

# (Casual) Causality Course 2025

## Session 1

Gherardo Varando

25 March 2025

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## Instructors

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## Schedule

- ▶ **week 1, Tuesday** Intro and causal inference (GV)
- ▶ **week 1, Thursday** Causal inference and robustness (GV)
- ▶ **week 2, Tuesday** Causal Discovery (ED)
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- ▶ **week 3** Intensive weeek with group projects!

# Learning outcomes

- ▶ Understand the fundamental goals and problems of causal methods

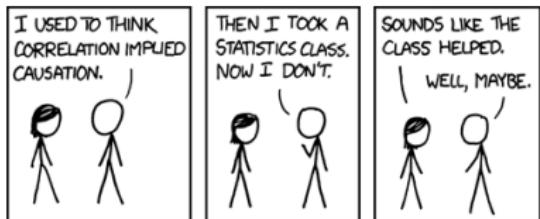


Figure: xkcd (CC BY-NC 2.5) <https://xkcd.com/552>

## Learning outcomes

- ▶ Understand the fundamental goals and problems of causal methods
- ▶ Familiarize with the vocabulary, definitions and basic concepts of causality

# causality noun

cau·sal·i·ty

kó·ža-lə-tē

plural **causalities**

[Synonyms of \*causality\*](#) >

Figure: <https://www.merriam-webster.com/dictionary/causality>

# Learning outcomes

- ▶ Understand the fundamental goals and problems of causal methods
- ▶ Familiarize with the vocabulary, definitions and basic concepts of causality
- ▶ Understand the fundamentals behind basic methodologies in causal inference and causal discovery

$$\begin{aligned} Y &= \theta(X) \cdot T + g(X, W) + \epsilon & \mathbb{E}[\epsilon | X, W] &= 0 \\ T &= f(X, W) + \eta & \mathbb{E}[\eta | X, W] &= 0 \\ && \mathbb{E}[\eta \cdot \epsilon | X, W] &= 0 \end{aligned}$$

Figure: <https://econml.azurewebsites.net>

# Learning outcomes

- ▶ Understand the fundamental goals and problems of causal methods
- ▶ Familiarize with the vocabulary, definitions and basic concepts of causality
- ▶ Understand the fundamentals behind basic methodologies in causal inference and causal discovery
- ▶ How causal methods and tools are relevant in ML?

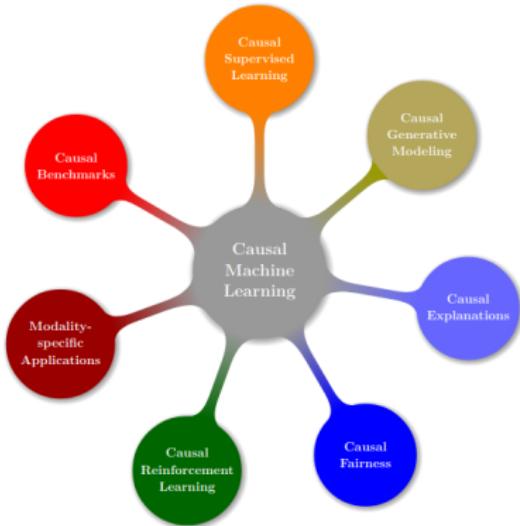


Figure: From Kaddour et al. [2022]

# Content week 1

## ► **Session 1** Tue 25/03

**Part I Intro to causality and causal methods**

**Part II Basics of causal inference**

## ► **Session 2** Thu 27/03

**Part I Causal inference methods**

**Part II Robustness to interventions**

# Content week 1

## ► Session 1 Tue 25/03

### Part I Intro to causality and causal methods

motivation, causal questions, what is causality?  
experiments, interventions and counterfactuals  
structural causal models and graphs

### Part II Basics of causal inference

causal effect, randomized experiments  
observational studies, identifiability conditions  
graphical representation, confounding, selection bias  
random variability and measurement error

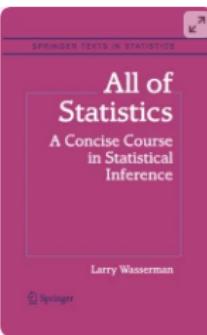
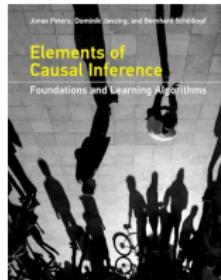
## ► Session 2 Thu 27/03

### Part I Causal inference methods

### Part II Robustness to interventions

# Basic references

- ▶ Elements of causal inference [Peters et al., 2017] [EC]
- ▶ Causal Inference: What If [Hernan and Robins, 2025] [Wif]
- ▶ All of Statistics [Wasserman, 2013] [AoS]



*Felix qui potuit rerum cognoscere causas . . .*

Publius Vergilius Maro  
Georgica Book Two



Figure: By Michael van der Gucht - [Public Domain](#)

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## Contents

- BkII:1-8 Introduction
- BkII:9-34 Methods of Propagation
- BkII:35-60 The Labour Required
- BkII:61-108 Treatment Of Individual Species
- BkII:109-135 The Effects of Climate and Location
- BkII:136-176 A Celebration of Italy
- BkII:177-225 The Nature of Various Soils
- BkII:226-258 The Recognition of Soil Types
- BkII:259-353 Planting A Vineyard
- BkII:354-420 Care of The Vineyard
- BkII:420-457 A Wealth of Trees And Plants
- BkII:458-542 The Joys Of The True Life



Figure: By Michael van der Gucht - [Public Domain](#)

# What is causality?

- ▶ Causality in Law

[https://en.wikipedia.org/wiki/Causation\\_\(law\)](https://en.wikipedia.org/wiki/Causation_(law))

Causation—Law of Tort playlist on youtube, first 3 videos

- ▶ Causality in Physics

[https://en.wikipedia.org/wiki/Causality\\_\(physics\)](https://en.wikipedia.org/wiki/Causality_(physics))

[https://www.youtube.com/watch?v=eG\\_eHDDMgCs](https://www.youtube.com/watch?v=eG_eHDDMgCs)

Rovelli [2022]



Figure: Karditsa Thinker at the National Archaeological Museum, Athens

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Figure: Karditsa Thinker at the National Archaeological Museum, Athens

- ▶ Work in groups and briefly discuss causality in physics or law [15 min], choose a speaker
- ▶ The two speakers present the main ideas [5 min] (whiteboard, slides)

# Non-causal questions ... still important!!



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- ▶ Estimate the tree diameters or age in a forest [West, 2021]



Figure: From <https://theforestguild.com/estimating-the-age-of-trees/>

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- ▶ Counts the number of trees in the desert [unexpectedly high Brandt et al., 2020]

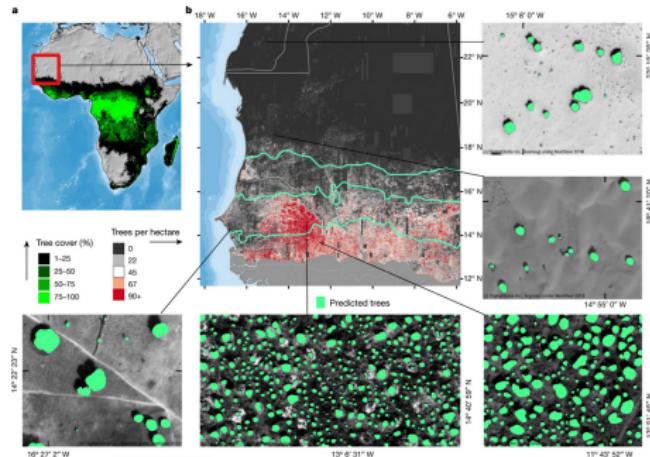


Figure: From Brandt et al. [2020] paper

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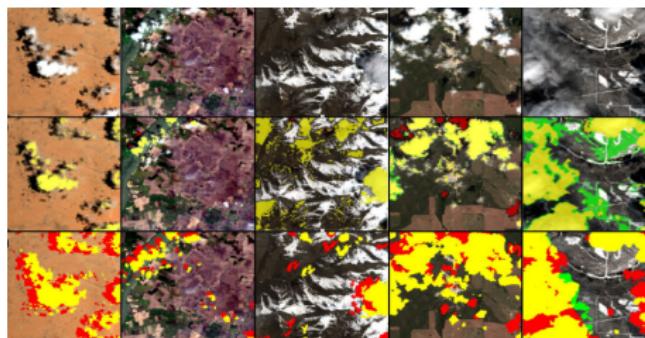


Figure: From Aybar et al. [2024] poster

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- ▶ Estimate the tree diameters or age in a forest [West, 2021]
- ▶ Counts the number of trees in the desert [unexpectedly high Brandt et al., 2020]
- ▶ Cloud detection [Aybar et al., 2024]
- ▶ Describing patterns of wood density [Yang et al., 2024]

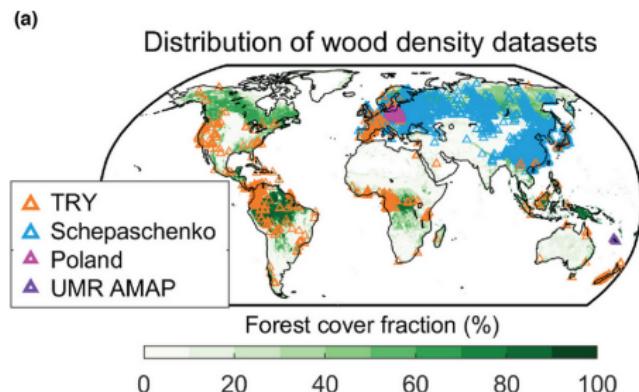


Figure: From Yang et al. [2024] paper

# Causal questions



# Causal questions

## ► How droughts affect yields

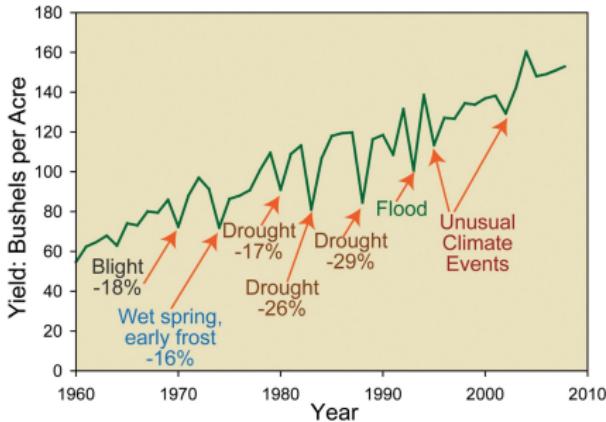


Figure: credit USDA FAS, from [ESA Website](#) (ESA Standard Licence)

# Causal questions

- ▶ How droughts affect yields
- ▶ Evaluate agricultural practice [Tsoumas et al., 2023, Giannarakis et al., 2022]

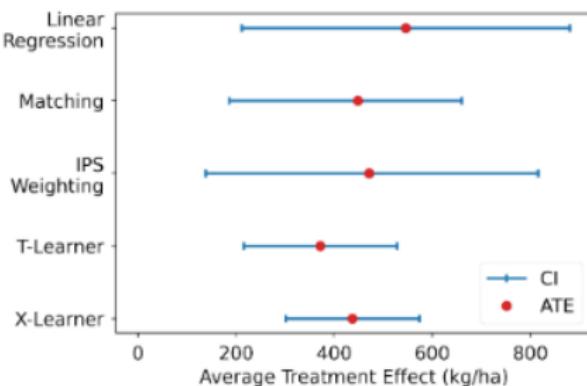


Figure: From Tsoumas et al. [2023]

# Causal questions

- ▶ How droughts affect yields
- ▶ Evaluate agricultural practice [Tsoumas et al., 2023, Giannarakis et al., 2022]
- ▶ How does armed conflict influence tropical forest loss? [Christiansen et al., 2022]

cumulative forest loss 2000–2018

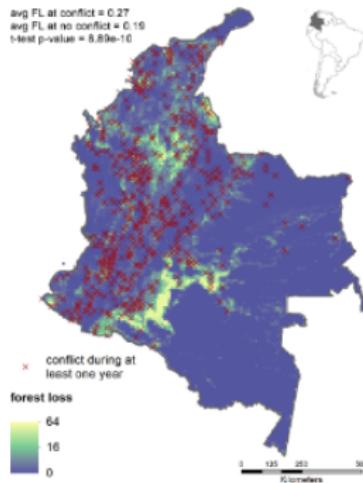


Figure: From Christiansen et al. [2022]

# Causal questions

- ▶ How droughts affect yields
- ▶ Evaluate agricultural practice [Tsoumas et al., 2023, Giannarakis et al., 2022]
- ▶ How does armed conflict influence tropical forest loss? [Christiansen et al., 2022]
- ▶ Estimate the effect of ENSO on vegetation [Le, 2023]

MODELS MEAN: ENSO - Leaf Area Index PERIOD 1915-2000 EXPERIMENT HISTC

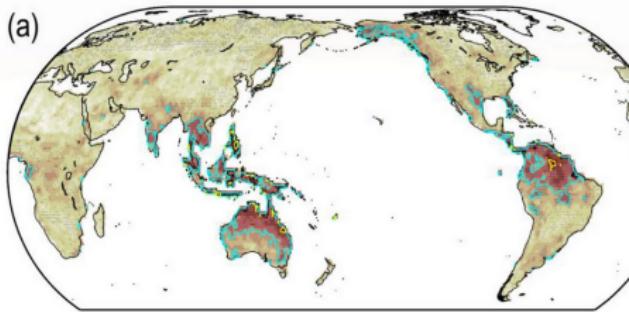


Figure: From Le [2023]

# Probabilistic causation

[Wasserman, 2013, AoS, Chapter 16]

*Roughly speaking, the statement “ $X$  causes  $Y$ ” means that changing the value of  $X$  will change the distribution of  $Y$ .*

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- ▶ What does it mean changing the value of  $X$ ?

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- ▶ How to make the previous *definition* precise and operative?
- ▶ What does it mean changing the value of  $X$ ?
- ▶ various solutions are possible, we will see two possibilities:  
**Structural Causal Models (SCM)** and the **Potential Outcome framework**

# Structural Causal Models

Definition [Peters et al., 2017]

A SCM over variables  $X_1, \dots, X_p$  with noise variables  $\varepsilon_1, \dots, \varepsilon_p$  is a collection of **structural assignments**:

$$X_i = f_i(X_{pa(i)}, \varepsilon_i)$$

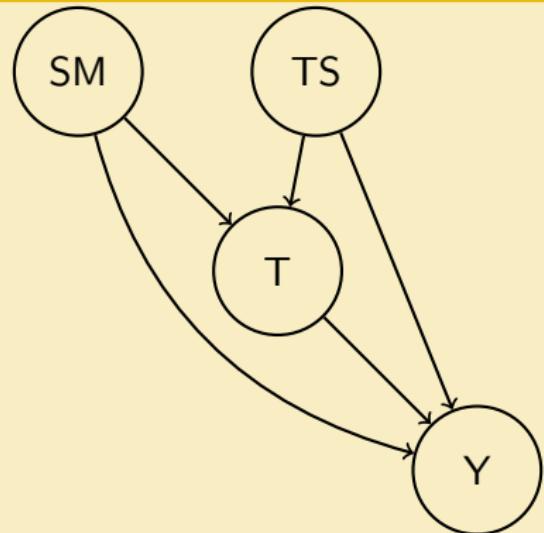
where  $\varepsilon_i$  are assumed jointly independent and  $f_i$  are fixed deterministic functions.

- ▶  $X_{pa(i)}$  are called the parents or the **direct causes** of  $X_i$
- ▶ we say  $X_i$  is a direct effect of its direct causes
- ▶ we assume the associated graph  $G$  to be a DAG

## Example

A simple SCM for variables

- ▶  $Y$  cotton yield
- ▶  $SM$  soil moisture on sowing
- ▶  $TS$  temperature on sowing
- ▶  $T$  sowing is performed on optimal day (1) or not (0)



$$SM = 0.3 + \epsilon_{SM}$$

$$TS = 25 + \epsilon_{TS}$$

$$T = \mathbf{1}_{>0} (1.2 + 0.8SM - 0.05TS + \epsilon_T > 0)$$

$$Y = \max\{0, 2.5 + 1.5SM - 0.7TS + 2.0T + \epsilon_Y\}$$

# SCMs as statistical models

Every SCM define a unique distribution over the considered variables.



Figure: From <https://www.quilting-in-america.com/cotton-cultivation-process.html>

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- ▶ How? ...
- ▶ We can sample from the SCM following a **topological order of the DAG**



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# SCMs as statistical models

Every SCM define a unique distribution over the considered variables.

- ▶ How? ...
- ▶ We can sample from the SCM following a **topological order of the DAG**
- ▶ If we can generate observations, we implicitly defined a distribution



Figure: From <https://www.quilting-in-america.com/cotton-cultivation-process.html>

# SCMs can model interventions or experiments

We define an experiment, or intervention when we **replace one or several of the structural assignments** to obtain a new SCM



Figure: From <https://www.cottonfarming.com/feature-story/field-day-highlights/>

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- ▶ The interventional distribution under this change is the new entailed probability distribution defined by the new SCM
- ▶ We indicate probabilities under interventions with e.g.  $P(Y \mid \text{do}(T = 1))$



Figure: From <https://www.cottonfarming.com/feature-story/field-day-highlights/>

## Definition (Total causal effect Peters et al. [2017])

We say there is a **total causal effect** from  $T$  to  $Y$  if (and only if) there exist  $\tilde{N}$  such that

$$T \perp\!\!\!\perp Y \text{ in } P^{do}(T=\tilde{N})$$

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- ▶ How we intuitive interpret the definition ?
- ▶ If there is no direct path between  $T$  and  $Y$  then there is no total causal effect
- ▶ Sometimes there is a direct path but no total causal effect

# With SCM we can reason about counterfactuals

We define an experiment, or intervention when we **replace one or several of the structural assignments** to obtain a new SCM

- ▶ Given some observed state/event, what would have happened if ... ?



Figure: From <https://www.nytimes.com/2011/07/12/us/12drought.html>

# With SCM we can reason about counterfactuals

We define an experiment, or intervention when we **replace one or several of the structural assignments** to obtain a new SCM

- ▶ Given some observed state/event, what would have happened if ... ?
- ▶ Counterfactual statements can be seen as intervention/do-statements in a counterfactual SCM which is obtained from an initial SCM by replacing the noise distribution with  $P(\varepsilon|X = x)$



Figure: From <https://www.nytimes.com/2011/07/12/us/12drought.html>

# Counterfactuals in SCM

Counterfactuals are different from interventions since we are not just doing an experiment, but rather imagining interventions for alternative *worlds*, that is alternative ways the system could have evolved but conditioning on everything else being the same

- ▶ SCMs allow the computation of counterfactual probabilities
- ▶ SCM can induce same observational and interventional distribution (Causal BN) but different counterfactuals (see example in Peters et al. [2017])

# Pearl Causal Hierarchy (PCH)

From Bareinboim et al. [2022]

Layer (Symbolic)	Typical Activity	Typical Question	Example	Machine Learning
$\mathcal{L}_1$ Associational $P(y x)$	Seeing	What is?  How would seeing $X$ change my belief in $Y$ ?	What does a symptom tell us about the disease?	Supervised / Unsupervised Learning
$\mathcal{L}_2$ Interventional $P(y do(x), c)$	Doing	What if?  What if I do $X$ ?	What if I take aspirin, will my headache be cured?	Reinforcement Learning
$\mathcal{L}_3$ Counterfactual $P(y_x x', y')$	Imagining	Why?  What if I had acted differently?	Was it the aspirin that stopped my headache?	

Table 1.1: Pearl's Causal Hierarchy.

# On the concept of *Harm*

- ▶ Sarvet and Stensrud [2023] interventionist vs counterfactual
- ▶ Mueller and Pearl [2024] response

## Personalized decision making

- ▶ Mueller and Pearl [2023]
- ▶ Tian and Pearl [2000]

# Potential outcomes - Counterfactual model

Hernan and Robins [2025]

Wasserman [2013]

- ▶ consider a binary **treatment variable**  $A$  (1: forest management practice (thinning, controlled burns,...) , 0: wild/uncontrolled forest)
- ▶ and a binary **outcome**  $Y$  (1: burned area, 0: not burned)
- ▶  $A, Y$  are random variables that take possible different values for each individual



Figure: From <https://www.kunc.org/2024-03-15/long-term-study-finds-combination-of-prescribed-burns>

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- ▶  $Y^{a=1}$  and  $Y^{a=0}$  are called **potential outcomes** or **counterfactual outcomes**
- ▶ for each individual, only one of the potential outcomes is actually observed/factual.

$$Y = Y^{a=A} \quad (\text{consistency equation})$$



Figure: From <https://www.kunc.org/2024-03-15/long-term-study-finds-combination-of-prescribed-burns>

# Causal effects

## Definition

Average causal effects An average causal effect of treatment  $A$  on outcome  $Y$  is present if

$$P(Y^{a=1} = 1) \neq P(Y^{a=0} = 1)$$

or equivalently (for binary outcomes)

$$\mathbb{E}[Y^{a=1}] \neq \mathbb{E}[Y^{a=0}]$$

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- ▶ causal risk ratio  $\frac{P(Y^{a=1}=1)}{P(Y^{a=0}=1)}$
- ▶ causal odds ratio  $\frac{P(Y^{a=1}=1)/P(Y^{a=1}=0)}{P(Y^{a=0}=1)/P(Y^{a=0}=0)}$

# Randomized experiments

- ▶ We collect data following a randomized control study: for each individual (forest unit/patch) we flip a coin and we assign the treatment variable to be  $a = 1$  if heads and  $a = 0$  if tails.



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- ▶ We collect data following a randomized control study: for each individual (forest unit/patch) we flip a coin and we assign the treatment variable to be  $a = 1$  if heads and  $a = 0$  if tails.
- ▶ We then collect the outcome variable  $Y$  (e.g. burned or not after 1 year) for all individuals in the study



# Randomized experiments

- ▶ assume no problem with the study, everybody is following instruction and there are no measurements problems  
*(ideal randomized experiment)*



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# Randomized experiments

- ▶ assume no problem with the study, everybody is following instruction and there are no measurements problems  
*(ideal randomized experiment)*
- ▶ can we say something about the causal effect of  $A$  on  $Y$  ?
- ▶ yes! we can compute the average causal effect ... formally because there is **exchangeability** between the treated ( $A = 1$ ) and untreated ( $A = 0$ ) groups





**New problem: the firefighters do not like your randomized study** they say that it is too dangerous not to manage some patches at all, and that some areas have a too high fire risk to be left completely untreated

## Conditional randomized experiments

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- ▶ In each stratum, we have exchangeability and we can compute average treatment effects
- ▶ this is called *stratification*
- ▶ moreover we say that this procedure ensure **conditional exchangeability**  $Y^a \perp\!\!\!\perp A|L$

## Computing ATE from conditionally randomized data

- ▶ From the data collected with a conditionally randomized experiment we can compute the ATE in all population.

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- ▶ From the data collected with a conditionally randomized experiment we can compute the ATE in all population.
- ▶ **Standardization** consists in computing the marginal counterfactual risk as the weighted average of the stratum-specific risk.

$$P(Y^a = 1) = \sum P(Y^a = 1 | L = l) P(L = l)$$

## Computing ATE from conditionally randomized data

- ▶ From the data collected with a conditionally randomized experiment we can compute the ATE in all population.
- ▶ **Standardization** consists in computing the marginal counterfactual risk as the weighted average of the stratum-specific risk.

$$P(Y^a = 1) = \sum P(Y^a = 1 | L = I)P(L = I)$$

- ▶ **Inverse Probability Weighting** is an alternative, but equivalent, procedure to compute  $P(Y^a = 1)$  by weighting each individual sample by  $w_I = 1/P(A = a | L = I)$  and then we compute  $P(Y^a = 1) = \sum w_I P(Y | A = a, L = I)$

## Observational studies

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- ▶ this conditions *assure that the observational study can be used somehow as a randomized trial*

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If we can assume this three conditions we can use the techniques such as IPW or standardization to compute ATE from observational data

## Effect modification

- ▶ We say that  $V$  is a modifier of the effect of  $A$  on  $Y$  when the average causal effect of  $A$  on  $Y$  varies across levels of  $V$
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- ▶ they require exchangeability and positivity
- ▶ Standardization (or IPW), stratification and matching measure different causal effects: Average effects in the entire population, conditional causal effects (stratification) and usually causal effects in the treated and untreated for matching

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