(Casual) Causality Course 2025
Session 1

Gherardo Varando

25 March 2025

(Casual) Causality Course 2025

Intructors

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- Vassilis vasileios.sitokonstantinou@uv.es

Schedule

- **week 1, Tuesday** Intro and causal inference (GV)
- **week 1, Thursday** Causal inference and robustness (GV)
- **week 2, Tuesday** Causal Discovery (ED)
- **week 2, Thursday** Causal Discovery (ED)
- **week 3** Intensive weeek with group projects!

Learning outcomes

- Understand the fundamental goals and problems of causal methods

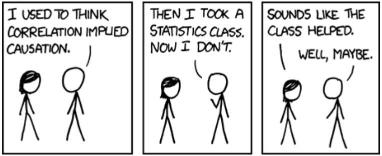


Figure: xkcd (CC BY-NC 2.5) https://xkcd.com/552

Learning outcomes

- Understand the fundamental goals and problems of causal methods
- Familiarize with the vocabulary, definitions and basic concepts of causality

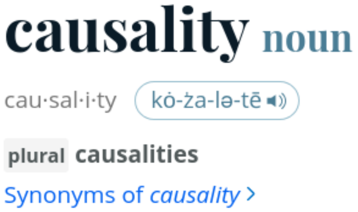


Figure: https://www.merriam-webster.com/dictionary/causality

Learning outcomes

- Understand the fundamental goals and problems of causal methods
- Familiarize with the vocabulary, definitions and basic concepts of causality
- Understand the fundamentals behind basic methodologies in causal inference and causal discovery

$$
\begin{aligned}
Y &= \theta(X) \cdot T + g(X, W) + \epsilon \qquad & \mathbb{E}[\epsilon | X, W] = 0 \\
T &= f(X, W) + \eta & \mathbb{E}[\eta \mid X, W] = 0 \\
& \mathbb{E}[\eta \cdot \epsilon | X, W] = 0
\end{aligned}
$$

Figure: https://econml.azurewebsites.net

Learning outcomes

- Understand the fundamental goals and problems of causal methods
- Familiarize with the vocabulary, definitions and basic concepts of causality
- Understand the fundamentals behind basic methodologies in causal inference and causal discovery
- How causal methods and tools are relevant in ML?

Figure: From Kaddour et al. [2022]

Content week 1

- **Session 1** Tue 25/03
  - Part I **Intro to causality and causal methods**
  - Part II **Basics of causal inference**

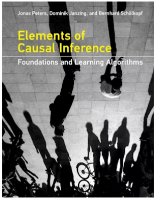
- **Session 2** Thu 27/03
  - Part I Causal inference methods
  - Part II Robustness to interventions

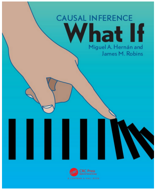
Content week 1

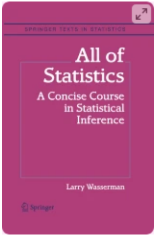
- **Session 1** Tue 25/03
  - Part I **Intro to causality and causal methods**
    motivation, causal questions, what is causality?
    experiments, interventions and counterfactuals
    structural causal models and graphs
  - Part II **Basics of causal inference**
    causal effect, randomized experiments
    observational studies, identifiability conditions
    graphical representation, confounding, selection bias
    random variability and measurement error
- **Session 2** Thu 27/03
  - Part I Causal inference methods
  - Part II Robustness to interventions

Basic references

- Elements of causal inference [Peters et al., 2017] [EC]
- Causal Inference: What If [Hernan and Robins, 2025] [Wif]
- All of Statistics [Wasserman, 2013] [AoS]







Felix qui potuit rerum cognoscere causas . . .

Publius Vergilius Maro

Georgica Book Two

Figure: By Michael van der Gucht - Public Domain

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Contents

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What is causality?

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  Causation—Low of Tort playlist on youtube, first 3 videos
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  Rovelli [2022]

Figure: Karditsa Thinker at the National Archaeological Museum, Athens

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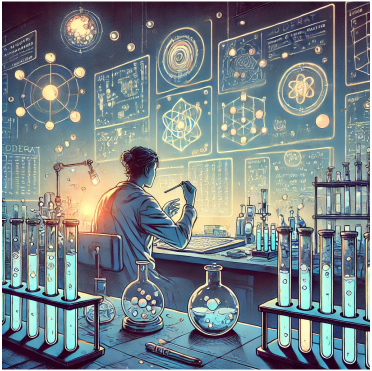
Figure: Karditsa Thinker at the National Archaeological Museum, Athens

- Work in groups and briefly discuss causality in physics or law **[15 min]**, choose a speaker
- The two speakers present the main ideas **[5 min]** (whiteboard, slides)

Non-causal questions … still important!!



Non-causal questions ... still important!!

- Estimate the tree diameters or age in a forest [West, 2021]

Figure: From https://theforestguild.com/estimating-the-age-of-trees/

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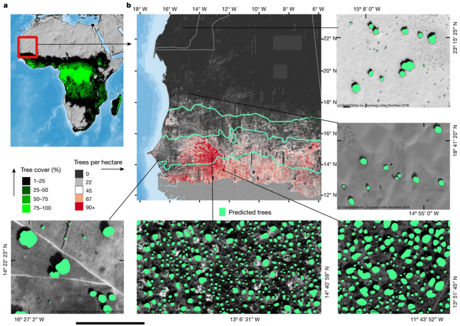

Figure: From Brandt et al. [2020] paper

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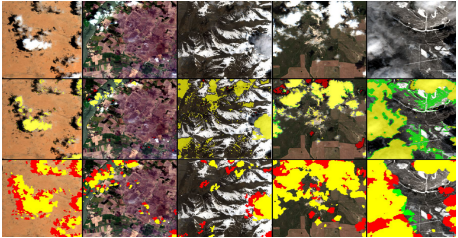

Figure: From Aybar et al. [2024] poster

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- Estimate the tree diameters or age in a forest [West, 2021]
- Counts the number of trees in the desert [unexpectedly high Brandt et al., 2020]
- Cloud detection [Aybar et al., 2024]
- Describing patterns of wood density [Yang et al., 2024]

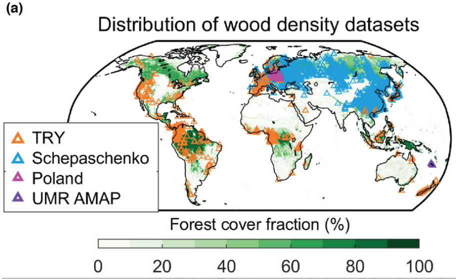
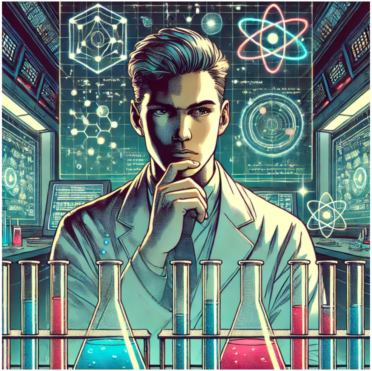


Figure: From Yang et al. [2024] paper

Causal questions



Causal questions

- How droughts affect yields

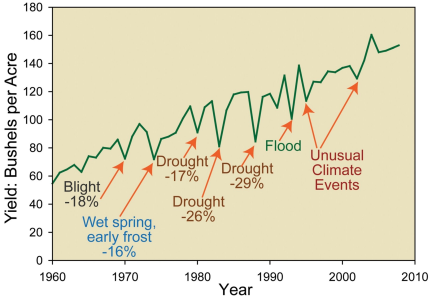


Figure: credit USDA FAS, from ESA Website (ESA Standard Licence)

Causal questions

- How droughts affect yields
- Evaluate agricultural practice [Tsoumas et al., 2023, Giannarakis et al., 2022]

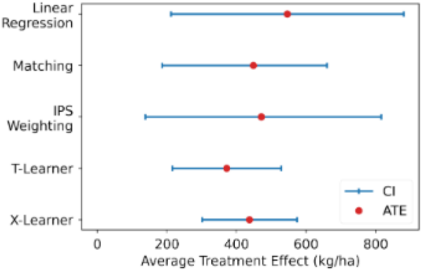


Figure: From Tsoumas et al. [2023]

Causal questions

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- Evaluate agricultural practice [Tsoumas et al., 2023, Giannarakis et al., 2022]
- How does armed conflict influence tropical forest loss? [Christiansen et al., 2022]

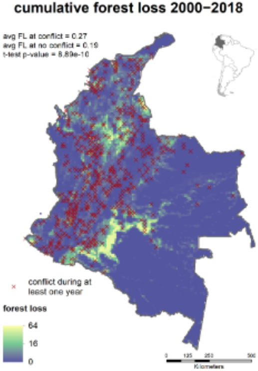

Figure: From Christiansen et al. [2022]

Causal questions

- How droughts affect yields
- Evaluate agricultural practice [Tsoumas et al., 2023, Giannarakis et al., 2022]
- How does armed conflict influence tropical forest loss? [Christiansen et al., 2022]
- Estimate the effect of ENSO on vegetation [Le, 2023]

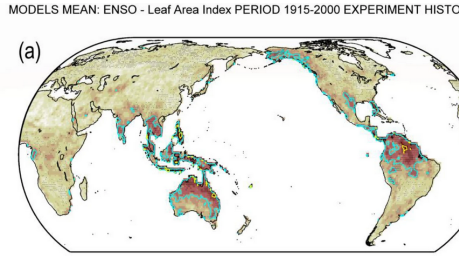

Figure: From Le [2023]

Probabilistic causation

[Wasserman, 2013, AoS, Chapter 16]

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- What does it mean changing the value of X?

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Roughly speaking, the statement "X causes Y" means that changing the value of X will change the distribution of Y.

- How to make the previous *definition* precise and operative?
- What does it mean changing the value of X?
- various solutions are possible, we will see two possibilities: **Structural Causal Models (SCM)** and the **Potential Outcome framework**

Structural Causal Models

Definition [Peters et al., 2017]

A SCM over variables $X_1, \ldots, X_p$ with noise variables $\varepsilon_1, \ldots, \varepsilon_p$ is a collection of **structural assignments**:

$$X_i = f_i(X_{pa(i)}, \varepsilon_i)$$

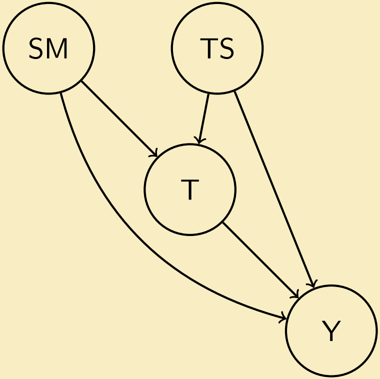
where ε_i are assumed jointly independent and f_i are fixed deterministic functions.

- $X_{pa(i)}$ are called the parents or the **direct causes** of X_i
- we say X_i is a direct effect of its direct causes
- we assume the associated graph G to be a DAG

Example

A simple SCM for variables

- Y cotton yield
- SM soil moisture on sowing
- TS temperature on sowing
- T sowing is performed on optimal day (1) or not (0)



$$
\begin{aligned}
SM &= 0.3 + \epsilon_{SM} \\
TS &= 25 + \epsilon_{TS} \\
T &= \mathbf{1}_{>0}\left(1.2 + 0.8SM - 0.05TS + \epsilon_T > 0\right) \\
Y &= \max\{0,\ 2.5 + 1.5SM - 0.07TS + 2.0T + \epsilon_Y\}
\end{aligned}
$$

SCMs as statistical models

Every SCM define a unique distribution over the considered variables.

Figure: From https://www.quilting-in-america.com/cotton-cultivation-process.html

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- How? ...
- We can sample from the SCM following **a topological order of the DAG**

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SCMs as statistical models

Every SCM define a unique distribution over the considered variables.

- How? ...
- We can sample from the SCM following **a topological order of the DAG**
- If we can generate observations, we implicitly defined a distribution

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SCMs can model interventions or experiments

We define an experiment, or intervention when we **replace one or several of the structural assignments** to obtain a new SCM

Figure: From https://www.cottonfarming.com/feature-story/field-day-highlights/

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- The interventional distribution under this change is the new entailed probability distribution defined by the new SCM
- We indicate probabilities under interventions with e.g. $P(Y|\,\mathrm{do}(T=1))$

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Definition (Total causal effect Peters et al. [2017])

We say there is a **total causal effect** from T to Y if (and only if) there exist $\tilde{N}$ such that

$$T \not\perp\!\!\!\perp Y \text{ in } P^{do(T=\tilde{N})}$$

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- How we intuitive interpret the definition ?
- If there is no direct path between T and Y then there is no total causal effect
- Sometimes there is a direct path but no total causal effect

With SCM we can reason about counterfactuals

We define an experiment, or intervention when we **replace one or several of the structural assignments** to obtain a new SCM

- Given some observed state/event,what would have happened if ... ?

Figure: From https://www.nytimes.com/2011/07/12/us/12drought.html

With SCM we can reason about counterfactuals

We define an experiment, or intervention when we **replace one or several of the structural assignments** to obtain a new SCM

- Given some observed state/event,what would have happened if ... ?
- Counterfactual statements can be seen as intervention/do-statements in a counterfactual SCM which is obtained from an initial SCM by replacing the noise distribution with $P(\varepsilon|X = x)$

Figure: From https://www.nytimes.com/2011/07/12/us/12drought.html

Counterfactuals in SCM

Counterfactuals are different from interventions since we are not just doing an experiment, but rather imagining interventions for alternative *worlds*, that is alternative ways the system could have evolved but conditioning on everything else being the same

- SCMs allow the computation of counterfactual probabilities
- SCM can induce same observational and interventional distribution (Causal BN) but different counterfactuals (see example in Peters et al. [2017])

Pearl Causal Hierarchy (PCH)

From Bareinboim et al. [2022]

	Layer (Symbolic)	Typical Activity	Typical Question	Example	Machine Learning
$\mathcal{L}_1$	Associational $P(y\|x)$	Seeing	What is? How would seeing X change my belief in Y?	What does a symptom tell us about the disease?	Supervised / Unsupervised Learning
$\mathcal{L}_2$	Interventional $P(y\|do(x),c)$	Doing	What if? What if I do X?	What if I take aspirin, will my headache be cured?	Reinforcement Learning
$\mathcal{L}_3$	Counterfactual $P(y_x\|x',y')$	Imagining	Why? What if I had acted differently?	Was it the aspirin that stopped my headache?	

Table 1.1: Pearl's Causal Hierarchy.

On the concept of *Harm*

- Sarvet and Stensrud [2023] interventionalist vs contrefactual
- Mueller and Pearl [2024] response

Personalized decision making

- Mueller and Pearl [2023]
- Tian and Pearl [2000]

Potential outcomes - Counterfactual model

Hernan and Robins [2025]
Wasserman [2013]

- consider a binary **treatment variable** A (1: forest management practice (thinning, controlled burns,...) , 0: wild/uncontrolled forest)
- and a binary **outcome** Y (1: burned area, 0: not burned)
- A, Y are random variables that take possible different values for each individual

Figure: From https://www.kunc.org/2024-03-15/long-term-study-finds-combination-of-prescribed-burns

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- denote with the outcome variable that would have been observed under treatment $a = 1$, and similarly $Y^{a=0}$

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- for each individual, only one of the potential outcomes is actually observed/factual.

Figure: From https://www.kunc.org/2024-03-15/long-term-study-finds-combination-of-prescribed-burns

$$Y = Y^{a=A} \quad \text{(consistency equation)}$$

Causal effects

Definition

Average causal effects An average causal effect of treatment A on outcome Y is present if

$$P(Y^{a=1} = 1) \neq P(Y^{a=0} = 1)$$

or equivalently (for binary outcomes)

$$\mathbb{E}[Y^{a=1}] \neq \mathbb{E}[Y^{a=0}]$$

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- causal odds ratio $\frac{P(Y^{a=1}=1)/P(Y^{a=1}=0)}{P(Y^{a=0}=1)/P(Y^{a=0}=0)}$

Randomized experiments

- We collect data following a randomized control study: for each individual (forest unit/patch) we flip a coin and we assign the treatment variable to be $a = 1$ if heads and $a = 0$ if tails.

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- We then collect the outcome variable Y (e.g. burned or not after 1 year) for all individuals in the study

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- assume no problem with the study, everybody is following instruction and there are no measurements problems (*ideal randomized experiment*)

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- can we say something about the causal effect of A on Y ?
- yes! we can compute the average causal effect ... formally because there is **exchangeability** between the treated ($A = 1$) and untreated ($A = 0$) groups

New problem: the firefighters do not like your randomized study they say that it is too dangerous not to manage some patches at all, and that some areas have a too high fire risk to be left completely untreated

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- this is called *stratification*
- moreover we say that this procedure ensure **conditional exchangeability** $Y^a \perp\!\!\!\perp A|L$

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- **Standardization** consists in computing the marginal counterfactual risk as the weighted average of the stratum-specific risk.

$$P(Y^a = 1) = \sum P(Y^a = 1 | L = l) P(L = l)$$

- **Inverse Probability Weighting** is an alternative, but equivalent, procedure to compute $P(Y^a = 1)$ by weighting each individual sample by $w_l = 1/P(A = a | L = l)$ and then we compute $P(Y^a = 1) = \sum w_l P(Y | A = a, L = l)$

Observational studies

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- Problems: confounding variables (e.g. forest are controlled more when we expect to have high risk of fire, patients are treated with more invasive/effective treatments when doctors thinks the case is more serious, ...)
- still under certain conditions and assumptions we can identify the causal effect
- this conditions *assure that the observational study can be used somehow as a randomized trial*

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If we can assume this three conditions we can use the techniques such as IPW or standardization to compute ATE from observational data

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- they require exchangeability and positivity
- Standardization (or IPW), stratification and matching measure different causal effects: Average effects in the entire population, conditional causal effects (startification) and usually causal effects in the treated and untreated for matching

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