- **Stright-Line Motion**

1. If a particle moves in the *xy*-plane so that at time $t > 0$ its position vector is $(t^3 - 1, \ln\sqrt{t^2+1})$, then at time $t = 1$, its velocity vector is

(A) $(0, \frac{1}{2})$ (B) $(1, \frac{1}{2})$ (C) $(3, \frac{1}{2})$ (D) $(3, \frac{1}{4})$

2. A particle moves in the *xy*-plane so that at any time t its coordinates are $x = t^3 - t^2$ and $y = t + \ln t$. At time $t = 2$, its acceleration vector is

(A) $(4, \frac{1}{2})$ (B) $(6, \frac{1}{4})$ (C) $(8, \frac{3}{4})$ (D) $(10, -\frac{1}{4})$

3. A particle moves in the *xy*-plane so that its position at time $t > 0$ is given by $x(t) = e^t \cos t$ and $y(t) = e^t \sin t$. What is the speed of the particle when $t = 2$?

(A) $\sqrt{2}e$ (B) $\sqrt{2}e^2$ (C) $2e$ (D) $2e^2$

4. If f is a vector-valued function defined by $f(t) = \left(\ln(\sin t),\ t^2 + e^{-t}\right)$, then the acceleration vector is

(A) $(-\csc^2 t,\ 2 + e^{-t})$

(B) $(\sec^2 t,\ 2 + e^{-t})$

(C) $(\csc^2 t,\ 2 - e^{-t})$

(D) $(-\csc^2 t \cdot \cot t,\ 2 + e^{-t})$

5. A particle moves on the curve $y = x + \sqrt{x}$ so that the *x*-component has velocity $x'(t) = \cos t$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1, 0)$. At time $t = \frac{\pi}{2}$, the particle is at the point

(A) $(0, 0)$ (B) $(1,\ 2)$ (C) $(\frac{\pi}{2},\ \frac{\pi}{2} + \sqrt{\frac{\pi}{2}})$ (D) $(2,\ 2 + \sqrt{2})$

6. In the xy- plane, a particle moves along the curve defined by the equation $y = 2x^4 - x$ with a constant speed of 20 units per second. If $\frac{dy}{dt} > 0$, what is the value of $\frac{dx}{dt}$ when the particle is at the point $(1,\ 1)$

(A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) 4

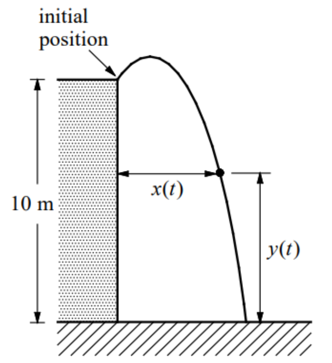
7. An object moving along a curve in the xy-plane is at position $(x(t),\ y(t))$ at time t, where $\dfrac{dx}{dt} = 1 + \cos(e^t)$. and $\dfrac{dy}{dt} = e^{(2-t^2)}$ for $t \geq 0$.

(a) At what time t is the speed of the object 3 units per second?

(b) Find the acceleration vector at time $t = 2$.

(c) Find the total distance traveled by the object over the time interval $1 \leq t \leq 4$.

(d) Find the magnitude of the displacement of the object over the time interval $1 \leq t \leq 4$.



Note: Figure not drawn to scale.

8. An object moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time $t \geq 0$, with $\dfrac{dx}{dt} = t - \sin(e^t)$. The derivative $\dfrac{dy}{dt}$ is not explicitly given. At time $t = 1$, the value of $\dfrac{dy}{dt}$ is 3 and the object is at position $(1, 4)$.

(a) Find the x-coordinate of the position of the object at time $t = 5$.

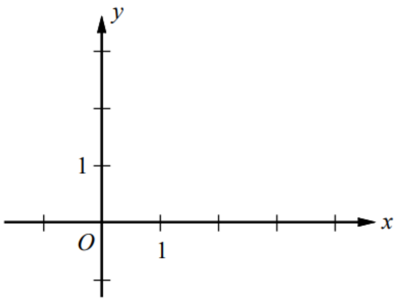
(b) Write an equation for the line tangent to the curve at the point $(x(1),\ y(1))$.

(c) Find the speed of the object at time $t = 1$.

(d) Suppose the line tangent to the curve at $(x(t),\ y(t))$ has a slope of $(t - 2)$ for $t \geq 0$. Find the acceleration vector of the object at time $t = 3$.

9. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t - \sin(\pi t)$ and $y(t) = 1 - \cos(\pi t)$ for $0 \le t \le 2$.

(a) On the axis provided below, sketch the graph of the path of the particle from $t = 0$ to $t = 2$. Indicate the direction of the particle along its path.



(b) Find the position of the particle when $t = 1$.

(c) Find the velocity vector for the particle at any time t.

(d) Write and evaluate an integral expression, in terms of sine and cosine, that gives the distance traveled of the particle from $t = 0$ to $t = 2$.

10. An object is thrown upward into the air 10 meters above the ground. The figure above shows the initial position of the object and the position at a later time. At time t seconds after the object is thrown upward, the horizontal distance from the initial position is given by $x(t)$ meters, and the vertical distance from the ground is given by $y(t)$ meters, where $\frac{dx}{dt} = 1.4$ and $\frac{dy}{dt} = 4.2 - 9.8t$, for $t \ge 0$.

(a) Find the time t when the object reaches its maximum height.

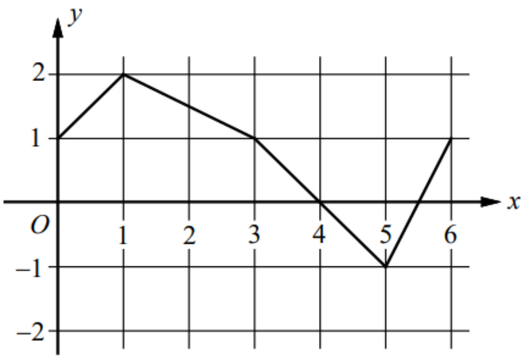
(b) Find the maximum vertical distance from the ground to the object.

(c) Find the time t when the object hit the ground.

(d) Find the total distance traveled by the object from time $t = 0$ until the object hit the ground.

(e) Find the magnitude of the displacement of the object from time $t = 0$ until the object hit the ground.

(f) Find the angle θ, $0 < \theta < \frac{\pi}{2}$, between the path of the object and the ground at the instance the object hit the ground.



11. At time t, the position of particle moving in the xy- plane is given by the parametric functions $(x(t),\ y(t))$, where $\frac{dx}{dt} = e^{\sqrt{x}} - \cos(x^2)$. The graph of y consisting of four line segments, is shown in the figure above. At time $t = 0$, the particle is at position $(2,1)$.

(a) Find the position of the particle at $t = 2$.

(b) Find the slope of the line tangent to the path of the particle at $t = 2$.

(c) Find the magnitude of the velocity vector at $t = 2$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 3$.

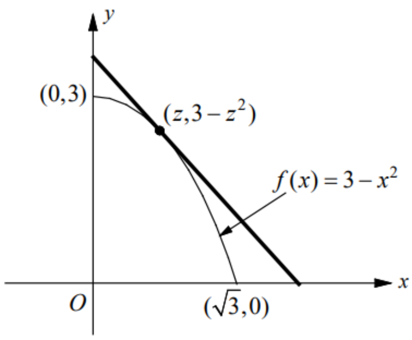
- **Optimization Problems**

1. If $y=\frac{1}{\sqrt{x}}-\sqrt{x}$, what is the maximum value of the product of xy ?

(A) $\frac{1}{9}$ (B) $\frac{\sqrt{3}}{9}$ (C) $\frac{2\sqrt{3}}{9}$ (D) $\frac{2}{3}$

2. If the maximum value of the function $y=\frac{\cos x-m}{\sin x}$ is at $x=\frac{\pi}{4}$, what the value of m ?

(A) $-\sqrt{2}$ (B) $\sqrt{2}$ (C) -1 (D) 1



3. The figure above shows the graph of the function $f(x)=3-x^2$. For $0<z<\sqrt{3}$, let $A(z)$ be the area of the triangle formed by the coordinate axes and the line tangent to the graph of f at the point $(z,3-z^2)$.

(a) Find the equation of the line tangent to the graph of f at the point $(z,3-z^2)$.

(b) For what values of z does the triangle bounded by the coordinate axis and tangent line have a minimum area?

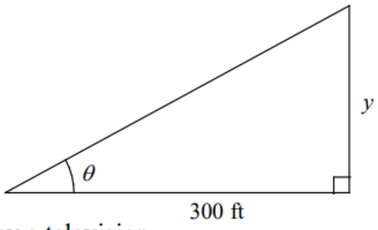
- **Related Rates**

1. The radius of a circle is changing at the rate of $1/\pi$ inches per second. At what rate, in square inches per second, is the circle's area changing when $r = 5$ in ?

   (A) $\frac{5}{\pi}$ (B) 10 (C) $\frac{10}{\pi}$ (D) 15

2. The volume of a cube is increasing at the rate of $12\ \text{in}^3/\text{min}$. How fast is the surface area increasing, in square inches per minute, when the length of an edge is 20 in?

   (A) 1 (B) $\frac{6}{5}$ (C) $\frac{4}{3}$ (D) $\frac{12}{5}$



3. In the figure shown above, a hot air balloon rising straight up from the ground is tracked by a television camera 300 ft from the liftoff point. At the moment the camera's elevation angle is $\pi/6$, the balloon is rising at the rate of 80 ft/min . At what rate is the angle of elevation changing at that moment?

   (A) 0.12 radian per minute
   (B) 0.16 radian per minute
   (C) 0.2 radian per minute
   (D) 0.4 radian per minute

4. A car is approaching a right-angled intersection from the north at 70 mph and a truck is traveling to the east at 60 mph. When the car is 1.5 miles north of the intersection and the truck is 2 miles to the east, at what rate, in miles per hour, is the distance between the car and truck is changing?

   (A) Decreasing 15 miles per hour
   (B) Decreasing 9 miles per hour
   (C) Increasing 6 miles per hour
   (D) Increasing 12 miles per hour

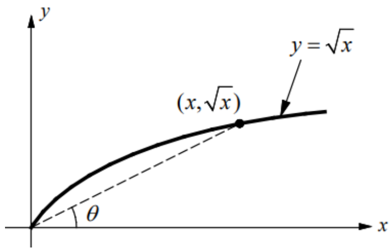
5. The radius r of a sphere is increasing at a constant rate. At the time when the surface area and the radius of sphere are increasing at the same numerical rate, what is the radius of the sphere? (The surface area of a sphere is $S = 4\pi r^2$.)

   (A) $\frac{1}{8\pi}$ (B) $\frac{1}{4\pi}$ (C) $\frac{1}{3\pi}$ (D) $\frac{\pi}{8}$

6. If the radius r of a cone is decreasing at a rate of 2 centimeters per minute while its height h is increasing at a rate of 4 centimeters per minute, which of the following must be true about the volume V of the cone?

   (A) V is always decreasing.
   (B) V is always increasing.
   (C) V is increasing only when $r > h$.
   (D) V is increasing only when $r < h$.

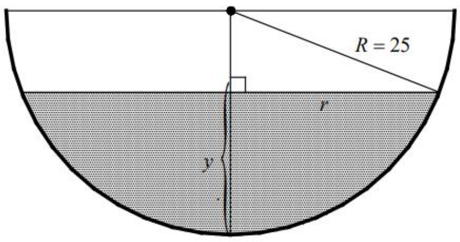
7. A particle moves along the curve $y=\sqrt{x}$. When $y=2$ the x-component of its position is increasing at the rate of 4 units per second.



(a) What is the value of $\frac{dy}{dt}$ when $y=2$?

(b) How fast is the distance from the particle to the origin changing when $y=2$?

(c) What is the value of $\frac{d\theta}{dt}$ when $y=2$?

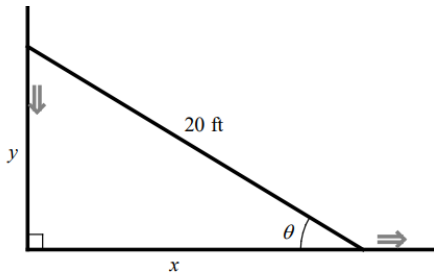


8. As shown in the figure above, water is draining at the rate of $12\ \text{ft}^3/\text{min}$ from a hemispherical bowl of radius 25 feet. The volume of water in a hemispherical bowl of radius R when the depth of the water is y meters is given as $V=\frac{\pi}{3}y^2(3R-y)$.

(a) Find the rate at which the depth of water is decreasing when the water is 18 meters deep. Indicate units of measure.

(b) Find the radius r of the water's surface when the water is y feet deep.

(c) At what rate is the radius r changing when the water is 18 meters deep. Indicate units of measure.



9. In the figure shown above, the top of a 20-foot ladder is sliding down a vertical wall at a constant rate of 2 feet per second.

   (a) When the top of the ladder is 12 feet from the ground, how fast is the bottom of the ladder moving away from the wall?

   (b) The triangle is formed by the wall, the ladder and the ground. At what rate is the area of the triangle is changing when the top of the ladder is 12 feet from the ground?

   (c) At what rate is the angle θ between the ladder and the ground is changing when the top of the ladder is 12 feet from the ground?

10. Consider the curve given by $2y^2 + 3xy = 1$.

    (a) Find $\dfrac{dy}{dx}$.

    (b) Find all points (x, y) on the curve where the line tangent to the curve has a slope of $-\dfrac{3}{4}$.

    (c) Let x and y be functions of time t that are related by the equation $2y^2 + 3xy = 1$. At time $t = 3$, the value of y is 2 and $\dfrac{dy}{dt} = -2$. Find the value of $\dfrac{dx}{dt}$ at time $t = 3$.

11. A man 6 feet tall walks at a rate of 3 feet per second away from a light that is 20 feet above the ground.

    (a) At what rate is the tip of his shadow moving when he is 12 feet from the base of the light.

    (b) At what rate is the length of his shadow changing when is 12 feet from the base of the light.

➢ Linear Approximation of a Function

1. For small values of h, the function $h(x) = \sqrt[3]{8+h}$ is best approximated by which of the following?

(A) $\frac{h}{12}$ (B) $2-\frac{h}{12}$ (C) $2+\frac{h}{12}$ (D) $3+\frac{h}{12}$

2. The approximate value of $y = \sqrt{1-\sin x}$ at $x = -0.1$, obtained from the line tangent to the graph at $x = 0$, is

(A) 0.9 (B) 0.95 (C) 1.01 (D) 1.05

3. Let $y = x^2 \ln x$. When $x = e$ and $dx = 0.1$, the value of dy is

(A) $\frac{e}{10}$ (B) $\frac{e}{5}$ (C) $\frac{3e}{10}$ (D) $\frac{2e}{5}$

4. Let f be a differentiable function such that $f(2) = \frac{5}{2}$ and $f'(2) = \frac{1}{2}$. If the line tangent to the graph of f at $x = 2$ is used to find an approximation of a zero of f, that approximation is

(A) -3 (B) -2.4 (C) -1.8 (D) -1.2

5. The approximate value of $y = \frac{1}{\sqrt{x}}$ at $x = 4.1$, obtained from the line tangent to the graph at $x = 4$ is

(A) $\frac{39}{80}$ (B) $\frac{79}{160}$ (C) $\frac{1}{2}$ (D) $\frac{81}{160}$

6. Let f be the function given by $f(x) = x^2 - 4x + 5$. If the line tangent to the graph of f at $x = 1$ is used to find an approximate value of f, which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?

(A) 1.5 (B) 1.6 (C) 1.7 (D) 1.8

7. The linear approximation to the function f at $x = a$ is $y = \frac{1}{2}x - 3$. What is the value of $f(a) + f'(a)$ in terms of a?

(A) $a-4$

(B) $a-\frac{5}{2}$

(C) $\frac{1}{2}a-4$

(D) $\frac{1}{2}a-\frac{5}{2}$

8. Let f be the function given by $f(x)=\dfrac{2}{e^{\sin x}+1}$.

(a) Write an equation for the line tangent to the graph of f at $x=0$.

(b) Using the tangent line to the graph of f at $x=0$, approximate $f(0.1)$.

(c) Find $f^{-1}(x)$.

x	-2	0	1	3	6
$f(x)$	-1	-4	-3	0	7

9. Let f be a twice differentiable function such that $f'(3)=\dfrac{9}{5}$. The table above gives values of f for selected points in the closed interval $-2 \le x \le 6$.

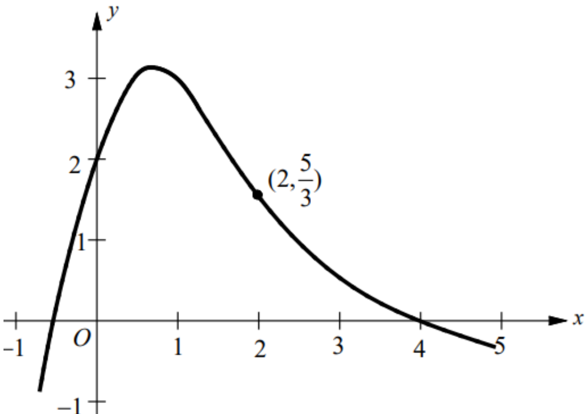
(a) Estimate $f'(0)$. Show the work that leads to your answer.

(b) Write an equation for the line tangent to the graph of f at $x=3$.

(c) Write an equation of the secant line for the graph of f on $1 \le x \le 6$.

(d) Suppose $f''(x)>0$ for all x in the closed interval $1 \le x \le 6$. Use the line tangent to the graph of f at $x=3$ to show $f(5) \ge \dfrac{18}{5}$.

(e) Suppose $f''(x)>0$ for all x in the closed interval $1 \le x \le 6$. Use the secant line for the graph of f on $1 \le x \le 6$ to show $f(5) \le 5$.



Graph of f'

10. Let f be twice differentiable function on the interval $-1 < x < 5$ with $f(1) = 0$ and $f(2) = 3$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x- axis at $x = -0.5$ and $x = 4$. Let h be the function given by $h(x) = f(\sqrt{x+1})$.

(a) Write an equation for the line tangent to the graph of h at $x = 3$.

(b) The second derivative of h is $h''(x) = \frac{1}{4}\left[\frac{\sqrt{x+1}f''(\sqrt{x+1}) - f'(\sqrt{x+1})}{(x+1)^{3/2}}\right]$. Is $h''(3)$ positive, negative, or zero? Justify your answer.

(c) Suppose $h''(x) < 0$ for all x in the closed interval $0 \le x \le 3$. Use the line tangent to the graph of h at $x = 3$ to show $h(2) \le \frac{31}{12}$. Use the secant line for the graph of h on $0 \le x \le 3$ to show $h(2) \ge 2$