

1 normal.net: Network Normal Regression for Continuous Proximity Matrix Dependent Variables

The Network Normal regression model is a close variant of the more standard least squares regression model (see `netlm`). Both models specify a continuous proximity matrix (a.k.a. sociomatrixes, adjacency matrices, or matrix representations of directed graphs) dependent variable as a linear function of a set of explanatory variables. The network Normal model reports maximum likelihood (rather than least squares) estimates. The two models differ only in their estimate for the stochastic parameter σ .

1.0.1 Syntax

```
> z.out <- zelig(y ~ x1 + x2, model = "normal.net", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

1.0.2 Additional Inputs

In addition to the standard inputs, `zelig()` takes the following additional options for network normal regression:

- **LF**: specifies the link function to be used for the network normal regression. Default is `LF="identity"`, but `LF` can also be set to `"log"` or `"inverse"` by the user.

1.0.3 Examples

1. Basic Example

Load the sample data (see `?friendship` for details on the structure of the network dataframe):

```
> data(friendship)
```

Estimate model:

```
> z.out <- zelig(perpower ~ friends + advice + prestige, model = "normal.net",
+               data = friendship)
> summary(z.out)
```

Setting values for the explanatory variables to their default values:

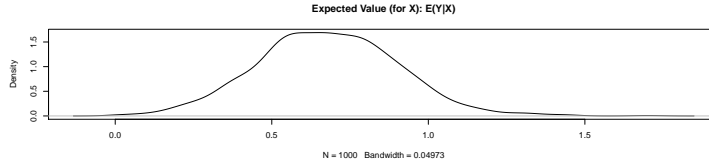
```
> x.out <- setx(z.out)
```

Simulate fitted values.

```

> s.out <- sim(z.out, x = x.out)
> summary(s.out)
> plot(s.out)

```



1.0.4 Model

The `normal.net` model performs a Normal regression of the proximity matrix \mathbf{Y} , a $m \times m$ matrix representing network ties, on a set of proximity matrices \mathbf{X} . This network regression model is directly analogous to standard Normal regression element-wise on the appropriately vectorized matrices. Proximity matrices are vectorized by creating Y , a $m^2 \times 1$ vector to represent the proximity matrix. The vectorization which produces the Y vector from the \mathbf{Y} matrix is performed by simple row-concatenation of \mathbf{Y} . For example, if \mathbf{Y} is a 15×15 matrix, the $\mathbf{Y}_{1,1}$ element is the first element of Y , and the $\mathbf{Y}_{2,1}$ element is the second element of Y and so on. Once the input matrices are vectorized, standard Normal regression is performed.

Let Y_i be the continuous dependent variable, produced by vectorizing a continuous proximity matrix, for observation i .

- The *stochastic component* is described by a univariate normal model with a vector of means μ_i and scalar variance σ^2 :

$$Y_i \sim \text{Normal}(\mu_i, \sigma^2).$$

- The *systematic component* is given by:

$$\mu_i = x_i \beta.$$

where x_i is the vector of k explanatory variables and β is the vector of coefficients.

1.0.5 Quantities of Interest

The quantities of interest for the network Normal regression are the same as those for the standard Normal regression.

- The expected value (`qi$ev`) for the `normal.net` model is the mean of simulations from the stochastic component,

$$E(Y) = \mu_i = x_i\beta,$$

given a draw of β from its posterior.

- The predicted value (`qi$pr`) is a draw from the distribution defined by the set of parameters (μ_i, σ^2) .
- The first difference (`qi$fd`) for the network Normal model is defined as

$$FD = \Pr(Y|x_1) - \Pr(Y|x)$$

1.0.6 Output Values

The output of each Zelig command contains useful information which you may view. For example, you run `z.out <- zelig(y ~ x, model = "normal.net", data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the coefficients by using `z.out$coefficients`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output stored in `z.out`, you may extract:
 - `coefficients`: parameter estimates for the explanatory variables.
 - `fitted.values`: the vector of fitted values for the systemic component λ .
 - `residuals`: the working residuals in the final iteration of the IWLS fit.
 - `linear.predictors`: fitted values. For the normal model, these are identical to fitted values.
 - `aic`: Akaike's Information Criterion (minus twice the maximized log-likelihood plus twice the number of coefficients).
 - `bic`: the Bayesian Information Criterion (minus twice the maximized log-likelihood plus the number of coefficients times $\log n$).
 - `df.residual`: the residual degrees of freedom.
 - `df.null`: the residual degrees of freedom for the null model.
 - `zelig.data`: the input data frame if `save.data = TRUE`
- From `summary(z.out)` (as well as from `zelig()`), you may extract:
 - `mod.coefficients`: the parameter estimates with their associated standard errors, p -values, and t statistics.
 - `cov.scaled`: a $k \times k$ matrix of scaled covariances.
 - `cov.unscaled`: a $k \times k$ matrix of unscaled covariances.

- From the `sim()` output stored in `s.out`, you may extract:
 - `qi$ev`: the simulated expected probabilities for the specified values of `x`.
 - `qi$pr`: the simulated predicted values drawn from the distribution defined by (μ_i, σ^2) .
 - `qi$fd`: the simulated first differences in the expected probabilities simulated from `x` and `x1`.

How to Cite Network Normal Regression

Matt Owen, Kosuke Imai, Olivia Lau, and Gary King. *normal.net: Network Normal Regression for Continuous Proximity Matrix Dependent Variables*, 2011

How to Cite the Zelig Software Package

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Imai, Kosuke, Gary King, and Olivia Lau. (2008). “Toward A Common Framework for Statistical Analysis and Development.” *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

See also

The network normal regression is part of the `netglm` package by Skyler J. Cranmer and is built using some of the functionality of the `sna` package by Carter T. Butts [1]. In addition, advanced users may wish to refer to `help(normal.net)`. Sample data are fictional.

References

- [1] C.T. Butts and K.M. Carley. Multivariate methods for interstructural analysis. Technical report, CASOS working paper, Carnegie Mellon University, 2001.
- [2] Matt Owen, Kosuke Imai, Olivia Lau, and Gary King. *normal.net: Network Normal Regression for Continuous Proximity Matrix Dependent Variables*, 2011.