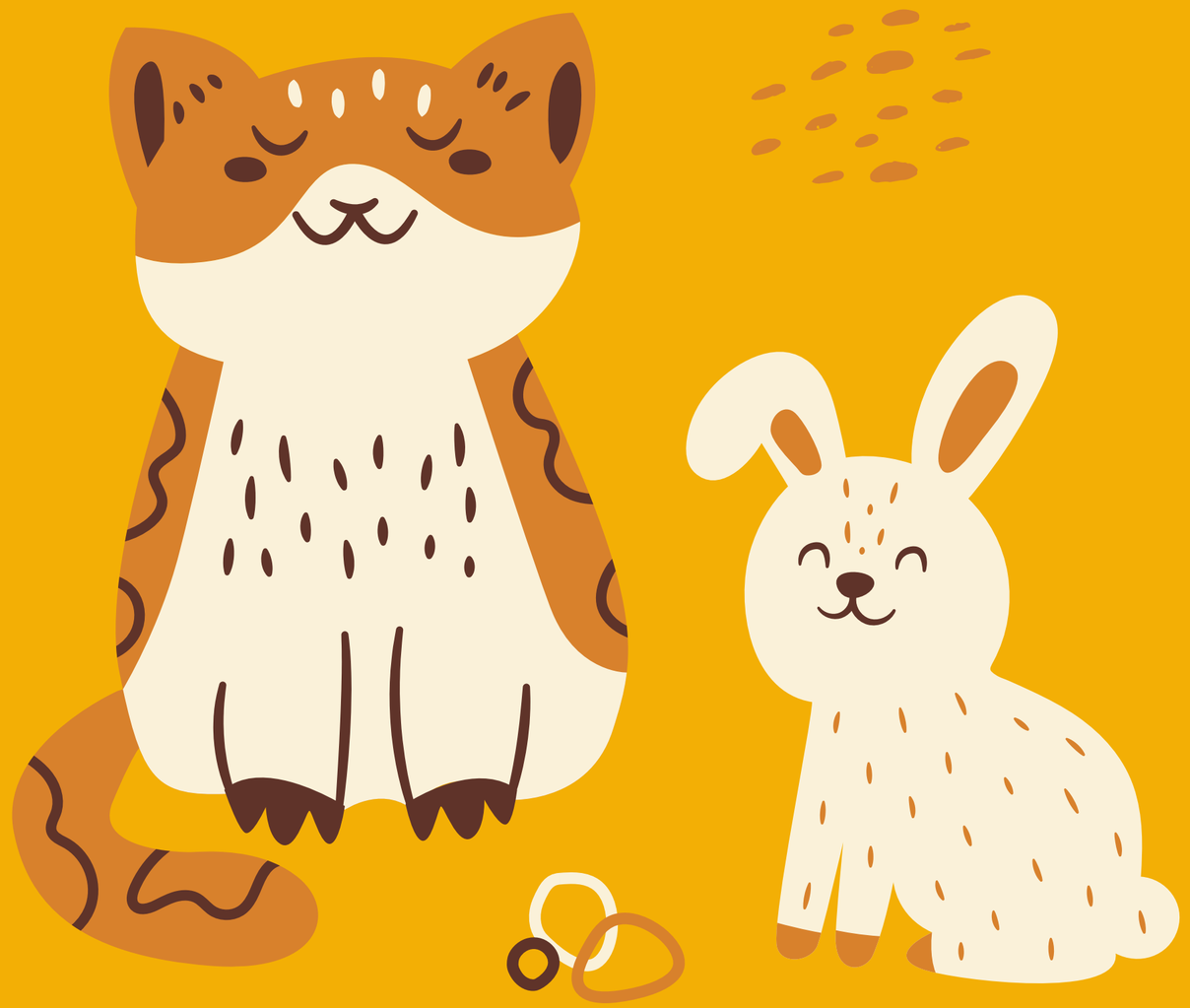
Lean 3 and mathlib

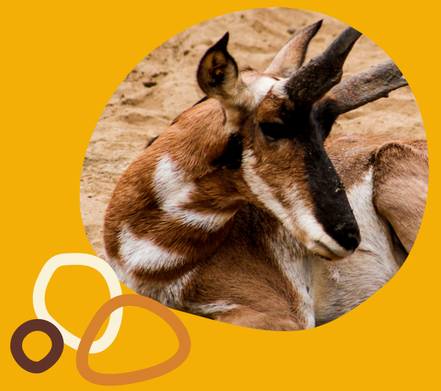
Markus Himmel



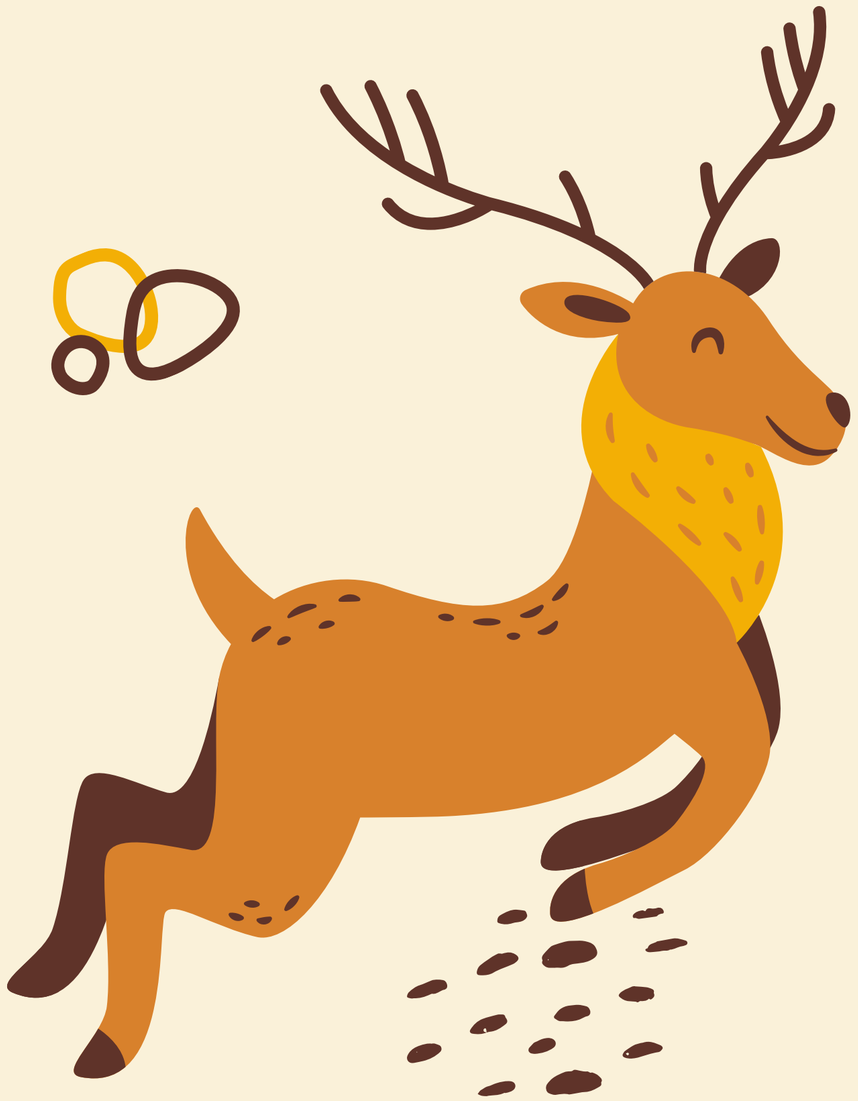
Lean 3 and Lean 4

Lean 3 and Lean 4 have similar underlying logic and syntax

Lean 4 has some very nice advantages over Lean 3, but it is only just becoming production-ready



Goal: port Lean 3 library and infrastructure to Lean 4



Interactive theorem proving is fun, but often also annoying

How can we reduce the friction of formalisation?

Better proofs

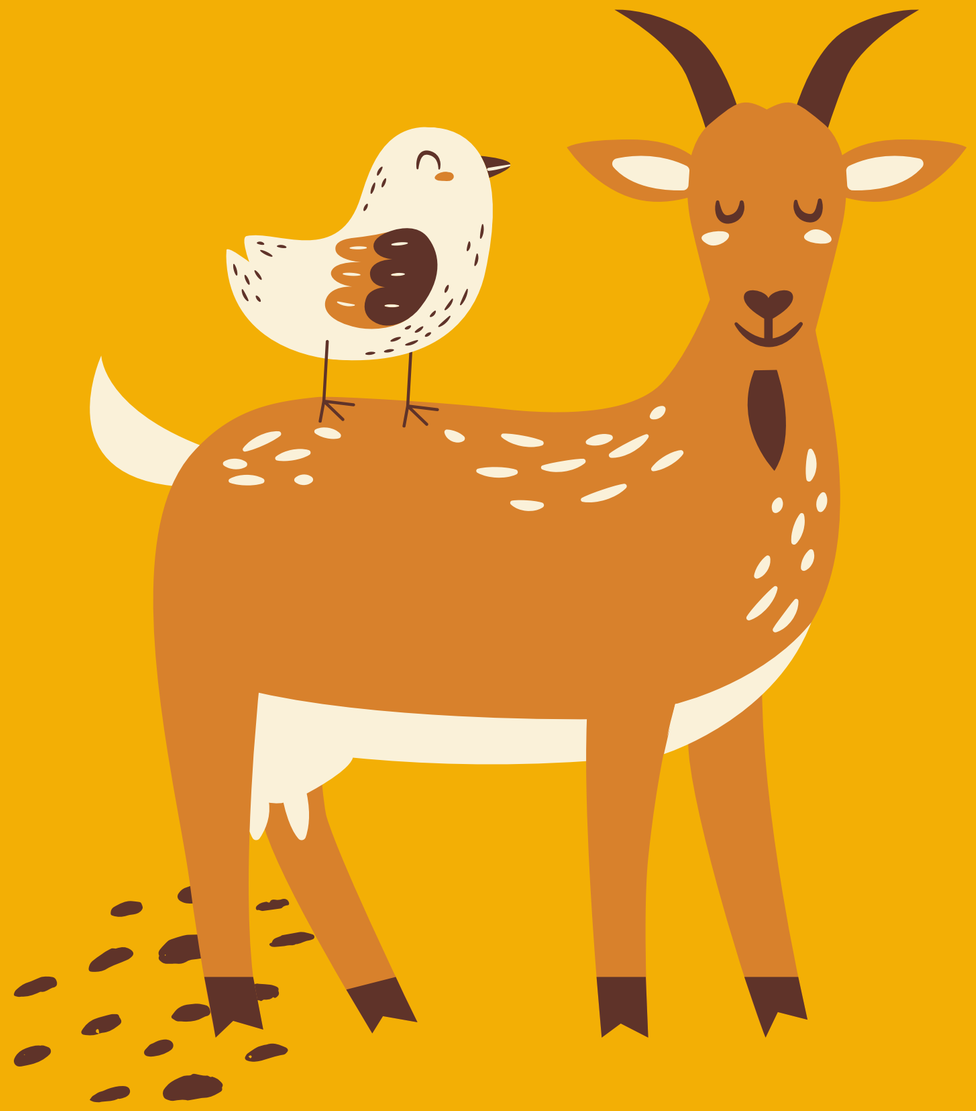
Can we find alternative approaches to theorems that are easier to formalize?

Automation

Can we make the system smart enough to solve all "maths-trivial" goals for us?

Libraries

Can we build a standard library that is so complete that we can jump right into our proof without worrying about fundamentals?



Let's see some of the automation in Lean 3

mathlib

Unlike most theorem provers, Lean has a large monolithic mathematics library

Algebra

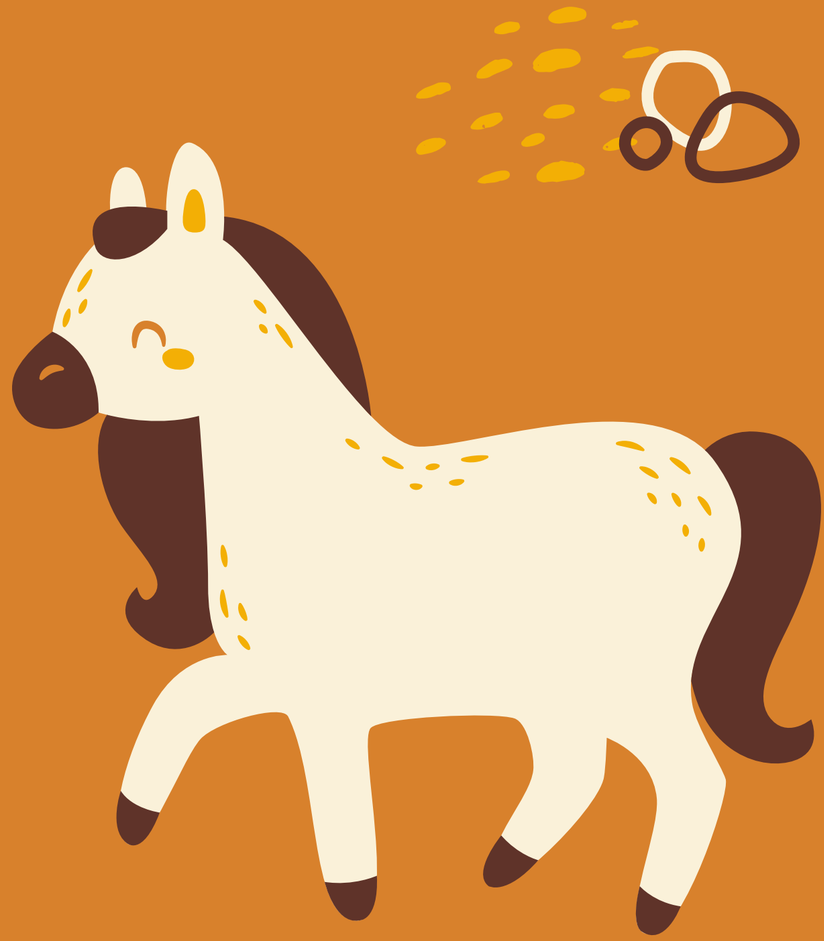
- Algebra hierarchy
- Linear algebra
- Groups, rings, modules

mathlib

Unlike most theorem provers, Lean has a large monolithic mathematics library

Analysis

- Real/complex numbers, quaternions
- Measure theory, integration theory
- Lebesgue measure, Bochner integral
- Calculus
- Calculation of integrals
- **Missing:** Differential equations, complex analysis



mathlib

Unlike most theorem provers, Lean has a large monolithic mathematics library

Category theory

- Limits, adjunctions, monads
- Monadicity theorems
- Adjoint functor theorems
- Enriched, triangulated, abelian, monoidal categories

mathlib

Unlike most theorem provers, Lean has a large monolithic mathematics library

Combinatorics

- Pigeonhole principles
- Hall's marriage theorem
- Ramsey theory
- Derangement formula
- Partition theorem
- Game theory

mathlib

Unlike most theorem provers, Lean has a large monolithic mathematics library

Computer Science

- DFAs, NFAs, regular expressions
- Turing machines, computability
- Halting problem
- Primitive/partial recursive functions

mathlib

Unlike most theorem provers, Lean has a large monolithic mathematics library

Geometry

- Analytic geometry
- Manifolds
- Schemes

mathlib

Unlike most theorem provers, Lean has a large monolithic mathematics library

Number theory

- Quadratic reciprocity
- Elementary number theory
- Lucas-Lehmer primality test
- p-adic numbers
- e is transcendental
- **Missing:** algebraic number theory

mathlib

Unlike most theorem provers, Lean has a large monolithic mathematics library

Numerical mathematics

- **Missing:** everything

mathlib

Unlike most theorem provers, Lean has a large monolithic mathematics library

Probability theory

- **Missing:** almost everything

mathlib

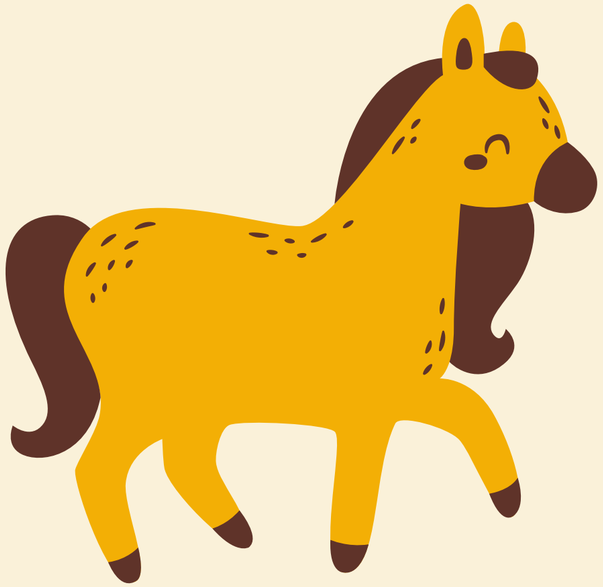
Unlike most theorem provers, Lean has a large monolithic mathematics library

Topology

- Stone-Čech compactification
- Urysohn's lemma
- Tychonoff's theorem
- Compact-open topology
- **Missing:** algebraic topology

If you liked the course and want to do more Lean

For now, Lean 3 is the way to go



Get involved!

1. Pick a topic/theorem you like and start formalising
2. Ask on Zulip if you get stuck or need inspiration
3. You will pick up Lean 3 syntax as you go

https://leanprover-community.github.io/

4. One URL for everything Lean 3
5. Contains links to installation instructions, learning resources and documentation
6. Any questions? Just message me or make a thread on Zulip