Causal frameworks and framing causal problems

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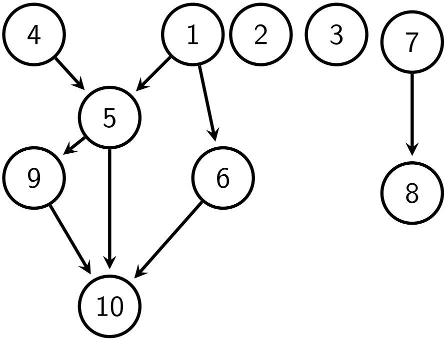
IPL

04 May 2023

Summary of last session

- statistical models
- BN/DAG, d-separation and conditional independences



Causal models desiderata

- Represent data, similar to a statistical model
- Model what happen when changes/experiment/interventions
- Reason on and explore the causal relationships
- Represent causal relationships

Causal regression models

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- in the sense that we imagine the physical/true generating process to be modeled by this equation
- If we make *experiments* by changing the value of X we know that the associated value for Y is generated accordingly to $f(X, \varepsilon)$

Structural Causal Models

Definition [Peters et al., 2017]

A SCM over variables $X_1, \ldots, X_p$ with noise variables $\varepsilon_1, \ldots, \varepsilon_p$ is a collection of **structural assignments**:

$$X_i = f_i(X_{pa(i)}, \varepsilon_i)$$

where ε_i are assumed jointly independent and f_i are fixed deterministic functions.

- $X_{pa(i)}$ are called the parents or the **direct causes** of X_i
- we say X_i is a direct effect of its direct causes
- we assume the associated graph G to be a DAG

Bareinboim et al. [2022]

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- (G, P) is a Bayesian network

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- sometimes you can find $P(\cdot|do())$
- we say there is a **total causal effect** from X to Y in a SCM if and only if there exist a random variable $\tilde{N}$ such that

$$X \not\perp\!\!\!\perp Y \text{ in } P^{do(X=\tilde{N})}$$

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- The set of observational and interventional distributions can also be represented with a Causal BN

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- The set of observational and interventional distributions can also be represented with a Causal BN
- but SCMs allow an additional causal reasoning step: **counterfactuals**

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- I have headache, I take aspirin and one hour later the pain disappear. Would the pain had disappeared if I had not taken aspirin?
- this is different from interventions since we are not just doing an experiment, but rather imagining interventions for alternative *worlds*, that is alternative ways the system could have evolved but conditioning on everything else being the same

Counterfactuals in SCM

- SCMs allow the computation of counterfactual probabilities
- Counterfactual statements can be seen as intervention/do-statements in a counterfactual SCM which is obtained from an initial SCM by replacing the noise distribution with $P(\varepsilon|X = x)$

Peters et al. [2017], Bareinboim et al. [2022]

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- SCMs allow the computation of counterfactual probabilities
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- SCM can induce same observational and interventional distribution (Causal BN) but different counterfactuals

Peters et al. [2017], Bareinboim et al. [2022]

Example from Peters et al. [2017]

Example 6.19 Let $N_1, N_2 \sim \mathrm{Ber}(0.5)$, and $N_3 \sim \mathrm{U}(\{0,1,2\})$, such that the three variables are jointly independent. That is, N_1, N_2 have a Bernoulli distribution with parameter 0.5 and N_3 is uniformly distributed on $\{0,1,2\}$. We define two different SCMs. First consider $\mathfrak{C}_A$:

$$\begin{aligned}
X_1 &:= N_1 \\
X_2 &:= N_2 \\
X_3 &:= (1_{N_3>0} \cdot X_1 + 1_{N_3=0} \cdot X_2) \cdot 1_{X_1 \neq X_2} + N_3 \cdot 1_{X_1 = X_2}.
\end{aligned}$$

If X_1 and X_2 have different values, depending on N_3 we either choose $X_3 = X_1$ or $X_3 = X_2$. Otherwise $X_3 = N_3$. Now, $\mathfrak{C}_B$ differs from $\mathfrak{C}_A$ only in the latter case:

$$\begin{aligned}
X_1 &:= N_1 \\
X_2 &:= N_2 \\
X_3 &:= (1_{N_3>0} \cdot X_1 + 1_{N_3=0} \cdot X_2) \cdot 1_{X_1 \neq X_2} + (2 - N_3) \cdot 1_{X_1 = X_2}.
\end{aligned}$$

Both SCMs entail the same observational distribution; and for any possible intervention they entail the same intervention distributions, too.[9] But the two models differ in a counterfactual statement. Suppose, we have made an observation $(X_1, X_2, X_3) = (1,0,0)$ and we are interested in the counterfactual question "what would X_3 have been if X_1 had been 0?" From both SCMs, it follows that $N_3 = 0$, and thus the two SCMs $\mathfrak{C}_A$ and $\mathfrak{C}_B$ "predict" different values for X_3 under a counterfactual change of X_1 (namely 0 and 2, respectively). $\square$

Pearl Causal Hierarchy (PCH)

	Layer (Symbolic)	Typical Activity	Typical Question	Example	Machine Learning
$\mathcal{L}_1$	Associational $P(y\|x)$	Seeing	What is? How would seeing X change my belief in Y?	What does a symptom tell us about the disease?	Supervised / Unsupervised Learning
$\mathcal{L}_2$	Interventional $P(y\|do(x),c)$	Doing	What if? What if I do X?	What if I take aspirin, will my headache be cured?	Reinforcement Learning
$\mathcal{L}_3$	Counterfactual $P(y_x\|x',y')$	Imagining	Why? What if I had acted differently?	Was it the aspirin that stopped my headache?	

Table 1.1: Pearl's Causal Hierarchy.

Bareinboim et al. [2022]

Potential outcomes - Counterfactual model

- consider a binary **treatment variable** A (1: treated, 0: untreated)
- and a binary **outcome** Y (1: death, 0: survival)
- A, Y are random variables that take possible different values for each individual

Hernán and Robins [2020], Wasserman [2004]

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- denote with $Y^{a=1}$ (Y under treatment $a = 1$) the outcome variable that would have been observed under treatment $a = 1$, and similarly $Y^{a=0}$
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- for each individual, only one of the potential outcomes if actually observed/factual.

$$Y = Y^{a=A} \quad \text{(consistency equation)}$$

Hernán and Robins [2020], Wasserman [2004]

Causal effects

Definition

Individual causal effects The treatment A has a causal effect on an individual's outcome Y if $Y^{a=1} \neq Y^{a=0}$

Definition

Average causal effects An average causal effect of treatment A on outcome Y is present if

$$P(Y^{a=1} = 1) \neq P(Y^{a=0} = 1)$$

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- Dynamic Causal Models (DCM) [Friston et al., 2003]
- Local Independence Graphs [Didelez, 2006, Mogensen and Hansen, 2020]

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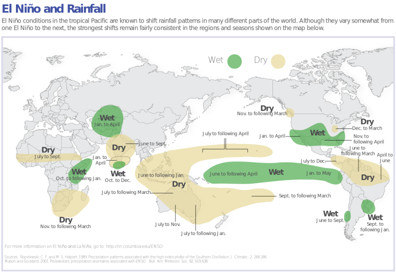
- Exploiting causal information in ML/prediction/forecasting/RL

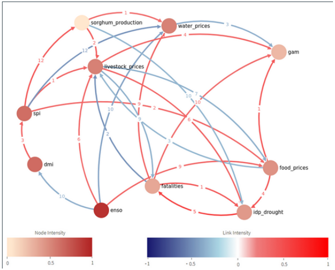
Causal Discovery

- Which are the regions most affected by ENSO?



Causal Discovery

- Which are the regions most affected by ENSO?
- Which are the causal drivers and the causal relationships involving food insecurity?



Causal Discovery

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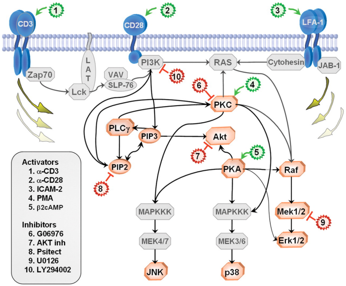
- Which are the causal drivers and the causal relationships involving food insecurity?
- Retrieve the reaction network between genes and proteins from single-cell data



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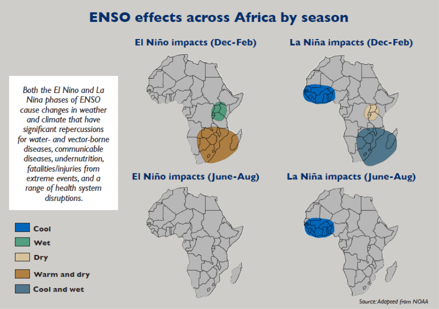
- If only two variables are considered: bivariate causal discovery
- it always require assumptions
- can use only observational data, or even interventions
- usually the output is not a complete description of the causal relationships (depends on the assumptions, the type of data etc..)

Causal Inference

- Estimate the strength of the effect of ENSO on vegetation greenness



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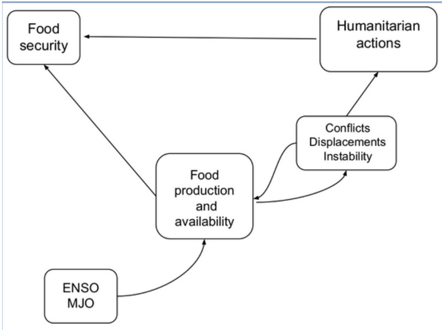
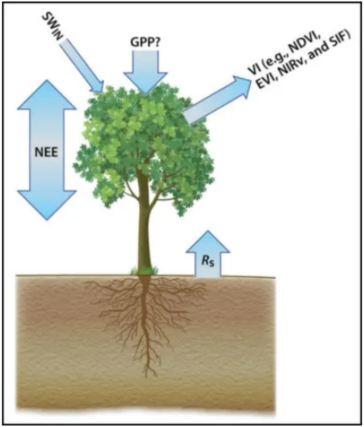


Causal Inference

- Estimate the strength of the effect of ENSO on vegetation greenness
- How effective are humanitarian actions to fight food insecurity?
- What is the effect of radiation on NEE/GPP ? Or the effect of temperature on RECO?



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- Usually provide confidence intervals and/or test hypothesis
- Consider possible biases: confounding, selection biases, estimation biases

Causal ML

- Spurious relationships
- Causal representation learning
- Causal feature extraction/ feature selection
- robustness to intervention
- causal XAI
- Causal RL

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Kaddour et al. [2022]

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