1. B

$$\int_1^3 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt \approx 8.268$$

$x'(t) \to 1-2t$, $\quad y'(t) \to 2t^{\frac{1}{2}}$

2. C

$$\int_0^{\pi} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = \frac{1}{2}a[t^2]_0^{\pi} = \frac{a\pi^2}{2}$$

$x'(t) \to a t\cos t$, $\quad y'(t) \to a t\sin t$

3. D $x'(t) = \cos t - \tan t$

$y'(t) = -\sin t$

$$(x'(t))^2 + (y'(t))^2 = \cos^2 t - 2\cos t \cdot \frac{\sin t}{\cos t} + \tan^2 t + \sin^2 t$$

$$= 1 - 2\sin t + \tan^2 t$$

$$= \sec^2 t - 2\sin t$$

C

1. If a particle moves in the xy-plane so that at time $t > 0$ its position vector is $(t^3 - 1, \ln\sqrt{t^2+1})$, then at time $t = 1$, its velocity vector is

$x(t)$ $y(t)$

(A) $(0, \frac{1}{2})$ (B) $(1, \frac{1}{2})$ (C) $(3, \frac{1}{2})$ (D) $(3, \frac{1}{4})$

$v(t) = \langle x'(t), y'(t) \rangle = \langle 3t^2, \frac{\frac{1}{2}(t^2+1)^{-\frac{1}{2}} \cdot 2t}{\sqrt{t^2+1}} \rangle$

$= \langle 3t^2, \frac{t}{t^2+1} \rangle$

$t=1$: $v(1) = \langle 3, \frac{1}{2} \rangle$

D

2. A particle moves in the xy-plane so that at any time t its coordinates are $x = t^3 - t^2$ and $y = t + \ln t$. At time $t = 2$, its acceleration vector is

$v(t) = \langle 3t^2 - 2t, 1 + \frac{1}{t} \rangle$

(A) $(4, \frac{1}{2})$ (B) $(6, \frac{1}{4})$ (C) $(8, \frac{3}{4})$ (D) $(10, -\frac{1}{4})$

$a(t) = \langle 6t - 2, -\frac{1}{t^2} \rangle$

$a(2) = \langle 10, -\frac{1}{4} \rangle$

B

3. A particle moves in the xy-plane so that its position at time $t > 0$ is given by $x(t) = e^t \cos t$ and $y(t) = e^t \sin t$. What is the speed of the particle when $t = 2$?

(A) $\sqrt{2}e$ (B) $\sqrt{2}e^2$ (C) $2e$ (D) $2e^2$

$v(t) = \langle x'(t), y'(t) \rangle = \langle e^t(\cos t - \sin t), e^t(\sin t + \cos t) \rangle$

$v(2) = \langle e^2(\cos 2 - \sin 2), e^2(\sin 2 + \cos 2) \rangle$

speed $= \sqrt{e^4[(\cos^2 2 - 2\sin 2 \cdot \cos 2 + \sin^2 2) + (\sin^2 2 + 2\sin 2 \cdot \cos 2 + \cos^2 2)]}$

$= \sqrt{e^4(2)} = e^2\sqrt{2}$

A

4. If f is a vector-valued function defined by $f(t) = (\ln(\sin t), t^2 + e^{-t})$, then the acceleration vector is

(A) $(-\csc^2 t, 2 + e^{-t})$

(B) $(\sec^2 t, 2 + e^{-t})$

(C) $(\csc^2 t, 2 - e^{-t})$

(D) $(-\csc^2 t \cdot \cot t, 2 + e^{-t})$

$v(t) = \left(\underbrace{\frac{\cos t}{\sin t}}_{\cot t}, 2t - e^{-t}\right)$

$a(t) = (-\cancel{\cot t}\ \csc^2 t, 2 + e^{-t})$

D

5. A particle moves on the curve $y = x + \sqrt{x}$ so that the x-component has velocity $x'(t) = \cos t$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1, 0)$. At time $t = \frac{\pi}{2}$, the particle is at the point

$x(0) = 1$

$y(0) = 0$

(A) $(0, 0)$ (B) $(1, 2)$ (C) $(\frac{\pi}{2}, \frac{\pi}{2} + \sqrt{\frac{\pi}{2}})$ (D) $(2, 2 + \sqrt{2})$

$x(\frac{\pi}{2}) = \int_0^{\frac{\pi}{2}} x'(t)\,dt + x(0)$

$= \int_0^{\frac{\pi}{2}} \cos t\,dt + 1$

$= \sin t \Big|_0^{\frac{\pi}{2}} + 1 = 2$

$y(\frac{\pi}{2}) = 2 + \sqrt{2}$

6. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$, with $\frac{dx}{dt} = t - \sin(e^t)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 1$, the value of $\frac{dy}{dt}$ is 3 and the object is at position $(1, 4)$. $x(1)=1 \quad y(1)=4$

   (a) Find the x-coordinate of the position of the object at time $t = 5$. (a) $x(5) = \int_1^5 x'(t)\,dt + x(1)$ $\approx 12.245 + 1 \approx 13.245$

   (b) Write an equation for the line tangent to the curve at the point $(x(1), y(1))$.

   (c) Find the speed of the object at time $t = 1$. (b) slope $= \frac{\frac{dy}{dt}\big|_{t=1}}{\frac{dx}{dt}\big|_{t=1}} = \frac{3}{1-\sin(e)} \approx 5.091$

   (d) Suppose the line tangent to the curve at $(x(t), y(t))$ has a slope of $(t-2)$ for $t \geq 0$. Find the acceleration vector of the object at time $t = 3$. $\therefore\ l:\ y - 4 = 5.091(x-1)$

(c) speed $= \sqrt{(x'(1))^2 + (y'(1))^2} = \sqrt{(1-\sin e)^2 + 3^2} \approx 3.057$

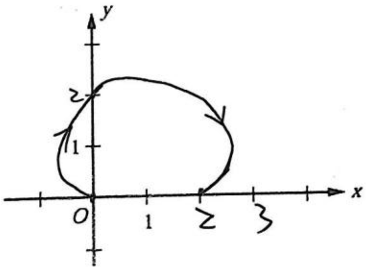
(d) slope $= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = t - 2 \qquad \therefore \frac{dy}{dt} = (t-2)\cdot(t - \sin(e^t))$

$a(t) = \langle \frac{d}{dt}(t - \sin(e^t)),\ \frac{d}{dt}((t-2)(t-\sin(e^t)))\rangle = \langle 1 - \cos(e^t)\cdot e^t,\ (t-\sin(e^t)) + (t-2)(1-\cos(e^t)\cdot e^t)\rangle$

$a(3) = \langle -5.600, -3.544\rangle$

7. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t - \sin(\pi t)$ and $y(t) = 1 - \cos(\pi t)$ for $0 \leq t \leq 2$.

   (a) On the axis provided below, sketch the graph of the path of the particle from $t = 0$ to $t = 2$. Indicate the direction of the particle along its path.



   (b) Find the position of the particle when $t = 1$. (b) $x(1) = 1 - \sin\pi = 1 \quad y(1) = 1 - \cos\pi = 2 \quad (1, 2)$

   (c) Find the velocity vector for the particle at any time t. (c) $v(t) = \langle 1 - \cos(\pi t)\cdot\pi,\ \sin(\pi t)\cdot\pi\rangle$

   (d) Write and evaluate an integral expression, in terms of sine and cosine, that gives the distance traveled of the particle from $t = 0$ to $t = 2$.

(d) $\int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

$= \int_0^2 \sqrt{(1 - \pi\cos(\pi t))^2 + (\pi\sin(\pi t))^2}\,dt$

≈ 6.443