AAE2004 Introduction to Aviation Systems
AAE
Design of Path Planning Algorithm for Aircraft Operation

Second Week

Dr Li-Ta Hsu and Dr Kam Hung NG

Assisted by

Miss Hiu Yi HO (Queenie), Miss Yan Tung LEUNG (Nikki)

Lecturer's Information

- **Instructor**: Dr Li-Ta HSU
- **Office**: QR828
- **Phone**: 3400-8061
- **Email**: lt.hsu@polyu.edu.hk
- Office Hour: by appointment

- **Expertise:** GPS navigation, Autonomous driving, Pedestrian localization using Smartphone, Sensor Integration

Ground Rules

For students

- Try to speak as much English as possible.
- Participate the class activates assigned.

For teaching staffs

- Reply your email with 3 working day.
- Open to any question regards to the subject

For us!

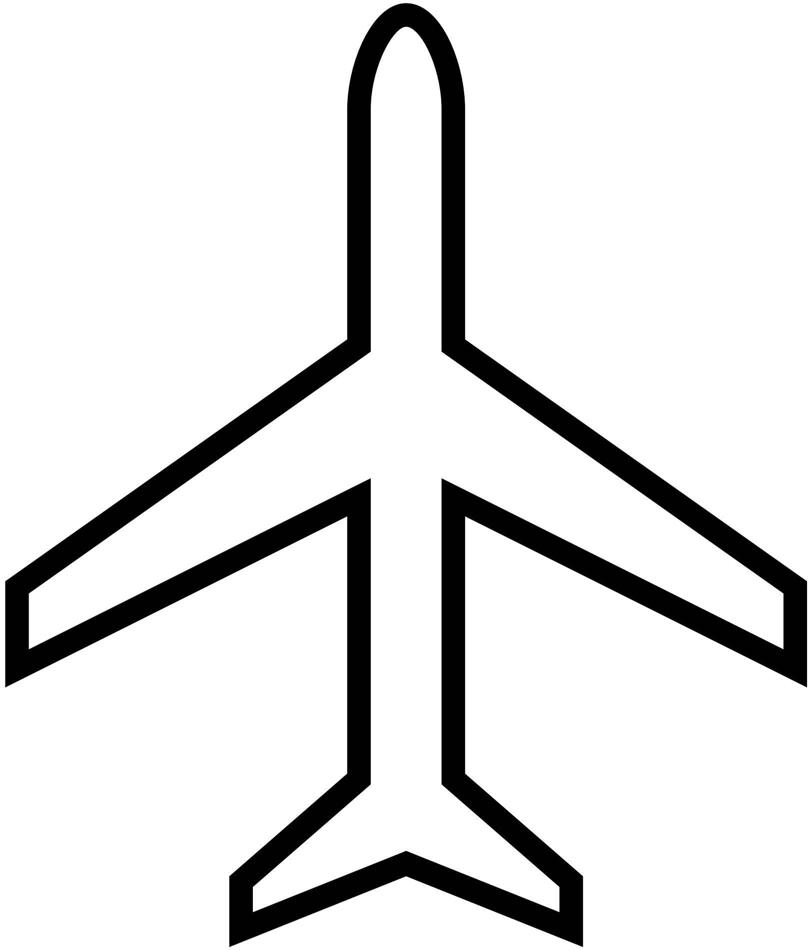
- Keep an open mind—enter the classroom dialogue with the expectation of learning something new. Look forward to learning about–and being challenged by–ideas, questions, and points of view that are different than your own.
- Arrive on time to the class and finish the class on time

Necessary Information

- Course Repository link: https://github.com/IPNL-POLYU/PolyU_AAE2004_Github_Project
- TA Information & Contact:
  - Group 1-5: Queenie Ho (hiu-yi.ho@connect.polyu.hk)
  - Group 6-10: Nikkie Leung (yan-tung.leung@connect.polyu.hk)

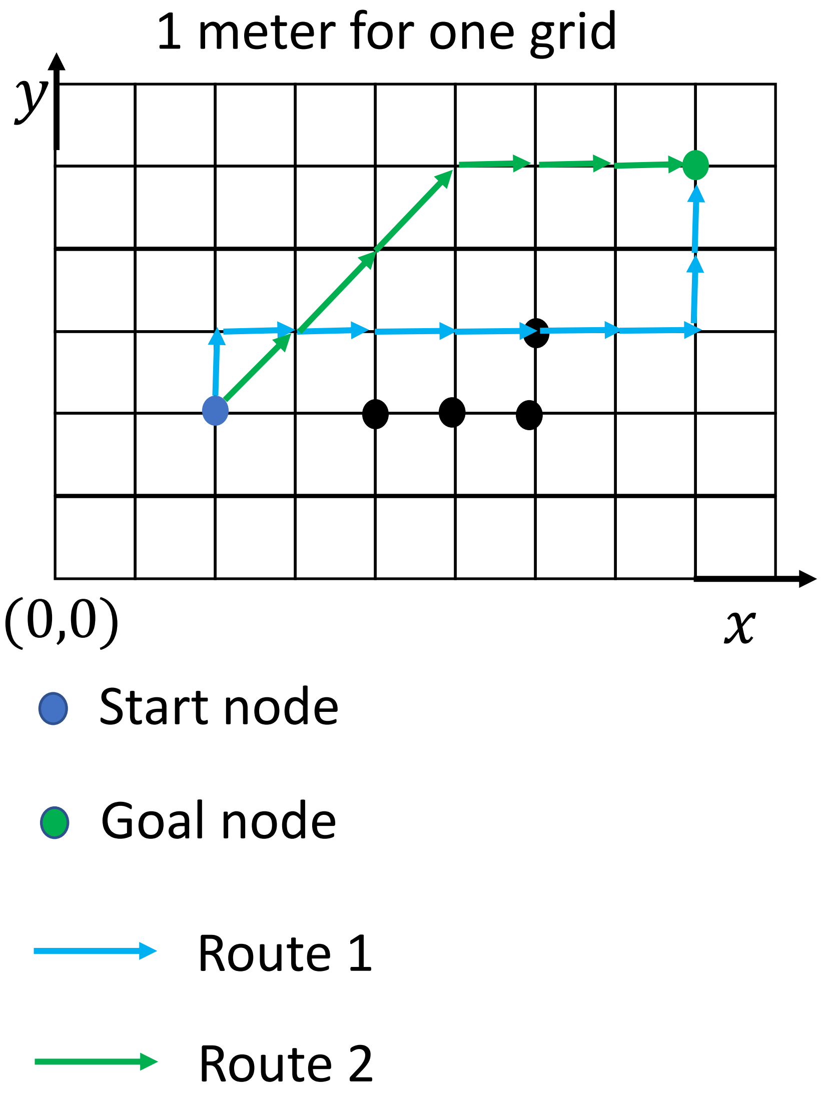
Week 2 Content

1. Introduction to A* Path Planning Algorithm
2. Cost Intensive Areas



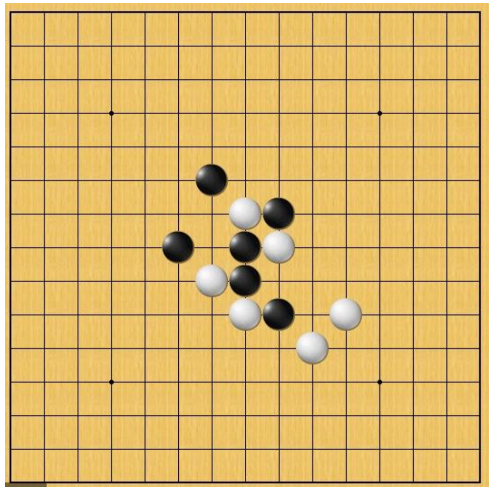
Introduction to A* Path Planning Algorithm

Definition of Path Planning



•**Node** — All potential position you can go across with a unique position (x, y)

•**Search Space** — A collection of nodes, like all board positions of a board game.

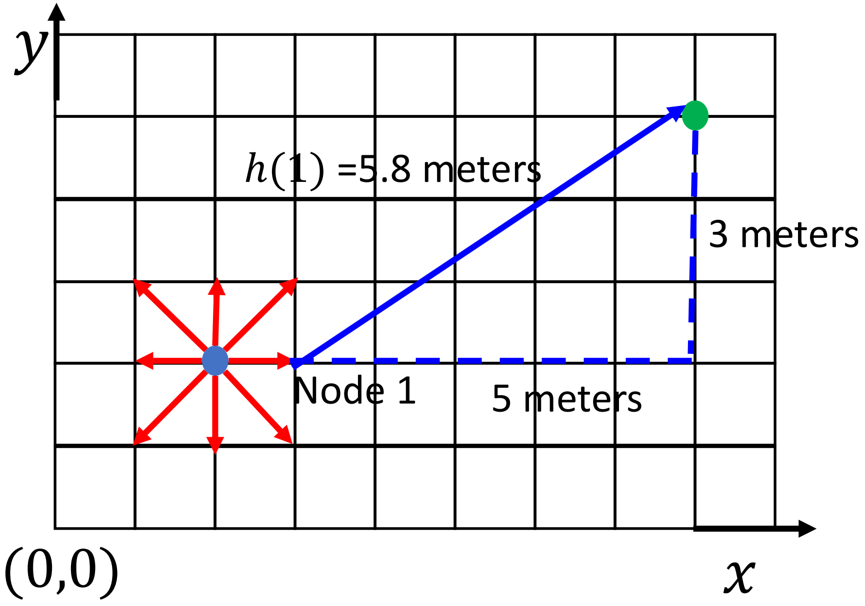


Gobang

•**Objective of path planning**— Find the shortest routes with smallest cost from start node to goal node.

How to find the shortest route!

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

•$\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)

•$\boldsymbol{h}(x,y)$ — this represents the heuristic ***estimated cost*** from node (x,y) to the goal node.

•$\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

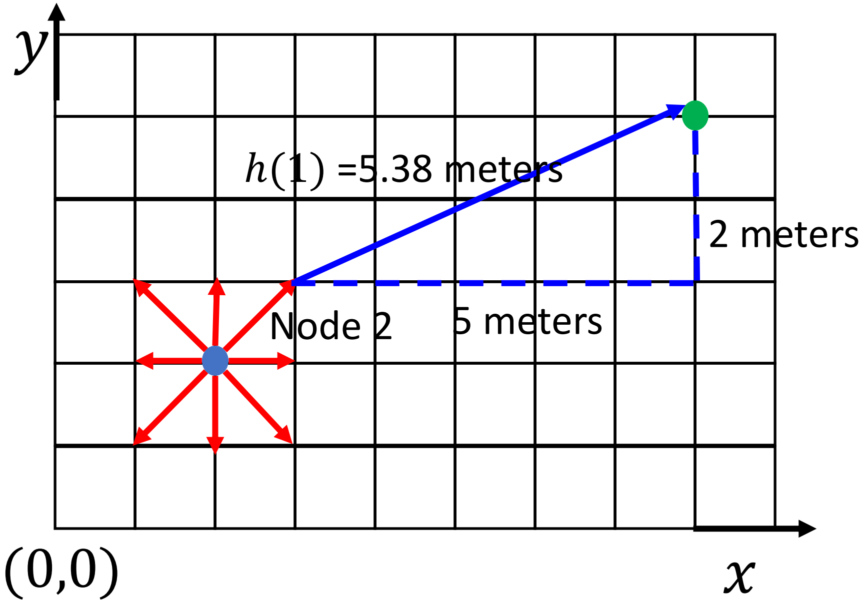
8 neighboring node and the cost can be calculated as follows!

Node 1:

$$f(3,2) = g(3,2) + h(3,2) = 6.8 \text{ meters}$$

with $g(3,2) = 1$ meter and $h(3,2) = 5.8$ meters

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

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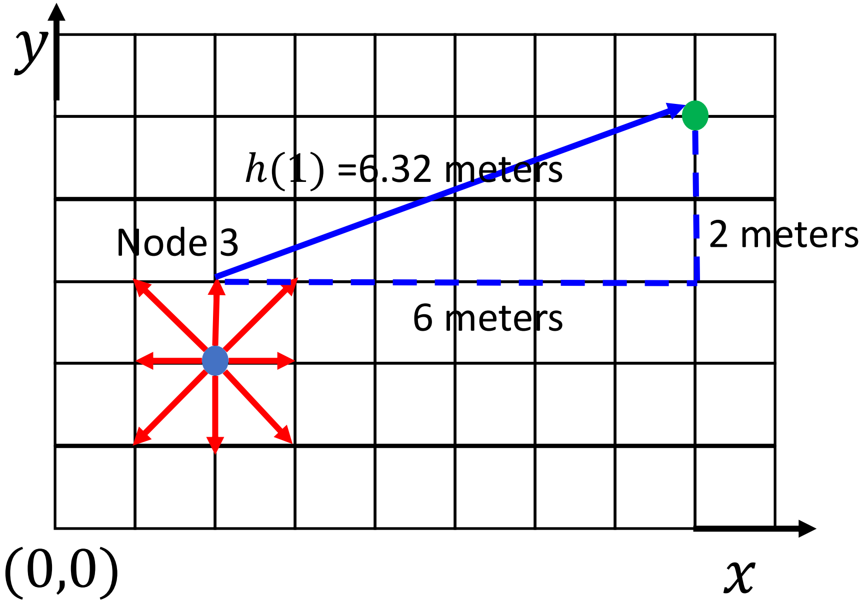
Node 2:

$$f(3,3) = g(3,3) + h(3,3)$$ =6.79 meters

with $g(3,3) = \sqrt{2}$ meter and $h(3,3) = 5.38$ meters

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

•$\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)

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8 neighboring node and the cost can be calculated as follows!

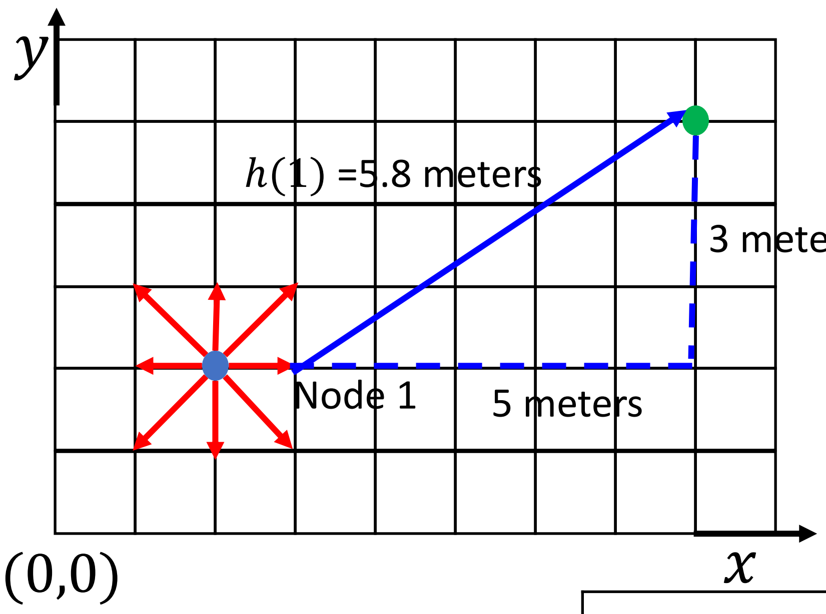
Node 3:

$$f(2,3) = g(2,3) + h(2,3) = 7.32 \text{ meters}$$

with $g(2,3) = 1$ meter and $h(2,3) = 6.32$ meters

Similar cost calculation method for other 5 nodes

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

- $\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)
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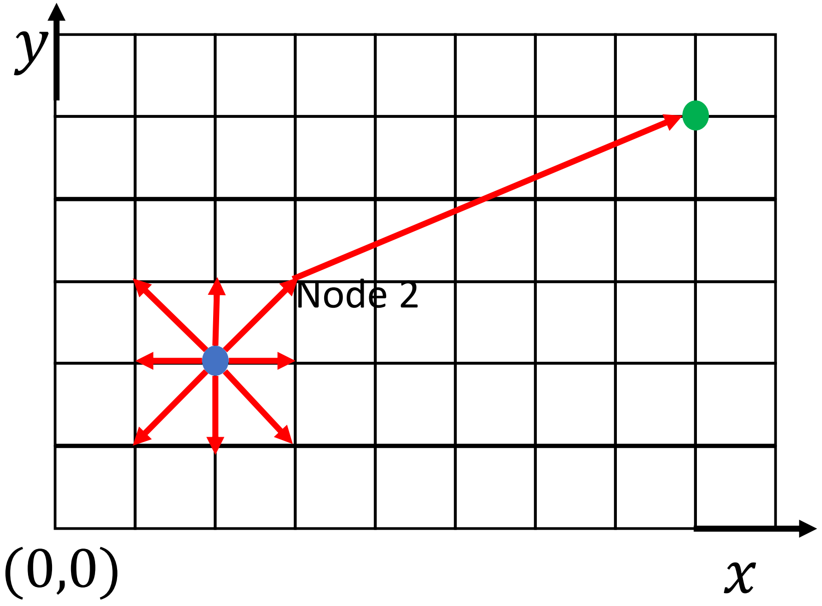
- $\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

Node (x,y)	Node 1 (x,y)	Node 2 (x,y)	Node 3 (x,y)	Node 4 (x,y)	Node 5 (x,y)	Node 6 (x,y)	Node 7 (x,y)	Node 8 (x,y)
$g(x,y)$	1	1.414	1	1.414	1	1.414	1	1.414
$h(x,y)$	5.8	5.38	6.32	7.28	7.62	8.06	7.21	6.40
$f(x,y)$	6.8	6.79	7.32	8.694	8.62	9.474	8.21	7.814

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

•$\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)

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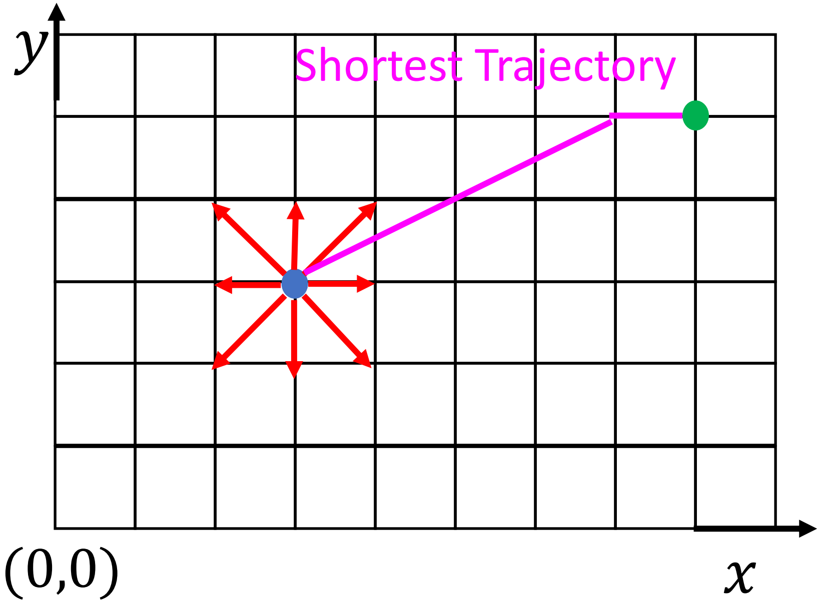
•$\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

8 neighboring node and the cost can be calculated as follows!

Node 2 leads to smallest cost

Calculate the cost of node



Start node

Goal node

1 meter for one grid

Definition of cost:

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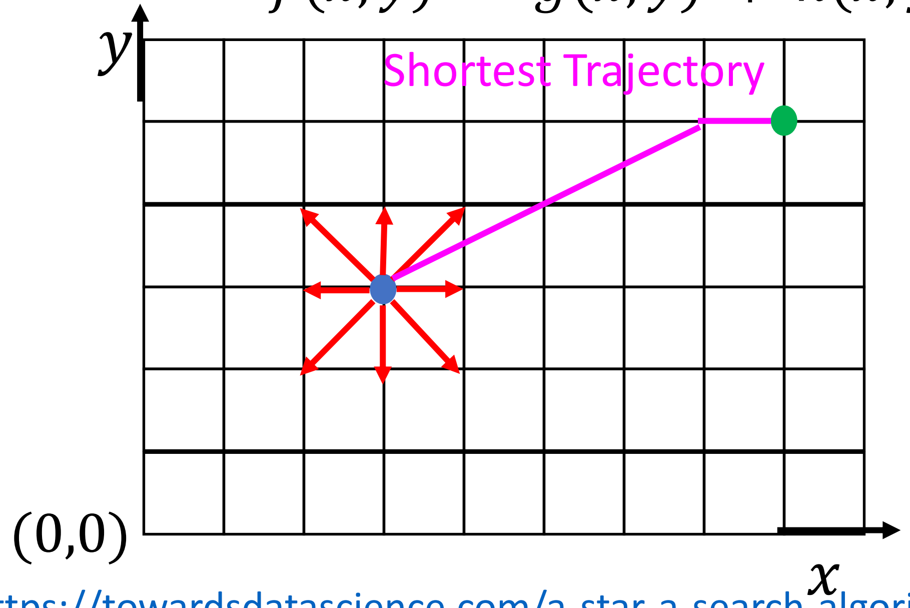
8 neighboring node and the cost can be calculated as follows!

Search from the neighbouring node with smallest cost until reaching the goal!

A star method example

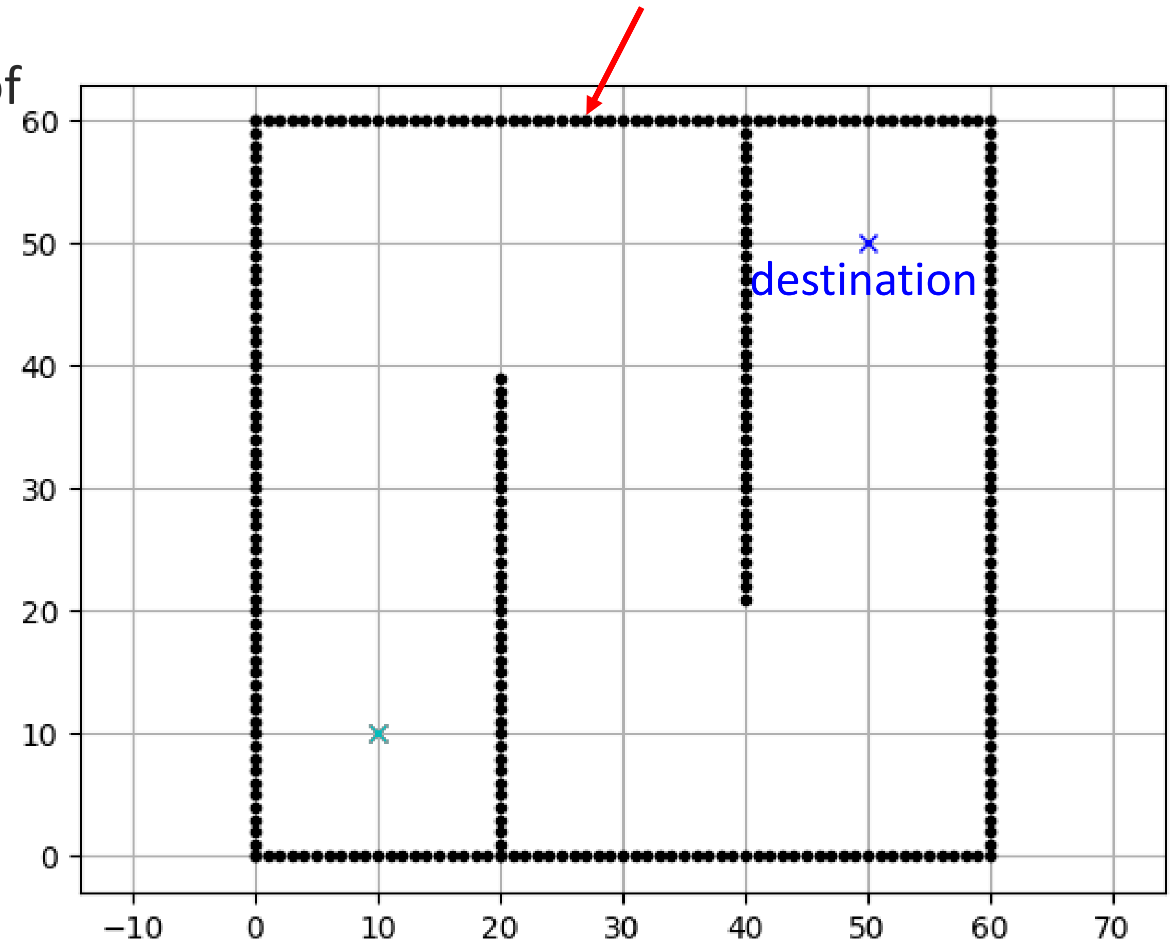
Each time A* enters a node, it calculates the cost, f(n)(n being the neighboring node), to travel to all of the neighboring nodes, and then enters the node with the lowest value of f(n).
These values we calculate using the following formula:

$$f(x,y) = g(x,y) + h(x,y)$$



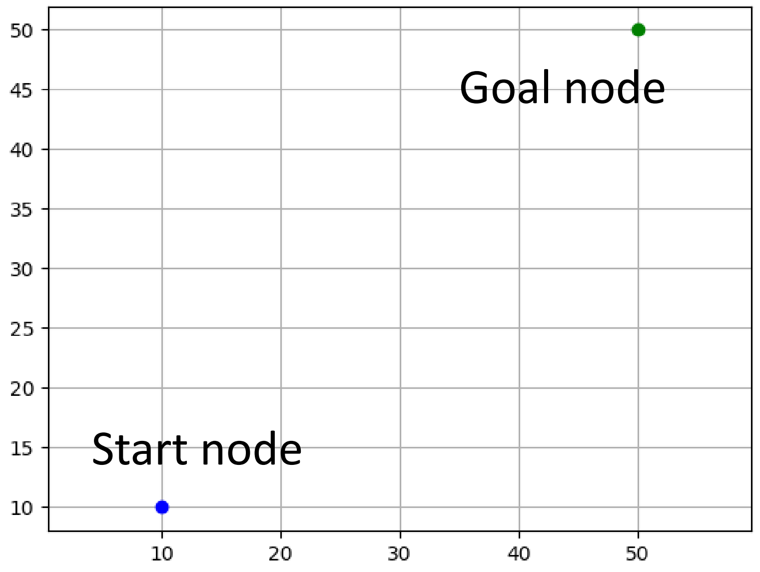
https://towardsdatascience.com/a-star-a-search-algorithm-eb495fb156bb

Wall (obstacles) cannot be touched!



Source: PythonRobotics

Code: set up start and goal node

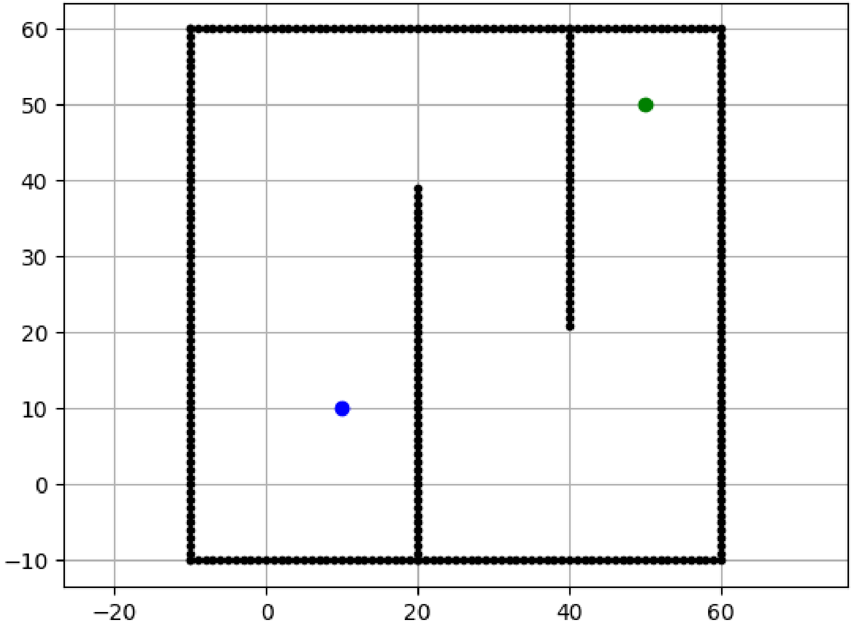


Set up the start and goal nodes using the code

```
# start and goal position
sx = 10.0  # [m]
sy = 10.0  # [m]
gx = 50.0  # [m]
gy = 50.0  # [m]
grid_size = 2  # [m]
```

- Start node
- Goal node

Code: set up obstacle

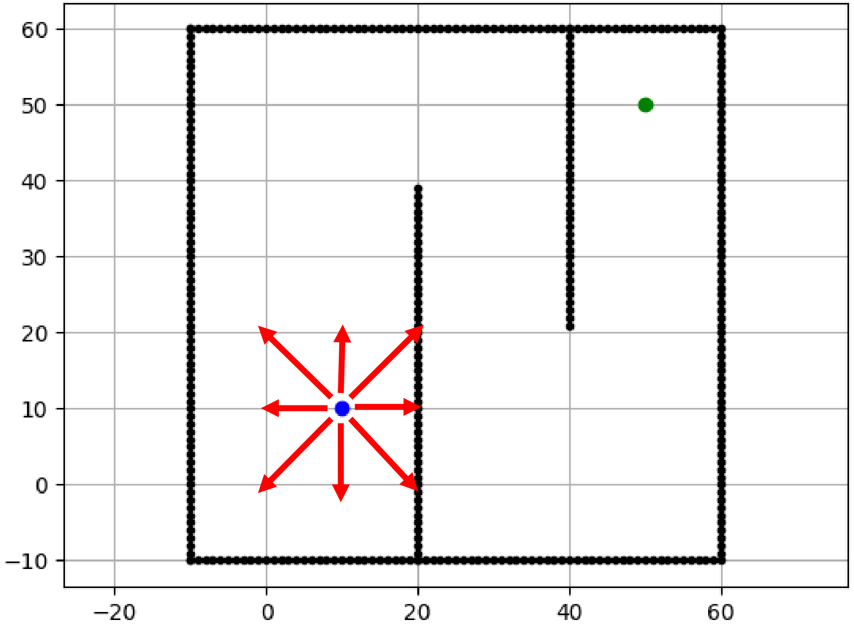


- Start node
- Goal node
- Obstacle (wall)

Set up the obstacle using the code

```python
# set obstacle positions
ox, oy = [], []
for i in range(-10, 60): # draw the button border
    ox.append(i)
    oy.append(-10.0)
for i in range(-10, 60):
    ox.append(60.0)
    oy.append(i)
for i in range(-10, 61):
    ox.append(i)
    oy.append(60.0)
for i in range(-10, 61):
    ox.append(-10.0)
    oy.append(i)
for i in range(-10, 40):
    ox.append(20.0)
    oy.append(i)
for i in range(0, 40):
    ox.append(40.0)
    oy.append(60.0 - i)
```

Code: neighboring node search



neighboring node search

```python
def get_neighbouring_node(): # the cost of the surrounding 8 points
    # dx, dy, cost
    motion = [[1, 0, 1],
              [0, 1, 1],
              [-1, 0, 1],
              [0, -1, 1],
              [-1, -1, math.sqrt(2)],
              [-1, 1, math.sqrt(2)],
              [1, -1, math.sqrt(2)],
              [1, 1, math.sqrt(2)]]

    return motion
```

- Start node
- Goal node
- Obstacle (wall)

Code: cost calculation

Heuristic cost$g(x, y)$ calculation

```
def calc_heuristic(n1, n2):
    w = 1.0  # weight of heuristic
    d = w * math.hypot(n1.x - n2.x, n1.y - n2.y)
    return d
```

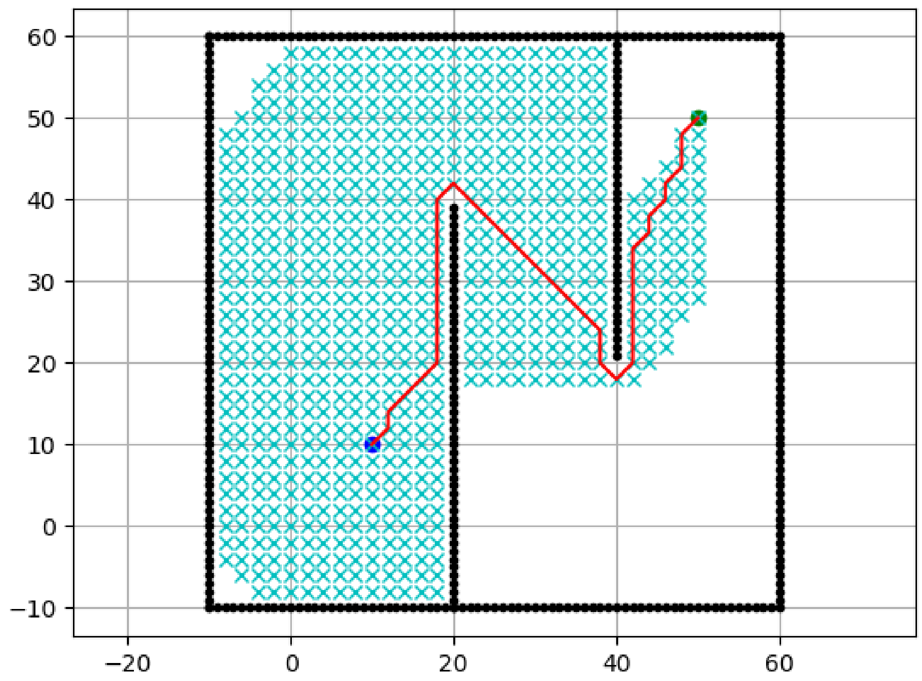
exact cost $g(x, y)$ calculation

```
node = self.Node(current.x + self.motion[i][0],
                 current.y + self.motion[i][1],
                 current.cost + self.motion[i][2], c_id)
```

Code: calculation of final path

```
def calc_final_path(self, goal_node, closed_set):
    # generate final course
    rx, ry = [self.calc_grid_position(goal_node.x, self.min_x)], [
        self.calc_grid_position(goal_node.y, self.min_y)] # save the goal node as the first point
    parent_index = goal_node.parent_index
    while parent_index != -1:
        n = closed_set[parent_index]
        rx.append(self.calc_grid_position(n.x, self.min_x))
        ry.append(self.calc_grid_position(n.y, self.min_y))
        parent_index = n.parent_index

    return rx, ry
```



Cost Intensive Areas

Flight planning considering trip cost

The fundamental rationale of the cost index concept is to achieve minimum trip cost by means of a trade-off between operating costs per hour and incremental fuel burn.

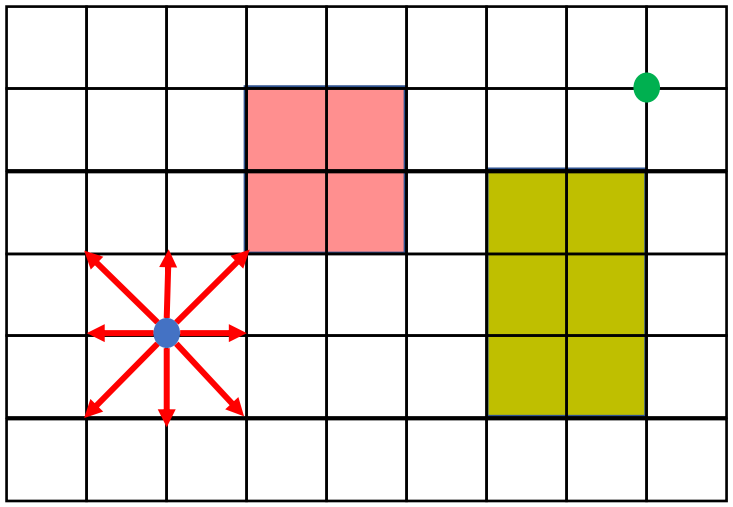
$$\mathrm{C} = C_F \cdot \Delta F + C_T \cdot \Delta T + C_c$$

With

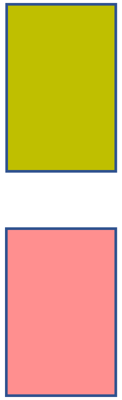
- C_F=cost of fuel per kg
- C_T=time related cost per minute of flight
- C_c=fixed cost independent of time
- C_T=time related cost per minute of flight
- ΔF=trip fuel (e.g. 3000kg/h)
- ΔT=trip Time (e.g. 8 hours from Hong Kong to Paris)

Can we consider this cost to our path planning to imitate the path planning for flights?

Flight planning considering trip cost



- Start node
- Goal node



Cost Intensive Areas: the cost for flying through such area is increased due to airflow, legal restrictions and other reasons. (additional cost ΔF_a, ΔT_a)

Cost can be calculated using the following formula:

$$f(x,y) = g(x,y) + h(x,y)$$

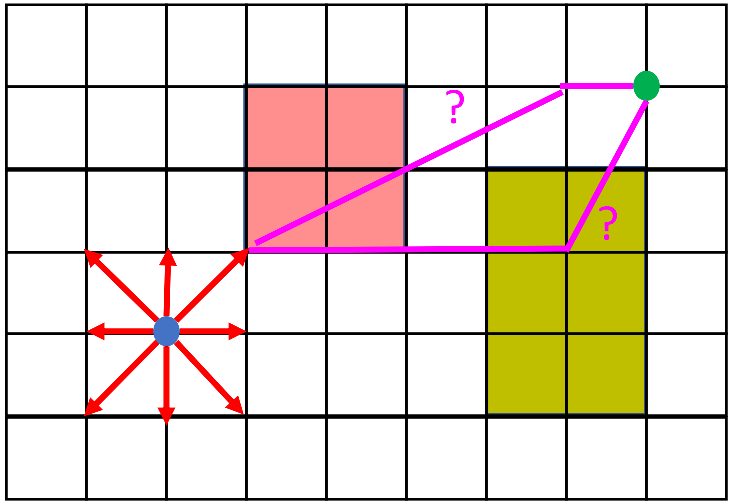
One white grid with cost as follows for $g(x,y)$&$h(x,y)$:

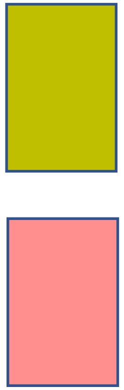
$$\mathrm{C} = C_F \cdot \Delta F + C_T \cdot \Delta T + C_c$$

One colored grid with cost as follows for $g(x,y)$&$h(x,y)$:

$$\mathrm{C} = C_F \cdot (\Delta F + \Delta F_a(x,y)) + C_T \cdot (\Delta T + \Delta T_a(x,y)) + C_c$$

How we choose the routes ?





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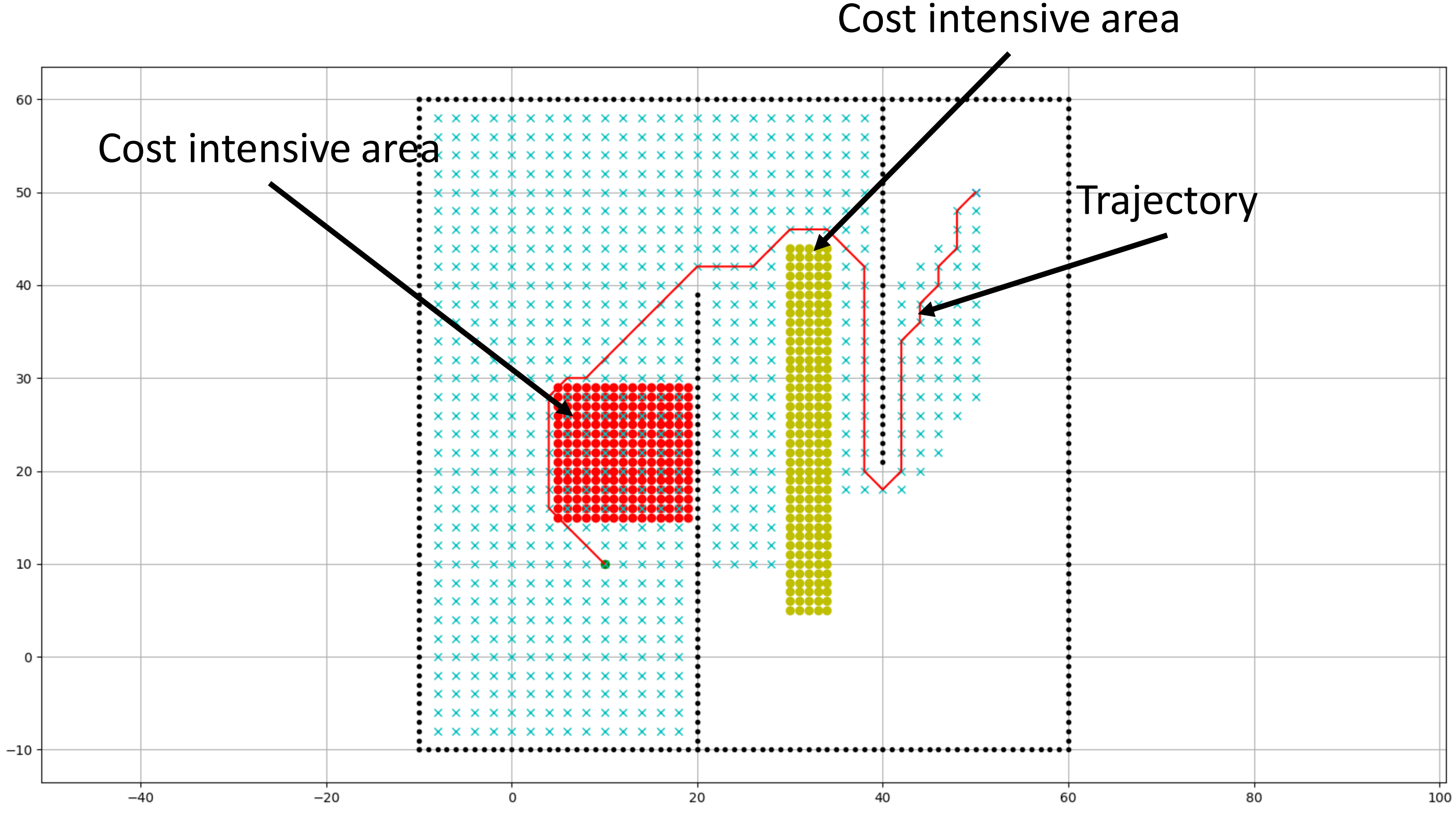
$$\mathrm{C} = C_F \cdot (\Delta F + \Delta F_a(x, y)) + C_T \cdot (\Delta T + \Delta T_a(x, y)) + C_c$$

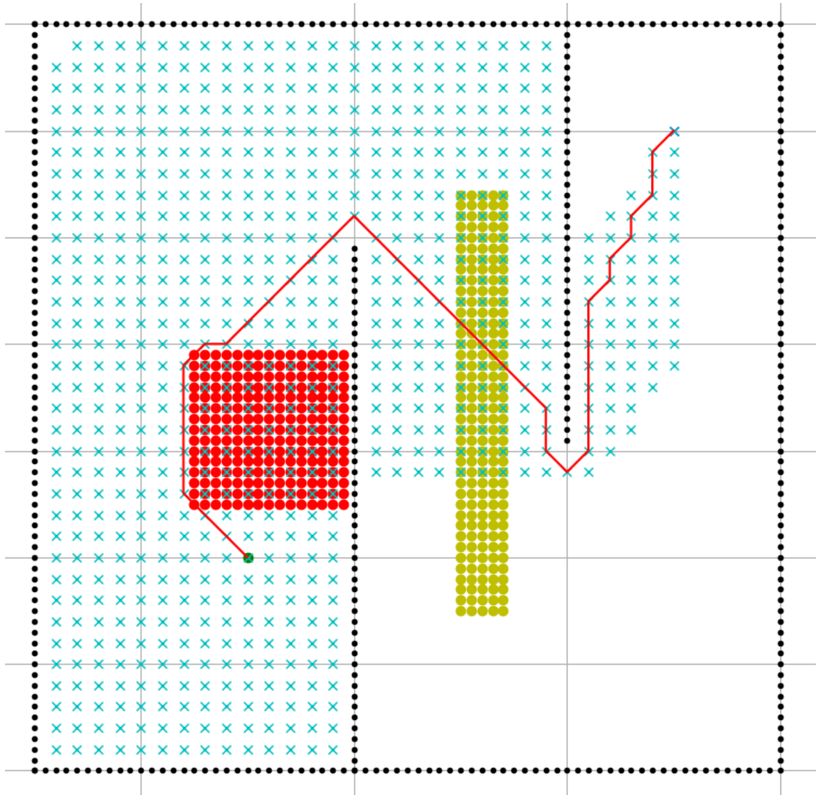
It depends on the ΔF_a and ΔT_a

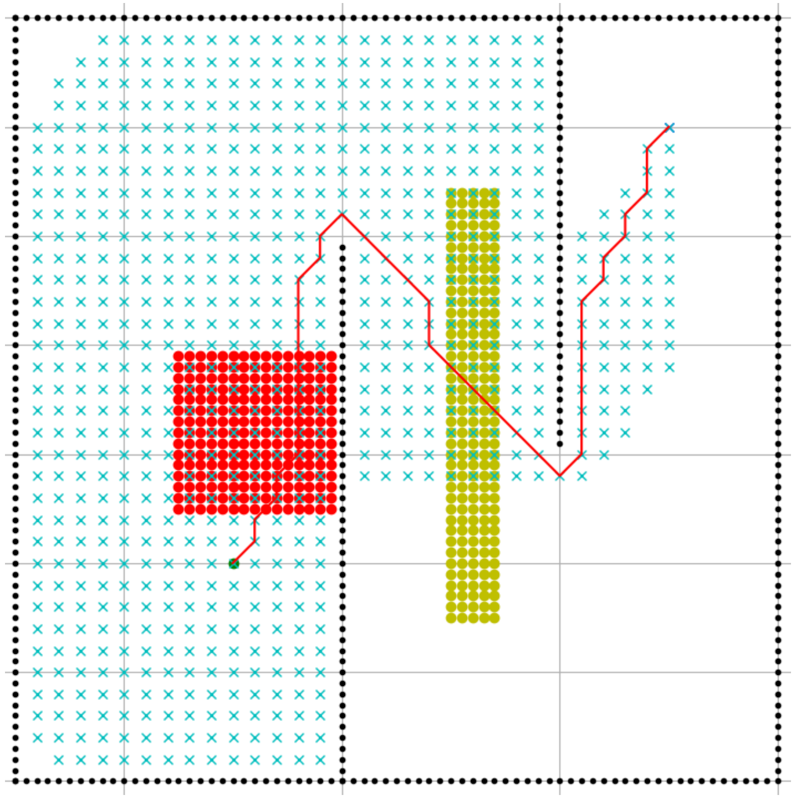
Example route planning



Avoiding the Cost intensive areas if their cost is too high?

Example route planning





Go through the Cost intensive area if their additional cost is quite small?

Path Planning Project

- You will be creating and completing your own path planning program based on groups
- You can find the project tasks / requirements in the Week 3 Slides
- Additional resources could be found inside the course GitHub repository
    - Video tutorial
    - Tutorial slides