Review – distribution of discrete random variable

Binomial distribution:

- n trials (n is fixed in advance)
- each trial: either success or failure
- Probability of success (p) remains the same throughout the trials
- All trials are independent

Binomial Random Variable:

X = the number of successes after n trials

$X \sim Binom(n,p)$ $\quad$ $P(X=k) = ?$

$E(X) = np$

$Var(X) = np(1-p)$

Review – distribution of discrete random variable

Geometric distribution:

- each trial: either success or failure
- Probability of success (p) remains the same throughout the trials
- All trials are independent

Geometric Random Variable:

X = the number of trials until the first success occurs

X~Geom(p)   P(X=k) = ?

E(X) = 1/p

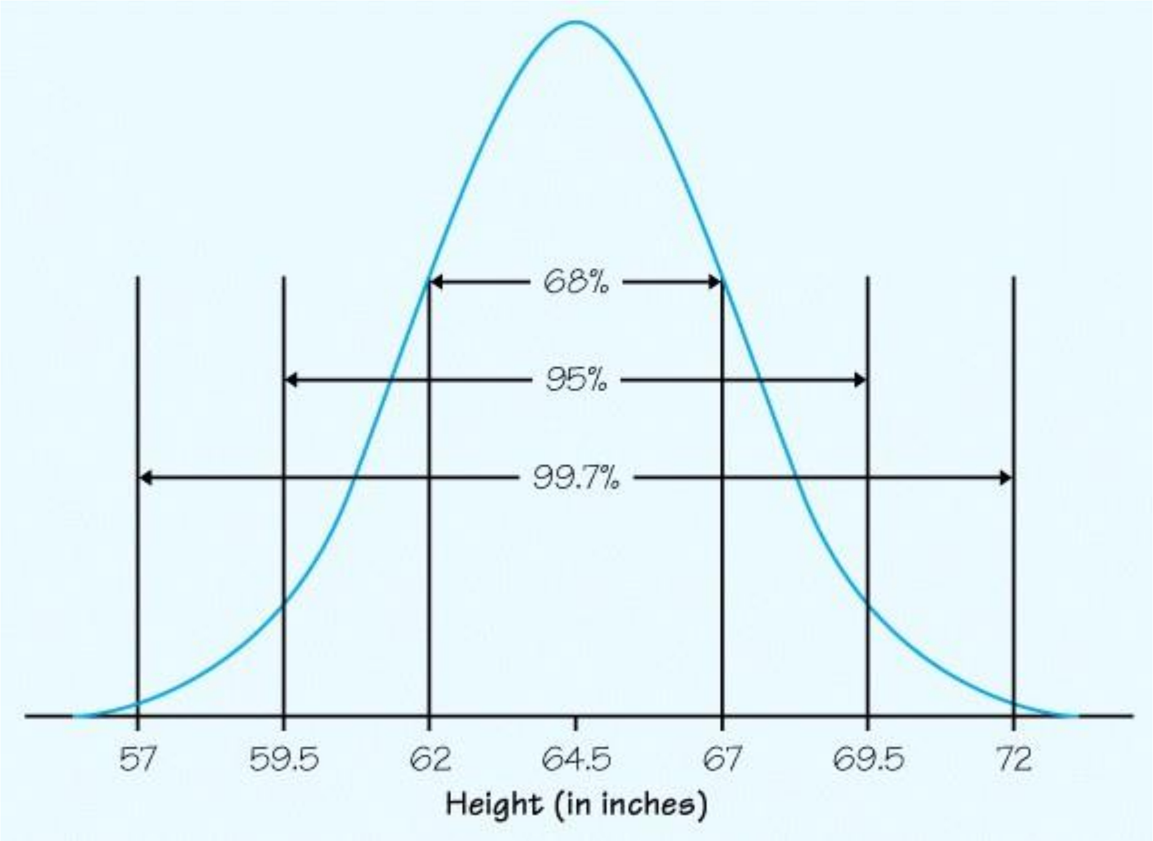
Var(X) = $\frac{1-p}{p^2}$

Distribution of

continuous

random variable

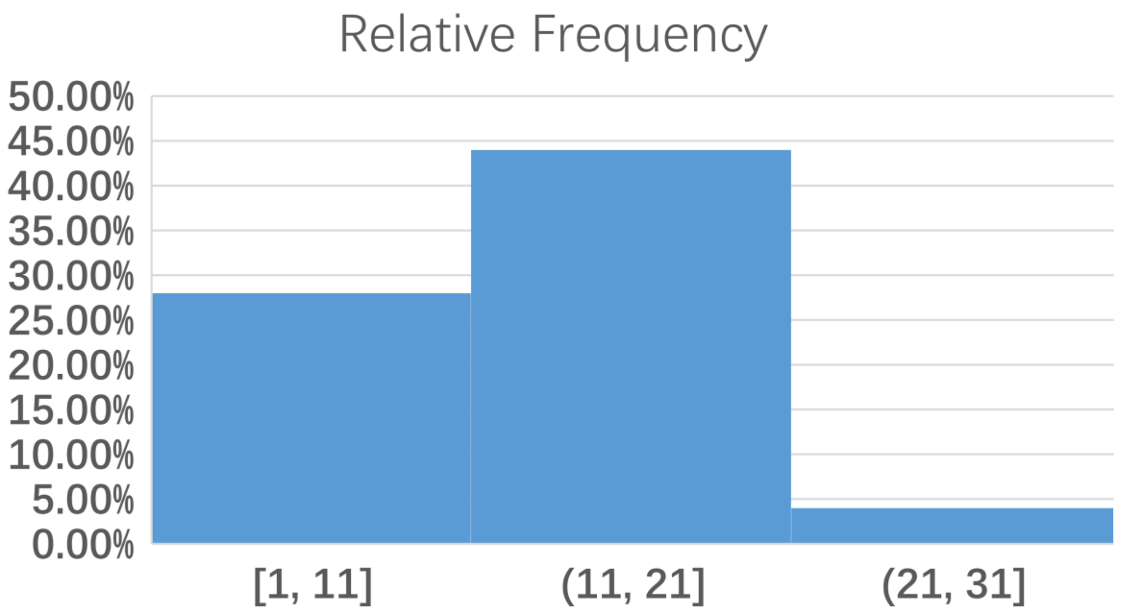
Density Curves

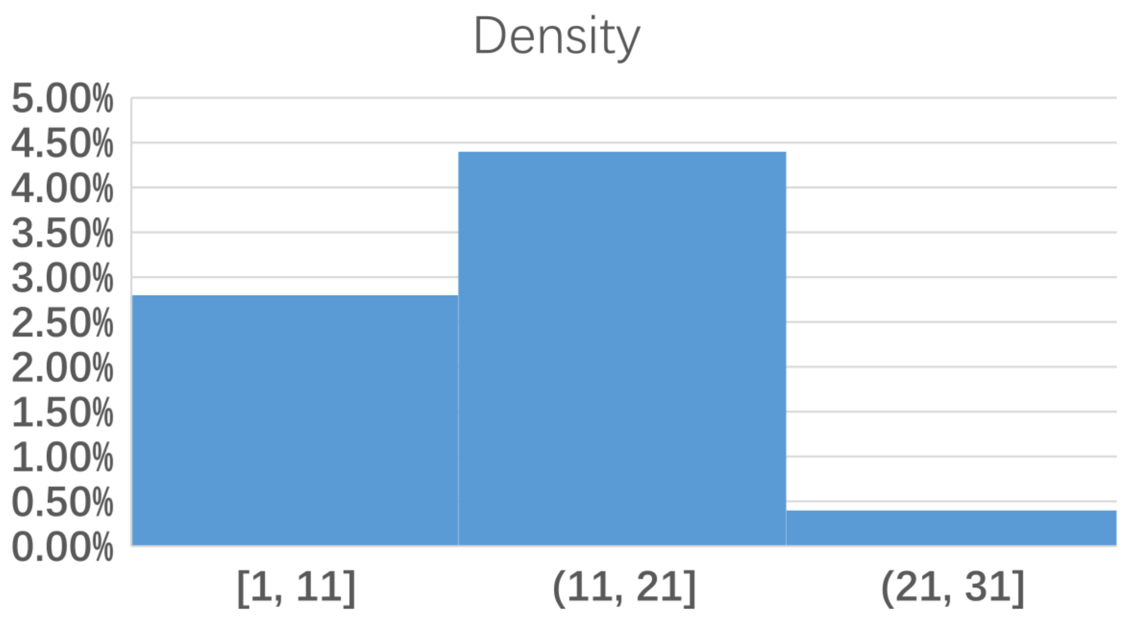


Density

$$\text{density} = \text{rectangle height} = \frac{\text{relative frequency of class interval}}{\text{class interval width}}$$

Relative Frequency = density * interval width



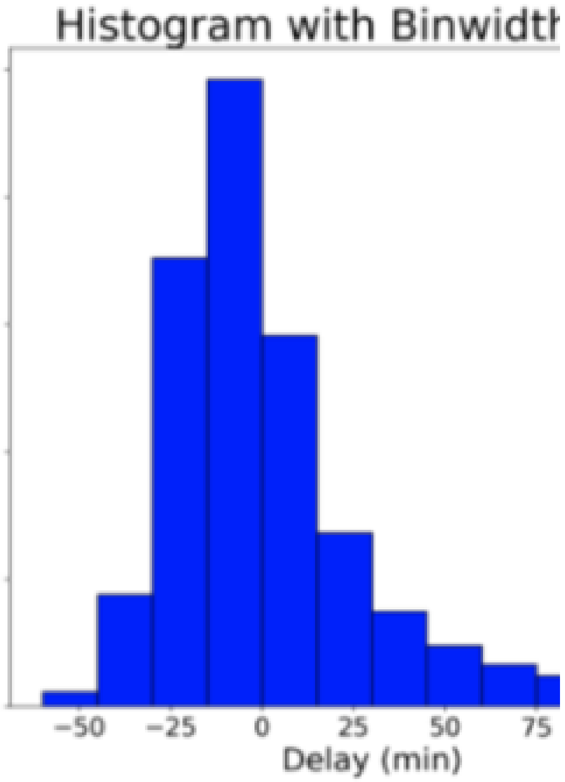


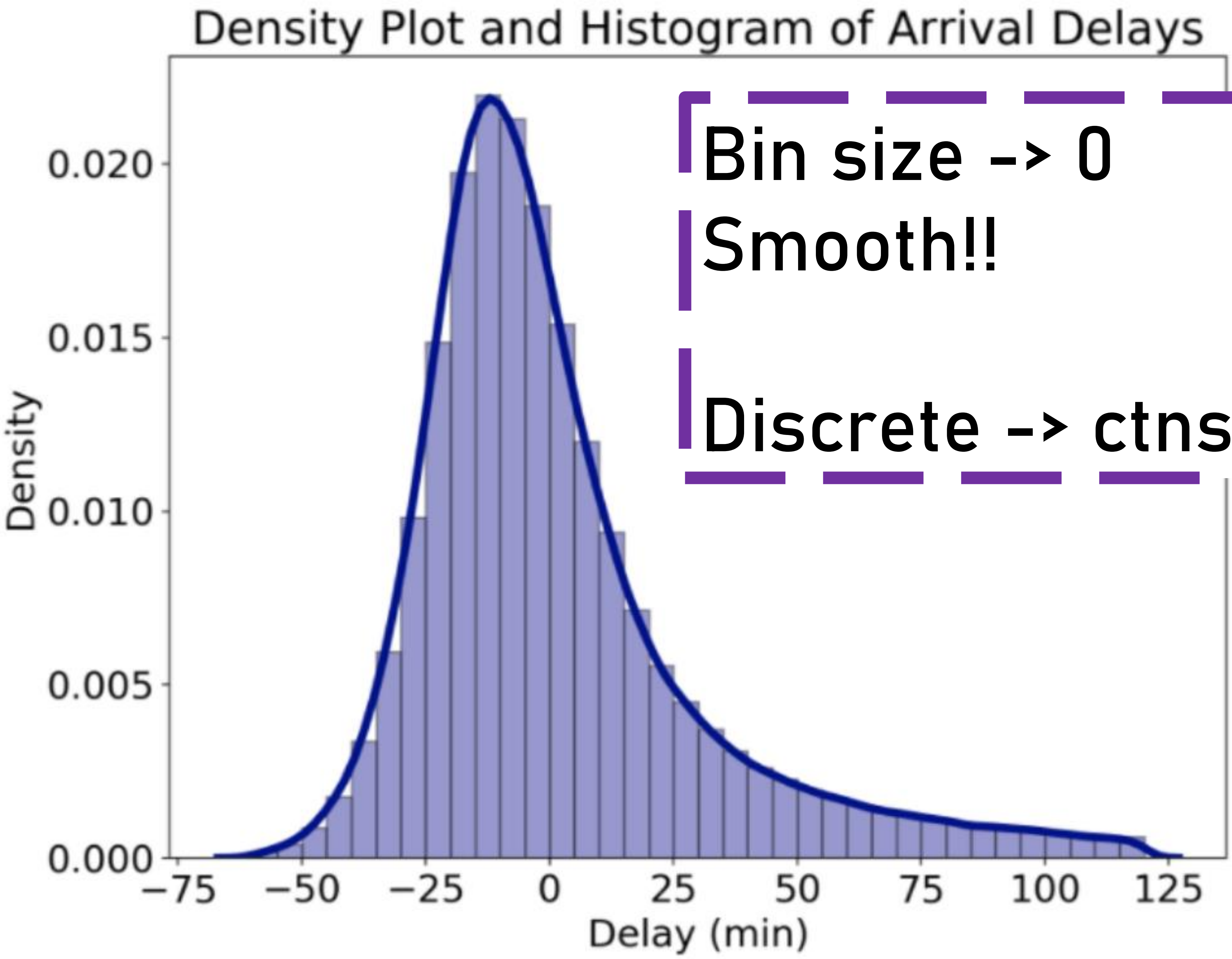
Density Curves

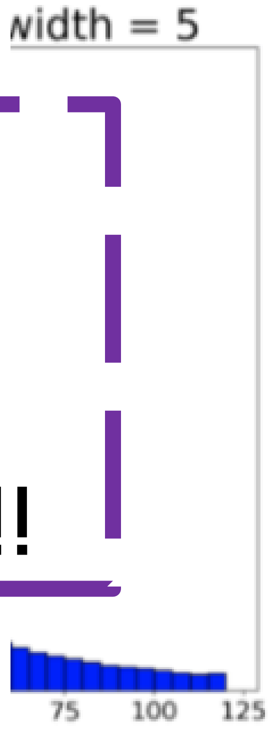
Histogram

working with smooth curves is much easier than jagged histograms

Density Curve







Bin size -> 0

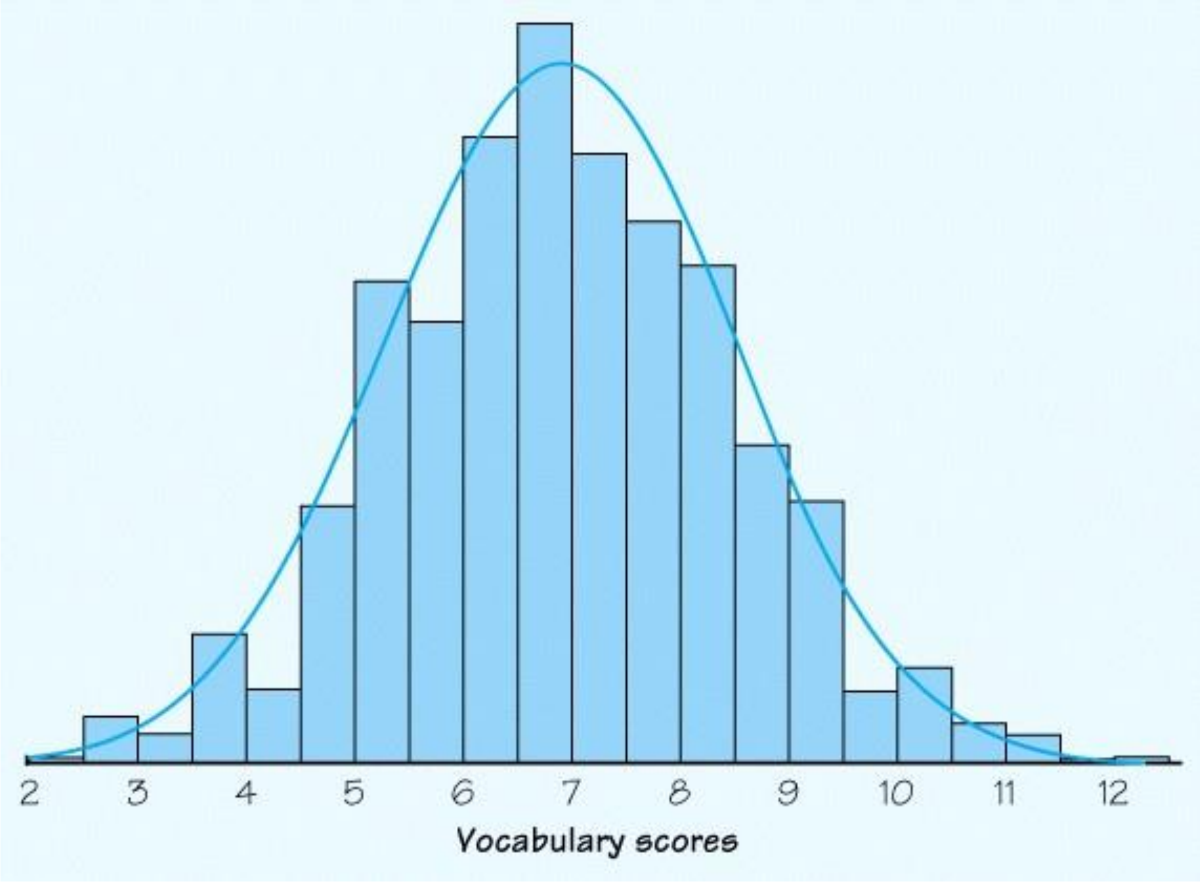
Smooth!!

Discrete -> ctns!!

Density Curves

A density curve is a **smooth curve** that describes the **probability distribution for a continuous random variable X.**

The function that defines this curve is denoted by f(x) and it is called the **density function.**

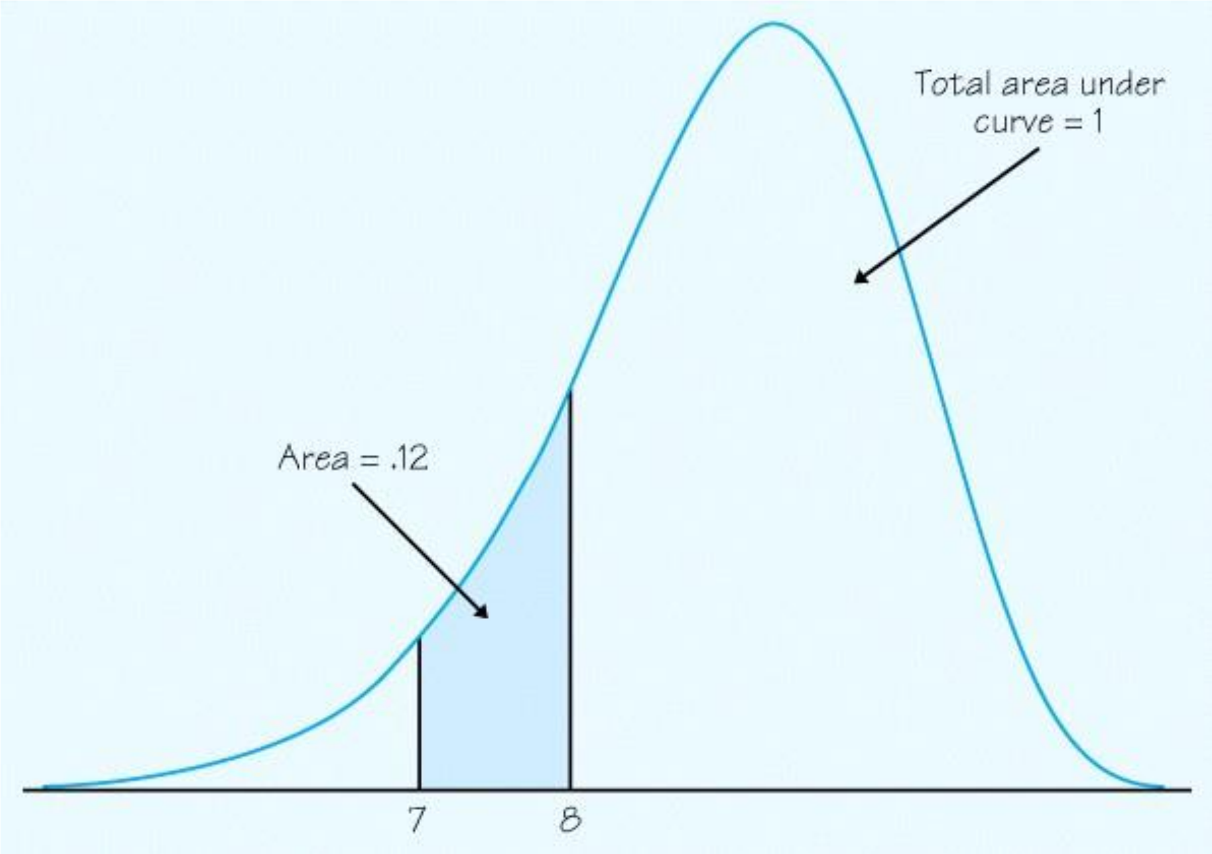


Density Curves -- Properties

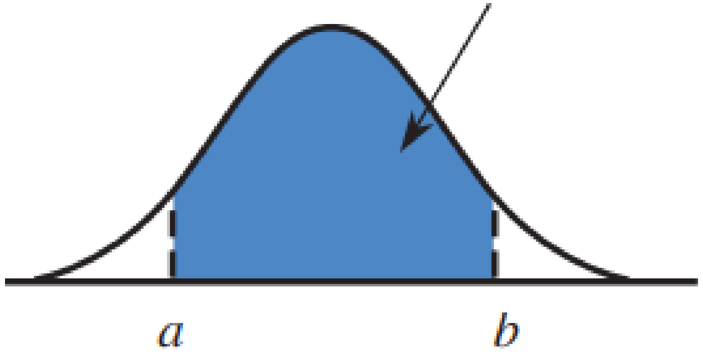
1. $f(x) \geq 0$ (the curve can**not** dip below the horizontal axis).
2. The total area under the density curve is equal to 1.

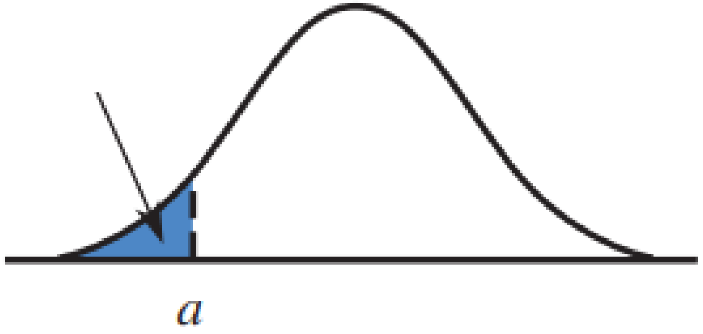
The probability that X falls in any particular interval is the area under the density curve and above the horizontal axis.

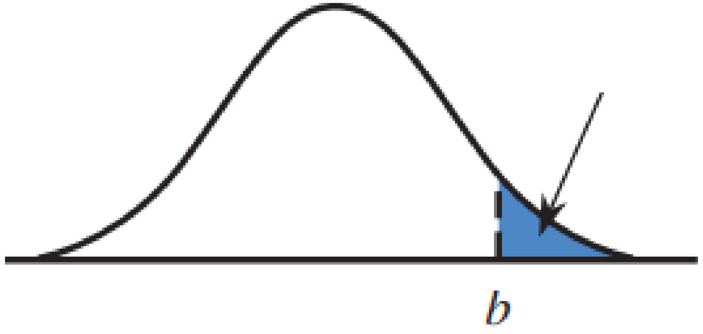
$$P(7 < X \leq 8) = 0.12$$



Density Curves -- Properties







The probability that a continuous random variable x lies between a lower limit a and an upper limit b is

$$
\begin{aligned}
P(a < x < b) &= (\text{cumulative area to the left of } b) - (\text{cumulative area to the left of } a) \\
&= P(x < b) - P(x < a)
\end{aligned}
$$

Density Curves -- Properties

$$P(x=a) = 0$$

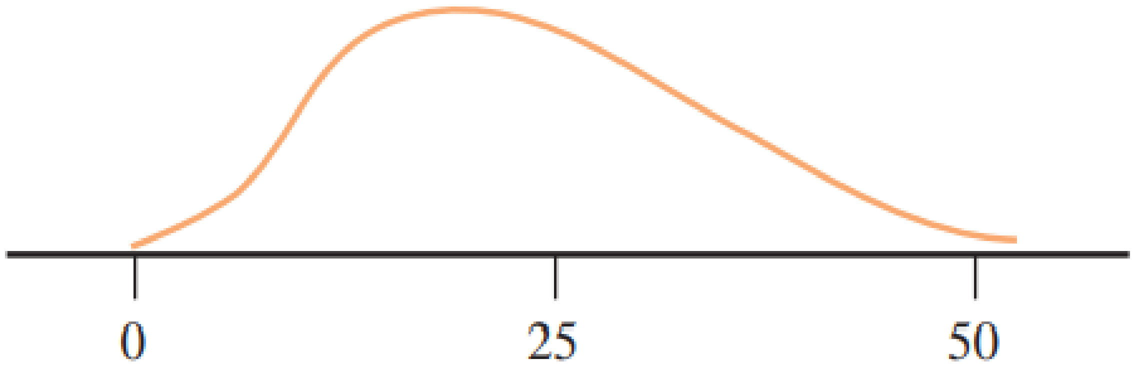
If x is a continuous random variable, then for any two numbers a and b with $a < b$,

$$P(a \leq x \leq b) = P(a < x \leq b) = P(a \leq x < b) = P(a < x < b)$$

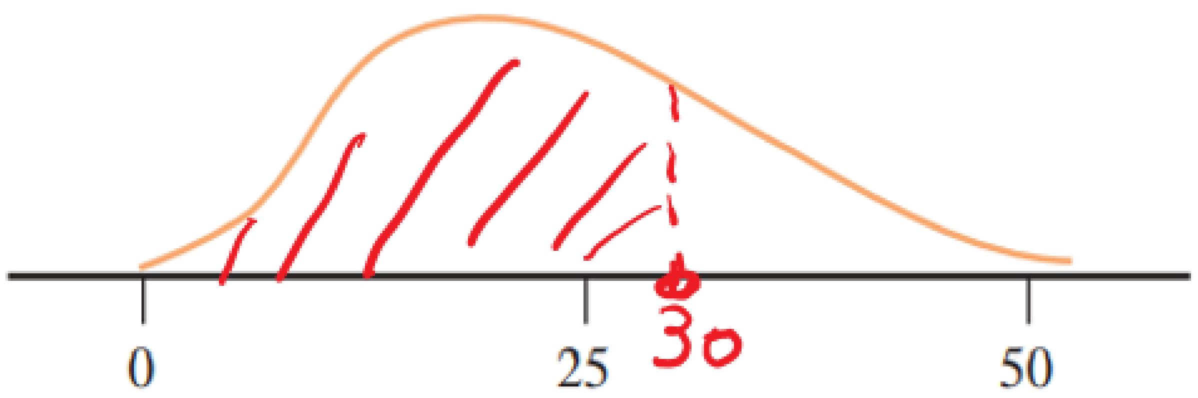
Practice:

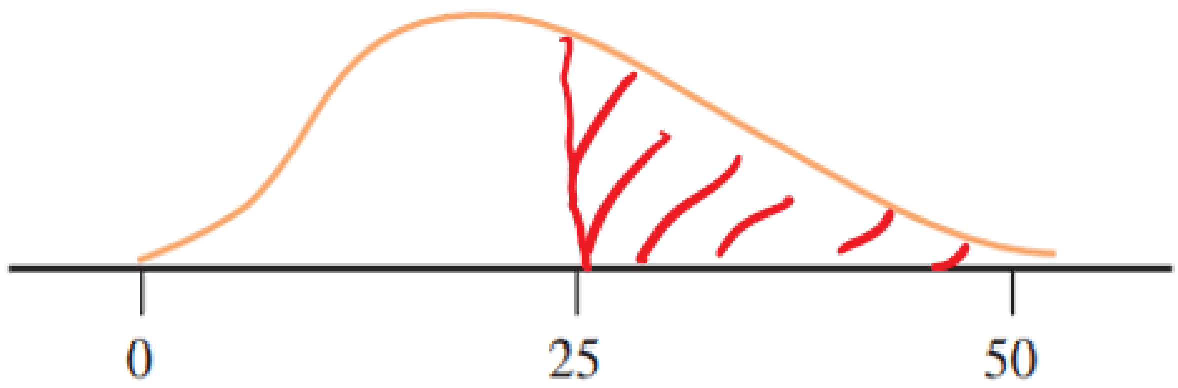
Let X denote the lifetime (in thousands of hours) of a certain type of fan used in diesel engines. The density curve of X is as pictured.

Shade the area under the curve corresponding to each of the following



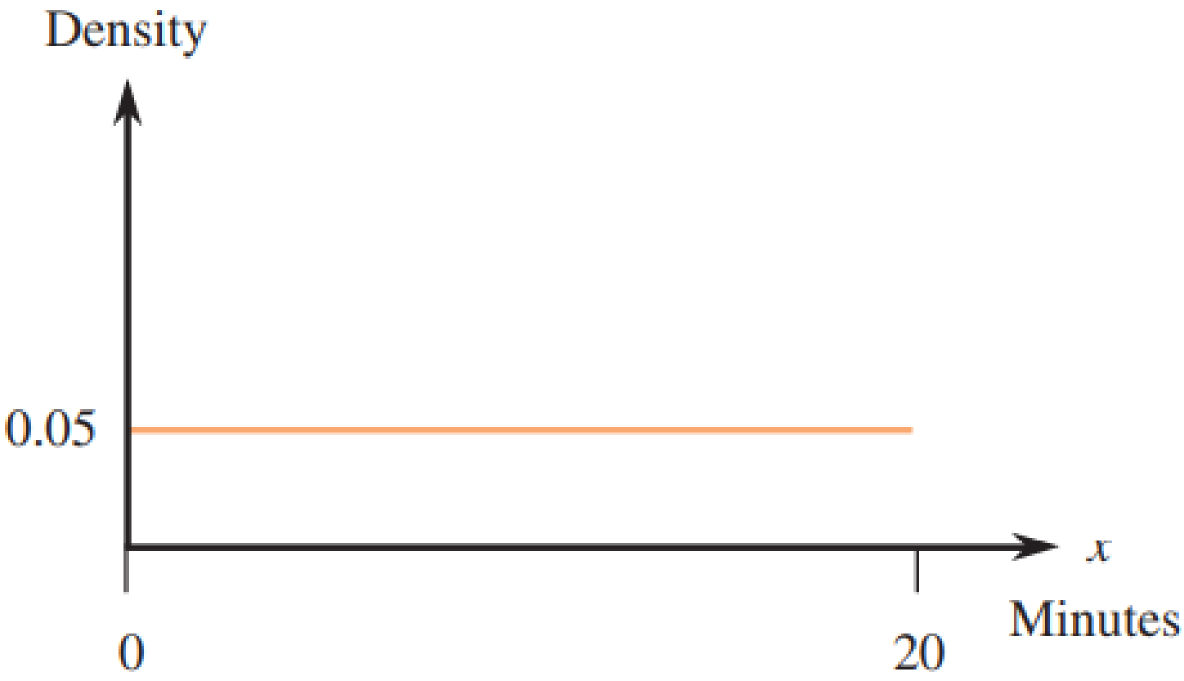
1. $P(X<30)$
2. The probability that the lifetime is at least 25,000 hours


0
25
30
50


0
25
50

Practice:

Let X be the amount of time (in minutes) that a particular San Francisco commuter must wait for a BART train. Suppose that the density curve is as pictured (a uniform distribution):



a. What is the probability that X is less than 10 minutes? more than 15 minutes?

b. What is the probability that X is between 7 and 12 minutes?

c. Find the value c for which $P(X<c) = 0.9$

Practice:

What is the probability that X is less than 10 minutes?

$P(X<10) = 0.5$

more than 15 minutes?

$P(X>15) = 0.25$

What is the probability that X is between 7 and 12 minutes?

$P(7<X<12) = 0.25$

Find the value c for which $P(X<c) = 0.9$

$c = 18$

7.24 Let x denote the amount of gravel sold (in tons) during a randomly selected week at a particular sales facility. Suppose that the density curve has height $f(x)$ above the value x, where

$$f(x) = \begin{cases} 2(1 - x) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a. $P\left(x < \frac{1}{2}\right)$

b. $P\left(x \leq \frac{1}{2}\right)$

c. $P\left(x < \frac{1}{4}\right)$

d. $P\left(\frac{1}{4} < x < \frac{1}{2}\right)$

Hint: Use the results of Parts (a)–(c)

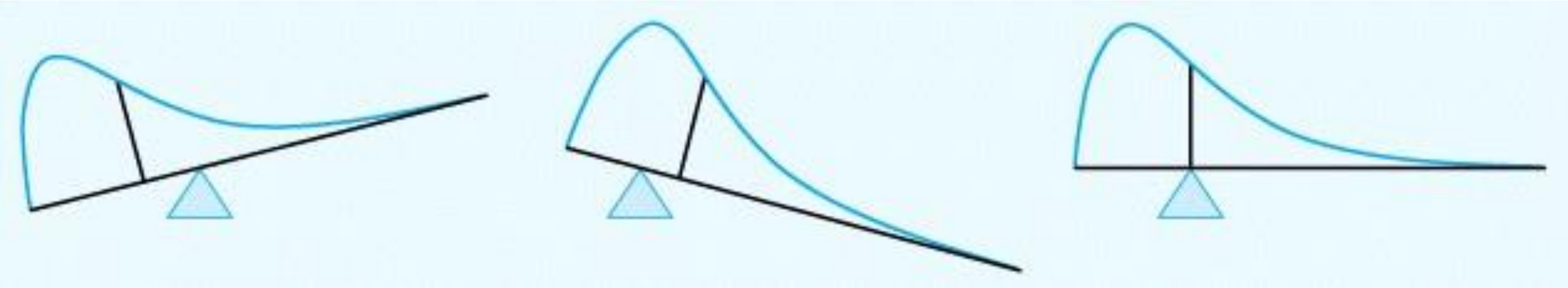
Mean and Median of Density Curves

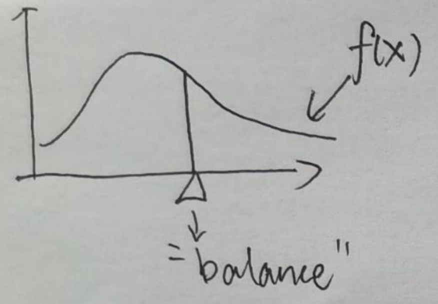
- <u>Median:</u>

The point divides the area under the curve in half

- <u>Mean:</u>

The point at which the curve would balance if made out of solid material





Assume the balance point is $x = \mu$.

G(solid on the left side) * moment of G left should be equal to G right * moment right

$$\int_{-\infty}^{\mu} |x-\mu| f(x)\, dx = \int_{\mu}^{+\infty} |x-\mu| f(x)\, dx$$

moment

$$\therefore \int_{-\infty}^{\mu} (\mu - x) f(x)\, dx = \int_{\mu}^{+\infty} (x-\mu) f(x)\, dx$$

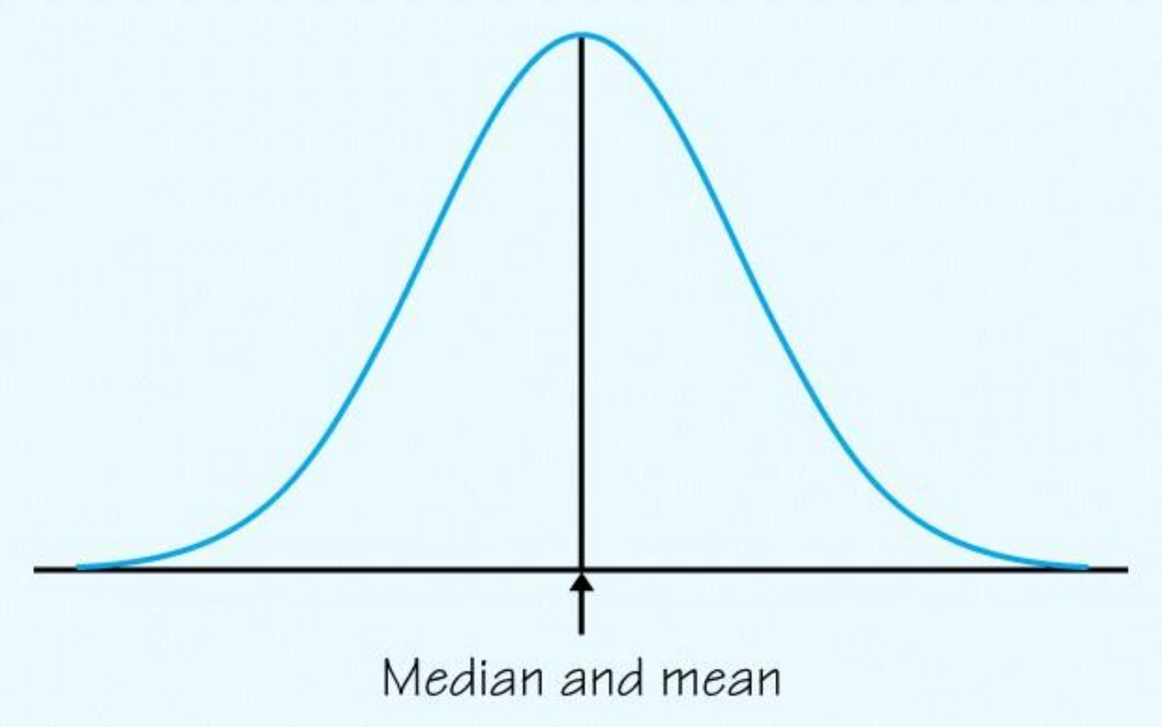
$$\therefore \int_{-\infty}^{\mu} x f(x)\, dx + \int_{\mu}^{+\infty} x f(x)\, dx = \int_{-\infty}^{\mu} f(x) \cdot \mu\, dx + \int_{\mu}^{+\infty} \mu f(x)\, dx$$

Left side $= \int_{-\infty}^{+\infty} x f(x)\, dx = E(x)$

Right side $= \mu \int_{-\infty}^{+\infty} f(x)\, dx = \mu * 1 = \mu$

Thus, $E(x)$ is the balance point of the solid material.

For a symmetric Density curve…



For a skewed density curve…