1. The graph of the function f is shown below. Find the limit or value of the function at a given point.

$\lim_{x\to 3^-} f(x) =$ 0

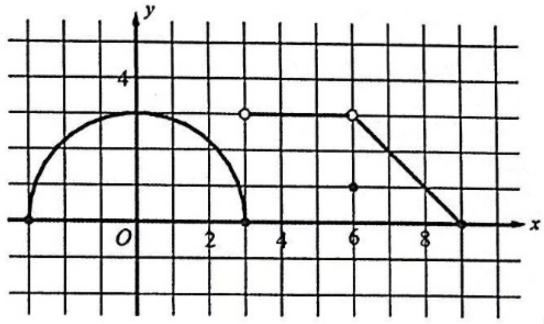
$\lim_{x\to 3^+} f(x) =$ 3

$\lim_{x\to 3} f(x) =$ DNE

$\lim_{x\to 6} f(x) =$ 3

$f(3) =$ 0

$f(6) =$ 1

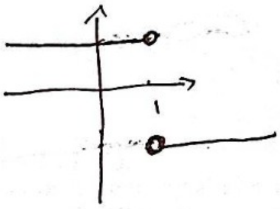


Graph of $y = f(x)$

2. $\lim_{x\to\frac{\pi}{6}} \cos^2 x =$ $\underline{\ \frac{3}{4}\ }$

3. If $f(x) = \begin{cases} x^2+3, x \neq 1 \\ 1, x = 1 \end{cases}$, then $\lim_{x\to 1} f(x) =$ $\underline{\ 4\ }$

4. $\lim_{x\to 1} \frac{|x-1|}{1-x} =$ $\underline{\text{DNE}}$ "Jump"



B

5. Let f be a function given by $f(x) = \begin{cases} 3-x^2, & \text{if } x<0 \\ 2-x, & \text{if } 0 \le x < 2 \\ \sqrt{x-2}, & \text{if } x > 2 \end{cases}$.

Which of the following statements are true about f?

I. $\lim_{x\to 0} f(x) = 2$ × $f(0^-) = 3 \neq f(0^+) = 2$ $\quad f(2^-) = 0 = f(2^+)$

II. $\lim_{x\to 2} f(x) = 0$ ✓

III. $\underbrace{\lim_{x\to 1} f(x)}_{1} = \underbrace{\lim_{x\to 6} f(x)}_{2}$ ×

(A) I only (B) II only (C) II and III only (D) I , II, and III

6. Let f be a function defined by $f(x) = \begin{cases} \frac{x^2-a^2}{x-a}, x \neq a \\ 4, x = a \end{cases}$. If f is continuous for all real numbers x, the condition should be satisfied is $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a) = 4$

The value of a is 2.

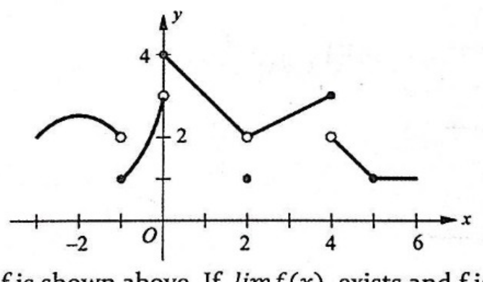
7. Let f be a function defined by $f(x) = \begin{cases} \frac{\pi \sin x}{x}, x < 0 \\ a - bx, 0 \leq x < 1 \\ \arctan x, x \geq 1 \end{cases}$.

If f is continuous for all real numbers x, what are the values of a and b?

Hint: $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0) \Rightarrow a = \pi$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1) \Rightarrow b = \frac{3}{4}\pi$



$x = -1, 0, 2, 4$

8. The graph of a function f is shown above. If $\lim_{x\to a} f(x)$ exists and f is not continuous at $x = a$, then $a =$ 2

$\frac{(\sqrt{3x-1}-\sqrt{2x})(\sqrt{3x-1}+\sqrt{2x})}{(x-1)\cdot(\sqrt{3x-1}+\sqrt{2x})} \to \frac{1}{2\sqrt{2}}$

9. If $f(x) = \begin{cases} \frac{\sqrt{3x-1}-\sqrt{2x}}{x-1}, x \neq 1 \\ a, x = 1 \end{cases}$, and if f is continuous at $x = 1$, then $a =$ $\frac{\sqrt{2}}{4}$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$a = \frac{\sqrt{2}}{4}$

B 10. $\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x} = \lim_{x\to 0}\frac{(\sqrt{4+x}-2)(\sqrt{4+x}+2)}{x(\sqrt{4+x}+2)} = \frac{1}{4}$

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) nonexistent

11. $\lim_{x\to 1}\frac{\sqrt{3+x}-2}{x^3-1} = \lim_{x\to 1}\frac{(\sqrt{3+x}-2)(\sqrt{3+x}+2)}{\cancel{(x-1)}(x^2+x+1)(\sqrt{3+x}+2)} = \lim_{x\to 1}\frac{1}{(x^2+x+1)(\sqrt{3+x}+2)} = \frac{1}{12}$

Hint: $x^3-1=(x-1)(x^2+x+1)$

12. Evaluate $\lim_{a\to 0}\frac{-1+\sqrt{1+a}}{a}$

$= \lim_{a\to 0}\frac{(\sqrt{1+a}-1)(\sqrt{1+a}+1)}{a(\sqrt{1+a}+1)} = \lim_{a\to 0}\frac{1}{\sqrt{1+a}+1} = \frac{1}{2}$

13. What is the value of a, if $\lim_{x\to 0}\frac{\sqrt{ax+9}-3}{x}=1$

$\lim_{x\to 0}\frac{(\sqrt{ax+9}-3)(\sqrt{ax+9}+3)}{x\cdot(\sqrt{ax+9}+3)} = \lim_{x\to 0}\frac{a}{\sqrt{ax+9}+3} = \frac{a}{6} = 1$

$\therefore a=6$

14. Find $\lim_{x\to 0}\frac{f(x)-g(x)}{\sqrt{g(x)+7}}$, if $\lim_{x\to 0}f(x)=2$ and $\lim_{x\to 0}g(x)=-3$.

$\frac{5}{2}$

15.

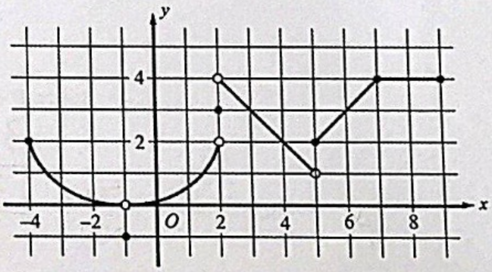
(1) $\lim_{x\to -1}\cos(f(x)) =$ $\cos(0)=1$

(2) $\lim_{x\to 2^-}f(x) =$ 2

(3) $\lim_{x\to 2}f(x) =$ DNE

(4) $\lim_{x\to 5^+}xf(x) =$ 10

(5) (Optional) $\lim_{x\to 5^-}\arctan(f(x)) =$ $\arctan 1 = \frac{\pi}{4}$



The figure above shows the graph of $y=f(x)$ on the closed interval $[-4,9]$.

D 16. $\lim_{x\to\pi/3}\dfrac{\sin(\frac{\pi}{3}-x)}{\frac{\pi}{3}-x}=$ $\lim_{u\to 0}\dfrac{\sin u}{u}=1$

$u=\frac{\pi}{3}-x$

(A) -1 (B) 0 (C) $\dfrac{\sqrt{3}}{2}$ (D) 1

C 17. $\lim_{x\to 0}\dfrac{\sin 3x}{\sin 2x}=$ $\lim_{x\to 0}\dfrac{\sin 3x}{3x}\cdot\dfrac{2x}{\sin 2x}\cdot\dfrac{3}{2}$

(A) $\dfrac{2}{3}$ (B) 1 (C) $\dfrac{3}{2}$ (D) nonexistent

D 18. $\lim_{\theta\to 0}\dfrac{\theta+\theta\cos\theta}{\sin\theta\cos\theta}=$ $\lim_{\theta\to 0}\dfrac{\frac{\theta}{\sin\theta}+\frac{\theta}{\sin\theta}\cos\theta}{\cos\theta}\to\dfrac{1+1\times 1}{1}=2$

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$ (C) 1 (D) 2

D 19. $\lim_{x\to 0}\dfrac{\tan 3x}{x}=$ $\lim_{x\to 0}\dfrac{\sin 3x}{3x}\cdot\dfrac{1}{\cos 3x}\cdot 3$

(A) 0 (B) $\dfrac{1}{3}$ (C) 1 (D) 3

A 20. $\lim_{x\to 3}\dfrac{\frac{1}{x}-\frac{1}{3}}{x-3}=$ $\lim_{x\to 3}\dfrac{\frac{3-x}{3x}}{x-3}=\lim_{x\to 3}\dfrac{1}{-3x}=-\dfrac{1}{9}$

(A) $-\dfrac{1}{9}$ (B) $\dfrac{1}{9}$ (C) -9 (D) 9

D

21. Let f be a continuous function on the closed interval $[-2,7]$. If $f(-2)=5$ and $f(7)=-3$, then the Intermediate Value Theorem guarantees that

(A) $f'(c)=0$ for at least one c between -2 and 7

(B) $f'(c)=0$ for at least one c between -3 and 5

(C) $f(c)=0$ for at least one c between -3 and 5

(D) $f(c)=0$ for at least one c between -2 and 7

B

22. $\lim\limits_{x\to\infty}\dfrac{3+2x^2-x^4}{3x^4-5}=$ $\lim\limits_{x\to\infty}\dfrac{-x^4}{3x^4}$

(A) -2 (B) $-\frac{1}{3}$ (C) $\frac{1}{5}$ (D) 1

C

23. What is $\lim\limits_{x\to-\infty}\dfrac{x^3+x-8}{2x^3+3x-1}=$

(A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 2

C

24. Which of the following lines is an asymptote of the graph of $f(x)=\dfrac{x^2+5x+6}{x^2-x-12}$?

I. $x=-3$ ✗ $\rightarrow$ Hole

II. $x=4$ ✓

III. $y=1$ ✓

$=\dfrac{(x+2)(x+3)}{(x-4)(x+3)}$

$\Rightarrow x=4$

$\lim\limits_{x\to\pm\infty}f(x)=1$

$\Rightarrow y=1$

(A) II only (B) III only (C) II and III only (D) I, II, and III

A

25. If the horizontal line $y=1$ is an asymptote for the graph of the function f, which of the following statements must be true?

(A) $\lim\limits_{x\to\infty}f(x)=1$ (严谨一為应該是 $\lim\limits_{x\to\infty}f(x)=1$ OR $\lim\limits_{x\to-\infty}f(x)=1$)

(B) $\lim\limits_{x\to1}f(x)=\infty$ $\Rightarrow$ $x=1$ is ~~a~~ vertical asymptote

(C) $f(1)$ is undefined

(D) $f(x)=1$ for all x

D

26. If $x=1$ is the vertical asymptote and $y=-3$ is the horizontal asymptote for the graph of the function f, which of the following could be the equation of the curve?

(A) $f(x)=\dfrac{-3x^2}{x-1}$ ✗

(B) $f(x)=\dfrac{-3(x-1)}{x+3}$ $\rightarrow -3$ as $x\to\infty$

(C) $f(x)=\dfrac{-3(x^2-1)}{x-1}$ $\rightarrow -\infty$ as $x\to\infty$ ✗

(D) $f(x)=\dfrac{-3(x^2-1)}{(x-1)^2}$ $\rightarrow -3$ as $x\to\infty$

$f(x)=-3\cdot\dfrac{(x+1)(x-1)}{(x-1)^2}$

$=(-3)\dfrac{x+1}{x-1}$

C

27. What are all horizontal asymptotes of the graph of $y = \dfrac{6+3e^x}{3-3e^x}$ in the xy-plane?

(A) $y=-1$ only

(B) $y=2$ only

(C) $y=-1$ and $y=2$

(D) $y=0$ and $y=2$

28. Let $f(x) = \dfrac{3x-1}{x^3-8} = \dfrac{3x-1}{(x^2+2x+4)(x-2)} = \dfrac{3x-1}{((x+1)^2+3)\ (x-2)}$

(a) Find the vertical asymptote(s) of f. Show the work that leads to your answer.

(b) Find the horizontal asymptote(s) of f. Show the work that leads to your answer.

(a) V.A. $x=2$ $\quad \lim\limits_{x\to 2^+} f(x) = +\infty$

$\lim\limits_{x\to 2^-} f(x) = -\infty$

(b) H.A. $y=0$ $\quad \lim\limits_{x\to \infty} f(x) = \lim\limits_{x\to -\infty} f(x) = 0$

29. Let $f(x) = \dfrac{\sin x}{x^2+2x} = \dfrac{\sin x}{x(x+2)}$

(a) Find the vertical asymptote(s) of f. Show the work that leads to your answer.

(b) Find the horizontal asymptote(s) of f. Show the work that leads to your answer.

(a) $x=-2$ $\quad \lim\limits_{x\to -2^+} f(x) = +\infty$

$\lim\limits_{x\to -2^-} f(x) = -\infty$

(b) $\lim\limits_{x\to \infty} f(x) = 0$ $\quad y=0$