AAE2004 Introduction to Aviation Systems
AAE
Design of Path Planning Algorithm for Aircraft Operation

Week 4: Path Planning Algorithm and Python Robotics

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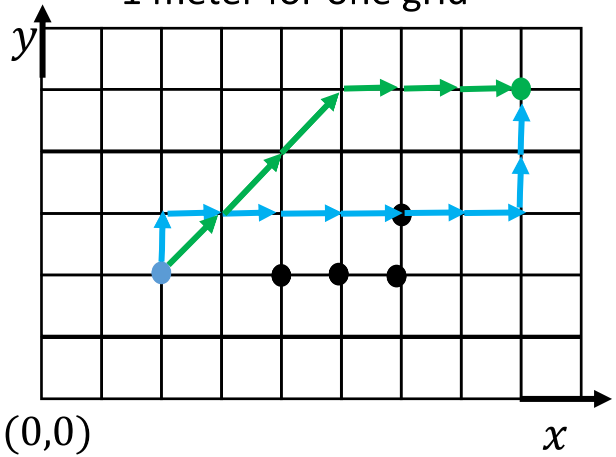
A* Path Planning Algorithm

Definition of Path Planning

1 meter for one grid



Start node

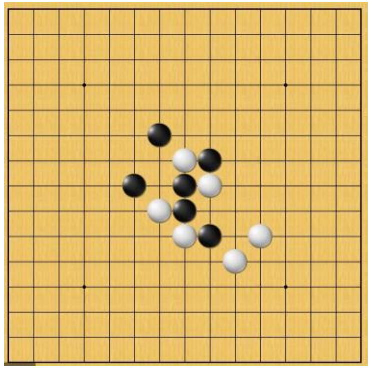
Goal node

Route 1

Route 2

•**Node** — All potential position you can go across with a unique position (x, y)

•**Search Space** — A collection of nodes, like all board positions of a board game.

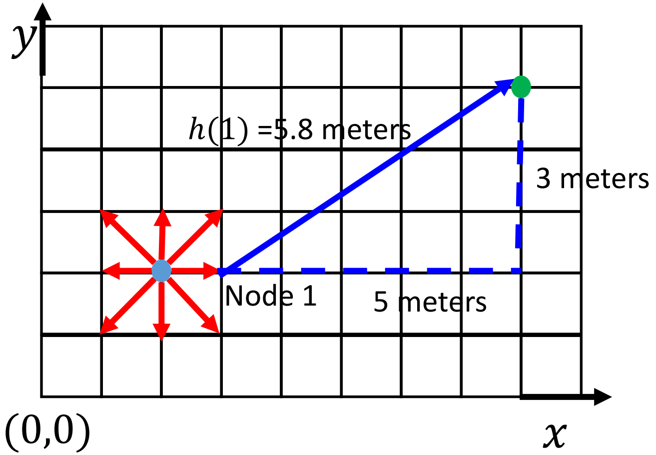


Gobang

•**Objective of path planning**— Find the shortest routes with smallest cost from start node to goal node.

How to find the shortest route!

Path Planning using A Star Method



- Start node
- Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

•$\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)

•$\boldsymbol{h}(x,y)$ — this represents the heuristic ***estimated cost*** from node (x,y) to the goal node.

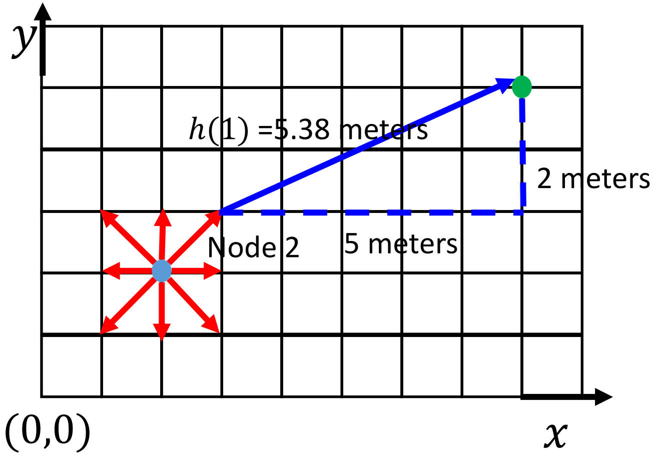
•$\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

8 neighboring node and the cost can be calculated as follows!

Node 1:

$$f(3,2) = g(3,2) + h(3,2) = 6.8 \text{ meters}$$
with $g(3,2) = 1$ meter and $h(3,2) = 5.8$ meters

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

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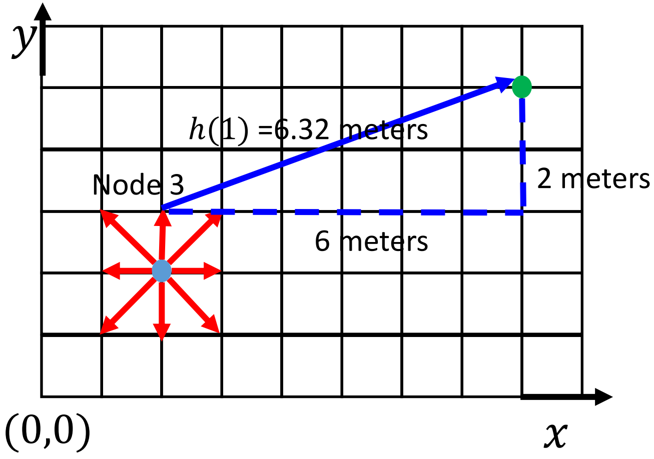
8 neighboring node and the cost can be calculated as follows!

Node 2:

$f(3,3) = g(3,3) + h(3,3)$ =6.79 meters

with $g(3,3) = \sqrt{2}$ meter and $h(3,3) = 5.38$ meters

Path Planning using A Star Method



Definition of cost:

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Start node

Goal node

1 meter for one grid

8 neighboring node and the cost can be calculated as follows!

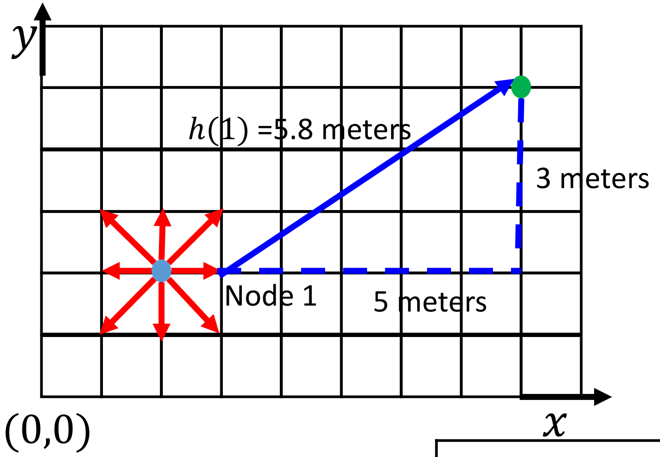
Node 3:

$$f(2,3) = g(2,3) + h(2,3) = 7.32 \text{ meters}$$

with $g(2,3) = 1$ meter and $h(2,3) = 6.32$ meters

Similar cost calculation method for other 5 nodes

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

$$f(x,y) = g(x,y) + h(x,y)$$

- $\boldsymbol{g}(x,y)$ — this represents the ***exact cost*** of the path from the **starting node** to node (x,y)
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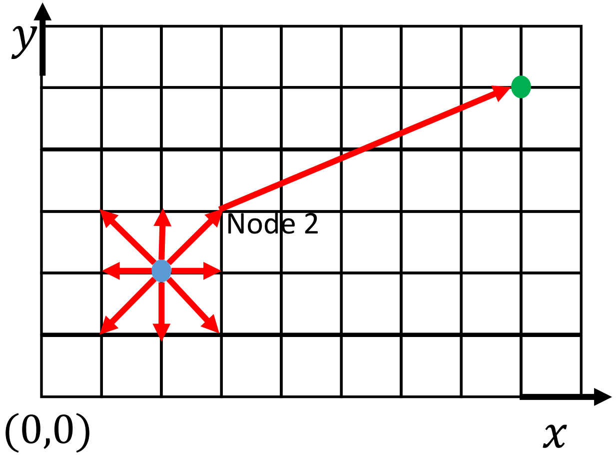
- $\boldsymbol{f}(x,y)$ —cost of the neighboring node (x,y)

Node (x,y)	Node 1 (x,y)	Node 2 (x,y)	Node 3 (x,y)	Node 4 (x,y)	Node 5 (x,y)	Node 6 (x,y)	Node 7 (x,y)	Node 8 (x,y)
$g(x,y)$	1	1.414	1	1.414	1	1.414	1	1.414
$h(x,y)$	5.8	5.38	6.32	7.28	7.62	8.06	7.21	6.40
$f(x,y)$	6.8	6.79	7.32	8.694	8.62	9.474	8.21	7.814

Path Planning using A Star Method



Start node

Goal node

1 meter for one grid

Definition of cost:

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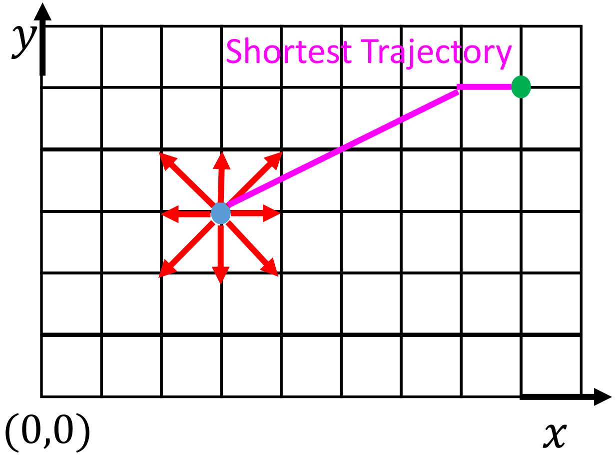
8 neighboring node and the cost can be calculated as follows!

Node 2 leads to smallest cost

Calculate the cost of node



Start node

Goal node

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8 neighboring node and the cost can be calculated as follows!

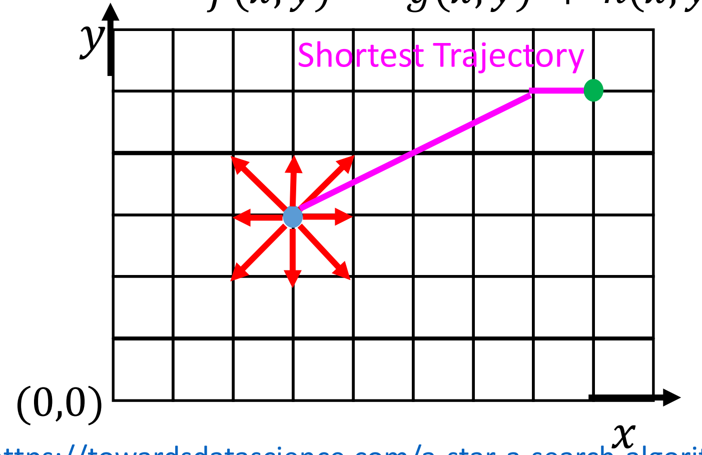
Search from the neighbouring node with smallest cost until reaching the goal!

A star method example

Each time A* enters a node, it calculates the cost, f(n)(n being the neighboring node), to travel to all of the neighboring nodes, and then enters the node with the lowest value of f(n).

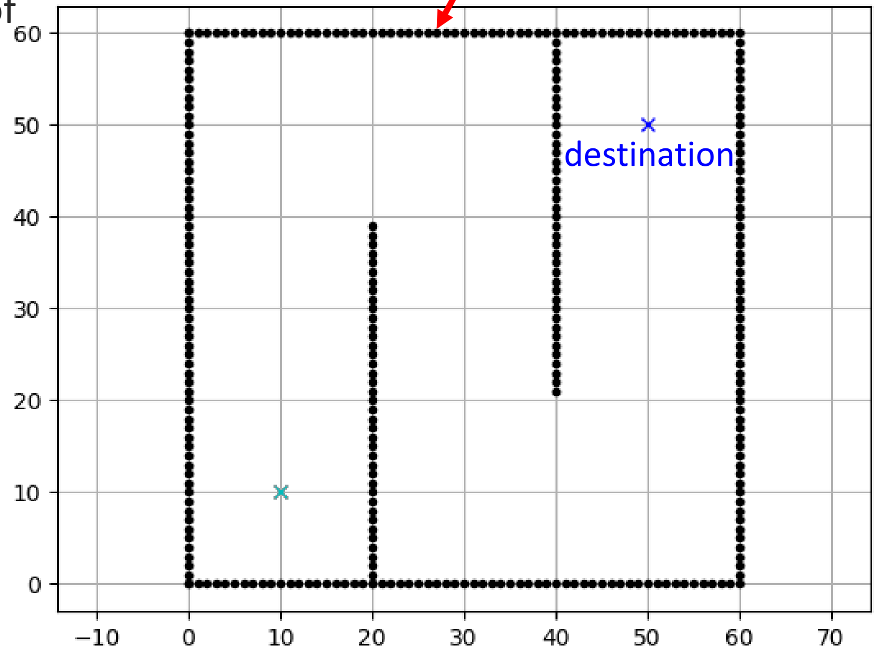
These values we calculate using the following formula:

$$f(x,y) = g(x,y) + h(x,y)$$



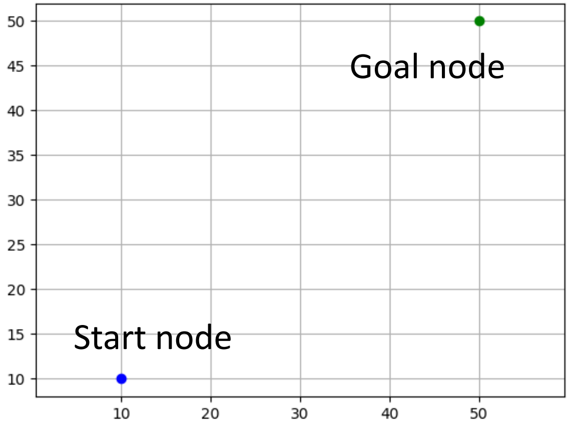
https://towardsdatascience.com/a-star-a-search-algorithm-eb495fb156bb

Wall (obstacles) cannot be touched!



Source: PythonRobotics

Code: set up start and goal node

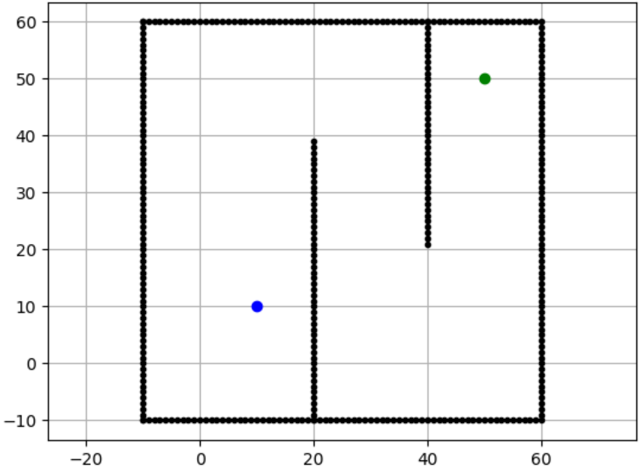


Set up the start and goal nodes using the code

```
# start and goal position
sx = 10.0  # [m]
sy = 10.0  # [m]
gx = 50.0  # [m]
gy = 50.0  # [m]
grid_size = 2  # [m]
```

- Start node
- Goal node

Code: set up obstacle

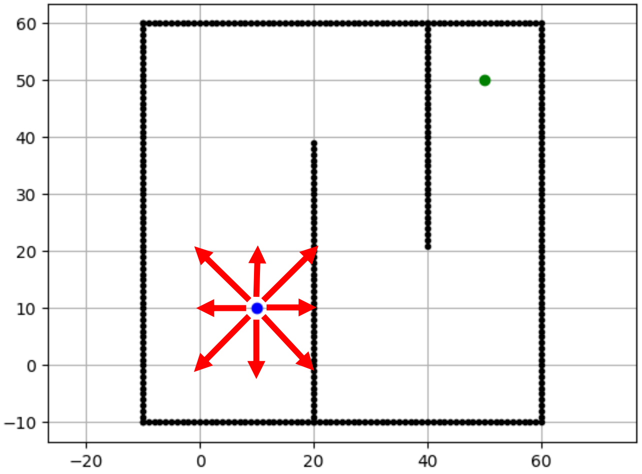


- Start node
- Goal node
- Obstacle (wall)

Set up the obstacle using the code

```
# set obstacle positions
ox, oy = [], []
for i in range(-10, 60): # draw the button border
    ox.append(i)
    oy.append(-10.0)
for i in range(-10, 60):
    ox.append(60.0)
    oy.append(i)
for i in range(-10, 61):
    ox.append(i)
    oy.append(60.0)
for i in range(-10, 61):
    ox.append(-10.0)
    oy.append(i)
for i in range(-10, 40):
    ox.append(20.0)
    oy.append(i)
for i in range(0, 40):
    ox.append(40.0)
    oy.append(60.0 - i)
```

Code: neighboring node search



neighboring node search

```python
def get_neighbouring_node(): # the cost of the surrounding 8 points
    # dx, dy, cost
    motion = [[1, 0, 1],
              [0, 1, 1],
              [-1, 0, 1],
              [0, -1, 1],
              [-1, -1, math.sqrt(2)],
              [-1, 1, math.sqrt(2)],
              [1, -1, math.sqrt(2)],
              [1, 1, math.sqrt(2)]]

    return motion
```

- Start node
- Goal node
- Obstacle (wall)

Code: cost calculation

Heuristic cost $g(x,y)$ calculation

```
def calc_heuristic(n1, n2):
    w = 1.0  # weight of heuristic
    d = w * math.hypot(n1.x - n2.x, n1.y - n2.y)
    return d
```

exact cost $g(x,y)$ calculation

```
node = self.Node(current.x + self.motion[i][0],
                 current.y + self.motion[i][1],
                 current.cost + self.motion[i][2], c_id)
```

Code: calculation of final path

```
def calc_final_path(self, goal_node, closed_set):
    # generate final course
    rx, ry = [self.calc_grid_position(goal_node.x, self.min_x)], [
        self.calc_grid_position(goal_node.y, self.min_y)] # save the goal node as the first point
    parent_index = goal_node.parent_index
    while parent_index != -1:
        n = closed_set[parent_index]
        rx.append(self.calc_grid_position(n.x, self.min_x))
        ry.append(self.calc_grid_position(n.y, self.min_y))
        parent_index = n.parent_index

    return rx, ry
```