AP

2024 AP DAILY: PRACTICE SESSIONS

AP Calculus AB Session 1 – MCQ (No Calculator)

Question 1

A

If $x\cos y = \pi - 2y$, then $\frac{dy}{dx} =$

A. $\frac{\cos y}{x\sin y - 2}$

B. $\frac{\cos y}{2 + x\sin y}$

C. $\frac{-\cos y}{2 + x\sin y}$

D. $\frac{x\sin y - \cos y}{2}$

$\cos y + x \cdot (-\sin y)\frac{dy}{dx} = -2\frac{dy}{dx}$

$\therefore \frac{dy}{dx} \cdot (2 - x\sin y) = -\cos y$

$\frac{dy}{dx} = \frac{\cos y}{x \cdot \sin y - 2}$

Question 2

A

$\lim_{x \to 0} \frac{2x^6 + 6x^3}{4x^5 + 3x^3}$ is

A. 0
B. ½
C. 2
D. nonexistent

$\frac{x^3 \cdot (2x^3 + 6)}{x^3 (4x^2 + 3)} = \frac{2x^3 + 6}{4x^2 + 3}$

$\lim_{x \to 0} \frac{6x^2}{8x} = 0$

Question 3

A

$\int \sec x \tan x \, dx =$

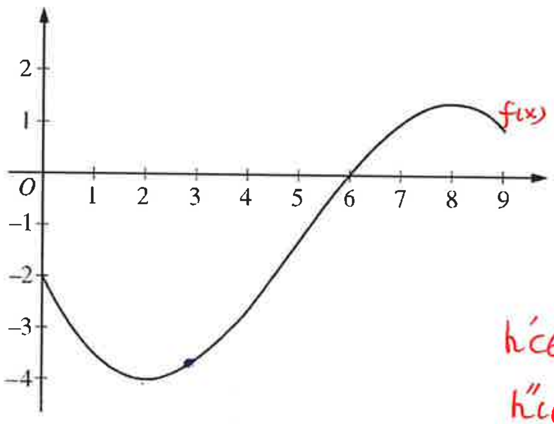
A. $\sec x + C$
B. $\tan x + C$
C. $\frac{\sec^2 x}{2} + C$
D. $\frac{\tan^2 x}{2} + C$

Question 4

The graph of a differentiable function f is shown.

If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

A



Graph of f

$h'(6) = f(6) = 0$

$h''(6) = f'(6) > 0$, f is increasing at $x=6$

$h(6) < 0$

A. $h(6) < h'(6) < h''(6)$

B. $h(6) < h''(6) < h'(6)$

C. $h'(6) < h(6) < h''(6)$

D. $h''(6) < h(6) < h'(6)$

Question 5

D

The function f is continuous on the closed interval $[0, 5]$ and differentiable on the open interval $(0, 5)$. Selected values of f are given in the table shown. The value $x = 4$ satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[0, 5]$. What is the slope of the line tangent to the graph of f at $x = 4$?

x	0	2	3	5
$f(x)$	2	3	6	12

A. 10

B. 4.5

C. 2.8

D. 2

$f'(4) = \dfrac{f(5)-f(0)}{5-0} = \dfrac{12-2}{5} = 2$

Mean Value Theorem: f is ctns on $[a,b]$,

$\exists c$, s.t. $f'(c) = $ AROC on $[a,b]$

$= \dfrac{f(b)-f(a)}{b-a}$

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2024 AP DAILY: PRACTICE SESSIONS

AP Calculus AB Session 2 – MCQ (Calculator Active)

Question 1

x	$f(x)$	$f'(x)$
0	1	1
1	3	4
2	11	13

B

The table above gives selected values for a differentiable and increasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(3)$?

A. 1/13
B. 1/4
C. 1
D. 4

$$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$$

$$= \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(1)} = \frac{1}{4}$$

Question 2

C

On a certain day, the total number of pieces of candy produced by a factory since it opened is modeled by C, a differentiable function of the number of hours since the factory opened. Which of the following is the best interpretation of $C'(3) = 500$?

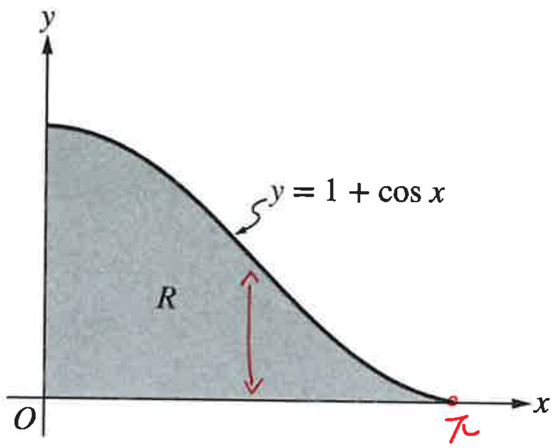
A. The factory produces 500 pieces of candy during its 3rd hour of operation.
B. The factory produces 500 pieces of candy in the first 3 hours after it opens.
C. The factory is producing candy at a rate of 500 pieces per hour, 3 hours after it opens.
D. The rate at which the factory is producing candy is increasing at a rate of 500 pieces per hour per hour, 3 hours after it opens.

$C''(3)$

Question 3

D

Let R be the shaded region in the first quadrant bounded by the x-axis, the y-axis, and the graph of $y = 1 + \cos x$, as shown in the figure. The shape of a chocolate treat can be modeled by rotating R around the x-axis. What is the volume of the chocolate treat?



$V = \int_0^{\pi} \pi \cdot (1+\cos x)^2 \, dx$

≈ 14.804

A. 3.142
B. 4.712
C. 9.870
D. 14.804

Question 4

B

The third derivative of the function f is continuous on the interval $(0, 4)$. Values for f and its first three derivatives at $x = 2$ are given in the table. What is $\lim_{x \to 2} \frac{f(x)}{(x-2)^2}$?

$: \frac{0}{0}$

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
2	0	0	5	7

$= \lim_{x \to 2} \frac{f'(x)}{2(x-2)} = \lim_{x \to 2} \frac{f''(x)}{2} = \frac{5}{2}$

A. 0
B. 5/2
C. 5
D. The limit does not exist.

Source: Released AP Exam; Taken from: AP Classroom

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2024 AP DAILY: PRACTICE SESSIONS

AP Calculus AB Session 3 – FRQ (Calculator Active)

$P(0)=100{,}000$

The penguin population on an island is modeled by a differentiable function P of time t, where $P(t)$ is the number of penguins and t is measured in years, for $0 \le t \le 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by $B(t) = 1000e^{0.06t}$ penguins per year and the death rate for the penguins on the island is modeled by $D(t) = 250e^{0.1t}$ penguins per year.

a. What is the rate of change of the penguin population on the island at time $t = 0$?

b. To the nearest whole number, what is the penguin population on the island at time $t = 40$?

c. To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \le t \le 40$?

d. To the nearest whole number, find the absolute minimum penguin population and the absolute maximum penguin population on the island for $0 \le t \le 40$. Show the analysis that leads to your answer.

a. $B(0)-D(0) = 750$ penguins per year

b. $P(0) + \int_0^{40} B(t)-D(t)\,dt \approx 133057.565$

Around 133058 penguines

c. $\dfrac{\int_0^{40} B(t)-D(t)\,dt}{40} \approx 826.45$ Around 826 penguines per year

d. $B(t)-D(t)=0$ when $t \approx 34.657$

$P(0) = 100{,}000$

$P(34.657) = 100{,}000 + \int_0^{34.657} B(t)-D(t)\,dt \approx 139166.667$

$P(40) \approx 133058$

Source: Released AP Exam; Taken from: AP Classroom

$\therefore$ abs min : 100,000 penguines

abs max : 139167 penguines

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2024 AP DAILY: PRACTICE SESSIONS

AP Calculus AB Session 4 – FRQ (Calculator Active)

The rate at which the number of bears in a region is changing is modeled by the differentiable function R, where $R(t)$ is measured in bears per year and t is measured in years. Selected values of $R(t)$ are shown in the table.

(a) $R'(7) \approx \frac{R(9)-R(4)}{9-4}$

$= \frac{3}{5}$

$= 0.6$ bears per year per year

t (years)	0	4	9	12
$R(t)$ (bears per year)	150	156	159	161

a. Approximate $R'(7)$ using the average rate of change of R over the interval $4 \le t \le 9$. Show the computations that lead to your answer, and indicate units of measure.

(b.) Average value of the number of bears in a region from $t=0$ years to $t=12$ years.

b. Interpret the meaning of $\frac{1}{12}\int_0^{12} R(t)\,dt$ in the context of the problem. Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\frac{1}{12}\int_0^{12} R(t)\,dt$.

$\int_0^{12} R(t)dt$

$\approx 4\times150+5\times156+3\times159$

$=1857$ bears

c. The rate at which the number of bears in the region is changing can also be modeled by the function P, defined by $P(t) = 150e^{0.02\sqrt{t}}$, where $P(t)$ is measured in bears per year and t is measured in years. There are 300 bears in the region at time $t = 0$.

Use this model to find the number of bears in the region at time $t = 12$. Show the setup for your calculations, and give your answer to the nearest whole number of bears.

d. Using the model defined in part (c), find the time t, for $0 \le t \le 12$, at which the number of bears in the region is a maximum. Justify your answer.

$\frac{1}{12}\int_0^{12} R(t)\,dt$

≈ 154.75 bears per year.

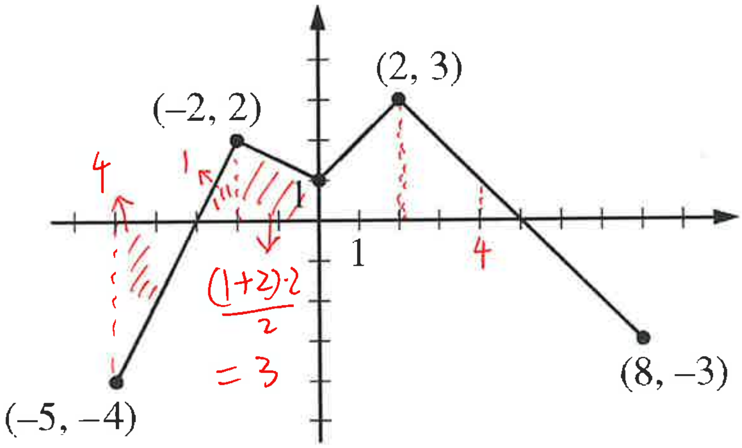
(c) $300+\int_0^{12} P(t)\,dt \approx 2185.339 \approx 2185$ bears.

(d) $P(t)>0$ for all t $\therefore$ the maximum value occurs at $t=12$

Source: Released AP Exam; Taken from AP Classroom

2024 AP DAILY: PRACTICE SESSIONS

AP Calculus AB Session 5 – FRQ (No Calculator)



Graph of f

The continuous function f is defined on the interval $-5 \le x \le 8$. The graph of f, which consists of four line segments, is shown in the figure above. Let g be the function given by $g(x) = 2x + \int_{-2}^{x} f(t)\,dt$.

a. Find $g(0)$ and $g(-5)$. $g(0) = \int_{-2}^{0} f(t)\,dt = 3$ $\quad g(-5) = -10 - \int_{-5}^{-2} f(t)\,dt = -7$

b. Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

c. On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

d. The function h is given by $h(x) = g(x^3+1)$.

Find $h'(1)$. Show the work that leads to your answer.

b. $g'(x) = 2 + f(x)$

$g''(x) = f'(x)$

$g''(4) = -1$

$g''(-2)$ DNE.

c. $g''(x) = f'(x) < 0 \Leftrightarrow g$ is concave down

$\therefore (-2, 0), (2, 8)$

d. $h'(x) = g'(x^3+1)\cdot(3x^2)$

$h'(1) = g'(2)\cdot 3$

$= [2 + f(2)]\cdot 3$

$= 15$

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2024 AP DAILY: PRACTICE SESSIONS

AP Calculus AB Session 6 – FRQ (No Calculator)

The functions $f(x)$ and $g(x)$ are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	3/2	−8	3	6
$g(x)$	1	2	−3	0
$g'(x)$	5	4	2	8

a. Let $h(x)$ be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

b. Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

c. Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)dt$. Find $m(2)$. Show the work that leads to your answer.

d. Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

a. $h'(x) = f'(g(x)) \cdot g'(x)$

$h'(7) = f'(g(7)) \cdot g'(7)$

$= f'(0) \cdot 8$

$= \frac{3}{2} \times 8 = 12$

c. $m(2) = 40 + f(2) - f(0) = 37$

d. $m'(x) = 15x^2 + f'(x)$

$m'(2) = 60 + f'(2) = 52 > 0$

$\therefore$ increasing.

b. $K''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$

$k''(4) = 2f(4)\, f'(4)\, g(4) + (f(4))^2\, g'(4)$

$= 8 \times 3 \times (-3) + 16 \times 2 = -40 < 0$

$\therefore$ concave down

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2024 AP DAILY: PRACTICE SESSIONS

AP Calculus AB Session 7 – FRQ (No Calculator)

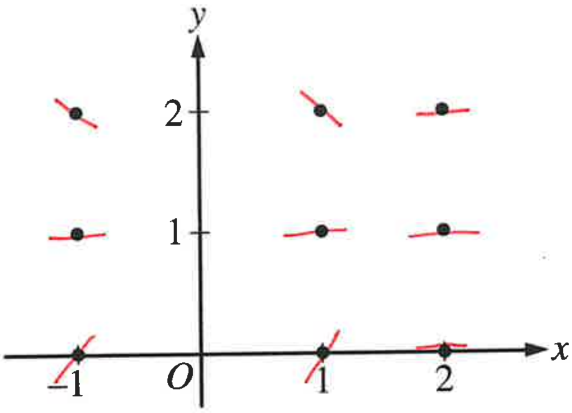
Consider the differential equation $\frac{dy}{dx}=\left(1-\frac{2}{x^2}\right)(y-1)$, where $x \neq 0$. Let $y=f(x)$ a be the particular solution to the differential equation with initial condition $f(1)=2$.

a. Find the slope of the line tangent to the graph of f at the point $(1, 2)$.

$\left.\frac{dy}{dx}\right|_{(1,2)} = (1-\frac{2}{1})\cdot(2-1) = -1$

b. On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

$\frac{dy}{dx}$	x	y
0	/	1
-1	±1	2
0	2	2
1	±1	0
0	2	0



c. Find the particular solution $y=f(x)$ to the differential equation $\frac{dy}{dx}=\left(1-\frac{2}{x^2}\right)(y-1)$, with initial condition $f(1)=2$.

$\int \frac{1}{y-1}\, dy = \int (1-\frac{2}{x^2})\, dx$

$\ln|y-1| = x + 2x^{-1} + C$

$y-1 = C\cdot e^{x+\frac{2}{x}}$

$y = 1 + C e^{x+\frac{2}{x}}$

$\because f(1)=2 \quad \therefore 2 = 1 + C\cdot e^{3}$

$\therefore C = e^{-3}$

Source: Released AP Exam; Taken from AP Classroom

$\therefore$ particular solution: $f(x) = 1 + e^{x+\frac{2}{x}-3}$

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2024 AP DAILY: PRACTICE SESSIONS

AP Calculus AB Session 8 – FRQ (No Calculator)

Two particles move along the x-axis.

For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

a. For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

b. For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

c. Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

d. Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

(a) $r'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3) > 0$ then $t \in [0,1), (3,6]$

(b) $p'(t) = -\frac{\pi}{2}\sin(\frac{\pi}{4}t)$ $\therefore p'(t) > 0$ when $\sin(\frac{\pi}{4}t) < 0$, $\frac{\pi}{4}t \in (\pi + 2k\pi, 2\pi + 2k\pi)$

$t \in (4+8k, 8+8k)$

$\therefore$ for $0 \le t \le 6$, $p'(t) > 0$ when $t \in (4,6]$

$p'(t) < 0$ when $t \in [0,4)$

$\therefore$ When $t \in (0,1), (3,4)$, two particles travel in opposite directions

(c) $a_p(t) = p''(t) = -\frac{\pi}{8}\cos(\frac{\pi}{4}t)$

$a_p(3) = \frac{\pi}{8}\cdot\frac{\sqrt{2}}{2} > 0$

$p'(3) = -\frac{\pi}{2}\cdot\frac{\sqrt{2}}{2} < 0$

$\therefore$ the particle P is slowing down since its acceleration and velocity are of opposite directions.

(d) $$\frac{\int_1^3 |p(t) - r(t)|\,dt}{3-1}$$

Source: Released AP Exam; Taken from AP Classroom