A minimal model for ICRF heating in a tokamak

R. Dumont (CEA / IRFM)

September 23, 2016

Appendix D

ICRF heating in a tokamak : a minimal model

The aim of this appendix is the derivation of a simple model for ICRF heating. The rationale is that during experiments, or in real-time contexts, running expensive numerical tools is not a sensible option. Therefore, it is always interesting to attempt to extract the most important physics features from the comprehensive models, and transpose them into fast and reliable numerical tools. Of course, it is also necessary to compare the outcome of such simplified models to more advanced ones, to check them as well as adjust any potentially tunable parameter. The followed procedure stems from the results presented in chapter 4, and essentially from three papers [2, 3, 142].

Our starting point is the quasi-local Fokker-Planck equation Eq. 4.65 discussed in details in chapter 4. Here, we only keep the collision and wave quasilinear terms, and assume that the ion distribution before heating is in thermal equilibrium with the other species, i.e.

$$\partial_\tau f_i = \hat{C} f_i + \hat{Q} f_i, \tag{D.1}$$

The time is normalized to a characteristic collision frequency denoted ν_i (see Eq. 4.55). The local collision operator is given by (see Eq. 4.56)

$$\hat{C} f_i = \frac{1}{u^2}\frac{\partial}{\partial u}\left[u^2\left(D_{uu}\frac{\partial f_i}{\partial u} - F_u f_i\right)\right] + \frac{1}{u^2}\left[\frac{\partial}{\partial \lambda}(1-\lambda^2)\frac{\Theta_c}{2u}\frac{\partial f_i}{\partial \lambda}\right], \tag{D.2}$$

The wave term is written as

$$\hat{Q} f_i = \frac{1}{u_\perp}\frac{\partial}{\partial u_\perp} u_\perp D_w \frac{\partial f_i}{\partial u_\perp}, \tag{D.3}$$

with D_w the quaslinear diffusion coefficient.

We will follow here a step-by-step procedure aimed at directly implementing the proposed model in real-time control systems or interpretative/predictive reduced codes.

D.1 Power density

We assume that the total RF power P_{RF}, frequency f and the dominant toroidal number n are known quantities[1]. We consider the local resonance condition

$$\omega = p\omega_{ci} + \frac{n}{R} v_{\|}, \tag{D.4}$$

where poloidal upshift effects are disregarded, i.e. $k_{\|} \approx n/R$. p is the cyclotron harmonic, assumed known. If we assume that the interaction involves ions with a parallel velocity ranging between 0 and $v_{th,\|}$, we deduce by differentiating Eq. D.4 that the interaction takes place in a spatial domain whose radial extension ΔR (see Fig. D.1) is approximately given by

$$\Delta R \approx \frac{n}{\omega} v_{th,\|} \approx \frac{n}{\omega}\sqrt{\frac{2T_i}{m_i}}, \tag{D.5}$$

where it was assumed that the parallel temperature remains close to the bulk temperature T_i.

Determining ΔZ is more complicated: it is fixed by the plasma and antenna geometry, ICRF scenario, plasma parameters (see Fig. D.1)... In fact, for a given set of parameters, it should be deduced from multi-dimensional simulations, as it clearly can have a large influence on the final result. Here, we introduce the corresponding tunable parameter f_z, so that

$$\Delta Z = 2a_0\epsilon_1 f_z, \tag{D.6}$$

with a_0 the minor radius and ϵ_1 the elongation. f_z represents the fraction of the vertical chord along which the absorption is significant.

By doing this, we have defined the toroidal interaction volume

$$V_{int} = 2\pi R \Delta R \Delta Z, \tag{D.7}$$

which will be used to obtain a rough estimate of the power density as

$$p_{abs} \equiv \frac{P_{RF} - P_{loss}}{V_{int}}, \tag{D.8}$$

with P_{loss} the lost power.

D.2 Quasilinear diffusion coefficient

We use the local quasilinear diffusion coefficient, Eq. 4.53, rewritten here as

$$D_w = D_p \left| J_{p-1}(k_\perp v_\perp/\omega_{ci}) + \frac{E_-}{E_+} J_{p+1}(k_\perp v_\perp/\omega_{ci}) \right|^2. \tag{D.9}$$

Notice that unlike in chapter 4, the dominant $|E_+|^2$ factor has been absorbed in the constant D_p. The unknowns in the previous expression are $k_\perp$, E_-/E_+ and D_p. Since

[1] n is usually either a unique "standard" value for a given tokamak/antenna/phasing, or a set of appropriately weighted values.

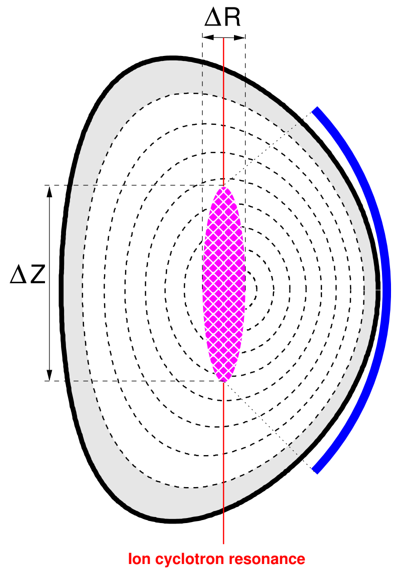

Figure D.1: Illustration of the approximate interaction (purple domain) zone between ICRF waves and plasma.

we only consider heating resulting from the fast magnetosonic wave damping, one can use the local dispersion relation [143] to deduce $k_\perp$. If we opt for a cold plasma model (a reasonable assumption for the fast wave propagation), we have

$$k_\perp^2 \approx -\frac{\omega^2}{c^2}\frac{(n_\parallel^2 - R)(n_\parallel^2 - L)}{n_\parallel^2 - S}. \tag{D.10}$$

and also

$$\frac{E_-}{E_+} \approx \left|\frac{n_\parallel^2 - L}{n_\parallel^2 - R}\right|, \tag{D.11}$$

with $n_\parallel \equiv k_\parallel c/\omega$. R, L, S and D are the Stix dielectric tensor elements [143].

The determination of D_p is more complicated. The general form for the absorbed power is

$$p_{abs,qlin} = \int d^3\mathbf{v}\frac{mv^2}{2}\hat{Q}f_i, \tag{D.12}$$

which, using Eq. D.3, may be rewritten as

$$p_{abs,qlin} = -4\pi T_i v_{th}^3 \int du_\parallel du_\perp u_\perp^2 D_w \frac{\partial f_i}{\partial u_\perp}. \tag{D.13}$$

This expression involves the solution to the Fokker-Planck equation which, at this stage, remains unknown. Assuming the "initial" distribution function[2] is a Maxwellian

[2]In other words, the distribution function prior ICRF heating.

characterized by density n_i and temperature T_i, we obtain the following expression for the linear power density

$$p_{abs,lin} = 8n_iT_iD_p \int_0^\infty du_\perp u_\perp^3 \left| J_{p-1}\left(\frac{k_\perp v_{th,i}}{\omega_{ci}}u_\perp\right) + \frac{E_-}{E_+}J_{p+1}\left(\frac{k_\perp v_{th,i}}{\omega_{ci}}u_\perp\right)\right|^2 e^{-u_\perp^2}. \quad \text{(D.14)}$$

It is possible to simplify the previous expression by assuming that $|E_-/E_+| \ll 1$. For low to moderate energy ions, fulfilling $k_\perp v_{th,i}/\omega_{ci} \ll 1$, we can then use the small argument expansion of the Bessel functions to obtain

$$p_{abs,lin}^{p=1} \approx 4n_iT_iD_p, \quad \text{(D.15)}$$

for fundamental (minority) heating and

$$p_{abs,lin}^{p=2} \approx 2n_iT_i\left(\frac{k_\perp v_{th}}{\Omega_{ci}}\right)^2 D_p, \quad \text{(D.16)}$$

for second harmonic heating.

However, we rather suggest to directly make use of Eq. D.14, whose numerical quadrature is quite easily performed on any personal computer. Eq. D.14 can be used to deduce D_p when the power density is known (e.g. given by Eq. D.8) as

$$D_p = \frac{p_{abs,lin}}{8n_iT_i \int_0^\infty du_\perp u_\perp^3 \left| J_{p-1}\left(\frac{k_\perp v_{th,i}}{\omega_{ci}}u_\perp\right) + \frac{E_-}{E_+}J_{p+1}\left(\frac{k_\perp v_{th,i}}{\omega_{ci}}u_\perp\right)\right|^2 e^{-u_\perp^2}}. \quad \text{(D.17)}$$

D.3 Low energy, isotropic distribution

We consider firstly the distribution function averaged over the pitch-angle. Introducing $\lambda \equiv v_\parallel/v$, we write

$$\langle f_i\rangle(r,v) \equiv \frac{1}{2}\int_{-1}^{1} d\lambda f_i(r,v,\lambda). \quad \text{(D.18)}$$

Likewise, the quasilinear diffusion coefficient in velocity is averaged according to

$$\langle D_w\rangle \equiv \frac{1}{2}\int_{-1}^{1} d\lambda (1-\lambda^2) D_w. \quad \text{(D.19)}$$

In the collision operator appearing in Eq. D.2, only the energy diffusion and friction survive the pitch-angle averaging procedure, so that

$$\langle \hat{C} f_i\rangle = \frac{1}{u^2}\frac{\partial}{\partial u}\left[u^2\left(D_{uu}\frac{\partial}{\partial u}\langle f_i\rangle - F_u\langle f_i\rangle\right)\right] \quad \text{(D.20)}$$

For the wave term, we use the relation

$$\frac{\partial}{\partial u_\perp} \equiv \frac{\sqrt{1-\lambda^2}}{u}\left[\frac{\partial}{\partial u}u - \frac{\partial}{\partial \lambda}\lambda\right]. \quad \text{(D.21)}$$

to approximate the pitch-angle averaged quasilinear operator in Eq. 4.52 as

$$\langle \hat{Q} f_i \rangle = \frac{1}{u^2} \frac{\partial}{\partial u} u^2 \langle D_w \rangle \frac{\partial \langle f_i \rangle}{\partial u}. \tag{D.22}$$

The quasilinear quantities can be obtained directly from the expressions already provided in chapter 4, i.e. Eqs. 4.82 or 4.83 for the energy content, and 4.85 and 4.88 for the dissipated power. Of course, in all these expressions, only the f_0 terms must be kept and the only needed moment of D_{ql} is Eq. 4.74, i.e.

$$D_{00}^{00}(u) \equiv \frac{1}{2} \int_{-1}^{1} d\lambda (1 - \lambda^2) D_w(u, \lambda) = \langle D_w \rangle. \tag{D.23}$$

The steady-state distribution is thus determined by

$$D_{uu} \frac{d\langle f_i \rangle}{du} - F_u \langle f_i \rangle + \langle D_w \rangle \frac{d\langle f_i \rangle}{du} = 0, \tag{D.24}$$

yielding

$$\langle f_i \rangle (u, r) = A_0(r) \exp\left(\int_0^u du \frac{F_u}{D_{uu} + \langle D_w \rangle} \right), \tag{D.25}$$

where A_0 is a constant determined by the local density.

Whereas the isotropy hypothesis would seem to indicate that this result is only valid at very low energies, Anderson et al. [2] assert that it can still be reliably used in the evaluation of velocity-space moments of the distribution function.

D.4 High-energy tail

At velocities much larger than the critical velocity, electron drag dominates the collision relaxation, so that pitch-angle scattering becomes very inefficient. According to Stix, this occurs when

$$v_\perp^3 > v_\gamma^3 / 4, \tag{D.26}$$

with

$$\frac{m_i v_\gamma^2}{2} \approx 14.810 T_e \left[\frac{2 A_i^{1/2}}{n_e} \sum_\beta n_\beta Z_\beta^2 \right]^{2/3}, \tag{D.27}$$

where the sum over β is carried over the background ions.

In this case, it makes more sense to reformulate the Fokker-Planck equation in terms of $(u_\parallel, u_\perp)$, and introduce the parallel energy-integrated distribution

$$F_\perp(u_\perp) \equiv \langle f_i \rangle_\perp \equiv \int_{-\infty}^{\infty} du_\parallel f_i(u_\parallel, u_\perp). \tag{D.28}$$

Following Stix [142], we assume

$$|u_\parallel| \ll u_\perp, \; u_\perp \sim u, \text{ and } u_\perp \frac{\partial}{\partial u_\perp} \sim u_\parallel \frac{\partial}{\partial u_\parallel}. \tag{D.29}$$

By doing this, one obtains the following approximate expression for the collision operator[3]

$$\langle \hat{C} f_i \rangle_\perp \equiv \hat{C} F_\perp \approx -\frac{1}{u_\perp}\frac{\partial}{\partial u_\perp}(u_\perp \alpha F_\perp) + \frac{1}{2u_\perp}\frac{\partial^2}{\partial u_\perp^2}(u_\perp \beta F_\perp) + \frac{1}{4u_\perp}\frac{\partial}{\partial u_\perp}(\gamma F_\perp), \qquad \text{(D.30)}$$

where for convenience, we have used the familiar Stix expressions for the functions appearing in the collision operators, which are related to our D_{uu}, F_u and Θ_c by the relations

$$\begin{cases} \alpha = & F_u + \dfrac{1}{u^2}\dfrac{\partial}{\partial u} u^2 D_{uu}, \\ \beta = & 2D_{uu}, \\ \gamma = & \dfrac{2\Theta_c}{u}. \end{cases} \qquad \text{(D.31)}$$

The wave term is more straightfoward to handle:

$$\langle D_w f_i \rangle_\perp = \frac{1}{u_\perp}\frac{\partial}{\partial u_\perp} u_\perp D_w \frac{\partial F_\perp}{\partial u_\perp}. \qquad \text{(D.32)}$$

Combining Eqs. D.30 and D.32, one can deduce the steady-state solution as

$$F_\perp(u_\perp) = B_0 \exp\left(-\int_0^{u_\perp} du_\perp \frac{-4\alpha u_\perp + 2\partial_{u_\perp}(u_\perp \beta) + \gamma}{2u_\perp \beta + 4u_\perp D_w(u_\perp)} \right). \qquad \text{(D.33)}$$

Although further analytical progress is possible by using approximate expressions [2,3, 142] for the quantities appearing in this expression, Eq. D.33 can be numerically computed at a quite modest cost.

Eq. 4.81 directly yields for the perpendicular energy content

$$W_\perp = 2\pi v_{th,i}^3 \int du_\perp u_\perp \frac{m v_\perp^2}{2} F_\perp(u_\perp). \qquad \text{(D.34)}$$

Of course, information on the distribution function in the parallel direction needs to be obtained as well. This is done by following Anderson et al. [3], i.e. introducing the reduced parallel temperature as

$$T_\parallel(u_\perp) \equiv \frac{\displaystyle\int_{-\infty}^{\infty} du_\parallel \frac{m v_\parallel^2}{2} f_i(u_\parallel, u_\perp)}{\displaystyle\int_{-\infty}^{\infty} du_\parallel f_i(u_\parallel, u_\perp)}. \qquad \text{(D.35)}$$

The advantage is that for both fundamental and second harmonic heating, $T_\parallel$ has the same asymptotic limit which can be written as

$$T_\parallel(u_\perp) \approx \frac{m_i v_\gamma^3}{4 v_\perp}, \qquad \text{(D.36)}$$

[3]The reader should beware that Eq. 35 in Ref. [142] has a missing $\partial/\partial v_\perp$ on the sixth line. Ref. [3] has a corrected expression.

with v_γ given by Eq. D.27.

Once $T_\parallel$ has been determined from Eq. D.36, it is possible to deduce from Eq. D.35 the parallel energy content as

$$W_\parallel = 2\pi v_{th,i}^3 \int_0^\infty du_\perp u_\perp T_\parallel(u_\perp) F_\perp(u_\perp). \tag{D.37}$$

The absorbed power, Eq. 4.84 is then given by

$$p_{abs,qlin} = -4\pi \nu_i v_{th,i}^3 T_i \int_0^\infty du_\perp u_\perp^2 D_w \frac{\partial F_\perp}{\partial u_\perp}, \tag{D.38}$$

whereas the power dissipated in collisions has the expression

$$p_{coll} = 4\pi \nu_i v_{th,i}^3 T_i \int_0^\infty du_\perp u_\perp \left[\left(\alpha u_\perp - \frac{\gamma}{4} \right) + \frac{\beta}{2} \right] F_\perp. \tag{D.39}$$

The repartition of this power between the various background species is obtained by using Eqs. D.31, and isolating the various terms in sums Eq. 4.57 and Eq. 4.58.

Chapter 4

Quasilinear plasma response

Arguably the most powerful aspect of the Hamiltonian framework presented in chapter 2 is the fast that it can be used to articulate a variational statement by which the electromagnetic field can be obtained, as was done in chapter 3, but also to obtain a global Fokker-Planck equation. The advantage is that both sides of the calculation are naturally self-consistent. This Fokker-Planck equation naturally includes potentially important effects such as finite orbit width effects [76, 80] or Hamiltonian chaos aspects [10, 108]. However, even codes working in terms of motion invariants (or combinations thereof) usually do not solve the global problem. An example of an advanced Fokker-Planck numerical solver is the orbit following Monte Carlo (OFMC) code SPOT [115]. We refer the interested reader to publications related to the ongoing effort of coupling SPOT with the wave code based on the Hamiltonian formalism discussed in chapter 3 [40].

In this chapter, we present how the Fokker-Planck equation is derived in the framework of the Hamiltonian description of chapter 2. How the energy conservation is automatically ensured is underlined. The goal here is to build a physics model able to capture the main features of the secular plasma response with minimal effort. Rather than assuming *a priori* that the interaction between the ions and the wave takes place at the finite locations defined by the local ion cyclotron resonance $\omega - p\Omega_{cs} - k_\parallel v_\parallel = 0$, as is often done, we derive the relevant local expressions for the quasilinear diffusion part from the global ones. This is done by performing quasi-local approximations of the various global quantites, following the procedure detailed in chapter 2. Note that this asymptotic limit allows one to recover the classical results obtained when the quasi-local approach is employed from the start [15, 139, 143] but the Hamiltonian approach lends itself to more refined studies (see, e.g., Refs. [9, 10, 98]).

4.1 Hamiltonian quasilinear theory

4.1.1 Fokker-Planck equation

In the framework of the Hamiltonian theory employed throughout this manuscript, the Fokker-Planck equation is deduced from the Vlasov equation (Eq. 2.31), written here in terms of action-angle variables

$$\frac{\partial f_s}{\partial t} - \frac{\partial H_s}{\partial \Phi_i}\frac{\partial f_s}{\partial J_i} + \frac{\partial H_s}{\partial J_i}\frac{\partial f_s}{\partial \Phi_i} = 0. \tag{4.1}$$

We use the same expansion as in chapter 2 (Eq. 2.35), namely

$$H_s = H_{s,0}(J_k, t) + \sum_{N_1,N_2,N_3} \delta h_{N_1,N_2,N_3}(J_k) e^{i(N_i\Phi_i - \omega t)} + \text{c.c.}, \tag{4.2}$$

and

$$f_s = f_{s,0}(J_k, t) + \sum_{N_1,N_2,N_3} \delta f_{N_1,N_2,N_3}(J_k) e^{i(N_i\Phi_i - \omega t)} + \text{c.c.}, \tag{4.3}$$

where c.c. designates the complex conjugate of the previous term. Note that the equilibrium quantities $H_{s,0}$ and $f_{s,0}$ depend on time in a secular fashion. δH_s and δf_s, on the other hand, feature an oscillatory time-dependence at the wave frequency ω.

The advantage of the angle-action formalism is that the quasilinear mode selection, which is performed by a space-time averaging operation over a finite volume-time period, cleanly reduces to a angle-time averaging, i.e.

$$\langle \ldots \rangle \equiv \frac{1}{(2\pi)^3} \int d\Phi_1 d\Phi_2 d\Phi_3 \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt \ldots. \tag{4.4}$$

The secular modification of f_s is then directly extracted by writing

$$f_{s,0}(t) = \langle f_s \rangle. \tag{4.5}$$

Averaging the Fokker-Planck equation and keeping the first order contributions of H_s and f_s yields

$$\frac{\partial f_{s,0}}{\partial t} - i \sum_{\mathbf{N}} N_i \frac{\partial}{\partial J_i} [\delta h_{\mathbf{N}} \delta f^*_{\mathbf{N}} - \delta h^*_{\mathbf{N}} \delta f_{\mathbf{N}}] = \left(\frac{\partial f_{s,0}}{\partial t}\right)_{\text{coll.}}, \tag{4.6}$$

where a collisional term has been added on the right-hand side to reflect the fact that the slow time variation of the equilibrium distribution is necessarily influenced by collisions.

Using the expression for the linear response to the wave (Eq. 2.36), we can rewrite the previous equation in the compact form

$$\frac{\partial f_{s,0}}{\partial t} = \frac{\partial}{\partial J_i} D_{ij}^{(QL)} \frac{\partial f_{s,0}}{\partial J_j} + \left(\frac{\partial f_{s,0}}{\partial t}\right)_{\text{coll.}}, \tag{4.7}$$

with the quasilinear diffusion operator

$$\bar{D}_{ij}^{(QL)} = \pi \sum_{N_1,N_2,N_3} N_i N_j |\delta h_{N_1,N_2,N_3}|^2 \delta(\omega - N_k \Omega_k), \tag{4.8}$$

At this stage, the Fokker-Planck equation (4.7) is global. This is clear from the quasilinear diffusion operator (Eq. 4.8), which features a global resonance: only those particles with an unperturbed motion in strict resonance with the wave can exchange energy. It must however be realized that the sum over (N_1, N_2, N_3) represents an infinite number of usually densily packed such resonances.

4.1.2 Quasilinear diffusion coefficient

It is more convenient to express the diffusion coefficient in terms of the invariants of the motion. As already discussed in section 2.1.2, a rather natural choice is $\mathbf{I} \equiv (E, \Lambda, P_\phi)$. The wave term in the Fokker-Planck equation then takes the form

$$\langle \mathcal{D}_w(f_s) \rangle = \frac{1}{g^{1/2}} \frac{\partial}{\partial I_i} g^{1/2} D_{ij}^{(QL)} \frac{\partial f_{s,0}}{\partial I_j}, \tag{4.9}$$

and

$$D_{ij}^{(QL)} = \frac{\partial I_i}{\partial J_k} \frac{I_j}{\partial J_l} \bar{D}_{kl}^{(QL)}. \tag{4.10}$$

The Jacobian of this transformation, $g^{1/2}$, is given by Eq. 2.22.

With the help of Eqs. 2.17, 2.18, 2.19 and 2.22, we obtain for the energy-energy element of the diffusion tensor

$$D_{EE}^{(QL)} = \pi \sum_{N_1,N_2,N_3} N_i \Omega_i N_j \Omega_j |\delta h_{N_1,N_2,N_3}|^2 \delta(\omega - N_k \Omega_k), \tag{4.11}$$

or

$$D_{EE}^{(QL)} = \pi \omega^2 \sum_{N_1,N_2,N_3} |\delta h_{N_1,N_2,N_3}|^2 \delta(\omega - N_k \Omega_k), \tag{4.12}$$

As shown in section 2.2.2, the wave-particle interaction dictates that only the Hamiltonian contributions with $N_1 = p$ and $N_3 = n$ survive, so that we may write, limiting ourselves to the interaction at harmonics p and toroidal wavenumber n (for the sake of concision, the indices p and n are omitted)

$$\begin{aligned} D_{EE}^{(QL)} =& \pi \omega^2 \sum_{N_2} |\delta h_{p,N_2,n}|^2 \delta(\omega - N_k \Omega_k) \\ =& \frac{\pi}{\omega_b} \omega^2 \sum_{N_2} |\delta h_{p,N_2,n}|^2 \delta\left(N_2 - \frac{\omega - p\Omega_1 - n\Omega_3}{\Omega_2}\right), \end{aligned} \tag{4.13}$$

$$D_{E\Lambda}^{(QL)} = D_{\Lambda E}^{(QL)} = \frac{\pi}{\omega_b} \omega \left(\frac{p\Omega_{cs}(0) - \Lambda\omega}{E}\right) \sum_{N_2} |\delta h_{p,N_2,n}|^2 \delta\left(N_2 - \frac{\omega - p\Omega_1 - n\Omega_3}{\Omega_2}\right), \tag{4.14}$$

$$D_{\Lambda\Lambda}^{(QL)} = \frac{\pi}{\omega_b} \left(\frac{p\Omega_{cs}(0) - \Lambda\omega}{E}\right)^2 \sum_{N_2} |\delta h_{p,N_2,n}|^2 \delta\left(N_2 - \frac{\omega - p\Omega_1 - n\Omega_3}{\Omega_2}\right), \tag{4.15}$$

$$D_{EP_\phi}^{(QL)} = D_{P_\phi E}^{(QL)} = \frac{\pi}{\omega_b} n\omega \sum_{N_2} |\delta h_{p,N_2,n}|^2 \delta\left(N_2 - \frac{\omega - p\Omega_1 - n\Omega_3}{\Omega_2}\right), \tag{4.16}$$

$$D_{\Lambda P_\phi}^{(QL)} = D_{P_\phi \Lambda}^{(QL)} = \frac{\pi}{\omega_b} n \left(\frac{p\Omega_{cs}(0) - \Lambda\omega}{E}\right) \sum_{N_2} |\delta h_{p,N_2,n}|^2 \delta\left(N_2 - \frac{\omega - p\Omega_1 - n\Omega_3}{\Omega_2}\right), \tag{4.17}$$

and

$$D^{(QL)}_{P_\phi P_\phi} = \frac{\pi}{\omega_b} n^2 \sum_{N_2} |\delta h_{p,N_2,n}|^2 \delta\left(N_2 - \frac{\omega - p\Omega_1 - n\Omega_3}{\Omega_2}\right). \tag{4.18}$$

Evidently

$$D^{(QL)}_{E\Lambda} = D^{(QL)}_{\Lambda E} = \frac{1}{E}\left(\frac{p\Omega_{cs}(0)}{\omega} - \Lambda\right) D^{(QL)}_{EE}, \tag{4.19}$$

$$D^{(QL)}_{\Lambda\Lambda} = \frac{1}{E^2}\left(\frac{p\Omega_{cs}(0)}{\omega} - \Lambda\right)^2 D^{(QL)}_{EE}, \tag{4.20}$$

$$D^{(QL)}_{EP_\phi} = D^{(QL)}_{P_\phi E} = \frac{\pi}{\omega_b}\frac{n}{\omega} D^{(QL)}_{EE}, \tag{4.21}$$

$$D^{(QL)}_{\Lambda P_\phi} = D^{(QL)}_{P_\phi \Lambda} = \frac{n}{\omega}\left(\frac{p\Omega_{cs}(0)}{\omega} - \Lambda\right) D^{(QL)}_{EE}, \tag{4.22}$$

and

$$D^{(QL)}_{P_\phi P_\phi} = \frac{\pi}{\omega_b}\frac{n^2}{\omega^2} D^{(QL)}_{EE}, \tag{4.23}$$

so that only the expression for $D^{(QL)}_{EE}$ is actually needed.

4.1.3 Energy conservation

As already discussed in section 3.1.4, energy conservation directly stems from the variational formulation [61]. We recall here the local Poynting theorem (3.78), which may be rewritten in the form

$$-i\omega W_{field}(\psi) + \mathcal{S}_{Poynting}(\psi) + \dot{W}_{dsp}(\psi) + \mathcal{P}_{abs}(\psi) + \mathcal{S}_{kin}(\psi) = -\dot{W}_{ant}(\psi). \tag{4.24}$$

As already discussed in chapter 3, a delicate task is to obtain the power irreversibly transfered from the wave to the particles, which requires the evaluation of the kinetic flux [17,116,118,141]. In the present approach, however, this step is unnecessary since the dissipated power is directly available from the particle functional $\mathcal{L}_{part}$. To demonstrate this point, it is necessary to evaluate the secular variation of the kinetic energy of the particles in interaction with the wave in the framework of the quasilinear theory, which is given by

$$W = \frac{m_s}{2}\int d^3\mathbf{r} d^3\mathbf{p} v^2 f_{s,0}(\mathbf{r},\mathbf{p},t), \tag{4.25}$$

so that the energy increase caused by the power transferred from the wave to the particles through non-collisional damping may be written as

$$\mathcal{P}_{abs} = \frac{\partial W}{\partial t} = \frac{m_s}{2}\int d^3\mathbf{r} d^3\mathbf{p} v^2 \frac{\partial}{\partial J_j} D^{(QL)}_{ij} \frac{\partial f_{s,0}}{\partial J_i}. \tag{4.26}$$

Using $d^3\mathbf{r}d^3\mathbf{p} = (2\pi)^3 d^3\mathbf{J}$ in the integral and integrating by parts yields

$$\mathcal{P}_{abs} = -\pi\frac{m_s}{2}(2\pi)^3 \int d^3\mathbf{J}\left(N_j\frac{\partial v^2}{\partial J_j}\right) N_i \frac{\partial f_{s,0}}{\partial J_i}\delta(\omega - N_k\Omega_k)|\delta h_{\mathbf{N}}|^2. \tag{4.27}$$

The quantity in parentheses may be rewritten as

$$N_j \frac{\partial v^2}{\partial J_j} = \frac{2}{m_s} N_j \frac{\partial E}{\partial J_j} = \frac{2}{m_s} N_j \Omega_j, \tag{4.28}$$

so that, using the global resonance condition imposed by the delta function

$$\mathcal{P}_{abs} = -\omega\pi(2\pi)^3 \int d^3\mathbf{J} N_i \frac{\partial f_{s,0}}{\partial J_i} \delta(\omega - N_k\Omega_k)|\delta h_{\mathbf{N}}|^2, \tag{4.29}$$

which is strictly identical to the imaginary part of the plasma functional (3.42), thereby demonstrating

$$\mathcal{P}_{abs} = \frac{\omega}{2} \Im(\mathcal{L}_{part}). \tag{4.30}$$

We also deduce from Eq. 3.75 that the kinetic flux is given by

$$\mathcal{S}_{kin}(\psi) = -\frac{1}{2}\Re\left\{ \int_\psi d^2\mathbf{S} \cdot \mathbf{j}_{part}\varphi^* \right\}. \tag{4.31}$$

Fig. 4.1 shows the power balance corresponding to the ^{3}He case in ITER discussed in Ref. [41]. After the field is reconstructed, the power coupled by the antenna is given by Eq. 3.72, the Poynting flux is available from Eq. 3.69, and the power absorbed on species, $\mathcal{P}_{abs}$, is directly deduced from the plasma functional (Eq 4.30). The kinetic flux is then deduced from the energy balance, Eq. 4.24.

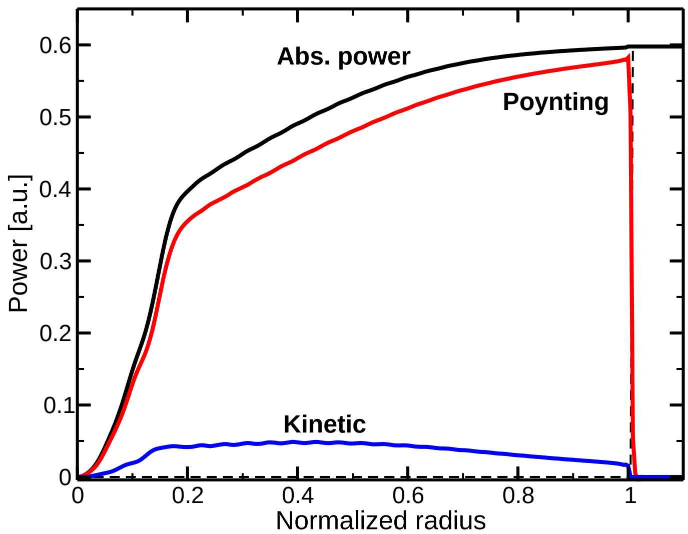

Figure 4.1: Power balance corresponding to a ^{3}He(DT) scenario in ITER [41]. Shown are the Poynting flux, the (cumulative) absorbed power, and the kinetic flux deduced from the energy balance. The cumulative power coupled by the antenna is also shown as a dashed line and matches the absorbed power on the vacuum vessel.

It is often convenient to separate the power absorbed by the various plasma species. Using the decomposition of Eq. 3.37 in terms of a sum over species, we may write

$$\mathcal{P}_{abs}(\psi) = \sum_s \mathcal{P}_{abs,s}(\psi), \tag{4.32}$$

with

$$\mathcal{P}_{abs,s}(\psi) = \frac{\omega}{2}\Im(\mathcal{L}_{part,s}(\psi)), \tag{4.33}$$

which is directly available from the wave calculation and corresponds to the power absorbed by species s inside magnetic surface ψ. An important quantity for experiments modeling is the power density absorbed by species s on magnetic surface ρ. It is obtained by writing

$$p_s(\psi) = \frac{1}{\mathcal{V}(\rho)} \frac{d\Im(\mathcal{L}_{parts,})}{d\rho}\bigg|_\psi, \tag{4.34}$$

with the volume element defined as

$$\mathcal{V}(\rho) = 2\pi \oint d\theta J(\rho,\theta). \tag{4.35}$$

4.2 Derivation of a quasi-local model

4.2.1 Quasilinear diffusion coefficient

We now turn to the calculation of $D_{EE}^{(QL)}$. Using the quasi-local expression for the Hamiltonian contributions, Eq. 2.68, we have

$$\delta h_{p,N_2,n} \approx \frac{1}{\tau_b} \sum_{m,t_0[m]} \Gamma_m(t_0)\delta H_{pmn}(t_0) e^{i[\gamma_m(t_0)-(p\Omega_1+N_2\Omega_2+n\Omega_3)]}, \tag{4.36}$$

We recall that t_0 is given by

$$\dot\gamma_{m_0}(t_0) = p\Omega_{cs}(t_0) + m_0\dot\theta(t_0) + n\dot\phi(t_0) = N_i\Omega_i, \tag{4.37}$$

which by virtue of the Dirac function which selects a given value of N_2 yields the usual quasi-local resonance

$$\omega = p\Omega_{cs}(t_0) + k_\parallel(t_0)v_\parallel(t_0). \tag{4.38}$$

Using the procedure detailed in chapter 3, we obtain

$$D_{EE}^{(QL)} = \frac{\pi}{\omega_b}\omega^2\left\{\frac{1}{\tau_b^2}\sum_{m_1,m_2}\sum_{t_0} |\Gamma_{m_0}(t_0)|^2 \delta H_{pm_1n}(t_0)\delta H^*_{pm_2n}(t_0)e^{i(m_1-m_2)\theta(t_0)}\right\}. \tag{4.39}$$

with $m_0 = (m_1+m_2)/2$.

We note that if the function γ_{m_0} only depends weakly on the poloidal wavenumber (which is typical of cyclotron interaction or Cerenkov interaction at sufficiently high toroidal numbers), we have

$$
\begin{aligned}
D_{EE}^{(QL)} \approx & \frac{\pi}{\omega_b}\omega^2\left\{\frac{1}{\tau_b^2}\sum_{m_1,m_2}\sum_{t_0}|\Gamma_{m_0}(t_0)|^2\delta H_{pm_1n}(t_0)\delta H^*_{pm_2n}(t_0)e^{i(m_1-m_2)\theta(t_0)}\right\} \\
= & \frac{\pi}{\omega_b}\omega^2\left\{\frac{1}{\tau_b^2}\sum_{t_0}|\Gamma_{m_0}(t_0)|^2\sum_{m_1,m_2}\delta H_{pm_1n}(t_0)\delta H^*_{pm_2n}(t_0)e^{i(m_1-m_2)\theta(t_0)}\right\} \\
= & \frac{\omega^2}{2\tau_b}\sum_{t_0}|\Gamma_{m_0}(t_0)\delta H_{pn}(t_0)|^2,
\end{aligned}
\tag{4.40}
$$

The quantity $|\Gamma_{m_0}(t_0)| \equiv \tau_{res}$ represents the time spent by the particle in resonance with the wave, allowing to rewrite the previous expression in a relatively transparent fashion as

$$
D_{EE}^{(QL)} \approx \frac{\omega^2}{2}\sum_{t_0}\left(\frac{\tau_{res}^2(t_0)}{\tau_b}\right)|\delta H_{pn}(t_0)|^2, \tag{4.41}
$$

From the local resonance condition, the resonance time is approximately given by

$$
\tau_{res} \equiv \frac{1}{|pv_\parallel\nabla_\parallel\Omega_{cs}|^{1/2}}, \tag{4.42}
$$

whereas the bounce time is

$$
\tau_b = \oint\frac{dl}{v_\parallel}, \tag{4.43}
$$

with dl the elemental arclength along the field line.

We can go one step further in simplyfying the problem if we assume, as Stix [142] did in his seminal paper, that the particle parallel motion is uniform along its unperturbed orbit (in a quasi-local sense). If we assume that τ_{res}^2/τ_b varies only slowly along the particle orbit as the particle passes the local resonance, then we have simply

$$
D_{EE}^{(QL)} \approx D_0\sum_{t_0}|\delta H_{pn}(t_0)|^2, \tag{4.44}
$$

where all quantities in the Hamiltonian are evaluated at the local resonance and the constant D_0 is determined imposing that the total absorbed power be equal to the imposed RF power.

In the framework of this quasi-local model, we may use a uniform approximation to the Hamiltonian contributions H_{pn} by employing a Kennel-Engelmann-type expression [97]. In fact, this is already available to us from our previous derivation of a WKB Hamiltonian (see section 2.3.2), using $\mathbf{k}_\perp$ from the fast wave dispersion relation, assuming the propagation angle β is zero. This yields

$$
\begin{aligned}
\delta H_{pn} = & -q_s\frac{v_\perp}{\sqrt{2}}A_+J_{p-1}(k_\perp v_\perp/\Omega_{cs}) - q_s\frac{v_\perp}{\sqrt{2}}A_-J_{p+1}(k_\perp v_\perp/\Omega_{cs}) \\
& + q_s(\varphi - v_\parallel A_\parallel)J_p(k_\perp v_\perp/\Omega_{cs}).
\end{aligned}
\tag{4.45}
$$

The last term in this equation corresponds to the interaction between the parallel electric field and the particles. Since we are interested in ion heating and we only consider the fast wave, the main contribution to the electric field is the perpendicular potential vector $\mathbf{A}_\perp$, so that may we may write

$$|\delta H_{pn}|^2 \approx q_s^2 \frac{v_\perp^2}{2\omega^2} |E_+ J_{p-1}(k_\perp v_\perp/\Omega_{cs}) + E_- J_{p+1}(k_\perp v_\perp/\Omega_{cs})|^2, \tag{4.46}$$

which is a well-known expression (see, e.g. [50, 143]).

Since we have adopted a quasi-local form for the Fokker-Planck equation, it is convenient to express the diffusion tensor in terms of the local velocity $(v_\parallel, v_\perp)$. We have

$$\Lambda \equiv \frac{m_i v_\perp^2}{2E} \frac{B_0(0)}{B_0} = \frac{u_\perp^2}{u^2} \frac{B_0(0)}{B_0}, \tag{4.47}$$

and

$$E \equiv \frac{m_i}{2}(v_\parallel^2 + v_\perp^2) = \frac{m_i v_{th,i}^2}{2} u^2, \tag{4.48}$$

from which we deduce

$$D_{v_\perp v_\perp}^{(QL)} = \frac{1}{m_i v_\perp^2} \left(p \frac{\Omega_{cs}}{\omega}\right)^2 D_{EE}^{(QL)}, \tag{4.49}$$

and

$$D_{v_\parallel v_\parallel}^{(QL)} = \frac{1}{m_i v_\parallel^2} \left(1 - p \frac{\Omega_{cs}}{\omega}\right)^2 D_{EE}^{(QL)}. \tag{4.50}$$

At the cyclotron resonance, we have $\omega \sim p\Omega_{cs}$, so that

$$|D_{v_\perp v_\perp}^{(QL)}| \gg |D_{v_\perp v_\parallel}^{(QL)}|, |D_{v_\parallel v_\perp}^{(QL)}| \gg |D_{v_\parallel v_\parallel}^{(QL)}|, \tag{4.51}$$

which is the justification for the subsequent neglecting of all terms but the perpendicular one in the quasilinear diffusion tensor. From expressions 4.44, 4.46 and 4.49, we see that the wave contribution to the distribution function evolution takes the simple form

$$\left(\frac{\partial f_i}{\partial t}\right)_{\text{wave}} \equiv \hat{Q} f_i = \frac{1}{u_\perp} \frac{\partial}{\partial u_\perp} u_\perp D_w \frac{\partial f_i}{\partial u_\perp}, \tag{4.52}$$

with

$$D_w = D_0 \sum_{t_0} |E_+ J_{p-1}(k_\perp v_\perp/\Omega_{cs}) + E_- J_{p+1}(k_\perp v_\perp/\Omega_{cs})|^2. \tag{4.53}$$

4.2.2 Coulomb collisions

The classical derivation of the Coulomb collision operator is by essence local in the velocity space [93, 99, 144]. If we denote "i" the heated specie, the collisional part of the Fokker-Planck equation at a given space location can be written as

$$\frac{\partial f_i}{\partial t} = \sum_s \hat{C}(f_i, f_s), \tag{4.54}$$

where the sum is performed over all plasma species. A complex problem is the correct handling of the $\hat{C}(f_i, f_i)$ term (self-collisions) [93, 146]. We choose here to disregard this issue by employing a linearized operator where self-collisions are assumed to remain negligeable, and assuming that the background species are all Maxwellians. The resulting operator can be written as the divergence of a quasilinear flux, i.e. $\hat{C}(f_i, f_s) \equiv -\nabla_{\mathbf{v}} \cdot \mathbf{S}_c$ with $\mathbf{S}_c = -\mathbf{D} \cdot \nabla_{\mathbf{v}} f_s + \mathbf{F}_c f_s$.

At this stage, it is convenient to introduce a reference frequency characterizing the collisions between test ions belonging to the heated specie and the background constituted by one of the bulk ion species (usually the majority ion, denoted here "M")

$$\nu_i \equiv \nu_{i/M} \equiv \frac{\Gamma^{i/M}}{v_{th,i}^3}, \tag{4.55}$$

where $v_{th,i}$ is a thermal velocity characterizing the heated species (for instance, the thermal velocity before heating). Introducting the normalized time $\tau \equiv \nu_i t$, the normalized velocity $u \equiv v/v_{th,i}$ and pitch angle cosine $\lambda \equiv v_{\|}/v$, we then obtain the collision term in the compact form [15]

$$\frac{\partial f_i}{\partial \tau} = \frac{1}{u^2}\frac{\partial}{\partial u}\left[u^2\left(D_{uu}\frac{\partial f_i}{\partial u} - F_u f_i\right)\right] + \frac{1}{u^2}\left[\frac{\partial}{\partial \lambda}(1-\lambda^2)\frac{\Theta_c}{2u}\frac{\partial f_i}{\partial \lambda}\right], \tag{4.56}$$

with

$$D_{uu} = \frac{1}{2u}\sum_\beta \frac{\nu_{i/\beta}}{\nu_i}\Psi(u_\beta), \tag{4.57}$$

$$F_u = -\sum_\beta \frac{\nu_{i/\beta}}{\nu_i}\frac{T_i}{T_\beta}\Psi(u_\beta) \tag{4.58}$$

and

$$\frac{\Theta_c}{2u} = \sum_\beta \frac{\nu_{i/\beta}}{\nu_i}\frac{\Theta(u_\beta)}{2u}, \tag{4.59}$$

where β designates the bulk species, i.e. all plasma species except the heated one. We introduce $u_\beta \equiv v/v_{th,\beta}$. Also

$$\Gamma^{a/b} \equiv \frac{n_b Z_a^2 Z_b^2 e^4 \log(\Lambda^{a/b})}{4\pi\epsilon_0^2 m_a^2}, \tag{4.60}$$

with $\log(\Lambda^{a/b})$ the Coulomb logarithm. Besides

$$\Psi(x) \equiv \frac{\mathrm{erf}(x) - x\,\mathrm{erf}'(x)}{x^2}, \tag{4.61}$$

and

$$\Theta(x) \equiv \frac{1}{x^2}\left[\left(x^2 - \frac{1}{2}\right)\mathrm{erf}(x) + \frac{x}{2}\mathrm{erf}'(x)\right]. \tag{4.62}$$

Note that we have

$$\frac{\nu_{i/\beta}}{\nu_i} = \frac{n_\beta}{n_M}\frac{Z_\beta^2}{Z_M^2}\frac{\log(\Lambda^{i/\beta})}{\log(\Lambda^{i/M})} \approx \frac{n_\beta}{n_M}\frac{Z_\beta^2}{Z_M^2}, \tag{4.63}$$

and also

$$u_\beta = u\frac{v_{th,i}}{v_{th,\beta}}. \tag{4.64}$$

4.2.3 A quasi-local Fokker-Planck solver

At this point, it would be useful to take advantage of the previously described model to predict the plasma response to a given wave-field. The idea is to be as numerically efficient as possible, including the following essential physics ingredients to simulate ICRF plasma heating :

- Building of the superthermal ion tail by RF-induced diffusion, balanced by collisions. For a given initial equilibrium distribution function $f_i(t_{\text{initial}})$, we impose that in the absence of RF source, f_i will tend towards a given f_{eq} at large t. This is essential to describe a distribution function which has been pre-heated by, e.g. neutral beam injection. Including a suitable model for NBI heating is beyond the scope of our study.

- Heating of the various thermal plasma species by collision relaxation of the ICRF-heated ions. This is essential because transport properties, which eventually determine the plasma performance, are dependent on this heat source term.

- Fast ions lost due to prompt losses, i.e. orbit widths comparable or exceeding the device size.

A generic form for a local Fokker-Planck equation fulfilling these requirements is

$$\partial_\tau f_i = \sum_\beta \hat{C}(f_i, f_\beta) - \sum_\beta \hat{C}(f_{eq}, f_\beta) + \hat{Q}f_i - \hat{L}f_i + \mathcal{S}_{\text{fuelling}}, \tag{4.65}$$

where $\hat{Q}f_i$ represents the wave quasilinear term, $\hat{L}f_i$ a fast ion loss term and $\mathcal{S}_{\text{fuelling}}$ is a particle source adjusted to compensate for losses caused either by the loss term, or by the choice of an insufficiently extended velocity grid.

In the presence of the quasi-local Coulomb operator, Eq. 4.56, a common procedure is to expand the distribution function in terms of Legendre polynomials

$$f_i(u,\lambda) \equiv \sum_n f_n(u)P_n(\lambda), \tag{4.66}$$

or conversely

$$f_n(u) \equiv \frac{2n+1}{2}\int_{-1}^{1} d\lambda f_i(u,\lambda)P_n(\lambda). \tag{4.67}$$

The rationale is that the pitch-angle scattering term appearing in Eq. 4.56 takes a diagonal form, i.e.

$$\frac{\partial}{\partial\lambda}(1-\lambda^2)\frac{\Theta_c}{2u}\frac{\partial f_i}{\partial\lambda} = -\sum_n n(n+1)\frac{\Theta_c}{2u}f_n(u)P_n(\lambda). \tag{4.68}$$

We assume a generic form for the loss term, i.e.

$$\hat{L} f_i = \frac{\Lambda_{fil}(u,\lambda)}{\tau_{fil}} f_i, \tag{4.69}$$

with τ_{fil} the typical fast ion loss time, and $\Lambda_{fil}(u,\lambda)$ a function of velocity and pitch-angle which cancels in regions where ions are confined, and tends toward 1 in regions of velocity space where particles are lost[1].

We impose that the total heated ion density is conserved, and that any lost ion must be reintroduced somehow as a thermal ion. This is done by setting

$$\mathcal{S}_{\text{fuelling}} = \frac{n_i}{\pi^{3/2} v_{th,i}} e^{-u^2} \mathcal{S}_0, \tag{4.71}$$

with $\mathcal{S}_0$ a constant. Since the employed forms of the collision and quasilinear diffusion operators conserve density, we immediately obtain

$$\mathcal{S}_0 = \frac{2\pi}{n_i \tau_{fil}} \int du d\lambda u^2 \Lambda_{fil}(u,\lambda) f_i(u,\lambda). \tag{4.72}$$

The correspoding term in the Fokker-Planck equation is straightforward to implement. If we project Eq. 4.65 using expansion 4.66 onto basis polynomial P_m, we obtain

$$\begin{aligned}
\sum_n \delta_{nm} \partial_\tau f_n = \sum_n \delta_{nm} \Big[& D_{uu} \partial_u^2 f_n + \left(\frac{1}{u^2} \partial_u (u^2 D_{uu}) - F_u \right) \partial_u f_n \\
& + \left(-\frac{1}{u^2} \partial_u (u^2 F_u) - n(n+1) \frac{\Theta_c}{2u^3} \right) f_n \Big] \\
+ \sum_n (2m+1) \Big[& D_{00}^{mn} \partial_u^2 f_n + \left(\frac{1}{u^2} \partial_u (u^2 D_{00}^{mn}) - \frac{1}{u} D_{01}^{mn} + \frac{1}{u} D_{10}^{mn} \right) \partial_u f_n \\
& - \left(\frac{1}{u^2} \partial_u (u D_{01}^{mn}) + \frac{1}{u^2} D_{11}^{mn} \right) f_n \Big] \\
& - \sum_n (2m+1) \frac{\Lambda_{mn}}{\tau_{fil}} f_n + u^2 \mathcal{S}_{\text{fuelling}} \delta_{m0} \\
+ \delta_{m0} \Big[& \frac{2}{u} \partial_u (u^2 D_{uu}) + \frac{1}{u^2} \partial_u (u^2 F_u) - 2(2u^2 - 1) D_{uu} - 2u F_u \Big] f_{eq}(u).
\end{aligned} \tag{4.73}$$

In the latter expression, f_{eq} is assumed to be an isotropic Maxwellian. Using more complicated equilibrium distribution function shapes does not pose any conceptual difficulty, but complexifies the source term somewhat.

[1] As an example of a most natural form for Λ_{fil}, one can employ the isotropic expression

$$\Lambda_{fil}(u) \equiv \frac{1}{1 + e^{-(E - E_{fil})/\Delta E}}, \tag{4.70}$$

with ΔE and adjustable parameter. This ensures that all ions with energy exceeding E_{fil} are virtually lost. Of course, more sophisticated expressions can be employed as well.

We have introduced the following moments

$$D_{00}^{mn}(u) \equiv \frac{1}{2}\int_{-1}^{1} d\lambda(1-\lambda^2)D_w(u,\lambda)P_m(\lambda)P_n(\lambda), \tag{4.74}$$

$$D_{01}^{mn}(u) \equiv \frac{1}{2}\int_{-1}^{1} d\lambda(1-\lambda^2)\lambda D_w(u,\lambda)P'_m(\lambda)P_n(\lambda), \tag{4.75}$$

$$D_{10}^{mn}(u) \equiv \frac{1}{2}\int_{-1}^{1} d\lambda(1-\lambda^2)\lambda D_w(u,\lambda)P_m(\lambda)P'_n(\lambda), \tag{4.76}$$

$$D_{11}^{mn}(u) \equiv \frac{1}{2}\int_{-1}^{1} d\lambda(1-\lambda^2)\lambda^2 D_w(u,\lambda)P'_m(\lambda)P'_n(\lambda), \tag{4.77}$$

and

$$\Lambda_{mn}(u) \equiv \frac{1}{2}\int_{-1}^{1} d\lambda \Lambda_{fil}(u,\lambda)P_m(\lambda)P_n(\lambda). \tag{4.78}$$

All these expressions lend themselves to implementation in an efficient numerical code, which was named AQL. An early version of AQL is described and extensivey employed in Ref. [46].

4.2.4 Derived quasilinear quantities

From the distribution function and various operators, it is possible to derive various quasilinear quantities of physical interest.

The heated ion density is defined as

$$n_i \equiv \int d^3\mathbf{v}\, f_i(\mathbf{v}), \tag{4.79}$$

and can be expressed in terms of the first Legendre moment of f_i (Eq. 4.66) as

$$n_i = 4\pi \int_0^\infty du u^2 f_0. \tag{4.80}$$

The perpendicular and parallel energy content can be directly obtained by writing

$$W_{\perp,\parallel} \equiv \int d^3\mathbf{v}\, \frac{m v_{\perp,\parallel}^2}{2} f_i(\mathbf{v}), \tag{4.81}$$

yielding

$$W_\parallel = T_i \int d^3\mathbf{v}\, u_\parallel^2 f_i = \frac{4\pi}{3} T_i \int_0^\infty du u^4 \left(f_0 + \frac{2}{5} f_2\right), \tag{4.82}$$

and

$$W_\perp = T_i \int d^3\mathbf{v}\, u_\perp^2 f_i = \frac{4\pi}{3} T_i \int_0^\infty du u^4 \left(2 f_0 - \frac{2}{5} f_2\right), \tag{4.83}$$

where f_2 and f_0 are the coefficients in the Legendre expansion of the distribution function (Eq. 4.66). It is readily seen that in the isotropic case, $W_\perp = 2W_\parallel$, as it must.

A crucial quantity, needed to perform the distribution function calculation is the absorbed power $p_{abs,qlin}$, which is deduced from Eq. 4.52 as

$$p_{abs,qlin} \equiv \int d^3\mathbf{v}\, \frac{mv^2}{2}\hat{Q} f_i, \tag{4.84}$$

and may be rewritten as

$$\begin{aligned} p_{abs,qlin} &= 4\pi\nu_i T_i v_{th,i}^3 \int dud\lambda u^2(1-\lambda^2)D_w(u,\lambda)\left(u\frac{\partial f_i}{\partial u} - \lambda\frac{\partial f_i}{\partial \lambda}\right) \\ &= 8\pi T_i v_{th,i}^3 \int duu^2 \sum_n \left[uD_{00}^{0n}\frac{\partial f_n}{\partial u} - D_{01}^{0n} f_n\right], \end{aligned} \tag{4.85}$$

where the moments of both the distribution function and quasilinear diffusion (Eqs. 4.74 and 4.75) are used. Ideally, one should compare both expressions above to ensure that the Legendre expansion retains a sufficient number of harmonics.

Another important quantity is the power transfered to the background plasma by collisions, which is given by

$$p_{coll} \equiv \int d^3\mathbf{v}\, \frac{mv^2}{2}\hat{C} f_i, \tag{4.86}$$

which may be written in terms of f_0 only

$$p_{coll} = 8\pi\nu_i T_i v_{th,i}^3 \int_0^\infty du\big[\partial_u(u^3 D_{uu}) + u^3 F_u\big] f_0, \tag{4.87}$$

where an integration by parts has been carried out in order to avoid involving the derivative of f_i in the calculations.

This expression may be used in the course of the calculation because of its simplicity and numerical efficiency. If one is interested in the power transfered from the heated ion to a given (Maxwellian) background specie β, then straightforward algebraic manipulations of Eq. 4.87 using Eqs. 4.57 and 4.58 yield

$$p_{coll}^{i\rightarrow\beta} = 8\pi\nu_i\left(\frac{\nu_{i/\beta}}{\nu_i}\right) v_{th,i}^3 \int_0^\infty duu\left[\Psi_d(u_\beta) - u^2\frac{T_i}{T_\beta}\Psi(u_\beta)\right] f_0, \tag{4.88}$$

with

$$\Psi_d(x) \equiv x\mathrm{erf}'(x). \tag{4.89}$$

Finally, the power lost because of fast ion losses is given by

$$p_{fil} \equiv \int d^3\mathbf{v}\, \frac{mv^2}{2}\hat{L} f_i. \tag{4.90}$$

Substituting Eq. 4.69 and using expansion Eq. 4.66 yields the following expression

$$p_{fil} = 4\pi\nu_i T_i v_{th,i}^3 \int duu^4 \frac{\Lambda_n}{\tau_{fil}} f_n, \tag{4.91}$$

with the Legendre moment of the loss function

$$\Lambda_n \equiv \frac{1}{2}\int_{-1}^1 d\lambda \Lambda_{fil}(u,\lambda) P_n(\lambda) = \Lambda_{0n}(u), \tag{4.92}$$

where Λ_{0n} refers to Eq. 4.78.

Bibliography

[1] S. Ali-Arshad and D. J. Campbell. Observation of tae activity in jet. *Plasma Physics and Controlled Fusion*, 37(7):715, 1995.

[2] D. Anderson, W. Core, L.-G. Eriksson, H. Hamnén, T. Hellsten, and M. Lisak. Distortion of ion velocity distributions in the presence of icrh: A semi-analytical analysis. *Nuclear Fusion*, 27(6):911, 1987.

[3] D. Anderson, L.-G. Eriksson, and M. Lisak. Analytical treatment of the distortion of velocity distributions in the presence of icrh. *Nuclear Fusion*, 25(12):1751, 1985.

[4] C. Angioni, F. J. Casson, P. Mantica, T. Pütterich, M. Valisa, E. A. Belli, R. Bilato, C. Giroud, P. Helander, and JET Contributors. The impact of poloidal asymmetries on tungsten transport in the core of jet h-mode plasmas. *Physics of Plasmas*, 22(5):055902, 2015.

[5] C. Angioni and P. Helander. Neoclassical transport of heavy impurities with poloidally asymmetric density distribution in tokamaks. *Plasma Physics and Controlled Fusion*, 56(12):124001, 2014.

[6] C. Angioni, P. Mantica, T. Pütterich, M. Valisa, M. Baruzzo, E. A. Belli, P. Belo, F. J. Casson, C. Challis, P. Drewelow, C. Giroud, N. Hawkes, T. C. Hender, J. Hobirk, T. Koskela, L. Lauro Taroni, C. F. Maggi, J. Mlynar, T. Odstrcil, M. L. Reinke, M. Romanelli, and JET EFDA Contributors. Tungsten transport in jet h-mode plasmas in hybrid scenario, experimental observations and modelling. *Nuclear Fusion*, 54(8):083028, 2014.

[7] T. M. Antonsen and K. R. Chu. Radio frequency current generation by waves in toroidal geometry. *Phys. Fluids*, 25:1295, 1982.

[8] J.-F. Artaud, V. Basiuk, F. Imbeaux, M. Schneider, J. Garcia, G. Giruzzi, P. Huynh, T. Aniel, F. Albajar, J.-M. Ané, A. Bécoulet, C. Bourdelle, A. Casati, L. Colas, J. Decker, R. Dumont, L.G. Eriksson, X. Garbet, R. Guirlet, P. Hertout, G. T. Hoang, W. Houlberg, G. Huysmans, E. Joffrin, S. H. Kim, F. Köchl, J. Lister, X. Litaudon, P. Maget, R. Masset, B. Pégourié, Y. Peysson, P. Thomas, E. Tsitrone, and F. Turco. The cronos suite of codes for integrated tokamak modelling. *Nuclear Fusion*, 50(4):043001, 2010.

[9] A. Bécoulet, D. Fraboulet, G. Giruzzi, D. Moreau, B. Saoutic, and J. Chinardet. Hamiltonian analysis of fast wave current drive in tokamak plasmas. *Phys. Plasmas*, 1:2908, 1994.

[10] A. Bécoulet, D. J. Gambier, and A. Samain. Hamiltonian theory of the ion cyclotron minority heating dynamics in tokamak plasmas. *Phys. Fluids B*, 3:137, 1991.

[11] A. Bécoulet, G.T. Hoang, J. Abiteboul, J. Achard, T. Alarcon, J. Alba-Duran, L. Allegretti, S. Allfrey, S. Amiel, J.M. Ané, T. Aniel, G. Antar, A. Argouarch, A. Armitano, J. Arnaud, D. Arranger, J.F. Artaud, D. Audisio, M. Aumeunier, E. Autissier, L. Azcona, A. Back, A. Bahat, X. Bai, B. Baiocchi, D. Balaguer, S. Balme, C. Balorin, O. Barana, D. Barbier, A. Barbuti, V. Basiuk, O. Baulaigue, P. Bayetti, C. Baylard, S. Beaufils, A. Beaute, M. Bécoulet, Z. Bej, S. Benkadda, F. Benoit, G. Berger-By, J.M. Bernard, A. Berne, B. Bertrand, E. Bertrand, P. Beyer, A. Bigand, G. Bonhomme, G. Borel, A. Boron, C. Bottereau, H. Bottollier-Curtet, C. Bouchand, F. Bouquey, C. Bourdelle, J. Bourg, S. Bourmaud, S. Brémond, F. Bribiesca Argomedo, M. Brieu, C. Brun, V. Bruno, J. Bucalossi, H. Bufferand, Y. Buravand, L. Cai, V. Cantone, B. Cantone, E. Caprin, T. Cartier-Michaud, A. Castagliolo, J. Belo, V. Catherine-Dumont, G. Caulier, J. Chaix, M. Chantant, M. Chatelier, D. Chauvin, J. Chenevois, B. Chouli, L. Christin, D. Ciazynski, G. Ciraolo, F. Clairet, R. Clapier, H. Cloez, M. Coatanea-Gouachet, L. Colas, G. Colledani, L. Commin, P. Coquillat, E. Corbel, Y. Corre, J. Cottet, P. Cottier, X. Courtois, I. Crest, R. Dachicourt, M. Dapena Febrer, C. Daumas, H.P.L. de Esch, B. De Gentile, C. Dechelle, J. Decker, P. Decool, V. Deghaye, J. Delaplanche, E. Delchambre-Demoncheaux, L. Delpech, C. Desgranges, P. Devynck, J. Dias Pereira Bernardo, G. Dif-Pradalier, L. Doceul, Y. Dong, D. Douai, H. Dougnac, N. Dubuit, J.-L. Duchateau, L. Ducobu, B. Dugue, N. Dumas, R. Dumont, et al. Science and technology research and development in support to iter and the broader approach at cea. *Nuclear Fusion*, 53(10):104023, 2013.

[12] D. Borba, G. D. Conway, S. Günter, G. T. A. Huysmans, S. Klose, M. Maraschek, A. Mück, I. Nunes, S. D. Pinches, F. Serra, and the ASDEX Upgrade Team. Destabilization of tae modes using icrh in asdex upgrade. *Plasma Physics and Controlled Fusion*, 46(5):809, 2004.

[13] C. J. Boswell, H. L. Berk, D. N. Borba, T. Johnson, S. D. Pinches, and S. E. Sharapov. Observation and explanation of the jet chirping mode. *Physics Letters A*, 358(2):154, 2006.

[14] C. Bourdelle, J.-F. Artaud, V. Basiuk, M. Bécoulet, S. Brémond, J. Bucalossi, H. Bufferand, G. Ciraolo, L. Colas, Y. Corre, X. Courtois, J. Decker, L. Delpech, P. Devynck, G. Dif-Pradalier, R. P. Doerner, D. Douai, R. Dumont, A. Ekedahl, N. Fedorczak, C. Fenzi, M. Firdaouss, J. Garcia, P. Ghendrih, C. Gil, G. Giruzzi, M. Goniche, C. Grisolia, A. Grosman, D. Guilhem, R. Guirlet, J. Gunn, P. Hennequin, J. Hillairet, T. Hoang, F. Imbeaux, I. Ivanova-Stanik, E. Joffrin, A. Kallenbach, J. Linke, T. Loarer, P. Lotte, P. Maget, Y. Marandet, M.-L. Mayoral, O. Meyer, M. Missirlian, P. Mollard, P. Monier-Garbet, P. Moreau, E. Nardon,

B. Pégourié, Y. Peysson, R. Sabot, F. Saint-Laurent, M. Schneider, J.M. Travère, E. Tsitrone, S. Vartanian, L. Vermare, M. Yoshida, R. Zagorski, and JET Contributors. West physics basis. *Nuclear Fusion*, 55(6):063017, 2015.

[15] M. Brambilla. *Kinetic Theory of Plasma Waves.* Clarendon Press, Oxford, 1998.

[16] M. Brambilla and R. Bilato. Advances in numerical simulations of ion cyclotron heating of non-maxwellian plasmas. *Nuclear Fusion*, 49(8):085004, 2009.

[17] M. Brambilla and T. Krücken. Numerical simulation of ion cyclotron heating of hot tokamak plasmas. *Nucl. Fusion*, 28:1813, 1988.

[18] M. Brambilla and M. Ottaviani. Mode conversion near ion-ion hybrid and ic harmonic resonances in tokamaks. *Plasma Physics and Controlled Fusion*, 27(1):1, 1985.

[19] K.G. Budden. *Radio Waves in the Ionosphere.* Cambridge Press, Cambridge, 1961.

[20] R. V. Budny, L. Berry, R. Bilato, P. Bonoli, M. Brambilla, R. J. Dumont, A. Fukuyama, R. Harvey, E. F. Jaeger, K. Indireshkumar, E. Lerche, D. McCune, C. K. Phillips, V. Vdovin, J. Wright, and members of the ITPA-IOS. Benchmarking icrf full-wave solvers for iter. *Nucl. Fusion*, 52:023023, 2012.

[21] B. Cambon, X. Leoncini, M. Vittot, R. Dumont, and X. Garbet. Chaotic motion of charged particles in toroidal magnetic configurations. *Chaos*, 24(3):033101, 2014.

[22] D. J. Campbell, D. F. H. Start, J. A. Wesson, D. V. Bartlett, V. P. Bhatnagar, M. Bures, J. G. Cordey, G. A. Cottrell, P. A. Dupperex, A. W. Edwards, C. D. Challis, C. Gormezano, C. W. Gowers, R. S. Granetz, J. H. Hammen, T. Hellsten, J. Jacquinot, E. Lazzaro, P. J. Lomas, N. Lopes Cardozo, P. Mantica, J. A. Snipes, D. Stork, P. E. Stott, P. R. Thomas, E. Thompson, K. Thomsen, and G. Tonetti. Stabilization of sawteeth with additional heating in the jet tokamak. *Phys. Rev. Lett.*, 60:2148–2151, May 1988.

[23] F. J. Casson, C. Angioni, E. A. Belli, R. Bilato, P. Mantica, T. Odstrcil, T. Pütterich, M. Valisa, L. Garzotti, C. Giroud, J. Hobirk, C. F. Maggi, J. Mlynar, and M. L. Reinke. Theoretical description of heavy impurity transport and its application to the modelling of tungsten in jet and asdex upgrade. *Plasma Physics and Controlled Fusion*, 57(1):014031, 2015.

[24] R. Cesario, A. Cardinali, C. Castaldo, M. Leigheb, M. Marinucci, V. Pericoli-Ridolfini, F. Zonca, G. Apruzzese, M. Borra, R. De Angelis, E. Giovannozzi, L. Gabellieri, H. Kroegler, G. Mazzitelli, P. Micozzi, L. Panaccione, P. Papitto, S. Podda, G. Ravera, B. Angelini, M. L. Apicella, E. Barbato, L. Bertalot, A. Bertocchi, G. Buceti, S. Cascino, C. Centioli, P. Chuilon, S. Ciattaglia, V. Cocilovo, F. Crisanti, F. De Marco, B. Esposito, G. Gatti, C. Gormezano, M. Grolli, F. Iannone, G. Maddaluno, G. Monari, P. Orsitto, D. Pacella, M. Panella, L. Pieroni, G. B. Righetti, F. Romanelli, E. Sternini, N. Tartoni, P. Trevisanutto, A. A. Tuccillo, O. Tudisco, V. Vitale, G. Vlad, and M. Zerbini. Reduction of the electron

thermal conductivity produced by ion bernstein waves on the frascati tokamak upgrade tokamak. *Physics of Plasmas*, 8(11):4721–4724, 2001.

[25] I. T. Chapman, S. D. Pinches, J. P. Graves, R. J. Akers, L. C. Appel, R. V. Budny, S. Coda, N. J. Conway, M. de Bock, L.-G. Eriksson, R. J. Hastie, T. C. Hender, G. T. A. Huysmans, T. Johnson, H. R. Koslowski, A. Krämer-Flecken, M. Lennholm, Y. Liang, S. Saarelma, S. E. Sharapov, I. Voitsekhovitch, the MAST, TEXTOR Teams, and JET EFDA Contributors. The physics of sawtooth stabilization. *Plasma Physics and Controlled Fusion*, 49(12B):B385, 2007.

[26] B. Chouli, C. Fenzi, X. Garbet, C. Bourdelle, J. Decker, T. Aniel, J.-F. Artaud, V. Basiuk, F. Clairet, G. Colledani, R. Dumont, D. Elbeze, C. Gil, P. Lotte, Y. Sarazin, and the Tore Supra Team. Co- and counter-current rotation in tore supra lower hybrid current drive plasmas. *Plasma Physics and Controlled Fusion*, 56(9):095018, 2014.

[27] J. Citrin, F. Jenko, P. Mantica, D. Told, C. Bourdelle, R. Dumont, J. Garcia, J. W. Haverkort, G. M. D. Hogeweij, T. Johnson, M. J. Pueschel, and JET-EFDA contributors. Ion temperature profile stiffness: non-linear gyrokinetic simulations and comparison with experiment. *Nuclear Fusion*, 54(2):023008, 2014.

[28] D. S. Clark and N. J. Fisch. The possibility of high amplitude driven contained modes during ion bernstein wave experiments in the tokamak fusion test reactor. *Physics of Plasmas*, 7(7):2923–2932, 2000.

[29] G. D. Conway, C. Angioni, F. Ryter, P. Sauter, and J. Vicente. Mean and oscillating plasma flows and turbulence interactions across the *l-h* confinement transition. *Phys. Rev. Lett.*, 106:065001, Feb 2011.

[30] Y. Corre, M. Lipa, G. Agarici, V. Basiuk, L. Colas, X. Courtois, G. Dunand, R. Dumont, A. Ekedahl, J.-L. Gardarein, C.C. Klepper, V. Martin, V. Moncada, C. Portafaix, F. Rigollet, R. Tawizgant, J.-M. Travère, and K. Vulliez. Heat flux calculation and problem of flaking of boron carbide coatings on the faraday screen of the icrh antennas during tore supra high power, long pulse operation. *Fusion Engineering and Design*, 86(45):429 – 441, 2011.

[31] G. G. Craddock and P. H. Diamond. Theory of shear suppression of edge turbulence by externally driven radio-frequency waves. *Phys. Rev. Lett.*, 67:1535–1538, Sep 1991.

[32] C. Darbos, R. Magne, S. Alberti, A. Barbuti, G. Berger-By, F. Bouquey, P. Cara, J. Clary, L. Courtois, R. Dumont, E. Giguet, D. Gil, G. Giruzzi, M. Jung, Y. Le Goff, F. Legrand, M. Lennholm, C. Liévin, Y. Peysson, D. Roux, M. Thumm, T. Wagner, M. Q. Tran, and X. Zou. The 118 ghz ecrh experiment on tore supra. *Fusion Engineering and Design*, 5657:605 – 609, 2001.

[33] L. Delpech, J. Achard, A. Armitano, J.-F. Artaud, Y. S. Bae, J. H. Belo, G. Berger-By, F. Bouquey, M. H. Cho, E. Corbel, J. Decker, H. Do, R. Dumont, A. Ekedahl,

P. Garibaldi, M. Goniche, D. Guilhem, J. Hillairet, G. T. Hoang, H. S. Kim, J. H. Kim, H. Kim, J. G. Kwak, R. Magne, P. Mollard, Y. S. Na, W. Namkung, Y. K. Oh, S. Park, H. Park, Y. Peysson, S. Poli, M. Prou, F. Samaille, H. L. Yang, and The Tore Supra Team. Advances in multi-megawatt lower hybrid technology in support of steady-state tokamak operation. *Nuclear Fusion*, 54(10):103004, 2014.

[34] R. O. Dendy, R. J. Hastie, K. G. McClements, and T. J. Martins. A model for ideal $m = 1$ internal kink stabilization by minority ion cyclotron resonant heating. *Phys. Plasmas*, 2(5):1623, 1995.

[35] Ph. Dennery and A. Krzywicki. *Mathematics for physicists.* Dover Publications, New York, NY, USA, 1996.

[36] P. H. Diamond, S.-I. Itoh, K. Itoh, and T. S. Hahm. Zonal flows in plasmaa review. *Plasma Physics and Controlled Fusion*, 47(5):R35, 2005.

[37] R. Dumont. Waves in plasmas. Master's lecture, 2016.

[38] R. Dumont and G. Giruzzi. Hot plasma effects on the polarization of electron cyclotron waves. *Physics of Plasmas*, 6(3):660–665, 1999.

[39] R. Dumont, G. Giruzzi, and E. Barbato. Combined kinetic and transport modeling of radiofrequency current drive. *Physics of Plasmas*, 7(12):4972–4982, 2000.

[40] R. Dumont, T. Mathurin, M. Schneider, L.-G. Eriksson, T. Johnson, and G. Steinbrecher. Advanced simulation of energetic ion populations in the presence of nbi and rf sources. In *Proceedings of the 41st EPS conference on Plasma Physics, Berlin (2014)*, page P2.038, 2015.

[41] R. J. Dumont. Variational approach to radiofrequency waves in magnetic fusion devices. *Nucl. Fusion*, 49:075033, 2009.

[42] R. J. Dumont and G. Giruzzi. Theory of synergy between electron cyclotron and lower hybrid waves. *Physics of Plasmas*, 11(7):3449, 2004.

[43] R. J. Dumont and G. Giruzzi. Synergy in rf current drive. *AIP Conference Proceedings*, 787(1):257–264, 2005.

[44] R. J. Dumont, M. Goniche, A. Ekedahl, B. Saoutic, J.-F. Artaud, V. Basiuk, C. Bourdelle, Y. Corre, J. Decker, D. Elbèze, G. Giruzzi, G.-T. Hoang, F. Imbeaux, E. Joffrin, X. Litaudon, Ph. Lotte, P. Maget, D. Mazon, E. Nilsson, and The Tore Supra Team. Multi-megawatt, gigajoule plasma operation in tore supra. *Plasma Physics and Controlled Fusion*, 56(7):075020, 2014.

[45] R. J. Dumont, C. K. Phillips, and D. N. Smithe. Effects of non-maxwellian species on ion cyclotron waves propagation and absorption in magnetically confined plasmas. *Physics of Plasmas*, 12(4):042508, 2005.

[46] R. J. Dumont and D. Zarzoso. Heating and current drive by ion cyclotron waves in the activated phase of iter. *Nucl. Fusion*, 53(1):013002, 2013.

[47] R. J. Dumont, D. Zarzoso, Y. Sarazin, X. Garbet, A. Strugarek, J. Abiteboul, T. Cartier-Michaud, G. Dif-Pradalier, Ph. Ghendrih, J.-B. Girardo, V. Grandgirard, G. Latu, C. Passeron, and O. Thomine. Interplay between fast ions and turbulence in magnetic fusion plasmas. *Plasma Physics and Controlled Fusion*, 55(12):124012, 2013.

[48] D. Edery, X. Garbet, J.-P. Roubin, and A. Samain. Variational formalism for kinetic-mhd instabilities in tokamaks. *Plasma Physics and Controlled Fusion*, 34(6):1089, 1992.

[49] A. Ekedahl, J. Bucalossi, V. Basiuk, S. Brémond, L. Colas, Y. Corre, E. Delchambre, D. Douai, R. Dumont, G. Dunand, G. Giruzzi, M. Goniche, S. Hong, F. Imbeaux, F. Kazarian, G. Lombard, L. Manenc, O. Meyer, L. Millon, R. Mitteau, P. Monier-Garbet, P. Moreau, B. Pégourié, F.G. Rimini, F. Saint-Laurent, F. Samaille, J.L. Schwob, E. Tsitrone, and the Tore Supra Team. Operational limits during high power long pulses with radiofrequency heating in tore supra. *Nuclear Fusion*, 49(9):095010, 2009.

[50] L.-G. Eriksson and P. Helander. Monte carlo operators for orbit-averaged fokker-planck equations. *Phys. Plasmas*, 1(2):308, 1994.

[51] A. Fasoli, D. Borba, C. Gormezano, R. Heeter, A. Jaun, J. Jacquinot, W. Kerner, Q. King, J. B. Lister, S. Sharapov, D. Start, and L. Villard. Alfvn eigenmode experiments in tokamaks and stellarators. *Plasma Physics and Controlled Fusion*, 39(12B):B287, 1997.

[52] N. Fedorczak, P. Monier-Garbet, T. Pütterich, S. Brezinsek, P. Devynck, R. Dumont, M. Goniche, E. Joffrin, E. Lerche, B. Lipschultz, E. de la Luna, G. Maddison, C. Maggi, G. Matthews, I. Nunes, F. Rimini, E.R. Solano, P. Tamain, M. Tsalas, and P. de Vries. Tungsten transport and sources control in {JET} iter-like wall h-mode plasmas. *Journal of Nuclear Materials*, 463:85, 2015.

[53] N. J. Fisch. Confining a tokamak plasma with rf-driven currents. *Phys. Rev. Lett.*, 41:873, 1978.

[54] N. J. Fisch. Theory of current drive in plasmas. *Rev. Mod. Physics*, 59:175, 1987.

[55] N. J. Fisch. Physics of alpha channelling and related tftr experiments. *Nuclear Fusion*, 40(6):1095, 2000.

[56] N. J. Fisch and M. C. Herrmann. Utility of extracting alpha particle energy by waves. *Nuclear Fusion*, 34(12):1541, 1994.

[57] N. J. Fisch and M. C. Herrmann. Alpha power channelling with two waves. *Nuclear Fusion*, 35(12):1753, 1995.

[58] N. J. Fisch and J.-M. Rax. Interaction of energetic alpha particles with intense lower hybrid waves. *Phys. Rev. Lett.*, 69:612–615, Jul 1992.

[59] J. P. Freidberg. *Ideal magnetohydrodynamics.* Modern Perspectives in Energy Series. Plenum Publishing Company Limited, 1987.

[60] G. Y. Fu. Energetic-particle-induced geodesic acoustic mode. *Phys. Rev. Lett.*, 101:185002, Oct 2008.

[61] D. J. Gambier and A. Samain. Variational theory of ion cyclotron resonance heating in tokamak plasmas. *Nucl. Fusion*, 25(3):283, 1985.

[62] X. Garbet, L. Laurent, F. Mourgues, J.P. Roubin, and A. Samain. Variational calculation of electromagnetic instabilities in tokamaks. *Journal of Computational Physics*, 87(2):249 – 269, 1990.

[63] J.-B. Girardo, S. Sharapov, J. Boom, R. Dumont, J. Eriksson, M. Fitzgerald, X. Garbet, N. Hawkes, V. Kiptily, I. Lupelli, M. Mantsinen, Y. Sarazin, M. Schneider, and JET Contributors. Stabilization of sawteeth with third harmonic deuterium icrf-accelerated beam in jet plasmas. *Physics of Plasmas*, 23(1):012505, 2016.

[64] J.-B. Girardo, D. Zarzoso, R. Dumont, X. Garbet, Y. Sarazin, and S. Sharapov. Relation between energetic and standard geodesic acoustic modes. *Physics of Plasmas*, 21(9):092507, 2014.

[65] G. Giruzzi, R. Abgrall, L. Allegretti, J.M. Ané, P. Angelino, T. Aniel, A. Argouarch, J.F. Artaud, S. Balme, V. Basiuk, P. Bayetti, A. Bécoulet, M. Bécoulet, L. Begrambekov, M.S. Benkadda, F. Benoit, G. Berger-by, B. Bertrand, P. Beyer, J. Blum, D. Boilson, H. Bottollier-Curtet, C. Bouchand, F. Bouquey, C. Bourdelle, F. Brémond, S. Brémond, C. Brosset, J. Bucalossi, Y. Buravand, P. Cara, S. Carpentier, A. Casati, O. Chaibi, M. Chantant, P. Chappuis, M. Chatelier, G. Chevet, D. Ciazynski, G. Ciraolo, F. Clairet, J. Clary, L. Colas, Y. Corre, X. Courtois, N. Crouseilles, G. Darmet, M. Davi, R. Daviot, H. De Esch, J. Decker, P. Decool, E. Delchambre, E. Delmas, L. Delpech, C. Desgranges, P. Devynck, L. Doceul, N. Dolgetta, D. Douai, H. Dougnac, J.L. Duchateau, R. Dumont, et al. Investigation of steady-state tokamak issues by long pulse experiments on tore supra. *Nuclear Fusion*, 49(10):104010, 2009.

[66] G. Giruzzi, J.-F. Artaud, R. J. Dumont, F. Imbeaux, P. Bibet, G. Berger-By, F. Bouquey, J. Clary, C. Darbos, A. Ekedahl G.-T. Hoang, M. Lennholm, P. Maget, R. Magne, J.-L. Ségui, A. Bruschi, and G. Granucci. Synergy of electron-cyclotron and lower-hybrid current drive in steady-state plasmas. *Phys. Rev. Lett.*, 93:255002, Dec 2004.

[67] H. Goldstein, C. Poole, and J. Safko. *Classical Mechanics.* Addison Wesley, Reading, MA, USA, 2002.

[68] M. Goniche, R. Dumont, C. Bourdelle, J. Decker, L. Delpech, A. Ekedahl, D. Guilhem, Z. Guimar aes Filho, X. Litaudon, Ph. Lotte, P. Maget, D. Mazon, B. Saoutic, and Tore Supra Team. Advances in long pulse operation at high radio frequency power in tore supra. *Physics of Plasmas*, 21(6):061515, 2014.

[69] N. N. Gorelenkov, N. J. Fisch, and E. Fredrickson. On the anomalous fast ion energy diffusion in toroidal plasmas due to cavity modes. *Plasma Physics and Controlled Fusion*, 52(5):055014, 2010.

[70] I. S. Gradshteyn and I. M. Ryzhik. *Table of Integrals, Series and Products.* Academic Press, San Diego, CA, USA, 1994.

[71] J. P. Graves, I. Chapman, S. Coda, L.-G. Eriksson, and T. Johnson. Sawtooth-control mechanism using toroidally propagating ion-cyclotron-resonance waves in tokamaks. *Phys. Rev. Lett.*, 102:065005, Feb 2009.

[72] J. P. Graves, I. T. Chapman, S. Coda, M. Lennholm, M. Albergante, and M. Jucker. Control of magnetohydrodynamic stability by phase space engineering of energetic ions in tokamak plasmas. *Nat. Commun.*, 3:624, Jan 2012.

[73] J. P. Graves, K. I. Hopcraft, R. O. Dendy, R. J. Hastie, K. G. McClements, and M. Mantsinen. Sawtooth evolution during jet ion-cyclotron-resonance-heated pulses. *Phys. Rev. Lett.*, 84:1204–1207, Feb 2000.

[74] J. P. Graves, M. Lennholm, I. T. Chapman, E. Lerche, M. Reich, B. Alper, V. Bobkov, R. Dumont, J. M. Faustin, P. Jacquet, F. Jaulmes, T. Johnson, D. L. Keeling, Yueqiang Liu, T. Nicolas, S. Tholerus, T. Blackman, I. S. Carvalho, R. Coelho, D. Van Eester, R. Felton, M. Goniche, V. Kiptily, I. Monakhov, M. F. F. Nave, C. Perez von Thun, R. Sabot, C. Sozzi, and M. Tsalas. Sawtooth control in jet with iter relevant low field side resonance ion cyclotron resonance heating and iter-like wall. *Plasma Physics and Controlled Fusion*, 57(1):014033, 2015.

[75] T. S. Hahm. Rotation shear induced fluctuation decorrelation in a toroidal plasma. *Physics of Plasmas*, 1(9):2940, 1994.

[76] J. Hedin, T. Hellsten, L.-G. Eriksson, and T. Johnson. The influence of finite drift orbit width on icrf heating in toroidal plasmas. *Nuclear Fusion*, 42(5):527, 2002.

[77] W. W. Heidbrink, E. J. Strait, E. Doyle, G. Sager, and R. T. Snider. An investigation of beam driven alfvn instabilities in the diii-d tokamak. *Nuclear Fusion*, 31(9):1635, 1991.

[78] P. Helander and D. J. Sigmar. *Collisional Transport in Magnetized Plasmas.* Cambridge Monographs on Plasma Physics. Cambridge University Press, 2005.

[79] T. Hellsten. Momentum transport by waveparticle interaction. *Plasma Physics and Controlled Fusion*, 53(5):054007, 2011.

[80] T. Hellsten, T. Johnson, J. Carlsson, L.-G. Eriksson, J. Hedin, M. Laxback, and M. Mantsinen. Effects of finite drift orbit width and rf-induced spatial transport on plasma heated by icrh. *Nuclear Fusion*, 44(8):892, 2004.

[81] T. C. Hender, P. Buratti, F. J. Casson, B. Alper, Y. Baranov, M. Baruzzo, C. D. Challis, F. Koechl, C. Marchetto, M. F. F. Nave, T. Pütterich, S. Reyes Cortes,

and J. Contributors. The role of MHD in causing impurity peaking in JET Hybrid plasmas. *ArXiv e-prints*, October 2015.

[82] M. C. Herrmann and N. J. Fisch. Cooling energetic α particles in a tokamak with waves. *Phys. Rev. Lett.*, 79:1495–1498, Aug 1997.

[83] J. Hillairet, A. Ekedahl, M. Goniche, Y.S. Bae, J. Achard, A. Armitano, B. Beckett, J. Belo, G. Berger-By, J.M. Bernard, E. Corbel, L. Delpech, J. Decker, R. Dumont, D. Guilhem, G.T. Hoang, F. Kazarian, H.J. Kim, X. Litaudon, R. Magne, L. Marfisi, P. Mollard, W. Namkung, E. Nilsson, S. Park, Y. Peysson, M. Preynas, P.K. Sharma, M. Prou, and the Tore Supra Team. Recent progress on lower hybrid current drive and implications for iter. *Nuclear Fusion*, 53(7):073004, 2013.

[84] D. Van Houtte, G. Martin, A. Bécoulet, J. Bucalossi, G. Giruzzi, G. T. Hoang, Th. Loarer, and B. Saoutic (on behalf of the Tore Supra Team). Recent fully non-inductive operation results in tore supra with 6min, 1gj plasma discharges. *Nucl. Fusion*, 44:L11, 2004.

[85] F. Imbeaux, M. Lennholm, A. Ekedahl, P. Pastor, T. Aniel, S. Brémond, J. Decker, P. Devynck, R. Dumont, G. Giruzzi, P. Maget, D. Mazon, A. Merle, D. Molina, P. Moreau, F. Saint-Laurent, J.L. Ségui, D. Zarzoso, and Tore Supra Team. Real-time control of the safety factor profile diagnosed by magneto-hydrodynamic activity on the tore supra tokamak. *Nuclear Fusion*, 51(7):073033, 2011.

[86] J. D. Jackson. *Classical Electrodymanics*. Wiley, New York, NY, USA, 1975. 2nd edition.

[87] E. F. Jaeger, L. A. Berry, S. D. Ahern, R. F. Barrett, D. B. Batchelor, M. D. Carter, E. F. D'Azevedo, R. D. Moore, R. W. Harvey, J. R. Myra, D. A. DIppolito, R. J. Dumont, C. K. Phillips, H. Okuda, D. N. Smithe, P. T. Bonoli, J. C. Wright, and M. Choi. Self-consistent full-wave and fokker-planck calculations for ion cyclotron heating in non-maxwellian plasmas. *Phys. Plasmas*, 13:056101, 2006.

[88] E. F. Jaeger, L. A. Berry, and D. B. Batchelor. Second-order radio frequency kinetic theory with applications to flow drive and heating in tokamak plasmas. *Physics of Plasmas*, 7(2):641–656, 2000.

[89] E. F. Jaeger, L. A. Berry, E. D'Azevedo, D. B. Batchelor, and M. D. Carter. All-orders spectral calculation of radio-frequency heating in two-dimensional toroidal plasmas. *Phys. Plasmas*, 8(5):1573, 2001.

[90] E. F. Jaeger, L. A. Berry, J. R. Myra, D. B. Batchelor, E. D'Azevedo, P. T. Bonoli, C. K. Phillips, D. N. Smithe, D. A. D'Ippolito, M. D. Carter, R. J. Dumont, J. C. Wright, and R. W. Harvey. Sheared poloidal flow driven by mode conversion in tokamak plasmas. *Phys. Rev. Lett.*, 90:195001, May 2003.

[91] E. F. Jaeger, R. W. Harvey, L. A. Berry, J. R. Myra, R. J. Dumont, C. K. Phillips, D. N. Smithe, R. F. Barrett, D. B. Batchelor, P. T. Bonoli, M. D. Carter, E. F.

D'azevedo, D. A. D'ippolito, R. D. Moore, and J. C. Wright. Global-wave solutions with self-consistent velocity distributions in ion cyclotron heated plasmas. *Nuclear Fusion*, 46(7):S397, 2006.

[92] E. Joffrin, O. Barana, D. Mazon, P. Moreau, F. Turco, J.F. Artaud, V. Basiuk, C. Bourdelle, S. Brémond, J. Bucalossi, F. Clairet, L. Colas, Y. Corre, R. Dumont, A. Ekedahl, G. Giruzzi, M. Goniche, F. Imbeaux, F. Kazarian, L. Laborde, P. Monier-Garbet, P. Maget, B. Pégourié, Y. Peysson, F. Rimini, F. Saint-Laurent, and E. Tsitrone. Integrated plasma controls for steady state scenarios. *Nuclear Fusion*, 47(12):1664, 2007.

[93] C. F. F. Karney. Fokker-planck and quasilinear codes. *Comp. Phys. Rep.*, 4:183, 1986.

[94] A. N. Kaufman. Quasilinear diffusion of an axisymmetric toroidal plasma. *Phys. Fluids*, 15(6):1063, 1972.

[95] Ye. O. Kazakov, D. Van Eester, R. Dumont, and J. Ongena. On resonant icrf absorption in three-ion component plasmas: a new promising tool for fast ion generation. *Nuclear Fusion*, 55(3):032001, 2015.

[96] Ye. O. Kazakov, J. Ongena, D. Van Eester, R. Bilato, R. Dumont, E. Lerche, M. Mantsinen, and A. Messiaen. A new ion cyclotron range of frequency scenario for bulk ion heating in deuterium-tritium plasmas: How to utilize intrinsic impurities in our favour. *Physics of Plasmas*, 22(8):082511, 2015.

[97] C. F. Kennel and F. Engelmann. Velocity space diffusion from weak plasma turbulence in a magnetic field. *Phys. Fluids*, 9:2377, 1966.

[98] P. U. Lamalle. On the radiofrequency response of a tokamak plasma. *Plasma Phys. Control. Fusion*, 39:1409, 1997.

[99] L. D. Landau. Kinetic equation for the coulomb effect. *Phys. Z. Sowjetunion*, 10:154, 1936.

[100] Ph. Lauber. Private communication, 2016.

[101] B. LeBlanc, S. Batha, R. Bell, S. Bernabei, L. Blush, E. de la Luna, R. Doerner, J. Dunlap, A. England, I. Garcia, D. Ignat, R. Isler, S. Jones, R. Kaita, S. Kaye, H. Kugel, F. Levinton, S. Luckhardt, T. Mutoh, M. Okabayashi, M. Ono, F. Paoletti, S. Paul, G. Petravich, A. PostZwicker, N. Sauthoff, L. Schmitz, S. Sesnic, H. Takahashi, M. Talvard, W. Tighe, G. Tynan, S. von Goeler, P. Woskov, and A. Zolfaghari. Active core profile and transport modification by application of ion bernstein wave power in the princeton beta experimentmodification. *Physics of Plasmas*, 2(3):741–751, 1995.

[102] B. P. LeBlanc, R. E. Bell, S. Bernabei, J. C. Hosea, R. Majeski, M. Ono, C. K. Phillips, J. H. Rogers, G. Schilling, C. H. Skinner, and J. R. Wilson. Direct observation of ion-bernstein-wave-induced poloidal flow in tftr. *Phys. Rev. Lett.*, 82:331–334, Jan 1999.

[103] M. Lennholm, T. Blackman, I.T. Chapman, L.-G. Eriksson, J.P. Graves, D.F. Howell, M. de Baar, G. Calabro, R. Dumont, M. Graham, S. Jachmich, M.L. Mayoral, C. Sozzi, M. Stamp, M. Tsalas, P. de Vries, and JET EFDA Contributors. Feedback control of the sawtooth period through real time control of the ion cyclotron resonance frequency. *Nuclear Fusion*, 51(7):073032, 2011.

[104] M. Lennholm, L-G. Eriksson, F. Turco, F. Bouquey, C. Darbos, R. Dumont, G. Giruzzi, M. Jung, R. Lambert, R. Magne, D. Molina, P. Moreau, F. Rimini, J-L. Segui, S. Song, and E. Traisnel. Closed loop sawtooth period control using variable eccd injection angles on tore supra. *Fusion Science and Technology*, 55(1):45, 2009.

[105] M. Lennholm, L.-G. Eriksson, F. Turco, F. Bouquey, C. Darbos, R. Dumont, G. Giruzzi, M. Jung, R. Lambert, R. Magne, D. Molina, P. Moreau, F. Rimini, J.-L. Segui, S. Song, and E. Traisnel. Demonstration of effective control of fast-ion-stabilized sawteeth by electron-cyclotron current drive. *Phys. Rev. Lett.*, 102:115004, 2009.

[106] M. Lennholm, D. Frigione, J. P. Graves, P. S. Beaumont, T. Blackman, I. S. Carvalho, I. Chapman, R. Dumont, R. Felton, L. Garzotti, M. Goniche, A. Goodyear, D. Grist, S. Jachmich, T. Johnson, P. Lang, E. Lerche, E. de la Luna, I. Monakhov, R. Mooney, J. Morris, M. F. F. Nave, M. Reich, F. Rimini, G. Sips, H. Sheikh, C. Sozzi, M. Tsalas, and JET Contributors. Real-time control of elm and sawtooth frequencies: similarities and differences. *Nuclear Fusion*, 56(1):016008, 2016.

[107] E. Lerche, D. Van Eester, J. Ongena, M.-L. Mayoral, M. Laxaback, F. Rimini, A. Argouarch, P. Beaumont, T. Blackman, V. Bobkov, D. Brennan, A. Brett, G. Calabro, M. Cecconello, I. Coffey, L. Colas, A. Coyne, K. Crombe, A. Czarnecka, R. Dumont, F. Durodie, R. Felton, D. Frigione, M. Gatu Johnson, C. Giroud, G. Gorini, M. Graham, C. Hellesen, T. Hellsten, S. Huygen, P. Jacquet, T. Johnson, V. Kiptily, S. Knipe, A. Krasilnikov, P. Lamalle, M. Lennholm, A. Loarte, R. Maggiora, M. Maslov, A. Messiaen, D. Milanesio, I. Monakhov, M. Nightingale, C. Noble, M. Nocente, L. Pangioni, I. Proverbio, C. Sozzi, M. Stamp, W. Studholme, M. Tardocchi, T. W. Versloot, V. Vdovin, M. Vrancken, A. Whitehurst, E. Wooldridge, V. Zoita, and JET EFDA Contributors. Optimizing ion-cyclotron resonance frequency heating for iter: dedicated jet experiments. *Plasma Physics and Controlled Fusion*, 54(6):069601, 2012.

[108] A. J. Lichtenberg and M. A. Lieberman. *Regular and chaotic dynamics*. Applied mathematical sciences. Springer, New York, Berlin, Heidelberg, 1992.

[109] Y. Lin, P. Mantica, T. Hellsten, V. Kiptily, E. Lerche, M. F. F. Nave, J. E. Rice, D. Van Eester, P. C. de Vries, R. Felton, C. Giroud, T. Tala, and JET EFDA Contributors. Ion cyclotron range of frequency mode conversion flow drive in d(3 he) plasmas on jet. *Plasma Physics and Controlled Fusion*, 54(7):074001, 2012.

[110] Y. Lin, J. E. Rice, S. J. Wukitch, M. J. Greenwald, A. E. Hubbard, A. Ince-Cushman, L. Lin, M. Porkolab, M. L. Reinke, and N. Tsujii. Observation of ion-

cyclotron-frequency mode-conversion flow drive in tokamak plasmas. *Phys. Rev. Lett.*, 101:235002, Dec 2008.

[111] Y. Lin, J. E. Rice, S. J. Wukitch, M. L. Reinke, M. J. Greenwald, A. E. Hubbard, E. S. Marmar, Y. Podpaly, M. Porkolab, N. Tsujii, and the Alcator C-Mod team. Icrf mode conversion flow drive on alcator c-mod. *Nuclear Fusion*, 51(6):063002, 2011.

[112] X. Litaudon, J.-M. Bernard, L. Colas, R. Dumont, A. Argouarch, H. Bottollier-Curtet, S. Brémond, S .Champeaux, Y. Corre, P. Dumortier, M. Firdaouss, D. Guilhem, J.P. Gunn, Ph. Gouard, G.T. Hoang, J. Jacquot, C.C. Klepper, M. Kubič, V. Kyrytsya, G. Lombard, D. Milanesio, A. Messiaen, P. Mollard, O. Meyer, and D. Zarzoso. Physics and technology in the ion-cyclotron range of frequency on tore supra and titan test facility: implication for iter. *Nuclear Fusion*, 53(8):083012, 2013.

[113] R. G. Littlejohn. Hamiltonian formulation of guiding center motion. *Phys. Fluids*, 24(9):1730, 1981.

[114] A. Lyssoivan, R. Koch, D. Douai, J.-M. Noterdaeme, V. Philipps, V. Rohde, F.C. Schller, G. Sergienko, D. Van Eester, T. Wauters, T. Blackman, V. Bobkov, S. Brémond, S. Brezinsek, E. de la Cal, R. Dumont, M. Garcia-Munoz, E. Gauthier, M. Graham, S. Jachmich, E. Joffrin, A. Kreter, P.U. Lamalle, E. Lerche, G. Lombard, F. Louche, M. Maslov, M.-L. Mayoral, V.E. Moiseenko, P. Mollard, I. Monakhov, J. Ongena, M.K. Paul, R.A. Pitts, V. Plyusnin, W. Suttrop, E. Tsitrone, M. Van Schoor, G. Van Wassenhove, and M. Vervier. Icrf physics aspects of wall conditioning with conventional antennas in large-size tokamaks. *Journal of Nuclear Materials*, 415(1, Supplement):S1029 – S1032, 2011. Proceedings of the 19th International Conference on Plasma-Surface Interactions in Controlled Fusion.

[115] V. Basiuk M. Schneider, L.-G. Eriksson and F. Imbeaux. On alpha particle effects in tokamaks with a current hole. *Plasma Phys. Control. Fusion*, 47:2087, 2005.

[116] B. D. McVey, R. S. Sund, and J. E. Scharer. Local power conservation for linear wave propagation in an inhomogeneous plasma. *Phys. Rev. Lett.*, 55:507, 1985.

[117] A. B. Mikhailovskii. Generalized mhd for numerical stability analysis of high-performance plasmas in tokamaks. *Plasma Phys. Control. Fusion*, 40:1907, 1998.

[118] J. R. Myra, L. A. Berry, D. A. D'Ippolito, and E. F. Jaeger. Nonlinear fluxes and forces from radio-frequency waves with application to driven flows in tokamaks. *Phys. Plasmas*, 11:1786, 2004.

[119] J. R. Myra, D. A. D'Ippolito, D. A. Russell, L. A. Berry, E. F. Jaeger, and M. D. Carter. Nonlinear icrf-plasma interactions. *Nuclear Fusion*, 46(7):S455, 2006.

[120] J. R. Myra and D. A. D'lppolito. Poloidal force generation by applied radio frequency waves. *Physics of Plasmas*, 7(9):3600–3609, 2000.

[121] R. Nazikian, G. Y. Fu, M. E. Austin, H. L. Berk, R. V. Budny, N. N. Gorelenkov, W. W. Heidbrink, C. T. Holcomb, G. J. Kramer, G. R. McKee, M. A. Makowski, W. M. Solomon, M. Shafer, E. J. Strait, and M. A. Van Zeeland. Intense geodesic acousticlike modes driven by suprathermal ions in a tokamak plasma. *Phys. Rev. Lett.*, 101:185001, Oct 2008.

[122] E. Nelson-Melby, M. Porkolab, P. T. Bonoli, Y. Lin, A. Mazurenko, and S. J. Wukitch. Experimental observations of mode-converted ion cyclotron waves in a tokamak plasma by phase contrast imaging. *Phys. Rev. Lett.*, 90:155004, Apr 2003.

[123] R. Neu, R. Dux, A. Geier, A. Kallenbach, R. Pugno, V. Rohde, D. Bolshukhin, J. C. Fuchs, O. Gehre, O. Gruber, J. Hobirk, M. Kaufmann, K. Krieger, M. Laux, C. Maggi, H. Murmann, J. Neuhauser, F. Ryter, A. C. C. Sips, A. Stäbler, J. Stober, W. Suttrop, H. Zohm, and the ASDEX Upgrade Team. Impurity behaviour in the asdex upgrade divertor tokamak with large area tungsten walls. *Plasma Physics and Controlled Fusion*, 44(6):811, 2002.

[124] E. Nilsson, J. Decker, Y. Peysson, R. S. Granetz, F. Saint-Laurent, and M. Vlainic. Kinetic modelling of runaway electron avalanches in tokamak plasmas. *Plasma Physics and Controlled Fusion*, 57(9):095006, 2015.

[125] H. Nordman, R. Singh, T. Flp, L.-G. Eriksson, R. Dumont, J. Anderson, P. Kaw, P. Strand, M. Tokar, and J. Weiland. Influence of the radio frequency ponderomotive force on anomalous impurity transport in tokamaks. *Physics of Plasmas*, 15(4):042316, 2008.

[126] B. Pégourié, V. Waller, R. J. Dumont, L.-G. Eriksson, L. Garzotti, A. Géraud, and F. Imbeaux. Modelling of pellet ablation in additionally heated plasmas. *Plasma Physics and Controlled Fusion*, 47(1):17, 2005.

[127] F. W. Perkins. Heating tokamaks via the ion-cyclotron and ion-ion hybrid resonances. *Nuclear Fusion*, 17(6):1197, 1977.

[128] C. K. Phillips, M. G. Bell, R. E. Bell, S. Bernabei, M. Bettenhausen, C. E. Bush, D. Clark, D. S. Darrow, E. D. Fredrickson, G. R. Hanson, J. C. Hosea, B. P. LeBlanc, R. P. Majeski, S. S. Medley, R. Nazikian, M. Ono, H. K. Park, M. P. Petrov, J. H. Rogers, G. Schilling, C. H. Skinner, D. N. Smithe, E. J. Synakowski, G. Taylor, and J. R. Wilson. Icrf heating and profile control techniques in tftr. *Nuclear Fusion*, 40(3Y):461, 2000.

[129] C. K. Phillips, J. Hosea, E. Marmar, M. W. Phillips, J. Snipes, J. Stevens, J. Terry, J. R. Wilson, M. Bell, M. Bitter, R. Boivin, C. Bush, C. Z. Cheng, D. Darrow, E. Fredrickson, R. Goldfinger, G. W. Hammett, K. Hill, D. Hoffman, W. Houlberg, H. Hsuan, M. Hughes, D. Jassby, D. McCune, K. McGuire, Y. Nagayama, D. K. Owens, H. Park, A. Ramsey, G. Schilling, J. Schivell, D. N. Smithe, B. Stratton, E. Synakowski, G. Taylor, H. Towner, R. White, S. Zweben, and the TFTR Group. Ion cyclotron range of frequencies stabilization of sawteeth on tokamak fusion test reactor. *Physics of Fluids B*, 4(7):2155–2164, 1992.

[130] T. Pütterich, R. Dux, R. Neu, M. Bernert, M. N. A. Beurskens, V. Bobkov, S. Brezinsek, C. Challis, J. W. Coenen, I. Coffey, A. Czarnecka, C. Giroud, P. Jacquet, E. Joffrin, A. Kallenbach, M. Lehnen, E. Lerche, E. de la Luna, S. Marsen, G. Matthews, M.-L. Mayoral, R. M. McDermott, A. Meigs, J. Mlynar, M. Sertoli, G. van Rooij, the ASDEX Upgrade Team, and JET EFDA Contributors. Observations on the w-transport in the core plasma of jet and asdex upgrade. *Plasma Physics and Controlled Fusion*, 55(12):124036, 2013.

[131] Z. Qiu, F. Zonca, and L. Chen. Nonlocal theory of energetic-particle-induced geodesic acoustic mode. *Plasma Physics and Controlled Fusion*, 52(9):095003, 2010.

[132] J.-M. Rax. *Physique des tokamaks.* Physique. Éd. de l'École Polytechnique, Palaiseau, 2011.

[133] J.M. Rax. *Physique des plasmas: Cours et applications.* Physique. Dunod, 2005.

[134] T. H. Rider. *Fundamental limitations on plasma fusion systems non in thermodynamical equilibrium.* PhD thesis, Massachusetts Institute of Technology., Dept. of Electrical Engineering and Computer Science, 1995.

[135] M. Saigusa, H. Kimura, S. Moriyama, Y. Neyatani, T. Fujii, Y. Koide, T. Kondoh, M. Sato, M. Nemoto, and Y. Kamada. Investigation of high-n tae modes excited by minority-ion cyclotron heating in jt-60u. *Plasma Physics and Controlled Fusion*, 37(3):295, 1995.

[136] A. Samain. Dynamic stabilization of a confined plasma. *Nuclear Fusion*, 10(3):325, 1970.

[137] B. Saoutic, J. Abiteboul, L. Allegretti, S. Allfrey, J.M. Ané, T. Aniel, A. Argouarch, J.-F. Artaud, M.-H. Aumenier, S. Balme, V. Basiuk, O. Baulaigue, P. Bayetti, A. Bécoulet, M. Bécoulet, M. S. Benkadda, F. Benoit, G. Berger-by, J.M. Bernard, B. Bertrand, P. Beyer, A. Bigand, J. Blum, D. Boilson, G. Bonhomme, H. Bottollier-Curtet, C. Bouchand, F. Bouquey, C. Bourdelle, S. Bourmaud, C. Brault, S. Brémond, C. Brosset, J. Bucalossi, Y. Buravand, P. Cara, V. Catherine-Dumont, A. Casati, M. Chantant, M. Chatelier, G. Chevet, D. Ciazynski, G. Ciraolo, F. Clairet, M. Coatanea-Gouachet, L. Colas, L. Commin, E. Corbel, Y. Corre, X. Courtois, R. Dachicourt, M. Dapena Febrer, M. Davi Joanny, R. Daviot, H. De Esch, J. Decker, P. Decool, P. Delaporte, E. Delchambre, E. Delmas, L. Delpech, C. Desgranges, P. Devynck, T. Dittmar, L. Doceul, D. Douai, H. Dougnac, J.L. Duchateau, B. Dugué, N. Dumas, R. Dumont, et al. Contribution of tore supra in preparation of iter. *Nuclear Fusion*, 51(9):094014, 2011.

[138] O. Sauter, E. Westerhof, M.-L. Mayoral, B. Alper, P. A. Belo, R. J. Buttery, A. Gondhalekar, T. Hellsten, T. C. Hender, D. F. Howell, T. Johnson, P. Lamalle, M. J. Mantsinen, F. Milani, M. F. F. Nave, F. Nguyen, A.-L. Pecquet, S. D. Pinches, S. Podda, and J. Rapp. Control of neoclassical tearing modes by sawtooth control. *Phys. Rev. Lett.*, 88:105001, Feb 2002.

[139] M. Schneider, L.-G. Eriksson, T. Johnson, R. Futtersack, J. F. Artaud, R. Dumont, B. Wolle, and ITM-TF Contributors. A rapid fast ion fokkerplanck solver for integrated modelling of tokamaks. *Nuclear Fusion*, 55(1):013003, 2015.

[140] M. Schneider, T. Johnson, R. Dumont, J. Eriksson, L.-G. Eriksson, L. Giacomelli, J.-B. Girardo, T. Hellsten, E. Khilkevitch, V. G. Kiptily, T. Koskela, M. Mantsinen, M. Nocente, M. Salewski, S. E. Sharapov, A. E. Shevelev, and JET contributors. Modelling third harmonic ion cyclotron acceleration of deuterium beams for jet fusion product studies experiments. Submitted to Nucl. Fusion.

[141] D. N. Smithe. Local full-wave energy and quasilinear analysis in nonuniform plasmas. *Plasma Physics and Controlled Fusion*, 31(7):1105, 1989.

[142] T. H. Stix. Fast wave heating of a two-component plasma. *Nucl. Fusion*, 15:737, 1975.

[143] T. H. Stix. *Waves in plasmas.* Springer-Verlag, New York, 1992.

[144] B. A. Trubnikov. *Reviews of Plasma Physics*, volume 1, page 105. Consultant Bureau, New York, m.a. leontovitch edition, 1965.

[145] F. Turco, F. Giruzzi, F. Imbeaux, V. S. Udintsev, J.-F. Artaud, O. Barana, R. Dumont, D. Mazon, and J.-L. Ségui. O-regime dynamics and modeling in tore supra. *Physics of Plasmas*, 16(6):062301, 2009.

[146] D. Van Eester and E. Lerche. Simple 1d fokkerplanck modelling of ion cyclotron resonance frequency heating at arbitrary cyclotron harmonics accounting for coulomb relaxation on non-maxwellian populations. *Plasma Phys. Control. Fusion*, 53(9):092001, 2011.

[147] J. Wesson. *Tokamaks.* Clarendon Press, Oxford, 1997.

[148] N. Winsor, J. L. Johnson, and J. M. Dawson. Geodesic acoustic waves in hydromagnetic systems. *Physics of Fluids*, 11(11):2448, 1968.

[149] K. L. Wong, R. J. Fonck, S. F. Paul, D. R. Roberts, E. D. Fredrickson, R. Nazikian, H. K. Park, M. Bell, N. L. Bretz, R. Budny, S. Cohen, G. W. Hammett, F. C. Jobes, D. M. Meade, S. S. Medley, D. Mueller, Y. Nagayama, D. K. Owens, and E. J. Synakowski. Excitation of toroidal alfvén eigenmodes in tftr. *Phys. Rev. Lett.*, 66:1874–1877, Apr 1991.

[150] J. C. Wright, L. A. Berry, P. T. Bonoli, D. B. Batchelor, E. F. Jaeger, M. D. Carter, E. D'Azevedo, C. K. Phillips, H. Okuda, R. W. Harvey, D. N. Smithe, J. R. Myra, D. A. D'Ippolito, M. Brambilla, and R. J. Dumont. Nonthermal particle and full-wave diffraction effects on heating and current drive in the icrf and lhrf regimes. *Nuclear Fusion*, 45(11):1411, 2005.

[151] D. Zarzoso, A. Biancalani, A. Bottino, Ph. Lauber, E. Poli, J.-B. Girardo, X. Garbet, and R. J. Dumont. Analytic dispersion relation of energetic particle driven geodesic acoustic modes and simulations with nemorb. *Nuclear Fusion*, 54(10):103006, 2014.

[152] D. Zarzoso, X. Garbet, Y. Sarazin, R. Dumont, and V. Grandgirard. Fully kinetic description of the linear excitation and nonlinear saturation of fast-ion-driven geodesic acoustic mode instability. *Physics of Plasmas*, 19(2):022102, 2012.

[153] D. Zarzoso, Y. Sarazin, X. Garbet, R. Dumont, A. Strugarek, J. Abiteboul, T. Cartier-Michaud, G. Dif-Pradalier, Ph. Ghendrih, V. Grandgirard, G. Latu, C. Passeron, and O. Thomine. Impact of energetic-particle-driven geodesic acoustic modes on turbulence. *Phys. Rev. Lett.*, 110:125002, 2013.

[154] W. Zhang, Z. Lin, and L. Chen. Transport of energetic particles by microturbulence in magnetized plasmas. *Phys. Rev. Lett.*, 101:095001, Aug 2008.

[155] F. Zonca, L. Chen, S. Briguglio, G. Fogaccia, A. V. Milovanov, Z. Qiu, G. Vlad, and X. Wang. Energetic particles and multi-scale dynamics in fusion plasmas. *Plasma Physics and Controlled Fusion*, 57(1):014024, 2015.

[156] X. L. Zou, G. Giruzzi, J.-F. Artaud, F. Bouquey, A. Clémençon, C. Darbos, R. J. Dumont, C. Guivarch, M. Lennholm, R. Magne, and J.L. Ségui. Electron heat transport and ecrh modulation experiments in tore supra tokamak. *Nuclear Fusion*, 43(11):1411, 2003.