

1 gamma.mixed: Mixed effects gamma regression

Use generalized multi-level linear regression if you have covariates that are grouped according to one or more classification factors. Gamma regression models a continuous, positive dependent variable.

While generally called multi-level models in the social sciences, this class of models is often referred to as mixed-effects models in the statistics literature and as hierarchical models in a Bayesian setting. This general class of models consists of linear models that are expressed as a function of both *fixed effects*, parameters corresponding to an entire population or certain repeatable levels of experimental factors, and *random effects*, parameters corresponding to individual experimental units drawn at random from a population.

1.0.1 Syntax

```
z.out <- zelig(formula= y ~ x1 + x2 + tag(z1 + z2 | g),
               data=mydata, model="gamma.mixed")

z.out <- zelig(formula= list(mu=y ~ x1 + x2 + tag(z1, delta | g),
                             delta= ~ tag(w1 + w2 | g)), data=mydata, model="gamma.mixed")
```

1.0.2 Inputs

zelig() takes the following arguments for mixed:

- **formula**: a two-sided linear formula object describing the systematic component of the model, with the response on the left of a `~` operator and the fixed effects terms, separated by `+` operators, on the right. Any random effects terms are included with the notation `tag(z1 + ... + zn | g)` with `z1 + ... + zn` specifying the model for the random effects and `g` the grouping structure. Random intercept terms are included with the notation `tag(1 | g)`.

Alternatively, **formula** may be a list where the first entry, **mu**, is a two-sided linear formula object describing the systematic component of the model, with the response on the left of a `~` operator and the fixed effects terms, separated by `+` operators, on the right. Any random effects terms are included with the notation `tag(z1, delta | g)` with `z1` specifying the individual level model for the random effects, `g` the grouping structure and `delta` references the second equation in the list. The **delta** equation is one-sided linear formula object with the group level model for the random effects on the right side of a `~` operator. The model is specified with the notation `tag(w1 + ... + wn | g)` with `w1 + ... + wn` specifying the group level model and `g` the grouping structure.

1.0.3 Additional Inputs

In addition, `zelig()` accepts the following additional arguments for model specification:

- **data:** An optional data frame containing the variables named in **formula**. By default, the variables are taken from the environment from which `zelig()` is called.
- **method:** a character string. The criterion is always the log-likelihood but this criterion does not have a closed form expression and must be approximated. The default approximation is "PQL" or penalized quasi-likelihood. Alternatives are "Laplace" or "AGQ" indicating the Laplacian and adaptive Gaussian quadrature approximations respectively.
- **na.action:** A function that indicates what should happen when the data contain NAs. The default action (**na.fail**) causes `zelig()` to print an error message and terminate if there are any incomplete observations.

Additionally, users may wish to refer to `lmer` in the package `lme4` for more information, including control parameters for the estimation algorithm and their defaults.

1.0.4 Examples

1. Basic Example with First Differences

Attach sample data:

```
> data(coalition2)
```

Estimate model using optional arguments to specify approximation method for the log-likelihood, and the log link function for the Gamma family:

```
> z.out1 <- zelig(duration ~ invest + fract + polar + numst2 +  
+   crisis + tag(1 | country), data = coalition2, model = "gamma.mixed",  
+   method = "PQL", family = Gamma(link = log))
```

Summarize regression coefficients and estimated variance of random effects:

```
> summary(z.out1)
```

Set the baseline values (with the ruling coalition in the minority) and the alternative values (with the ruling coalition in the majority) for X:

```
> x.high <- setx(z.out1, numst2 = 1)  
> x.low <- setx(z.out1, numst2 = 0)
```

Simulate expected values (`qi$ev`) and first differences(`qi$fd`):

```
> s.out1 <- sim(z.out1, x = x.high, x1 = x.low)
> summary(s.out1)
```

1.0.5 Mixed effects gamma regression Model

Let Y_{ij} be the continuous, positive dependent variable, realized for observation j in group i as y_{ij} , for $i = 1, \dots, M$, $j = 1, \dots, n_i$.

- The *stochastic component* is described by a Gamma model with scale parameter α .

$$Y_{ij} \sim \text{Gamma}(y_{ij} | \lambda_{ij}, \alpha)$$

where

$$\text{Gamma}(y_{ij} | \lambda_{ij}, \alpha) = \frac{1}{\alpha^{\lambda_{ij}} \Gamma \lambda_{ij}} y_{ij}^{\lambda_{ij}-1} \exp(-\{ \frac{y_{ij}}{\alpha} \})$$

for $\alpha, \lambda_{ij}, y_{ij} > 0$.

- The q -dimensional vector of *random effects*, b_i , is restricted to be mean zero, and therefore is completely characterized by the variance covariance matrix Ψ , a $(q \times q)$ symmetric positive semi-definite matrix.

$$b_i \sim \text{Normal}(0, \Psi)$$

- The *systematic component* is

$$\lambda_{ij} \equiv \frac{1}{X_{ij}\beta + Z_{ij}b_i}$$

where X_{ij} is the $(n_i \times p \times M)$ array of known fixed effects explanatory variables, β is the p -dimensional vector of fixed effects coefficients, Z_{ij} is the $(n_i \times q \times M)$ array of known random effects explanatory variables and b_i is the q -dimensional vector of random effects.

1.0.6 Quantities of Interest

- The predicted values (`qi$pr`) are draws from the gamma distribution for each given set of parameters (α, λ_{ij}) , for

$$\lambda_{ij} = \frac{1}{X_{ij}\beta + Z_{ij}b_i}$$

given X_{ij} and Z_{ij} and simulations of β and b_i from their posterior distributions. The estimated variance covariance matrices are taken as correct and are themselves not simulated.

- The expected values (`qi$ev`) are simulations of the mean of the stochastic component given draws of α , β from their posteriors:

$$E(Y_{ij}|X_{ij}) = \alpha\lambda_{ij} = \frac{\alpha}{X_{ij}\beta}.$$

- The first difference (`qi$fd`) is given by the difference in expected values, conditional on X_{ij} and X'_{ij} , representing different values of the explanatory variables.

$$FD(Y_{ij}|X_{ij}, X'_{ij}) = E(Y_{ij}|X_{ij}) - E(Y_{ij}|X'_{ij})$$

- In conditional prediction models, the average predicted treatment effect (`qi$att.pr`) for the treatment group is given by

$$\frac{1}{\sum_{i=1}^M \sum_{j=1}^{n_i} t_{ij}} \sum_{i=1}^M \sum_{j:t_{ij}=1}^{n_i} \{Y_{ij}(t_{ij}=1) - Y_{ij}(\widehat{t_{ij}}=0)\},$$

where t_{ij} is a binary explanatory variable defining the treatment ($t_{ij} = 1$) and control ($t_{ij} = 0$) groups. Variation in the simulations is due to uncertainty in simulating $Y_{ij}(t_{ij} = 0)$, the counterfactual predicted value of Y_{ij} for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_{ij} = 0$.

- In conditional prediction models, the average expected treatment effect (`qi$att.ev`) for the treatment group is given by

$$\frac{1}{\sum_{i=1}^M \sum_{j=1}^{n_i} t_{ij}} \sum_{i=1}^M \sum_{j:t_{ij}=1}^{n_i} \{Y_{ij}(t_{ij}=1) - E[Y_{ij}(t_{ij}=0)]\},$$

where t_{ij} is a binary explanatory variable defining the treatment ($t_{ij} = 1$) and control ($t_{ij} = 0$) groups. Variation in the simulations is due to uncertainty in simulating $E[Y_{ij}(t_{ij} = 0)]$, the counterfactual expected value of Y_{ij} for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_{ij} = 0$.

1.0.7 Output Values

The output of each Zelig command contains useful information which you may view. You may examine the available information in `z.out` by using `slotNames(z.out)`, see the fixed effect coefficients by using `summary(z.out)$coefs`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output stored in `summary(z.out)`, you may extract:

- **fixef**: numeric vector containing the conditional estimates of the fixed effects.
 - **ranef**: numeric vector containing the conditional modes of the random effects.
 - **frame**: the model frame for the model.
- From the `sim()` output stored in `s.out`, you may extract quantities of interest stored in a data frame:
 - **qi\$pr**: the simulated predicted values drawn from the distributions defined by the expected values.
 - **qi\$ev**: the simulated expected values for the specified values of `x`.
 - **qi\$fd**: the simulated first differences in the expected values for the values specified in `x` and `x1`.
 - **qi\$ate.pr**: the simulated average predicted treatment effect for the treated from conditional prediction models.
 - **qi\$ate.ev**: the simulated average expected treatment effect for the treated from conditional prediction models.

How to Cite the Multi-level Gamma Model

Matthew Owen, Olivia Lau, Kosuke Imai, and Gary King. *gamma.mixed: Mixed Effects Gamma Regression*, 2011

How to Cite the Zelig Software Package

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Imai, Kosuke, Gary King, and Olivia Lau. (2008). “Toward A Common Framework for Statistical Analysis and Development.” *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

See also

Mixed effects gamma regression is part of `lme4` package by Douglas M. Bates [1].

References

- [1] Douglas Bates. *lme4: Fit linear and generalized linear mixed-effects models*, 2007.
- [2] Matthew Owen, Olivia Lau, Kosuke Imai, and Gary King. *gamma.mixed: Mixed Effects Gamma Regression*, 2011.