Lecture 2

Conditional Probability

Students in a college statistics class wanted to find out how common it is for young adults to have their ears pierced. They recorded data on two variables—gender and whether or not the student had a pierced ear—for all 178 people in the class. The two-way table summarizes the data

		Gender		
		Male	Female	Total
Pierced ear	Yes	19	84	103
	No	71	4	75
	Total	90	88	178

Suppose we choose a student from the class at random. Define event A as getting a male student and event B as getting a student with a pierced ear.

a. Find P(B).

b. Find P(A and B). Interpret this value in context.

c. Find P(A or B).

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a. Find P(B). $P(\mathrm{B}) = P(\text{pierced ear}) = \frac{103}{178} = 0.579$

b. Find P(A and B). Interpret this value in context.

c. Find P(A or B).

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b. Find P(A and B). Interpret this value in context.

$$P(\text{A and B}) = P(\text{male and pierced ear}) = \frac{19}{178} = 0.107$$

There's about an 11% chance that a randomly selected student from this class is male and has a pierced ear.

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c. Find P(A or B).

$$\begin{aligned} P(\text{A or B}) &= P(\text{male or pierced ear}) \\ &= \frac{71+19+84}{178} = \frac{174}{178} = 0.978 \end{aligned}$$

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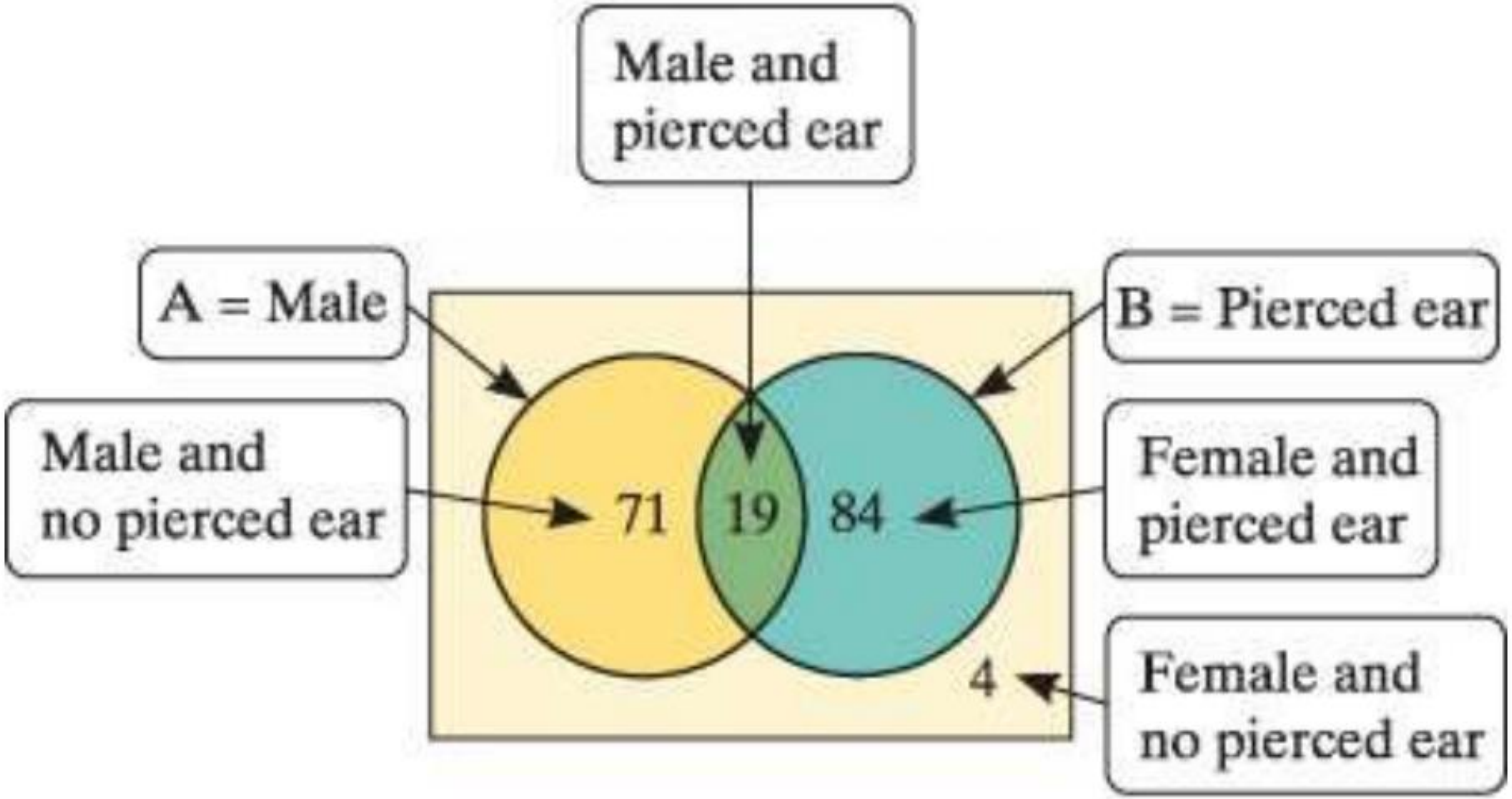
AP® EXAM TIP

Many probability problems involve simple computations that you can do on your calculator. It may be tempting to just write down your final answer without showing the supporting work. Don't do it! A "naked answer," even if it's correct, will usually be penalized on a free-response question.

getting a male student and event B as getting a student with a pierced ear.

c. Find P(A or B).

$$\begin{aligned} P(\text{male or pierced ear}) &= P(\text{male}) + P(\text{pierced ear}) - P(\text{male and pierced ear}) \\ &= 90/178 + 103/178 - 19/178 \\ &= 174/178 \end{aligned}$$


Male and
pierced ear
A = Male
B = Pierced ear
Male and
no pierced ear
71
19
84
Female and
pierced ear
4
Female and
no pierced ear

MALE	FEMALE	
19	84 U	YES
71	4	NO

		Gender		
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	Total	90	88	178

If we know that a randomly selected student has a pierced ear, what is the probability that the student is male?

$$P(\text{male given pierced ear})=19/103=0.184=18.4\%$$

		Gender		
		Male	Female	Total
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	No	71	4	75
	Total	90	88	178

If we know that a randomly selected student is male, what’s the probability that the student has a pierced ear?

$$P(\text{pierced ear given male}) = 19/90 = 0.211 = 21.1\%$$

Def. Conditional Probability

The probability that one event happens given that another event is known to have happened is called a conditional probability.

The conditional probability that event A happens given that event B has happened is denoted by $P(A|B)$.

Formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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P(male given pierced ear)=19/103=0.184=18.4%

$$\frac{P(\text{male and pierced ear})}{P(\text{pierced ear})}=\frac{\frac{19}{178}}{\frac{103}{178}}=\frac{19}{103}=P\,(\text{male}\,|\,\text{pierced ear})$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

General Multiplication Rule for Two Events

For any two events E and F,

$$P(E \cap F) = P(E \mid F)\,P(F)$$

Practice:

The following table gives information on DVD players sold by a certain electronics store:

	Percentage of Customers Purchasing	Of Those Who Purchase, Percentage Who Purchase Extended Warranty
Brand 1	70	20
Brand 2	30	40

A purchaser is randomly selected from among all those who bought a DVD player from the store. What is the probability that the selected customer purchased a Brand 1 model and an extended warranty?

	Percentage of Customers Purchasing	Of Those Who Purchase, Percentage Who Purchase Extended Warranty
Brand 1	70	20
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A purchaser is randomly selected from among all those who bought a DVD player from the store. What is the probability that the selected customer purchased a Brand 1 model and an extended warranty?

Define the events as follows.

B_1 = event that Brand 1 is purchased

B_2 = event that Brand 2 is purchased

E = event that an extended warranty is purchased

From the table, we have $P(\text{Brand 1 purchased}) = P(B_1) = .70$

$$P(\text{extended warranty}|\text{Brand 1 purchased}) = P(E|B_1) = .20$$

Hence, we have the probability that the selected customer purchased a Brand 1 model and an extended warranty is P(B1 and E) = P(E|B1)P(B1)=0.2*0.7=0.14

Tree Diagram

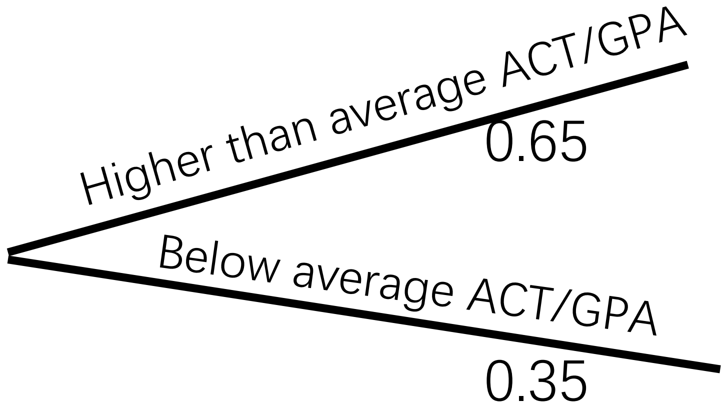
Tree Diagram

Models multiple **dependent** or successive events (events that depend on each other or that happen right after each other).

Each **branch** represents a different event

Tree Diagram (Dream School)

1. You have a 0.65 probability of getting higher than average GPA and ACT scores.
2. If you have a higher than average GPA/ACT, you have a 0.83 chance of being admitted.
3. If you have a below average GPA/ACT score, you have a 0.39 chance of being admitted.



Event 1: GPA/ACT Scores

Tree Diagram (Dream School)

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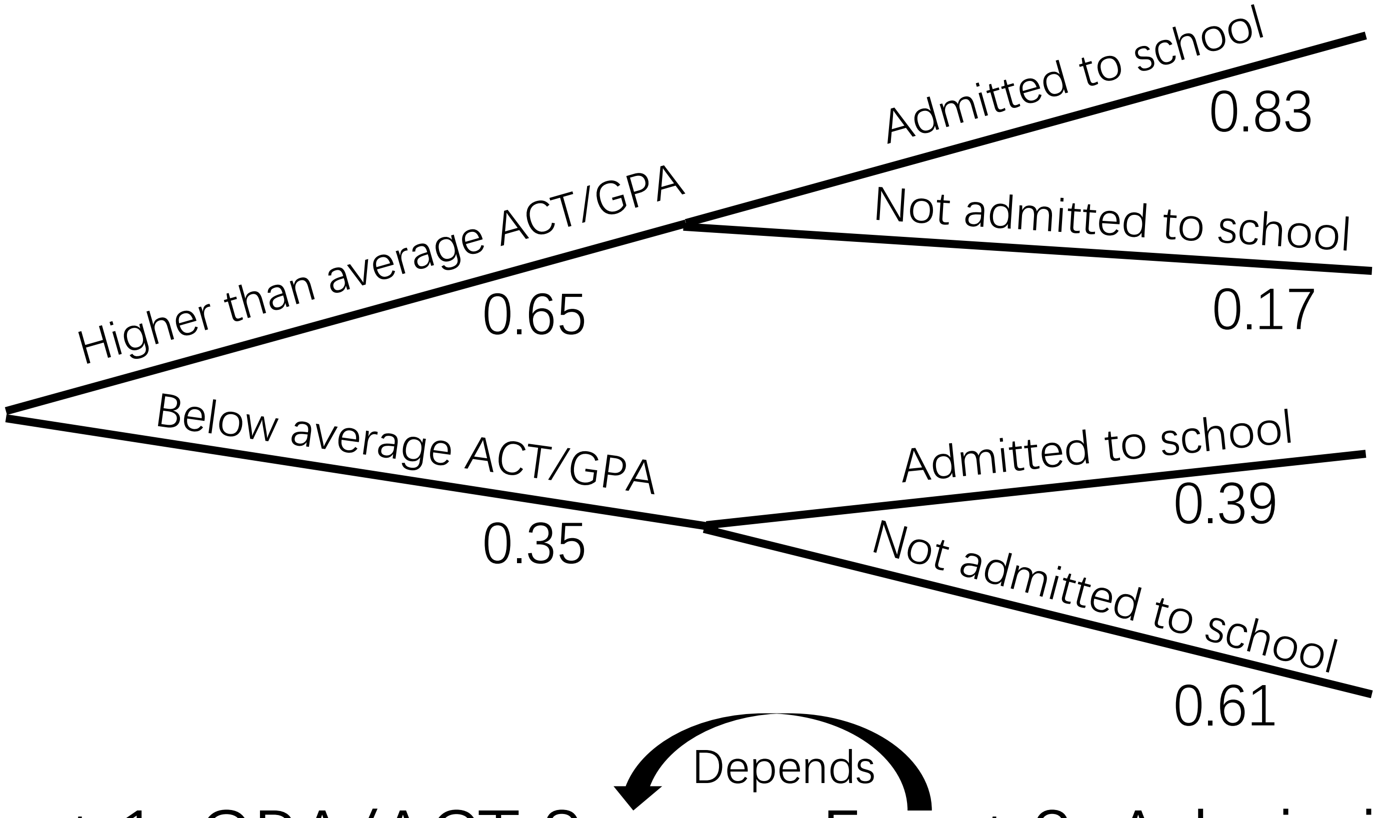
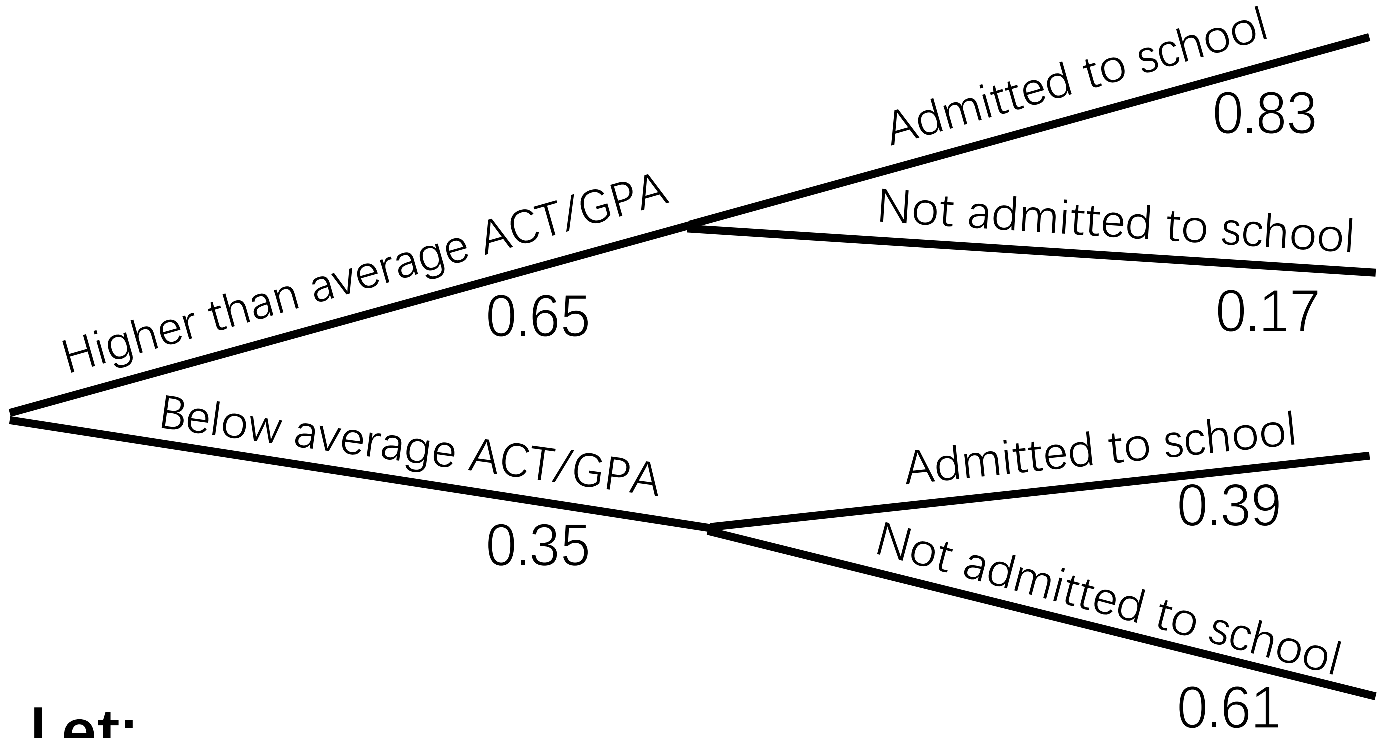


Event 1: GPA/ACT Scores Event 2: Admission to College

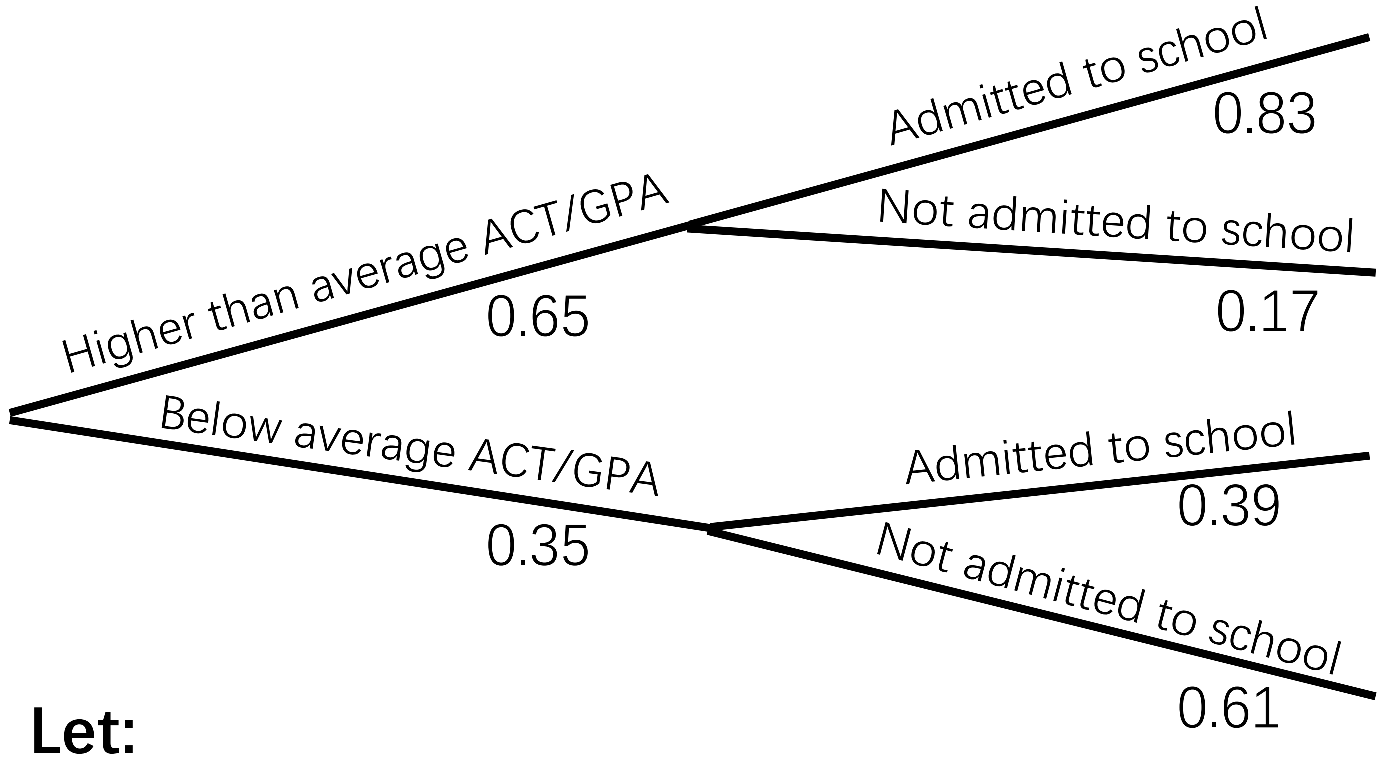
Tree Diagram



Let:
H = Event of getting higher than average ACT/GPA

A = Event of being admitted to your dream school

Tree Diagram



Let:

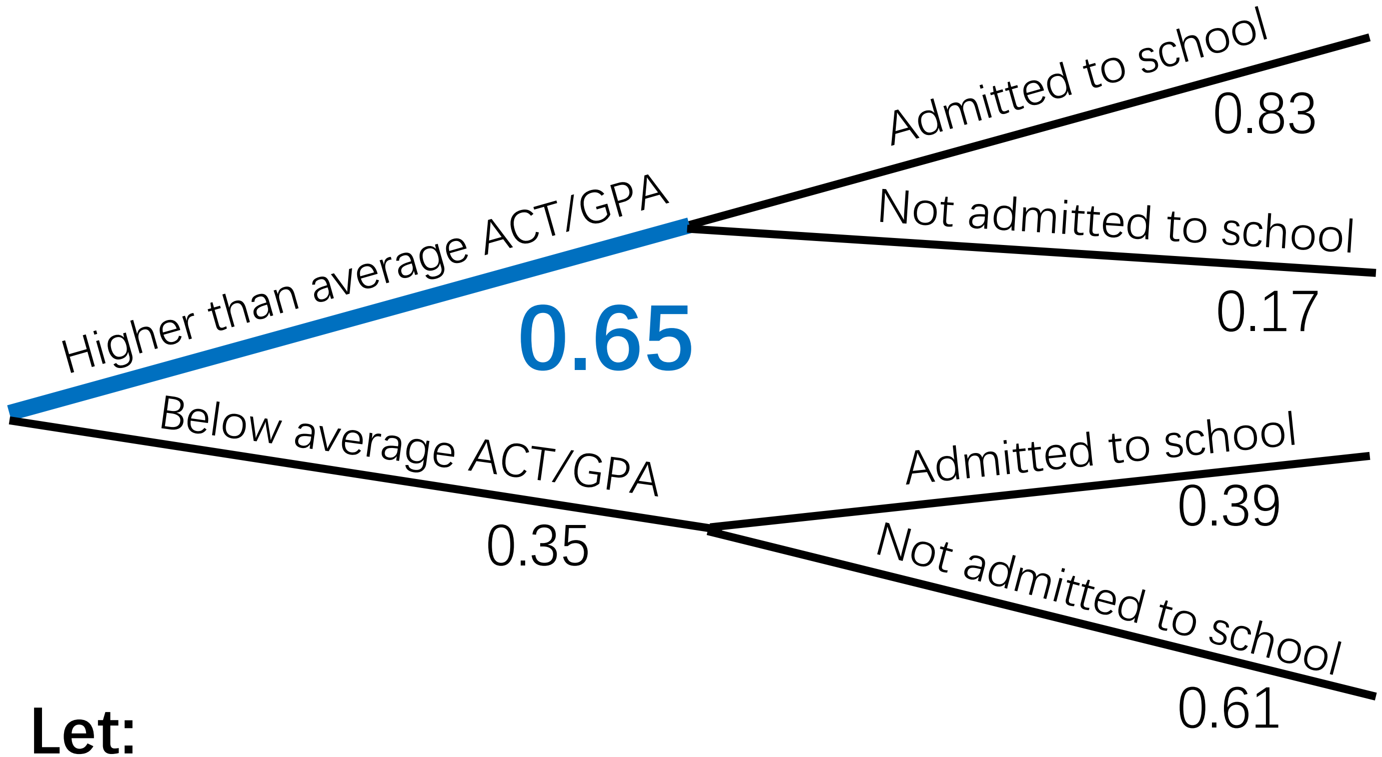
H = Event of getting higher than average ACT/GPA

A = Event of being admitted to your dream school

Find:

1. $P(H \cap A) =$

Tree Diagram



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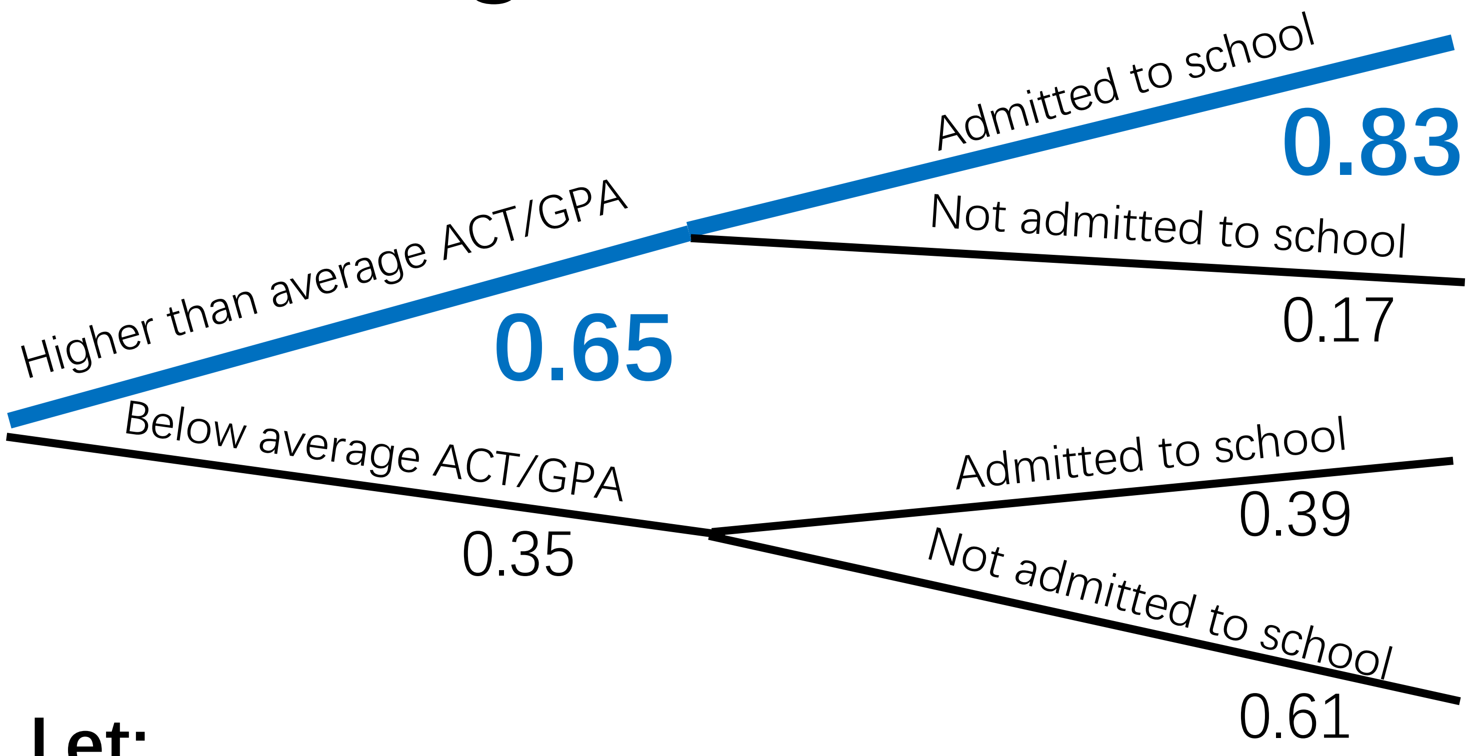

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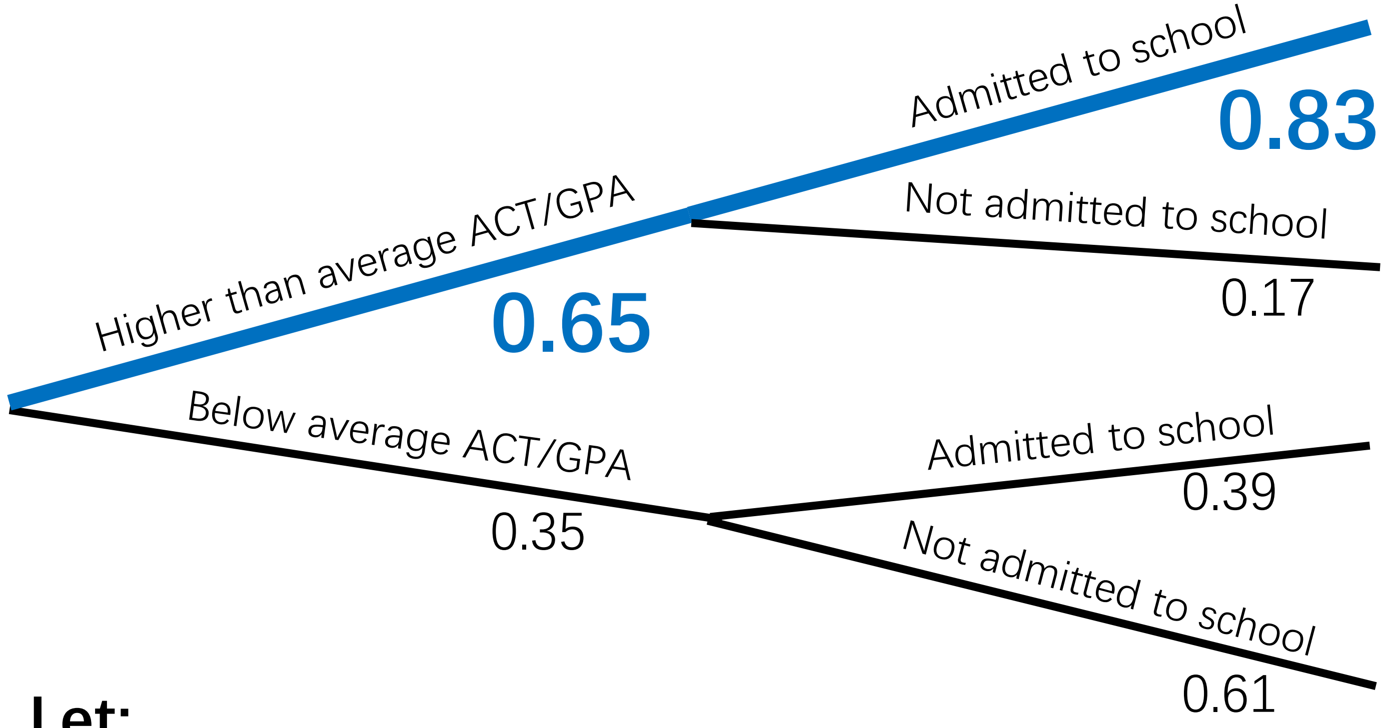

“and” means multiply!

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Find:

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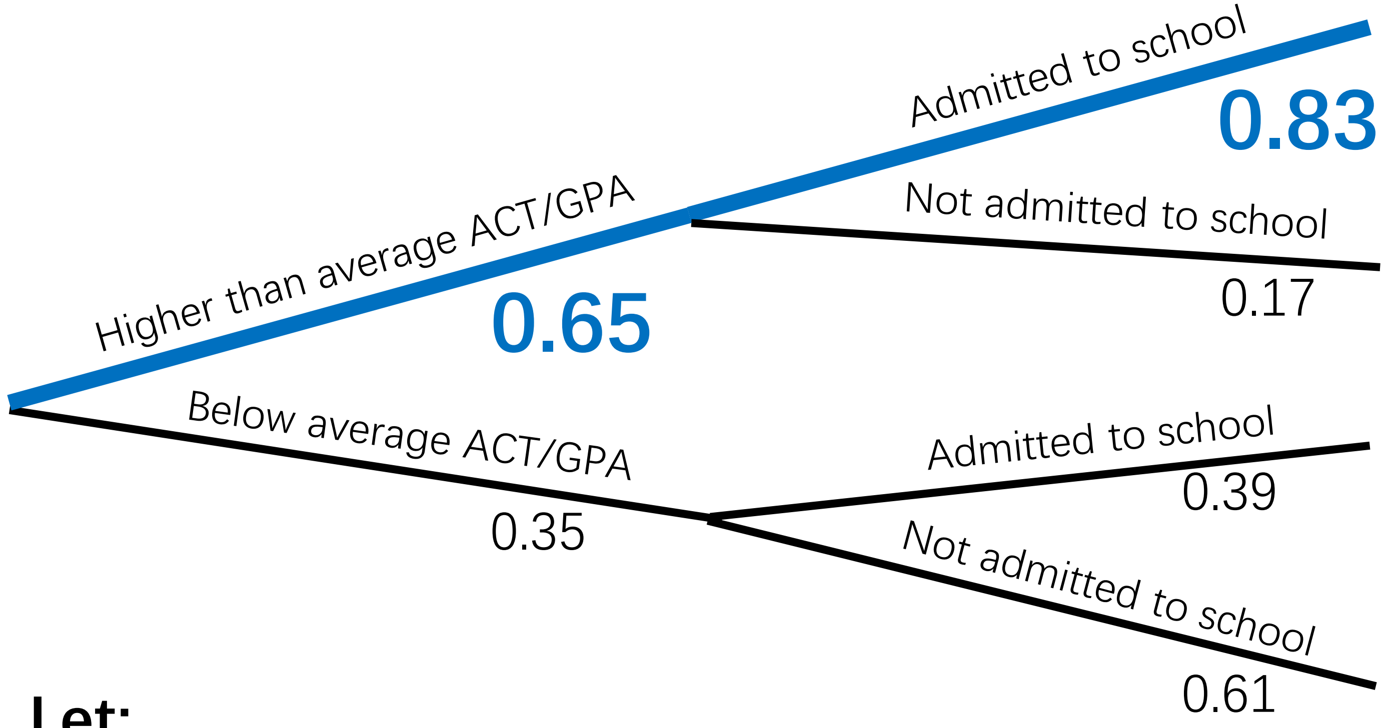
Tree Diagram



$$\mathbf{0.65 * 0.83 = 0.54}$$

"and" means multiply!

Let:

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Find:

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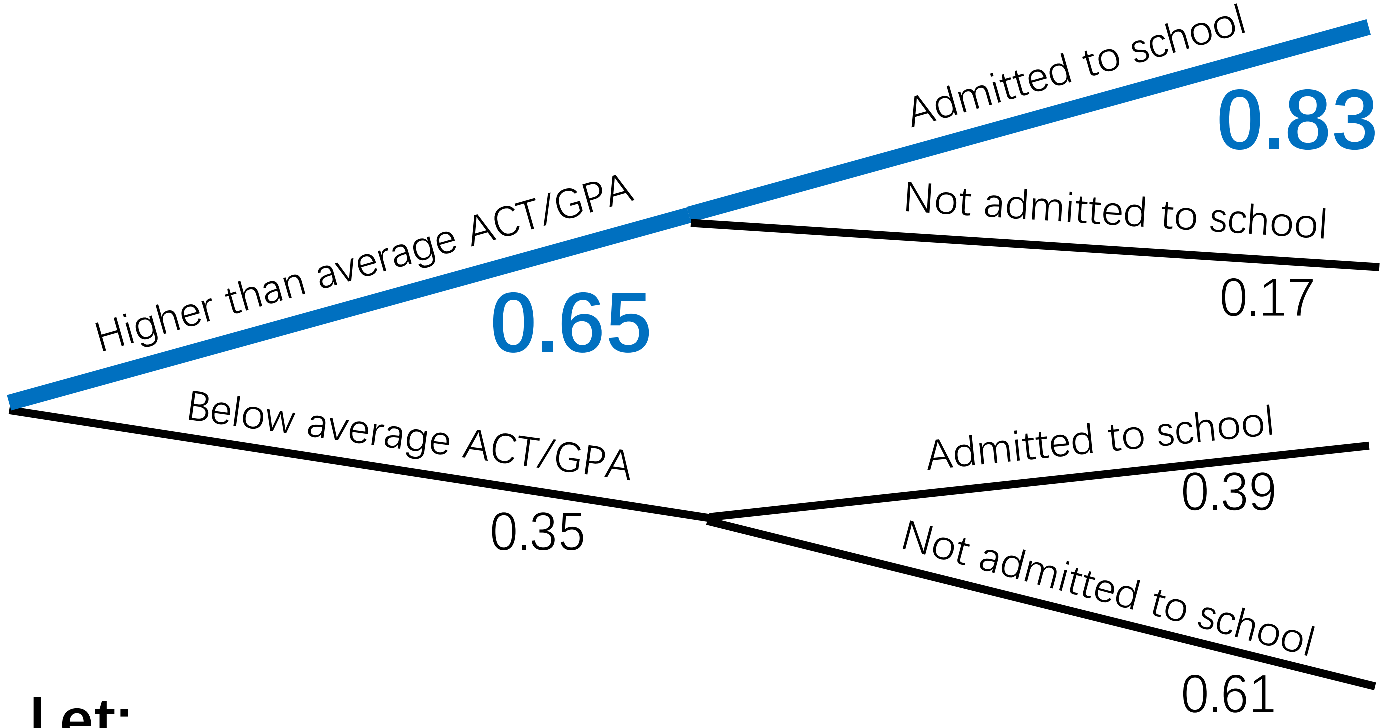
Tree Diagram



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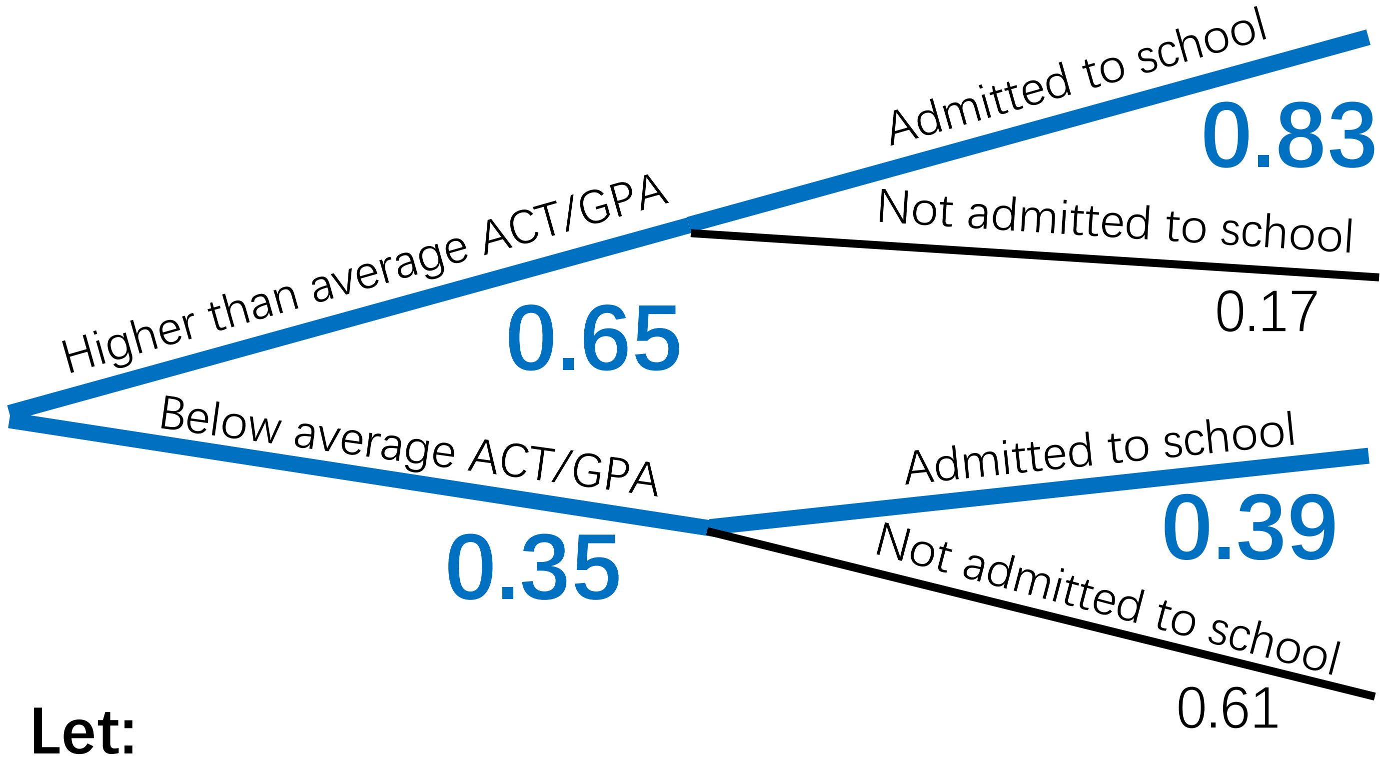
Tree Diagram



Path 1

$$0.65 * 0.83 = 0.54$$

Or

Path 2

$$0.35 * 0.39 = 0.14$$

"or" means add!

Let:

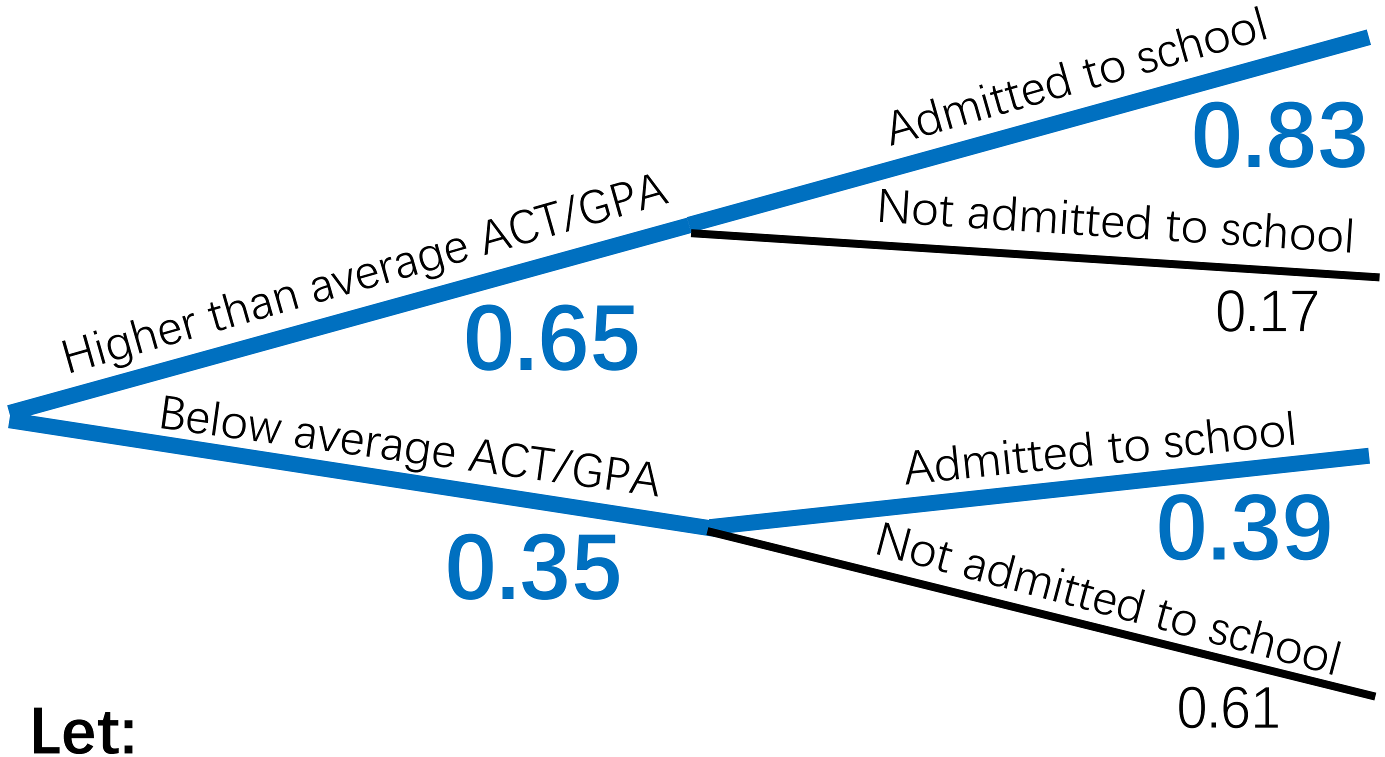
H = Event of getting higher than average ACT/GPA

A = Event of being admitted to your dream school

Find:

2. P(A) =

Tree Diagram



Path 1

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Or

+

Path 2

$$0.35 * 0.39 = 0.14$$

“or” means add!

Let:

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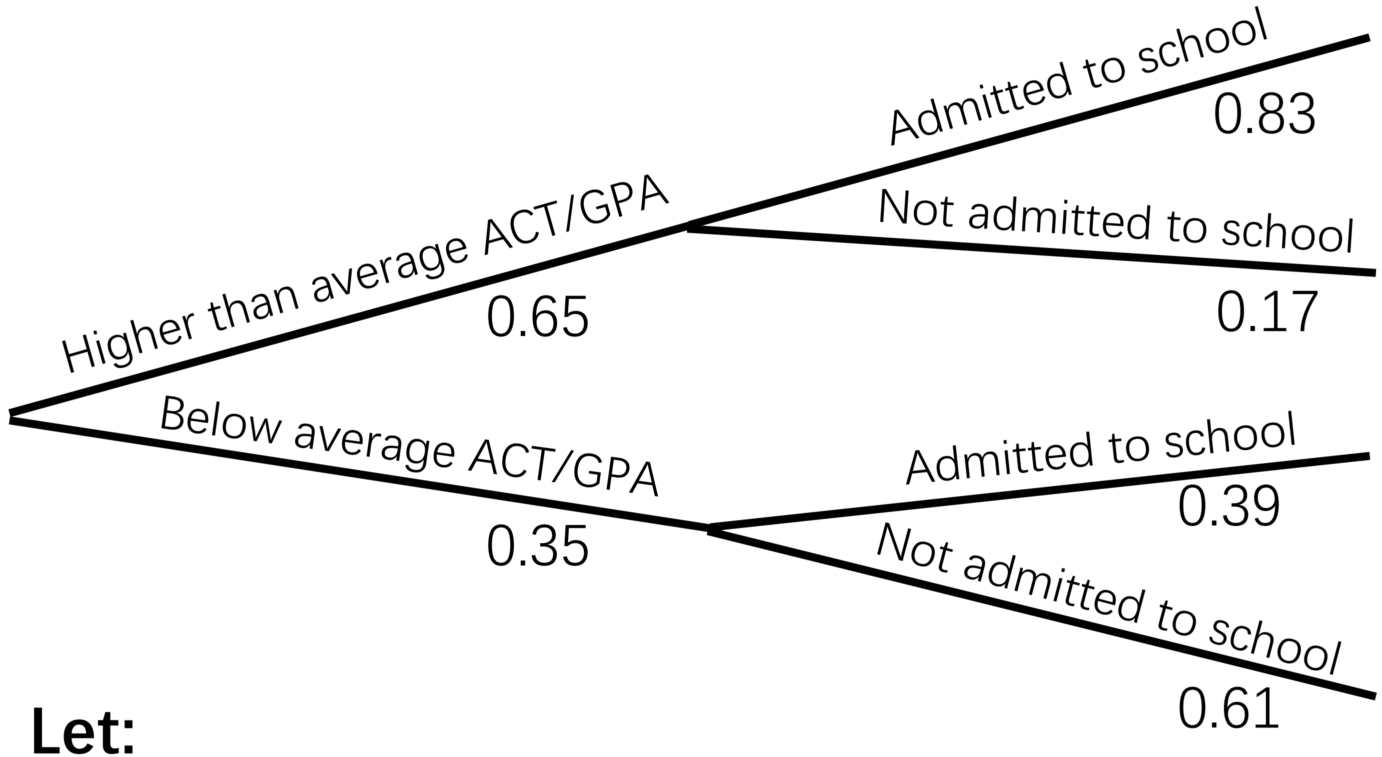
A = Event of being admitted to your dream school

Find:

2. P(A) = **0.68**

Tree Diagram



Let:

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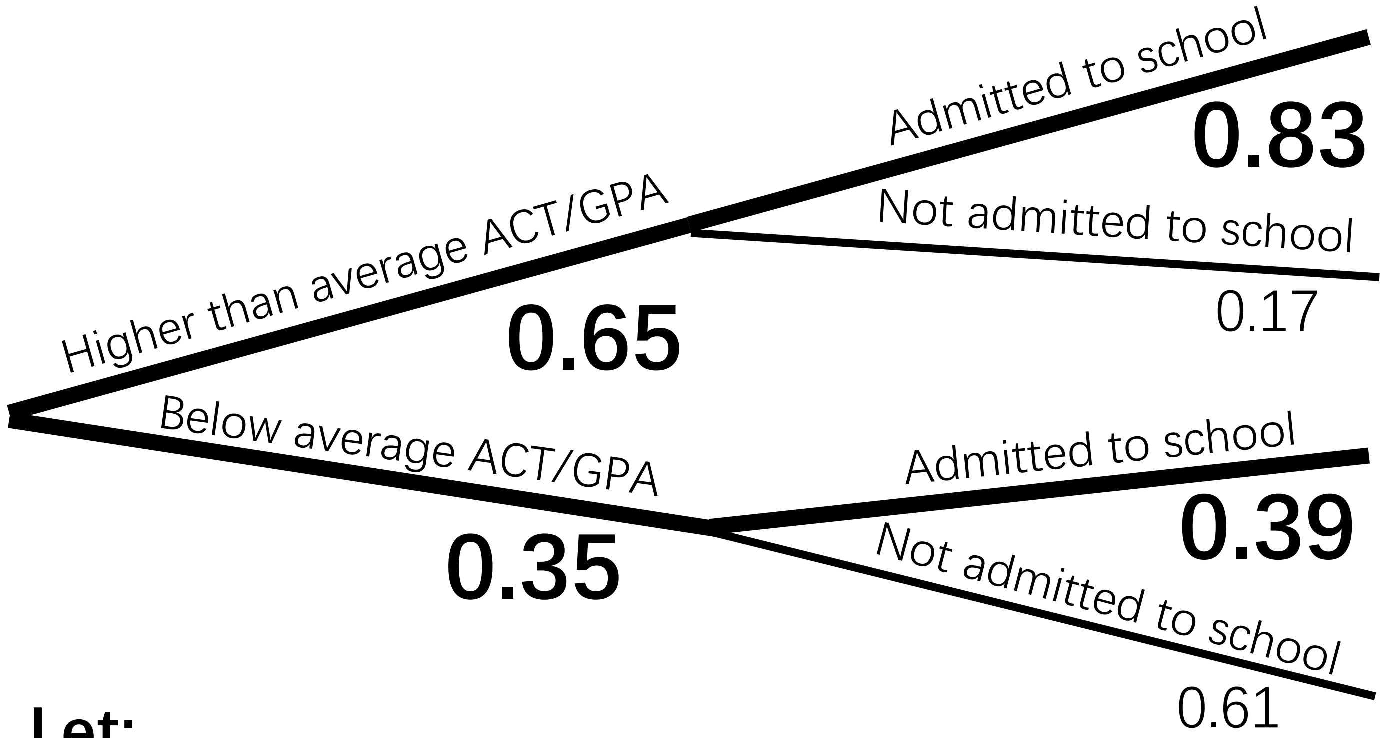
A = Event of being admitted to your dream school

Find:

3. P(H|A) =

Tree Diagram



Path 1

$0.65 * 0.83 = 0.54$

+

Path 2

$0.35 * 0.39 = 0.14$

P(A) = 0.68

Let:

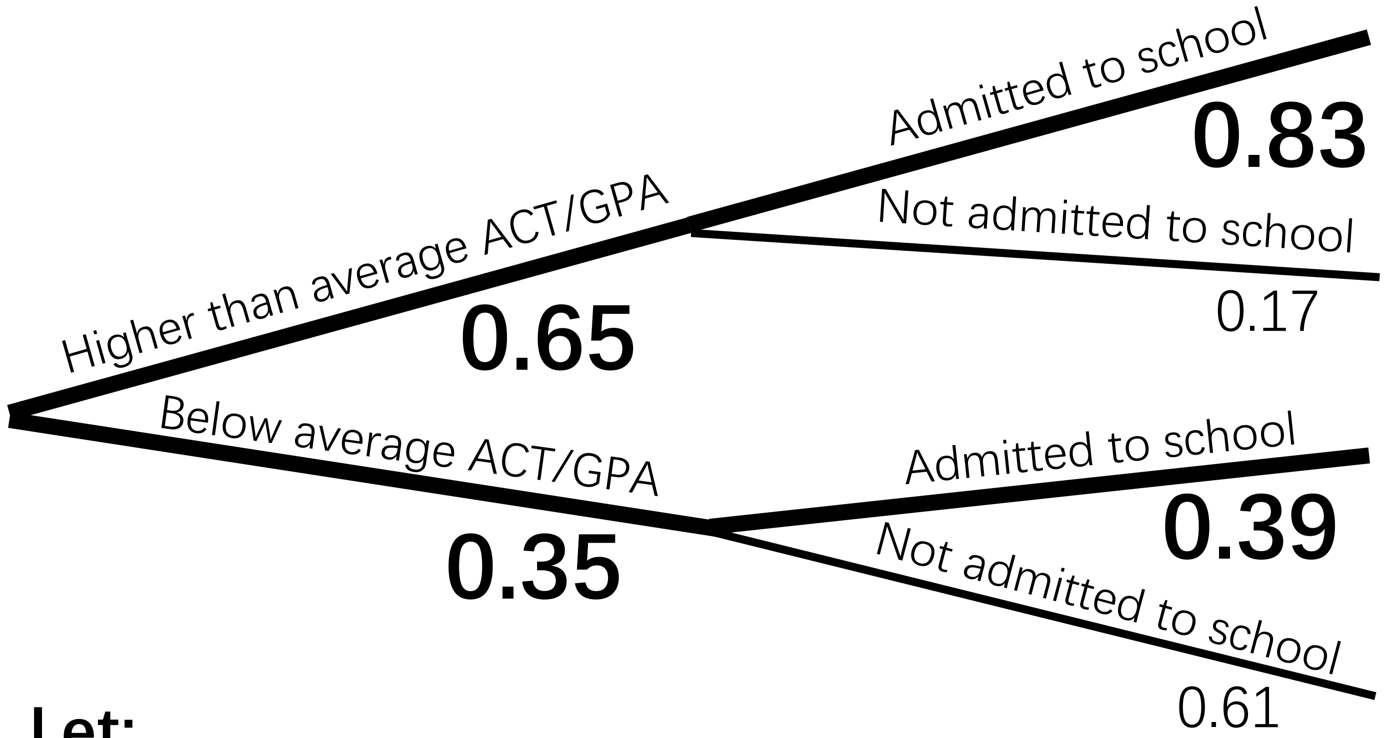
H = Event of getting higher than average ACT/GPA

A = Event of being admitted to your dream school

Find:

3. $P(H|\mathbf{A}) =$

Tree Diagram



Path 1

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+

Path 2

$0.35 * 0.39 = 0.14$

P(A) = 0.68

Let:

H = Event of getting higher than average ACT/GPA

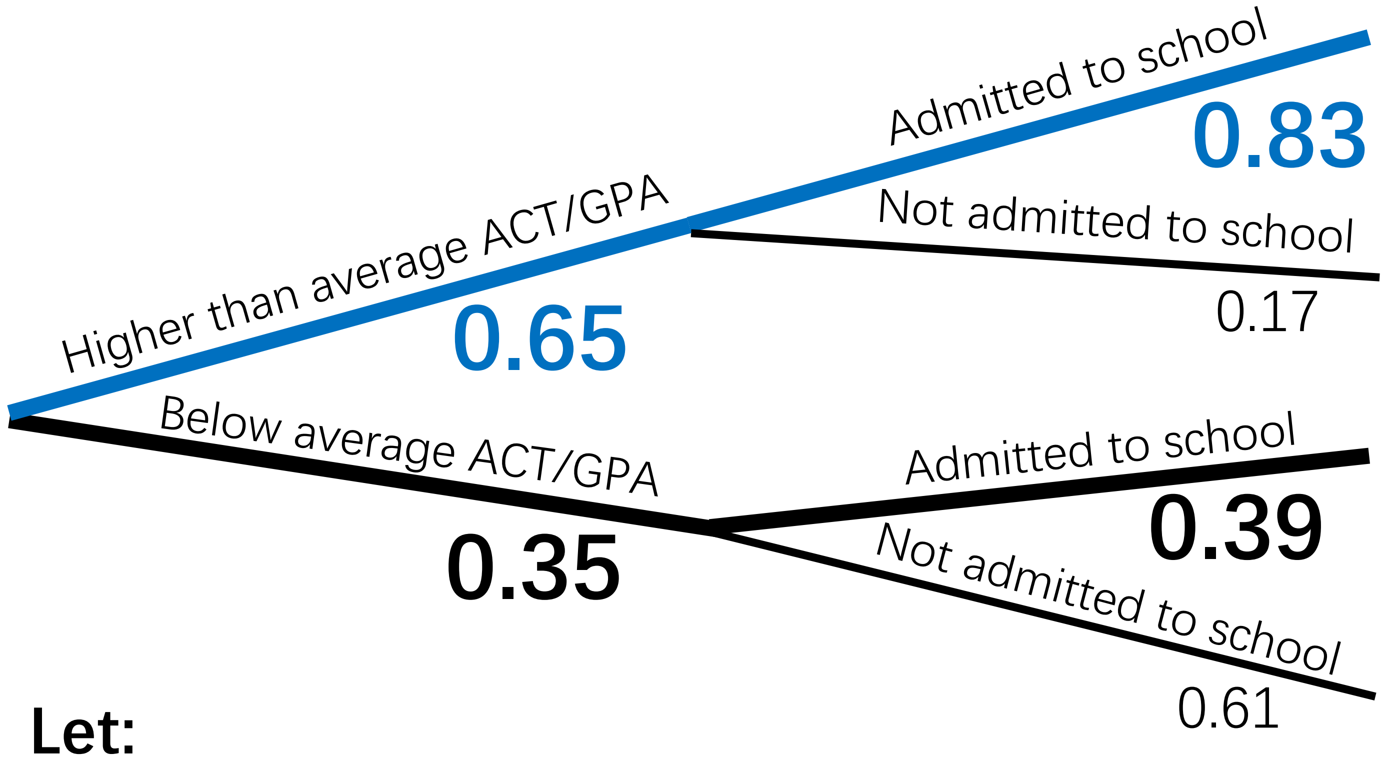
A = Event of being admitted to your dream school

Find:

3. P(H|**A**) =

“given” means divide by the given!

Tree Diagram



Path 1

$$0.65 * 0.83 = 0.54$$

Path 2

$$0.35 * 0.39 = 0.14$$

Let:

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Find:

$$3.\ P(H|A) = \frac{0.54}{0.68} = 0.79$$

Tree Diagram

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Brand 1	70	20
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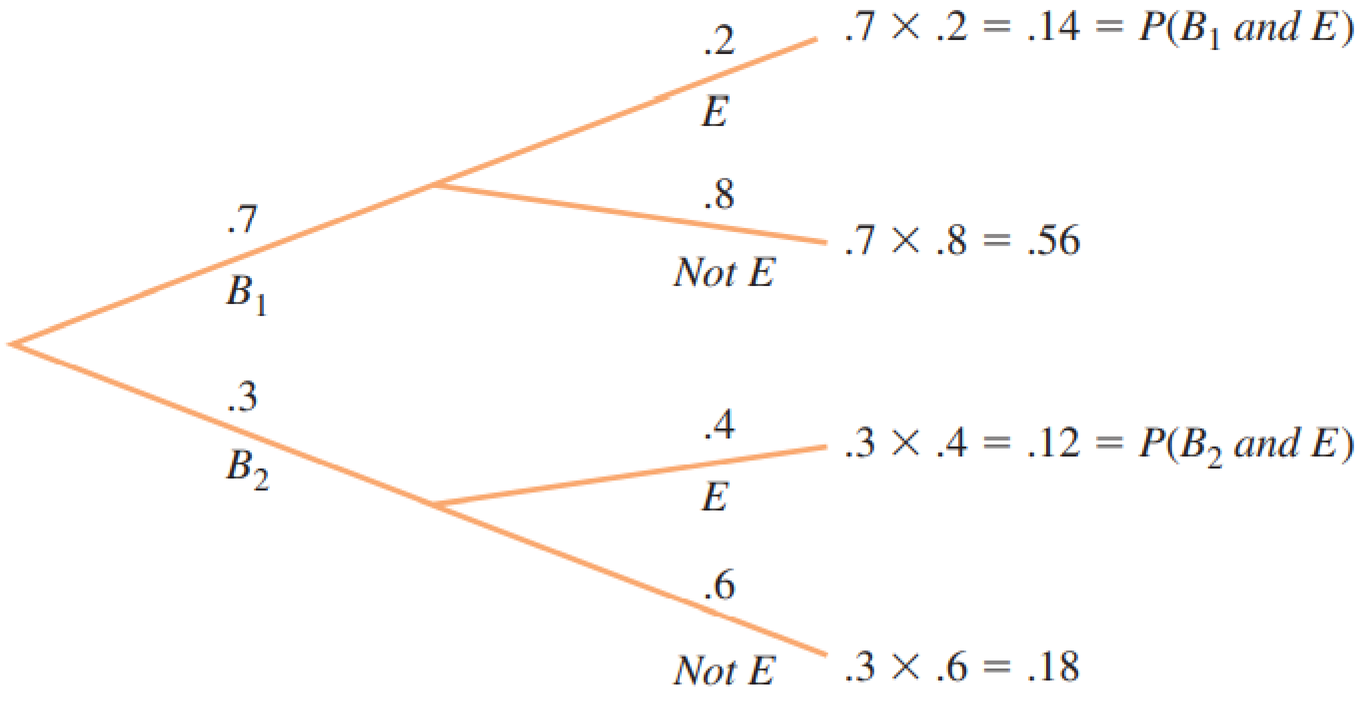
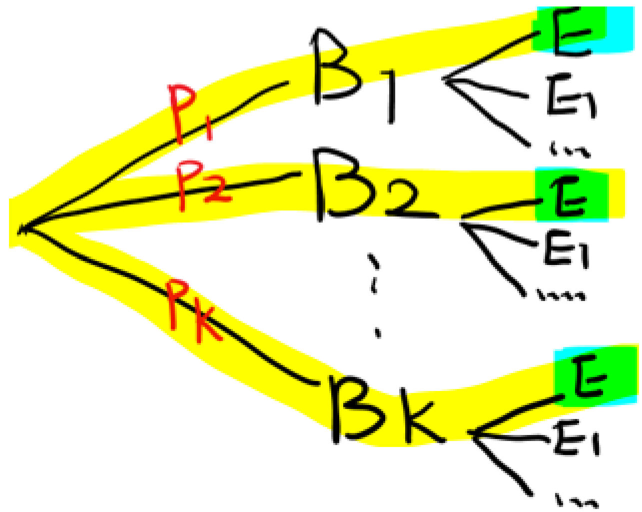


The Law of Total Probability

If B_1 and B_2 are disjoint events with $P(B_1) + P(B_2) = 1$, then for any event E

$$\begin{aligned} P(E) &= P(E \cap B_1) + P(E \cap B_2) \\ &= P(E|B_1)P(B_1) + P(E|B_2)P(B_2) \end{aligned}$$



【TP】

L = the event that the goalkeeper jumps to the left
C = the event that the goalkeeper stays in the center
R = the event that the goalkeeper jumps to the right
B = the event that the penalty kick is blocked

$P(B|L) = .142 \qquad P(B|C) = .333 \qquad P(B|R) = .126$

$P(L) = .493 \qquad P(C) = .063 \qquad P(R) = .444$

What pr

$$\begin{aligned} P(B) &= P(B \cap L) + P(B \cap C) + P(B \cap R) \\ &= P(B|L)P(L) + P(B|C)P(C) + P(B|R)P(R) \\ &= (.142)(.493) + (.333)(.063) + (.126)(.444) \\ &= .070 + .021 + .056 \\ &= .147 \end{aligned}$$

Bayes' Rule

If B_1 and B_2 are disjoint events with $P(B_1) + P(B_2) = 1$, then for any event E

$$P(B_1|E) = \frac{P(E|B_1)P(B_1)}{P(E|B_1)P(B_1) + P(E|B_2)P(B_2)}$$

More generally, if $B_1, B_2, \ldots, B_k$ are disjoint events with $P(B_1) + P(B_2) + \cdots + P(B_k) = 1$ then for any event E,

$$P(B_i|E) = \frac{P(E|B_i)P(B_i)}{P(E|B_1)P(B_1) + P(E|B_2)P(B_2) + \cdots + P(E|B_k)P(B_k)}$$

Lyme disease is the leading tick-borne disease in the United States and Europe. Diagnosis of the disease is difficult and is aided by a test that detects particular antibodies in the blood. The article "Laboratory Considerations in the Diagnosis and Management of Lyme Borreliosis" (*American Journal of Clinical Pathology* [1993]: 168–174) used the following notation:

$+$	represents a positive result on the blood test
$-$	represents a negative result on the blood test
L	represents the event that the patient actually has Lyme disease
L^C	represents the event that the patient actually does not have Lyme disease

The following probabilities were reported in the article:

$P(L) = .00207$	The prevalence of Lyme disease in the population; .207% of the population actually has Lyme disease.
$P(L^C) = .99793$	99.793% of the population does not have Lyme disease.
$P(+\mid L) = .937$	93.7% of those with Lyme disease test positive.
$P(-\mid L) = .063$	6.3% of those with Lyme disease test negative.
$P(+\mid L^C) = .03$	3% of those who do not have Lyme disease test positive.
$P(-\mid L^C) = .97$	97% of those who do not have Lyme disease test negative.

$+$ represents a positive result on the blood test
$-$ represents a negative result on the blood test
L represents the event that the patient actually has Lyme disease
L^C represents the event that the patient actually does not have Lyme disease

$$P(L) = .00207$$
$$P(L^C) = .99793$$
$$P(+|L) = .937$$
$$P(-|L) = .063$$
$$P(+|L^C) = .03$$
$$P(-|L^C) = .97$$

Given that a person tests positive for the disease, what is the probability that he or she actually has Lyme disease?

$$P(L|+) = \frac{P(+|L)P(L)}{P(+|L)P(L) + P(+|L^C)P(L^C)}$$
$$= \frac{(.937)(.00207)}{(.937)(.00207) + (.03)(.99793)} = \frac{.0019}{.0319} = .0596$$

Independence

Two events (A & B) are independent if knowing the outcome of one event **does not** affect the probability that the other event will occur.

$$P(A|B) = P(A)$$

Formal Multiplication Rule

The formal multiplication rule (all events)…

$$P(A \cap B) = P(A) * P(B|A)$$

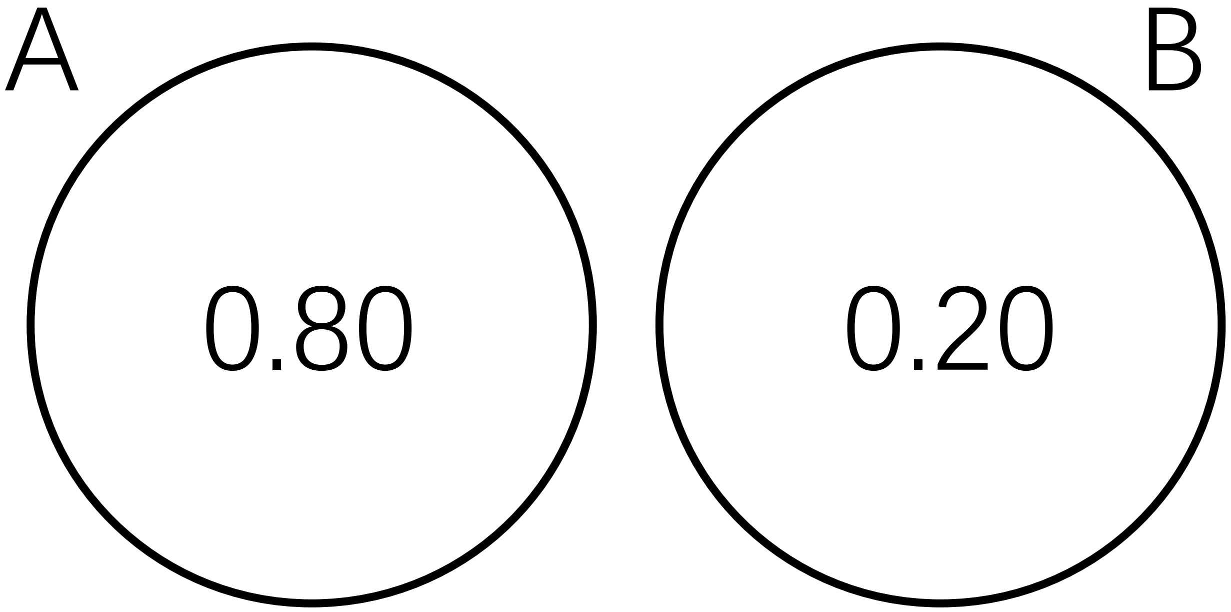
Independence:
P(B) = P(B|A)

For independent events only…

$$P(A \cap B) = P(A) * P(B)$$

Independent vs. Mutually Exclusive

Mutually exclusive: when events that have no intersection (i.e. they cannot both occur)



$P(A) = 0.80$

$P(A|B) = ?$

Independent vs. Mutually Exclusive

Mutually exclusive: when events that have no intersection (i.e. they cannot both occur)

$P(A) = 0.80$

$P(A|B) = 0.00$

Mutually exclusive events are **not independent**. Knowing that one event occurs greatly affects the probability of the other event (**lowers it to 0**).