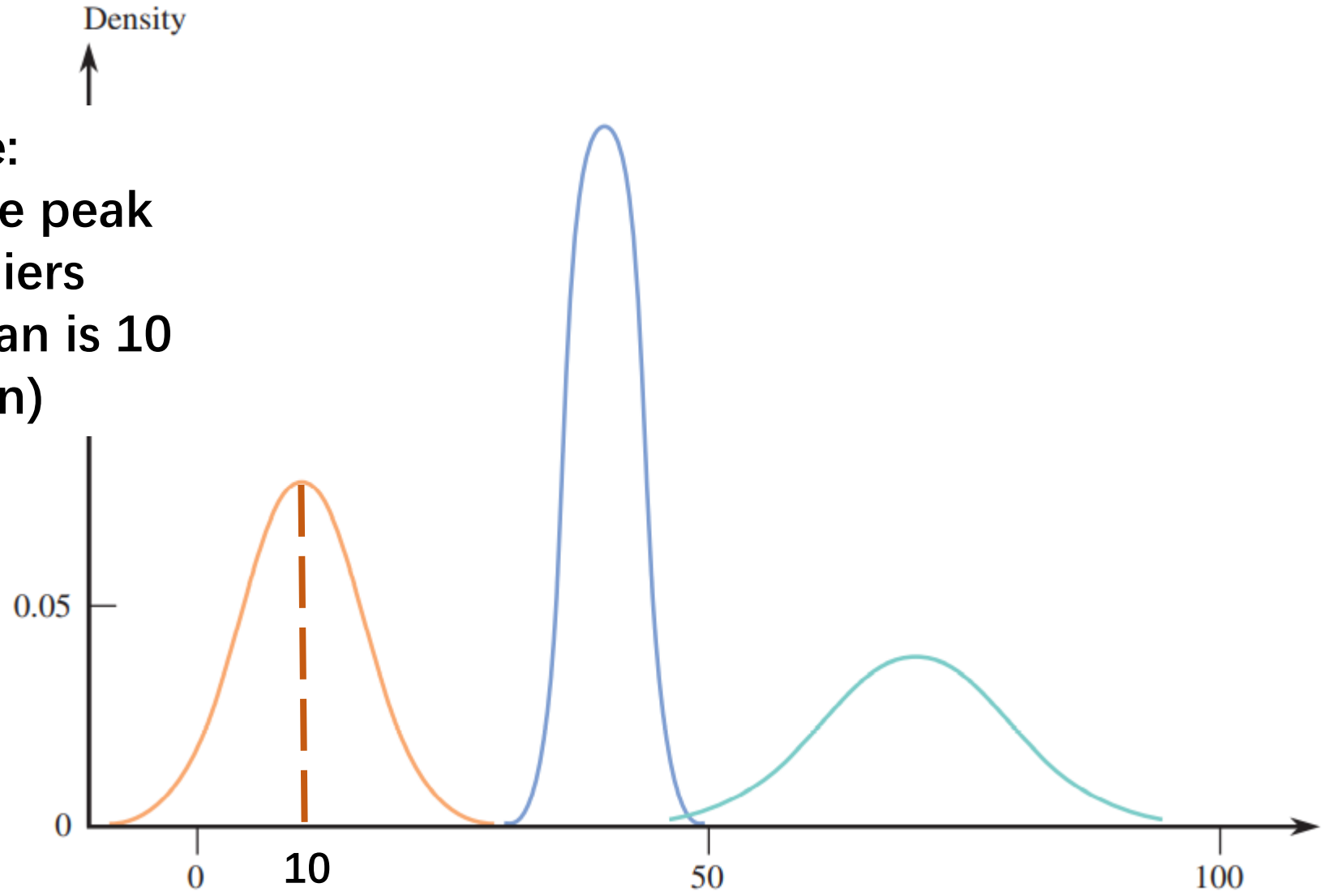
Normal Distribution

Normal Curves

Describe the density curve:

- **Shape: symmetric, single peak**
- **Outlier: no obvious outliers**
- **Center: median and mean is 10**
- **Spread: sigma (unknown)**



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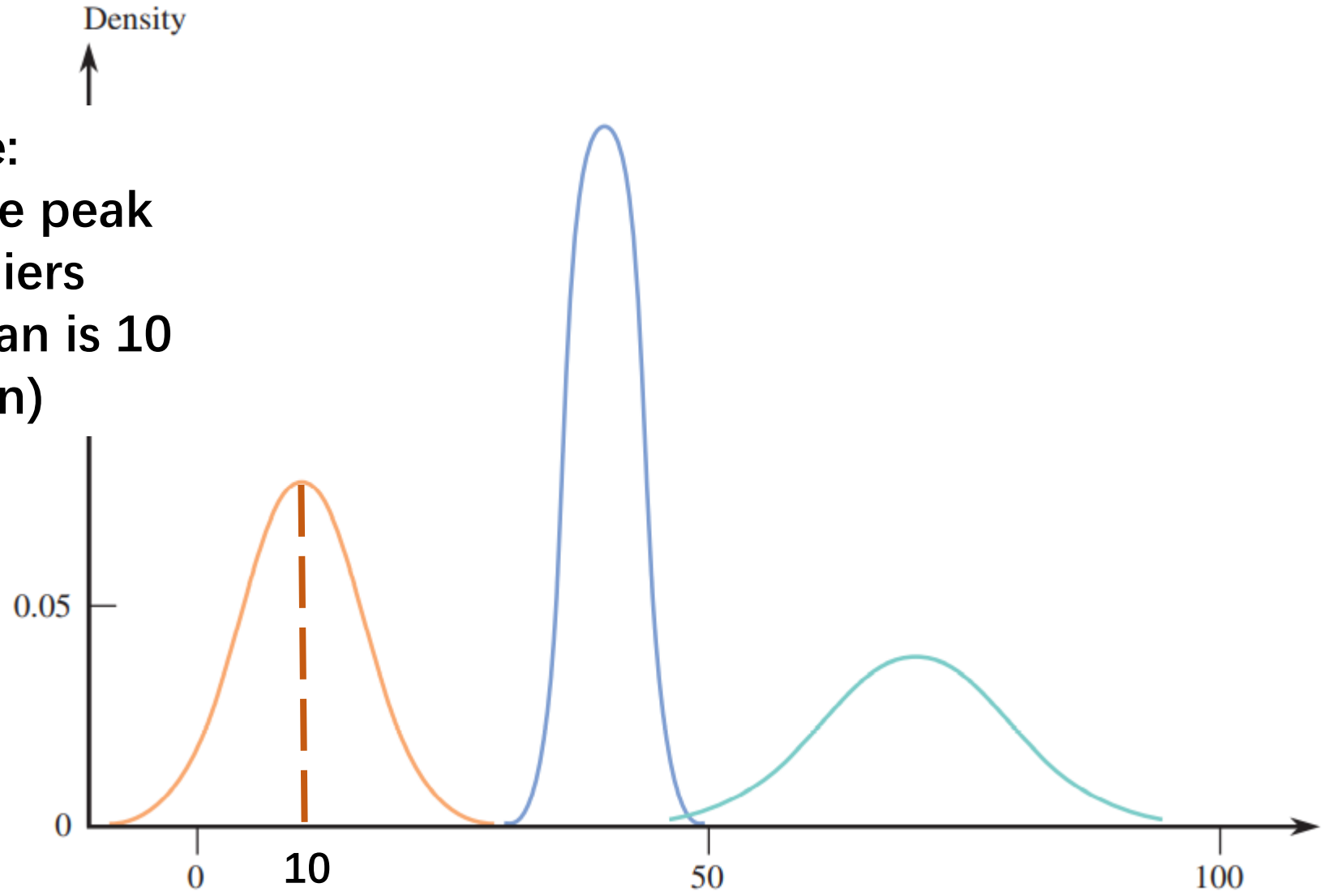
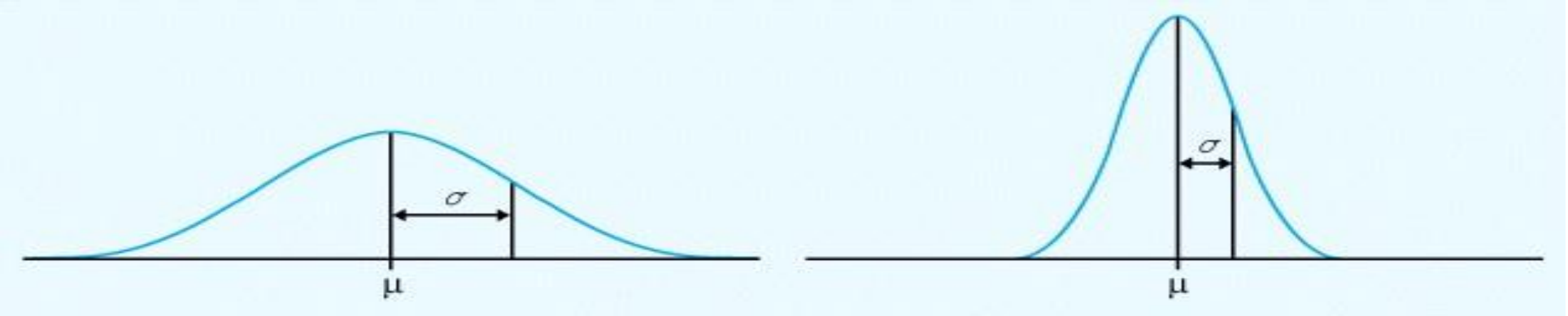


Normal Curves

- A Normal distribution is described by a **symmetric, single-peaked, bell-shaped** density curve called a Normal curve.
- Any Normal distribution is completely specified by two numbers:

  its mean μ and standard deviation σ.
- All Normal distributions have **the same overall shape**. $X \sim N\left(\mu, \sigma^2\right)$

  Any differences can be explained by μ and σ.



Property

If X follows the normal distribution, aX+b still follows the normal distribution

$X \sim N(\mu, \sigma^2)$ aX+b~?

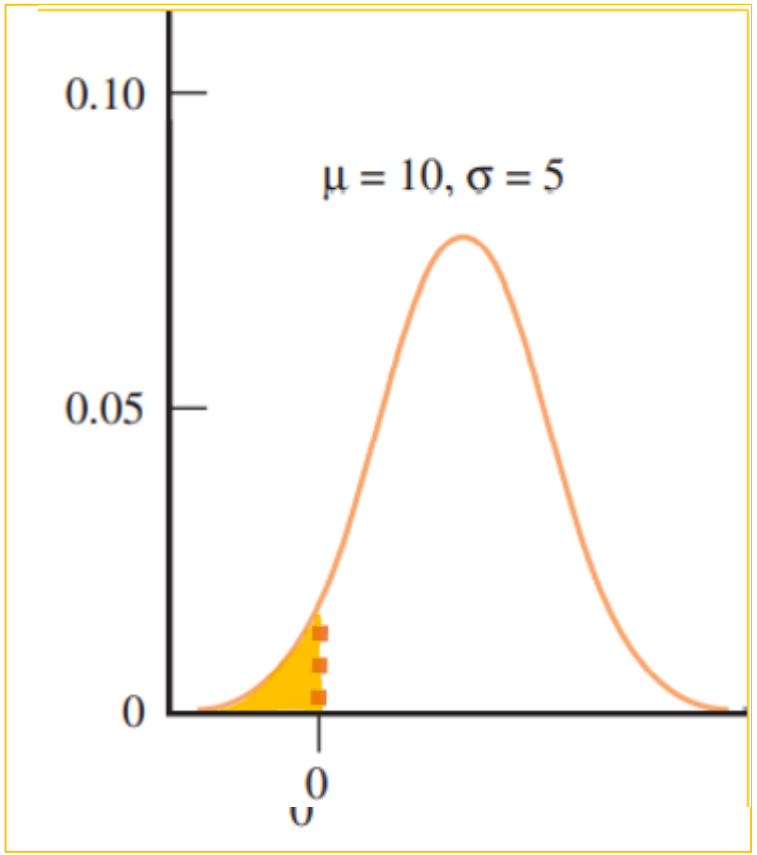
All Normal distributions have **the same overall shape**.

Any differences can be explained by μ and σ.

E(aX+b) = ? $a\mu + b$

Var(aX+b) = ? $a^2\sigma^2$

aX+b~N($a\mu + b, a^2\sigma^2$)



What is the value of P(X<0)?

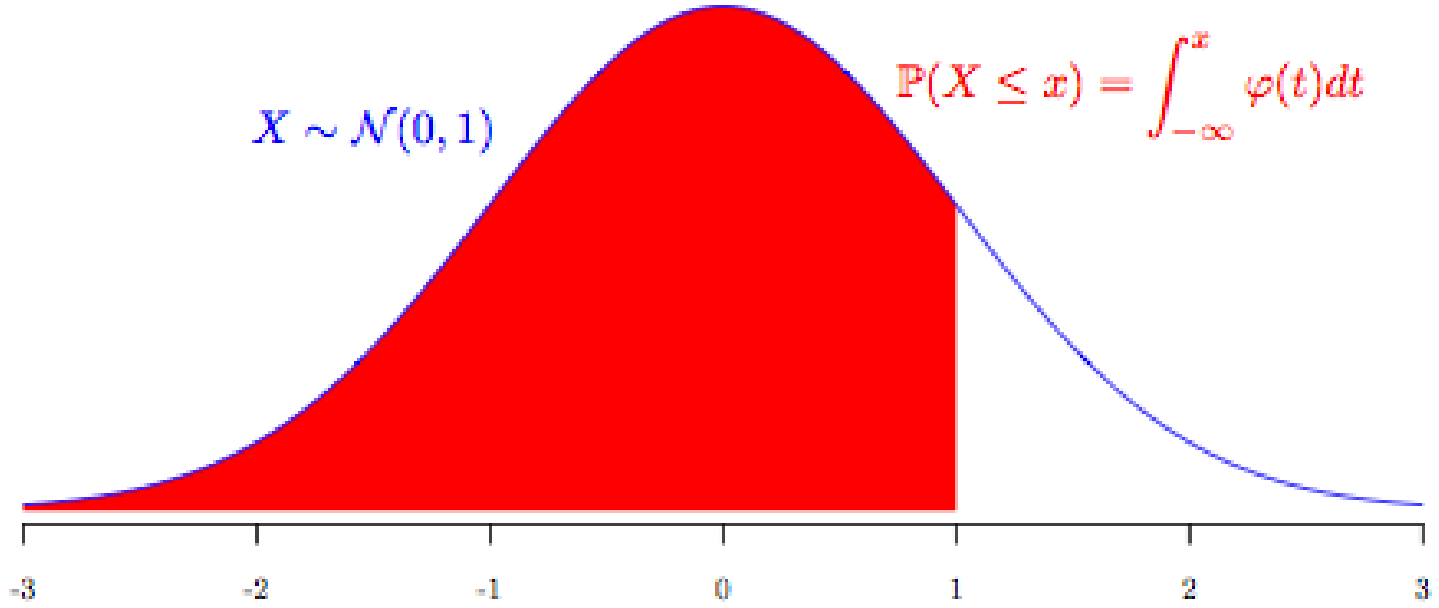
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

【Not required】

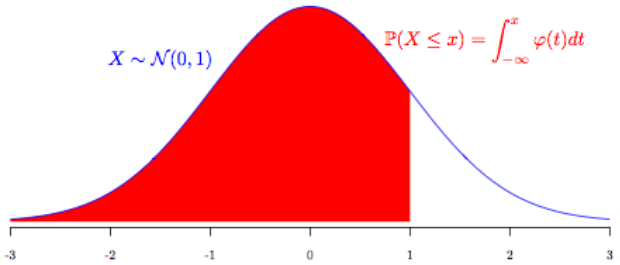
☹

DEFINITION **Standard Normal distribution**

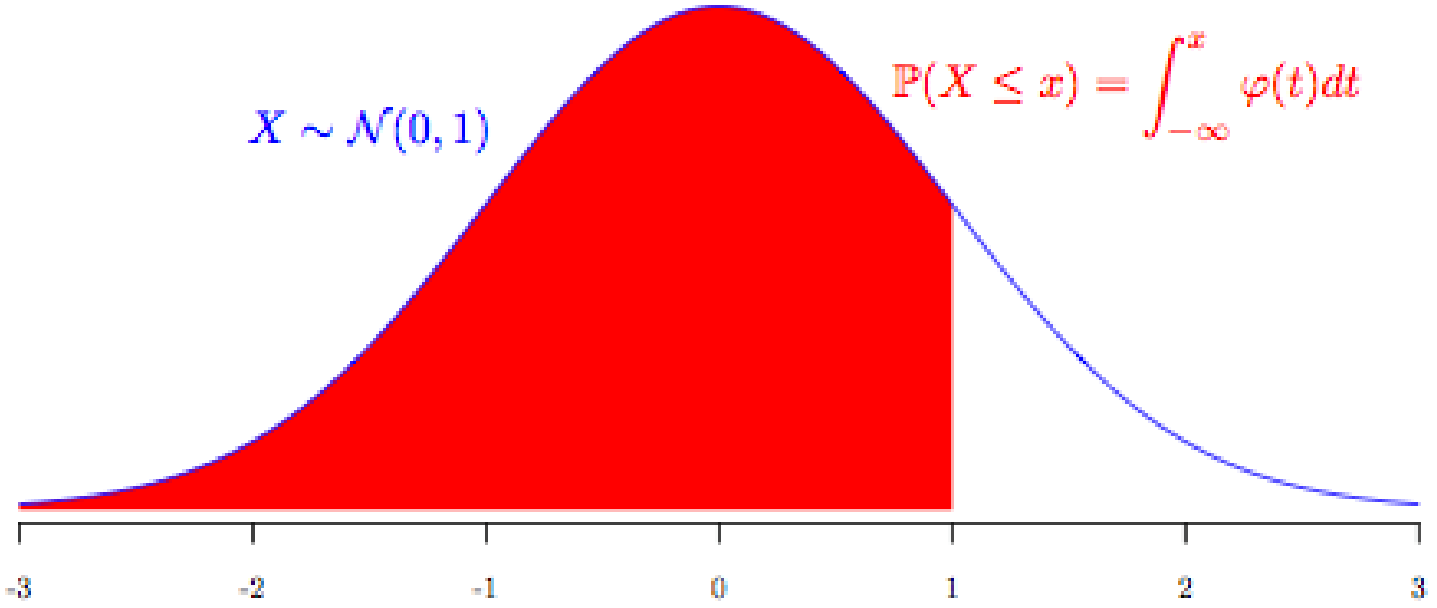
The **standard Normal distribution** is the Normal distribution with mean 0 and standard deviation 1.

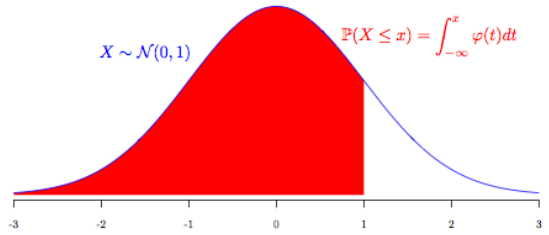


BUT we only have the distribution table of the STANDARD NORMAL DISTRIBUTION!!!



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990





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2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
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2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
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2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

$$P(X \leq \mathbf{1.33}) = 0.9082$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
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2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

Use the table of Standard Normal Curve Areas to find

P(-1.76<Z<0.58)

P(-1.76<Z<0.58)

= P(Z<0.58) – P(Z<-1.76)

= P(Z<0.58) – [1-P(Z<1.76)]

= 0.7190 – 0.0392

= 0.6798

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
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1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633

$X \sim N(\mu, \sigma^2)$ X $\overset{?}{\Leftrightarrow}$ standard normal random variable

$X-\mu \sim ?$ $N(0, \sigma^2)$

$\frac{X-\mu}{\sigma} \sim ?$ $N(0,1)$

Z-value (Z-score)

$$Z = \frac{X - \mu}{\sigma}$$

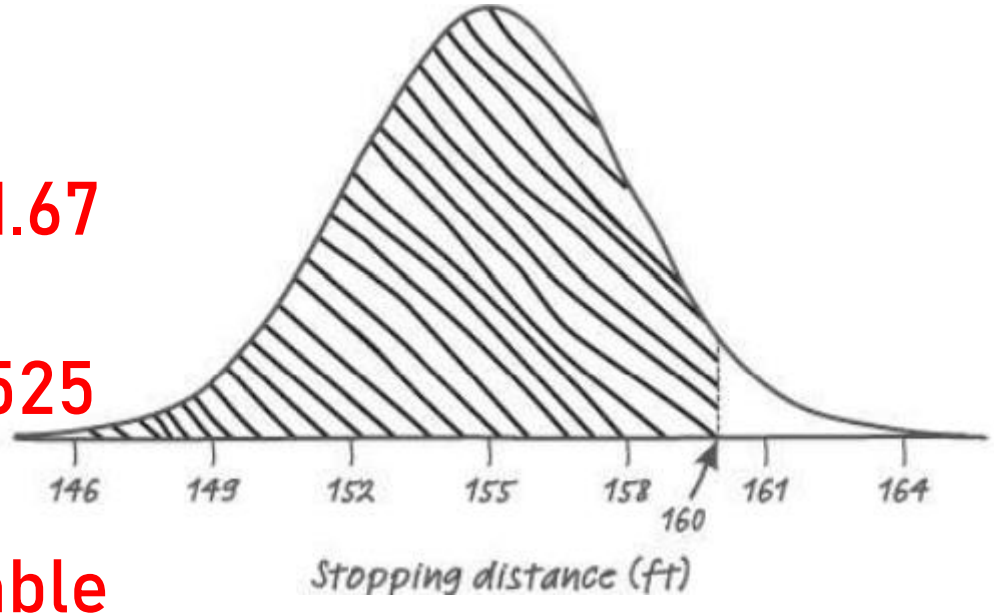
$P(X<x) = P(\frac{X-\mu}{\sigma}<\frac{x-\mu}{\sigma}) = P(Z<\frac{x-\mu}{\sigma})$ Then use the table!

Studies on automobile safety suggest that stopping distances follow an approximately Normal distribution. For one model of car traveling at 62 mph, the mean stopping distance is $\mu=155$ ft with a standard deviation of $\sigma=3$ ft . Danielle is driving one of these cars at 62 mph when she spots a wreck 160 feet in front of her and needs to make an emergency stop. About what percent of cars of this model when going 62 mph would be able to make an emergency stop in less than 160 feet? Is Danielle likely to stop safely?

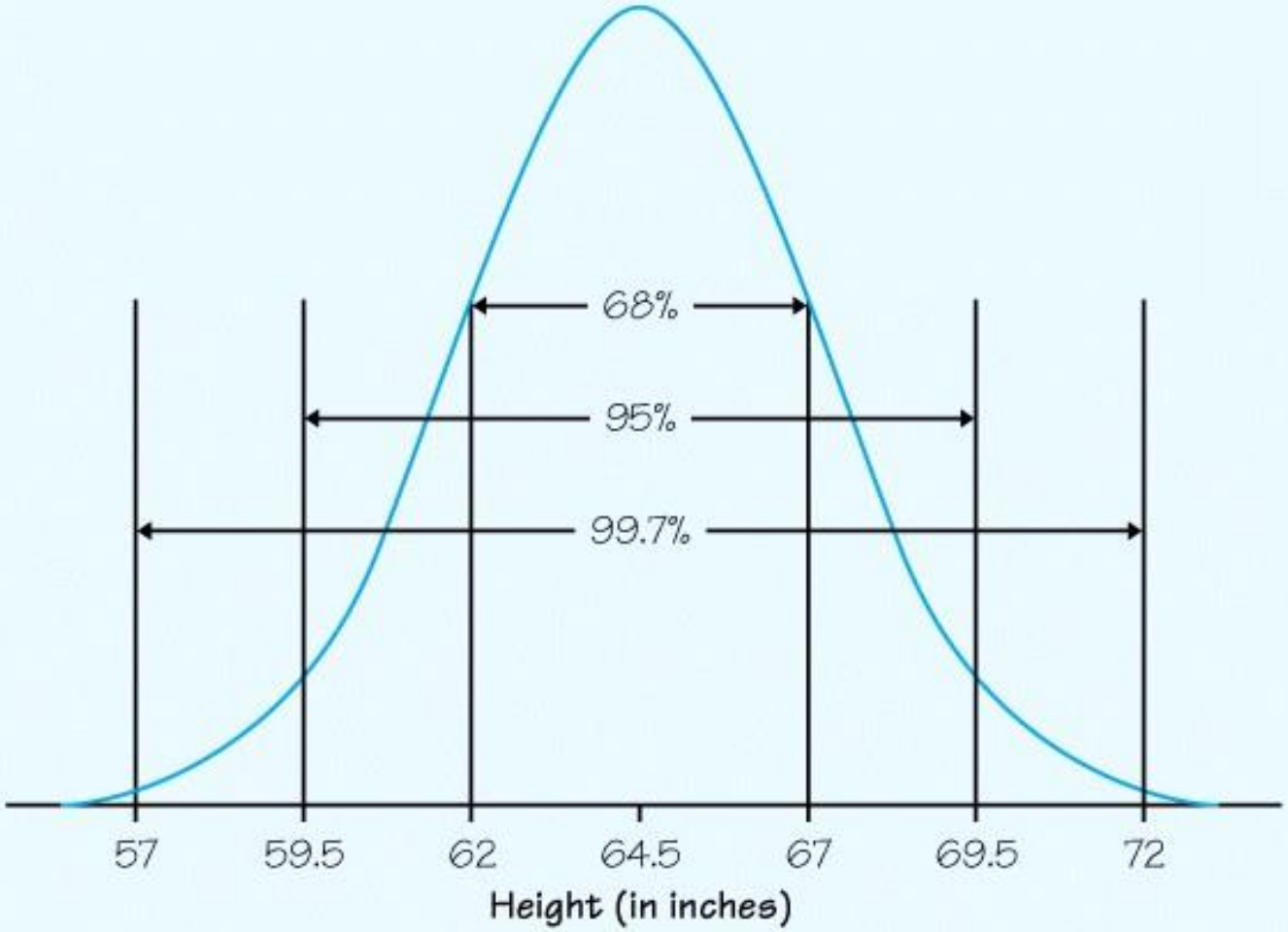
$Z = (160 - 155)/3 = 1.67$

Using the table, we have the area for $Z < 1.67$ is 0.9525.

$P(\text{stopping distance}<160) = P(Z<1.67) = 0.9525$



About 95% of cars of this model would be able to make an emergency stop within 160 feet. So Danielle is likely to be able to stop safely.



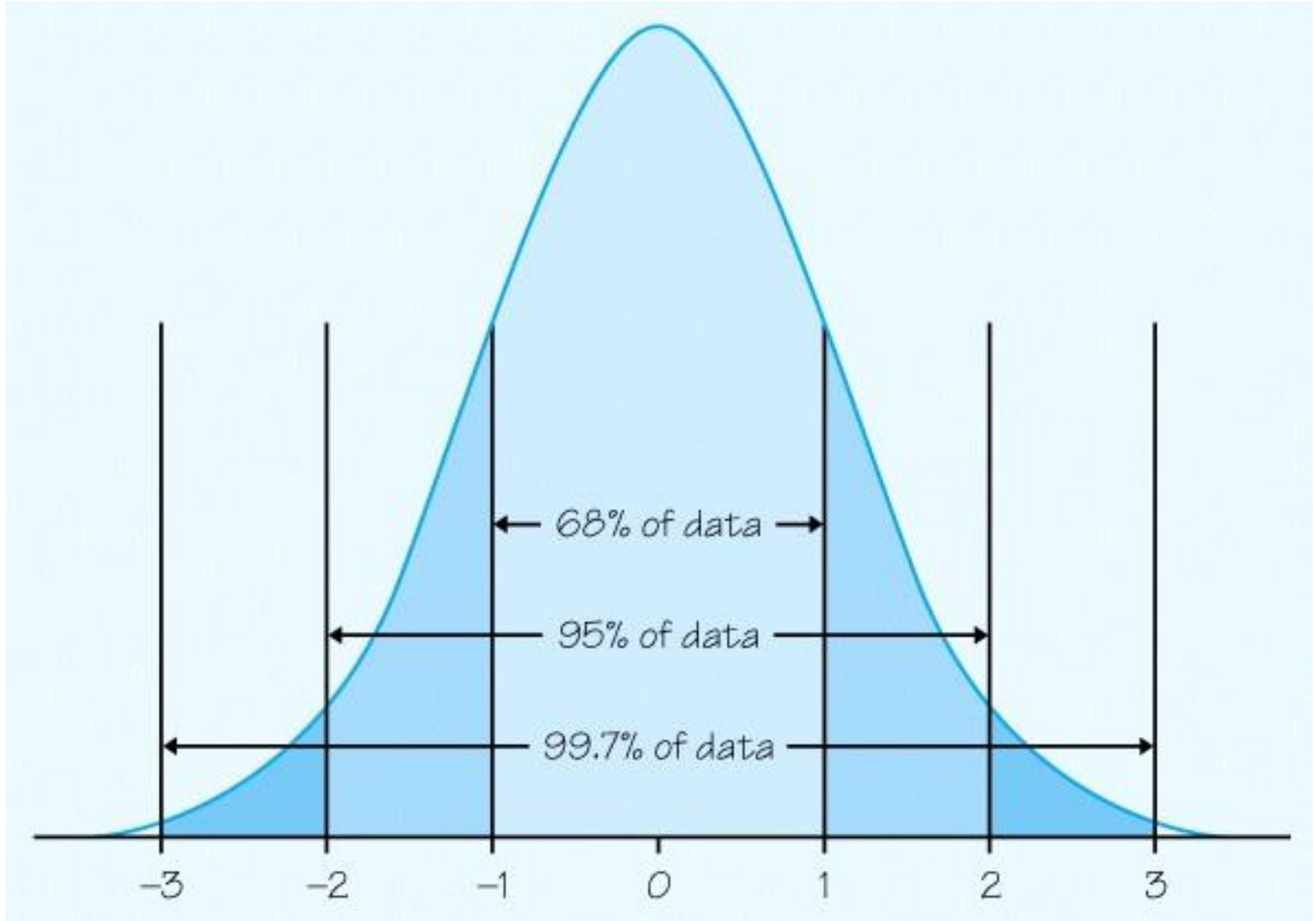
The 68–95–99.7 RULE

By remembering these three numbers, you can quickly estimate proportions of observations using Normal distributions and recognize when an observation is unusual.

In a Normal distribution with mean μ and standard deviation σ:

- Approximately **68%** of the observations fall within σ of the mean μ.
- Approximately **95%** of the observations fall within 2σ of the mean μ.
- Approximately **99.7%** of the observations fall within 3σ of the mean μ.

This result is known as the **68–95–99.7 rule.**


68% of data
95% of data
99.7% of data
−3
−2
−1
0
1
2
3

Percentiles

The *percentile* of a score, x, is the percentage of scores which fall at or below the score.

The kth percentile P_k:

$$\int_{-\infty}^{P_k} f(x)\,dx = \text{k\%}$$

Determine the value of each of the following percentiles for the standard normal distribution (Hint: If the cumulative area that you must look for does not appear in the z table, use the closest entry):

a. The 91st percentile

b. The 77th percentile

c. The 50th percentile

d. The 9th percentile

e. What is the relationship between the 70th z percentile and the 30th z percentile?

Determine the value of each of the following percentiles for the standard normal distribution (Hint: If the cumulative area that you must look for does not appear in the z table, use the closest entry):

a. The 91st percentile **1.341**

b. The 77th percentile **0.737**

c. The 50th percentile **0**

d. The 9th percentile **-1.34**

e. What is the relationship between the 70th z percentile and the 30th z percentile?

70th z percentile = - 30th z percentile

Data from the paper “Fetal ⋯ Composition” suggest that a normal distribution with mean 3500 grams and standard deviation 600 grams is a reasonable model for the probability distribution of the continuous numerical variable X = birth weight of a randomly selected full-term baby. What proportion of birth weights are between 2900 and 4700 grams?

$$a^* = \frac{a - \mu}{\sigma} = \qquad P(2900 < x < 4700) = P(-1.00 < z < 2.00)$$

$$= (z \text{ curve area to the left of } 2.00)$$

$$-(z \text{ curve area to the left of } -1.00)$$

$$b^* = \frac{b - \mu}{\sigma} = \qquad = .9772 - .1587$$

$$= .8185$$

Garbage trucks entering a particular waste management facility are weighed and then they offload garbage into a landfill. Data from the paper “Estimating ··· GPS” suggest that a normal distribution with mean 13 minutes and standard deviation 3.9 minutes is a reasonable model for the probability distribution of the random variable X = total processing time for a garbage truck at this waste management facility (total proce d to weigh to descr 0% with the lo times corres ation.

For the standard normal distribution, the largest 10% are those with z values greater than $z^* = 1.28$ (from Appendix Table 2, based on a cumulative area of .90). Then

$$\begin{aligned} x^* &= \mu + z^*\sigma \\ &= 13 + 1.28(3.9) \\ &= 13 + 4.992 \\ &= 17.992 \end{aligned}$$

About 10% of the garbage trucks using this facility would have a total processing time of more than 17.992 minutes.

Determine the value of z* such that

a.-z* and z* separate the middle 95% of all z values from the most extreme 5%

b.-z* and z* separate the middle 90% of all z values from the most extreme 10%

c.-z* and z* separate the middle 98% of all z values from the most extreme 2%

d.-z* and z* separate the middle 92% of all z values from the most extreme 8%

Determine the value of z* such that

a.-z* and z* separate the middle 95% of all z values from the most extreme 5% z*=1.96

b.-z* and z* separate the middle 90% of all z values from the most extreme 10% z*=1.645

c.-z* and z* separate the middle 98% of all z values from the most extreme 2% z*=2.33

d.-z* and z* separate the middle 92% of all z values from the most extreme 8% z*=1.75