# 1 Introduction

Logistic regression specifies a dichotomous dependent variable as a function of a set of explanatory variables.

# 2 Syntax

```
> z.out <- zelig(Y ~ X1 + X2, model = "logit", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out, x1 = NULL)
```

# 3 Additional Inputs

In addition to the standard inputs, `zelig()` takes the following additional options for logistic regression:

- `robust`: defaults to `FALSE`. If `TRUE` is selected, `zelig()` computes robust standard errors via the `sandwich` package (see [8]). The default type of robust standard error is heteroskedastic and autocorrelation consistent (HAC), and assumes that observations are ordered by time index.

  In addition, `robust` may be a list with the following options:

  - `method`: Choose from
    - `"vcovHAC"`: (default if `robust = TRUE`) HAC standard errors.
    - `"kernHAC"`: HAC standard errors using the weights given in [1].
    - `"weave"`: HAC standard errors using the weights given in [5].
  - `order.by`: defaults to `NULL` (the observations are chronologically ordered as in the original data). Optionally, you may specify a vector of weights (either as `order.by = z`, where `z` exists outside the data frame; or as `order.by = ~z`, where `z` is a variable in the data frame) The observations are chronologically ordered by the size of `z`.
  - `...`: additional options passed to the functions specified in `method`. See the `sandwich` library and [8] for more options.

# 4 Examples

1. Basic Example

   Attaching the sample turnout dataset:

   ```
   > data(turnout)
   ```

   Estimating parameter values for the logistic regression:

```
>  z.out1 <- zelig(vote ~ age + race,  model = "logit", data = turnout)
>
```

Setting values for the explanatory variables:

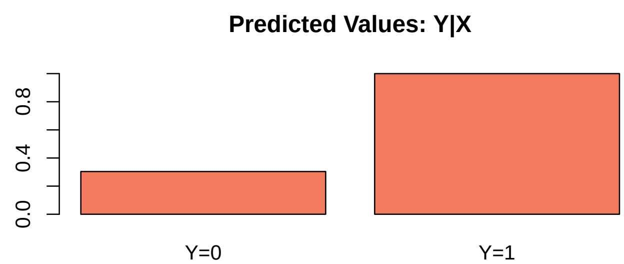
```
>  x.out1 <- setx(z.out1, age = 36, race = "white")
```

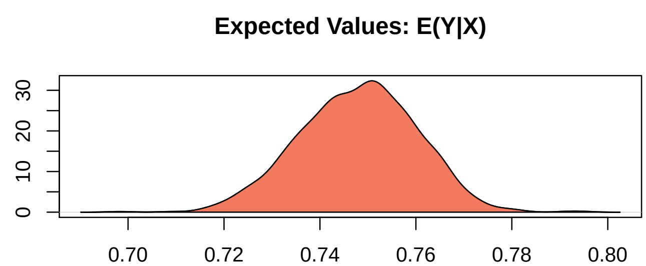
Simulating quantities of interest from the posterior distribution.

```
>  s.out1 <- sim(z.out1, x = x.out1)
```

```
>  summary(s.out1)
```

```
>  plot(s.out1)
```




2. Simulating First Differences

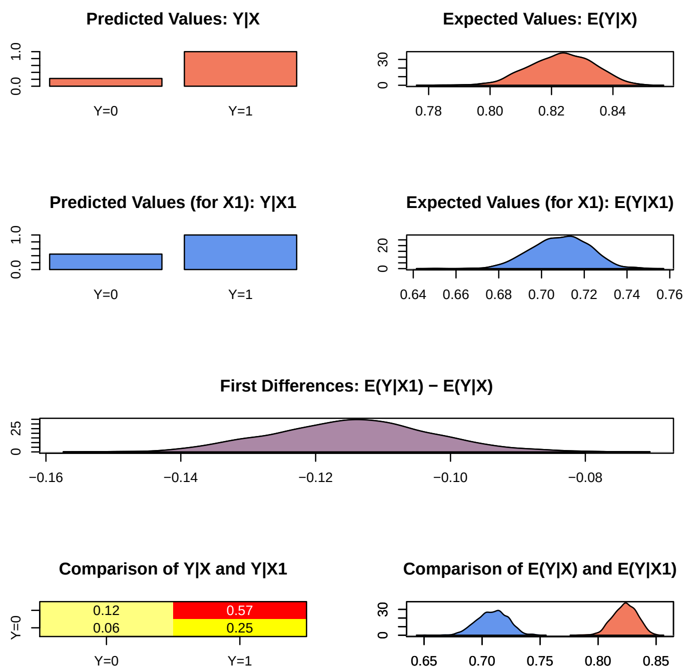
   Estimating the risk difference (and risk ratio) between low education (25th percentile) and high education (75th percentile) while all the other variables held at their default values.

```
>  z.out2 <- zelig(vote ~ race + educate, model = "logit", data = turnout)
>  x.high <- setx(z.out2, educate = quantile(turnout$educate, prob = 0.75))
>  x.low <- setx(z.out2, educate = quantile(turnout$educate, prob = 0.25))
```

```
> s.out2 <- sim(z.out2, x = x.high, x1 = x.low)
```

```
> summary(s.out2)
```

```
> plot(s.out2)
```

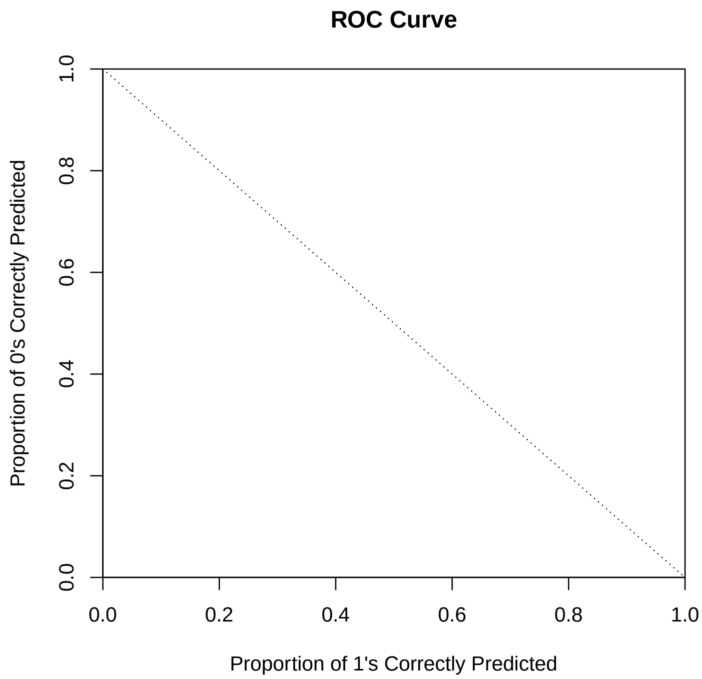



3. Presenting Results: An ROC Plot

   One can use an ROC plot to evaluate the fit of alternative model specifications. (Use `demo(roc)` to view this example, or see King and Zeng (2002).)

```
> z.out1 <- zelig(vote ~ race + educate + age, model = "logit",
+                 data = turnout)
> z.out2 <- zelig(vote ~ race + educate, model = "logit", data = turnout)
```

```
> rocplot(z.out1$y, z.out2$y, fitted(z.out1), fitted(z.out2))
```




# 5 Model

Let $Y_i$ be the binary dependent variable for observation $i$ which takes the value of either 0 or 1.

- The *stochastic component* is given by

$$\begin{aligned} Y_i &\sim \text{Bernoulli}(y_i \mid \pi_i) \\ &= \pi_i^{y_i}(1-\pi_i)^{1-y_i} \end{aligned}$$

  where $\pi_i = \Pr(Y_i = 1)$.

- The *systematic component* is given by:

$$\pi_i \;=\; \frac{1}{1+\exp(-x_i\beta)}.$$

  where $x_i$ is the vector of $k$ explanatory variables for observation $i$ and $\beta$ is the vector of coefficients.

# 6 Quantities of Interest

- The expected values (`qi$ev`) for the logit model are simulations of the predicted probability of a success:

$$E(Y) = \pi_i = \frac{1}{1 + \exp(-x_i \beta)},$$

  given draws of $\beta$ from its sampling distribution.

- The predicted values (`qi$pr`) are draws from the Binomial distribution with mean equal to the simulated expected value $\pi_i$.

- The first difference (`qi$fd`) for the logit model is defined as

$$\text{FD} = \Pr(Y = 1 \mid x_1) - \Pr(Y = 1 \mid x).$$

- The risk ratio (`qi$rr`) is defined as

$$\text{RR} = \Pr(Y = 1 \mid x_1) \ / \ \Pr(Y = 1 \mid x).$$

- In conditional prediction models, the average expected treatment effect (`att.ev`) for the treatment group is

$$\frac{1}{\sum_{i=1}^{n} t_i} \sum_{i:t_i=1}^{n} \left\{ Y_i(t_i = 1) - E[Y_i(t_i = 0)] \right\},$$

  where $t_i$ is a binary explanatory variable defining the treatment ($t_i = 1$) and control ($t_i = 0$) groups. Variation in the simulations are due to uncertainty in simulating $E[Y_i(t_i = 0)]$, the counterfactual expected value of $Y_i$ for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_i = 0$.

- In conditional prediction models, the average predicted treatment effect (`att.pr`) for the treatment group is

$$\frac{1}{\sum_{i=1}^{n} t_i} \sum_{i:t_i=1}^{n} \left\{ Y_i(t_i = 1) - Y_i(\widehat{t_i = 0}) \right\},$$

  where $t_i$ is a binary explanatory variable defining the treatment ($t_i = 1$) and control ($t_i = 0$) groups. Variation in the simulations are due to uncertainty in simulating $Y_i(\widehat{t_i = 0})$, the counterfactual predicted value of $Y_i$ for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_i = 0$.

# 7 Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run `z.out <- zelig(y ~ x, model = "logit", data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the `coefficients` by using `z.out$coefficients`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:
  - `coefficients`: parameter estimates for the explanatory variables.
  - `residuals`: the working residuals in the final iteration of the IWLS fit.
  - `fitted.values`: the vector of fitted values for the systemic component, $\pi_i$.
  - `linear.predictors`: the vector of $x_i\beta$
  - `aic`: Akaike's Information Criterion (minus twice the maximized log-likelihood plus twice the number of coefficients).
  - `df.residual`: the residual degrees of freedom.
  - `df.null`: the residual degrees of freedom for the null model.
  - `data`: the name of the input data frame.
- From `summary(z.out)`, you may extract:
  - `coefficients`: the parameter estimates with their associated standard errors, $p$-values, and $t$-statistics.
  - `cov.scaled`: a $k \times k$ matrix of scaled covariances.
  - `cov.unscaled`: a $k \times k$ matrix of unscaled covariances.
- From the `sim()` output object `s.out`, you may extract quantities of interest arranged as matrices indexed by simulation $\times$ `x`-observation (for more than one `x`-observation). Available quantities are:
  - `qi$ev`: the simulated expected probabilities for the specified values of `x`.
  - `qi$pr`: the simulated predicted values for the specified values of `x`.
  - `qi$fd`: the simulated first difference in the expected probabilities for the values specified in `x` and `x1`.
  - `qi$rr`: the simulated risk ratio for the expected probabilities simulated from `x` and `x1`.
  - `qi$att.ev`: the simulated average expected treatment effect for the treated from conditional prediction models.
  - `qi$att.pr`: the simulated average predicted treatment effect for the treated from conditional prediction models.

### How to Cite the Logit Model

Kosuke Imai, Olivia Lau, and Gary King. *logit: Logistic Regression for Dichotomous Dependent*, 2011

### How to Cite the Zelig Software Package

To cite Zelig as a whole, please reference these two sources:

> Kosuke Imai, Gary King, and Olivia Lau. 2007. "Zelig: Everyone's Statistical Software," `http://GKing.harvard.edu/zelig`.
>
> Imai, Kosuke, Gary King, and Olivia Lau. (2008). "Toward A Common Framework for Statistical Analysis and Development." Journal of Computational and Graphical Statistics, Vol. 17, No. 4 (December), pp. 892-913.

### See also

The logit model is part of the stats package by **(author?)** [7]. Advanced users may wish to refer to `help(glm)` and `help(family)`, as well as [6]. Robust standard errors are implemented via the sandwich package by **(author?)** [8]. Sample data are from [3].

## References


[1] Donald W.K. Andrews. Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica*, 59(3):817–858, May 1991.

[2] Kosuke Imai, Olivia Lau, and Gary King. *logit: Logistic Regression for Dichotomous Dependent*, 2011.

[3] Gary King, Michael Tomz, and Jason Wittenberg. Making the most of statistical analyses: Improving interpretation and presentation. *American Journal of Political Science*, 44(2):341–355, April 2000. http://gking.harvard.edu/files/abs/making-abs.shtml.

[4] Gary King and Langche Zeng. Improving forecasts of state failure. *World Politics*, 53(4):623–658, July 2002. http://gking.harvard.edu/files/abs/civil-abs.shtml.

[5] Thomas Lumley and Patrick Heagerty. Weighted empirical adaptive variance estimators for correlated data regression. *jrssb*, 61(2):459–477, 1999.

[6] Peter McCullagh and James A. Nelder. *Generalized Linear Models*. Number 37 in Monograph on Statistics and Applied Probability. Chapman & Hall, 2nd edition, 1989.

[7] William N. Venables and Brian D. Ripley. *Modern Applied Statistics with S*. Springer-Verlag, 4th edition, 2002.

[8] Achim Zeileis. Econometric computing with hc and hac covariance matrix estimators. *Journal of Statistical Software*, 11(10):1–17, 2004.