Lyme disease is the leading tick-borne disease in the United States and Europe. Diagnosis of the disease is difficult and is aided by a test that detects particular antibodies in the blood. The article "Laboratory Considerations in the Diagnosis and Management of Lyme Borreliosis" (*American Journal of Clinical Pathology* [1993]: 168–174) used the following notation:

$+$	represents a positive result on the blood test
$-$	represents a negative result on the blood test
L	represents the event that the patient actually has Lyme disease
L^C	represents the event that the patient actually does not have Lyme disease

The following probabilities were reported in the article:

$P(L) = .00207$	The prevalence of Lyme disease in the population; .207% of the population actually has Lyme disease.
$P(L^C) = .99793$	99.793% of the population does not have Lyme disease.
$P(+\mid L) = .937$	93.7% of those with Lyme disease test positive.
$P(-\mid L) = .063$	6.3% of those with Lyme disease test negative.
$P(+\mid L^C) = .03$	3% of those who do not have Lyme disease test positive.
$P(-\mid L^C) = .97$	97% of those who do not have Lyme disease test negative.

$+$ represents a positive result on the blood test
$-$ represents a negative result on the blood test
L represents the event that the patient actually has Lyme disease
L^C represents the event that the patient actually does not have Lyme disease

$$P(L) = .00207$$
$$P(L^C) = .99793$$
$$P(+|L) = .937$$
$$P(-|L) = .063$$
$$P(+|L^C) = .03$$
$$P(-|L^C) = .97$$

Given that a person tests positive for the disease, what is the probability that he or she actually has Lyme disease?

$$P(L|+) = \frac{P(+|L)P(L)}{P(+|L)P(L) + P(+|L^C)P(L^C)}$$
$$= \frac{(.937)(.00207)}{(.937)(.00207) + (.03)(.99793)} = \frac{.0019}{.0319} = .0596$$

Independence

Two events (A & B) are independent if knowing the outcome of one event **does not** affect the probability that the other event will occur.

$$P(A|B) = P(A)$$

Formal Multiplication Rule

The formal multiplication rule (all events)…

$$P(A \cap B) = P(A) * P(B|A)$$

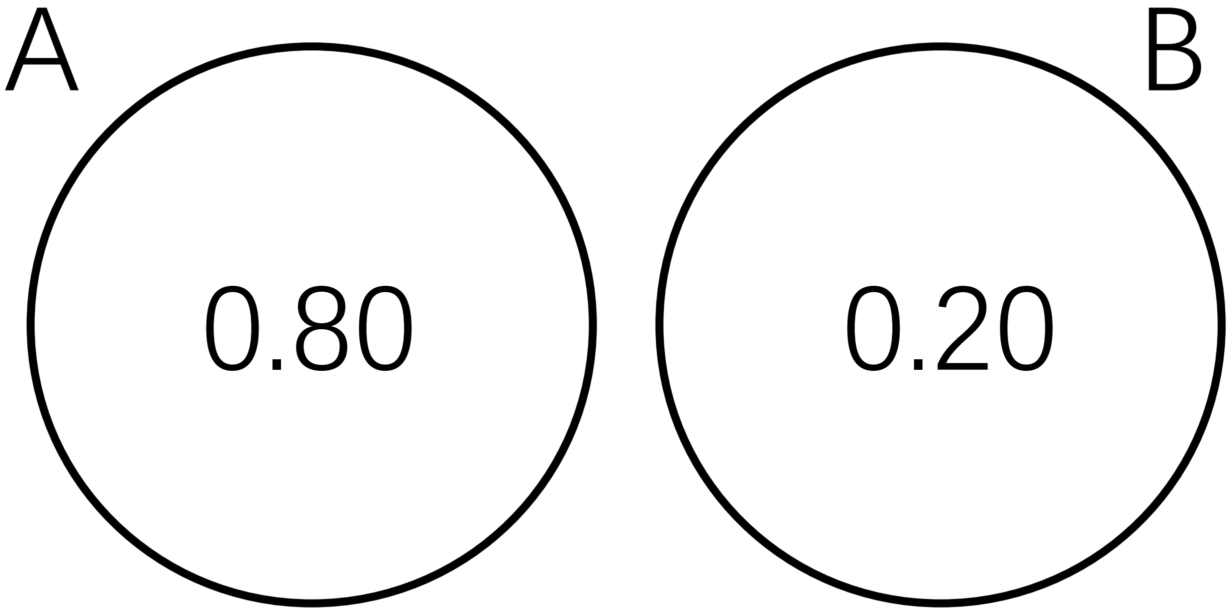
Independence:
P(B) = P(B|A)

For independent events only…

$$P(A \cap B) = P(A) * P(B)$$

Independent vs. Mutually Exclusive

<u>Mutually exclusive</u>: when events that have no intersection (i.e. they cannot both occur)



$P(A) = 0.80$

$P(A|B) = ?$

Independent vs. Mutually Exclusive

Mutually exclusive: when events that have no intersection (i.e. they cannot both occur)

$P(A) = 0.80$

$P(\mathbf{A}|\mathbf{B}) = 0.00$

Mutually exclusive events are **not independent**. Knowing that one event occurs greatly affects the probability of the other event (**lowers it to 0**).