(Casual) Causality Course 2025
Session 2

Gherardo Varando

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Intructors

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Schedule

- **week 1, Tuesday** Intro and causal inference (GV)
- **week 1, Thursday** Causal inference and robustness (GV)
- **week 2, Tuesday** Causal Discovery (ED)
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- **week 3** Intensive weeek with group projects!

Content week 1

- **Session 1** Tue 25/03

  Part I **Intro to causality and causal methods**

  Part II **Basics of causal inference**

- **Session 2** Thu 27/03
  - Causal inference methods

Content week 1

- **Session 1** Tue 25/03

  Part I **Intro to causality and causal methods**
  motivation, causal questions, what is causality?
  experiments, interventions and counterfactuals
  structural causal models and graphs

  Part II **Basics of causal inference**
  causal effect, randomized experiments
  observational studies, identifiability conditions
  graphical representation, confounding, selection bias
  random variability and measurement error

- **Session 2** Thu 27/03
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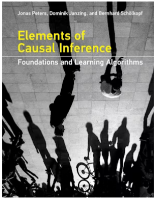
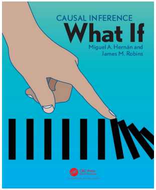
Basic references

- Elements of causal inference [Peters et al., 2017] [EC]
- Causal Inference: What If [Hernan and Robins, 2025] [Wif]
- All of Statistics [Wasserman, 2013] [AoS]





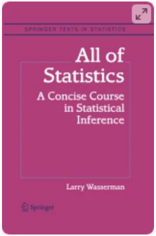


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Figure: Karditsa Thinker at the National Archaeological Museum, Athens

- Work in groups and briefly discuss causality in physics or law **[15 min]**, choose a speaker
- The two speakers present the main ideas **[5 min]** (whiteboard, slides)

Potential outcomes - Counterfactual model

Hernan and Robins [2025]
Wasserman [2013]

- consider a binary **treatment variable** A (1: forest management practice (thinning, controlled burns,...) , 0: wild/uncontrolled forest)
- and a binary **outcome** Y (1: burned area, 0: not burned)
- A, Y are random variables that take possible different values for each individual

Figure: From https://www.kunc.org/2024-03-15/long-term-study-finds-combination-of-prescribed-burns

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- for each individual, only one of the potential outcomes is actually observed/factual.

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$$Y = Y^{a=A} \quad \text{(consistency equation)}$$

Causal effects

Definition

Average causal effects An average causal effect of treatment A on outcome Y is present if

$$P(Y^{a=1} = 1) \neq P(Y^{a=0} = 1)$$

or equivalently (for binary outcomes)

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- causal odds ratio $\frac{P(Y^{a=1}=1)/P(Y^{a=1}=0)}{P(Y^{a=0}=1)/P(Y^{a=0}=0)}$

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- We then collect the outcome variable Y (e.g. burned or not after 1 year) for all individuals in the study

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- can we say something about the causal effect of A on Y ?
- yes! we can compute the average causal effect ... formally because there is **exchangeability** between the treated $(A = 1)$ and untreated $(A = 0)$ groups

New problem: the firefighters do not like your randomized study they say that it is too dangerous not to manage some patches at all, and that some areas have a too high fire risk to be left completely untreated

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- moreover we say that this procedure ensure **conditional exchangeability** $Y^a \perp\!\!\!\perp A | L$

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- **Inverse Probability Weighting** is an alternative, but equivalent, procedure to compute $P(Y^a = 1)$ by weighting each individual sample by $w_l = 1/P(A = a|L = l)$ and then we compute $P(Y^a = 1) = \sum w_l P(Y|A = a, L = l)$

Observational studies

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- still under certain conditions and assumptions we can identify the causal effect
- this conditions *assure that the observational study can be used somehow as a randomized trial*

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If we can assume this three conditions we can use the techniques such as IPW or standardization to compute ATE from observational data

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- Standardization (or IPW), stratification and matching measure different causal effects: Average effects in the entire population, conditional causal effects (startification) and usually causal effects in the treated and untreated for matching

Computing interventional distributions in SCM

truncated factorization [Pearl, 1993], G-computation formula [Robins, 1986], manipulation theorem [Spirtes et al., 2000]

Given an SCM $\mathcal{C}$ and an intervened SCM $\tilde{\mathcal{C}}$, obtained from $\mathcal{C}$ by intervening on some X_k with $k \neq j$, we have that

$$P^{\tilde{\mathcal{C}}}(X_j | X_{pa(j)}) = P^{\mathcal{C}}(X_j | X_{pa(j)})$$

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- This requires the *knowledge of the causal graph*

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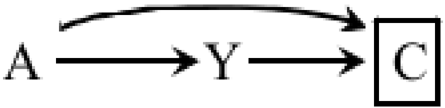
- viceversa there are techniques that can handle confounding problems without relying on exchangeability: e.g. difference-in-differences, instrumental variables and the front door criterion

Selection bias



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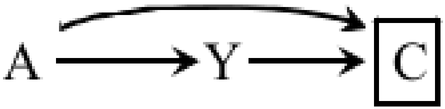

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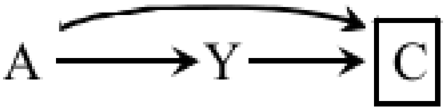

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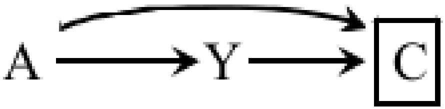

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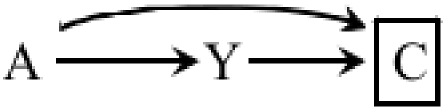

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- IPW or stratification can be used to control for selection bias
- randomization does not protect from selection bias

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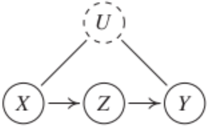
- do-caluclus is complete, every identifiable interventional distribution can be obtained
- one corollary of the do-calculus theorem is the **front-door adjustment**

Example 6.46 (Front-door adjustment) Let $\mathfrak{C}$ be an SCM with corresponding graph



If we do not observe U, we cannot apply the backdoor criterion. In fact, there is no valid adjustment set. But still, provided that $p^{\mathfrak{C}}(x,z) > 0$, the *do*-calculus provides us with

$$p^{\mathfrak{C};do(X:=x)}(y) = \sum_{z} p^{\mathfrak{C}}(z \mid x) \sum_{\tilde{x}} p^{\mathfrak{C}}(y \mid \tilde{x}, z)\, p^{\mathfrak{C}}(\tilde{x}). \tag{6.23}$$

The fact that observing Z in addition to X and Y here reveals causal information nicely shows that causal relations can also be explored by observing the "channel" (here Z) that carries the "signal" from X to Y. □

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