Interdisciplinary Division of Aeronautical and Aviation Engineering
航空工程跨領域學部

THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

AAE2004 Introduction to Aviation Systems
AAE
Design of Path Planning Algorithm for Aircraft Operation

Second Week

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Assisted by

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Lecturer's Information

- **Instructor**: Dr Li-Ta HSU
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- **Expertise:** GPS navigation, Autonomous driving, Pedestrian localization using Smartphone, Sensor Integration

Ground Rules

For students

- Try to speak as much English as possible.
- Participate the class activates assigned.

For teaching staffs

- Reply your email with 3 working day.
- Open to any question regards to the subject

For us!

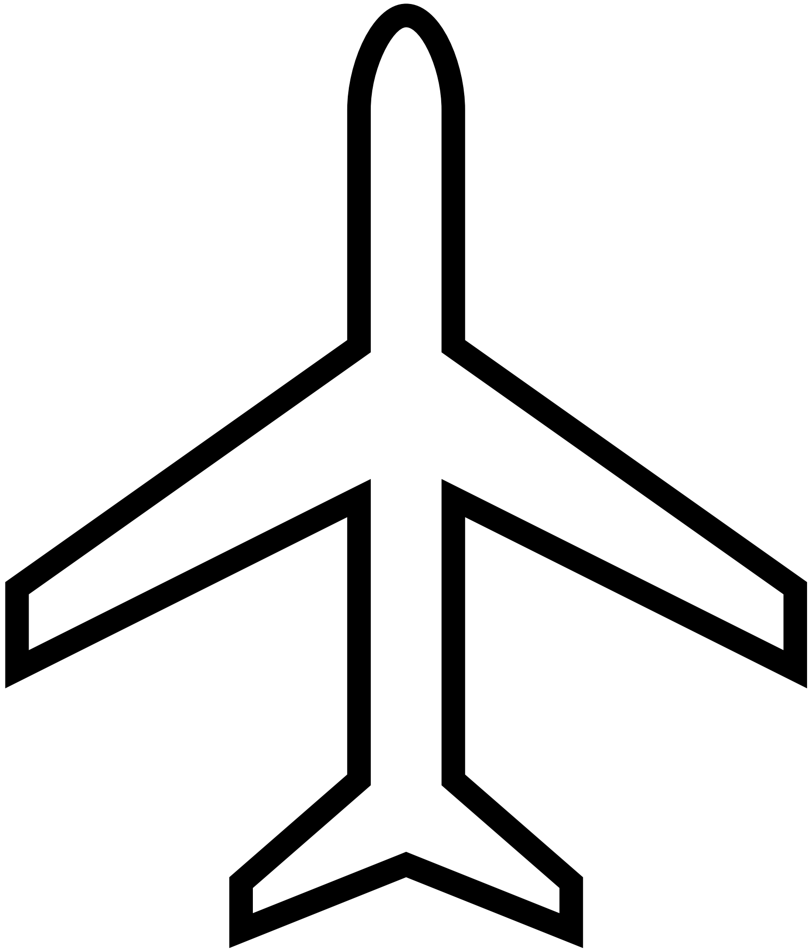
- Keep an open mind—enter the classroom dialogue with the expectation of learning something new. Look forward to learning about–and being challenged by–ideas, questions, and points of view that are different than your own.
- Arrive on time to the class and finish the class on time

Necessary Information

- Course Repository link: https://github.com/IPNL-POLYU/PolyU_AAE2004_Github_Project
- TA Information & Contact:
  - Group 1-5: Queenie Ho (hiu-yi.ho@connect.polyu.hk)
  - Group 6-10: Nikkie Leung (yan-tung.leung@connect.polyu.hk)

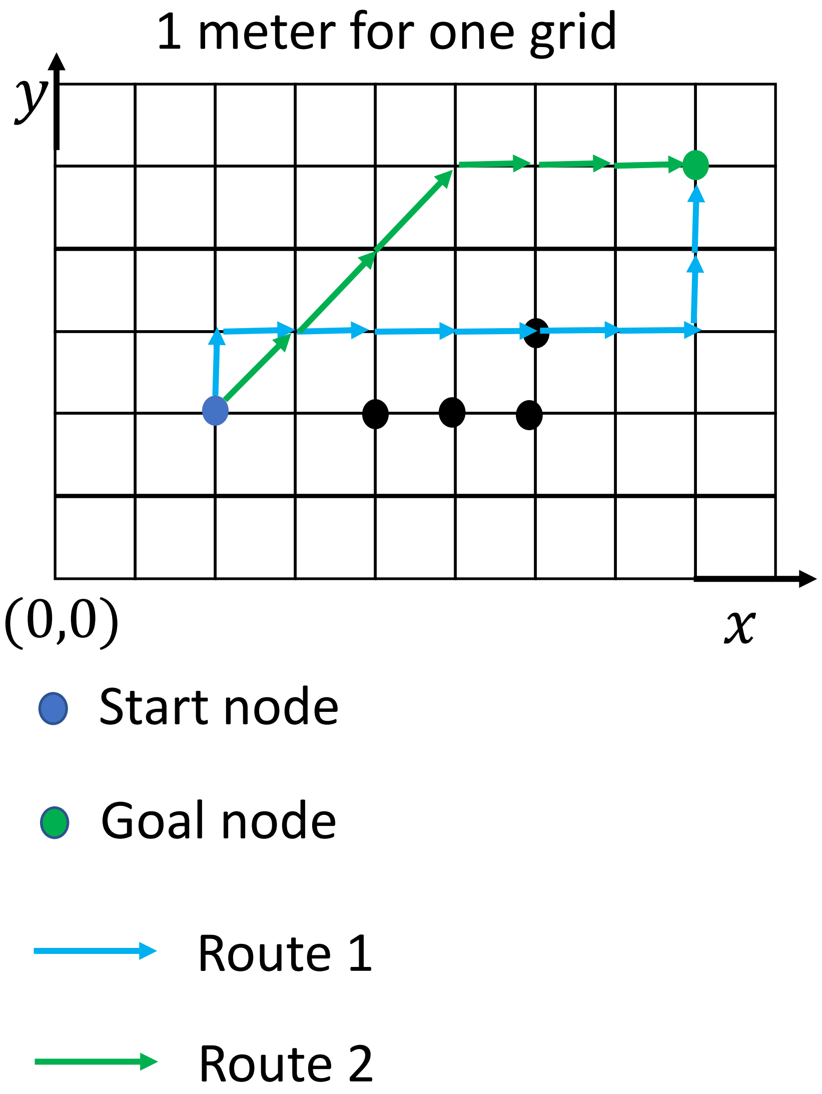
Week 2 Content

1. Introduction to A* Path Planning Algorithm
2. Cost Intensive Areas
3. Path Planning Programming Guide



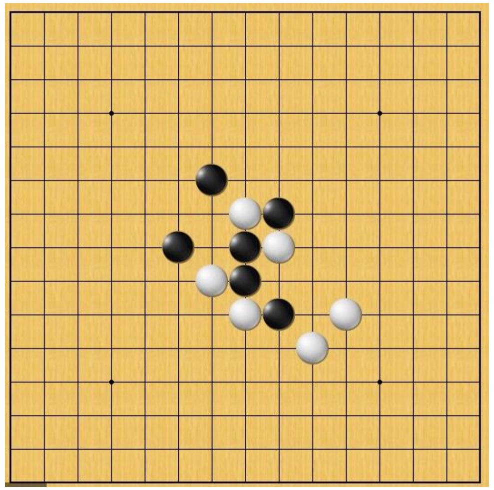
Introduction to A* Path Planning Algorithm

Definition of Path Planning



•**Node** — All potential position you can go across with a unique position (x, y)

•**Search Space** — A collection of nodes, like all board positions of a board game.

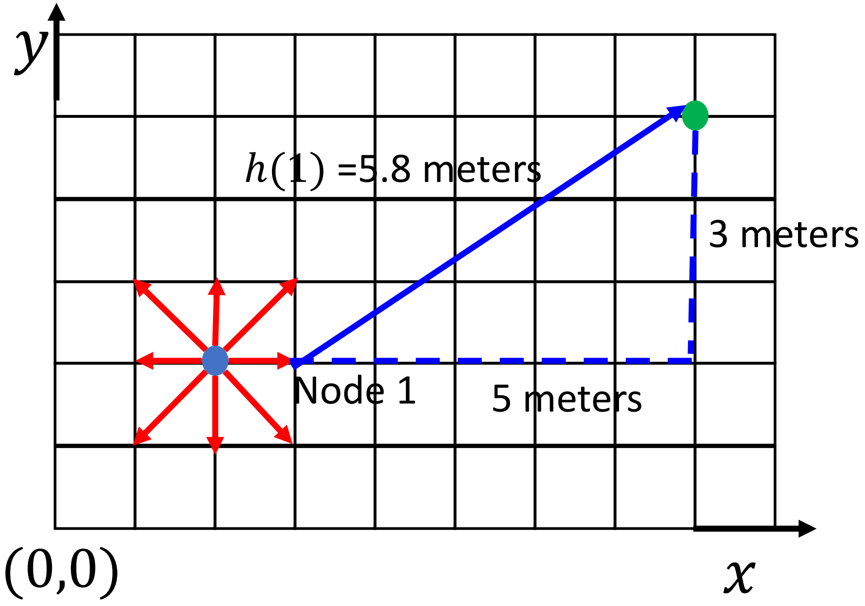


Gobang

•**Objective of path planning**— Find the shortest routes with smallest cost from start node to goal node.

How to find the shortest route!

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

•$\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)

•$\boldsymbol{h}(x,y)$ — this represents the heuristic ***estimated cost*** from node (x,y) to the goal node.

•$\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

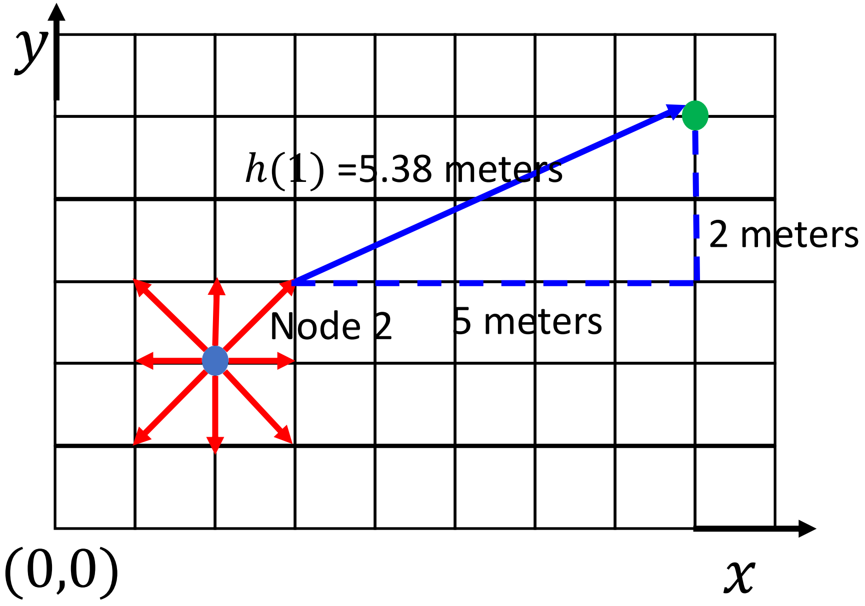
8 neighboring node and the cost can be calculated as follows!

Node 1:

$f(3,2) = g(3,2) + h(3,2)$ =6.8 meters

with $g(3,2) = 1$ meter and $h(3,2) = 5.8$ meters

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

•$\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)

•$\boldsymbol{h}(x,y)$ — this represents the heuristic ***estimated cost*** from node (x,y) to the goal node.

•$\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

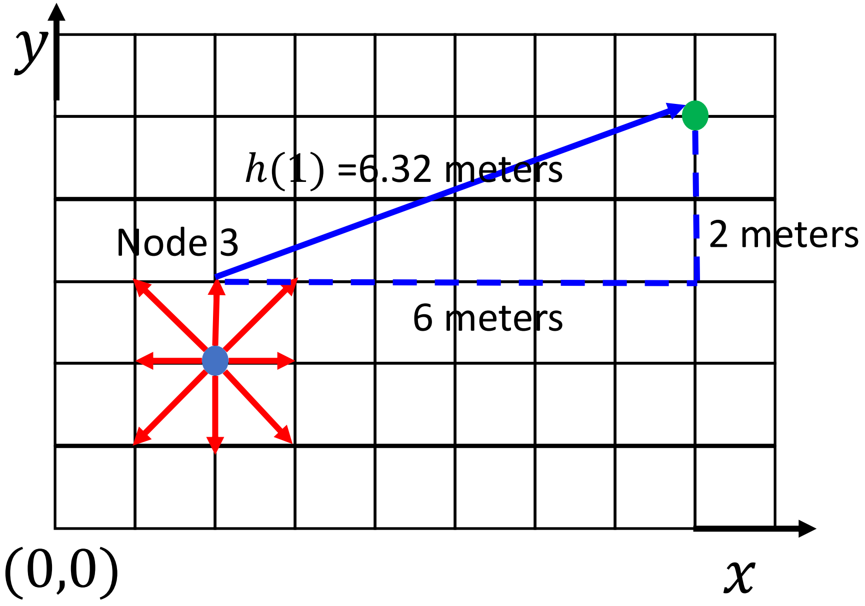
8 neighboring node and the cost can be calculated as follows!

Node 2:

$$f(3,3) = g(3,3) + h(3,3) \text{ =6.79 meters}$$

with $g(3,3) = \sqrt{2}$ meter and $h(3,3) = 5.38$ meters

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

- $\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)
- $\boldsymbol{h}(x,y)$ — this represents the heuristic ***estimated cost*** from node (x,y) to the goal node.
- $\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

8 neighboring node and the cost can be calculated as follows!

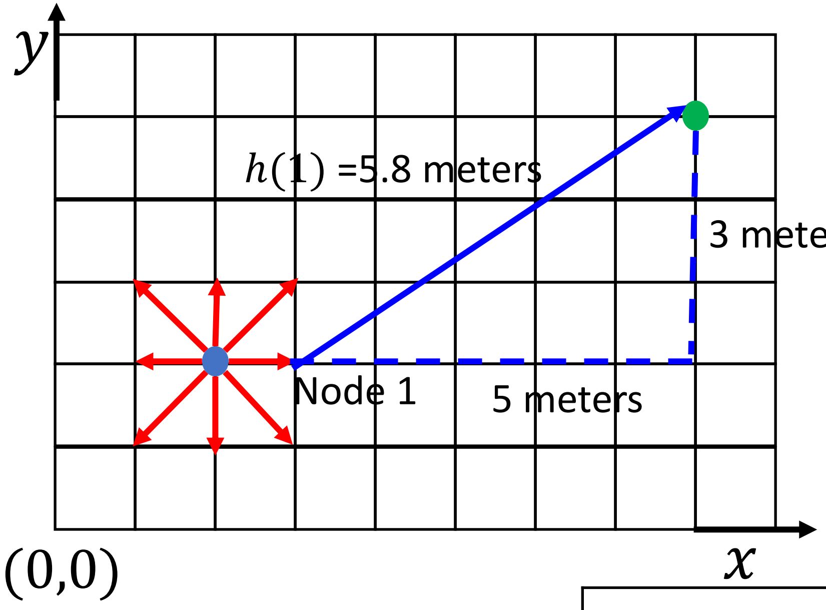
Node 3:

$$f(2,3) = g(2,3) + h(2,3) = 7.32 \text{ meters}$$

with $g(2,3) = 1$ meter and $h(2,3) = 6.32$ meters

Similar cost calculation method for other 5 nodes

Path Planning using A Star Method



Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

• $\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)

• $\boldsymbol{h}(x,y)$ — this represents the heuristic ***estimated cost*** from node (x,y) to the goal node.

• $\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

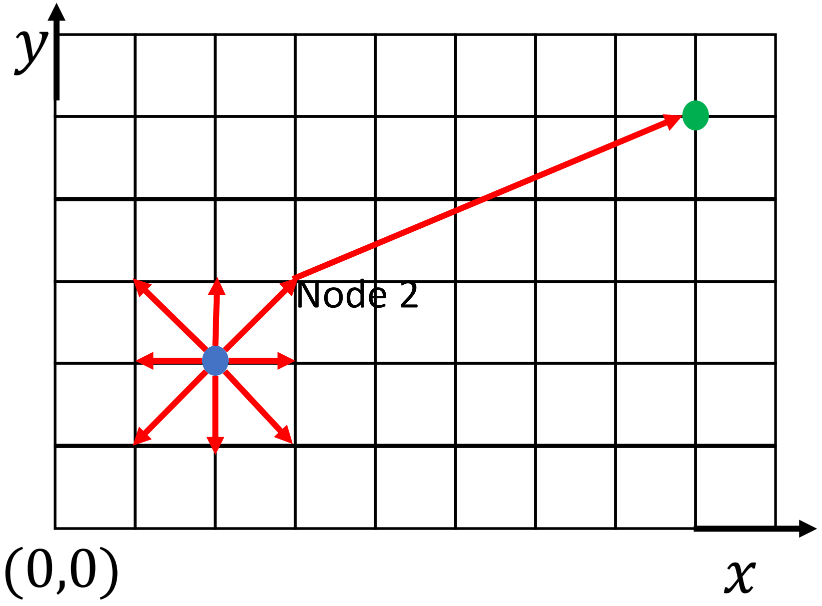
● Start node

● Goal node

1 meter for one grid

Node (x,y)	Node 1 (x,y)	Node 2 (x,y)	Node 3 (x,y)	Node 4 (x,y)	Node 5 (x,y)	Node 6 (x,y)	Node 7 (x,y)	Node 8 (x,y)
$g(x,y)$	1	1.414	1	1.414	1	1.414	1	1.414
$h(x,y)$	5.8	5.38	6.32	7.28	7.62	8.06	7.21	6.40
$f(x,y)$	6.8	6.79	7.32	8.694	8.62	9.474	8.21	7.814

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

•$\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)

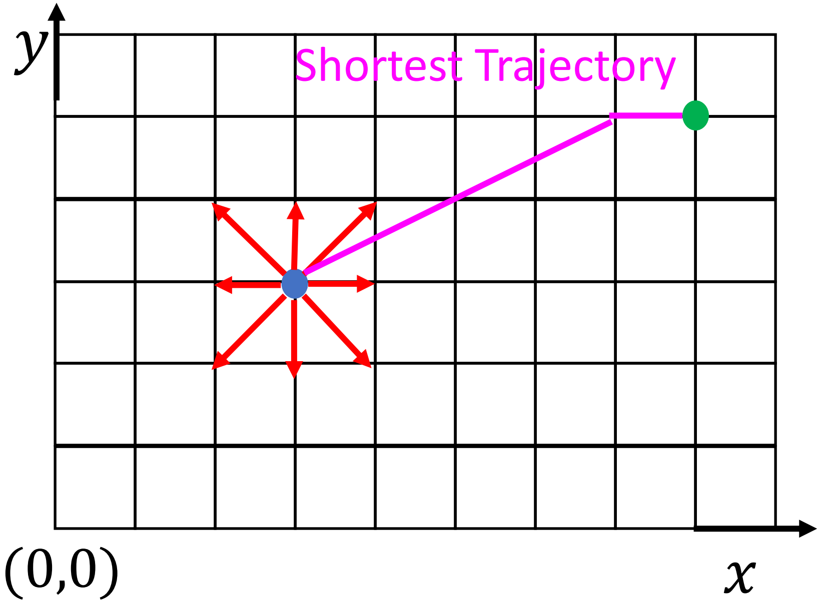
•$\boldsymbol{h}(x,y)$ — this represents the heuristic ***estimated cost*** from node (x,y) to the goal node.

•$\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

8 neighboring node and the cost can be calculated as follows!

Node 2 leads to smallest cost

Calculate the cost of node



- Start node
- Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

•$\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)

•$\boldsymbol{h}(x,y)$ — this represents the heuristic ***estimated cost*** from node (x,y) to the goal node.

•$\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

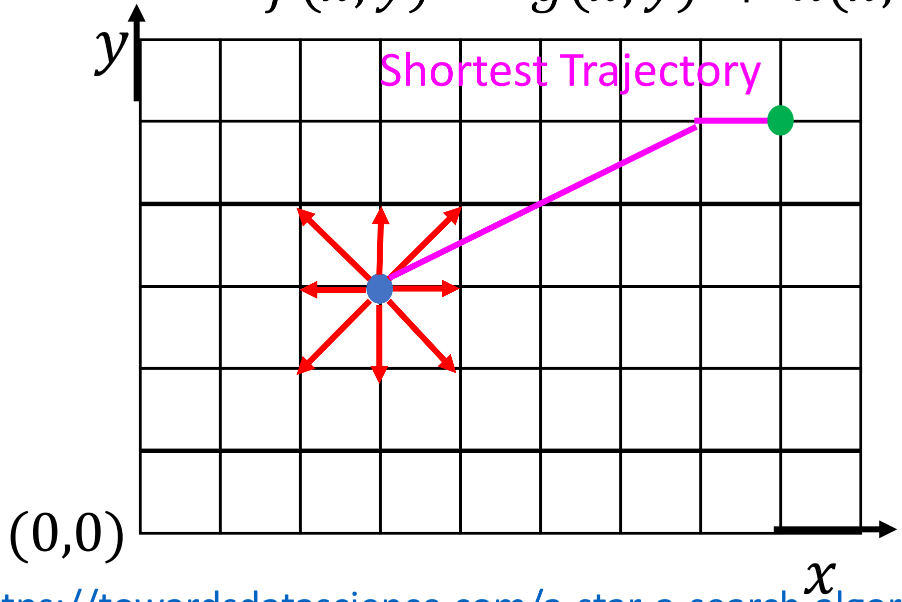
8 neighboring node and the cost can be calculated as follows!

Search from the neighbouring node with smallest cost until reaching the goal!

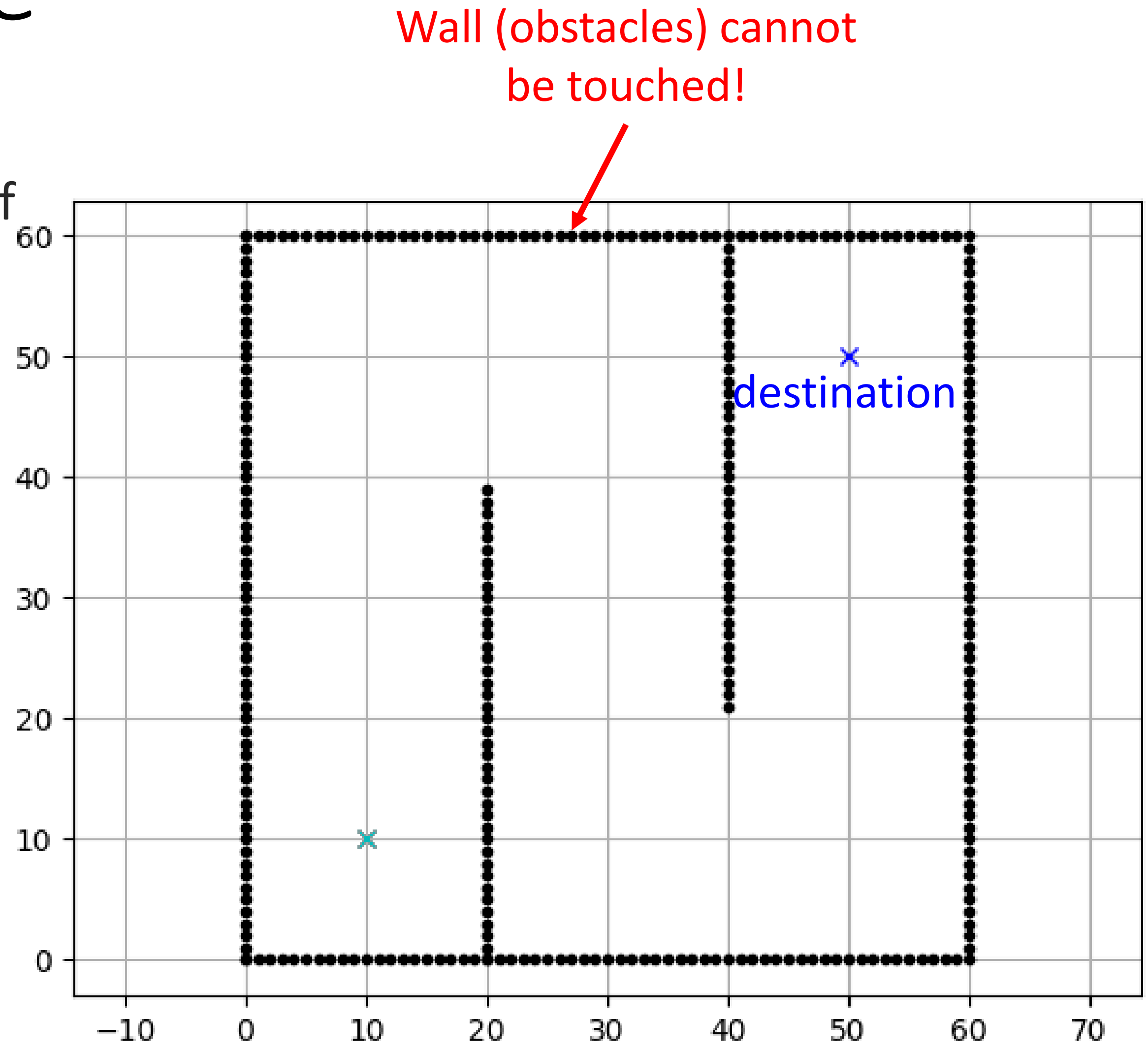
A star method example

Each time A* enters a node, it calculates the cost, f(n)(n being the neighboring node), to travel to all of the neighboring nodes, and then enters the node with the lowest value of f(n).
These values we calculate using the following formula:

$$f(x,y) = g(x,y) + h(x,y)$$

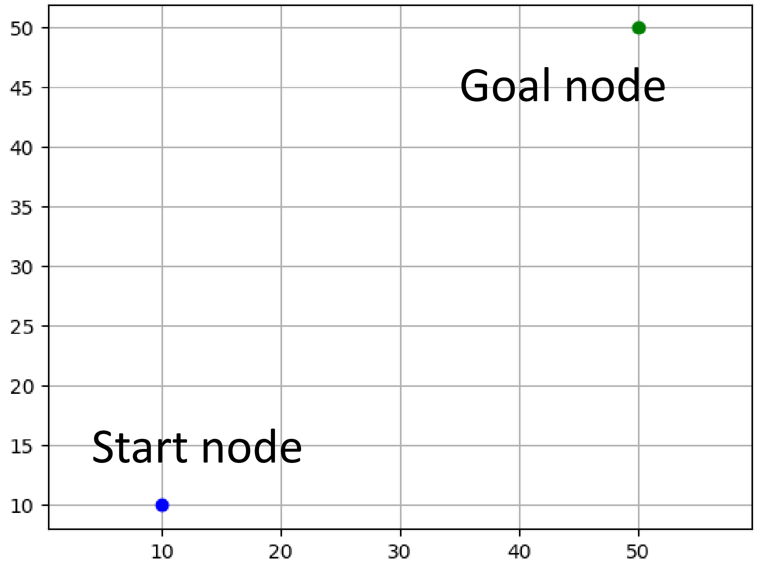


https://towardsdatascience.com/a-star-a-search-algorithm-eb495fb156bb



Source: PythonRobotics

Code: set up start and goal node

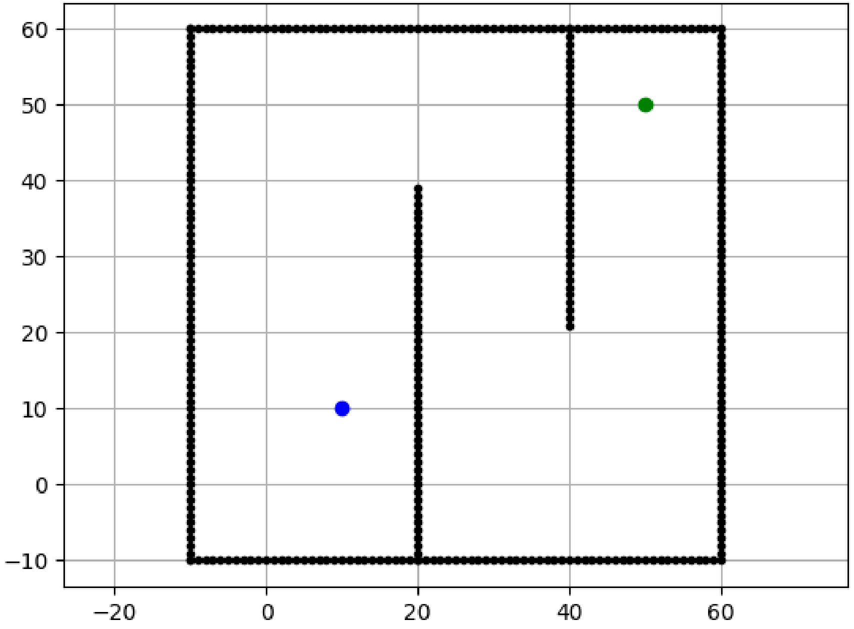


Set up the start and goal nodes using the code

```
# start and goal position
sx = 10.0  # [m]
sy = 10.0  # [m]
gx = 50.0  # [m]
gy = 50.0  # [m]
grid_size = 2  # [m]
```

- Start node
- Goal node

Code: set up obstacle



Start node

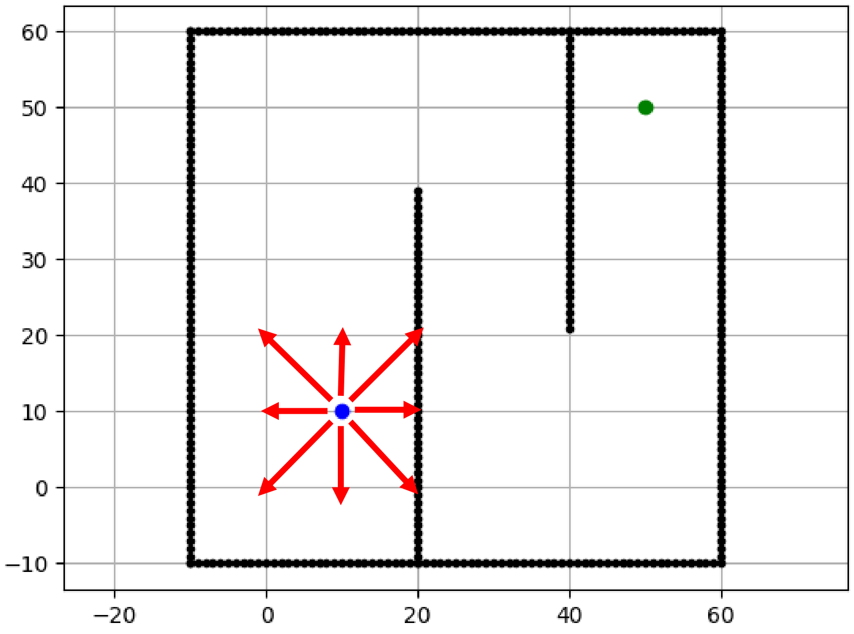
Goal node

Obstacle (wall)

Set up the obstacle using the code

```
# set obstacle positions
ox, oy = [], []
for i in range(-10, 60): # draw the button border
    ox.append(i)
    oy.append(-10.0)
for i in range(-10, 60):
    ox.append(60.0)
    oy.append(i)
for i in range(-10, 61):
    ox.append(i)
    oy.append(60.0)
for i in range(-10, 61):
    ox.append(-10.0)
    oy.append(i)
for i in range(-10, 40):
    ox.append(20.0)
    oy.append(i)
for i in range(0, 40):
    ox.append(40.0)
    oy.append(60.0 - i)
```

Code: neighboring node search



neighboring node search

```python
def get_neighbouring_node(): # the cost of the surrounding 8 points
    # dx, dy, cost
    motion = [[1, 0, 1],
              [0, 1, 1],
              [-1, 0, 1],
              [0, -1, 1],
              [-1, -1, math.sqrt(2)],
              [-1, 1, math.sqrt(2)],
              [1, -1, math.sqrt(2)],
              [1, 1, math.sqrt(2)]]

    return motion
```

- Start node
- Goal node
- Obstacle (wall)

Code: cost calculation

Heuristic cost$g(x, y)$ calculation

```
def calc_heuristic(n1, n2):
    w = 1.0  # weight of heuristic
    d = w * math.hypot(n1.x - n2.x, n1.y - n2.y)
    return d
```

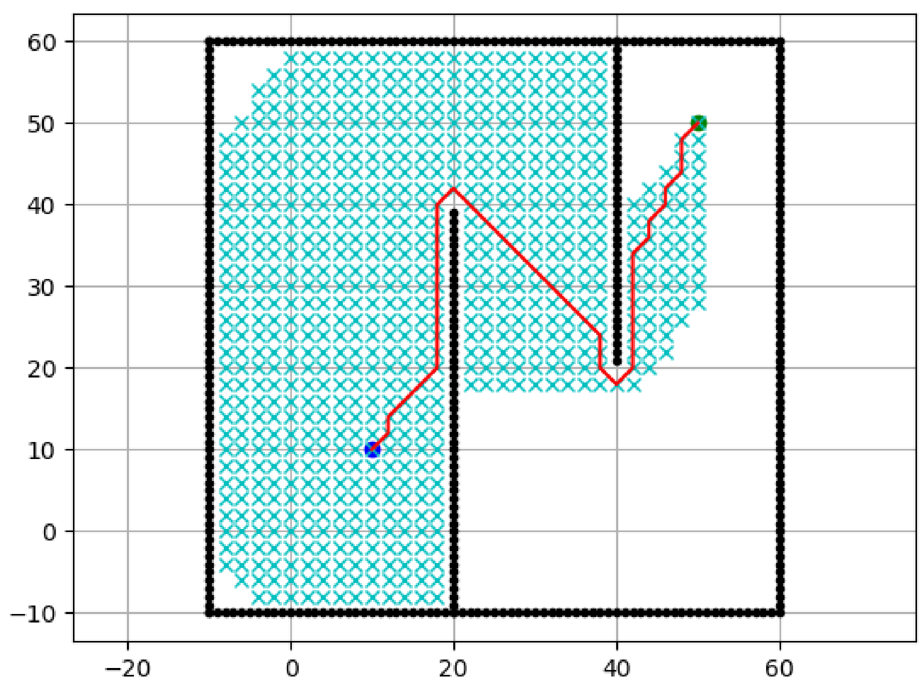
exact cost $g(x, y)$ calculation

```
node = self.Node(current.x + self.motion[i][0],
                 current.y + self.motion[i][1],
                 current.cost + self.motion[i][2], c_id)
```

Code: calculation of final path

```
def calc_final_path(self, goal_node, closed_set):
    # generate final course
    rx, ry = [self.calc_grid_position(goal_node.x, self.min_x)], [
        self.calc_grid_position(goal_node.y, self.min_y)] # save the goal node as the first point
    parent_index = goal_node.parent_index
    while parent_index != -1:
        n = closed_set[parent_index]
        rx.append(self.calc_grid_position(n.x, self.min_x))
        ry.append(self.calc_grid_position(n.y, self.min_y))
        parent_index = n.parent_index

    return rx, ry
```



Cost Intensive Areas

Flight planning considering trip cost

The fundamental rationale of the cost index concept is to achieve minimum trip cost by means of a trade-off between operating costs per hour and incremental fuel burn.

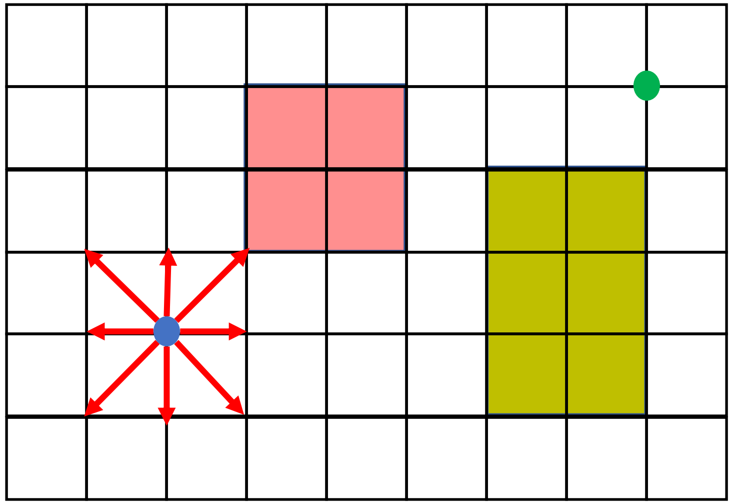
$$\mathrm{C} = C_F \cdot \Delta F + C_T \cdot \Delta T + C_c$$

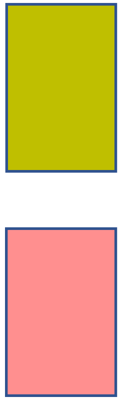
With

- C_F=cost of fuel per kg
- C_T=time related cost per minute of flight
- C_c=fixed cost independent of time
- C_T=time related cost per minute of flight
- ΔF=trip fuel (e.g. 3000kg/h)
- ΔT=trip Time (e.g. 8 hours from Hong Kong to Paris)

Can we consider this cost to our path planning to imitate the path planning for flights?

Flight planning considering trip cost





Cost Intensive Areas: the cost for flying through such area is increased due to airflow, legal restrictions and other reasons. (additional cost ΔF_a, ΔT_a)

Cost can be calculated using the following formula:

$$f(x, y) = g(x, y) + h(x, y)$$

One white grid with cost as follows for $g(x, y)\&h(x, y)$:

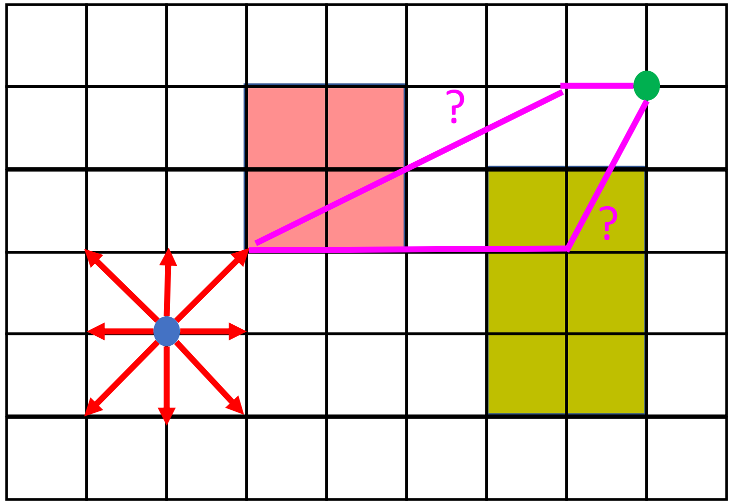
$$\mathrm{C} = C_F \cdot \Delta F + C_T \cdot \Delta T + C_c$$

One colored grid with cost as follows for$g(x, y)\&h(x, y)$:

$$\mathrm{C} = C_F \cdot (\Delta F + \Delta F_a(x, y)) + C_T \cdot (\Delta T + \Delta T_a(x, y)) + C_c$$

- Start node
- Goal node

How we choose the routes ?



Cost Intensive Areas: the cost for flying through such area is increased due to airflow, legal restrictions and other reasons. (additional cost ΔF_a, ΔT_a)

Start node

Goal node

It depends on the ΔF_a and ΔT_a

Cost can be calculated using the following formula:

$$f(x,y) = g(x,y) + h(x,y)$$

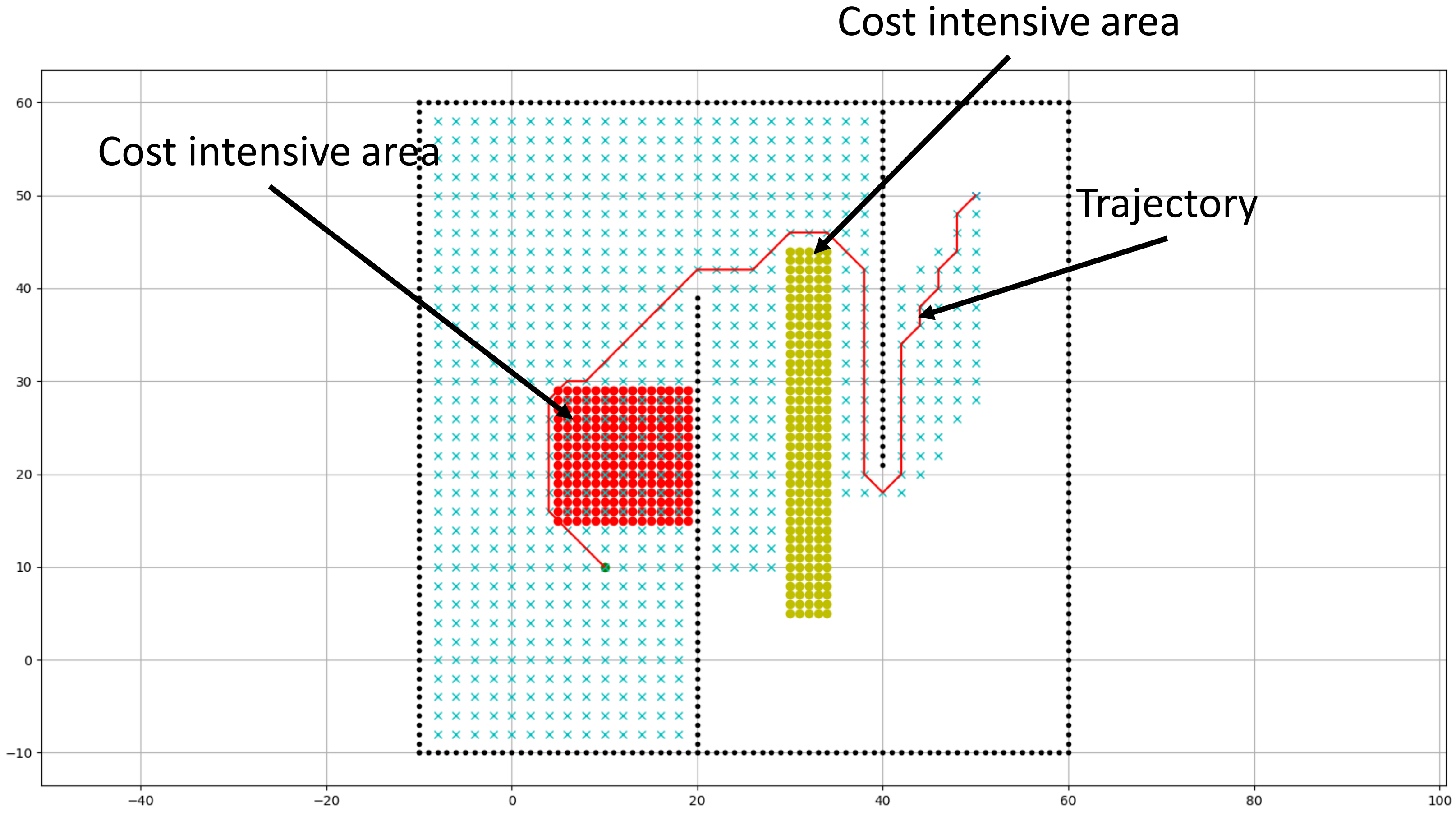
One white grid with cost as follows for $g(x,y)\&h(x,y)$:

$$\mathrm{C} = C_F \cdot \Delta F + C_T \cdot \Delta T + C_c$$

One colored grid with cost as follows for $g(x,y)\&h(x,y)$:

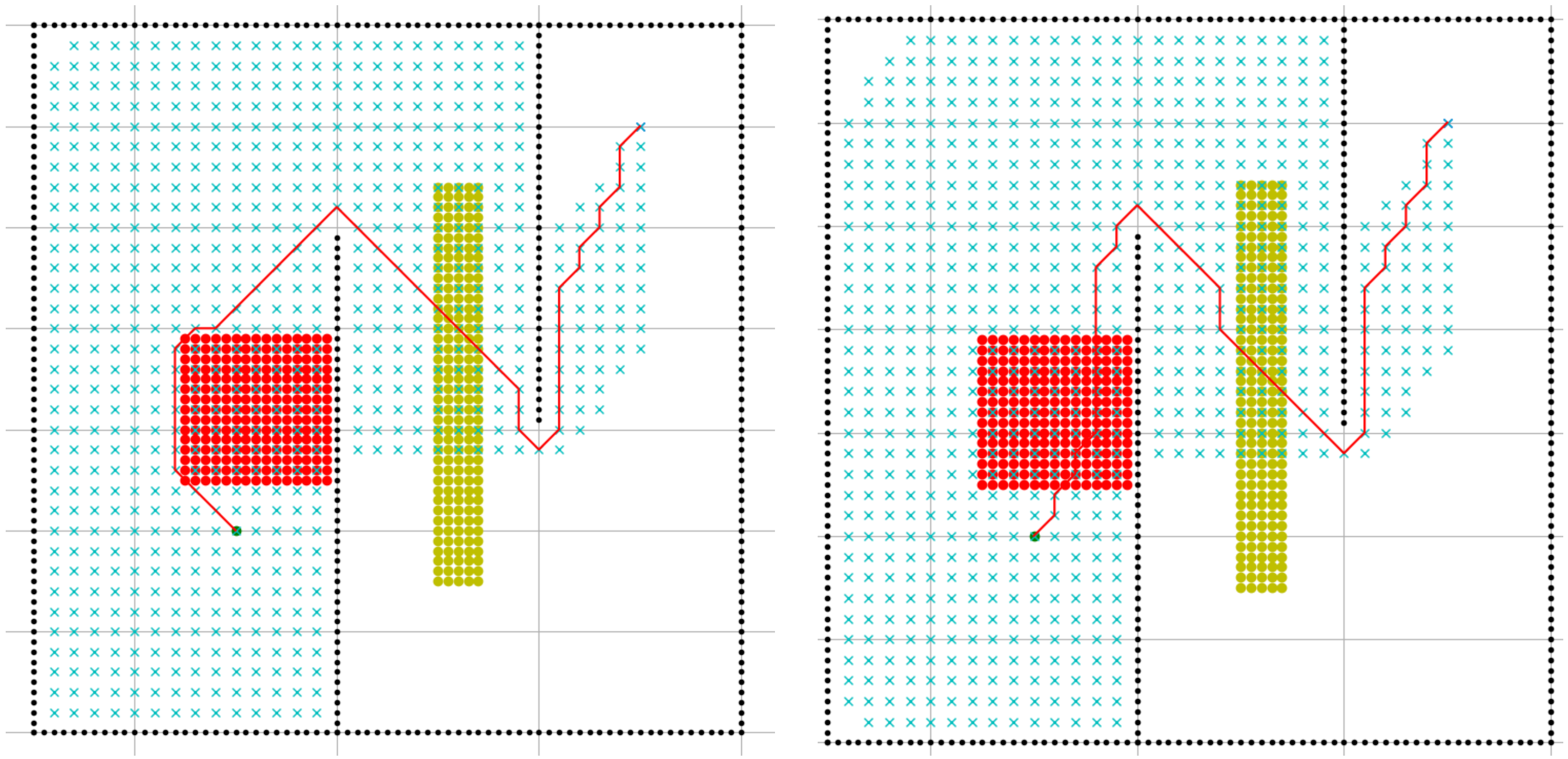
$$\mathrm{C} = C_F \cdot (\Delta F + \Delta F_a(x,y)) + C_T \cdot (\Delta T + \Delta T_a(x,y)) + C_c$$

Example route planning



Avoiding the Cost intensive areas if their cost is too high?

Example route planning



Go through the Cost intensive area if their additional cost is quite small?

Path Planning Project

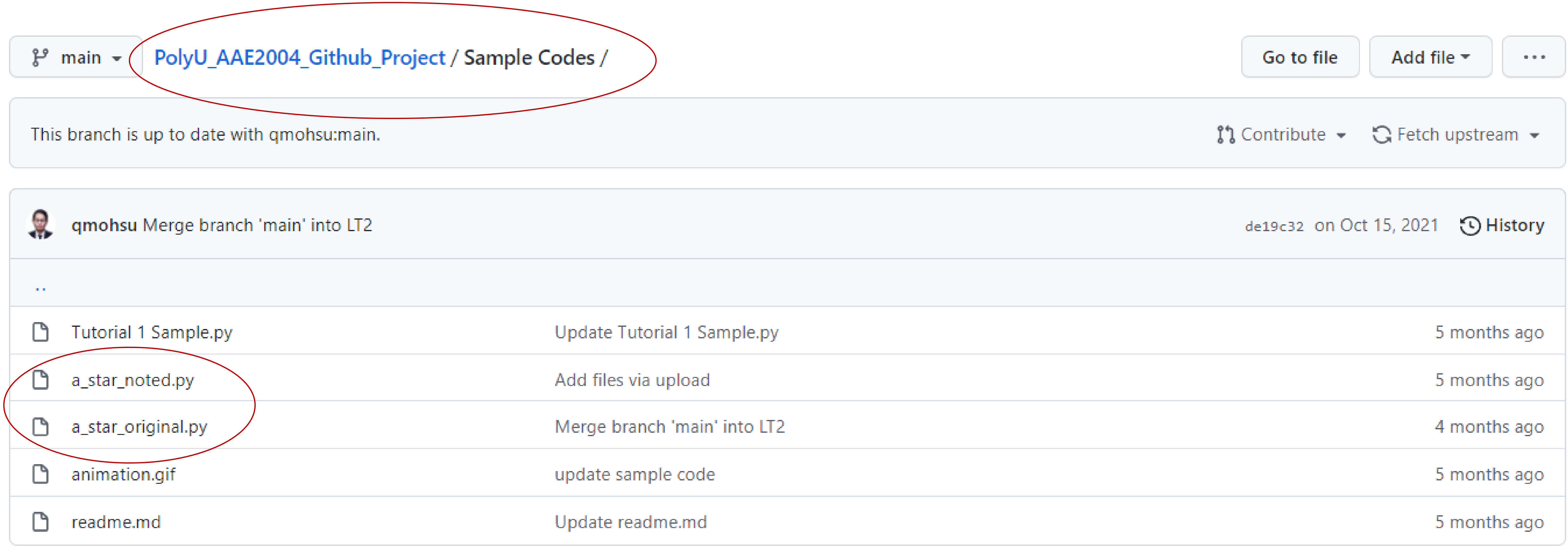
- You will be creating and completing your own path planning program based on groups
- You can find the project tasks / requirements in the Week 3 Slides
- Additional resources could be found inside the course GitHub repository
  - Video tutorial
  - Tutorial slides

Path Planning Programming Guide

The Path Planning Code

- You can find the path planning code inside the course GitHub repository
- There are 2 set of codes:
  - A default one
  - A noted one
- The default one is a basic A* path planning code without any extra information and features
- The noted one provides an example of what your code should look like after modifications (Remember each group should complete a different set of obstacles and requirements)
- Repository link: https://github.com/IPNL-POLYU/PolyU_AAE2004_Github_Project

Where you can find the code



Noted Version Guide

- Line 50,51: Declaration of cost intensive area cost modifier
- Line 53: Declare cost per grid

```
34
35         self.resolution = resolution # get resolution of the grid
36         self.rr = rr # robot radis
37         self.min_x, self.min_y = 0, 0
38         self.max_x, self.max_y = 0, 0
39         self.obstacle_map = None
40         self.x_width, self.y_width = 0, 0
41         self.motion = self.get_motion_model() # motion model for grid search expansion
42         self.calc_obstacle_map(ox, oy)
43
44         self.fc_x = fc_x
45         self.fc_y = fc_y
46         self.tc_x = tc_x
47         self.tc_y = tc_y
48
49
50         self.Delta_C1 = 0.2 # cost intensive area 1 modifier
51         self.Delta_C2 = 0.4 # cost intensive area 2 modifier
52
53         self.costPerGrid = 1
54
```

Noted Version Guide

- Line 115: Showing the final calculation of total trip time
- Line 135-144: Adding additional cost during cost intensive area

```
103             if show_animation:  # pragma: no cover
104                 plt.plot(self.calc_grid_position(current.x, self.min_x),
105                          self.calc_grid_position(current.y, self.min_y), "xc")
106                 # for stopping simulation with the esc key.
107                 plt.gcf().canvas.mpl_connect('key_release_event',
108                                              lambda event: [exit(
109                                                  0) if event.key == 'escape' else None])
110                 if len(closed_set.keys()) % 10 == 0:
111                     plt.pause(0.001)
112 
113             # reaching goal
114             if current.x == goal_node.x and current.y == goal_node.y:
115                 print("Total Trip time required -> ",current.cost )
116                 goal_node.parent_index = current.parent_index
117                 goal_node.cost = current.cost
118                 break
119 
120             # Remove the item from the open set
121             del open_set[c_id]
122 
123             # Add it to the closed set
124             closed_set[c_id] = current
125 
126             # print(len(closed_set))
127 
128             # expand_grid search grid based on motion model
129             for i, _ in enumerate(self.motion): # tranverse the motion matrix
130                 node = self.Node(current.x + self.motion[i][0],
131                                  current.y + self.motion[i][1],
132                                  current.cost + self.motion[i][2] * self.costPerGrid, c_id)
133 
134                 ## add more cost in cost intensive area 1
135                 if self.calc_grid_position(node.x, self.min_x) in self.tc_x:
136                     if self.calc_grid_position(node.y, self.min_y) in self.tc_y:
137                         # print("cost intensive area!!")
138                         node.cost = node.cost + self.Delta_C1 * self.motion[i][2]
139 
140                 # add more cost in cost intensive area 2
141                 if self.calc_grid_position(node.x, self.min_x) in self.fc_x:
142                     if self.calc_grid_position(node.y, self.min_y) in self.fc_y:
143                         # print("cost intensive area!!")
144                         node.cost = node.cost + self.Delta_C2 * self.motion[i][2]
145                     # print()
146 
```

Noted Version Guide

- Line 263-270: Declaring motions for the aircraft
- Line 279-284: Declaring starting point and end point

```
260     @staticmethod
261     def get_motion_model(): # the cost of the surrounding 8 points
262         # dx, dy, cost
263         motion = [[1, 0, 1],
264                   [0, 1, 1],
265                   [-1, 0, 1],
266                   [0, -1, 1],
267                   [-1, -1, math.sqrt(2)],
268                   [-1, 1, math.sqrt(2)],
269                   [1, -1, math.sqrt(2)],
270                   [1, 1, math.sqrt(2)]]
271 
272         return motion
273 
274 
275 def main():
276     print(__file__ + " start the A star algorithm demo !!") # print simple notes
277 
278     # start and goal position
279     sx = 0.0  # [m]
280     sy = 0.0  # [m]
281     gx = 50.0  # [m]
282     gy = 0.0  # [m]
283     grid_size = 1  # [m]
284     robot_radius = 1.0  # [m]
```

Noted Version Guide

- Line 309-329: Adding obstacles
- Line 337-348, Adding cost intensive areas (Hint: Refer to this part for your task 2!)

```python
308     # set obstacle positions for group 9
309     ox, oy = [], []
310     for i in range(-10, 60): # draw the button border
311         ox.append(i)
312         oy.append(-10.0)
313     for i in range(-10, 60): # draw the right border
314         ox.append(60.0)
315         oy.append(i)
316     for i in range(-10, 60): # draw the top border
317         ox.append(i)
318         oy.append(60.0)
319     for i in range(-10, 60): # draw the left border
320         ox.append(-10.0)
321         oy.append(i)
322
323     for i in range(-10, 30): # draw the free border
324         ox.append(20.0)
325         oy.append(i)
326
327     for i in range(0, 20):
328         ox.append(i)
329         oy.append(-1 * i + 10)
330
331     # for i in range(40, 45): # draw the button border
332     #     ox.append(i)
333     #     oy.append(30.0)
334
335
336     # set cost intesive area 1
337     fc_x, fc_y = [], []
338     for i in range(30, 40):
339         for j in range(0, 40):
340             fc_x.append(i)
341             fc_y.append(j)
342
343     # set cost intesive area 1
344     tc_x, tc_y = [], []
345     for i in range(10, 20):
346         for j in range(20, 50):
347             tc_x.append(i)
348             tc_y.append(j)
```

Noted Version Guide

- If you wish to do the calculation using the program, you should add the calculation function under line 117, inside the reaching goal condition
- It would be even better if the program could distinguish viable and non-viable aircraft types!
- Use the noted version as your sample to modify your own code!

Program Calculation for Task 1

- When you add in a cost calculation function, the output should look something like this, it should be able to:

1. Calculate each aircraft types' operating costs
2. Mention which type might not be viable for certain scenarios

```
min_x: -10
min_y: -10
max_x: 60
max_y: 60
x_width: 70
y_width: 70
Total travelling time ->  93.35575746753788
A321 not viable!
Total cost of operating A330 in this scenario:  27360.167918740684
Total cost of operating A350 in this scenario:  30752.648960130347
```