A Minimal Book Example

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2018-06-16

Contents

Chapter 1

Prerequisites

This is a *sample* book written in **Markdown**. You can use anything that Pandoc's Markdown supports, e.g., a math equation $a^2 + b^2 = c^2$.

The **bookdown** package can be installed from CRAN or Github:

```r
install.packages("bookdown")
# or the development version
# devtools::install_github("rstudio/bookdown")
```

Remember each Rmd file contains one and only one chapter, and a chapter is defined by the first-level heading `#`.

To compile this example to PDF, you need XeLaTeX. You are recommended to install TinyTeX (which includes XeLaTeX): https://yihui.name/tinytex/.

Chapter 2

Introduction

You can label chapter and section titles using `{#label}` after them, e.g., we can reference Chapter 2. If you do not manually label them, there will be automatic labels anyway, e.g., Chapter 4.

Figures and tables with captions will be placed in `figure` and `table` environments, respectively.

```
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

Reference a figure by its code chunk label with the `fig:` prefix, e.g., see Figure 2.1. Similarly, you can reference tables generated from `knitr::kable()`, e.g., see Table 2.1.

```
knitr::kable(
  head(iris, 20), caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

You can write citations, too. For example, we are using the **bookdown** package (Xie, 2018) in this sample book, which was built on top of R Markdown and **knitr** (Xie, 2015).

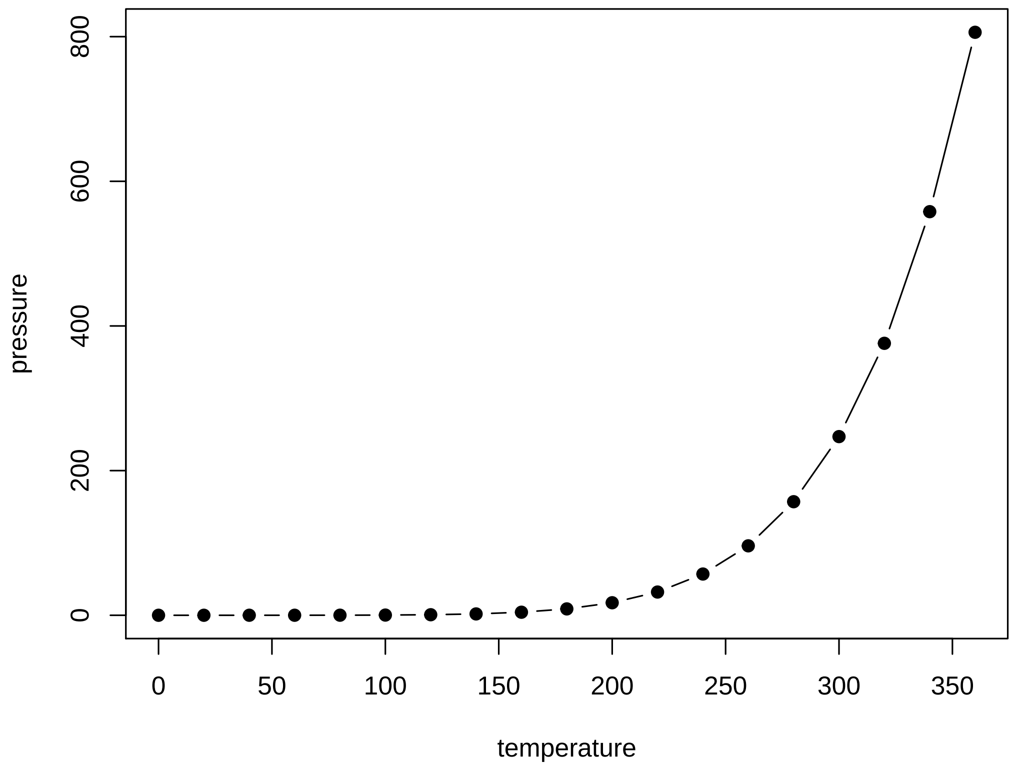

Figure 2.1: Here is a nice figure!

Table 2.1: Here is a nice table!

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa
4.6	3.4	1.4	0.3	setosa
5.0	3.4	1.5	0.2	setosa
4.4	2.9	1.4	0.2	setosa
4.9	3.1	1.5	0.1	setosa
5.4	3.7	1.5	0.2	setosa
4.8	3.4	1.6	0.2	setosa
4.8	3.0	1.4	0.1	setosa
4.3	3.0	1.1	0.1	setosa
5.8	4.0	1.2	0.2	setosa
5.7	4.4	1.5	0.4	setosa
5.4	3.9	1.3	0.4	setosa
5.1	3.5	1.4	0.3	setosa
5.7	3.8	1.7	0.3	setosa
5.1	3.8	1.5	0.3	setosa

Chapter 3

Functions and Notation

Topics[1]:\

Dimensionality; Interval Notation for $\mathbf{R}^1$; Neighborhoods: Intervals, Disks, and Balls; Introduction to Functions; Domain and Range; Some General Types of Functions; Log, Ln, and e; Other Useful Functions; Graphing Functions; Solving for Variables; Finding Roots; Limit of a Function; Continuity; Sets, Sets, and More Sets.

3.1 Dimensionality

- $\mathbf{R}^1$ is the set of all real numbers extending from $-\infty$ to $+\infty$ — i.e., the real number line.
- $\mathbf{R}^n$ is an n-dimensional space (often referred to as Euclidean space), where each of the n axes extends from $-\infty$ to $+\infty$.
  - $\mathbf{R}^1$ is a one dimensional line. $\mathbf{R}^2$ is a two dimensional plane. $\mathbf{R}^3$ is a three dimensional space. $\mathbf{R}^4$ could be 3-D plus time (or temperature, etc).
- Points in $\mathbf{R}^n$ are ordered n-tuples, where each element of the n-tuple represents the coordinate along that dimension.
  - $\mathbf{R}^1$: (3)
  - $\mathbf{R}^2$: (-15, 5)
  - $\mathbf{R}^3$: (86, 4, 0)\

3.2 Interval Notation for $\mathbf{R}^1$

Open interval:

$$(a, b) \equiv \{x \in \mathbf{R}^1 : a < x < b\}$$

x is a one-dimensional element in which x is greater than a and less than b

Closed interval:

$$[a, b] \equiv \{x \in \mathbf{R}^1 : a \leq x \leq b\}$$

x is a one-dimensional element in which x is greater or equal to than a and less than or equal to b

Half open, half closed:

$$(a, b] \equiv \{x \in \mathbf{R}^1 : a < x \leq b\}$$

x is a one-dimensional element in which x is greater than a and less than or equal to b

[1]Much of the material and examples for this lecture are taken from Simon & Blume (1994) *Mathematics for Economists*, Boyce & Diprima (1988) *Calculus*, and Protter & Morrey (1991) *A First Course in Real Analysis*

3.3 Neighborhoods: Intervals, Disks, and Balls

In many areas of math, we need a formal construct for what it means to be "near" a point **c** in $\mathbf{R}^n$. This is generally called the **neighborhood** of **c**. It's represented by an open interval, disk, or ball, depending on whether $\mathbf{R}^n$ is of one, two, or more dimensions, respectively.

Given the point c, these are defined as

- {ϵ**-interval} in $\mathbf{R}^1$:}** $\{x : |x - c| < \epsilon\}$**.** x **is in the neighborhood of {c} if it is in the open interval** $(c - \epsilon, c + \epsilon)$**.**
- **{**ϵ**-disk} in $\mathbf{R}^2$:** $\{x : ||x - c|| < \epsilon\}$**.** x **is in the neighborhood of {c} if it is inside the circle or disc with center c and radius** ϵ**.**
- ϵ**-ball in $\mathbf{R}^n$:}** $\{x : ||x - c|| < \epsilon\}$**.** x **is in the neighborhood of {c} if it is inside the sphere or ball with center c and radius** ϵ**.**

3.4 Introduction to Functions

A {**function**} **(in $\mathbf{R}^1$) is a mapping, or transformation, that relates members of one set to members of another set. For instance, if you have two sets: set** A **and set** B**, a function from** A **to** B **maps every value** a **in set** A **such that** $f(a) \in B$**. Functions can be** `many-to-one", where many values or combinations of values from set $A$ produce a single output in set $B$, or they can be`**one-to-one", where each value in set** A **corresponds to a single value in set** B**.**

Examples: Mapping notation

- **Function of one variable:** $f : \mathbf{R}^1 \to \mathbf{R}^1 \backslash \; f(x) = x + 1 \backslash$ **For each** x **in** $\mathbf{R}^1$**,** $f(x)$ **assigns the number** $x + 1$**.**
- **Function of two variables:** $f : \mathbf{R}^2 \to \mathbf{R}^1 \backslash \; f(x, y) = x^2 + y^2 \backslash$ **For each ordered pair** (x, y) **in** $\mathbf{R}^2$**,** $f(x, y)$ **assigns the number** $x^2 + y^2$**.**

We often use variable x **as input and another** y **as output, e.g.** $y = x + 1$

3.5 Domain and Range/Image

Some functions are defined only on proper subsets of $\mathbf{R}^n$. * {Domain}: the set of numbers in X **at which** $f(x)$ **is defined. * {Range}: elements of** Y **assigned by** $f(x)$ **to elements of** X**, or**

$$f(X) = \{y : y = f(x), x \in X\}$$

Most often used when talking about a function $f : \mathbf{R}^1 \to \mathbf{R}^1$. * {Image}: same as range, but more often used when talking about a function $f : \mathbf{R}^n \to \mathbf{R}^1$.

```
<!-- \begin{samepage}    -->
<!-- \begin{framed} -->
<!-- \item[] Examples: -->
```

–>

–>

–>

3.6 Some General Types of Functions

```
\begin{itemize}
```

```
\item {\bf Monomials}:  $f(x)=a x^k$\\
    $a$ is the coefficient.  $k$ is the degree.\\
    Examples: $y=x^2$, $y=-\frac{1}{2}x^3$

\item {\bf Polynomials}: sum of monomials.\\
    Examples: $y=-\frac{1}{2}x^3+x^2$, $y=3x+5$\\
    The degree of a polynomial is the highest degree of its monomial terms.  Also, it's often a good ide

\item {\bf Rational Functions}: ratio of two polynomials.\\
    Examples: $y=\frac{x}{2}$, $y=\frac{x^2+1}{x^2-2x+1}$

\item {\bf Exponential Functions}: Example: $y=2^x$

\item {\bf Trigonometric Functions}: Examples: $y=\cos(x)$, $y=3\sin(4x)$

\item \parbox[c]{4.75in}{{\bf Linear}: polynomial of degree 1.\\
    <!-- Example: $y=m x + b$, where $m$ is the slope and $b$ is the $y$-intercept.}\epsfxsize=1in \parb

\item \parbox[c]{4.75in}{{\bf Nonlinear}: anything that isn't constant or polynomial of degree 1.\\
    Examples:  $y=x^2+2x+1$, $y=\sin(x)$, $y=\ln(x)$, $y=e^x$}
    <!-- \parbox{1in}{\,  {\includegraphics[width=.9in, angle = 270]{nonlin.eps}}} -->
\item[]
```

\end{itemize}

3.7 Log, Ln, and e

Relationship of logarithmic and exponential functions:

$$y = \log_a(x) \iff a^y = x$$

The log function can be thought of as an inverse for exponential functions. a **is referred to as the "base" of the logarithm.**

Common Bases: The two most common logarithms are base 10 and base e**.**

1. **Base 10:** $y = \log_{10}(x) \iff 10^y = x$. **The base 10 logarithm is often simply written as "**$\log(x)$**" with no base denoted.**
2. **Base** e**:** $y = \log_e(x) \iff e^y = x$. **The base** e **logarithm is referred to as the** `natural" logarithm and is written as`$\ln(x)$**“.**

Properties of exponential functions:

- $a^x a^y = a^{x+y}$
- $a^{-x} = 1/a^x$
- $a^x/a^y = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $a^0 = 1$

Properties of logarithmic functions (any base):

Generally, when statisticians or social scientists write $\log(x)$ **they mean** $\log_e(x)$**. In other words:** $\log_e(x) \equiv \ln(x) \equiv \log(x)$

$$\log_a(a^x) = x$$

and

$$a^{\log_a(x)} = x$$

- $\log(xy) = \log(x) + \log(y)$
- $\log(x^y) = y\log(x)$
- $\log(1/x) = \log(x^{-1}) = -\log(x)$
- $\log(x/y) = \log(x \cdot y^{-1}) = \log(x) + \log(y^{-1}) = \log(x) - \log(y)$
- $\log(1) = \log(e^0) = 0$

Change of Base Formula: Use the change of base formula to switch bases as necessary:

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Example:

$$\log_{10}(x) = \frac{\ln(x)}{\ln(10)}$$

- $\log_{10}(\sqrt{10}) = \log_{10}(10^{1/2}) =$
- $\log_{10}(1) = \log_{10}(10^0) =$
- $\log_{10}(10) = \log_{10}(10^1) =$
- $\log_{10}(100) = \log_{10}(10^2) =$
- $\ln(1) = \ln(e^0) =$
- $\ln(e) = \ln(e^1) =$

3.8 Other Useful Functions

Factorials!:

$$x! = x \cdot (x-1) \cdot (x-2) \cdots (1)$$

Modulo: Tells you the remainder when you divide one number by another. Can be extremely useful for programming.

- `x mod y or x % y`
- $17 \mod 3 = 2$
- $100 \ \% \ 30 = 10$

Summation:

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \cdots + x_n$$

- $\sum_{i=1}^{n} cx_i = c\sum_{i=1}^{n} x_i$
- $\sum_{i=1}^{n} (x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$
- $\sum_{i=1}^{n} c = nc$

Product:

$$\prod_{i=1}^{n} x_i = x_1 x_2 x_3 \cdots x_n$$

Properties:

- $\prod\limits_{i=1}^{n} cx_i = c^n \prod\limits_{i=1}^{n} x_i$
- $\prod\limits_{i=1}^{n} (x_i + y_i) =$ **a total mess**
- $\prod\limits_{i=1}^{n} c = c^n$

You can use logs to go between sum and product notation. This will be particularly important when you're learning maximum likelihood estimation.

```
\begin{eqnarray*}
    \log \bigg(\prod\limits_{i=1}^n x_i \bigg) &=& \log(x_1 \cdot x_2 \cdot x_3 \cdots \cdot x_n)\\
    &=& \log(x_1) + \log(x_2) + \log(x_3) + \cdots + \log(x_n)\\
    &=& \sum\limits_{i=1}^n \log (x_i)
\end{eqnarray*}

Therefore, you can see that the log of a product is equal to the sum of the logs. We can write this

\begin{eqnarray*}
    \log \bigg(\prod\limits_{i=1}^n c x_i\bigg) &=& \log(cx_1 \cdot cx_2 \cdots cx_n)\\
    &=& \log(c^n \cdot x_1 \cdot x_2 \cdots x_n)\\
    &=& \log(c^n) + \log(x_1) + \log(x_2) + \cdots + \log(x_n)\\\\
    &=& n \log(c) +  \sum\limits_{i=1}^n \log (x_i)\\
\end{eqnarray*}
```

3.9 Graphing Functions

What can a graph tell you about a function?

1. **Is the function increasing or decreasing? Over what part of the domain?**
2. **How "fast" does it increase or decrease?**
3. **Are there global or local maxima and minima? Where?**
4. **Are there inflection points?**
5. **Is the function continuous?**
6. **Is the function differentiable?**
7. **Does the function tend to some limit?**
8. **Other questions related to the substance of the problem at hand.**

3.10 Solving for Variables and Finding Inverses

Sometimes we're given a function $y = f(x)$ and we want to find how x varies as a function of y. If f is a one-to-one mapping, then it has an inverse.

Use algebra to move x to the left hand side (LHS) of the equation and so that the right hand side (RHS) is only a function of y.

Examples: (we want to solve for x)

1. $y = 3x + 2 \quad \Longrightarrow \quad -3x = 2 - y \quad \Longrightarrow \quad 3x = y - 2 \quad \Longrightarrow \quad x = \frac{1}{3}(y - 2)$
2. $y = 3x - 4z + 2 \quad \Longrightarrow \quad y + 4z - 2 = 3x \quad \Longrightarrow \quad x = \frac{1}{3}(y + 4z - 2)$

3. $y = e^x + 4 \quad \Longrightarrow \quad y - 4 = e^x \quad \Longrightarrow \quad \ln(y-4) = \ln(e^x) \quad \Longrightarrow \quad x = \ln(y-4)$

Sometimes (often?) the inverse does not exist. For example, we're given the function $y = x^2$ (a parabola). Solving for x, we get $x = \pm\sqrt{y}$. For each value of y, there are two values of x.

3.11 Finding the Roots or Zeroes of a Function

Solving for variables is especially important when we want to find the {roots} of an equation: those values of variables that cause an equation to equal zero. Especially important in finding equilibria and in doing maximum likelihood estimation.

Procedure: Given $y = f(x)$, set $f(x) = 0$. Solve for x.

Multiple Roots:

$$f(x) = x^2 - 9 \quad \Longrightarrow \quad 0 = x^2 - 9 \quad \Longrightarrow \quad 9 = x^2 \quad \Longrightarrow \quad \pm\sqrt{9} = \sqrt{x^2} \quad \Longrightarrow \quad \pm 3 = x$$

Quadratic Formula: For quadratic equations $ax^2 + bx + c = 0$, use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

}

Examples:

1. $f(x) = 3x + 2$

2. $f(x) = e^{-x} - 10$

3. $f(x) = x^2 + 3x - 4 = 0$

3.12 The Limit of a Function

`We're often interested in determining if a function $f$ approaches some number $L$ as its independent va`

For a limit L to exist, the function $f(x)$ must approach L from both the left and right.

Limit of a function. Let $f(x)$ be defined at each point in some open interval containing the point c. Then L equals $\lim\limits_{x \to c} f(x)$ if for any (small positive) number ϵ, there exists a corresponding number $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.

Note: f(x) does not necessarily have to be defined at c for $\lim\limits_{x \to c}$ to exist.

Uniqueness: $\lim\limits_{x \to c} f(x) = L$ and $\lim\limits_{x \to c} f(x) = M \Longrightarrow L = M$

Properties: Let f and g be functions with $\lim\limits_{x \to c} f(x) = A$ and $\lim\limits_{x \to c} g(x) = B$.

1. $\lim\limits_{x \to c} [f(x) + g(x)] = \lim\limits_{x \to c} f(x) + \lim\limits_{x \to c} g(x)$
2. $\lim\limits_{x \to c} \alpha f(x) = \alpha \lim\limits_{x \to c} f(x)$
3. $\lim\limits_{x \to c} f(x)g(x) = [\lim\limits_{x \to c} f(x)][\lim\limits_{x \to c} g(x)]$
4. $\lim\limits_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, **provided** $\lim\limits_{x \to c} g(x) \neq 0$.

5. Note: In a couple days we'll talk about L'Hôpital's Rule, which uses simple calculus to help find the limits of functions like this.

Examples:

1. $\lim_{x \to c} k =$
2. $\lim_{x \to c} x =$
3. $\lim_{x \to 2} (2x - 3) =$
4. $\lim_{x \to c} x^n =$

Types of limits:

1. Right-hand limit: The value approached by $f(x)$ when you move from right to left.
2. Left-hand limit: The value approached by $f(x)$ when you move from left to right.\
3. Infinity: The value approached by $f(x)$ as x grows infinitely large. Sometimes this may be a number; sometimes it might be ∞ or $-\infty$.\
4. Negative infinity: The value approached by $f(x)$ as x grows infinitely negative. Sometimes this may be a number; sometimes it might be ∞ or $-\infty$.\

3.13 Continuity

Continuity: Suppose that the domain of the function f includes an open interval containing the point c. Then f is continuous at c if $\lim_{x \to c} f(x)$ exists and if $\lim_{x \to c} f(x) = f(c)$. Further, f is continuous on an open interval (a, b) if it is continuous at each point in the interval.

- Examples: Continuous functions.

Properties:

1. If f and g are continuous at point c, then $f + g$, $f - g$, $f \cdot g$, $|f|$, and αf are continuous at point c also. f/g is continuous, provided $g(c) \neq 0$.
2. Boundedness: If f is continuous on the closed bounded interval $[a, b]$, then there is a number K such that $|f(x)| \leq K$ for each x in $[a, b]$.
3. Max/Min: If f is continuous on the closed bounded interval $[a, b]$, then f has a maximum and a minimum on $[a, b]$. They may be located at the end points.

3.14 Sets

Interior Point: The point x is an interior point of the set S if x is in S and if there is some ϵ-ball around x that contains only points in S. The {interior} of S is the collection of all interior points in S. The interior can also be defined as the union of all open sets in S.

- If the set S is circular, the interior points are everything inside of the circle, but not on the circle's rim.
- Example: The interior of the set $\{(x, y) : x^2 + y^2 \leq 4\}$ is $\{(x, y) : x^2 + y^2 < 4\}$.

Boundary Point: The point x is a boundary point of the set S if every ϵ-ball around x contains both points that are in S and points that are outside S. The {boundary} is the collection of all boundary points.

- If the set S is circular, the boundary points are everything on the circle's rim.
- Example: The boundary of $\{(x, y) : x^2 + y^2 \leq 4\}$ is $\{(x, y) : x^2 + y^2 = 4\}$.

Open: A set S is open if for each point x in S, there exists an open ϵ-ball around x completely contained in S.

- **If the set S is circular and open, the points contained within the set get infinitely close to the circle's rim, but do not touch it.**
- **Example:** $\{(x,y) : x^2 + y^2 < 4\}$

Closed: A set S is closed if it contains all of its boundary points. * If the set S is circular and closed, the set contains all points within the rim as well as the rim itself. * Example: $\{(x,y) : x^2 + y^2 \leq 4\}$ *** Note: a set may be neither open nor closed. Example:** $\{(x,y) : 2 < x^2 + y^2 \leq 4\}$

Complement: The complement of set S is everything outside of S.\ * If the set S is circular, the complement of S is everything outside of the circle. * Example: The complement of $\{(x,y) : x^2 + y^2 \leq 4\}$ **is** $\{(x,y) : x^2 + y^2 > 4\}$**.**

Closure: The closure of set S is the smallest closed set that contains S. Example: The closure of $\{(x,y) : x^2 + y^2 < 4\}$ **is** $\{(x,y) : x^2 + y^2 \leq 4\}$

Bounded: A set S is bounded if it can be contained within an ϵ-ball.

- **Examples: Bounded: any interval that doesn't have ∞ or $-\infty$ as endpoints; any disk in a plane with finite radius.**
- **Unbounded: the set of integers in $\mathbf{R}^1$; any ray.**

Compact: A set is compact if and only if it is both closed and bounded.

Empty: The empty (or null) set is a unique set that has no elements, denoted by {} or ø. [2]

- **Examples: The set of squares with 5 sides; the set of countries south of the South Pole.**

[2]The set, S, denoted by $\{ø\}$ is technically *not* empty. That is because this set contains the empty set within it, so S is not empty.

Chapter 4

Methods

We describe our methods in this chapter.

Chapter 5

Applications

Some *significant* applications are demonstrated in this chapter.

5.1 Example one

5.2 Example two

Chapter 6

Final Words

We have finished a nice book.

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Xie, Y. (2018). ***bookdown: Authoring Books and Technical Documents with R Markdown.*** **R package version 0.7.**