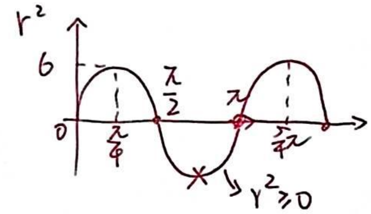
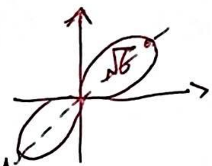
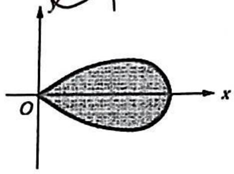
C 1. The area of the region enclosed by the polar curve $r^2 = 6\sin(2\theta)$ is

(A) 2 (B) 4 (C) 6 (D) 12





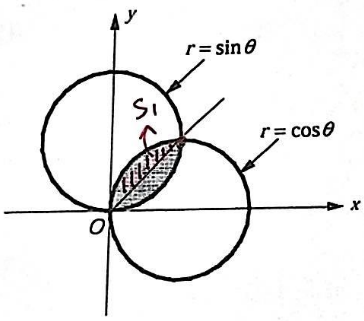
$$2\cdot\int_0^{\frac{\pi}{2}} \frac{1}{2} r^2 \, d\theta$$
$$= \int_0^{\frac{\pi}{2}} 6\sin(2\theta)\, d\theta$$
$$= \left[-3\cos 2\theta\right]_0^{\frac{\pi}{2}}$$
$$= 3+3 = 6$$



B 2. What is the area of the region enclosed by the loop of the graph of the polar curve $r = 2\cos(2\theta)$ $\Rightarrow r=0: \theta = -\frac{\pi}{4}, \frac{\pi}{4}$
shown in the figure above?

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}\cdot(2\cos 2\theta)^2 \, d\theta$$
$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 2\cdot\frac{\cos 4\theta + 1}{2}\, d\theta$$
$$= \left[\frac{1}{4}\sin 4\theta + \theta\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$
$$= \frac{\pi}{2}$$



Intersection: $\sin\theta = \cos\theta$
$\theta = \frac{\pi}{4}$

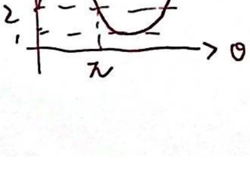
A 3. The area of the shaded region that lies inside the polar curves $r = \sin\theta$ and $r = \cos\theta$ is

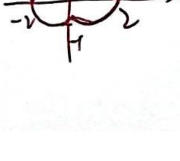
(A) $\frac{1}{8}(\pi-2)$ (B) $\frac{1}{4}(\pi-2)$ (C) $\frac{1}{2}(\pi-2)$ (D) $\frac{1}{8}(\pi-1)$

$$S_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}\cdot\cos^2\theta \, d\theta$$
$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos 2\theta + 1}{4}\, d\theta$$
$$= \left[\frac{\sin 2\theta}{8} + \frac{\theta}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$
$$= \frac{\pi}{8} - \left(\frac{1}{8} + \frac{\pi}{16}\right)$$
$$= \frac{\pi}{16} - \frac{1}{8}$$
$$\therefore S = 2S_1 = \frac{\pi}{8} - \frac{1}{4}$$

D 4. The area of the region enclosed by the polar curve $r = 2+\sin\theta$ is

(A) 3π (B) $\frac{7\pi}{2}$ (C) 4π (D) $\frac{9\pi}{2}$

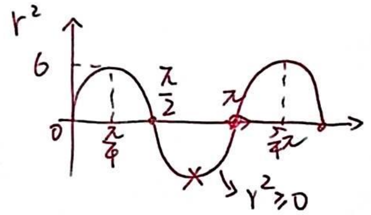


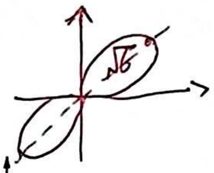


$$\int_0^{2\pi} \frac{1}{2}(2+\sin\theta)^2 \, d\theta$$
$$= \int_0^{2\pi} \frac{1}{2}\cdot(4 + 4\sin\theta + \sin^2\theta)\, d\theta$$
$$= \int_0^{2\pi} 2 + 2\sin\theta + \frac{1-\cos 2\theta}{4}\, d\theta$$
$$= \left[\frac{9}{4}\theta + (-2\cos\theta) - \frac{\sin 2\theta}{8}\right]_0^{2\pi} = \frac{9}{2}\pi$$

C 1. The area of the region enclosed by the polar curve $r^2 = 6\sin(2\theta)$ is

(A) 2 (B) 4 (C) 6 (D) 12





$$2 \cdot \int_0^{\frac{\pi}{2}} \frac{1}{2} r^2 \, d\theta$$
$$= \int_0^{\frac{\pi}{2}} 6\sin(2\theta) \, d\theta$$
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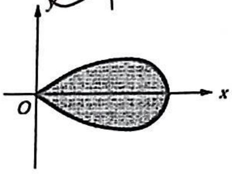

B 2. What is the area of the region enclosed by the loop of the graph of the polar curve $r = 2\cos(2\theta)$ shown in the figure above?

$\Rightarrow r = 0: \theta = -\frac{\pi}{4}, \frac{\pi}{4}$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} \cdot (2\cos 2\theta)^2 \, d\theta$$
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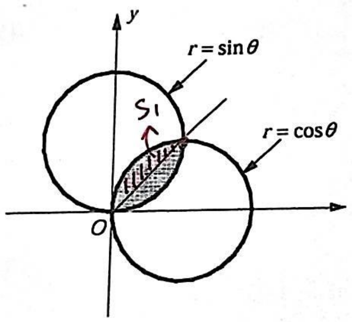

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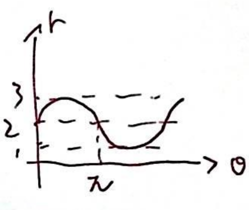
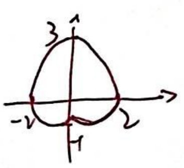
(A) $\frac{1}{8}(\pi - 2)$ (B) $\frac{1}{4}(\pi - 2)$ (C) $\frac{1}{2}(\pi - 2)$ (D) $\frac{1}{8}(\pi - 1)$

$$S_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} \cdot \cos^2\theta \, d\theta$$
$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos 2\theta + 1}{4} \, d\theta$$
$$= \left[\frac{\sin 2\theta}{8} + \frac{\theta}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$
$$= \frac{\pi}{8} - \left(\frac{1}{8} + \frac{\pi}{16}\right)$$
$$= \frac{\pi}{16} - \frac{1}{8}$$
$$\therefore S = 2S_1 = \frac{\pi}{8} - \frac{1}{4}$$

D 4. The area of the region enclosed by the polar curve $r = 2 + \sin\theta$ is

(A) 3π (B) $\frac{7\pi}{2}$ (C) 4π (D) $\frac{9\pi}{2}$





$$\int_0^{2\pi} \frac{1}{2} (2+\sin\theta)^2 \, d\theta$$
$$= \int_0^{2\pi} \frac{1}{2} \cdot (4 + 4\sin\theta + \sin^2\theta) \, d\theta$$
$$= \int_0^{2\pi} 2 + 2\sin\theta + \frac{1-\cos 2\theta}{4} \, d\theta$$
$$= \left[\frac{9}{4}\theta + (-2\cos\theta) - \frac{\sin 2\theta}{8}\right]_0^{2\pi} = \frac{9}{2}\pi$$

5. (a) Let $r = 2 + \cos(2\theta) = 2$

$\therefore 2\theta = \frac{\pi}{2} + k\pi$

$\therefore \theta = \frac{\pi}{4} + \frac{k\pi}{2}$

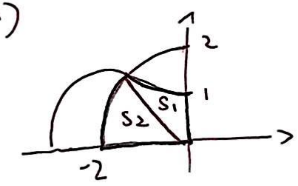
P: $\theta = \frac{\pi}{4}$. $r = 2 + \cos(2 \cdot \frac{\pi}{4}) = 2$

$\therefore (2, \frac{\pi}{4})$

(b) $\frac{1}{2}\int_0^{\frac{\pi}{4}} [2+\cos(2\theta)]^2 - 2^2 \, d\theta$

(c) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2^2 - [2+\cos(2\theta)]^2 \, d\theta$

(d)



intersection: $(2, \frac{3}{4}\pi)$

$S_1 = \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3}{4}\pi} [2+\cos(2\theta)]^2 \, d\theta$

$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3}{4}\pi} 4 + 4\cos 2\theta + \frac{\cos 4\theta + 1}{2} \, d\theta$

$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3}{4}\pi} \frac{9}{2} + 4\cos 2\theta + \frac{1}{2}\cos 4\theta \, d\theta$

$= \left[\frac{9}{4}\theta + \sin 2\theta + \frac{1}{16}\sin 4\theta\right]_{\frac{\pi}{2}}^{\frac{3}{4}\pi}$

$= \frac{9}{4} \cdot (\frac{3}{4}\pi - \frac{\pi}{2}) + (-1) = \frac{9}{16}\pi - 1$

$S_2 = \pi \cdot 2^2 \cdot \frac{1}{8} = \frac{\pi}{2}$

$\therefore R_3 = S_1 + S_2 = \frac{17}{16}\pi - 1$

(e) $dis = (2 + \cos 2\theta) - 2 = \cos 2\theta$

$\frac{d(dis)}{d\theta} = -2\sin 2\theta$ when $\theta = \frac{\pi}{6}$: $\frac{d(dis)}{d\theta} = -\sqrt{3}$

6.

$S = S_1 - S_2$

(a) $S_1 = \int_{2\pi}^{\frac{5}{2}\pi} \frac{1}{2} r(\theta)^2 \, d\theta$

$= \int_{2\pi}^{\frac{5}{2}\pi} \frac{1}{2} \theta^2 \sin^2\theta \cos^2\theta \, d\theta$

$= \int_{2\pi}^{\frac{5}{2}\pi} \frac{1}{8}\theta^2 \sin^2 2\theta \, d\theta$

$= \int_{2\pi}^{\frac{5}{2}\pi} \frac{1}{8} \theta^2 \frac{1-\cos 4\theta}{2} \, d\theta$

$= \int_{2\pi}^{\frac{5}{2}\pi} \frac{1}{16}\theta^2 - \frac{1}{16}\theta^2 \cos 4\theta \, d\theta$

$\int \theta^2 \cos 4\theta \, d\theta = \frac{1}{4}\theta^2 \sin 4\theta - 2\theta \cdot \left(-\frac{1}{16}\cos 4\theta\right) + 2 \cdot \left(-\frac{1}{64}\sin 4\theta\right) + C$

$= \frac{1}{4}\theta^2 \sin 4\theta + \frac{1}{8}\theta \cos 4\theta - \frac{1}{32}\sin 4\theta + C$

u	dv
θ^2	$\cos 4\theta$
2θ	$\frac{1}{4}\sin 4\theta$
2	$-\frac{1}{16}\cos 4\theta$
0	$-\frac{1}{64}\sin 4\theta$

$\therefore I = \left[\frac{1}{48}\theta^3 - \frac{1}{16}\cdot\left(\frac{1}{4}\theta^2 \sin 4\theta + \frac{1}{8}\theta\cos 4\theta - \frac{1}{32}\sin 4\theta\right)\right]_{2\pi}^{\frac{5}{2}\pi}$

$= \frac{1}{48}\cdot\frac{61}{8}\pi^3 - \frac{1}{16}\cdot\frac{1}{8}\left(\frac{5}{2}\pi - 2\pi\right)$

≈ 4.913

$S_2 = \frac{1}{2}\int_0^{\frac{\pi}{2}} \theta^2 \sin^2\theta \cos^2\theta \, d\theta$

$\therefore S \approx 4.845$

(b) $\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2} r^2 \, d\theta = \left[\frac{1}{48}\theta^3 - \frac{1}{16}\cdot\left(\frac{1}{4}\theta^2 \sin 4\theta + \frac{1}{8}\theta \cos 4\theta - \frac{1}{32}\sin 4\theta\right)\right]_{\frac{\pi}{2}}^{\pi}$

≈ 0.553

(c) $|r|$ is increasing on this interval. which means the curve is getting farther to the origin as θ grows.