

Programare funcțională

Introducere în programarea funcțională folosind Haskell
C11

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Functori - recap

Clasa de tipuri Functor

Definiție

```
class Functor f where
```

```
    fmap :: (a -> b) -> f a -> f b
```

Data fiind o funcție $f :: a \rightarrow b$ și $ca :: f\ a$, `fmap` produce $cb :: f\ b$ obținută prin transformarea rezultatelor produse de computația `ca` folosind funcția `f` (și doar atât!)

Instanță pentru liste

```
instance Functor [] where
```

```
    fmap = map
```

Clasa de tipuri Functor

```
class Functor f where
```

```
  fmap :: (a -> b) -> f a -> f b
```

Instanță pentru tipul optiune

```
fmap :: (a -> b) -> Maybe a -> Maybe b
```

```
instance Functor Maybe where
```

```
  fmap f Nothing = Nothing
```

```
  fmap f (Just x) = Just (f x)
```

Proprietăți ale functorilor

- Argumentul `f` al lui `Functor f` definește o transformare de tipuri
 - `f a` este tipul `a` transformat prin functorul `f`
- `fmap` definește transformarea corespunzătoare a funcțiilor
 - `fmap :: (a -> b) -> (f a -> f b)`

Contractul lui `fmap`

- `fmap f ca` e obținută prin transformarea rezultatelor produse de computația `ca` folosind funcția `f` (și doar atât!)
- Abstractizat prin două legi:
 - identitate** `fmap id == id`
 - compunere** `fmap (g . h) == fmap g . fmap h`

Functori applicativi

Problemă

- Folosind fmap putem transforma o funcție $h :: a \rightarrow b$ într-o funcție între colecții/computații $\text{fmap } h :: m\ a \rightarrow m\ b$
- Dar ce se întâmplă dacă avem o funcție cu mai multe argumente?
E.g., cum trecem de la $h :: a \rightarrow b \rightarrow c$ la
 $h' :: m\ a \rightarrow m\ b \rightarrow m\ c$
- putem încerca să folosim fmap

Problemă

- Folosind fmap putem transforma o funcție $h :: a \rightarrow b$ într-o funcție între colecții/computații $\text{fmap } h :: m\ a \rightarrow m\ b$
- Dar ce se întâmplă dacă avem o funcție cu mai multe argumente?
E.g., cum trecem de la $h :: a \rightarrow b \rightarrow c$ la $h' :: m\ a \rightarrow m\ b \rightarrow m\ c$
- putem încerca să folosim fmap
- Dar, deoarece $h :: a \rightarrow (b \rightarrow c)$, avem că $\text{fmap } h :: m\ a \rightarrow m\ (b \rightarrow c)$
- Putem aplica $\text{fmap } h$ la o valoare $ca :: m\ a$ și obținem $\text{fmap } h\ ca :: m\ (b \rightarrow c)$

Cum transformăm un obiect din $m(b \rightarrow c)$ într-o funcție $m b \rightarrow m c$?

- **ap** :: $m(b \rightarrow c) \rightarrow (m b \rightarrow m c)$, sau, ca operator
- $(\langle * \rangle)$:: $m(b \rightarrow c) \rightarrow m b \rightarrow m c$

Merge pentru funcții cu oricâte argumente

Problemă

Data fiind o funcție

$$f :: a_1 \rightarrow a_2 \rightarrow a_3 \rightarrow \dots \rightarrow a_n \rightarrow a$$

și computațiile

$$ca_1 :: m\ a_1, ca_2 :: m\ a_2, \dots, ca_n :: m\ a_n,$$

vrem să „aplicăm” funcția f pe rând computațiilor ca_1, \dots, ca_n pentru a obține o computație finală $ca :: m\ a$.

Merge pentru funcții cu oricâte argumente

Date fiind

- $f :: a1 \rightarrow a2 \rightarrow a3 \rightarrow \dots \rightarrow a_n \rightarrow a$
- $ca1 :: m\ a1, ca2 :: m\ a2, \dots, can :: m\ a_n,$
- $fmap :: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$
- $(<*>) :: m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$ cu „proprietăți bune”

Merge pentru funcții cu oricâte argumente

Date fiind

- $f :: a1 \rightarrow a2 \rightarrow a3 \rightarrow \dots \rightarrow a_n \rightarrow a$
- $ca1 :: m\ a1, ca2 :: m\ a2, \dots, ca_n :: m\ a_n,$
- $fmap :: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$
- $(<*>) :: m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$ cu „proprietăți bune”

Atunci

$$fmap\ f :: m\ a1 \rightarrow m\ (a2 \rightarrow a3 \rightarrow \dots \rightarrow a_n \rightarrow a)$$

Merge pentru funcții cu oricâte argumente

Date fiind

- $f :: a1 \rightarrow a2 \rightarrow a3 \rightarrow \dots \rightarrow a_n \rightarrow a$
- $ca1 :: m\ a1, ca2 :: m\ a2, \dots, ca_n :: m\ a_n,$
- $fmap :: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$
- $(<*>) :: m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$ cu „proprietăți bune”

Atunci

$fmap\ f :: m\ a1 \rightarrow m\ (a2 \rightarrow a3 \rightarrow \dots \rightarrow a_n \rightarrow a)$

$fmap\ f\ ca1 :: m\ (a2 \rightarrow a3 \rightarrow \dots \rightarrow a_n \rightarrow a)$

Merge pentru funcții cu oricâte argumente

Date fiind

- $f :: a_1 \rightarrow a_2 \rightarrow a_3 \rightarrow \dots \rightarrow a_n \rightarrow a$
- $ca_1 :: m\ a_1, ca_2 :: m\ a_2, \dots, ca_n :: m\ a_n,$
- $fmap :: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$
- $(<*>) :: m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$ cu „proprietăți bune”

Atunci

$fmap\ f :: m\ a_1 \rightarrow m\ (a_2 \rightarrow a_3 \rightarrow \dots \rightarrow a_n \rightarrow a)$

$fmap\ f\ ca_1 :: m\ (a_2 \rightarrow a_3 \rightarrow \dots \rightarrow a_n \rightarrow a)$

$fmap\ f\ ca_1\ <*>\ ca_2 :: m\ (a_3 \rightarrow \dots \rightarrow a_n \rightarrow a)$

Merge pentru funcții cu oricâte argumente

Date fiind

- $f :: a1 \rightarrow a2 \rightarrow a3 \rightarrow \dots \rightarrow a_n \rightarrow a$
- $ca1 :: m\ a1, ca2 :: m\ a2, \dots, can :: m\ a_n,$
- $fmap :: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$
- $(<*>) :: m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$ cu „proprietăți bune”

Atunci

$fmap\ f :: m\ a1 \rightarrow m\ (a2 \rightarrow a3 \rightarrow \dots \rightarrow a_n \rightarrow a)$

$fmap\ f\ ca1 :: m\ (a2 \rightarrow a3 \rightarrow \dots \rightarrow a_n \rightarrow a)$

$fmap\ f\ ca1\ <*>\ ca2 :: m\ (a3 \rightarrow \dots \rightarrow a_n \rightarrow a)$

...

$fmap\ f\ ca1\ <*>\ ca2\ <*>\ ca3\ \dots\ <*>\ can :: m\ a$

Clasa de tipuri Applicative

```
class Functor m => Applicative m where  
  pure  :: a -> m a  
  (<*>) :: m (a -> b) -> m a -> m b
```


Clasa de tipuri Applicative

```
class Functor m => Applicative m where  
  pure  :: a -> m a  
  (<*>) :: m (a -> b) -> m a -> m b
```

- Orice instanță a lui **Applicative** trebuie să fie instanță a lui **Functor**
- `pure` transformă o valoare într-o computație minimală care are acea valoare ca rezultat, și nimic mai mult!
- `(<*>)` ia o computație care produce funcții și o computație care produce argumente pentru funcții și obține o computație care produce rezultatele aplicării funcțiilor asupra argumentelor

Clasa de tipuri Applicative

```
class Functor m where
```

```
  fmap :: (a -> b) -> m a -> m b
```

```
class Functor m => Applicative m where
```

```
  pure  :: a -> m a
```

```
  (<*>) :: m (a -> b) -> m a -> m b
```

Proprietate importantă

- $\text{fmap } f \ x == \text{pure } f \ \text{<*>} \ x$
- Se definește operatorul ($\text{<}\$>$) prin $\text{(<}\$>) = \text{fmap}$

Instante – Maybe

```
class Functor m where
```

```
  fmap :: (a -> b) -> m a -> m b
```

```
class Functor m => Applicative m where
```

```
  pure :: a -> m a
```

```
  (<*>) :: m (a -> b) -> m a -> m b
```

```
instance Applicative Maybe where
```

```
  pure = Just
```

```
  Nothing <*> _ = Nothing
```

```
  Just f <*> x = fmap f x
```

Instante – Maybe

```
Prelude> pure "Hey" :: Maybe String  
Just "Hey"
```

Instante – Maybe

```
Prelude> pure "Hey" :: Maybe String  
Just "Hey"
```

```
Prelude> (++) <$> (Just "Hey ") <*> (Just "You!")  
Just "Hey You!"
```

Instante – Maybe

```
Prelude> pure "Hey" :: Maybe String  
Just "Hey"
```

```
Prelude> (++) <$> (Just "Hey ") <*> (Just "You!")  
Just "Hey You!"
```

- **(++) :: String -> (String -> String)**

Instante – Maybe

```
Prelude> pure "Hey" :: Maybe String  
Just "Hey"
```

```
Prelude> (++) <$> (Just "Hey ") <*> (Just "You!")  
Just "Hey You!"
```

- **(++) :: String -> (String -> String)**
- **Just "Hey_" :: Maybe String**
- **(<\$>) :: (a -> b) -> m a -> m b** (este fmap)

Instante – Maybe

```
Prelude> pure "Hey" :: Maybe String  
Just "Hey"
```

```
Prelude> (++) <$> (Just "Hey ") <*> (Just "You!")  
Just "Hey You!"
```

- **(++) :: String -> (String -> String)**
- **Just "Hey_" :: Maybe String**
- **(<\$>) :: (a -> b) -> m a -> m b** (este fmap)
- **(++) <\$> (Just "Hey_") :: Maybe (String -> String)**

Instante – Maybe

```
Prelude> pure "Hey" :: Maybe String  
Just "Hey"
```

```
Prelude> (++) <$> (Just "Hey ") <*> (Just "You!")  
Just "Hey You!"
```

- **(++) :: String -> (String -> String)**
- **Just "Hey_" :: Maybe String**
- **(<\$>) :: (a -> b) -> m a -> m b** (este fmap)
- **(++) <\$> (Just "Hey_") :: Maybe (String -> String)**
- **Just "You!" :: Maybe String**
- **(<*>) :: m (b -> c) -> m b -> m c**

Instante – Maybe

```
Prelude> pure "Hey" :: Maybe String  
Just "Hey"
```

```
Prelude> (++) <$> (Just "Hey ") <*> (Just "You!")  
Just "Hey You!"
```

- **(++) :: String -> (String -> String)**
- **Just "Hey_" :: Maybe String**
- **(<\$>) :: (a -> b) -> m a -> m b** (este fmap)
- **(++) <\$> (Just "Hey_") :: Maybe (String -> String)**
- **Just "You!" :: Maybe String**
- **(<*>) :: m (b -> c) -> m b -> m c**
- **Just "Hey_You!" :: Maybe String**

Instante – Maybe

```
mDiv x y = if y == 0 then Nothing  
           else Just (x 'div' y)  
mF x = (+) <$> pure 4 <*> mDiv 10 x
```

Instante – Maybe

```
mDiv x y = if y == 0 then Nothing  
           else Just (x 'div' y)
```

```
mF x = (+) <$> pure 4 <*> mDiv 10 x
```

- `(+)` :: `Int -> Int -> Int`

Instante – Maybe

```
mDiv x y = if y == 0 then Nothing  
           else Just (x 'div' y)
```

```
mF x = (+) <$> pure 4 <*> mDiv 10 x
```

- $(+)$:: **Int** \rightarrow **Int** \rightarrow **Int**
- `pure 4` :: **Maybe Int**
- $(\<\$>)$:: $(a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este `fmap`)

Instante – Maybe

```
mDiv x y = if y == 0 then Nothing  
           else Just (x 'div' y)
```

```
mF x = (+) <$> pure 4 <*> mDiv 10 x
```

- $(+)$:: **Int** \rightarrow **Int** \rightarrow **Int**
- `pure 4` :: **Maybe Int**
- $(\<\$>)$:: $(a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este `fmap`)
- $(+) \<\$> \text{pure } 4$:: **Maybe (Int \rightarrow Int)**

Instante – Maybe

```
mDiv x y = if y == 0 then Nothing  
           else Just (x 'div' y)
```

```
mF x = (+) <$> pure 4 <*> mDiv 10 x
```

- $(+)$:: **Int** \rightarrow **Int** \rightarrow **Int**
- `pure 4` :: **Maybe Int**
- $(\<\$>)$:: $(a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este fmap)
- $(+) \<\$> \text{pure } 4$:: **Maybe (Int \rightarrow Int)**
- `mDiv` :: **Int** \rightarrow **Int** \rightarrow **Maybe Int**
- `mDiv 10 x` :: **Maybe Int**
- $(\<*>)$:: $m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$

Instante – Maybe

```
mDiv x y = if y == 0 then Nothing
           else Just (x 'div' y)
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```
mF x = (+) <$> pure 4 <*> mDiv 10 x
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- $(+)$:: **Int** \rightarrow **Int** \rightarrow **Int**
- `pure 4` :: **Maybe Int**
- $(\<\$>)$:: $(a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este `fmap`)
- $(+) \<\$> \text{pure } 4$:: **Maybe (Int \rightarrow Int)**
- `mDiv` :: **Int** \rightarrow **Int** \rightarrow **Maybe Int**
- `mDiv 10 x` :: **Maybe Int**
- $(\<*>)$:: $m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$

```
Prelude> mF 2
```

```
Just 9
```


Instante – Maybe

```
mDiv x y = if y == 0 then Nothing  
          else Just (x 'div' y)
```

```
mF x = (+) <$> pure 4 <*> mDiv 10 x
```

- $(+)$:: **Int** \rightarrow **Int** \rightarrow **Int**
- `pure 4` :: **Maybe Int**
- $(\<\$>)$:: $(a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este `fmap`)
- $(+) \<\$> \text{pure } 4$:: **Maybe (Int \rightarrow Int)**
- `mDiv` :: **Int** \rightarrow **Int** \rightarrow **Maybe Int**
- `mDiv 10 x` :: **Maybe Int**
- $(\<*>)$:: $m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$

```
Prelude> mF 2
```

```
Just 9
```

```
Prelude> let f x = 4 + 10 'div' x
```

```
Prelude> fmap f (Just 2)
```

```
Just 9
```

```
class Functor m where
```

```
  fmap :: (a -> b) -> m a -> m b
```

```
class Functor m => Applicative m where
```

```
  pure :: a -> m a
```

```
  (<*>) :: m (a -> b) -> m a -> m b
```

```
instance Applicative (Either a) where
```

```
  pure = Right
```

```
  Left e   <*> _ = Left e
```

```
  Right f <*> x = fmap f x
```

Instante – Either

```
Prelude> pure "Hey" :: Either a String  
Right "Hey"
```

Instante – Either

```
Prelude> pure "Hey" :: Either a String  
Right "Hey"
```

```
Prelude> (++) <$> (Right "Hey ") <*> (Right "You!")  
Right "Hey You!"
```

Instante – Either

```
Prelude> pure "Hey" :: Either a String  
Right "Hey"
```

```
Prelude> (++) <$> (Right "Hey ") <*> (Right "You!")  
Right "Hey You!"
```

- **(++) :: String -> (String -> String)**

Instante – Either

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Prelude> pure "Hey" :: Either a String  
Right "Hey"
```

```
Prelude> (++) <$> (Right "Hey ") <*> (Right "You!")  
Right "Hey You!"
```

- **(++) :: String -> (String -> String)**
- **Right "Hey_" :: Either a String**
- **(<\$>) :: (a -> b) -> m a -> m b** (este fmap)

Instante – Either

```
Prelude> pure "Hey" :: Either a String  
Right "Hey"
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Prelude> (++) <$> (Right "Hey ") <*> (Right "You!")  
Right "Hey You!"
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- **(++) :: String -> (String -> String)**
- **Right "Hey_" :: Either a String**
- **(<\$>) :: (a -> b) -> m a -> m b** (este fmap)
- **(++) <\$> (Right "Hey_") :: Either a (String -> String)**

Instante – Either

```
Prelude> pure "Hey" :: Either a String  
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Prelude> (++) <$> (Right "Hey ") <*> (Right "You!")  
Right "Hey You!"
```

- **(++) :: String -> (String -> String)**
- **Right "Hey_" :: Either a String**
- **(<\$>) :: (a -> b) -> m a -> m b** (este fmap)
- **(++) <\$> (Right "Hey_") :: Either a (String -> String)**
- **Right "You!" :: Either a String**
- **(<*>) :: m (b -> c) -> m b -> m c**

Instante – Either

```
Prelude> pure "Hey" :: Either a String  
Right "Hey"
```

```
Prelude> (++) <$> (Right "Hey ") <*> (Right "You!")  
Right "Hey You!"
```

- **(++) :: String -> (String -> String)**
- **Right "Hey_" :: Either a String**
- **(<\$>) :: (a -> b) -> m a -> m b** (este fmap)
- **(++) <\$> (Right "Hey_") :: Either a (String -> String)**
- **Right "You!" :: Either a String**
- **(<*>) :: m (b -> c) -> m b -> m c**
- **Right "Hey_You!" :: Either a String**

Instante – Either

```
eDiv x y = if y == 0 then Left "Division by 0!"  
           else Right (x 'div' y)  
eF x = (+) <$> pure 4 <*> eDiv 10 x
```

Instante – Either

```
eDiv x y = if y == 0 then Left "Division by 0!"
```

```
         else Right (x 'div' y)
```

```
eF x = (+) <$> pure 4 <*> eDiv 10 x
```

- $(+) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$

Instante – Either

```
eDiv x y = if y == 0 then Left "Division by 0!"  
           else Right (x 'div' y)
```

```
eF x = (+) <$> pure 4 <*> eDiv 10 x
```

- `(+)` :: **Int** -> **Int** -> **Int**
- `pure 4` :: **Either String Int**
- `(<$>)` :: `(a -> b) -> m a -> m b` (este `fmap`)

Instante – Either

```
eDiv x y = if y == 0 then Left "Division by 0!"  
           else Right (x 'div' y)  
eF x = (+) <$> pure 4 <*> eDiv 10 x
```

- `(+)` :: **Int** -> **Int** -> **Int**
- `pure 4` :: **Either String Int**
- `(<$>)` :: `(a -> b) -> m a -> m b` (este `fmap`)
- `(+) <$> pure 4` :: **Either String (Int -> Int)**

Instante – Either

```
eDiv x y = if y == 0 then Left "Division by 0!"  
           else Right (x 'div' y)  
eF x = (+) <$> pure 4 <*> eDiv 10 x
```

- $(+)$:: **Int** \rightarrow **Int** \rightarrow **Int**
- $\text{pure } 4$:: **Either String Int**
- $(\text{<}\$ \text{>})$:: $(a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este fmap)
- $(+) \text{ <}\$ \text{>} \text{ pure } 4$:: **Either String (Int \rightarrow Int)**
- eDiv :: **Int** \rightarrow **Int** \rightarrow **Either String Int**
- $\text{eDiv } 10\ x$:: **Either String Int**
- $(\text{<} * \text{>})$:: $m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$

Instante – Either

```
eDiv x y = if y == 0 then Left "Division by 0!"  
           else Right (x 'div' y)  
eF x = (+) <$> pure 4 <*> eDiv 10 x
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- $(+)$:: **Int** \rightarrow **Int** \rightarrow **Int**
- `pure 4` :: **Either String Int**
- $(\<\$>)$:: $(a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este `fmap`)
- $(+) \<\$> \text{pure } 4$:: **Either String (Int \rightarrow Int)**
- `eDiv` :: **Int** \rightarrow **Int** \rightarrow **Either String Int**
- `eDiv 10 x` :: **Either String Int**
- $(\<*>)$:: $m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$

```
Prelude> eF 2
```

```
Right 9
```

Instante – Either

```
eDiv x y = if y == 0 then Left "Division by 0!"  
           else Right (x 'div' y)  
eF x = (+) <$> pure 4 <*> eDiv 10 x
```

- $(+)$:: **Int** \rightarrow **Int** \rightarrow **Int**
- `pure 4` :: **Either String Int**
- $(\<\$>)$:: $(a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este `fmap`)
- $(+) \<\$> \text{pure } 4$:: **Either String (Int \rightarrow Int)**
- `eDiv` :: **Int** \rightarrow **Int** \rightarrow **Either String Int**
- `eDiv 10 x` :: **Either String Int**
- $(\<*>)$:: $m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$

```
Prelude> eF 2
```

```
Right 9
```

```
Prelude> let f x = 4 + 10 'div' x
```

```
Prelude> fmap f (Right 2)
```

```
Right 9
```



```
class Functor m where
```

```
  fmap :: (a -> b) -> m a -> m b
```

```
class Functor m => Applicative m where
```

```
  pure :: a -> m a
```

```
  (<*>) :: m (a -> b) -> m a -> m b
```

```
instance Applicative [] where
```

```
  pure x = [x]
```

```
  fs  <*> xs = [f x | f <- fs, x <- xs]
```

```
Prelude> pure "Hey" :: [String]  
[ "Hey" ]
```

```
Prelude> pure "Hey" :: [String]  
["Hey"]
```

```
Prelude> (++) <$> ["Hello ", "Goodbye "] <*> ["world"  
    , "happiness"]  
["Hello world", "Hello happiness", "Goodbye world", "  
    Goodbye happiness"]
```

Instante – Liste

```
Prelude> pure "Hey" :: [String]  
["Hey"]
```

```
Prelude> (++) <$> ["Hello ", "Goodbye "] <*> ["world"  
    , "happiness"]  
["Hello world", "Hello happiness", "Goodbye world", "  
    Goodbye happiness"]
```

- **(++) :: String -> (String -> String)**

Instante – Liste

```
Prelude> pure "Hey" :: [String]  
["Hey"]
```

```
Prelude> (++) <$> ["Hello ", "Goodbye "] <*> ["world"  
    , "happiness"]  
["Hello world", "Hello happiness", "Goodbye world", "  
    Goodbye happiness"]
```

- $(++) :: \text{String} \rightarrow (\text{String} \rightarrow \text{String})$
- $["Hello_", "Goodbye_"] :: [\text{String}]$
- $(\<\$>) :: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este fmap)

Instante – Liste

```
Prelude> pure "Hey" :: [String]  
["Hey"]
```

```
Prelude> (++) <$> ["Hello ", "Goodbye "] <*> ["world"  
    , "happiness"]  
["Hello world", "Hello happiness", "Goodbye world", "  
    Goodbye happiness"]
```

- $(++) :: \text{String} \rightarrow (\text{String} \rightarrow \text{String})$
- $["\text{Hello}_\perp", "\text{Goodbye}_\perp"] :: [\text{String}]$
- $(\<\$>) :: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este fmap)
- $(++) \<\$> ["\text{Hello}_\perp", "\text{Goodbye}_\perp"] :: [\text{String} \rightarrow \text{String}]$

Instante – Liste

```
Prelude> pure "Hey" :: [String]  
["Hey"]
```

```
Prelude> (++) <$> ["Hello ", "Goodbye "] <*> ["world"  
    , "happiness"]  
["Hello world", "Hello happiness", "Goodbye world", "  
    Goodbye happiness"]
```

- $(++) :: \text{String} \rightarrow (\text{String} \rightarrow \text{String})$
- $["\text{Hello}_\perp", "\text{Goodbye}_\perp"] :: [\text{String}]$
- $(\<\$>) :: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este fmap)
- $(++) \<\$> ["\text{Hello}_\perp", "\text{Goodbye}_\perp"] :: [\text{String} \rightarrow \text{String}]$
- $["\text{world}", "\text{happiness}] :: [\text{String}]$
- $(\<*>) :: m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$

```
Prelude> [(+) , (*)] <*> [1,2] <*> [3,4]  
[4,5,5,6,3,4,6,8]
```



```
Prelude> [(+) , (*)] <*> [1,2] <*> [3,4]  
[4,5,5,6,3,4,6,8]
```

- $(+), (*) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$
- $[(+), (*)] :: [\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}]$

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- $(+), (*) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$
- $[(+), (*)] :: [\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}]$
- $[1,2] :: [\text{Int}]$
- $(<*>) :: m (b \rightarrow c) \rightarrow m b \rightarrow m c$
- $[(+), (*)] <*> [1,2] :: [\text{Int} \rightarrow \text{Int}]$

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- $[(+), (*)] <*> [1,2] <*> [3,4] :: [\text{Int}]$

```
Prelude> filter (>50) $ (*) <$> [2,5,10] <*> [8,10,11]  
[55,80,100,110]
```

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Prelude> filter (>50) $ (*) <$> [2,5,10] <*> [8,10,11]  
[55,80,100,110]
```

- $(*) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$
- $[2,5,10] :: [\text{Int}]$
- $(< \$ >) :: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este fmap)

```
Prelude> filter (>50) $ (*) <$> [2,5,10] <*> [8,10,11]  
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- $(*) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$
- $[2,5,10] :: [\text{Int}]$
- $(<\$>) :: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b$ (este fmap)
- $(*) <\$> [2,5,10] :: [\text{Int} \rightarrow \text{Int}]$
- $(<*>) :: m\ (b \rightarrow c) \rightarrow m\ b \rightarrow m\ c$
- $(*) <\$> [2,5,10] <*> [8,10,11] :: [\text{Int}]$

```
Prelude> (*) <$> [2,5,10] <*> [8,10,11]  
[16,20,22,40,50,55,80,100,110]
```


Proprietăți ale functorilor aplicativi

```
class Functor m where
```

```
  fmap :: (a -> b) -> m a -> m b
```

```
class Functor m => Applicative m where
```

```
  pure :: a -> m a
```

```
  (<*>) :: m (a -> b) -> m a -> m b
```

- **identitate** `pure id <*> v = v`
- **compoziție** `pure (.) <*> u <*> v <*> w = u <*> (v <*> w)`
- **homomorfism** `pure f <*> pure x = pure (f x)`

Consecință: `fmap f x == f <$> x == pure f <*> x`

$$\begin{aligned}(\$) &:: (a \rightarrow b) \rightarrow a \rightarrow b \\(<\$>) &:: (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b \\(<*>) &:: m\ (a \rightarrow b) \rightarrow m\ a \rightarrow m\ b\end{aligned}$$

Quiz time!

Seria 23: <https://www.questionpro.com/t/AT4qgZqjts>

Seria 24: <https://www.questionpro.com/t/AT4NiZqjWQ>

Seria 25: <https://www.questionpro.com/t/AT4qgZqjt3>

Pe săptămâna viitoare!