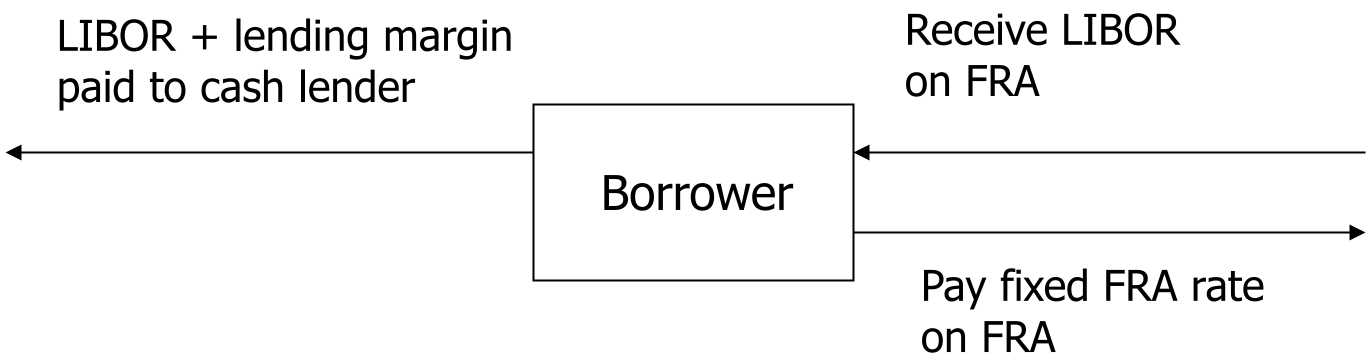
Interest Rate Swaps (IRS)

Definitions

- A **swap** is a derivative in which two counterparties agree to exchange one stream of cash flows against another stream.
- These streams are called the **legs** of the swap.
- An **interest rate swap** is a derivative in which one party exchanges a stream of interest payments for another party's stream of cash flows.

Hedging with an FRA



Hedging with an IRS

Very similar to an FRA, but is applied to a series of cashflows

Characteristics of IRS

Similar to FRA

- No exchange of principal
- Only interest flows are exchanged and netted

Different from FRA

- Settlement amount paid at the end of relevant period

Motivation

Company A has access to

floating-rates borrowing at LIBOR+0.1%

fixed rate borrowing at 8.0%

Company A would prefer floating-rate borrowing

Company B has access to

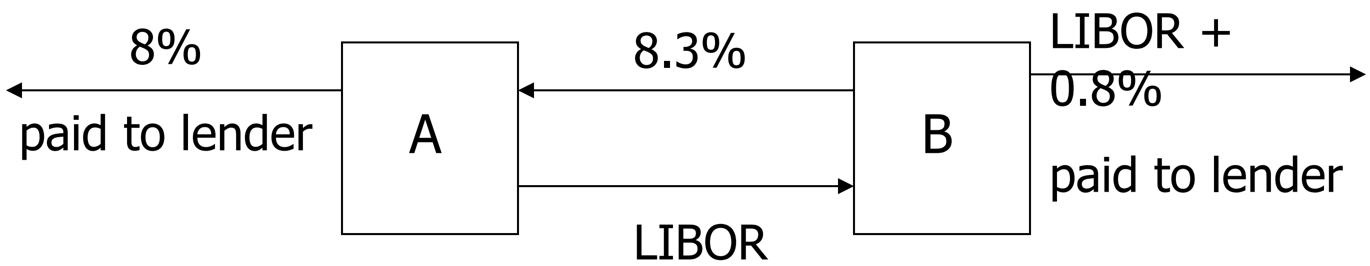
floating-rate borrowing at LIBOR+0.8%

fixed rate borrowing at 9.5%

Company B would prefer fixed rate borrowing

How to create a win-win?

Win-Win

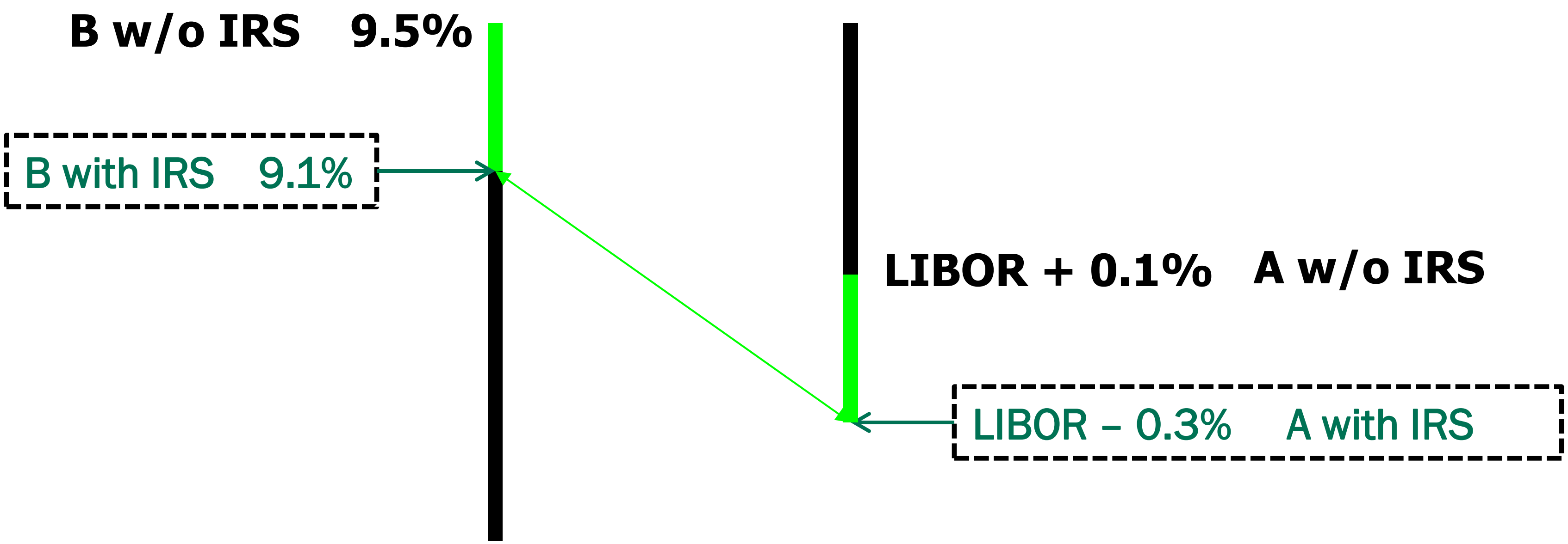


$$\text{Net cost for A is}: -8.0\% + 8.3\% - \text{LIBOR} = -(\text{LIBOR} - 0.3\%)$$

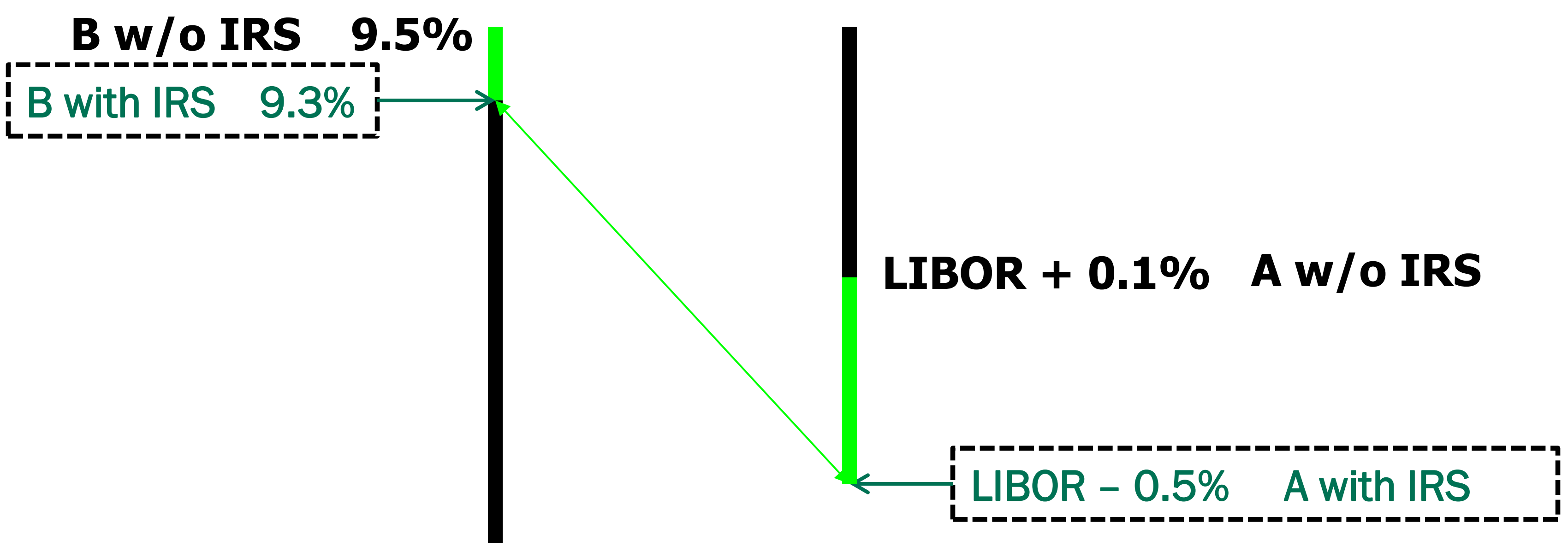
$$\text{Net cost for B is}: -(\text{LIBOR} + 0.8\%) - 8.3\% + \text{LIBOR} = -9.1\%$$

Borrowing Cost Comparison:	**Co. A (Floating)**	**Co.B (Fixed)**
➢ Borrowing at market	LIBOR+0.1%	9.5%
➢ Borrowing in association with IRS	LIBOR – 0.3%	9.1%
➢ Cost Saving	-0.4%	-0.4%

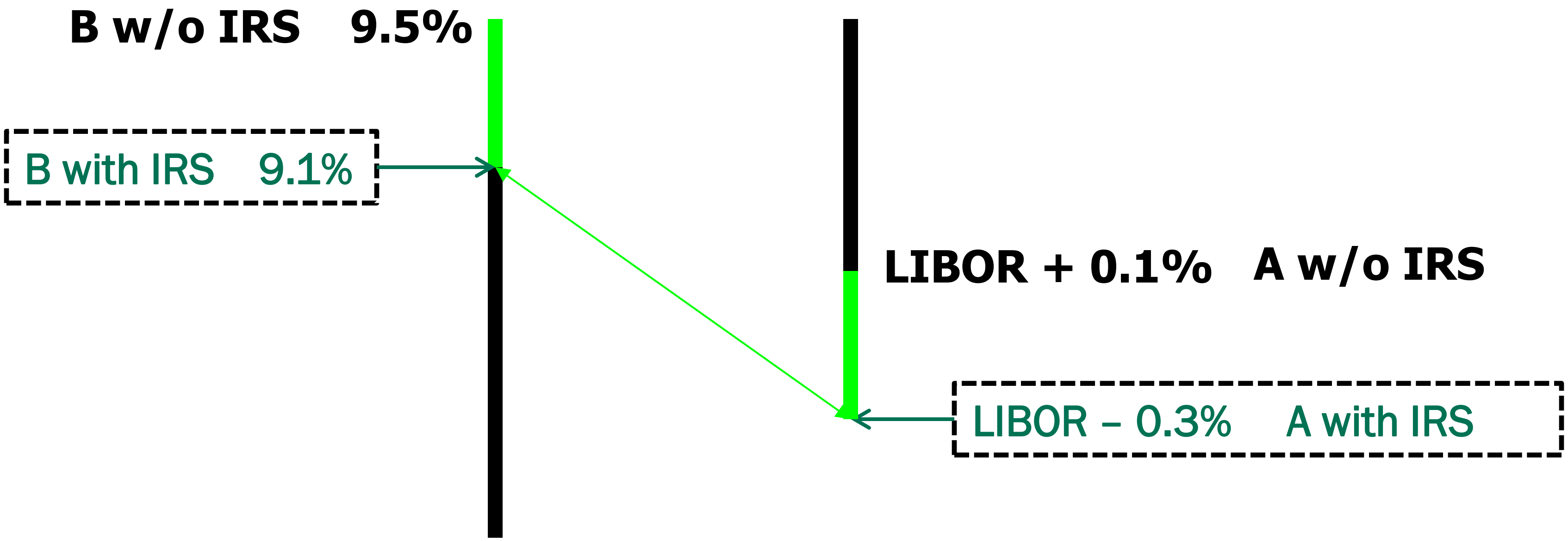
Relative Advantage



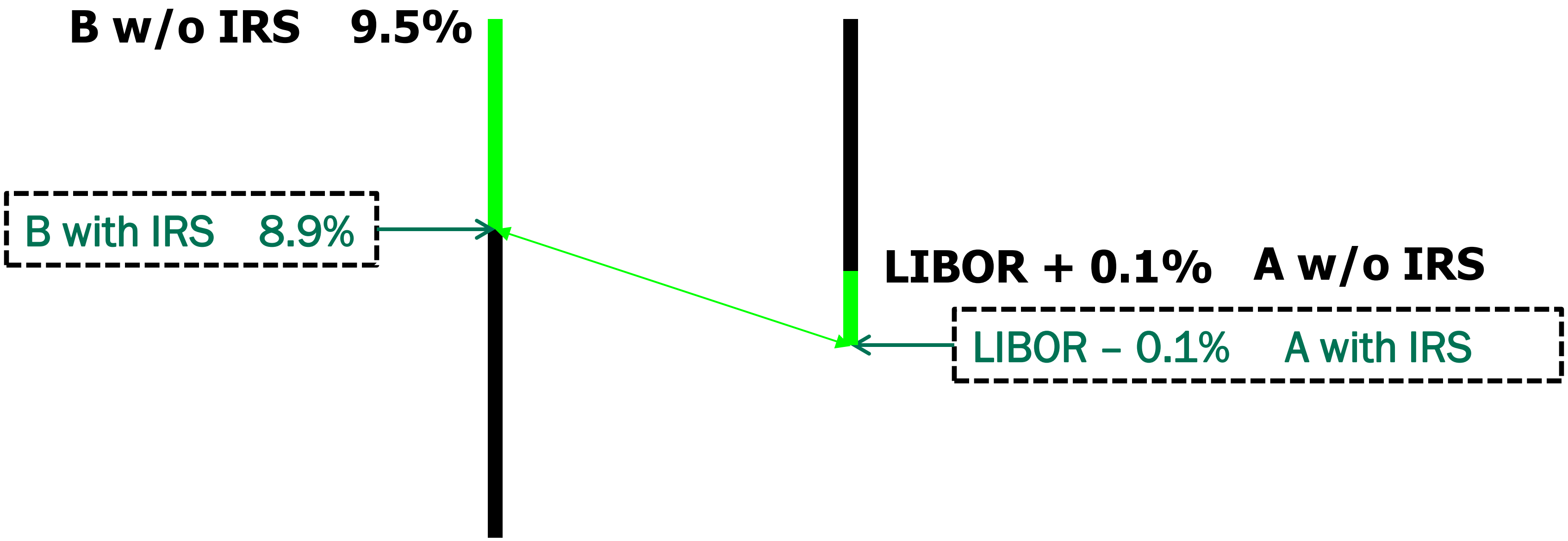
Relative Advantage



Relative Advantage



Relative Advantage



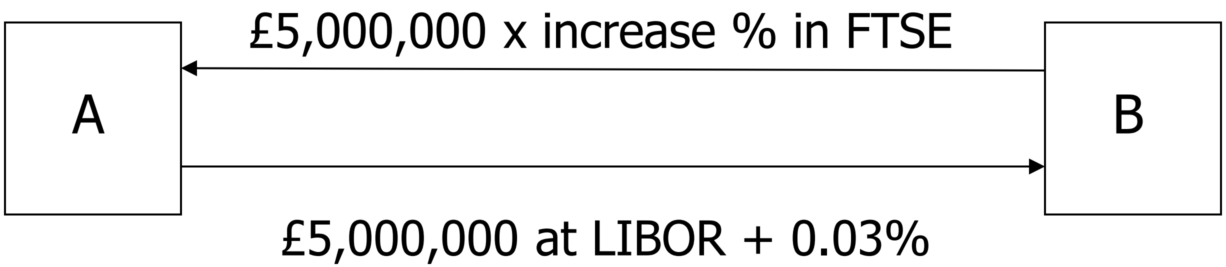
Type of Swap

- Coupon Swap
  - Party A pay fixed interest rate and receive floating interest rate from party B
- Basis Swap
  - Floating vs floating but on different rate basis
- Index Swap
  - the flow in one / other direction are based on index

Basis Swap

An example of a user of basis swaps would be bank that is lending at the six month Libor rate but its funding cost is based on the 3M Libor curve.

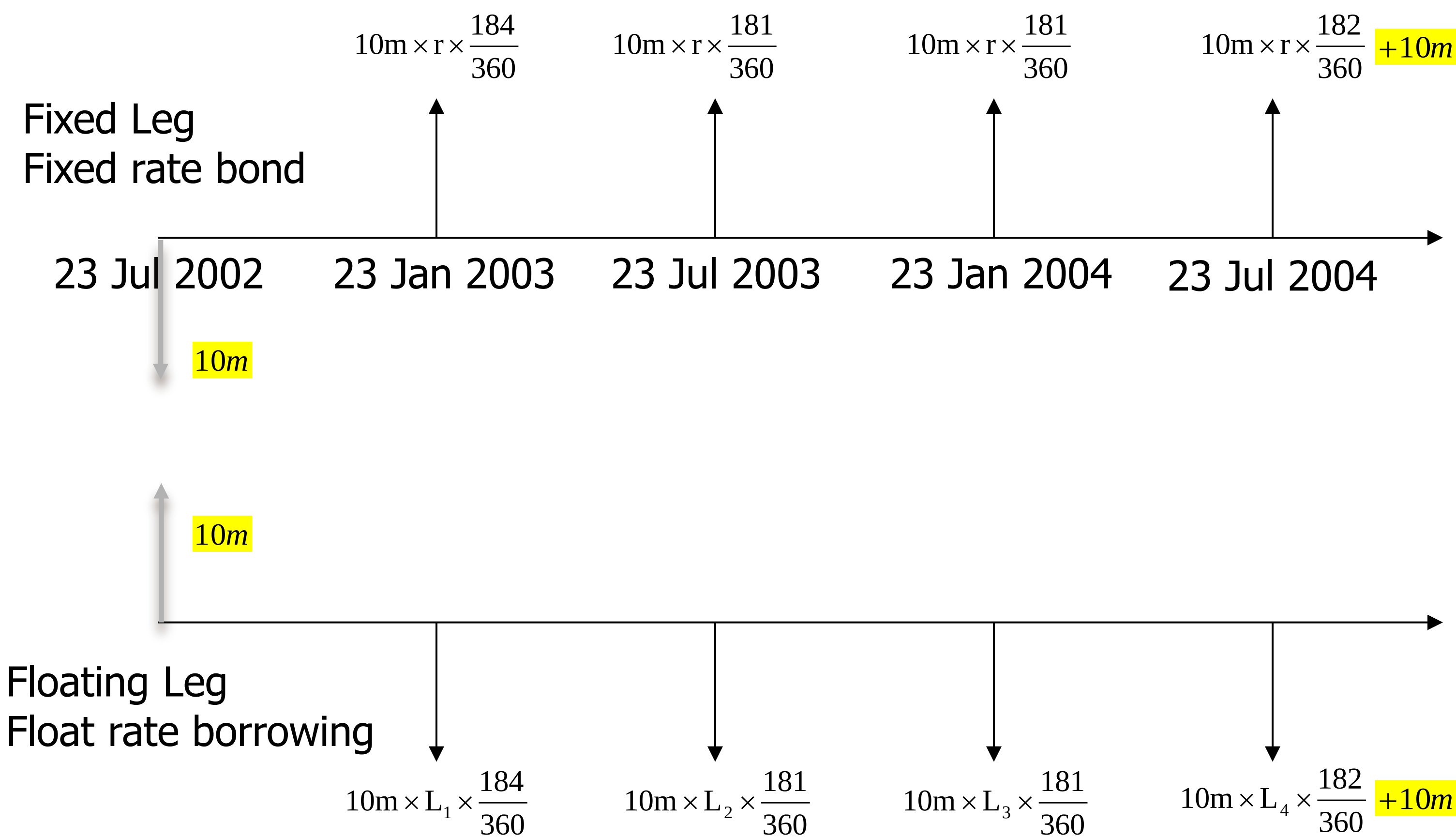
Index Swap



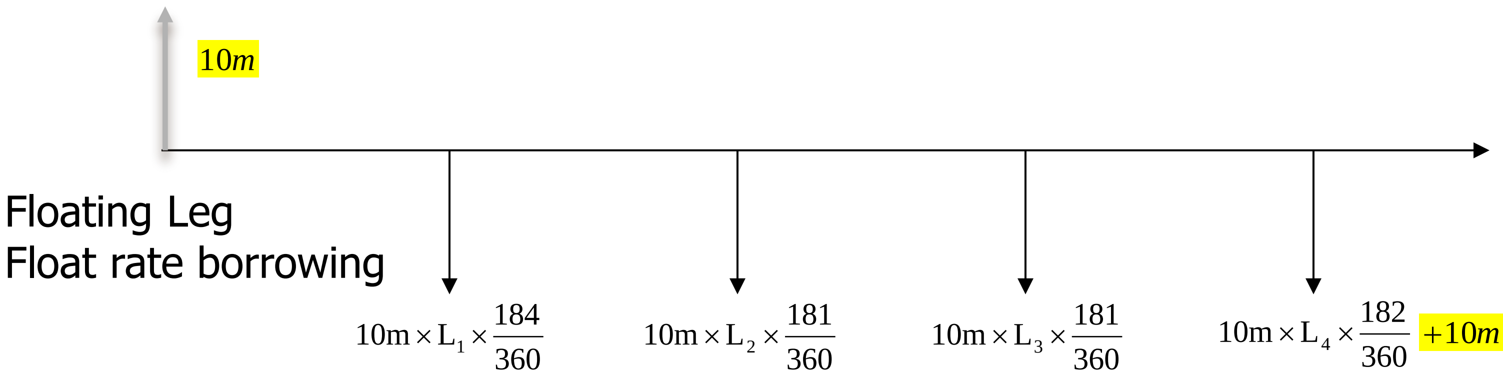
Valuation of Swap – Basic Concept

- Long Swap = + Long a Fixed rate bond – Floating rate borrowing
- At inception, the NPV of the floating leg cashflow is zero. This is assumed that all future period floating rate are discounted at the then market floating borrowing rate, hence the NPV of floating leg = 0 at inception.
- Swap Value = + PV of Fixed leg cashflow – PV of Floating leg cashflow
- Hence, Swap Value = NPV of the Fixed leg cashflow
- At inception, NPV of all the cashflow = 0
- i.e. NPV of the Fixed leg cashflow = 0

+ Long a Fixed rate bond – Floating rate borrowing



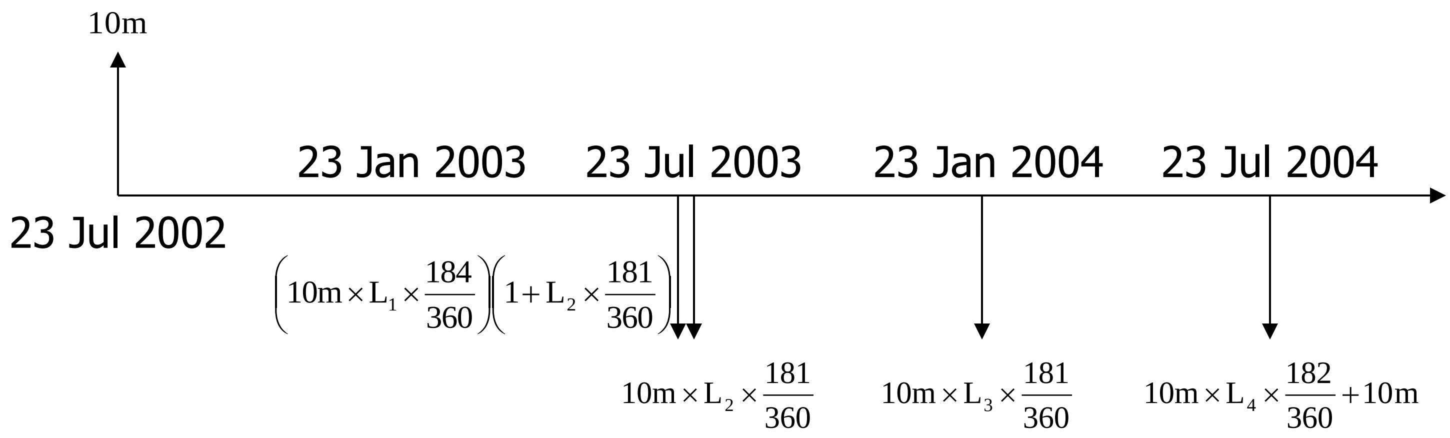
NPV of Floating Leg Cashflow



$$\text{NPV} = 10\text{m} - \frac{10\text{m}\times L_1\times\frac{184}{360}}{\left(1+L_1\times\frac{184}{360}\right)} - \frac{10\text{m}\times L_2\times\frac{181}{360}}{\left(1+L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)} - \frac{10\text{m}\times L_3\times\frac{181}{360}}{\left(1+L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)\left(1+L_3\times\frac{181}{360}\right)}$$

$$- \frac{10\text{m}\times L_4\times\frac{182}{360}+10\text{m}}{\left(1+L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)\left(1+L_3\times\frac{181}{360}\right)\left(1+L_4\times\frac{182}{360}\right)}$$

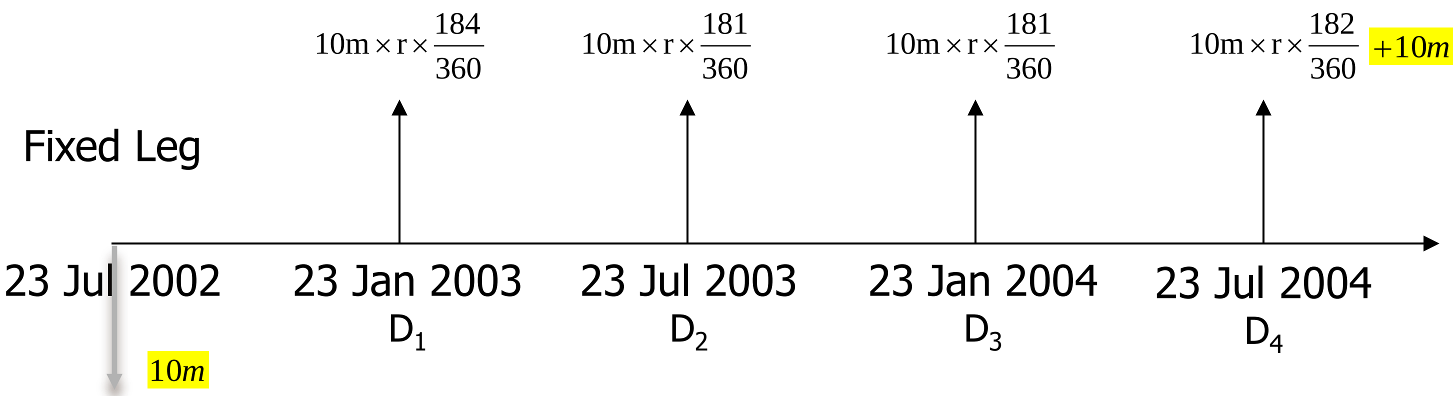
NPV of Floating Leg Cashflow



$$\text{NPV} = 10\text{m} - \frac{\left(10\text{m}\times L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)+10\text{m}\times L_2\times\frac{181}{360}}{\left(1+L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)} - \frac{10\text{m}\times L_3\times\frac{181}{360}}{\left(1+L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)\left(1+L_3\times\frac{181}{360}\right)}$$

$$-\frac{10\text{m}\times L_4\times\frac{182}{360}+10\text{m}}{\left(1+L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)\left(1+L_3\times\frac{181}{360}\right)\left(1+L_4\times\frac{182}{360}\right)}$$

NVP of Fixed Leg



$$-10m + 10m \times \left(r \times \frac{184}{360}\right) D_1 + 10m \times \left(r \times \frac{181}{360}\right) D_2 + 10m \times \left(r \times \frac{181}{360}\right) D_3 + 10m \times \left(1 + r \times \frac{182}{360}\right) D_4 = 0$$

$$\left(r \times \frac{184}{360}\right) D_1 + \left(r \times \frac{181}{360}\right) D_2 + \left(r \times \frac{181}{360}\right) D_3 + \left(1 + r \times \frac{182}{360}\right) D_4 = 1$$

Valuation of Swap (Cont)

where

P = hypothetical principal notional

t_i = day count fraction of each interest payment period i

C_i = cashflow at time period i

= $P*r*t_i$

D_i = discount factor at time i

D_n = discount factor at time n (i.e. at maturity)

r = swap par rate (fixed leg)

$$\mathrm{NPV} = -P + \sum_{i=1}^{n} C_i D_i + P D_n$$

$$\mathrm{NPV} = 0$$

$$D_n = \frac{\left(1 - r \sum_{i=1}^{n-1} t_i D_i\right)}{1 + r t_n}$$

$$r = \frac{(1 - D_n)}{\sum_{i=1}^{n} t_i D_i}$$

Pricing IRS from Futures or FRAs

- For each successive futures maturity, create a strip to generate a discount factor
- Use the series of discount factors to calculate the yield of a par swap

Pricing IRS from FRAs

3-month LIBOR		14.0625%	(91 days)
FRA	3 v 6	12.42%	(91 days)
	6 v 9	11.57%	(91 days)
	9 v 12	11.25%	(92 days)

What is the 1 year IRS rate, which pay fix and receive floating LIBOR on a quarterly basis?

Consider the following:

- Borrow USD 1 now for 3 months. At end of 3 months, repay:

$$\text{USD}\left(1+0.140625\times\frac{91}{360}\right)=\text{USD }1.03555$$

- Borrow USD 1.03555 and use FRA 3 v 6. Assume repayment at the end of 6 months:

$$\text{USD }1.03555\times\left(1+0.1242\times\frac{91}{360}\right)=\text{USD }1.06806$$

Pricing IRS from FRAs

- Borrow USD 1.03555 and use FRA 6 v 9. Assume repayment at the end of 9 months:

$$\text{USD } 1.06806 \times \left(1 + 0.1157 \times \frac{91}{360}\right) = \text{USD } 1.09929$$

- Borrow USD 1.03555 and use FRA 9 v 12. Assume repayment at the end of 12 months:

$$\text{USD } 1.09929 \times \left(1 + 0.1125 \times \frac{92}{360}\right) = \text{USD } 1.13090$$

Valuing the cashflows

$$1 = \left(\mathrm{i} \times \frac{91}{360} \times \frac{1}{1.03555}\right) + \left(\mathrm{i} \times \frac{91}{360} \times \frac{1}{1.06806}\right) + \left(\mathrm{i} \times \frac{91}{360} \times \frac{1}{1.09929}\right) + \left(\left(1 + \mathrm{i} \times \frac{92}{360}\right) \times \frac{1}{1.13090}\right)$$

$$\mathrm{i} = 12.35\%$$

Example

Base date = 17-7-2006

Day to Spot is 2, Start date = 19-7-2006

Pricing of a 4 year Interest Rate Swap

Maturity 4 years

Receive Fixed rate = ?, quarterly, Act / 365

Pay Floating rate = 3month HIBOR, quarterly, 30/365

Notional = $1mio

Example – Swap Pricing

Swap Pricing = Rec Fixed vs Pay Floating

INPUT		
Base Date	17-Jul-2006	
Day to Spot	2	
No. of Years	4	
Fixed Pymt Frq (no. of months)	3	
Notional (P)	1,000,000	
Start Date	19-Jul-2006	**(Spot Date)**
End Date	19-Jul-2010	

$$r = \frac{(1 - D_n)}{\sum_{i=1}^{n} t_i D_i}$$

OUTPUT

Calculate the Swap Rate

From	To	No. of days	Daycount fraction ti (Act/365)	Spot DF (extracted from Yield Curve)	ti*Di
19/07/2006	19/10/2006	92	0.252055	0.99204	0.250047
19/10/2006	19/01/2007	92	0.252055	0.98302	0.247774
19/01/2007	19/04/2007	90	0.246575	0.97403	0.240173
19/04/2007	19/07/2007	91	0.249315	0.96439	0.240436
19/07/2007	19/10/2007	92	0.252055	0.95522	0.240768
19/10/2007	19/01/2008	92	0.252055	0.94599	0.238441
19/01/2008	19/04/2008	91	0.249315	0.93679	0.233556
19/04/2008	19/07/2008	91	0.249315	0.92752	0.231246
19/07/2008	19/10/2008	92	0.252055	0.91826	0.231451
19/10/2008	19/01/2009	92	0.252055	0.90896	0.229108
19/01/2009	19/04/2009	90	0.246575	0.89984	0.221879
19/04/2009	19/07/2009	91	0.249315	0.89060	0.222040
19/07/2009	19/10/2009	92	0.252055	0.88131	0.222138
19/10/2009	19/01/2010	92	0.252055	0.87201	0.219795
19/01/2010	19/04/2010	90	0.246575	0.86291	0.212772
19/04/2010	19/07/2010	91	0.249315	0.85370	0.212840
				Σ ti*Di	3.694463
				Swap Rate ==>	**3.9600%**

r = (1-Dn)/(Σti*Di)

Example – Valuation of Swap

Validation of Swap at Inception : NPV = 0

INPUT

Fix Rate (r)	3.9600%
Notional (P)	1,000,000

$$NPV = -P + \sum_{i=1}^{n} C_i D_i + PD_n$$

Base Date	17-Jul-2006	
Start Date	19-Jul-2006	**(Spot Date)**
End Date	19-Jul-2010	

*** Di is spot Discount Factor*

OUTPUT

**** Valuation of Swaps with DF to Spot date**

From	To	No. of days	Fix Rate	Daycount fraction ti (Act/365)	Ci = P*r*t	Spot DF (extracted from Yield Curve)	PV
19/07/2006	19/07/2006	0	Initial Notional Exch (Hypothetical) =>		-1,000,000.00	1.00000	-1,000,000
19/07/2006	19/10/2006	92	3.9600%	0.252055	9,981.37	0.99204	9,902
19/10/2006	19/01/2007	92	3.9600%	0.252055	9,981.37	0.98302	9,812
19/01/2007	19/04/2007	90	3.9600%	0.246575	9,764.38	0.97403	9,511
19/04/2007	19/07/2007	91	3.9600%	0.249315	9,872.88	0.96439	9,521
19/07/2007	19/10/2007	92	3.9600%	0.252055	9,981.37	0.95522	9,534
19/10/2007	19/01/2008	92	3.9600%	0.252055	9,981.37	0.94599	9,442
19/01/2008	19/04/2008	91	3.9600%	0.249315	9,872.88	0.93679	9,249
19/04/2008	19/07/2008	91	3.9600%	0.249315	9,872.88	0.92752	9,157
19/07/2008	19/10/2008	92	3.9600%	0.252055	9,981.37	0.91826	9,165
19/10/2008	19/01/2009	92	3.9600%	0.252055	9,981.37	0.90896	9,073
19/01/2009	19/04/2009	90	3.9600%	0.246575	9,764.38	0.89984	8,786
19/04/2009	19/07/2009	91	3.9600%	0.249315	9,872.88	0.89060	8,793
19/07/2009	19/10/2009	92	3.9600%	0.252055	9,981.37	0.88131	8,797
19/10/2009	19/01/2010	92	3.9600%	0.252055	9,981.37	0.87201	8,704
19/01/2010	19/04/2010	90	3.9600%	0.246575	9,764.38	0.86291	8,426
19/04/2010	19/07/2010	91	3.9600%	0.249315	9,872.88	0.85370	8,428
19/07/2010	19/07/2010	0	Final Notional Exch (Hypothetical) =>		1,000,000.00	0.85370	853,699
						NPV	**0**

Valuing Swaps

Value the following IRS on 27 March 2002

Notional amount:	10 million
Start of swap:	23 July 2001
Maturity of swap:	23 July 2004
Receive:	7.4% (annual 30/360)
Pay:	LIBOR (semi-annual ACT/360)
Previous LIBOR fixing:	9.3% from 23 Jan 2002 to 23 Jul 2002

Zero-coupon discount factors from 27 Mar 2002:

23 Jul 2002:	0.9703	23 Jan 2003:	0.9249
23 Jul 2003:	0.8825	23 Jan 2004:	0.8415
23 Jul 2004:	0.8010		

Swaps Cashflows

Dates	Fixed Leg	Floating Leg	
23 Jul 2002 :	$+10\text{m}\times 7.4\%$	$-10\text{m}\times 9.3\%\times\frac{181}{360}$	-10m
23 Jan 2003 :		$-10\text{m}\times L_1\times\frac{184}{360}$	$+10\text{m}\times L_1\times\frac{184}{360}$
23 Jul 2003 :	$+10\text{m}\times 7.4\%$	$-10\text{m}\times L_2\times\frac{181}{360}$	$+10\text{m}\times L_2\times\frac{181}{360}$
23 Jan 2004 :		$-10\text{m}\times L_3\times\frac{181}{360}$	$+10\text{m}\times L_3\times\frac{181}{360}$
23 Jul 2004 :	$+10\text{m}\times 7.4\%$	$-10\text{m}\times L_4\times\frac{182}{360}$	$+10\text{m}\times L_4\times\frac{182}{360}+10\text{m}$

where L_1, L_1, L_1 and L_1 are LIBOR

Reversing a Swap

- To close out a previous position
- Transact another swap in the opposite direction for the remaining term of the existing swap
- Fixed rate unlikely to be the same
- Net receipt or payment on each future payment date

Example

The original swap details:
Notional amount = 10 mio
Start date of swap = 23 Jul 2001
Maturity of swap = 23 Jul 2004
Receive Leg = 7.4% (annual 30/360) Fixed
Pay Leg = LIBOR (semi-annual ACT/360) Floating

On 23 Jul 2002, we decide to reverse the swap. The same counterparty quotes a swap rate of 8.25% for the remaining 2 years.

Discount factors from 27 Mar 2002:
23 Jul 2003: 0.9250 23 Jul 2004: 0.8530

LIBOR-based flows on the two swaps offset each other exactly. The remaining flows are:

Date	Original swap	Reverse swap	Net cashflows
23 Jul 2003	$+10\text{m} \times 7.4\%$	$-10\text{m} \times 8.25\%$	-85,000
23 Jul 2004	$+10\text{m} \times 7.4\%$	$-10\text{m} \times 8.25\%$	-85,000

The NPV of the net cashflows is ? -151,130

Example – Further Explain

In other words, the cashflow leg that relevant in swap unwind on 23-Jul-2002 ...

Start Date	23-Jul-2001	(Spot Date)				
End Date	23-Jul-2004					
OUTPUT	(Original Swap)					
From	To	No. of days	Fix Rate (Original Swap)	Daycount fraction ti (30/360)	Ci = P*r*t	
23/07/2001	23/07/2001	0	Initial Notional Exch (Hypothetical) =>		-10,000,000.00	
23/07/2001	23/07/2002	360	7.4000%	1.000000	740,000.00	Realised Cashflow
23/07/2002	23/07/2003	360	7.4000%	1.000000	740,000.00	Unrealised Cashflow
23/07/2003	23/07/2004	360	7.4000%	1.000000	740,000.00	Unrealised Cashflow
23/07/2004	23/07/2004	0	Final Notional Exch (Hypothetical) =>		10,000,000.00	

The NPV of the net cashflows is ?

Notional (P)	10,000,000						
From	To	Fix Rate (Original Swap)	Fix Rate (Unwind Swap)	Fix Rate Diff. (rd)	Net Cash Flow Ci = P*rd*t	Spot DF	PV
23/07/2002	23/07/2003	7.4000%	8.2500%	-0.85%	-85,000	0.925	-78,625
23/07/2003	23/07/2004	7.4000%	8.2500%	-0.85%	-85,000	0.853	-72,505
				-1.70%	-170,000		-151,130

Hence, we have to pay 151,130 to the same counterparty to close out the swap.