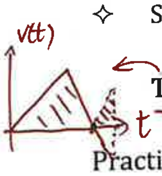
➢ Motion, Distance, Displacement...

✧ If a particle moves along a straight line with position function $s(t)$, then its velocity is $v(t) =$ $S'(t)$.

✧ So, its **displacement** (change of the position) from $t = a$ to $t = b$ is $S(b)-S(a) = \int_a^b V(t)\,dt$

✧ The **average velocity** over the time interval from $t = a$ to $t = b$ is $\frac{displacement}{time} = \frac{S(b)-S(a)}{b-a} = \frac{\int_a^b V(t)\,dt}{b-a}$

✧ Similarly, the **average acceleration** from $t = a$ to $t = b$ is $\frac{\int_a^b a(t)\,dt}{b-a}$

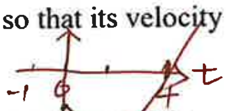


Total distance traveled = $\int_a^b |v(t)|\,dt$ **Average speed** = $\frac{\int_a^b |V(t)|\,dt}{b-a}$

⇒ (P3) Average value of a function is $\frac{\int_a^b f(x)\,dx}{b-a}$

Practice.

1. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t^2 - 3t - 4$.



(a) In which direction (left or right) is the particle moving at time $t = 5$?

(b) Find the acceleration of the particle at time $t = 5$.

(c) Given that $x(t)$ is the position of the particle at time t and that $x(0) = 12$, find $x(3)$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 6$.

(e) Find the average speed of the particle from $t = 0$ to $t = 6$.

(f) For what values of t, $0 \leq t \leq 6$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0,6]$?

(a) $V(5) > 0$ right

(b) $a(t) = V'(t) = 2t - 3$

$a(5) = 7$

(c) $x(3) - x(0) = \int_0^3 V(t)\,dt$

$= \left[\frac{1}{3}t^3 - \frac{3}{2}t^2 - 4t\right]_0^3$

$= -12 - \frac{9}{2}$

$\therefore x(3) = -\frac{9}{2} = -4.5$

(d) $\int_0^6 |V(t)|\,dt = \int_0^4 -(t^2-3t-4)\,dt + \int_4^6 t^2-3t-4\,dt$

$= \frac{94}{3}$

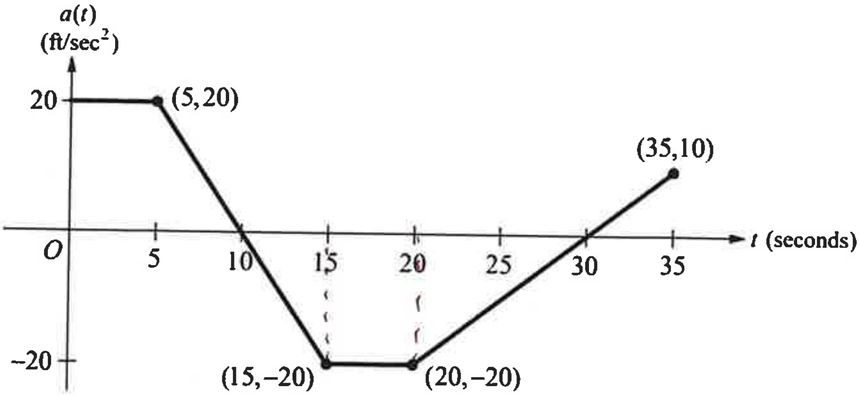
(e) $\frac{94}{3} \cdot \frac{1}{6} = \frac{94}{18} = \frac{47}{9}$

(f) $\bar{V}(t) = \frac{\int_0^6 V(t)\,dt}{6} = -1$

$V(t) = t^2 - 3t - 4 = -1$

$\therefore t = \frac{3+\sqrt{21}}{2} \approx 3.791$

2.



A car is traveling on a straight road with velocity 80 ft / sec at time $t = 0$. For $0 \le t \le 35$ seconds, the cars acceleration $a(t)$, in ft / sec^2, is the piecewise linear function defined by the graph above.

$V(0) = 80$ ft/sec

(a) Find $a(15)$ and $v(15)$.

(b) At what time, other than $t = 0$, on the interval $0 \le t \le 35$, is the velocity of the car 80 ft / sec ?

(c) On the time interval $0 \le t \le 35$, what is the car's absolute maximum velocity, in ft / sec, and at what time does it occur?

(d) On the time interval $0 \le t \le 35$, what is the car's absolute minimum velocity, in ft / sec, and at what time does it occur?

(a) $a(15) = -20$ ft/sec^2 $V(15) - V(0) = \int_0^{15} a(t)\,dt = 100$ ft/sec

$\therefore V(15) = 180$ ft/sec

(b) $\int_0^t a(x)\,dx = 0$

$\therefore t = 20$

(c) $t = 10$

$\int_0^{10} a(t)\,dt = 150$ ft/sec

$\therefore V(10) = V(0) + 150$

$= 230$ ft/sec

(d) $t = 30$

$V(t)_{min} = V(30) = V(0) + \int_0^{30} a(t)\,dt$

$= 80 - 100$

$= -20$ ft/sec

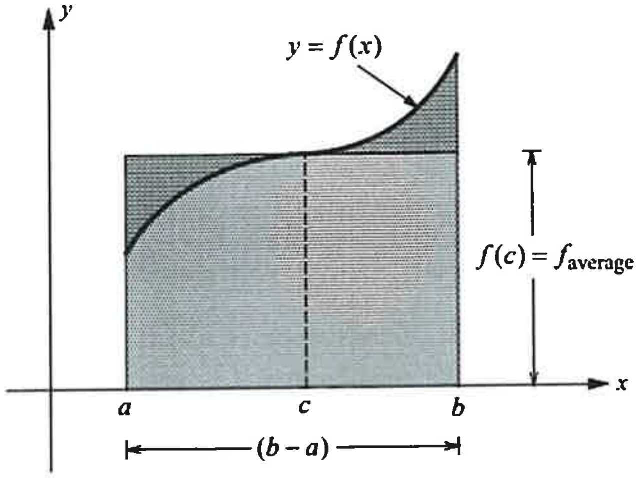
- **Average Value of a Function**

  - **Average value of f**

    If f is integrable on $[a, b]$, then the average value of f on the interval is $\dfrac{\int_a^b f(x)dx}{b-a}$

  - **The Mean Value Theorem for Definite Integrals**

    If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$, such that $\int_a^b f(x)\,dx = f(c)(b-a)$



MVT f: ctns on $[a,b]$ diff on (a,b)

then $\exists\, c \in (a,b)$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$\Downarrow$

AROC

Q1. Find the average value of $f(x) = \frac{1}{2}x\cos(x^2) + x$ on the interval $[0, \sqrt{2\pi}]$.

$$\frac{\int_0^{\sqrt{2\pi}} \frac{1}{2}x\cos(x^2) + x \; dx}{\sqrt{2\pi}} = \frac{1}{\sqrt{2\pi}} \cdot \left[\int_0^{\sqrt{2\pi}} \frac{1}{4}\cos(x^2)\, d(x^2) + \left[\frac{x^2}{2}\right]_0^{\sqrt{2\pi}}\right]$$

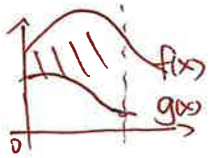
$$= \frac{1}{\sqrt{2\pi}}\left[\left[\frac{1}{4}\sin(x^2)\right]_0^{\sqrt{2\pi}} + \pi\right]$$

$$= \frac{\pi}{\sqrt{2\pi}} = \frac{\sqrt{2\pi}}{2}$$

Q2. (Calculator) Let f be the function given by $f(x) = x^3 - 2x + 4$ on the interval [-1,2]. Find c such that the average value of f on the interval is equal to f(c).

$$\frac{\int_{-1}^{2} f(x)\,dx}{3} = \frac{1}{3}\left[\frac{1}{4}x^4 - x^2 + 4x\right]_{-1}^{2} = 4.25 = f(c)$$

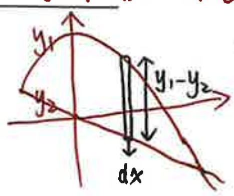
Calculator: $c = -0.126$ or 1.473



➢ Area

If f and g are continuous on $[a, b]$ and $g(x) \le f(x)$ for all x on $[a, b]$, then the area of the region bounded by two curves and the vertical lines on $x = a$ and $x = b$ is: $\int_a^b f(x) - g(x)\, dx$ （大面积－小面积）

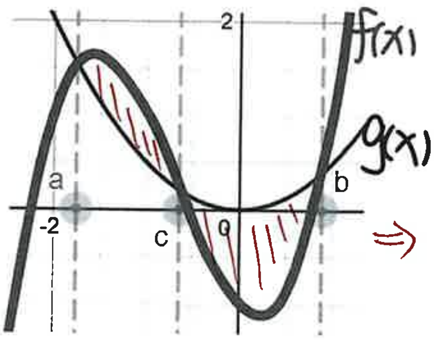
另一个角度理解:



Total Area $= \sum$ area of strips.

$= \sum$ area of lines.

$= \int_{-1}^{2} y_1 - y_2\, dx$



$\Rightarrow \int_a^c f(x) - g(x)\, dx + \int_c^b g(x) - f(x)\, dx$

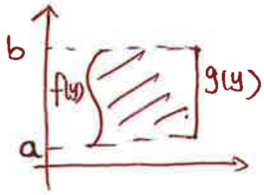
Sometimes, the upper and the lower functions switch. In this case, area=_

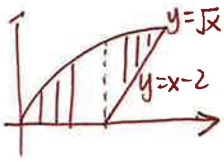
In general, if the curves are defined by the function of x, the area between two curves can be determined by

$$A = \int_{x_1}^{x_2} [top\ curve - bottom\ curve]\ dx$$

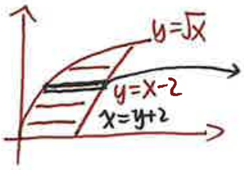
If the curves are defined by function of y, $A =$ ______ $\int_a^b g(y) - f(y)\, dy$

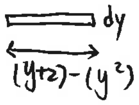
$= \int_a^b (right - left)\, dy$





integrate along the x-axis : not easy



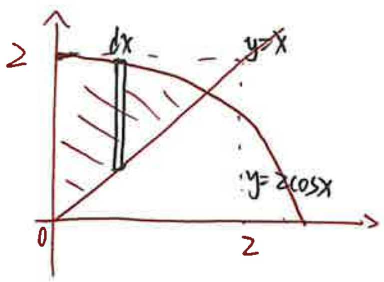


Area $= \int_0^2 (y+2) - y^2\, dy$

Practice.

(cal)

1. Find the area of the region in the first quadrant enclosed by the graphs of $f(x) = 2\cos x$, $g(x) = x$ and the y-axis.



① Sketch

② vertical strips

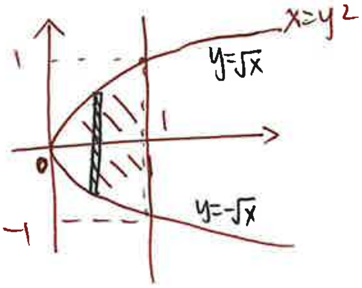
③ $(2\cos x - x)\, dx$

④ $2\cos x = x$

$x = 1.030$

⑤ Area $= \int_0^{1.030} (2\cos x - x)\, dx = 0.85$

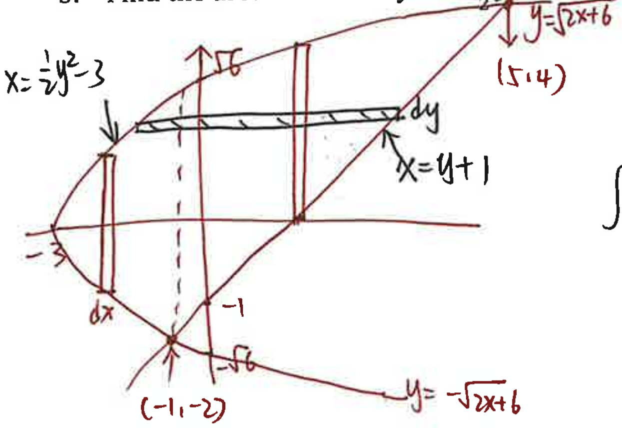
2. Find the area bounded by $x = y^2$ and $x = 1$



$$\int_0^1 \sqrt{x}-(-\sqrt{x})\, dx = 2\cdot\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^1 = \frac{4}{3}$$

OR $$\int_{-1}^{1} 1-y^2\, dy = \left[y-\frac{1}{3}y^3\right]_{-1}^{1} = \frac{4}{3}$$

3. Find the area bounded by $x = \frac{1}{2}y^2 - 3$ and $y = x - 1$



$$\int_{-3}^{-1} 2\sqrt{2x+6}\, dx + \int_{-1}^{5} \sqrt{2x+6}-(x-1)\, dx$$

$$\int_{-2}^{4} (y+1)-\left(\frac{1}{2}y^2-3\right) dy$$

$$= \int_{-2}^{4} -\frac{1}{2}y^2+y+4\, dy$$

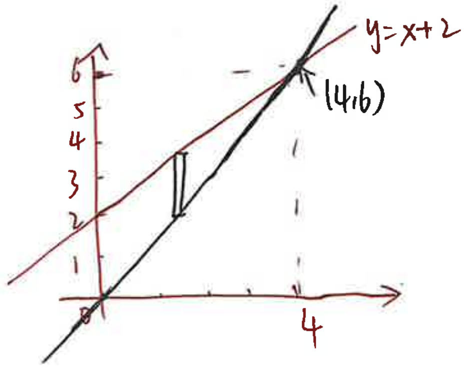
$$= \left[-\frac{1}{6}y^3+\frac{1}{2}y^2+4y\right]_{-2}^{4}$$

$$= -\frac{1}{6}(64+8)+\frac{1}{2}(16-4)+4(4+2)$$

$$= -12+6+24$$

$$= 18$$

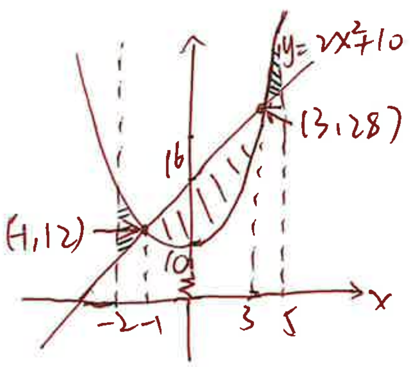
4. Find the area bounded by $y = x + 2, x = 0$ and $y = \frac{3}{2}x$



$$\int_0^4 (x+2)-\frac{3}{2}x\, dx$$

$$= 4$$

5. Determine the area of the region bounded by $y = 4x + 16,\ y = 2x^2 + 10,\ x = -2$ and $x = 5$



$$\begin{cases} y=4x+16 \\ y=2x^2+10 \end{cases} \Rightarrow \text{intersection } (3,28)\ (-1,12)$$

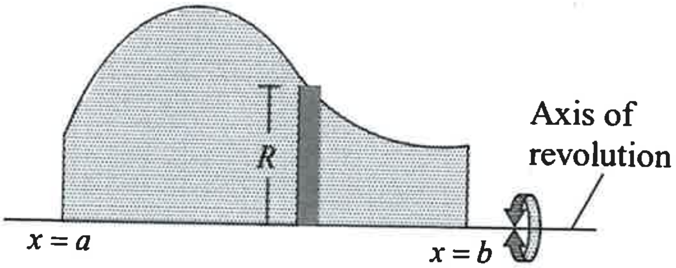
$$\int_{-2}^{-1} (2x^2+10)-(4x+16)\, dx + \int_{-1}^{3} (4x+16)-(2x^2+10)\, dx$$

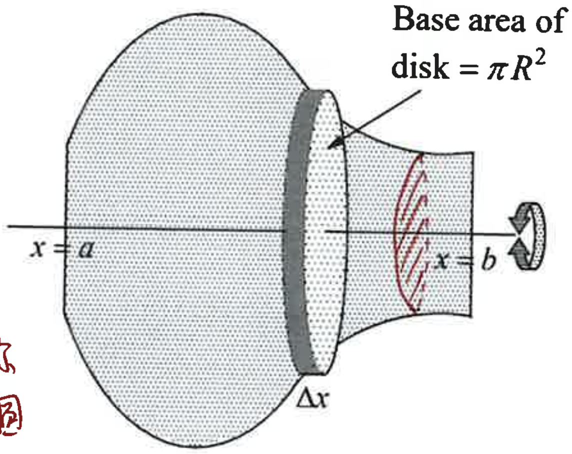
$$+\int_{3}^{5} (2x^2+10)-(4x+16)\, dx$$

Volume of a solid revolution – Disk

- Volume of a disk = Base area of disk * width of disk = $\pi R^2 dx$

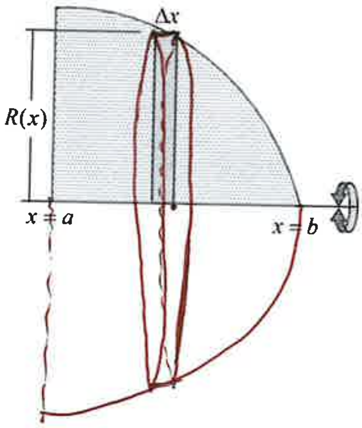
  So, the volume of a solid revolution is V= $\int_a^b \pi R^2 dx$





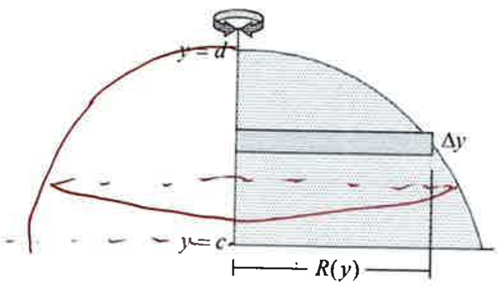
画图:①对称
②椭圆

- Horizontal Axis of Revolution (revolve the region about the x-axis)



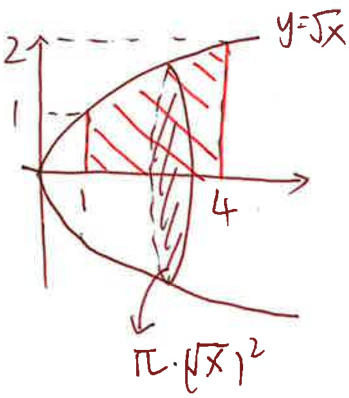
$$\int_a^b \pi \, R(x)^2 \, dx$$

- Vertical Axis of Revolution (revolve about the y-axis)



$$\int_c^d R(y)^2 \cdot \pi \, dy$$

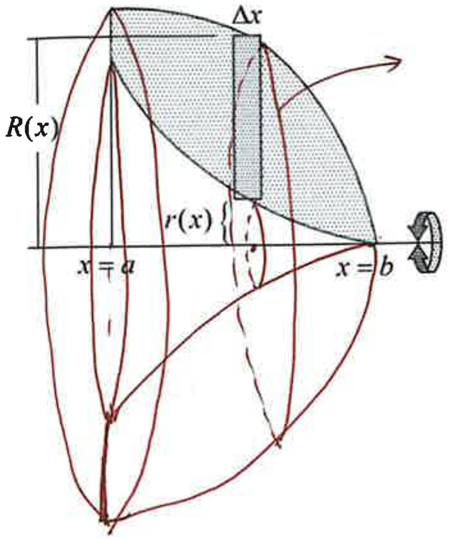
Practice: Find the volume of the solid generated by revolving the region bounded by the graphs of $y = \sqrt{x}$, x = 1, and x = 4 about the x-axis.



$$\int_1^4 \pi(\sqrt{x})^2 dx$$

$$= \frac{15}{2}\pi$$

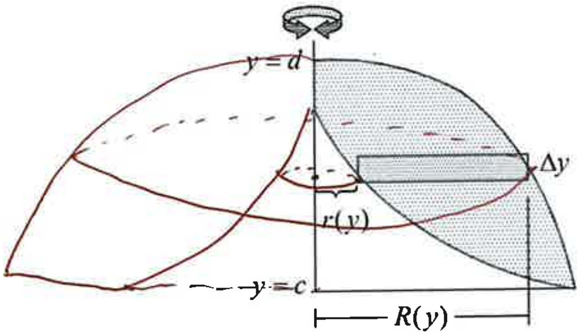
- **Volume of a solid revolution –Washer**
- Horizontal Axis of Revolution (revolve the region about the x-axis)



$\pi R^2 - \pi r^2$

$$\int_a^b \pi\left[(R(x))^2 - (r(x))^2\right] dx$$

- Vertical Axis of Revolution (revolve about the y-axis)



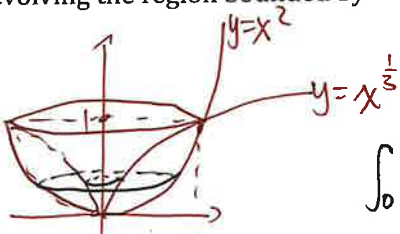
$$\int_c^d \pi\left[(R(y))^2 - (r(y))^2\right] dy$$

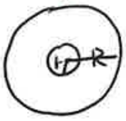
Practice:

1. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^{\frac{1}{3}}, y = x^2$

(a) About the x-axis

(b) About the y-axis





$r: x = y^3$

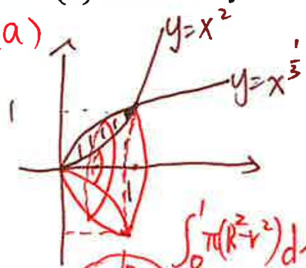
$R: x = y^{\frac{1}{2}}$

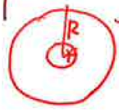
$$\int_0^1 \pi(y - y^6)\, dy$$

$$= \left[\frac{1}{2}y^2 - \frac{1}{7}y^7\right]_0^1 \cdot \pi$$

$$= \frac{5}{14}\pi$$

(a)



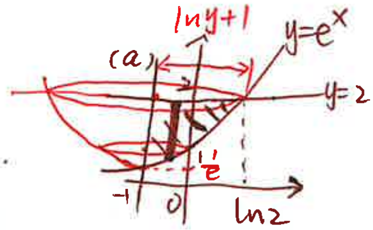


$$\int_0^1 \pi(R^2 - r^2)\, dx = \pi\int_0^1 (x^{\frac{1}{3}})^2 - (x^2)^2\, dx$$

$$= \pi\int_0^1 x^{\frac{2}{3}} - x^4\, dx$$

$$= \pi\left[\frac{3}{5}x^{\frac{5}{3}} - \frac{1}{5}x^5\right]_0^1$$

$$= \frac{2}{5}\pi$$



2. Let R be the region between the graphs of $y=e^x$, $y=2$ and $x=-1$.

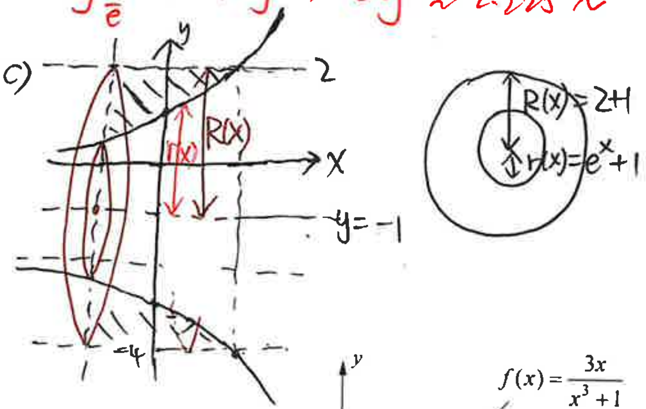
(a) Find the area of R. (a) $\int_{-1}^{\ln 2} 2-e^x \, dx = [2x-e^x]_{-1}^{\ln 2} = (2\ln 2-2)-(-2-e^{-1}) = 2\ln 2+e^{-1}$

(Cal) (b) Find the volume of the solid generated when R is revolved about the line $x=-1$.

(c) Find the volume of the solid generated when R is revolved about the line $y=-1$.

(b) $\int_{\frac{1}{e}}^{2} \pi \cdot (\ln y+1)^2 \, dy \approx 2.225\pi$

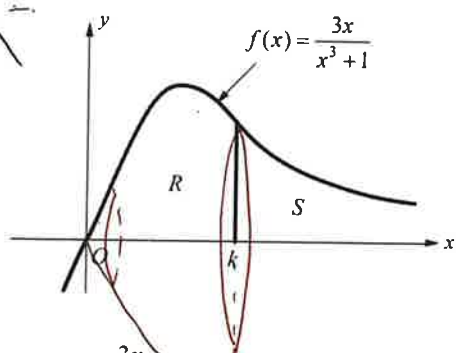
(c)



$\int_{-1}^{\ln 2} \pi \cdot [3^2-(e^x+1)^2] \, dx$

$\approx 8.349\pi$

3.



Let f be the function given by $f(x)=\dfrac{3x}{x^3+1}$. Let R be the region bounded by the graph of f, the x-axis, and the vertical line $x=k$, where $k>0$. $\boxed{BC}$

(a) Find the volume of the solid generated when R is revolved about the x-axis in terms of k.

(b) Let S be the unbounded region in the first quadrant to the right of the vertical line $x=k$ and below the graph of f, as shown in the figure above. Find the value of k such that the volume of the solid generated when S is revolved about the x-axis is equal to the volume of the solid found in part (a).

(a) $V_R = \int_0^k \pi \left(\frac{3x}{x^3+1}\right)^2 dx$

$= \pi \int_0^k \frac{3x \cdot 3x^2}{(x^3+1)^2} \, dx$

$= \pi \int_0^k \frac{3}{(x^3+1)^2} \, d(x^3+1)$

$= 3\pi \cdot [-(x^3+1)^{-1}]_0^k$

$= 3\pi \cdot \left[-\frac{1}{k^3+1}+1\right]$

$= 3\pi - \frac{3\pi}{k^3+1}$

(b) $V_S = \int_k^{\infty} \left(\frac{3x}{x^3+1}\right)^2 \cdot \pi \, dx$

$= \lim_{t\to\infty} \int_k^t \pi \left(\frac{3x}{x^3+1}\right)^2 dx$

$= -3\pi \cdot [(x^3+1)^{-1}]_k^t$

$= -3\pi\left(\frac{1}{1+t^3}-\frac{1}{1+k^3}\right)$

$= \frac{3\pi}{1+k^3}$

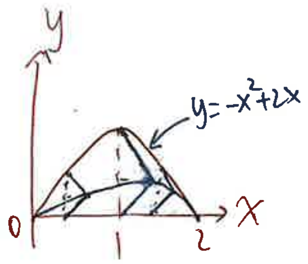
$= V_R$

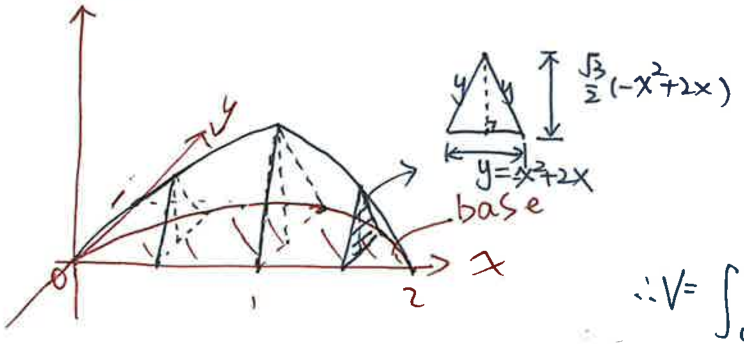
$\therefore k=1$

➢ **Volume of a solid revolution – Known Cross Sections**

$= x(-x+2)$

Example. The base of a solid is the region in the first quadrant enclosed by the graph of $y = -x^2 + 2x$ and the x-axis. If every cross section of the solid perpendicular to the x-axis is an equilateral triangle, what is the volume of the solid?





$S = \frac{1}{2}\cdot(-x^2+2x)^2\cdot\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4}(-x^2+2x)^2$

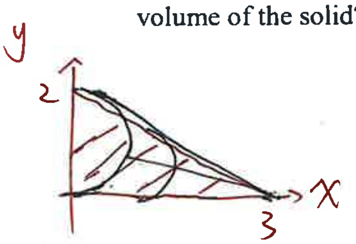
$\therefore V = \int_0^2 \frac{\sqrt{3}}{4}(-x^2+2x)^2\,dx$

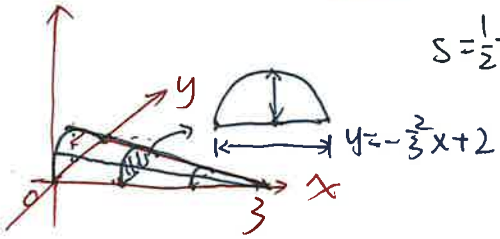
$= \frac{\sqrt{3}}{4}\int_0^2 x^4 - 4x^3 + 4x^2\,dx$

$= \frac{\sqrt{3}}{4}\left[\frac{1}{5}x^5 - x^4 + \frac{4}{3}x^3\right]_0^2$

$= \frac{4}{15}\sqrt{3}$

Q1. The base of a solid is the region in the first quadrant bounded by the coordinate axes, and the line $2x+3y=6$. If the cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?





$S = \frac{1}{2}\pi\cdot\left(\frac{-\frac{2}{3}x+2}{2}\right)^2 = \frac{\pi(-\frac{2}{3}x+2)^2}{8}$

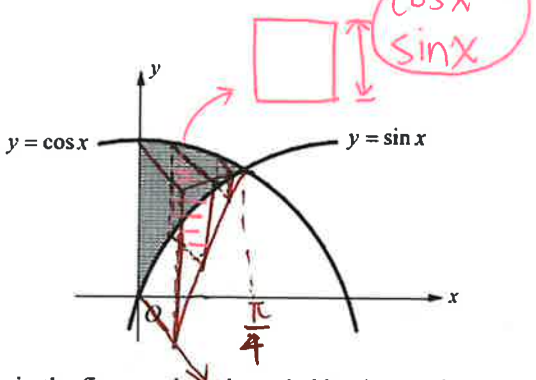
$V = \int_0^3 \frac{\pi}{8}(-\frac{2}{3}x+2)^2\,dx$

$= \frac{\pi}{8}\int_0^3 \frac{4}{9}x^2 - \frac{8}{3}x + 4\,dx$

$= \frac{\pi}{8}\left[\frac{4}{27}x^3 - \frac{4}{3}x^2 + 4x\right]_0^3$

$= \frac{\pi}{2}$

Q2.



$S = (\cos x - \sin x)^2$

The base of a solid is the region in the first quadrant bounded by the y-axis and the graphs of $y = \cos x$ and $y = \sin x$, as shown in the figure above. If the cross sections of the solid perpendicular to the x-axis are squares, what is the volume of the solid?

$$V = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)^2 \, dx$$

$$= \int_0^{\frac{\pi}{4}} 1 - 2\sin x \cos x \, dx$$

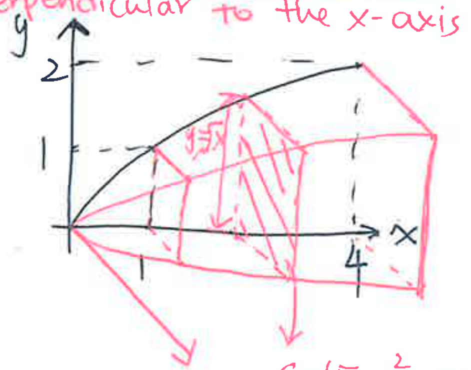
$$= [x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sin 2x \, dx$$

$$= \frac{\pi}{4} + \left[\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

Q3. The base of a solid is the region bounded by the graph of $y = \sqrt{x}$, the x-axis and the line $x = 4$. If the cross sections of the solid perpendicular to the y-axis are squares, the volume of the solid is given by

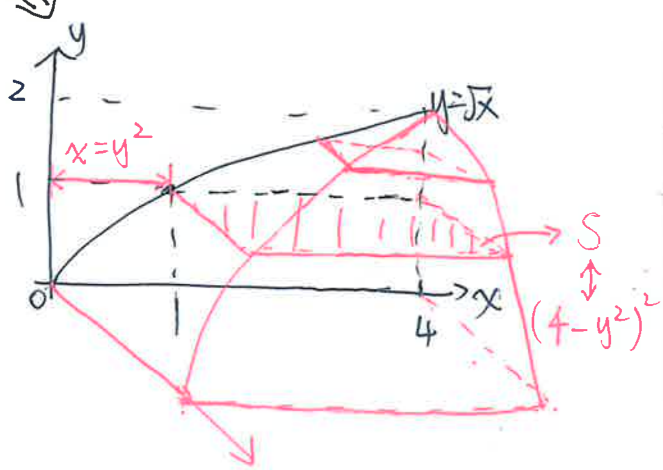
* perpendicular to the x-axis are squares.



$S = (\sqrt{x})^2 = x$

$$V = \int_0^4 x \, dx$$

$$= \left[\frac{1}{2}x^2\right]_0^4 = 2$$

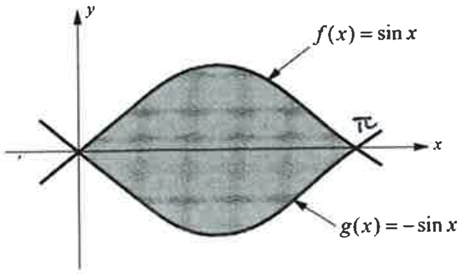


$$V = \int_0^2 (4-y^2)^2 \, dy$$

$$= \int_0^2 16 - 8y^2 + y^4 \, dy = \left[16y - \frac{8}{3}y^3 + \frac{1}{5}y^5\right]_0^2$$

$$= 32 - \frac{224}{15} \approx 17.067$$

Q4.



Let $f(x) = \sin x$ and $g(x) = -\sin x$ for $0 \le x \le \pi$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 3$.

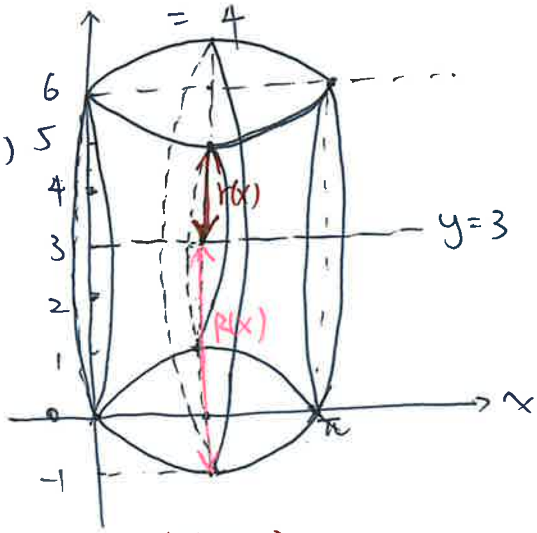
(c) Let h be the function given by $h(x) = k \sin x$ for $0 \le x \le \pi$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. If the volume of the solid is equal to 8π, what is the value of k?

(a) $\int_0^{\pi} \sin x - (-\sin x)\, dx$

$= \left[-2\cos x\right]_0^{\pi}$

$= 4$

(b)



$r(x) = 3 - \sin x$

$R(x) = 3 + \sin x$

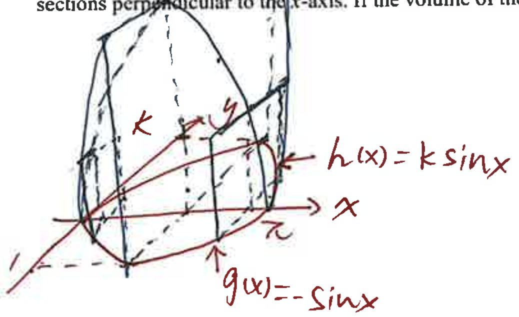
$$V = \int_0^{\pi} \pi\left[(3+\sin x)^2 - (3-\sin x)^2\right] dx$$

$$= \pi\int_0^{\pi} 12\sin x\, dx$$

$$= \pi\left[-12\cos x\right]_0^{\pi}$$

$$= 24\pi$$

(c)



$$S = (k\sin x + \sin x)^2$$

$$= \sin^2 x \cdot (k+1)^2$$

$$V = \int_0^{\pi} \sin^2 x\, (k+1)^2\, dx$$

$$= (k+1)^2 \int_0^{\pi} \frac{1-\cos 2x}{2}\, dx$$

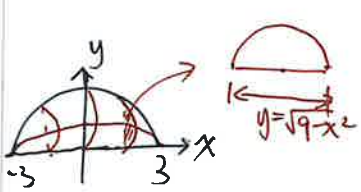
$$= \frac{(k+1)^2}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi}$$

$$= \frac{(k+1)^2}{2}\,\pi = 8\pi$$

$$\therefore k = 3$$

 $x^2+y^2=9 \quad (y\geq 0)$

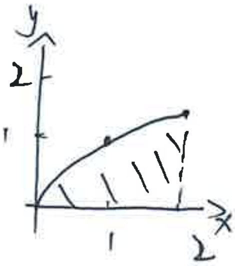
Q5. The base of a solid S is the semicircular region enclosed by $y=\sqrt{9-x^2}$ and the x-axis. The cross sections of S perpendicular to the x-axis are semicircles. Find the volume.

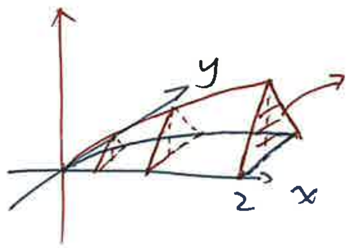


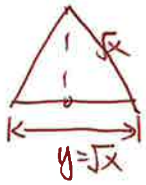
$$S=\frac{1}{2}\cdot\pi\cdot\left(\frac{\sqrt{9-x^2}}{2}\right)^2$$
$$=\frac{\pi}{8}(9-x^2)$$

$$V=2\int_0^3 \frac{\pi}{8}(9-x^2)\,dx$$
$$=\frac{\pi}{4}\left[9x-\frac{1}{3}x^3\right]_0^3$$
$$=\frac{\pi}{4}\cdot(27-9)=\frac{9}{2}\pi$$

Q6. The base of a solid S is the region enclosed by the graph of $y=\sqrt{x}$ the x-axis, and the line $x=2$. If each cross section perpendicular to the x-axis is an equilateral triangle, what is the volume of the solid?



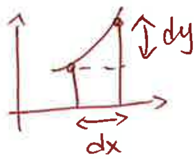




$$S=\frac{1}{2}\cdot\sqrt{x}\cdot\frac{\sqrt{3}}{2}\sqrt{x}$$
$$=\frac{\sqrt{3}}{4}x$$

$$V=\int_0^2\frac{\sqrt{3}}{4}x\,dx$$
$$=\frac{\sqrt{3}}{8}\left[x^2\right]_0^2$$
$$=\frac{\sqrt{3}}{2}$$

➢ Length of a curve



If f' is continuous on $[a,b]$, then the length of the curve $y = f(x)$ from $x = a$ to $x = b$ is

$$L = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

If g' is continuous on $[c,d]$, then the length of the curve $x = g(y)$ from $y = c$ to $y = d$ is

$$L = \int_c^d \sqrt{1+(g'(y))^2}\,dy$$

length of arc

$$\approx \sqrt{(dy)^2+(dx)^2}$$

$$= dy\cdot\sqrt{1+\underbrace{\left(\frac{dx}{dy}\right)^2}_{(g'(y))^2}}$$

$$= dx\sqrt{\underbrace{\left(\frac{dy}{dx}\right)^2}_{(f'(x))^2}+1}$$

(Cal)

1. Find the length of the curve $y = 2x^{\frac{3}{2}} + 1$, from $x = 1$ to $x = 3$.

$$\int_1^3 \sqrt{1+(y')^2}\,dx$$

$$= \int_1^3 \sqrt{1+\left(2\cdot\frac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx$$

$$= \int_1^3 \sqrt{1+9x}\ \frac{1}{9}\,d(1+9x)$$

$$= \frac{1}{9}\cdot\frac{2}{3}\left[(1+9x)^{\frac{3}{2}}\right]_1^3 = \frac{2}{27}\left(28^{\frac{3}{2}} - 10^{\frac{3}{2}}\right) \approx 8.633$$

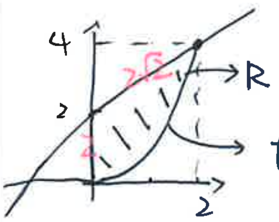
(Cal)

2. Let R be the region bounded by the y-axis and the graphs of $y = x^2$ and $y = x + 2$. Find the perimeter of the region R.

$\parallel$

$2+2\sqrt{2}+4.647$

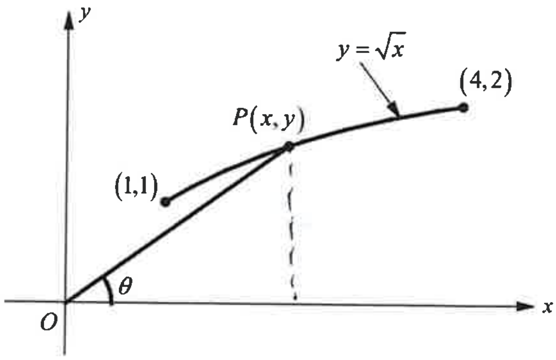
≈ 9.475



$$l = \int_0^2 \sqrt{1+(y')^2}\,dx$$

$$= \int_0^2 \sqrt{1+(2x)^2}\,dx \approx 4.647$$

3.



The figure above shows a point, $P(x, y)$, moving on the curve of $y=\sqrt{x}$, from the point $(1,1)$ to the point $(4,2)$. Let θ be the angle between $\overline{OP}$ and the positive x-axis.

(a) Find the x- and y-coordinates of point P in terms of $\cot\theta$.

(Cal) (b) Find the length of the curve from the point $(1,1)$ to the point $(4,2)$.

(c) If the angle θ is changing at the rate of -0.1 radian per minute, how fast is the point P moving along the curve at the instant it is at the point $(3,\sqrt{3})$?

(a) $\cot\theta = \frac{x}{y} = \frac{x}{\sqrt{x}} = \sqrt{x}$

$\therefore\ x = \cot^2\theta$

$\therefore\ y = \cot\theta$

$\therefore\ (\cot^2\theta, \cot\theta)$

(b) $\int_1^4 \sqrt{1+(y')^2}\, dx$

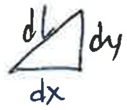
$= \int_1^4 \sqrt{1+(\frac{1}{2}x^{-\frac{1}{2}})^2}\, dx$

$= \int_1^4 \sqrt{1+\frac{1}{4}\cdot\frac{1}{x}}\, dx$

≈ 3.168

(c) $\frac{d\theta}{dt} = -0.1$ rad/min

$\left.\frac{dl}{dt}\right|_{x=3,\, y=\sqrt{3}} = ?$



$dl = \sqrt{(dx)^2+(dy)^2}$

$= \sqrt{(\frac{dx}{d\theta})^2+(\frac{dy}{d\theta})^2}\ d\theta$

$= \sqrt{(2\cot\theta\cdot(-\csc^2\theta))^2+(-\csc^2\theta)^2}\ d\theta$

$= \sqrt{\csc^4\theta\cdot(4\cot^2\theta+1)}\ d\theta$

$\therefore\ \frac{dl}{dt} = \sqrt{\csc^4\theta\cdot(4\cot^2\theta+1)}\ \frac{d\theta}{dt}$

When $x=3,\ y=\sqrt{3}$, we have

$\cot\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$.

$\csc\theta = 2$

$\therefore\ \left.\frac{dl}{dt}\right|_{x=3,\, y=\sqrt{3}} = -1.442$ unit/min