Studies on automobile safety suggest that stopping distances follow an approximately Normal distribution. For one model of car traveling at 62 mph, the mean stopping distance is $\mu=155$ ft with a standard deviation of $\sigma=3$ ft . Danielle is driving one of these cars at 62 mph when she spots a wreck 160 feet in front of her and needs to make an emergency stop. About what percent of cars of this model when going 62 mph would be able to make an emergency stop in less than 160 feet? Is Danielle likely to stop safely?

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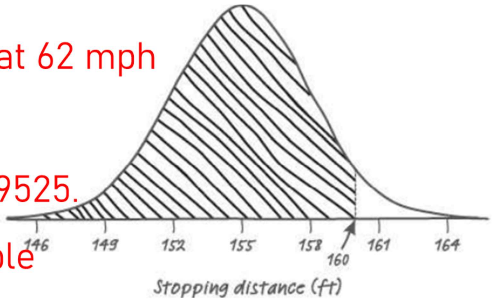
X = stopping distance when a car traveling at 62 mph

X~N(155, 9)

$P(X<160)=P(Z < (160 - 155)/3) = P(Z<1.67) = 0.9525.$



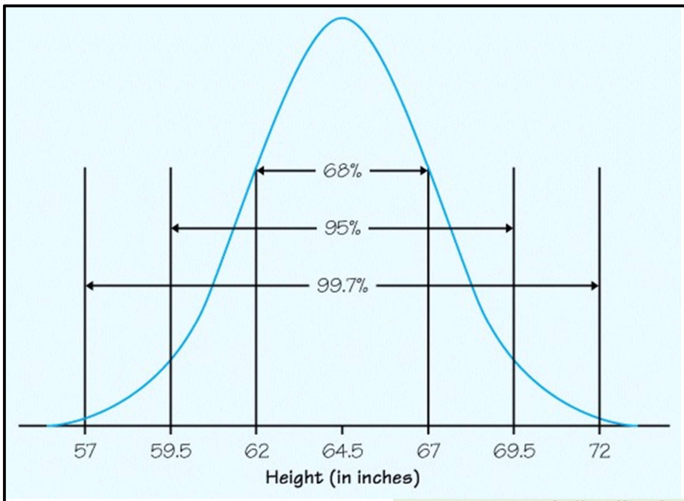
About 95% of cars of this model would be able to make an emergency stop within 160 feet.
So Danielle is likely to be able to stop safely.

Calculator

$X \sim N(155, 3^2)$

$P(X < 160) = ?$

注意 lower bound 千万不要设置成0！！ Normal有可能取到负无穷



The 68–95–99.7 RULE

In a Normal distribution with mean μ and standard deviation σ:

- Approximately **68%** of the observations fall within σ of the mean μ.
- Approximately **95%** of the observations fall within 2σ of the mean μ.
- Approximately **99.7%** of the observations fall within 3σ of the mean μ.

This result is known as the **68–95–99.7 rule.**

大家对68 95 99.7还有没有印象，我们上周也提到过
这个其实是源于normal Distribution
对于正态分布来说，
均值+-一倍的标准差，会包含68%的数据
均值+-2倍的标准差，会包含95%的数据
均值+-3倍的标准差，会包含99.7%的数据

Percentiles

The *percentile* of a score, x, is the percentage of scores which fall at or below the score.

The kth percentile P_k:

$$\int_{-\infty}^{P_k} f(x)\, dx = \text{k\%}$$

$$P_{90} = ?$$

Percentile 百分位数，Pk对应的应该就是排在k%这个位置的值，所以：

如果我们现在想求 90^{th} percentile,怎么求？
先看查表的方法怎么求：

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633

大概是1.285

看一下用计算器怎么找？
InvN，也就是反函数，已知概率，想去求对应的随机变量的区间取值

Determine the value of each of the following percentiles for the standard normal distribution (Hint: If the cumulative area that you must look for does not appear in the z table, use the closest entry):

a. The 91st percentile

b. The 77th percentile

c. The 50th percentile

d. The 9th percentile

e. What is the relationship between the 70th z percentile and the 30th z percentile?

练习一下:

注意：如果用查表的方法的话，如果没有对应的概率，就找最近的进行估算

Determine the value of each of the following percentiles for the standard normal distribution (Hint: If the cumulative area that you must look for does not appear in the Z table, use the closest entry):

a. The 91st percentile 1.341

b. The 77th percentile 0.737

c. The 50th percentile 0

d. The 9th percentile -1.34

e. What is the relationship between the 70th z percentile and the 30th Z percentile?

70th z percentile = - 30th z percentile

Data from the paper “Fetal ⋯ Composition” suggest that a normal distribution with mean 3500 grams and standard deviation 600 grams is a reasonable model for the probability distribution of the continuous numerical variable X = birth weight of a randomly selected full-term baby. What proportion of birth weights are between 2900 and 4700 grams?

$$X \sim \mathrm{N}(3500, 600^2)$$

$$Z = \frac{X-3500}{600} \sim N(0,1)$$

$$P(2900 < X < 4700) = P\left(\frac{2900-3500}{600} < Z < \frac{2900-3500}{600}\right)$$

$$= P(-1 < Z < 2) = 0.8185$$

不用自己去算2900-3500/600的值，计算器会自己帮我们算出来

Garbage trucks entering a particular waste management facility are weighed and then they offload garbage into a landfill. Data from the paper "Estimating … GPS" suggest that a normal distribution with mean 13 minutes and standard deviation 3.9 minutes is a reasonable model for the probability distribution of the random variable X = total processing time for a garbage truck at this waste management facility (total processing time includes waiting time as well as the time required to weigh the truck and offload the garbage). Suppose that we want to describe the total processing times of the trucks making up the 10% with the longest processing times.

X = total processing time for a garbage truck at this waste management facility

$X \sim \mathrm{N}(13, 3.9^2)$

$Z = \frac{X-13}{3.9} \sim N(0,1)$

$X = 3.9 \times \mathrm{Z} + 13$

We want to find the value of $X^* = 3.9 \times Z^* + 13$ such that $P(Z \geq Z^*) = 0.1$

$$Z^* = 1.28$$

$X^* = 13 + 3.9 \times 1.28 = 17.992\, minutes$

About 10% of the garbage trucks using this facility would have a total processing time of more than 17.992 minutes.

Determine the value of z* such that

a. -z* and z* separate the middle 95% of all z values from the most extreme 5%

b. -z* and z* separate the middle 90% of all z values from the most extreme 10%

c. -z* and z* separate the middle 98% of all z values from the most extreme 2%

d. -z* and z* separate the middle 92% of all z values from the most extreme 8%

0.975 0.025

0.95 0.05

0.99 0.01

0.96 0.04

Determine the value of z^* such that

a. $-z^*$ and z^* separate the middle 95% of all z values from the most extreme 5% $z^*=1.96$

b. $-z^*$ and z^* separate the middle 90% of all z values from the most extreme 10% $z^*=1.645$

c. $-z^*$ and z^* separate the middle 98% of all z values from the most extreme 2% $z^*=2.33$

d. $-z^*$ and z^* separate the middle 92% of all z values from the most extreme 8% $z^*=1.75$