1. Find $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ using the definition of derivative，if $f(x) = \sqrt{2x+1}$

2. $\lim_{h \to 0} \frac{\sqrt[3]{8+h}-2}{h} =$

3. $\lim_{h \to 0} \frac{(2+h)^5 - 32}{h}$ is

(A) $f'(5)$, where $f(x) = x^2$

(B) $f'(2)$, where $f(x) = x^5$

(C) $f'(5)$, where $f(x) = 2^x$

(D) $f'(2)$, where $f(x) = 2^x$

4. If f is a differentiable function, then $f'(1)$ is given by which of the following?

I. $\lim_{h \to 0} \frac{f(1+h)-f(1)}{h}$

II. $\lim_{x \to 1} \frac{f(x)-f(1)}{x-1}$

III. $\lim_{x \to 0} \frac{f(x+h)-f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only

5. What is the instantaneous rate of change at $x = -1$ of the function $f(x) = -\sqrt[3]{x^2}$?

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

6. $\lim_{h \to 0} \frac{\frac{1}{2}[\ln(e+h)-1]}{h}$ is

(A) $f'(1)$, where $f(x) = \ln\sqrt{x}$

(B) $f'(1)$, where $f(x) = \ln\sqrt{x+e}$

(C) $f'(e)$, where $f(x) = \ln\sqrt{x}$

(D) $f'(e)$, where $f(x) = \ln(\frac{x}{2})$

7. Let f be the function defined by $f(x)=\begin{cases} mx^2-2 & \text{if } x\le 1 \\ k\sqrt{x} & \text{if } x>1 \end{cases}$. If f is differentiable at $x=1$, what are the values of k and m?

$$f(x)=\begin{cases} 1-2x, & \text{if } x\le 1 \\ -x^2, & \text{if } x>1 \end{cases}$$

8. Let f be the function given above. Which of the following must be true?

I. $\lim_{x\to 1} f(x)$ exists.

II. f is continuous at $x=1$.

III. f is differentiable at $x=1$.

(A) I only
(B) I and II only
(C) II and III only
(D) I, II, and III

9. Let f be the function defined by

$$f(x)=\begin{cases} x+2 & \text{for } x\le 0 \\ \frac{1}{2}(x+2)^2 & \text{for } x>0. \end{cases}$$

(a) Find the left-hand derivative of f at $x=0$.

(b) Find the right-hand derivative of f at $x=0$.

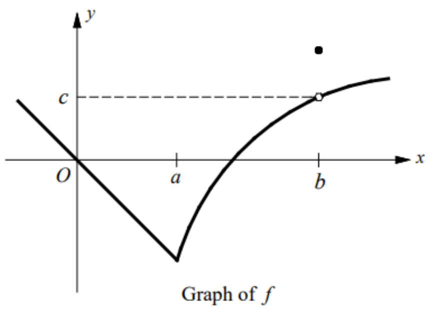
(c) Is the function f differentiable at $x=0$? Explain why or why not.

(d) Suppose the function g is defined by

$$g(x)=\begin{cases} x+2 & \text{for } x\le 0 \\ a(x+b)^2 & \text{for } x>0, \end{cases}$$

where a and b are constants. If g is differentiable at $x=0$, what are the values of a and b?

10.



Graph of f

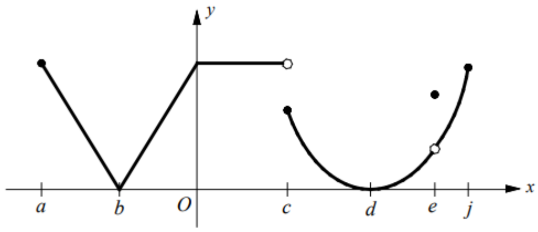
The graph of a function f is shown in the figure above. Which of the following statements must be false?

(A) $f(x)$ is defined for $0 \le x \le b$.

(B) $f(b)$ exists.

(C) $f'(b)$ exists.

(D) $\lim_{x \to a^-} f'(x)$ exists.



Graph of f

11. The graph of a function f is shown in the figure above. At how many points in the interval $a < x < j$ is f' not defined?

(A) 3 (B) 4 (C) 5 (D) 6

12. The equation of the line tangent to the graph of $y = x\sqrt{3} + x^2$ at the point $(1,2)$ is _______

13. If $f(x)=(x^3-2x+5)(x^{-2}+x^{-1})$, then $f'(1)=$

(A) -10 (B) -6 (C) $-\frac{9}{2}$ (D) $\frac{7}{2}$

14. If $f(x)=\frac{\sqrt{x}-1}{\sqrt{x}+1}$ then $f'(x)=$

(A) $\frac{\sqrt{x}}{(\sqrt{x}+1)^2}$

(B) $\frac{x}{(\sqrt{x}+1)^2}$

(C) $\frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$

(D) $\frac{\sqrt{x}-1}{\sqrt{x}(\sqrt{x}+1)^2}$

15. If $g(2)=3$ and $g'(2)=-1$, what is the value of $\frac{d}{dx}\left(\frac{g(x)}{x^2}\right)$ at $x=2$?

16. If $f(x)=\frac{x}{x-\frac{a}{x}}$ and $f'(1)=\frac{1}{2}$, what is the value of a?

17. If $y=x^2\cdot f(x)$, then $y''=$

(A) $x^2 f''(x)+x f'(x)+2f(x)$

(B) $x^2 f''(x)+x f'(x)+f(x)$

(C) $x^2 f''(x)+2x f'(x)+f(x)$

(D) $x^2 f''(x)+4x f'(x)+2f(x)$

18. Let $h(x)=x\cdot f(x)\cdot g(x)$. Find $h'(1)$, if $f(1)=-2$, $g(1)=3$, $f'(1)=1$, and $g'(1)=\frac{1}{2}$.

19. Let $g(x)=\frac{x}{\sqrt{x}-1}$. Find $g''(4)$.

20. If $f(x) = (x^2 - 3x)^{\frac{3}{2}}$, then $f'(4) =$

21.

If $f(x) = (3 - \sqrt{x})^{-1}$, then $f''(4) =$

(A) $\frac{3}{32}$ (B) $\frac{3}{16}$ (C) $\frac{3}{4}$ (D) $\frac{9}{4}$

22.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	1	-1
2	-2	1	-1	3
3	1	4	2	3
4	5	2	1	-2

The table above gives values of f, f', g, and g' at selected values of x.

(1) Find $h'(1)$, if $h(x) = f(g(x))$.

(2) Find $h'(2)$, if $h(x) = x f(x^2)$.

(3) Find $h'(3)$, if $h(x) = \frac{f(x)}{\sqrt{g(x)}}$.

(4) Find $h'(2)$, if $h(x) = [f(2x)]^2$

(5) Find $h'(1)$, if $h(x) = (x^9 + f(x))^{-2}$

23. Let $f(x)=xe^x$ and $f^{(n)}(x)$ be the nth derivative of f with respect to x. If $f^{(10)}(x)=(x+n)\,e^x$, what is the value of n?

24. If $y=x^x$, then $y'=$

(A) $x^x \ln x$ (B) $x^x(1+\ln x)$ (C) $x^x(x+\ln x)$ (D) $\dfrac{x^x \ln x}{x}$

25. If $y=e^{\sqrt{x^2+1}}$, then $y'=$

(A) $\sqrt{x^2+1}\, e^{\sqrt{x^2+1}}$

(B) $2x\sqrt{x^2+1}\, e^{\sqrt{x^2+1}}$

(C) $\dfrac{e^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$

(D) $\dfrac{xe^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$

26. If $y=x^{\ln\sqrt{x}}$, then $y'=$

(A) $\dfrac{x^{\ln\sqrt{x}}\ln x}{2x}$

(B) $\dfrac{x^{\ln\sqrt{x}}\ln x}{x}$

(C) $\dfrac{2x^{\ln\sqrt{x}}\ln x}{x}$

(D) $\dfrac{x^{\ln\sqrt{x}}(1+\ln x)}{x}$

27. If $3xy+x^2-2y^2=2$, then the value of $\dfrac{dy}{dx}$ at the point $(1,1)$ is

(A) 5 (B) $\dfrac{7}{2}$ (C) $-\dfrac{1}{2}$ (D) $-\dfrac{7}{2}$

28. If $3x^4-x^2-y^2=0$, then the value of $\dfrac{dy}{dx}$ at the point $(1,\sqrt{2})$ is

(A) $\dfrac{\sqrt{2}}{2}$ (B) $\dfrac{3\sqrt{2}}{2}$ (C) $\dfrac{5\sqrt{2}}{2}$ (D) $\dfrac{7\sqrt{2}}{2}$

29.

If $x^2y+2xy^2=5x$, then $\dfrac{dy}{dx}=$

(A) $\dfrac{5-4xy-4y}{x^2+4xy}$

(B) $\dfrac{5-2xy-2y^2}{x^2+4xy}$

(C) $\dfrac{5-2xy-y^2}{x^2+2xy}$

(D) $\dfrac{5-xy-2y}{x^2-2xy}$

30.

An equation of the line tangent to the graph of $3y^2-x^3-xy^2=7$ at the point $(1,2)$ is

(A) $y=\dfrac{3}{4}x-\dfrac{3}{8}$ (B) $y=\dfrac{3}{4}x+\dfrac{1}{2}$ (C) $y=-\dfrac{7}{8}x+\dfrac{3}{2}$ (D) $y=\dfrac{7}{8}x+\dfrac{9}{8}$

31.

An equation of the line normal to the graph of $2x^2+3y^2=5$ at the point $(1,1)$ is

(A) $y=\dfrac{3}{2}x+1$ (B) $y=\dfrac{3}{2}x-\dfrac{1}{2}$ (C) $y=-\dfrac{2}{3}x+\dfrac{5}{3}$ (D) $y=-\dfrac{2}{3}x+\dfrac{3}{2}$

32. Consider the curve given by $x^3-xy+y^2=3$.

(a) Find $\dfrac{dy}{dx}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is horizontal.

33. Consider the curve $x^2+y^2-xy=7$.

(a) Find $\dfrac{dy}{dx}$.

(b) Find all points on the curve whose x-coordinate is 2, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

34. If $y=(\sin x)^{1/x}$, then $y'=$

(A) $(\sin x)^{\frac{1}{x}}\left[\dfrac{\ln(\sin x)}{x}\right]$

(B) $(\sin x)^{\frac{1}{x}}\left[\dfrac{x-\ln(\sin x)}{x^2}\right]$

(C) $(\sin x)^{\frac{1}{x}}\left[\dfrac{x\sin x-\ln(\sin x)}{x^2}\right]$

(D) $(\sin x)^{\frac{1}{x}}\left[\dfrac{x\cot x-\ln(\sin x)}{x^2}\right]$

35. If $f(x) = e^{\tan x}, then\ f'\left(\frac{\pi}{4}\right) =$

36. If $f(x) = \ln(\cos x)\,, then\ f'(x) =$

37. If $f(x)=\ln\left[\sec(\ln x)\right]$, then $f'(e)=$

(A) $\dfrac{\cos 1}{e}$ (B) $\dfrac{\sin 1}{e}$ (C) $\dfrac{\tan 1}{e}$ (D) $\dfrac{\cot 1}{e}$

38. $\lim\limits_{h\to 0}\dfrac{\cos(\frac{\pi}{3}+h)-\frac{1}{2}}{h}=$

(A) $-\dfrac{1}{2}$ (B) $-\dfrac{\sqrt{3}}{2}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{\sqrt{3}}{2}$

39. $\lim\limits_{h\to 0}\dfrac{\sin 2(x+h)-\sin 2x}{h}=$

(A) $2\sin 2x$ (B) $-2\sin 2x$ (C) $2\cos 2x$ (D) $-2\cos 2x$

40. If $f(x)=\sin(\cos 2x)$, then $f'(\frac{\pi}{4})=$

(A) 0 (B) -1 (C) 1 (D) -2

41. If $y=a\sin x+b\cos x$, then $y+y''=$

(A) 0 (B) $2a\sin x$ (C) $2b\cos x$ (D) $-2a\sin x$

42. $\frac{d}{dx}\sec^2(\sqrt{x}) =$

(A) $\frac{2\sec(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

(B) $\frac{2\sec^2(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

(C) $\frac{\sec^2(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

(D) $\frac{\sec(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

43. $\frac{d}{dx}\left[x^2\cos 2x\right] =$

(A) $-2x\sin 2x$

(B) $2x(-x\sin 2x + \cos 2x)$

(C) $2x(x\sin 2x - \cos 2x)$

(D) $2x(x\sin 2x - \cos 2x)$

44. If $f(\theta) = \cos\pi - \frac{1}{2\cos\theta} + \frac{1}{3\tan\theta}$, then $f'(\frac{\pi}{6}) =$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	$-1/2$	$3/2$	4	$\sqrt{2}$
$\pi/4$	-2	1	2	3

45. The table above gives values of f, f', g, and g' at selected values of x.

Find $h'(\frac{\pi}{4})$, if $h(x) = f(x)\cdot g(\tan x)$.

46. If $xy + \tan(xy) = \pi$, then $\frac{dy}{dx} =$

(A) $-y\sec^2(xy)$ (B) $-y\cos^2(xy)$ (C) $-x\sec^2(xy)$ (D) $-\frac{y}{x}$

47.

Find the value of the constants a and b for which the function

$f(x)=\begin{cases}\sin x, & x<\pi \\ ax+b, & x\geq \pi\end{cases}$ is differentiable at $x=\pi$.

48.

An equation of the line normal to the graph of $y=\tan x$, at the point $(\frac{\pi}{6},\frac{1}{\sqrt{3}})$ is

(A) $y-\frac{1}{\sqrt{3}}=-\frac{1}{4}(x-\frac{\pi}{6})$

(B) $y-\frac{1}{\sqrt{3}}=\frac{1}{4}(x-\frac{\pi}{6})$

(C) $y-\frac{1}{\sqrt{3}}=-\frac{3}{4}(x-\frac{\pi}{6})$

(D) $y-\frac{1}{\sqrt{3}}=\frac{3}{4}(x-\frac{\pi}{6})$

49.

If $x+\sin y=y+3$, then $\frac{d^2y}{dx^2}=$

(A) $\frac{-\sin y}{(1-\cos y)^2}$ (B) $\frac{-\sin y}{(1+\cos y)^2}$ (C) $\frac{-\sin y}{(1-\cos y)^3}$ (D) $\frac{-\sin y}{(1+\cos y)^3}$

50.

Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(3)=4$ and $f'(4)=\frac{3}{2}$, then $g'(3)=$

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

51.

If $f(-3)=2$ and $f'(-3)=\frac{3}{4}$, then $(f^{-1})'(2)=$

(A) $\frac{1}{2}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{4}$

52.

If $f(x)=x^3-x+2$, then $(f^{-1})'(2)=$

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 4 (D) 6

53.

If $f(x)=\sin x$, then $(f^{-1})'(\frac{\sqrt{3}}{2})=$

(A) $\frac{1}{2}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) 2

54.

If $f(x)=1+\ln x$, then $(f^{-1})'(2)=$

(A) $-\frac{1}{e}$ (B) $\frac{1}{e}$ (C) $-e$ (D) e

55.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	3	-2	2	6
0	-2	-1	0	-3
1	0	1	-1	2
2	-1	4	3	-1

The functions f and g are differentiable for all real numbers. The table above gives the values of the functions and their first derivatives at selected values of x.

(a) If f^{-1} is the inverse function of f, write an equation for the line tangent to the graph of $y=f^{-1}(x)$ at $x=-1$.

(b) Let h be the function given by $h(x)=f(g(x))$. Find $h(1)$ and $h'(1)$.

(c) Find $(h^{-1})'(3)$, if h^{-1} is the inverse function of h.

56. $\frac{d}{dx}(\arcsin x^2) =$

57. If $f(x) = \arctan(e^{-x})$, *then* $f'(-1) =$

58. If $f(x) = \arctan(\sin x)$, *then* $f'\left(\frac{\pi}{3}\right) =$

59. If $f(x) = \cos(\sin^{-1} x)$, *then* $f'(x) =$

60. Let f be the function given by $f(x) = x^{\tan^{-1} x}$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = 1$.

61. Some values of differentiable function f are shown in the table below.
What is the approximation value of $f'(3.5)$?

x	3.0	3.3	3.8	4.2	4.9
$f(x)$	21.8	26.1	32.5	38.2	48.7

(A) 8 (B) 10 (C) 13 (D) 16

62.

Month	1	2	3	4	5	6
Temperature	−8	0	25	50	72	88

The normal daily maximum temperature F for a certain city is shown in the table above.

(a) Use data in the table to find the average rate of change in temperature from $t = 1$ to $t = 6$.

(b) Use data in the table to estimate the rate of change in maximum temperature at $t = 4$.

(c) The rate at which the maximum temperature changes for $1 \le t \le 6$ is modeled by $F(t) = 40 - 52\sin(\frac{\pi t}{6} - 5)$ degrees per minute. Find $F'(4)$ using the given model.