Coupled Mode Theory and 50/50 Splitter

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***Abstract*—In this work, we aimed to design a 50/50 splitter composed by a two-waveguide coupled system. First, we started with a pre-analysis with COMSOL to optimize the material of the waveguides, their width and thickness, alongside with a resonable gap to obtain a short splitter. We followed by theoretically modeling the problem based on Coupled Mode Theory, ranging from the derivation of the coupling constant formula, the coupled mode equations, and a numerical method to calculate the coupling rate. We then analysed two cases of beamplitter, the first with identical waveguides, and the second with one waveguide with doubled dimensions. We found that the second case allows for a more compact splitter.**

***Index Terms*—Waveguides, Coupled Mode Theory, Beamsplitter.**

I. Introduction

In this section, we aim to describe the investigation to choose the material and the geometrical parameters of the waveguides belonging the coupled system. We considered SiO_2, TiO_2, Si_3N_4, and Ag_3AsS_3 as the possible materials due to their renowned applications in photonic integrated circuits (PIC). Fig. 1 displays the dispersion of each one of these materials.

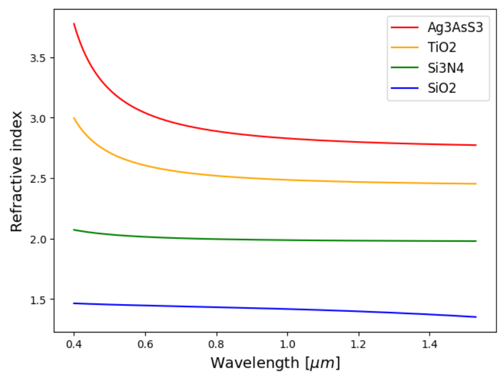

Fig. 1: Dispersion relation for the SiO_2, TiO_2, Si_3N_4, and Ag_3AsS_3.

Motivated by the common use of SiO_2 and TiO_2 in waveguides and because they present distinctive dispersion relations, TiO_2 presenting the highest values, we will focus on these materials as the candidates for the beam splitter. Their dispersion relations are described by Eq. (1).

$$n_{SiO_2} = \sqrt{1 + \frac{0.6961663\lambda^2}{\lambda^2 - 0.0684043^2} + \frac{0.4079426\lambda^2}{\lambda^2 - 0.1162414^2} + \frac{0.8974794\lambda^2}{\lambda^2 - 9.896161}} \quad \text{(1a)}$$

$$n_{TiO_2} = \sqrt{5.913 + \frac{0.2441}{\lambda^2 - 0.0803}} \quad \text{(1b)}$$

To understand the number of modes in a waveguide, we plotted the characteristic equations for SiO_2 for the infinite dielectric slab case, which has a easier mathematical description. The generic characteristic equation is described by Eq. (2) where d is the thickness of the slab, and the plot of the functions describing this equation, f_1 and f_2, are plotted in Fig. 3.

$$\begin{aligned} f_1(\sin(\theta)) &\equiv \tan\left[\pi\left(\frac{\sin(\theta)d}{\lambda} - \frac{m}{2}\right)\right] \\ &= \sqrt{\frac{\sin^2(\overline{\theta}_c)}{\sin^2(\theta)} - 1} \equiv f_2(\sin(\theta)) \end{aligned} \quad (2)$$

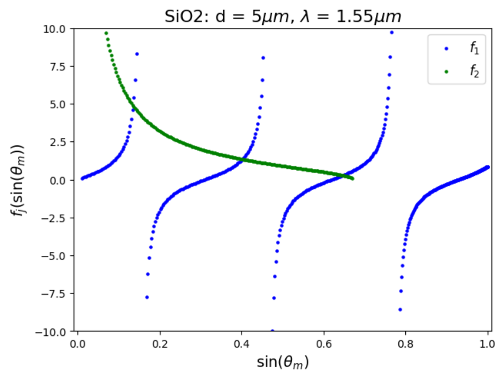

Fig. 2: SiO_2.

Fig. 3: Plot of the characteristic equations for the SiO_2 and TiO_2 infinite slab case.

From the graphical solution in Fig. 3 we observe that f_1 and f_2 intersect at three points, therefore this waveguide presents exactly three modes. As we increase the thickness of the slab, we make room for more modes to appear. We also plotted the characteristic functions for the rectangular dielectric case for SiO_2 and TiO_2, considering the approximation $E(x, y) = X(x)Y(y)$. Considering these components independent, we can solve the characteristic equation for x and y dimensions separately, find θ_{mx} and θ_{my}, and latter combine them into the solution described by Eq. (3), where $k_j = n_1 k_0 \sin(\theta_{m_j}),\ \forall j \in \{1, 2\}$.

$$\beta_{m_x,m_y} = \sqrt{(n_1 k_0)^2 - k_x^2 - k_y^2} \tag{3}$$

With the results above, we could note the waveguide made of SiO_2 presented a smaller number of modes when compared to the TiO_2 case. Since we are concerned with a waveguide with a few guided modes only, we decided to explore the coupled system made of SiO_2 cores and the geometrical parameters $t_{core,j} = 2\mu m$, (i) $w_{core,j} = 6\mu m$, and (ii) $w_{core,2} = 2w_{core,1} = 6\mu m$ and $t_{core,j} = 2\mu m$. Another observation, is that we are considering the two cores for the coupled systems to be aligned at the center of the cartesian coordiniate system, in a sense the gap in the y-direction is $d_y = 0$, otherwise it would distort the overlapping region and give similar results. Thus, we name the gap in the x-direction simply d.

II. Theoretical Development

A. Coupling coefficient

B. Coupled equations

III. 50/50 Splitter

In this section, we aim to effectively model the 50/50 splitter describe at the end of Sec. I with COMSOL with the cores made by SiO_2 and the cladding made of air with the following geometrical parameters $t_{clad} = 15\mu m$ and $w_{clad} = 30\mu m$. We discuss the case of two identical cores with $t_{core} = 2\mu m$ and $w_{core} = 6\mu mu$ in Subsec. III-A and in Subsec. III-B we analyse the changes when we let the first core to have width $w_{core,1} = w_{core,2}/2 = 3\mu m$, that is, core 2 presents the double width of core 1.

A. Identical cores

For this subsection and the next, we are consider $1.55\mu m$ as the reference input wavelength due its vast applicability in telecommunication. Our analysis is also limited to the first even and odd supermodes. Also, we are separating the even supermodes from the odd ones based on thedirection of $\mathbf{E}_x$ vs. $\mathbf{E}_y$ as indicated in Fig. 4. The even modes are in the same direction for both cores, while the odd ones are in the opposite direction.

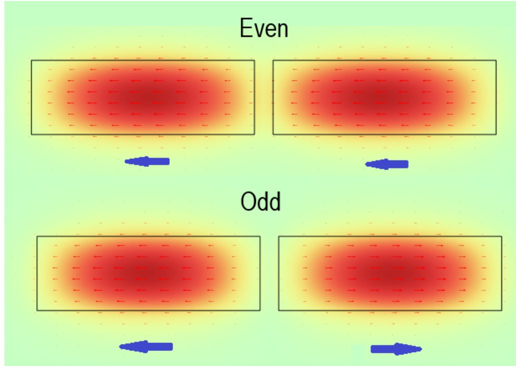

Fig. 4: Distinction between even and odd supermodes SiO_2 core with $w_{core,j} = 6\mu m$ and $t_{core,j} = 2\mu m$, and the gap between cores given by $d = 0.5\mu m$.

We varied the gap between the cores and decided to use $d = 0.5mum$ due to its considerable overlapping region and because lower gaps might be difficult to frabricate with techniques such as photolitography.

Aiming to explore the sensitivity of this splitter with the wavelength, we repeated the simulation for the range $\lambda \in [1.40\mu m, 1.70\mu m]$ and obtained from COMSOL the effective refractive index n_{eff} for each supermode. Using $\beta = k_0 n_{eff}$, we could obtain the propagation constant for each supermode mode as well.

Once we now the order of the even and odd supermodes for a given wavelength, we can compute the coupling length L_c, the length needed for the light being initally guided through the first core to be entirely coulpled into the second core. To do so, we used Eq. (4), where $\Delta\beta \equiv \beta_{even} - \beta_{odd}$. These values are place in Tab. I, which also displays the coupling efficiency η from the first core into the second one considering an interaction length of $L_{int} = 484.4\mu m$.

$$L_c = \frac{\pi}{\Delta\beta} \tag{4}$$

$\lambda[\mu m]$	L_c	$\eta[\%]$
1.40	1400.0	34.6
1.45	1450.0	33.4
1.50	1071.4	45.2
1.55	968.7	50.0
1.60	888.9	54.5
1.65	750.0	64.59
1.70	653.8	74.09

TABLE I: Coupling length and coupling efficiency coefficient for the first supermode making $L_{int} = 484.4\mu m$ varying the input wavelength for the identical cores.

As seen from Table I, the operational range of this 50/50 beamplitter is short, it can operate within a 5% margin of η in the range $\lambda \in [1.50\mu m, 1.60\mu m]$, a span of only $100nm$. Also, if we consider a fabrication error in the interaction length of 10%, we would have a coupling interval in $1.55\mu m$ of $\eta \in [45\%, 55\%]$.

B. First core with half width

For the case of the first core to present half the widht of the second, we repeated the same analysis of Subsec. III-A.

The wavelength sensitivity is analysed in Table II. As you can see, the η vary much less with the wavelength than the previous case. For the range $\lambda \in [1.40\mu m, 1.70\mu m]$, a span of $300nm$, η has a margin less than 2%. Thus, if we accept a margin of 5%, we could extend even further the wavelength range. Again, if we consider a fabrication error in the interaction length of 10% for $1.55\mu m$ input wavelength, we would obtain a efficiency interval of $[45\%, 55\%]$. In this case, the interaction length error would contribute more to the deviation of the 50/50 coupling when compared to the wavelength variation.

$\lambda[\mu m]$	L_c	$\eta[\%]$
1.40	179.5	49.1
1.45	176.8	49.8
1.50	174.4	50.5
1.55	176.1	50.0
1.60	170.2	51.8
1.65	175.5	50.2
1.70	177.1	49.7

TABLE II: Coupling length and coupling efficiency coefficient for the first supermode making $L_{int} = 88.1\mu m$ varying the input wavelength for different size cores.

IV. Conclusion

In summary, we began our study with a investigation of the material of cores for the coupled system, their geometry and the gap distance between them. In sequence, we derived the coupling coefficient form and the coupled mode equations, followed by the discussion of a numerical method to compute the coupling rate. At last, we projected two cases of a 50/50 beamplitter varying the geometry of the cores, and analyse their sensitivity with the wavelength and the length interaction error.