1. $\sum_{n=1}^{\infty} \frac{(3)^{n+1}}{5^n} =$

(A) $\frac{3}{5}$ (B) $\frac{5}{2}$ (C) $\frac{9}{2}$ (D) The series diverges

2. If $f(x) = \sum_{n=1}^{\infty} (\tan x)^n$, then $f(1) =$

(A) -2.794 (B) -0.61 (C) 0.177 (D) The series diverges

3. $\sum_{n=2}^{\infty} \frac{2}{n^2 - 1} =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$

4. The sum of the geometric series $\frac{2}{21} + \frac{4}{63} + \frac{8}{189} + \ldots$ is

(A) $\frac{5}{21}$ (B) $\frac{2}{7}$ (C) $\frac{4}{7}$ (D) The series diverges

5. If $S_n = \left(\frac{3^{n-1}}{(4+n)^{20}} \right) \left(\frac{(7+n)^{20}}{3^n} \right)$, to what number does the sequence $\{S_n\}$ converge?

(A) $\frac{1}{3}$ (B) $\frac{7}{4}$ (C) $\left(\frac{7}{4}\right)^{20}$ (D) Diverges

6. Which of the following <u>sequences</u> converge?

I. $\left\{ \frac{\cos^2 n}{(1.1)^n} \right\}$ II. $\left\{ \frac{e^n - 3}{3^n} \right\}$ III. $\left\{ \frac{n}{9 + \sqrt{n}} \right\}$

(A) I only (B) II only (C) III only (D) I and II only

7. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{10(n+1)}$ II. $\sum_{n=1}^{\infty} \arctan n$ III. $\sum_{n=1}^{\infty} \frac{-6}{(-5)^n}$

(A) I only (B) II only (C) III only (D) II and III only

8. Find the sum of the series $\sum_{n=1}^{\infty} \left(\frac{3}{n(n+3)} + \frac{1}{7^n} \right)$.

9. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$

10. Determine whether the series is convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{n}{2n+3}$ (b) $\sum_{n=1}^{\infty} 2^{-n} 5^n$

1. If $\int_1^\infty \frac{dx}{x^2+1} = \frac{\pi}{4}$, then which of the following must be true?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ diverges.

II. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges.

III. $\sum_{n=1}^{\infty} \frac{1}{n^2+1} = \frac{\pi}{4}$

(A) none (B) I only (C) II only (D) II and III only

2. What are all values of p for which $\int_1^\infty \frac{1}{\sqrt[3]{x^p}}$ converges?

(A) $P < -3$

(B) $P < -1$

(C) $P > 1$

(D) $P > 3$

3. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ II. $\sum_{n=1}^{\infty} ne^{-n^2}$ III. $\sum_{n=2}^{\infty} \frac{1}{x \ln x}$

(A) I only (B) II only (C) III only (D) I and II only

4. What are all values of p for which $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^p+1}$ converges?

(A) $p > 0$ (B) $p > \frac{1}{2}$ (C) $p > 1$ (D) $p > \frac{3}{2}$

5. What are all values of k for which the series $1+(\sqrt{2})^k+(\sqrt{3})^k+(\sqrt{4})^k+\cdots+(\sqrt{n})^k+\cdots$ converges?

(A) $k<-2$ (B) $k<-1$ (C) $k>1$ (D) $k>2$

6. Determine whether the following series converge or diverge.

(a) $1+\frac{1}{8}+\frac{1}{27}+\frac{1}{64}+\cdots$

(b) $1+\frac{1}{(\sqrt[3]{2})^2}+\frac{1}{(\sqrt[3]{3})^2}+\frac{1}{(\sqrt[3]{4})^2}+\cdots$

7. Determine whether the series is convergent or divergent. $\sum_{n=1}^{\infty} n^{1-\pi}$

1. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2+n+3}$ II. $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+2}$ III. $\sum_{n=1}^{\infty} \frac{1+4^n}{3^n}$

(A) I only (B) II only (C) III only (D) I and II only

2. Which of the following series diverge?

I. $\sum_{n=1}^{\infty} \frac{1}{n!}$ II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2}$ III. $\sum_{n=1}^{\infty} \sin(\frac{1}{\mathrm{n}})$

(A) I only (B) II only (C) II and III only (D) I, II, and III

3.Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n^{3/2}}{3n^3+7}$ II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4+1}}$ III. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(A) I only (B) I and II only (C) I and III only (D) I, II, and III

4. Which of the following series cannot be shown to converge using the limit comparison test with the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$?

(A) $\sum_{n=1}^{\infty} \frac{1}{2^n-1}$

(B) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

(C) $\sum_{n=1}^{\infty} \frac{2n}{2^{n+1}\sqrt{n^2+1}}$

(D) $\sum_{n=1}^{\infty} \frac{2n^2-3n}{2^n(n^2+n-100)}$

5. Determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{\cos(2n)}{1+(1.6)^n}$

(b) $\sum_{n=1}^{\infty} \frac{4^n}{2^n+3^n}$

6. Determine whether the series is convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+4}}$

(b) $\sum_{n=3}^{\infty} \frac{2^n}{3^n+1}$

1. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n}$ II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$ III. $\sum_{n=1}^{\infty} \cos(n\pi)$

(A) I only (B) II only (C) III only (D) I and II only

2. Which of the following series converge?

I. $\sum_{n=1}^{\infty} (-1)^n \cos(\frac{\pi}{n})$ II. $\sum_{n=1}^{\infty} \sin(\frac{2n-1}{2})\pi$ III. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n^2+1}$

(A) I only (B) II only (C) III only (D) I and II only

3. For what integer k, $k>1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{n^2\sqrt{n}}{n^k+1}$ converge?

(A) 3 (B) 4 (C) 5 (D) 6

4. Let $s = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ and s_n be the sum of the first n terms of the series. If $|s-s_n| < \frac{1}{500}$ what is the smallest value of n?

(A) 6 (B) 7 (C) 8 (D) 9

5. Which of the following series converge?

I. $\sum_{n=2}^{\infty} (-1)^n \sqrt[n]{3}$ II. $\sum_{n=1}^{\infty} \frac{3^{n+1}}{\pi^n}$ III. $\sum_{n=1}^{\infty} (\tan^{-1}(n+1) - \tan^{-1}(n))$

(A) I only (B) II only (C) III only (D) II and III only

6. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+3)}{n^2}$ is true?

(A) The series converges conditionally.

(B) The series converges absolutely.

(C) The series converges but neither conditionally nor absolutely.

(D) The series diverges.

7. Which of the following series is absolutely convergent?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n}$

(B) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2\sqrt{n}}$

(C) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{n^2-\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n^2+1)}{n^3}$

8. An alternating series is given by $S=\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+3}$. Let S_3 be the sum of the first three terms of the given alternating series. Of the following, which is the smallest number M for which the alternating series error bound guarantees that $|S-S_3| \le M$?

(A) $\frac{1}{4}$ (B) $\frac{1}{7}$ (C) $\frac{1}{19}$ (D) $\frac{1}{28}$

9. Let $f(x)=1-\frac{3x}{2!}+\frac{9x^2}{4!}-\frac{27x^3}{6!}+\cdots+\frac{(-1)^n(3x)^n}{(2n)!}+\cdots$.

Use the alternating series error bound to show that $1-\frac{3}{2!}+\frac{9}{4!}$ approximates $f(1)$ with an error less than $\frac{1}{20}$.

Test	Series	Conditions of Convergence or Divergence
n th-Term	$\sum_{n=1}^{\infty} a_n$	The series is divergent if $\lim_{x\to\infty} a_n \neq 0$. Test is inconclusive if $\lim_{x\to\infty} a_n = 0$.
Geometric Series	$\sum_{n=1}^{\infty} ar^{n-1}$	The series is convergent if $\lvert r\rvert < 1$, divergent if $\lvert r\rvert \geq 1$. $S = \dfrac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (a_n - a_{n+1})$	The series is convergent if $\lim_{x\to\infty} a_n = L$. $S = a_1 - L$
p-Series	$\sum_{n=1}^{\infty} \dfrac{1}{n^p}$	The series is convergent if $p > 1$, divergent if $p \leq 1$.
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	The series is convergent if $\lim_{x\to\infty} a_n = 0$ and $0 < a_{n+1} \leq a_n$.
Integral	$\sum_{n=1}^{\infty} a_n$	If f is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$, then the series converges if $\int_1^{\infty} f(x)\ dx$ converges, diverges if $\int_1^{\infty} f(x)\ dx$ diverges.
Ratio	$\sum_{n=1}^{\infty} a_n$	The series is convergent if $\lim_{n\to\infty} \left\lvert \dfrac{a_{n+1}}{a_n} \right\rvert < 1$, divergent if $\lim_{n\to\infty} \left\lvert \dfrac{a_{n+1}}{a_n} \right\rvert > 1$, inconclusive if $\lim_{n\to\infty} \left\lvert \dfrac{a_{n+1}}{a_n} \right\rvert = 1$.
Direct Comparison	$\sum_{n=1}^{\infty} a_n$	Let $0 < a_n \leq b_n$ for all n. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.
Limit Comparison	$\sum_{n=1}^{\infty} a_n$	Suppose that $a_n > 0$, $b_n > 0$, and $\lim_{n\to\infty} (\frac{a_n}{b_n}) = L$. Then $\sum_{n=1}^{\infty} a_n$ converges if $\sum_{n=1}^{\infty} b_n$ converges and $\sum_{n=1}^{\infty} a_n$ diverges if $\sum_{n=1}^{\infty} b_n$ diverges.

1. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n!}{2^n}$ II. $\sum_{n=1}^{\infty} \frac{n}{3^n}$ III. $\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n$

(A) I only (B) II only (C) II and III only (D) I, II, and III

2. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ II. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ III. $\sum_{n=1}^{\infty} \frac{n^9}{9^n}$

(A) I only (B) II only (C) I and II only (D) I, II, and III

3. Determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{n!}{n\,2^n}$

(b) $\sum_{n=0}^{\infty} \frac{\cos^n x}{2^n}$

(c) $\sum_{k=1}^{\infty} \frac{3^k k!}{(k+3)!}$