Practice Questions

1. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n^3}$ converges?

(A) $-1 < x < 1$ (B) $-1 \le x \le 1$ (C) $-1 < x \le 1$ (D) $-1 \le x < 1$

2. What are all values of x for which the series $\sum_{n=0}^{\infty} \frac{n(x-2)^n}{3^n}$ converges?

(A) $-1 < x < 5$ (B) $-1 < x \le 5$ (C) $-2 \le x < 4$ (D) $-2 < x \le 4$

3. What are all values of x for which the series $\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!}$ converges?

(A) $0 < x < 2$ (B) $0 \le x < 2$ (C) $-1 < x \le 2$ (D) All real x

4. What are all values of x for which the series $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{2^n \sqrt{n}}$ converges?

(A) $-2 < x < 2$ (B) $-2 \le x < 2$ (C) $-2 < x \le 2$ (D) All real x

5. What are all values of x for which the series $\sum_{n=1}^{\infty} n!(3x-2)^n$ converges?

(A) No values of x (B) $(-\infty, \frac{2}{3}]$ (C) $x = \frac{2}{3}$ (D) $[\frac{2}{3}, \infty)$

Practice Questions

1. The power series expansion for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{1}{1+x^3}$?

(A) $1+x^2+x^4+x^6+\cdots$

(B) $1-x^3+x^6-x^9+\cdots$

(C) $1+\frac{x^3}{3}+\frac{x^6}{6}+\frac{x^9}{9}+\cdots$

(D) $1-\frac{x^3}{3}+\frac{x^6}{6}-\frac{x^9}{9}+\cdots$

2. The power series expansion for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{1}{2-x}$?

(A) $1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+\cdots$

(B) $1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{8}+\cdots$

(C) $\frac{1}{2}+\frac{x}{4}+\frac{x^2}{8}+\frac{x^3}{16}+\cdots$

(D) $\frac{1}{2}-\frac{x}{4}+\frac{x^2}{8}-\frac{x^3}{16}+\cdots$

4. A power series expansion for $f(x)=\frac{1}{1-x}$ can be obtained from the sum of the geometric series $\sum_{n=0}^{\infty} ar^n=\frac{a}{1-r}$, if you let $a=1$ and $r=x.$. Let $g(x)$ be defined as $g(x)=\frac{1}{1+x}$.

(a) Write the first four terms and the general term of the power series expansion of $g(x)$.

(b) Write the first four terms and the general term of the power series expansion of $g(x^2)$.

(c) Write the first four terms and the general term of the power series expansion of h, where $h(x)=\int g(x^2)\,dx$ and $h(0)=0$.

(d) Find the value of $h(1)$.

Practice Questions

1. Let $P(x)=\frac{1}{3}-\frac{2}{3}x+\frac{2}{3}x^2-\frac{4}{9}x^3+\frac{2}{9}x^4$ be the fourth-degree Taylor polynomial for the function f about $x=0$. What is the value of $f^{(4)}(0)$?

(A) $-\frac{32}{3}$ (B) $-\frac{4}{3}$ (C) $\frac{8}{9}$ (D) $\frac{16}{3}$

3. Let f be a function that has derivatives of all orders for all real numbers. If $f(1)=2$, $f'(1)=-3$, $f''(1)=4$, and $f'''(1)=-9$, which of the following is the third-degree Taylor polynomial for f about $x=1$?

(A) $P(x)=2-3(x-1)+2(x-1)^2-\frac{3}{2}(x-1)^3$

(B) $P(x)=2-3(x+1)+2(x+1)^2-\frac{3}{2}(x+1)^3$

(C) $P(x)=2-3(x-1)+4(x-1)^2-9(x-1)^3$

(D) $P(x)=2-3(x+1)+2(x+1)^2-3(x+1)^3$

4. The third-degree Taylor polynomial of xe^x about $x=0$ is

(A) $P_3(x)=x-\frac{1}{2}x^2+\frac{1}{6}x^3$

(B) $P_3(x)=x+x^2+\frac{1}{2}x^3$

(C) $P_3(x)=x+x^2-\frac{1}{3}x^3$

(D) $P_3(x)=1-x+\frac{1}{2}x^2-\frac{1}{6}x^3$

7. Let $P(x) = 3 - 2(x-2) + 5(x-2)^2 - 12(x-2)^3 + 3(x-2)^4$ be the fourth-degree Taylor polynomial for the function f about $x = 2$. Assume f has derivatives of all orders for all real numbers.

(a) Find $f(2)$ and $f'''(2)$.

(b) Write the third-degree Taylor polynomial for f' about 2 and use it to approximate $f'(2.1)$.

(c) Write the fourth-degree Taylor polynomial for $g(x) = \int_2^x f(t)\,dt$ about 2.

(d) Can $f(1)$ be determined from the information given? Justify your answer.

8. Let f be the function given by $f(x) = \sin(2x) + \cos(2x)$, and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$.

(a) Find $P(x)$.

(b) Find the coefficient of x^{19} in the Taylor series for f about $x = 0$.

(c) Use the Lagrange error bound to show that $\left| f(\frac{1}{5}) - P(\frac{1}{5}) \right| < \frac{1}{100}$

(d) Let h be the function given by $h(x) = \int_0^x f(t)\,dt$. Write the third-degree Taylor polynomial for h about $x = 0$.

1. A series expansion of $\dfrac{\arctan x}{x}$ is

(A) $1-\dfrac{x}{3}+\dfrac{x^3}{5}-\dfrac{x^5}{7}+\cdots$

(B) $1-\dfrac{x^2}{3}+\dfrac{x^4}{5}-\dfrac{x^6}{7}+\cdots$

(C) $1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\cdots$

(D) $x-\dfrac{x^3}{3}+\dfrac{x^4}{5}-\dfrac{x^6}{7}+\cdots$

2. The coefficient of x^3 in the Taylor series for e^{-2x} about $x=0$ is

(A) $-\dfrac{4}{3}$ (B) $-\dfrac{2}{3}$ (C) $-\dfrac{1}{3}$ (D) $\dfrac{4}{3}$

3. A function f has a Maclaurin series given by $-\dfrac{x^4}{3!}+\dfrac{x^6}{5!}-\dfrac{x^8}{7!}+\cdots+\dfrac{(-1)^n x^{2n+2}}{(2n+1)!}+\cdots$

Which of the following is an expression for $f(x)$?

(A) $x^3e^x - x^2$

(B) $x\ln x - x^2$

(C) $\tan^{-1} x - x$

(D) $x\sin x - x^2$

4. A series expansion of $\dfrac{x-\sin x}{x^2}$ is

(A) $\dfrac{1}{2!}-\dfrac{x^2}{4!}+\dfrac{x^4}{6!}+\cdots+\dfrac{(-1)^{n+1} x^{2n-2}}{(2n)!}+\cdots$

(B) $\dfrac{x}{2!}-\dfrac{x^3}{4!}+\dfrac{x^5}{6!}+\cdots+\dfrac{(-1)^{n+1} x^{2n+1}}{(2n)!}+\cdots$

(C) $\dfrac{x}{3!}-\dfrac{x^3}{5!}+\dfrac{x^5}{7!}+\cdots+\dfrac{(-1)^{n+1} x^{2n-1}}{(2n+1)!}+\cdots$

(D) $\dfrac{x^2}{3!}-\dfrac{x^4}{5!}+\dfrac{x^6}{7!}+\cdots+\dfrac{(-1)^{n+1} x^{2n}}{(2n+1)!}+\cdots$

5. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+2}}{n!}$ is the Taylor series about zero for which of the following functions?

(A) $x\sin x$ (B) $x\cos x$ (C) x^2e^{-x} (D) $x\ln(x+1)$

6. The graph of the function represented by the Maclaurin series $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots+\frac{(-1)^n x^{2n+1}}{2n+1}+\cdots$ intersects the graph of $y=e^{-x}$ at $x=$

(A) 0.495 (B) 0.607 (C) 1.372 (D) 2.166

7. What is the coefficient of x^4 in the Taylor series for $\cos^2 x$ about $x=0$?

(A) $\frac{1}{12}$ (B) $\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$

8. The fifth-degree Taylor polynomial for $\tan x$ about $x=0$ is $x+\frac{1}{3}x^3+\frac{2}{15}x^5$. If f is a function such that $f'(x)=\tan(x^2)$, then the coefficient of x^7 for $f(x)$ about $x=0$ is

(A) $\frac{1}{21}$ (B) $\frac{3}{42}$ (C) 0 (D) $\frac{1}{7}$

9. The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty}\frac{(-2)^{n-1}}{n+1}x^n=\frac{1}{2}x-\frac{2}{3}x^2+x^3-\cdots+\frac{(-2)^{n-1}}{n+1}x^n+\cdots$. Which of the following is the third-degree Taylor polynomial for $g(x)=\cos x\cdot f(x)$ about $x=0$?

(A) $x-\frac{1}{2}x^2-\frac{2}{3}x^3$

(B) $1-\frac{1}{2}x^2+\frac{2}{3}x^3$

(C) $\frac{1}{2}x-\frac{2}{3}x^2+\frac{3}{4}x^3$

(D) $\frac{1}{2}x-\frac{11}{12}x^2+x^3$

10. The Maclaurin series for the function f is given by

$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}=1-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots+\frac{(-1)^n x^{2n}}{(2n+1)!}+\cdots$ on its interval of convergence.

Which of the following statements about f must be true?

(A) f has a relative minimum at $x=0$.

(B) f has a relative maximum at $x=0$.

(C) f does not have a relative maximum or a relative minimum at $x=0$.

(D) f has a point of inflection at $x=0$.

11. Let f be the function given by $f(x) = e^{-x}$.

(a) Write the first four terms and the general term of the Taylor series for f about $x = 0$.

(b) Use the result from part (a) to write the first four nonzero terms and the general term of the series expansion about $x = 0$ for $g(x) = \dfrac{1 - x - f(x)}{x}$.

(c) For the function g in part (b), find $g'(-1)$ and use it to show that $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1$.

12. The Maclaurin series for $f(x)$ is given by $f(x) = \dfrac{x}{2!} - \dfrac{x^3}{4!} + \dfrac{x^5}{6!} + \cdots + \dfrac{(-1)^{n+1} x^{2n-1}}{(2n)!} + \cdots$.

The Maclaurin series for $g(x)$ is given by $g(x) = 1 - \dfrac{x}{2} + \dfrac{x^2}{3} - \dfrac{x^3}{4} + \cdots + \dfrac{(-1)^n x^n}{n+1} + \cdots$.

(a) Find $f'''(0)$ and $f^{(15)}(0)$.

(b) Find the interval of convergence of the Maclaurin series for $g(x)$.

(c) The graph of $y = f(x) + g(x)$ passes through the point $(0,1)$. Find $y'(0)$ and $y''(0)$ and determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.