Theorembeweiserpraktikum

Aesop: A General Proof Search Tactic

Jakob von Raumer, Sebastian Ullrich | SS 2022

Recursive Simplification Revisited

We learned that `simp` can be quite powerful

```
example (h1 : y = 0 → x = 0) (h2 : p → 0 = y) (h3 : p) : x = 0 := by simp [h1, h2, h3]
```

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Limitations:

- depth-first search with very low default max depth (2)
- supports simple *backward reasoning* only

```
example (h1 : ∀ y, p y → y = 0) (h2 : p x) : 2 * x = 0 := sorry  -- by simp [*]
example : ∃ x, x = 0 := sorry  -- by simp [Exists.intro]
example : match n with | 0 => True | n + 1 => True := sorry  -- by simp
```

- usability: must inspect trace to found out what went wrong!

Aesop

Aesop (https://github.com/JLimperg/aesop) is "a work-in-progress proof search tactic for Lean 4"

```
import Aesop
import TBA.AesopExts  -- temporary

example (h1 : ∀ x, p x → x = 0) (h2 : p x) : 2 * x = 0 := by aesop (add safe forward h1)
example : ∃ x, x = 0 := by aesop
example : match n with | 0 => True | n + 1 => True := by aesop
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- *best-first* search explores different branches in turn

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- *best-first* search explores different branches in turn
- applies *safe* rules exhaustively followed by *unsafe* rules, shows state after safe rules on error

```
example (h : p ∧ q) : r ∨ s := by aesop
-- After applying safe rules, Aesop tried to solve these goals:
-- (unprovable)
-- p q r s : Prop
-- : p
-- : q
-- ⊢ r ∨ s
```

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- *best-first* search explores different branches in turn
- applies *safe* rules exhaustively followed by *unsafe* rules, shows state after safe rules on error
  Builtin safe/unsafe examples:

```
safe Eq.refl
unsafe 30% constructors Exists

[safe cases, unsafe 50% constructors] Or
```

Adding Rules

Add globally:

```
@[aesop safe constructors] inductive Foo ...
attribute [aesop safe cases] Foo  -- post hoc
```

or locally:

```
by aesop (add safe constructors Foo, ...)
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More rule examples; see https://github.com/JLimperg/aesop#rule-builders for everything

```
(cases (patterns := [⟨.skip, _⟩ ⇓ _ : _])) Bigstep  -- apply rule inversion on any `skip` hypothesis
apply f
elim g    -- like `forward`, but remove used hypotheses
simp h
unfold f
```

Custom Tactic Rules from Our Repo

- enabled by default as safe: `substVars` , `split` , `splitAt`
- disabled by default: `simpAll` (*probably* safe), `elimAny` (*might* be safe)

Demo