Yield Optimization on DeFi Protocols.

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Abstract

In these notes we propose a method to optimize a portfolio focused on lending rates from yield generating protocols in Decentralized Finance

We optimize the portfolio with respect to its risk-return taking into account market impact of different pools.

We accomplish this by writing the investment decision as a Quadratic optimization Problem.

Keywords: DeFi, Fixed income, Quadratic Optimization Problem, Quantitative finance.

1 Introduction

In these notes we propose a method for portfolio optimization for lending rates in yield generating pools in Decentralized Finance (DeFi).

Yield generating pools are protocols in DeFi that offer borrowing of assets via collateralization of crypto assets, returning most of the borrowing rates to the liquidity providers (LPs) as the lending rates (often described as annual percentage yield or APY). Some of the most popular are Compound ([1]), Morpho ([2]), Aave ([3]), IPOR ([4]), among others.

The yields of these protocols have been historically volatile, making it hard to receive a competitive rate without some level of continuous monitoring.

Another problem one should keep in mind when providing liquidity to yield pools is the market impact. Due to their relative low volume, an unwary LP might have suboptimal yields.

This problem of capital allocation given the constraints above is a perfect fit for Quadratic programing (QP) optimization.

In these notes we propose a framework to answer the following question:

1. How to frame the capital allocation on DeFi protocols as a Quadratic optimization problem?

2 Quadratic Programing Optimization

QP is concerned with finding the maximum (or minimum) of quadratic functions under given constraints. We can define the problem as follows:

Definition 1 (QP Problem) *Find* $\boldsymbol{w}^* \in \mathbb{R}^n$ *such that for* $f : \mathbb{R}^n \to \mathbb{R}$ *and under the constraints (u.c.) below, the following holds:*

$$\boldsymbol{w}^* = argmin\ f(w)$$

$$(u.c.)\ \begin{cases} Aw = B \\ Gw \preceq h \end{cases}$$

In order to apply the QP toolkit to our problems we will need to find a quadratic function f and the matrixes and vectors A, B, G

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and h from definition 1 in such a way that the equality and element-wise inequality ($\preceq$) hold.

The link between QP and portfolio management was first done by Markowitz in his seminal paper ([5]). The leap proposed by Markowitz was to identify the function f from definition 1 as a mean variance trade-off.

Mean-Variance Tradeoff: Given a universe of n assets, a portfolio is a sequence $w_n \in \mathbb{R}^n$ such that $\sum_i^n w_i = 1$. If the returns of each asset i is given by R_i, the return of the portfolio $R(w)$ is given by

$$R(w) = \sum_i^n w_i \cdot R_i = w^\top R$$

and if the expected return of each asset i is given by $E[R_i] = \mu_i$, the expected return of the portfolio w is given by

$$E[R(w)] = \mu(w) = \sum_i^n E[w_i \cdot R_i] = w^\top \mu$$

Finally, the variance of the portfolio w is given by

$$\sigma_w^2 = E[R(w) - E[R[w]] = w^\top Q w$$

where Q is the covariances matrix between the assets. For a detailed introduction to QP see [6].

With the notation above, we can rewrite the QP problem in terms of portfolio optimization as follows:

Definition 2 (Portfolio Optimization) *For an investor with risk aversion λ, the portfolio $\boldsymbol{w}^*$ has the best risk-return trade-off for the investor if the following holds:*

$$\boldsymbol{w}^* = argmin\ \frac{\lambda}{2} w^\top Q w - w^\top \mu$$

$$(u.c.)\ \begin{cases} Aw = B \\ Gw \preceq h \end{cases}$$

Our challenge is to frame our problems into the QP framework. These challenges will become clear as we add restrictions in the next sections, specially when adding market impact or rebalancing.

Next we will solve a number of problems using the equation in definition 2.

3 Solving problems

In this section we will have a look on some actual market data. Then we will propose 3 QP problems and their solutions.

3.1 The Data

Figure 1 displays a snapshot of 7 protocols that offered borrow rates on DeFi for the DAI digital asset. Figure 2 displays the time series of the total value locked (TVL) of each pool. The importance of TVL will become clear when dealing with market impact.

Note the volatility of the rates over time. Morpho protocol is often on top, but it is also the most volatile as we will see later. On the TVL, see how Maker DAO dominates on the liquidity from August of 2023.

3.2 Scenario: No Constraints

The problem without any constraints despite being infeasible in practice, its solution is a useful benchmark for the problems to come.

If we have no restrictions, we assume that market participants can borrow at the same rate as they can lend, which we know is not the case (borrowing rates are sometimes not available – for example spark– but often just higher than lend rates – like Aave or Compound).

QP Problem The equation in definition 2 becomes:

$$\mathbf{w}^* = argmin\ \frac{1}{2} w^\top Q w - w^\top \mu$$

$$\text{(u.c.)}\ \left\{ \mathbf{1}_n^\top w = 1 \right.$$

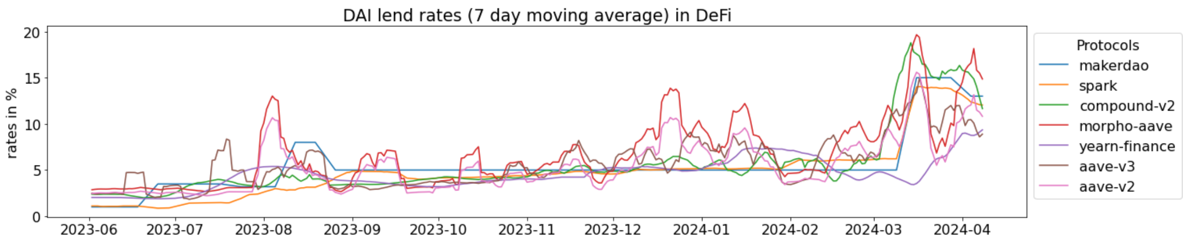

Figure 1: Borrow rates for DAI in seven different pools. MakerDao, Spark, Compound V2, Yearn-Finance, Aave V2 and V3 and Morpho

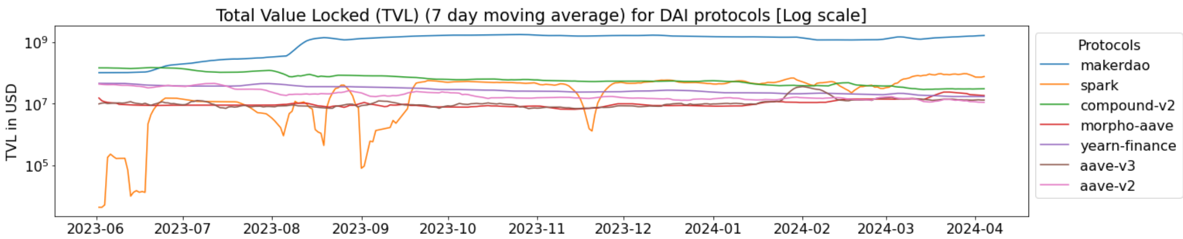

Figure 2: TVL of DAI pool in log scale. Note the dominance of MakerDao pool.

Solution Since this problem has no inequalities, we can solve it analytically using Lagrange multipliers. The solution for σ_P for a portfolio P is a function of the expected return σ_P is a well known solution and it is given by:

$$\sigma_P = \frac{1}{ac - b^2}(a\mu_P^2 - 2\mu_P + c)$$

Where $a = \mathbf{1}_n^\top Q^{-1}\mathbf{1}_n, b = \mu^\top Q^{-1}\mathbf{1}_n$ and $c = \mu^\top Q^{-1}\mu$.

Figure 3 shows the solution for this problem along each asset's historical return and standard deviation. Note the interesting trend of the pools risk-return hinting on the impossibility of the theoretical solution of this section's problem. Also, see how Morpho is the protocol that exhibits the highest historical average returns but also the highest volatility.

3.3 Scenario: Lend Only

The problem in this scenario is closer to our use case because LPs looking for yield rates are often limited to lend rates when choosing protocols due to spreads between lending and borrowing. Hence, each individual weights cannot be negative and since we started on the restrictions let us also go ahead and limit individuals weights to be at most 1 as well.

In this case, the constraints are simply $0 \geq w_i \geq 1$ for all weight i, i.e.,

QP Problem The equation in definition 2 becomes:

$$\mathbf{w}^* = argmin \ \frac{1}{2}w^\top Q w - w^\top \mu$$

$$\text{(u.c.)} \begin{cases} \mathbf{1}_n^\top w = 1 \\ \begin{pmatrix} -\mathbf{1}_n^\top \\ \mathbf{1}_n^\top \end{pmatrix} w \preceq \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{cases}$$

The solution for DAI data can be seen on figure 4. This solution highlights a few interesting portfolios during the span of the data. The first one is a portfolio with 70% of the weights to Compound and 30% to Morpho, this portfolio had a 5.9% yield and a 3.7% standard deviation. A lower standard deviation than Compound alone, but 0.4% higher yield.

The second interesting portfolio is the 63% Morpho, 14% Compound, and 23% Aave V3 portfolio, which has a 6.3% yield and a 3.9%

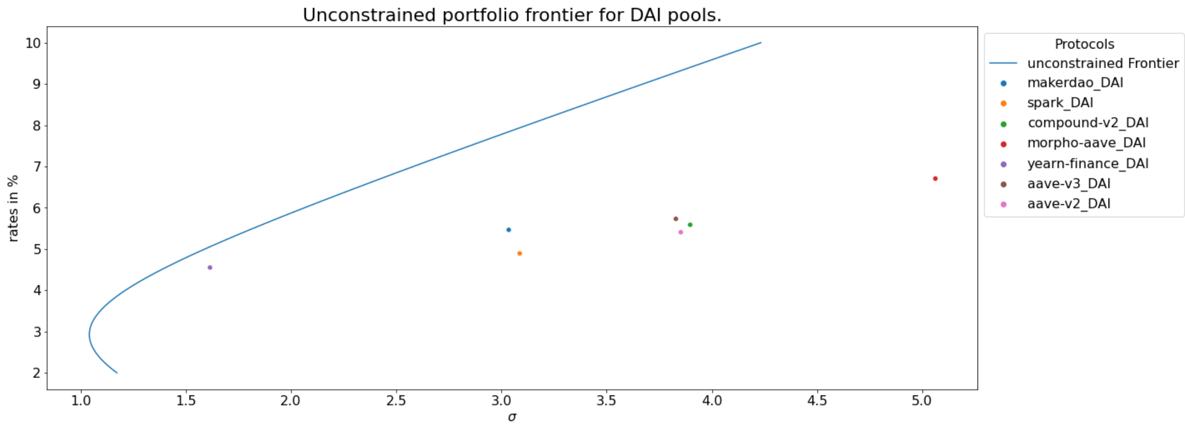

Figure 3: Solution frontier for unconstrained scenario.

standard deviation. This portfolio had a 0.8% higher yield than Compound alone, but only 0.1% higher standard deviation.

See the summary of a few portfolios in table 1.

4 Scenario: Market Impact

Now, let us add a constraint that is more realistic. The market impact constraint is a way to model the fact that the more liquidity you provide to a pool, the lower the yield you will get. This is a common feature in DeFi protocols, where the yield is a function of the total value locked (TVL) in the pool.

First, let's define the market impact and estimate it for the pools. In this section we will again use DAI data as a case study, but the approach can be generalized to any other asset.

4.1 Market Impact

Our goal in this section is to find a slope α such that the yield is a function of the TVL in the pool and the amount of capital X deployed. In this case we will assume that the yield will decrease by $\alpha\frac{X}{TVL}$.

This linear approach has a number of tradeoffs that we also discuss in this section.

A common interest rate pricing model in DeFi has the following form:

$$y(U_r) = \begin{cases} \alpha_1 \cdot U_r \text{ if } U_r \leq U^* \\ \alpha_1 \cdot U^* + \alpha_2 \cdot (U_r - U^*) \text{ if } U_r > U^* \end{cases}$$

Where U_r is the utilization rate of the pool as a ratio of total lending amount and TVL, U^* is the target utilization rate.

For these notes, we will approximate the step function above with a linear function. Despite the error this approximation introduces, it is a good starting point for our analysis as it will fit the QP framework. We conjecture that it is possible to write a QP problem using the step function above, but we leave this for future work.

Our process to estimate the slope α is as follows:

1. Collect the historical TVl, lending amounts and lend rates for a pool.
2. Calculate the Utilization rate (U_r) and fit a line on the $U_r\times$ (Lend Rate) axis per section.
3. Extract the slope α from the line.

Figure 5 shows the estimated slopes for Aave V2 DAI Pool up to 80% utilization rate. There are two points to note in this figure.

The first issue here is that the model of Aave V2 for DAI changed over the period of the data (2023). This is a common problem that we need to address when estimating the market impact. In this case it is simple, we

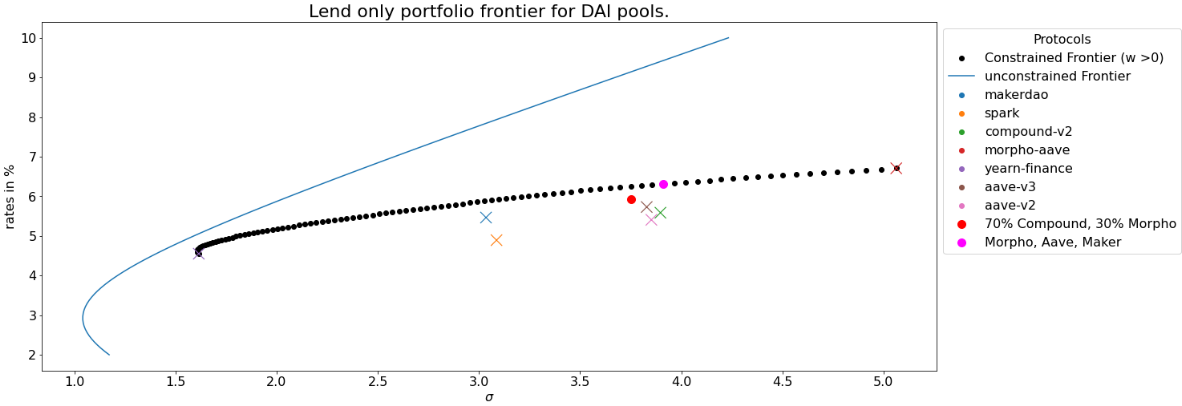

Figure 4: Solution frontier for borrow only scenario.

Weights					
Compound	Morpho	Maker	AaveV3	μ	σ
100%	0	0	0	5.5%	3.8%
0	100%	0	0	6.7%	5.0%
70%	30%	0	0	5.9%	3.7%
0	63%	14%	23%	6.3%	3.9%

Table 1: Highlighted portfolios for lend only scenario. For example, note how the last portfolio had a higher yield than Compound alone with similar standard deviation (higher returns with similar risk).

just need to fit the line on a more recent period. We added this artifact plot stress the importance of this point.

The second point is how to read the slope. In the plot we got a slope of 6.6, that means that each 1% decrease in utilization rate the yield decrease by 0.066%.

For the remainder of these notes we will fix this slope to 6.6, but in practice this slope should be estimated for each pool and for each period.

Ultimately, the tradeoff we are doing with the linear estimation is gaining all the speed and simplicity of the QP framework with a good market impact estimation most of the time at the cost of a under or overestimation of the impact at particular periods. The key decision factor here, is if the period we are apply our QP toolkit is calibrated to the most recent model and state of the market.

4.2 QP Problem

The main change we need to incorporate is to the returns as it gets penalized by the market impact. The new returns are given by

$$\mu_i - \alpha_i \frac{C}{TVL_i},$$

where TVL_i is the total value locked in pool i and C is the total of capital in the portfolio.

With that, our solution will be dependent on our capital amount, since the more capital is deployed the lower the yield.

QP Problem Let C be the total capital for an LP and let $X = (\frac{C_1}{TVL_1}, \ldots, \frac{C_n}{TVL_n})^\top$, then for the market impact QP problem the equation in definition 2 becomes:

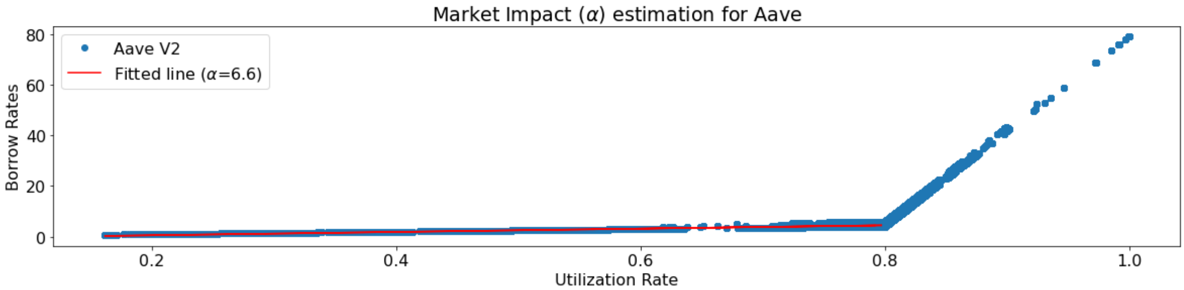

Figure 5: Aave V2 DAI pool market impact estimation. Dates: 2023-01 ∼ 2024-03

$$\mathbf{w}^* = argmin\ \frac{1}{2} w^\top Q w - w^\top(\mu - \alpha^\top X)$$

$$\text{(u.c.)} \begin{cases} \mathbf{1}_n^\top w = 1 \\ \begin{pmatrix} -\mathbf{1}_n^\top \\ \mathbf{1}_n^\top \end{pmatrix} w \preceq \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{cases}$$

Figure 6 shows the solution for the lend only scenario with market impact. Note how the solution depends on the amount of capital deployed. The more capital, the lower the yield.

5 Conclusion

In these notes we proposed a method to optimize a portfolio of yield generating protocols in Decentralized Finance with respect to risk-reward taking into account market impact of different pools.

We accomplished this by writing the investment decision as a Quadratic optimization Problem. We solved a number of problems using the equation in definition 2.

The main takeaway from these notes is that the QP framework is a powerful tool to optimize portfolios in DeFi and it can be used as such. The main challenge is to frame the problem in such a way that the QP toolkit can be applied. In this case, we used the market impact as a constraint to the problem.

The main limitation of this approach is the linear approximation of the market impact. This approximation is a tradeoff between speed and accuracy. The key decision factor here is if the period we are applying our QP toolkit is calibrated to the most recent model and state of the market. But we conjecture that it is possible to write a QP problem using the step function above, but we leave this for future work.

5.1 Future Work

In future work we wish to address the following points: 1) Write a QP problem using the step function above. 2) Add rebalancing constraints to the QP problem. 3) Benchmark the performance of the QP solution against a simple strategy like Compound alone. 4) Add more assets to the portfolio.

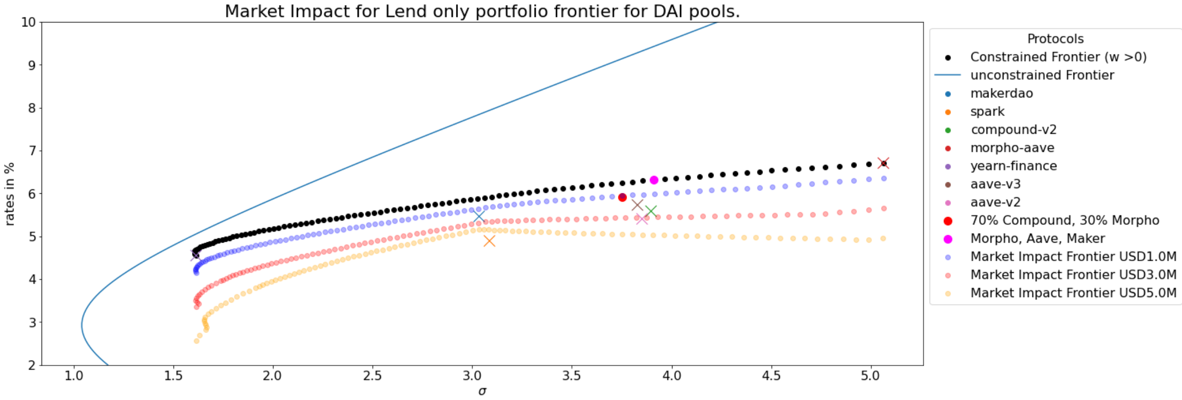

Figure 6: Market Impact for Lend only portfolio frontier for DAI pools.

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