazurile:

~~1 - Primul pește are rest 1 $\Rightarrow$ numerotăm 3) sunt descărcători~~

~~2 $\Rightarrow$ 11 - descărcători~~

~~3 $\Rightarrow$ 11 - descărcători~~

7 $\Rightarrow$ 1 caz: (7, 8, 9, 10)

8 $\Rightarrow$ 0

9 $\Rightarrow$ 0

10 $\Rightarrow$ 0

2)

X abc

↑

1-10 $a<b<c$ și $a, b, c \neq X$

~~Sunt 210 cazuri posibile~~

$X = 1 \Rightarrow a = 2 \Rightarrow b = 3 \Rightarrow C = \underbrace{4-10}_{7 \text{ cazuri}}$

$b = 4 \Rightarrow C = \underbrace{5-10}_{6 \text{ cazuri}}$

$\vdots$

$b = 10 \Rightarrow C = - \Rightarrow 0$ cazuri

$a = 3 \Rightarrow b = 4 \Rightarrow C = \underbrace{5-10}_{6}$

$\vdots \qquad \vdots$

$a = 4$

$\vdots$

$a = 8 \Rightarrow b = 9 \Rightarrow C = 10$

$a \geq 9 \Rightarrow -$

$a = 10 \Rightarrow -$

$\Rightarrow X = 1 \Rightarrow \underbrace{\frac{7 \cdot 8}{2}}_{a=2} + \underbrace{\frac{6 \cdot 7}{2}}_{a=3} + \underbrace{\frac{5 \cdot 6}{2}}_{a=4} + \frac{4 \cdot 5}{2} + \frac{3 \cdot 4}{2} + \frac{2 \cdot 3}{2} + 1 =$

$= 28 + 21 + 15 + 10 + 6 + 3 + 1 = 49 + 15 + 20 = 84$ cazuri

3)

Dc. $x=2 \Rightarrow$ Tot 84 cazuri

$x=3$ —11—

⋮

$x=10 \Rightarrow$ Tot 84 cazuri

$\Rightarrow$ Cazuri favorabile $= 10 \cdot 84 = 840$

Cazuri posibile $=$ [illegible] $10 \cdot 9 \cdot 8 \cdot 7 = 5040$ cazuri

$\Rightarrow P(G) = \frac{840}{5040} = \frac{84}{504} = \frac{42}{252}$

Ex 2

$X \backslash Y$	1	2	3
1	0,22	0,11	0,02
2	0,2	0,15	0,1
3	0,06	0,07	0,07

a) $X \sim \begin{pmatrix} 1 & 2 & 3 \\ 0,35 & 0,45 & 0,20 \end{pmatrix}$ $Y \sim \begin{pmatrix} 1 & 2 & 3 \\ 0,48 & 0,33 & 0,19 \end{pmatrix}$

b) $E(X) = 1 \cdot 0,35 + 2 \cdot 0,45 + 3 \cdot 0,20 = 0,35 + 0,90 + 0,60 = 1,85$

4) $E(Y) = 1,71$

$$Var(X) = E(X^2) - (E(X))^2 =$$

$$= (0,35 + 1,00 + 1,80) - (1,85)^2 =$$

$$= 0,53$$

$$Var(Y) = (0,48 + 4 \cdot 0,33 + 9 \cdot 0,19) - (1,71)^2 =$$

$$= 3,51 - 2,92 = 0,59$$

c) $$Cov(X,Y) = \sum_{i,j} p(x_i y_j)(x_i - 1)(y_j - 1) =$$

$$= 0 + 0 + 0 + 0 + (0,15)(1)(1) + (0,1) \cdot (1) \cdot (2) +$$

$$+ 0 + (0,07)(1)(1) + (0,07)(1)(2) =$$

$$= 0,15 + 0,2 + 0,07 + 0,14 = 0,35 + 0,21 =$$

$$= 0,56$$

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var X \times Var Y}} = \frac{0,56}{\sqrt{(0,53)(0,59)}}$$

d/e) $$E(X \mid Y = 2) = 1 \cdot (0,11) + 2 \cdot (0,15) + 3 \cdot (0,07) =$$

$$= 0,11 + 0,30 + 0,21 = 0,62$$

$$E(Y \mid X = 2) = 1 \cdot (0,2) + 2(0,15) + 3(0,1) = 0,2 + 0,3 + 0,3 =$$

$$= 0,80$$

5)

$X/Y=2 \sim \begin{pmatrix} 1 & 2 & 3 \\ 0,11 & 0,15 & 0,07 \end{pmatrix}$; $Y/X=2 \sim \begin{pmatrix} 1 & 2 & 3 \\ 0,2 & 0,5 & 0,1 \end{pmatrix}$

$\text{Var}(X/Y=2) = (0,11 + 4 \cdot 0,15 + 9 \cdot 0,07) - (0,62)^2 =$

$= 1,34 - 0,38 = 0,96$

$\text{Var}(Y/X=2) = (0,2 + 4 \cdot 0,5 + 9 \cdot 0,1) - (0,80)^2 =$

$= (0,2 + 0,6 + 0,9) - 0,64 =$

$= 1,7 - 0,64 = 1,06$

Ex 3) $X \sim N(m, \sigma)$; $\begin{cases} P(X<22) = \frac{91}{100} \\ P(X>28) = \frac{6}{100} \end{cases}$

$\Phi(1,35) = 0,91$

$\Phi(1,56) = 0,94$

$P(X<22) = \Phi\left(\frac{22-m}{\sigma}\right) = 0,91 \\ \Phi(1,35) = 0,91 \Big\} \Rightarrow$

$\Rightarrow \frac{22-m}{\sigma} = 1,35 \Rightarrow 22 - m = \sigma \cdot 1,35 \quad (1)$

6) $P(X<28) = 1 - P(X>28) = 1 - \frac{6}{100} = \frac{94}{100}$

$$P(X<28) = \Phi\left(\frac{28-m}{\sigma}\right) = 0{,}94 \Rightarrow \Phi(1{,}56) = 0{,}94 \Rightarrow 28-m = \sigma \cdot 1{,}94 \ (2)$$

$$\Rightarrow \begin{matrix}(1)\\(2)\end{matrix} \Rightarrow \begin{matrix}28-m = 1{,}56\cdot\sigma\\ 22-m = 1{,}35\cdot\sigma\end{matrix} \Big| \ominus \Rightarrow 6 = \sigma(1{,}56-1{,}35) \Rightarrow$$

$$\Rightarrow \sigma = \frac{6}{0{,}21} \cong 28{,}57$$

$$\Rightarrow m = 28 - 1{,}56 \cdot 28{,}57 \cong 28 - 44{,}56 \cong -16{,}56$$

Ex 4

$$f(x) = \begin{cases} \alpha x^2 e^{-kx}, & x \geq 0 \\ 0, & x < 0 \end{cases}, \ k > 0$$

a) $$\underbrace{\int_{-\infty}^{0} 0 \, dx}_{0} + \int_{0}^{\infty} \alpha x^2 e^{-kx} \, dx = 1 \Rightarrow$$

$$\Rightarrow \frac{2\alpha}{k^3} = 1 \Rightarrow k^3 = 2\alpha \Rightarrow \alpha = \frac{k^3}{2}$$

7)

b) $F(x) = \begin{cases} -\alpha\dfrac{(kx(kx+2)+2)\cdot e^{-kx}}{k^3}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

c) $P(0 < x < k^{-1}) = F(k^{-1}) - F(0) =$

$= \dfrac{-\alpha\cdot\left(\frac{k}{k}\left(\frac{k}{k}+2\right)+2\right)\cdot e^{-\frac{k}{k}}}{k^3} - \left(\dfrac{-\alpha\cdot(0+2)\cdot \overbrace{e^{0}}^{1}}{k^3}\right) =$

$= \dfrac{-\alpha\,(3+2)\cdot\frac{1}{e}}{k^3} + \dfrac{\alpha\cdot 2\cdot 1}{k^3} =$

$= \dfrac{-\frac{5\alpha}{e}+2\alpha}{k^3} = \dfrac{-5\alpha+2\alpha e}{k^3 e} = \dfrac{\alpha(-5+2e)}{k^3 e}$

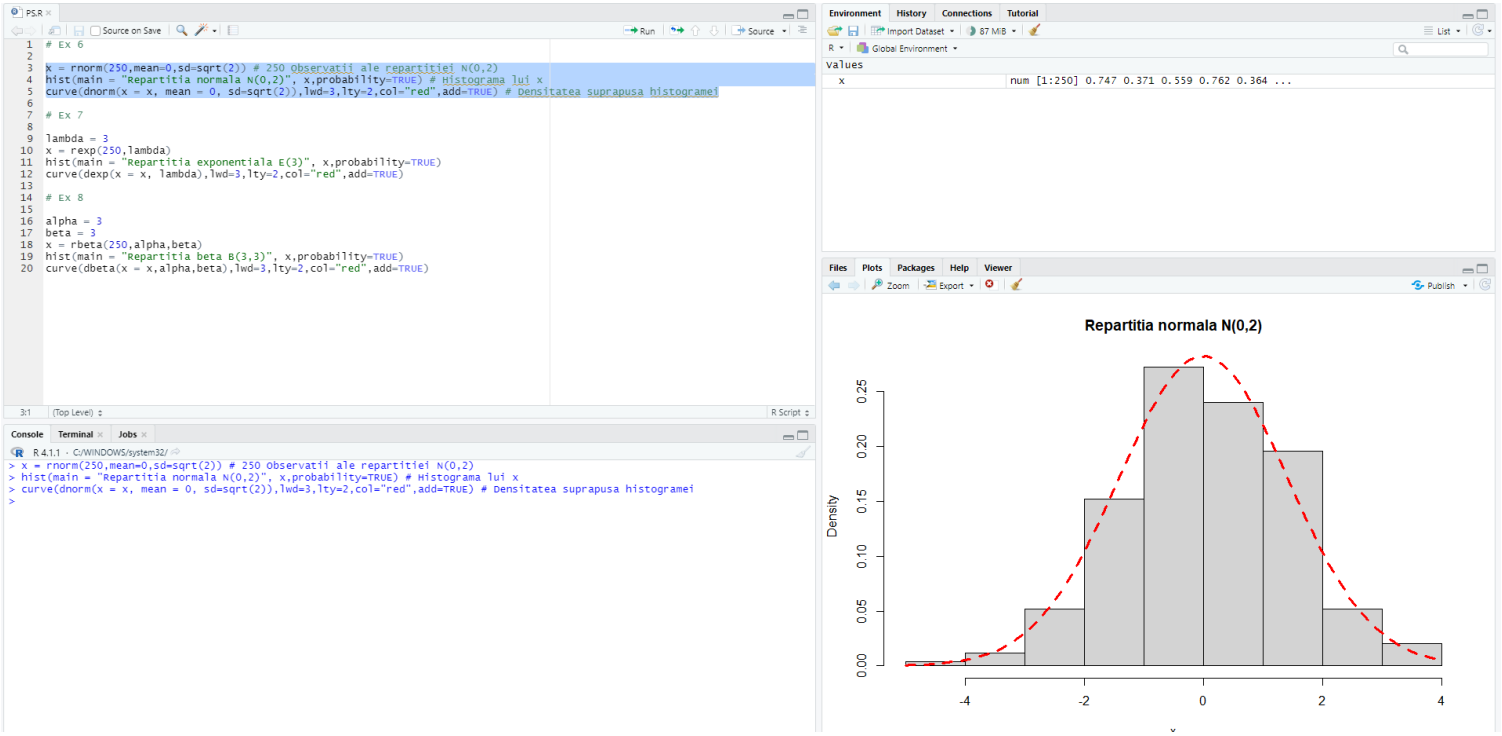
p)

Ex 6:

x = rnorm(250,mean=0,sd=sqrt(2)) # 250 Observatii ale repartitiei N(0,2)

hist(main = "Repartitia normala N(0,2)", x,probability=TRUE) # Histograma lui x

curve(dnorm(x = x, mean = 0, sd=sqrt(2)),lwd=3,lty=2,col="red",add=TRUE) # Densitatea suprapusa histogramei



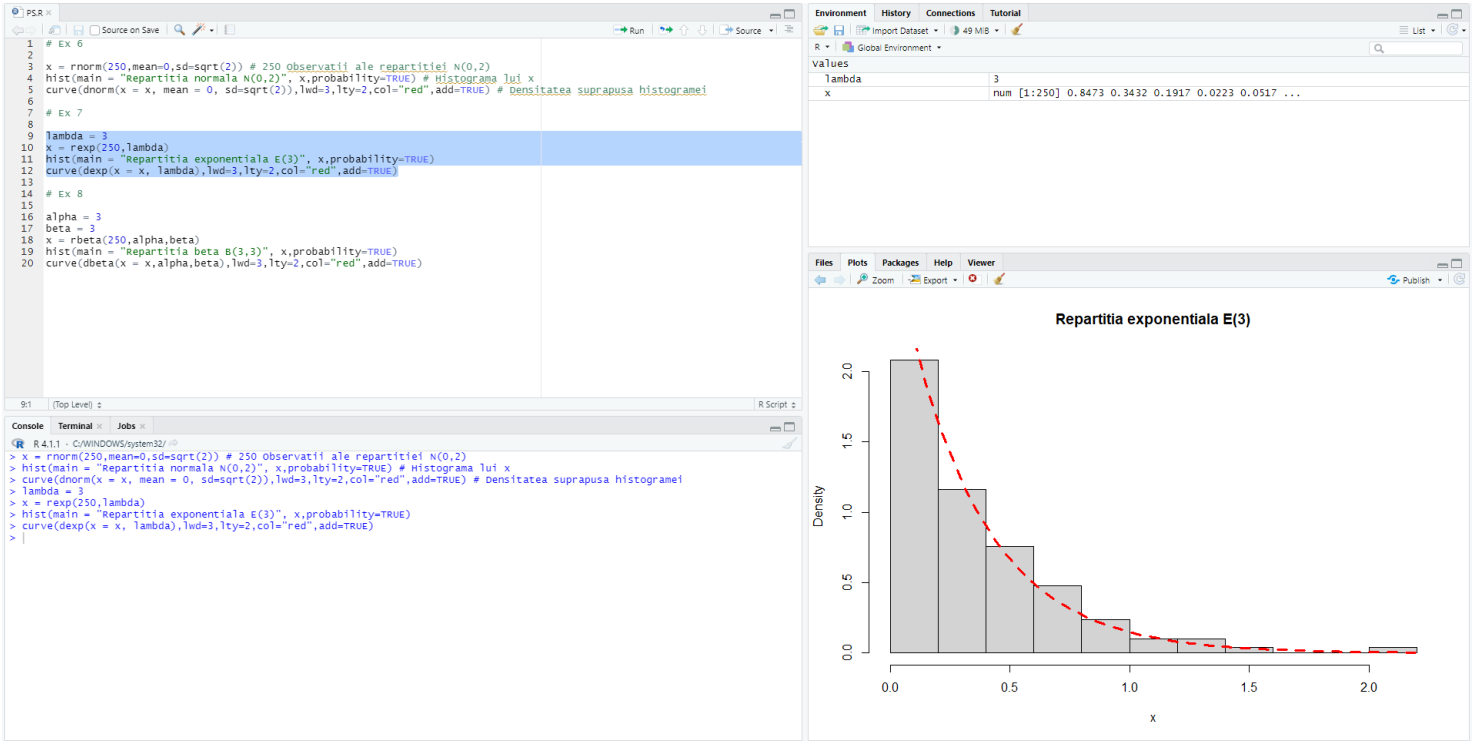
Ex 7:

lambda = 3

x = rexp(250,lambda)

hist(main = "Repartitia exponentiala E(3)", x,probability=TRUE)

curve(dexp(x = x, lambda),lwd=3,lty=2,col="red",add=TRUE)



Ex 8:

alpha = 3

beta = 3

x = rbeta(250,alpha,beta)

hist(main = "Repartitia beta B(3,3)", x,probability=TRUE)

curve(dbeta(x = x,alpha,beta),lwd=3,lty=2,col="red",add=TRUE)