ENG1003 Freshman Seminar for Engineering AAE Design of Path Planning Algorithm for Aircraft Operation

Week 4: Path Planning Algorithm and Python Robotics

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Assisted by

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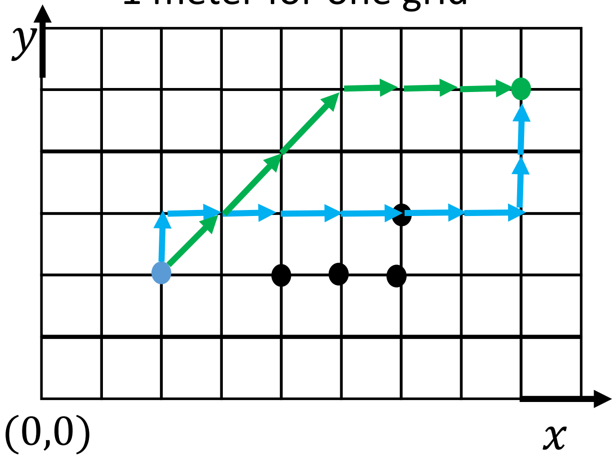
A* Path Planning Algorithm

Definition of Path Planning

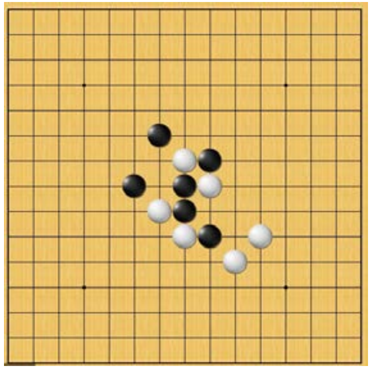
1 meter for one grid



- Start node
- Goal node
- Route 1
- Route 2

•**Node** — All potential position you can go across with a unique position (x, y)

•**Search Space** — A collection of nodes, like all board positions of a board game.

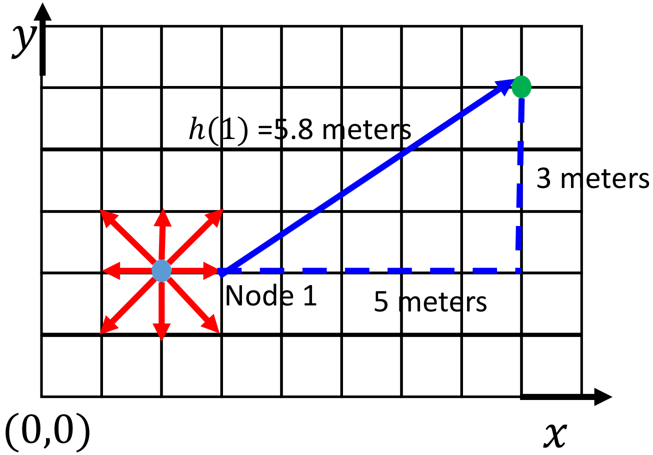


Gobang

•**Objective of path planning**— Find the shortest routes with smallest cost from start node to goal node.

How to find the shortest route!

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

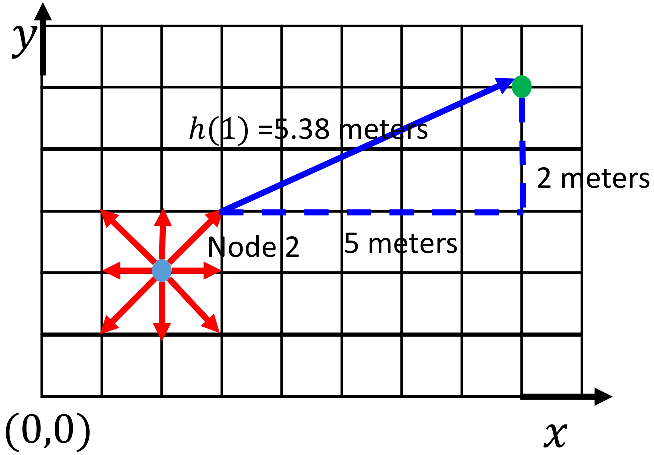
- $g(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)
- $h(x,y)$ — this represents the heuristic ***estimated cost*** from node (x,y) to the goal node.
- $f(x,y)$ —cost of the neighboring node (x,y)

8 neighboring node and the cost can be calculated as follows!

Node 1:

$$f(3,2) = g(3,2) + h(3,2) = 6.8 \text{ meters}$$
with $g(3,2) = 1$ meter and $h(3,2) = 5.8$ meters

Path Planning using A Star Method



- Start node
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•$\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

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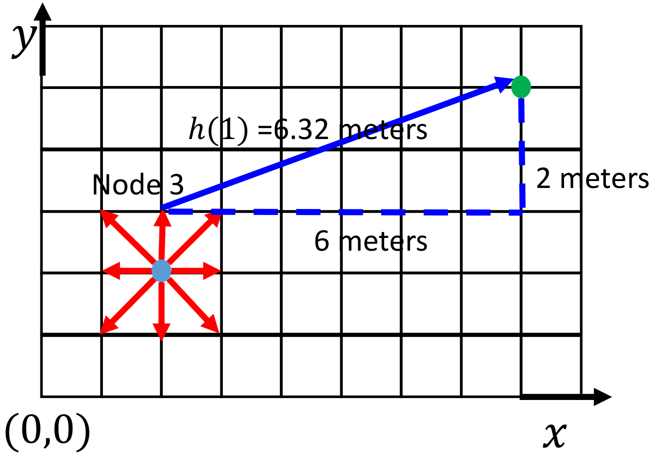
Node 2:

$f(3,3) = g(3,3) + h(3,3)$ =6.79 meters

with $g(3,3) = \sqrt{2}$ meter and $h(3,3) = 5.38$ meters

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

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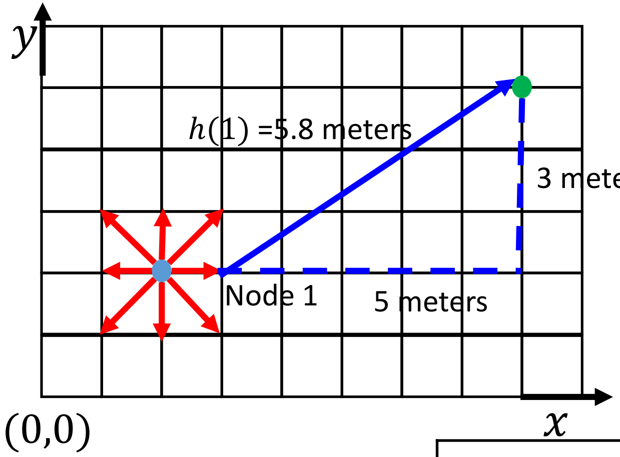
8 neighboring node and the cost can be calculated as follows!

Node 3:

$f(2,3) = g(2,3) + h(2,3)$ =7.32 meters
with $g(2,3) = 1$ meter and $h(2,3) = 6.32$ meters

Similar cost calculation method for other 5 nodes

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

- $\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)
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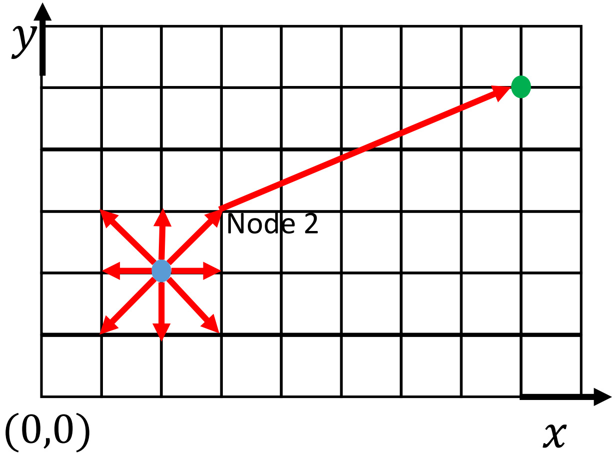
Node (x,y)	Node 1 (x,y)	Node 2 (x,y)	Node 3 (x,y)	Node 4 (x,y)	Node 5 (x,y)	Node 6 (x,y)	Node 7 (x,y)	Node 8 (x,y)
$g(x,y)$	1	1.414	1	1.414	1	1.414	1	1.414
$h(x,y)$	5.8	5.38	6.32	7.28	7.62	8.06	7.21	6.40
$f(x,y)$	6.8	6.79	7.32	8.694	8.62	9.474	8.21	7.814

Path Planning using A Star Method



Start node

Goal node

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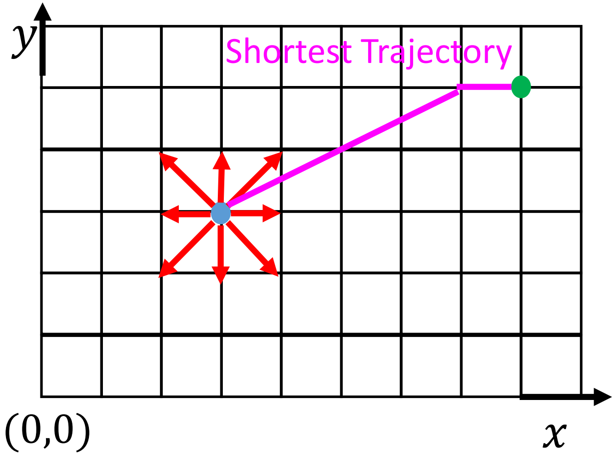
8 neighboring node and the cost can be calculated as follows!

Node 2 leads to smallest cost

Calculate the cost of node



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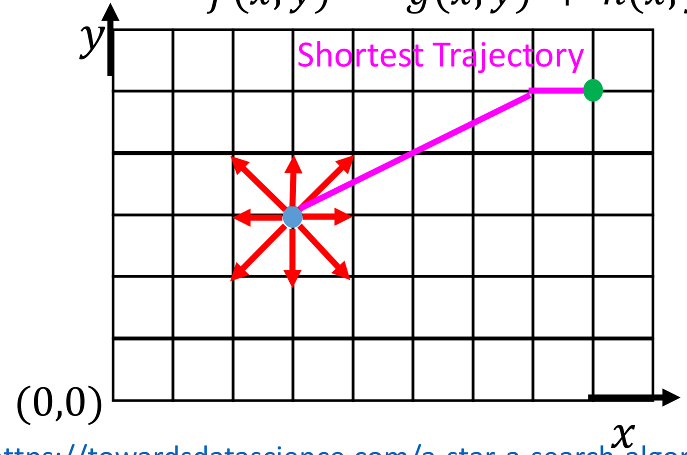
8 neighboring node and the cost can be calculated as follows!

Search from the neighbouring node with smallest cost until reaching the goal!

A star method example

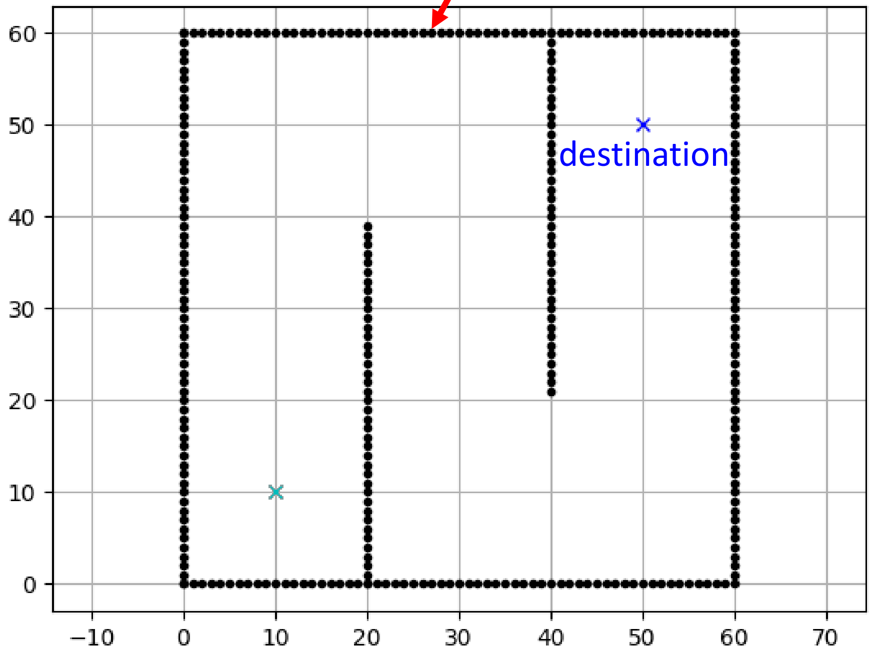
Each time A* enters a node, it calculates the cost, f(n)(n being the neighboring node), to travel to all of the neighboring nodes, and then enters the node with the lowest value of f(n). These values we calculate using the following formula:

$$f(x,y) = g(x,y) + h(x,y)$$



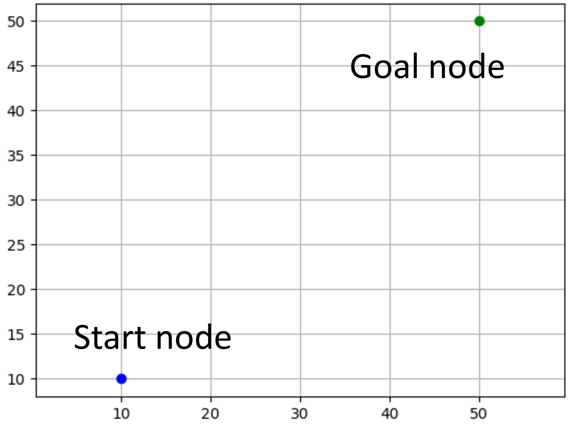
https://towardsdatascience.com/a-star-a-search-algorithm-eb495fb156bb

Wall (obstacles) cannot be touched!



Source: PythonRobotics

Code: set up start and goal node

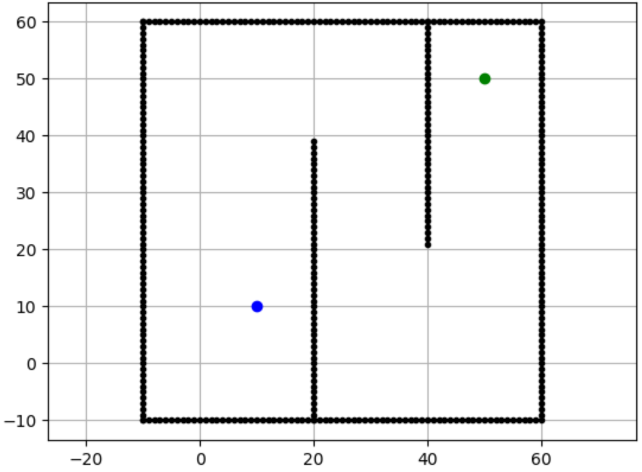


Set up the start and goal nodes using the code

```
# start and goal position
sx = 10.0  # [m]
sy = 10.0  # [m]
gx = 50.0  # [m]
gy = 50.0  # [m]
grid_size = 2  # [m]
```

- Start node
- Goal node

Code: set up obstacle

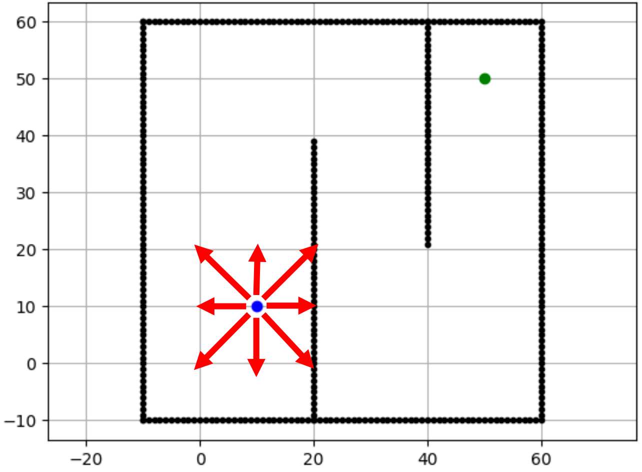


- Start node
- Goal node
- Obstacle (wall)

Set up the obstacle using the code

```
# set obstacle positions
ox, oy = [], []
for i in range(-10, 60): # draw the button border
    ox.append(i)
    oy.append(-10.0)
for i in range(-10, 60):
    ox.append(60.0)
    oy.append(i)
for i in range(-10, 61):
    ox.append(i)
    oy.append(60.0)
for i in range(-10, 61):
    ox.append(-10.0)
    oy.append(i)
for i in range(-10, 40):
    ox.append(20.0)
    oy.append(i)
for i in range(0, 40):
    ox.append(40.0)
    oy.append(60.0 - i)
```

Code: neighboring node search



- Start node
- Goal node
- Obstacle (wall)

neighboring node search

```python
def get_neighbouring_node(): # the cost of the surrounding 8 points
    # dx, dy, cost
    motion = [[1, 0, 1],
              [0, 1, 1],
              [-1, 0, 1],
              [0, -1, 1],
              [-1, -1, math.sqrt(2)],
              [-1, 1, math.sqrt(2)],
              [1, -1, math.sqrt(2)],
              [1, 1, math.sqrt(2)]]

    return motion
```

Code: cost calculation

Heuristic cost $g(x, y)$ calculation

```python
def calc_heuristic(n1, n2):
    w = 1.0  # weight of heuristic
    d = w * math.hypot(n1.x - n2.x, n1.y - n2.y)
    return d
```

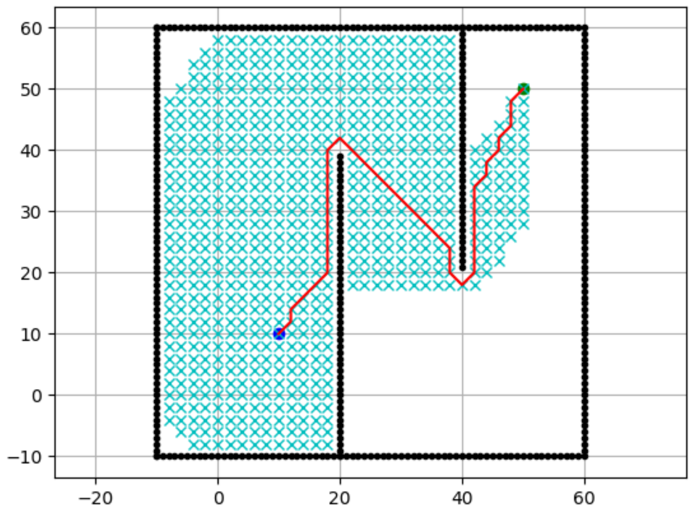
exact cost $g(x, y)$ calculation

```python
node = self.Node(current.x + self.motion[i][0],
                 current.y + self.motion[i][1],
                 current.cost + self.motion[i][2], c_id)
```

Code: calculation of final path

```python
def calc_final_path(self, goal_node, closed_set):
    # generate final course
    rx, ry = [self.calc_grid_position(goal_node.x, self.min_x)], [
        self.calc_grid_position(goal_node.y, self.min_y)] # save the goal node as the first point
    parent_index = goal_node.parent_index
    while parent_index != -1:
        n = closed_set[parent_index]
        rx.append(self.calc_grid_position(n.x, self.min_x))
        ry.append(self.calc_grid_position(n.y, self.min_y))
        parent_index = n.parent_index

    return rx, ry
```



Additional Cost.

Flight planning considering trip cost

The fundamental rationale of the cost index concept is to achieve minimum trip cost by means of a trade-off between operating costs per hour and incremental fuel burn.

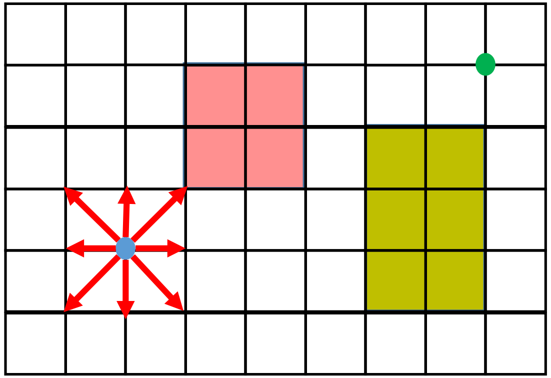
$$\mathrm{C} = C_F \cdot \Delta F + C_T \cdot \Delta T + C_c$$

With

- C_F=cost of fuel per kg
- C_T=time related cost per minute of flight
- C_c=fixed cost independent of time
- C_T=time related cost per minute of flight
- ΔF=trip fuel (e.g. 3000kg/h)
- ΔT=trip Time (e.g. 8 hours from Hong Kong to Paris)

Can we consider this cost to our path planning to imitate the path planning for flights?

Flight planning considering trip cost



- Start node
- Goal node

Fuel-consuming area: the volume of fuel consumption is twice larger than other area duet to unstable airflow. (additional cost ΔF_a)

Time-consuming area: the flying speed is limited due to the air traffic control. (additional cost ΔT_a)

Cost can be calculated using the following formula:

$$f(x,y) = g(x,y) + h(x,y)$$

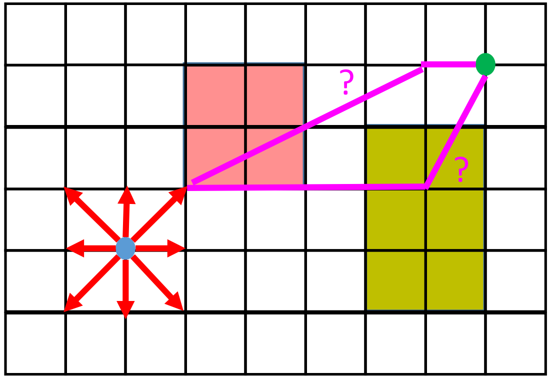
One white grid with cost as follows for $g(x,y)\&h(x,y)$:

$$\mathrm{C} = C_F \cdot \Delta F + C_T \cdot \Delta T + C_c$$

One colored grid with cost as follows for$g(x,y)\&h(x,y)$:

$$\mathrm{C} = C_F \cdot (\Delta F + \Delta F_a(x,y)) + C_T \cdot (\Delta T + \Delta T_a(x,y)) + C_c$$

How we choose the routes ?



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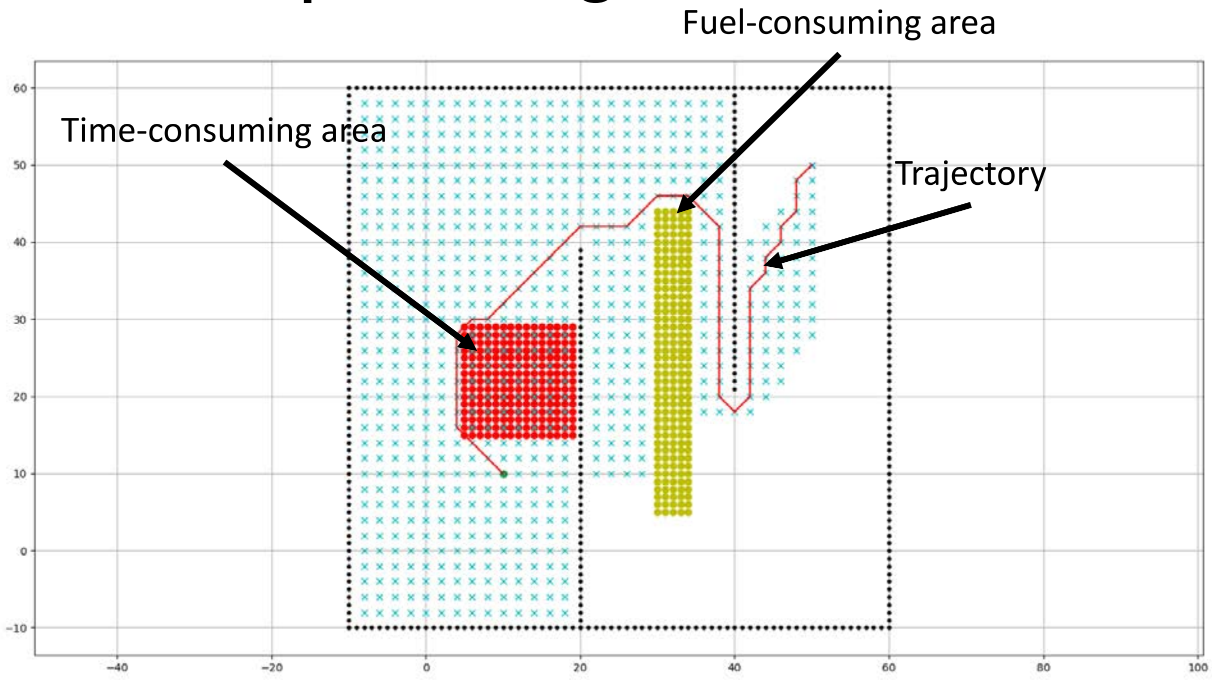
- Start node
- Goal node

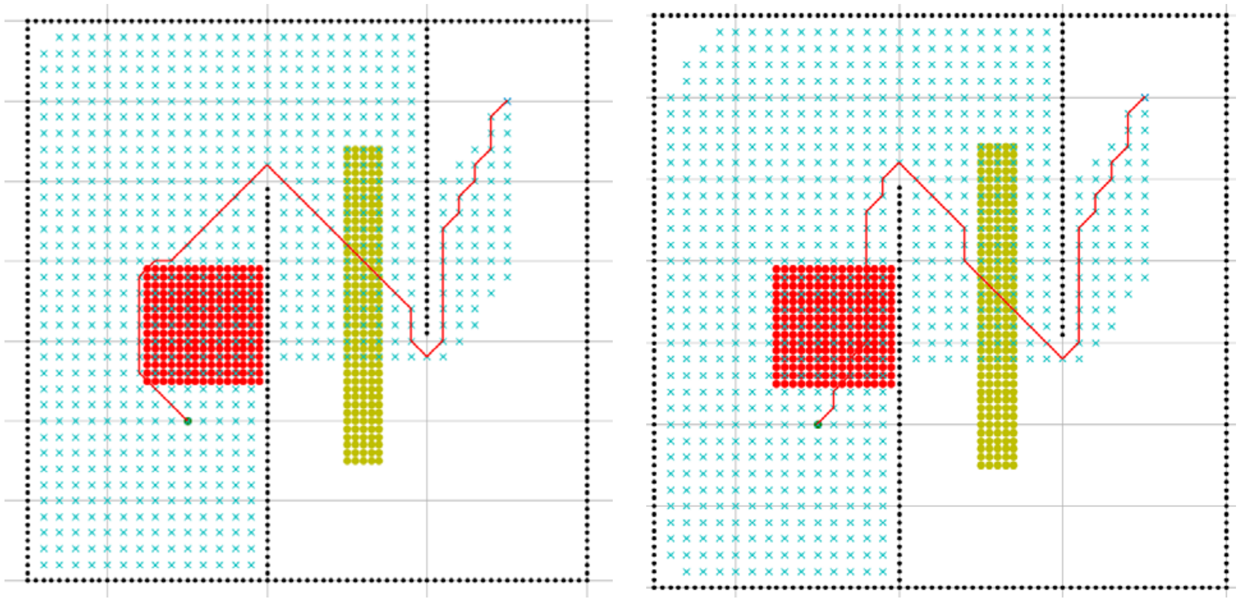
It depends on the ΔF_a and ΔT_a

Example route planning



Avoiding the Fuel-consuming and time-consuming area if their cost is too high?

Example route planning



Go through the fuel-consuming and time-consuming area if their additional cost is quite small?

Design your route

Aircraft Model	C_F	ΔF	C_T	ΔT	C_c	ΔF_a	ΔT_a
PolyU-A380	1	1	2	5	10	0.2	0.2
PolyU-A381	1	1.5	3	5	10	0.3	0.4
PolyU-A382	1	2.0	4	5	10	0.4	0.5
PolyU-A383	1	2.5	5	5	10	0.5	0.1

$$\mathrm{C} = C_F \cdot \Delta F + C_T \cdot \Delta T + C_c$$

With

- C_F=cost of fuel per kg
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