Exploring data #3

forcats

FORCATS

Hadley Wickham has developed a package called `forcats` that helps you work with categorical variables (factors). I'll show some examples of its functions using the `worldcup` dataset:

```
library(forcats)
library(faraway)
data(worldcup)
```

FORCATS

The `fct_recode` function can be used to change the labels of a function (along the lines of using `factor` with `levels` and `labels` to reset factor labels).

One big advantage is that `fct_recode` lets you change labels for some, but not all, levels. For example, here are the team names:

```
worldcup %>%
  filter(str_detect(Team, "^US")) %>%
  slice(1:3) %>% select(Team, Position, Time)
```

```
##   Team   Position Time
## 1  USA Midfielder   10
## 2  USA   Defender  390
## 3  USA   Defender  200
```

forcats

If you just want to change "USA" to "United States", you can run:

```
worldcup <- worldcup %>%
  mutate(Team = fct_recode(Team, `United States` = "USA"))
worldcup %>%
  filter(str_detect(Team, "^Un")) %>%
  slice(1:3) %>% select(Team, Position, Time)
```

```
##              Team   Position Time
## 1 United States Midfielder   10
## 2 United States   Defender  390
## 3 United States   Defender  200
```

FORCATS

You can use the fct_lump function to lump uncommon factors into an "Other" category. For example, to lump the two least common positions together, you can run (n specifies how many categories to keep outside of "Other"):

```
worldcup %>%
  mutate(Position = fct_lump(Position, n = 2)) %>%
  count(Position)
```

```
## # A tibble: 3 × 2
##     Position     n
##       <fctr> <int>
## 1   Defender   188
## 2 Midfielder   228
## 3      Other   179
```

forcats

You can use the `fct_infreq` function to reorder the levels of a factor from most common to least common:

```
levels(worldcup$Position)
```

```
## [1] "Defender"   "Forward"    "Goalkeeper" "Midfielder"
```

```
worldcup <- worldcup %>%
  mutate(Position = fct_infreq(Position))
levels(worldcup$Position)
```

```
## [1] "Midfielder" "Defender"   "Forward"    "Goalkeeper"
```

FORCATS

If you want to reorder one factor by another variable (ascending order), you can use `fct_reorder` (e.g., homework 3). For example, to relevel `Position` by the average shots on goals for each position, you can run:

```
levels(worldcup$Position)
```

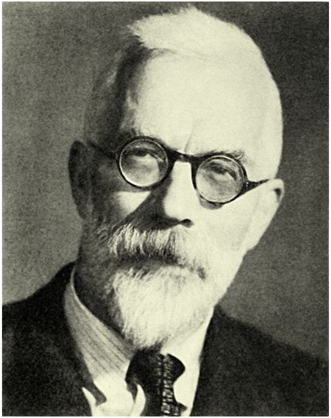
```
## [1] "Midfielder" "Defender"   "Forward"    "Goalkeeper"
```

```
worldcup <- worldcup %>%
  group_by(Position) %>%
  mutate(ave_shots = mean(Shots)) %>%
  ungroup() %>%
  mutate(Position = fct_reorder(Position, ave_shots))
levels(worldcup$Position)
```

```
## [1] "Goalkeeper" "Defender"   "Midfielder" "Forward"
```

Simulations

The lady tasting tea



Source: Flikr commons, https://www.flickr.com/photos/internetarchivebookimages/20150531109/

The lady tasting tea

"Dr. Muriel Bristol, a colleague of Fisher's, claimed that when drinking tea she could distinguish whether milk or tea was added to the cup first (she preferred milk first). To test her claim, Fisher asked her to taste eight cups of tea, four of which had milk added first and four of which had tea added first." — Agresti, Categorical Data Analysis, p.91

The lady tasting tea

Question:

- If she just guesses, what is the probability she will get all cups right?
- What if more or fewer cups are used in the experiment?

THE LADY TASTING TEA

One way to figure this out is to run a *simulation*.

In R, `sample` can be a very helpful function for simulations. It lets you randomly draw values from a vector, with or without replacement.

```
## Generic code
sample(x = [vector to sample from],
       size = [number of samples to take],
       replace = [logical-- should values in the
                  vector be replaced?],
       prob = [vector of probability weights])
```

THE LADY TASTING TEA

Create vectors of the true and guessed values, in order, for the cups of tea:

```
n_cups <- 8
cups <- sample(rep(c("milk", "tea"), each = n_cups / 2))
cups
```

```
## [1] "milk" "milk" "tea"  "tea"  "tea"  "milk" "tea"  "milk"
```

```
guesses <- sample(rep(c("milk", "tea"), each = n_cups / 2))
guesses
```

```
## [1] "tea"  "milk" "milk" "milk" "tea"  "milk" "tea"  "tea"
```

THE LADY TASTING TEA

For this simulation, determine how many cups she got right (i.e., guess equals the true value):

```
cup_results <- cups == guesses
cup_results
```

```
## [1] FALSE  TRUE FALSE FALSE  TRUE  TRUE  TRUE FALSE
```

```
n_right <- sum(cup_results)
n_right
```

```
## [1] 4
```

The lady tasting tea

Right a function that will run one simulation. It takes the argument n_cups— in real life, they used eight cups (n_cups = 8). Note that this function just wraps the code we just walked through.

```
sim_null_tea <- function(n_cups){
  cups <- sample(rep(c("milk", "tea"), each = n_cups / 2))
  guesses <- sample(rep(c("milk", "tea"), each = n_cups / 2))
  cup_results <- cups == guesses
  n_right <- sum(cup_results)
  return(n_right)
}
sim_null_tea(n_cups = 8)
```

```
## [1] 2
```

The lady tasting tea

Now, we need to run a lot of simulations, to see what happens on average if she guesses. You can use the replicate function to do that.

```
## Generic code
replicate(n = [number of replications to run],
          eval = [code to replicate each time])
```

```
tea_sims <- replicate(5, sim_null_tea(n_cups = 8))
tea_sims
```

```
## [1] 8 2 4 6 2
```

The lady tasting tea

This call gives a vector with the number of cups she got right for each simulation. You can replicate the simulation many times to get a better idea of what to expect if she just guesses, including what percent of the time she gets all cups right.

```
tea_sims <- replicate(1000, sim_null_tea(n_cups = 8))
mean(tea_sims)
```

```
## [1] 3.998
```

```
quantile(tea_sims, probs = c(0.025, 0.975))
```

```
##  2.5% 97.5%
##     2     6
```

```
mean(tea_sims == 8)
```

```
## [1] 0.015
```

The lady tasting tea

Now we'd like to know, for different numbers of cups of tea, what is the probability that the lady will get all right?

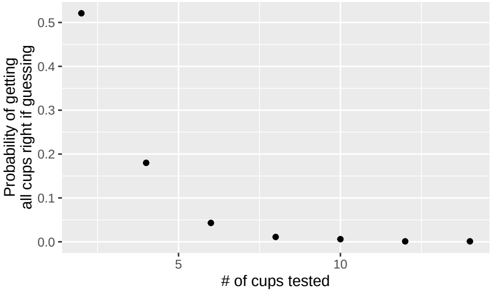
For this, we can apply the replication code across different values of `n_cups`:

```
n_cups <- seq(from = 2, to = 14, by = 2)
perc_all_right <- sapply(n_cups, FUN = function(n_cups){
  cups_right <- replicate(1000, sim_null_tea(n_cups))
  out <- mean(cups_right == n_cups)
  return(out)
})
perc_all_right
```

```
## [1] 0.521 0.180 0.043 0.011 0.006 0.001 0.001
```

The lady tasting tea

```
tea_sims <- data_frame(n_cups, perc_all_right)
ggplot(tea_sims, aes(x = n_cups, y = perc_all_right)) +
  geom_point() + xlab("# of cups tested") +
  ylab("Probability of getting\nall cups right if guessing")
```



The lady tasting tea

You can answer this question analytically using the hypergeometric distribution:

$$P(n_{11} = t) = \frac{\binom{n_{1+}}{t}\binom{n_{2+}}{n_{+1}-t}}{\binom{n}{n_{+1}}}$$

	Guessed milk	Guessed tea	Total
Really milk	n_{11}	n_{12}	$n_{1+} = 4$
Really tea	n_{21}	n_{22}	$n_{2+} = 4$
Total	$n_{+1} = 4$	$n_{+2} = 4$	$n = 8$

The lady tasting tea

In R, you can use `dhyper` to get the density of the hypergeometric function:

```
dhyper(x = [# of cups she guesses have milk first that do],
       m = [# of cups with milk first],
       n = [# of cups with tea first],
       k = [# of cups she guesses have milk first])
```

The lady tasting tea

Probability she gets three "milk" cups right if she's just guessing and there are eight cups, four with milk first and four with tea first:

```
dhyper(x = 3, m = 4, n = 4, k = 4)
```

```
## [1] 0.2285714
```

Probability she gets three or more "milk" cups right if she's just guessing:

```
dhyper(x = 3, m = 4, n = 4, k = 4) +
  dhyper(x = 4, m = 4, n = 4, k = 4)
```

```
## [1] 0.2428571
```

The lady tasting tea

Other density functions:

- `dnorm`: Normal
- `dpois`: Poisson
- `dbinom`: Binomial
- `dchisq`: Chi-squared
- `dt`: Student's t
- `dunif`: Uniform

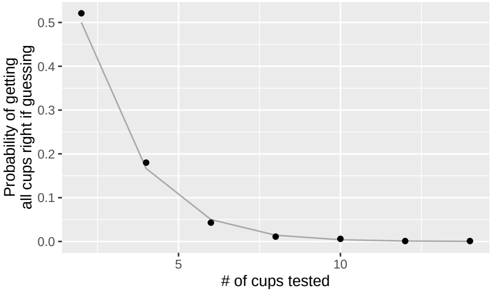
The lady tasting tea

You can get the analytical result for each of the number of cups we simulated and compare those values to our simulations:

```
analytical_results <- data_frame(n_cups = seq(2, 14, 2)) %>%
  mutate(perc_all_right = dhyper(x = n_cups / 2,
                                 m = n_cups / 2,
                                 n = n_cups / 2,
                                 k = n_cups / 2))
```

The lady tasting tea

```
ggplot(analytical_results, aes(x = n_cups, y = perc_all_right))
  geom_line(color = "darkgray") +
  geom_point(data = tea_sims) + xlab("# of cups tested") +
  ylab("Probability of getting\nall cups right if guessing")
```



The lady tasting tea

The Lady Tasting Tea: How Statistics Revolutionized Science in the Twentieth Century. David Salsburg.

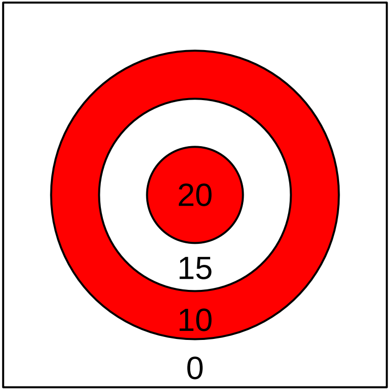
The Design of Experiments. Ronald Fisher.

https://priceonomics.com/why-the-father-of-modern-statistics-didnt-believe/

Playing darts

Research question: Is a person skilled at playing darts?

Here's our dart board– the numbers are the number of points you win for a hit in each area.



Playing darts

First, what would we expect to see if the person we test has no skill at playing darts?

Questions to consider:

- *What would the dart board look like under the null (say the person throws 20 darts for the experiment)?*
- *About what do you think the person's mean score would be if they had no skill at darts?*
- *What are some ways to estimate or calculate the expected mean score under the null?*

Playing darts

Let's use R to answer the first question: what would the null look like?

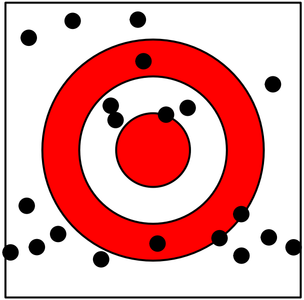
First, create some random throws (the square goes from -1 to 1 on both sides):

```
n.throws <- 20
throw.x <- runif(n.throws, min = -1, max = 1)
throw.y <- runif(n.throws, min = -1, max = 1)
head(cbind(throw.x, throw.y))
```

```
##          throw.x    throw.y
## [1,] -0.06494184  0.6029856
## [2,] -0.84198600  0.7620486
## [3,] -0.25394754  0.2038871
## [4,]  0.45040536 -0.5976734
## [5,]  0.23447420  0.2866149
## [6,] -0.35185288 -0.7411755
```

Playing darts

```
plot(c(-1, 1), c(-1,1), type = "n", asp=1,
     xlab = "", ylab = "", axes = FALSE)
rect( -1, -1, 1, 1)
draw.circle( 0, 0, .75, col = "red")
draw.circle( 0, 0, .5, col = "white")
draw.circle( 0, 0, .25, col = "red")
points(throw.x, throw.y, col = "black", pch = 19)
```



Playing darts

Next, let's tally up the score for this simulation of what would happen under the null.

To score each throw, we calculate how far the point is from (0, 0), and then use the following rules:

- **20 points**: $0.00 \leq \sqrt{x^2 + y^2} \leq .25$
- **15 points**: $0.25 < \sqrt{x^2 + y^2} \leq .50$
- **10 points**: $0.50 < \sqrt{x^2 + y^2} \leq .75$
- **0 points**: $0.75 < \sqrt{x^2 + y^2} \leq 1.41$

Playing darts

Use these rules to "score" each random throw:

```
throw.dist <- sqrt(throw.x^2 + throw.y^2)
head(throw.dist)
```

```
## [1] 0.6064727 1.1356313 0.3256675 0.7483839 0.3703056 0.82045
```

```
throw.score <- cut(throw.dist,
                   breaks = c(0, .25, .5, .75, 1.5),
                   labels = c("20", "15", "10", "0"),
                   right = FALSE)
head(throw.score)
```

```
## [1] 10 0  15 10 15 0
## Levels: 20 15 10 0
```

Playing darts

Now that we've scored each throw, let's tally up the total:

```
table(throw.score)
```

```
## throw.score
## 20 15 10  0
##  0  4  4 12
```

```
mean(as.numeric(as.character(throw.score)))
```

```
## [1] 5
```

Playing darts

So, this just showed *one* example of what might happen under the null. If we had a lot of examples like this (someone with no skill throwing 20 darts), what would we expect the mean scores to be?

Questions to consider:

- *How can you figure out the expected value of the mean scores under the null (that the person has no skill)?*
- *Do you think that 20 throws will be enough to figure out if a person's mean score is different from this value, if he or she is pretty good at darts?*
- *What steps do you think you could take to figure out the last question?*
- *What could you change about the experiment to make it easier to tell if someone's skilled at darts?*

Playing darts

How can we figure this out?

- **Theory.** Calculate the expected mean value using the expectation formula
- **Simulation.** Simulate a lot of examples using R and calculate the mean of the mean score from these.

Playing darts

The expected value of the mean, $E[\bar{X}]$, is the expected value of X, $E[X]$. To calculate the expected value of X, use the formula:

$$E[X] = \sum_x x p(x)$$

$$E[X] = 20 * p(X = 20) + 15 * p(X = 15) + 10 * p(X = 10) + 0 * p(X = 0)$$

So we just need to figure out $p(X = x)$ for $x = 20, 15, 10$.

Playing darts

(In all cases, we're dividing by 4 because that's the area of the full square, 2^2.)

- $p(X = 20)$: Proportional to area of the smallest circle, $(\pi * 0.25^2)/4 = 0.049$
- $p(X = 15)$: Proportional to area of the middle circle minus area of the smallest circle, $\pi(0.50^2 - 0.25^2)/4 = 0.147$
- $p(X = 10)$: Proportional to area of the largest circle minus area of the middle circle, $\pi(0.75^2 - 0.50^2)/4 = 0.245$
- $p(X = 0)$: Proportional to area of the square minus area of the largest circle, $(2^2 - \pi * 0.75^2)/4 = 0.558$

As a double check, if we've done this write, the probabilities should sum to 1:

$$0.049 + 0.147 + 0.245 + 0.558 = 0.999$$

Playing darts

$$E[X] = \sum_x xp(x)$$

$$E[X] = 20 * 0.049 + 15 * 0.147 + 10 * 0.245 + 0 * 0.558$$

$$E[X] = 5.635$$

Remember, this also gives us $E[\bar{X}]$.

Playing darts

Now it's pretty easy to also calculate $var(X)$ and $var(\bar{X})$:

$$Var(X) = E[(X - \mu)^2] = E[X^2] - E[X]^2$$

$$E[X^2] = 20^2 * 0.049 + 15^2 * 0.147 + 10^2 * 0.245 + 0^2 * 0.558 = 77.18$$

$$Var(X) = 77.175 - (5.635)^2 = 45.42$$

$$Var(\bar{X}) = \sigma^2 / n = 45.42/20 = 2.27$$

Playing darts

Now that we can use the Central Limit Theorem to calculate a 95% confidence interval for the mean score when someone with no skill (null hypothesis) throws 20 darts:

```
5.635 + c(-1, 1) * qnorm(.975) * sqrt(2.27)
```

```
## [1] 2.682017 8.587983
```

Playing darts

We can check our math by running simulations– we should get the same values of $E[\bar{X}]$ and $Var(\bar{X})$ (which we can calculate directly from the simulations using R).

```
n.throws <- 20
n.sims <- 10000

x.throws <- matrix(runif(n.throws * n.sims, -1, 1),
                   ncol = n.throws, nrow = n.sims)
y.throws <- matrix(runif(n.throws * n.sims, -1, 1),
                   ncol = n.throws, nrow = n.sims)
dist.throws <- sqrt(x.throws^2 + y.throws^2)
score.throws <- apply(dist.throws, 2, cut,
                   breaks = c(0, .25, .5, .75, 1.5),
                   labels = c("20", "15", "10", "0"),
                   right = FALSE)
```

Playing darts

```
dist.throws[1:3,1:5]
```

```
##              [,1]      [,2]      [,3]      [,4]      [,5]
## [1,] 0.4155520 0.9032675 0.0275598 0.7911705 0.5746545
## [2,] 0.1693246 0.4574311 0.4701004 0.5953042 1.0076644
## [3,] 0.9347008 1.2608088 0.1671307 0.7965368 1.0024044
```

```
score.throws[1:3,1:5]
```

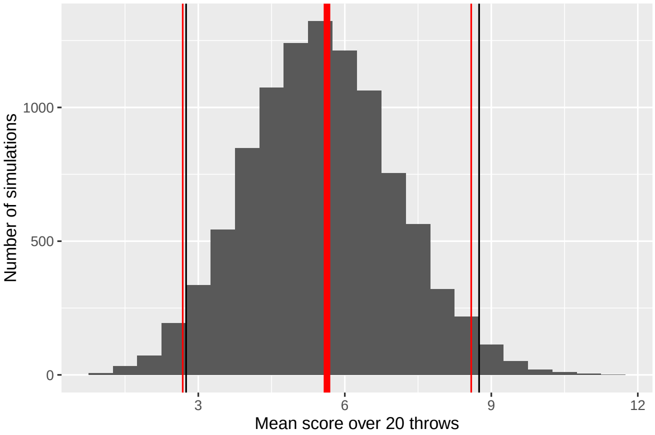
```
##      [,1] [,2] [,3] [,4] [,5]
## [1,] "15" "0"  "20" "0"  "10"
## [2,] "20" "15" "15" "10" "0"
## [3,] "0"  "0"  "20" "0"  "0"
```

PLAYING DARTS

```
mean.scores <- apply(score.throws, MARGIN = 1,
                     function(x){
                             out <- mean(as.numeric(
                                     as.character(x)))
                             return(out)
                     })
head(mean.scores)
```

```
## [1] 5.75 6.00 7.00 6.00 5.75 7.00
```

Playing darts



Playing darts

Let's check the simulated mean and variance against the theoretical values:

```
mean(mean.scores) ## Theoretical: 5.635
```

```
## [1] 5.654575
```

```
var(mean.scores) ## Theoretical: 2.27
```

```
## [1] 2.301981
```

Simulations in research

Simulations in the wild (just a few examples):

- The Manhattan Project
- US Coast Guard search and rescue
- Infectious disease modeling

Other computationally-intensive approaches

Bootstrap and friends

- **Bootstraping:** Sample the dataset with replacement and reestimate the statistical parameter(s) each time.
- **Jackknifing:** Rake out one observation at a time and reestimate the statistical parameter(s) with the rest of the data.
- **Permutation tests:** See how unusual the result from the data is compared to if you shuffle your data (and so remove any relationship in observed data between variables).
- **Cross-validation:** See how well your model performs if you pick a subset of the data, build the model just on that subset, and then test how well it predicts for the rest of the data, and repeat that many times.

Bayesian analysis

Suggested books for learning more about Bayesian analysis in R:

- *Doing Bayesian Data Analysis, Second Edition: A Tutorial with R, JAGS, and Stan.* John Kruschke.
- *Statistical Rethinking: A Bayesian Course with Examples in R and Stan.* Richard McElreath.
- *Bayesian Data Analysis, Third Edition.* Andrew Gelman et al.

R can tap into software for Bayesian analysis:

- BUGS
- JAGS
- STAN

Ensemble models and friends

- **Bagging:** Sample data with replacement and build a tree model. Repeat many times. To predict, predict from all models and take the majority vote.
- **Random forest:** Same as bagging, for picking each node of a tree, only consider a random subset of variables.
- **Boosting:** Same as bagging, but "learn" from previous models as you build new models.
- **Stacked models:** Build many different models (e.g., generalized linear regression, Naive Bayes, k-nearest neighbors, random forest, ...), determine weights for each, and predict using weighted predictions combined from all models

Ensemble models and friends

For more on these and other machine learning topics, see:

An Introduction to Statistical Learning. Gareth James, Robert Tibshirani, and Trevor Hastie.

The `caret` package: `http://topepo.github.io/caret/index.html`

For many examples of predictive models like this built with R (and Python): `https://www.kaggle.com`

Speeding up R

Working with large datasets