1. Find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ using the definition of derivative, if $f(x) = \sqrt{2x+1}$

$\lim_{h\to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} = \lim_{h\to 0} \frac{2h}{h(\sqrt{2(x+h)+1} + \sqrt{2x+1})}$

$= \lim_{h\to 0} \frac{2}{\sqrt{2x+1+2h} + \sqrt{2x+1}}$

$= (2x+1)^{-\frac{1}{2}}$

2. $\lim_{h\to 0} \frac{\sqrt[3]{8+h}-2}{h} = f'(8)$

$f(x) = \sqrt[3]{x}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$ $\quad \therefore f'(8) = \frac{1}{12}$

B 3. $\lim_{h\to 0} \frac{(2+h)^5 - 32}{h}$ is $\quad f(x) = x^5$

(A) $f'(5)$, where $f(x) = x^2$

(B) $f'(2)$, where $f(x) = x^5$

(C) $f'(5)$, where $f(x) = 2^x$

(D) $f'(2)$, where $f(x) = 2^x$

C 4. If f is a differentiable function, then $f'(1)$ is given by which of the following?

I. $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ ✓

II. $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$ ✓

III. $\lim_{x\to 0} \frac{f(x+h)-f(x)}{h}$ ✗

(A) I only (B) II only (C) I and II only (D) I and III only

D 5. What is the instantaneous rate of change at $x = -1$ of the function $f(x) = -\sqrt[3]{x^2}$? $\quad f'(-1)$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

C 6. $\lim_{h\to 0} \frac{\frac{1}{2}[\ln(e+h) - 1]}{h}$ is $\quad$ (1: $\ln e$)

$f(x) = \frac{1}{2}\ln x = \ln\sqrt{x}$

(A) $f'(1)$, where $f(x) = \ln\sqrt{x}$

(B) $f'(1)$, where $f(x) = \ln\sqrt{x+e}$

(C) $f'(e)$, where $f(x) = \ln\sqrt{x}$

(D) $f'(e)$, where $f(x) = \ln(\frac{x}{2})$

So $f(1^-)=f(1^+)=f(1)$. $\underline{k=m-2}$ ②

f is ctns at $x=1$

7. Let f be the function defined by $f(x)=\begin{cases} mx^2-2 & \text{if } x\le 1 \\ k\sqrt{x} & \text{if } x>1 \end{cases}$. If f is differentiable at $x=1$, what are the values of k and m?

$f'(x)=\begin{cases} 2mx & , x<1 \\ \frac{k}{2}\cdot x^{-\frac{1}{2}} & , x>1 \end{cases}$

$f'(1^-)=f'(1^+)$

$\therefore 2m=\frac{k}{2}$ ①

①,② $\Rightarrow \begin{cases} m=-\frac{2}{3} \\ k=-\frac{8}{3} \end{cases}$

$$f(x)=\begin{cases} 1-2x, & \text{if } x\le 1 \\ -x^2, & \text{if } x>1 \end{cases}$$

D 8. Let f be the function given above. Which of the following must be true?

I. $\lim_{x\to 1} f(x)$ exists. I. $f(1^-)=-1=f(1^+)=-1$

II. f is continuous at $x=1$. II. $\lim_{x\to 1} f(x)=-1=f(1)$

III. f is differentiable at $x=1$.

III. $f'(x)=\begin{cases} -2 & , x<1 \\ -2x & , x>1 \end{cases}$

$f'(1^-)=f'(1^+)=-2$

$\therefore f'(1)=2$

(A) I only
(B) I and II only
(C) II and III only
(D) I, II, and III

9. Let f be the function defined by

$$f(x)=\begin{cases} x+2 & \text{for } x\le 0 \\ \frac{1}{2}(x+2)^2 & \text{for } x>0. \end{cases}$$

$f'(x)=\begin{cases} 1 & , x<0 \\ x+2 & , x>0 \end{cases}$

(a) Find the left-hand derivative of f at $x=0$. (a) $f'(0^-)=1$

(b) Find the right-hand derivative of f at $x=0$. b) $f'(0^+)=2$

(c) Is the function f differentiable at $x=0$? Explain why or why not.

(c) $\lim_{x\to 0^+} f'(x)=2\ne \lim_{x\to 0^-} f(x)$

$\therefore$ DNE.

(d) Suppose the function g is defined by

$$g(x)=\begin{cases} x+2 & \text{for } x\le 0 \\ a(x+b)^2 & \text{for } x>0, \end{cases}$$

where a and b are constants. If g is differentiable at $x=0$, what are the values of a and b?

$\Downarrow$

g is ctns at $x=0$

ctns: $g(0)=\lim_{x\to 0^-} g(x)=\lim_{x\to 0^+} g(x)$

$ab^2=2$ ①

diff: $g'(0^-)=g'(0^+)$

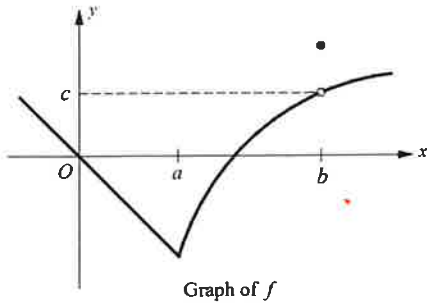
$g'(x)=\begin{cases} 1 & , x<0 \\ 2a(x+b) & , x>0 \end{cases}$

$\therefore 1=2ab$ ②

①,② $\Rightarrow \begin{cases} a=\frac{1}{8} \\ b=4 \end{cases}$

C

10.



Graph of f

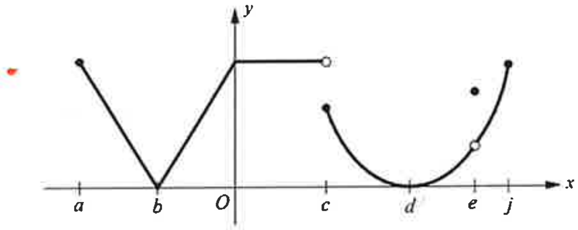
The graph of a function f is shown in the figure above. Which of the following statements must be false?

(A) $f(x)$ is defined for $0 \le x \le b$.

(B) $f(b)$ exists.

(C) $f'(b)$ exists.

(D) $\lim_{x \to a^-} f'(x)$ exists.



Graph of f

B

11. The graph of a function f is shown in the figure above. At how many points in the interval $a < x < j$ is f' not defined?

(A) 3 (B) 4 (C) 5 (D) 6

12. The equation of the line tangent to the graph of $y = x\sqrt{3} + x^2$ at the point $(1,2)$ is ______

$y' = \sqrt{3} + 2x$

$y'(1) = 2+\sqrt{3}$ = slope

$y - 2 = (2+\sqrt{3})(x-1)$

A 13. If $f(x)=(x^3-2x+5)(x^{-2}+x^{-1})$, then $f'(1)=$

(A) -10 (B) -6 (C) $-\frac{9}{2}$ (D) $\frac{7}{2}$

C 14. If $f(x)=\dfrac{\sqrt{x}-1}{\sqrt{x}+1}$ then $f'(x)=$

(A) $\dfrac{\sqrt{x}}{(\sqrt{x}+1)^2}$

(B) $\dfrac{x}{(\sqrt{x}+1)^2}$

(C) $\dfrac{1}{\sqrt{x}(\sqrt{x}+1)^2}$

(D) $\dfrac{\sqrt{x}-1}{\sqrt{x}(\sqrt{x}+1)^2}$

15. If $g(2)=3$ and $g'(2)=-1$, what is the value of $\dfrac{d}{dx}\left(\dfrac{g(x)}{x^2}\right)$ at $x=2$?

$$= \left[\frac{g'x^2-2xg}{x^4}\right]_{x=2} = -1$$

16. If $f(x)=\dfrac{x}{x-\frac{a}{x}}$ and $f'(1)=\dfrac{1}{2}$, what is the value of a?

$$f(x)=\frac{x}{\frac{x^2-a}{x}} = \frac{x^2}{x^2-a} = \frac{x^2-a+a}{x^2-a} = 1+\frac{a}{x^2-a}$$

$$f'(x)= a\cdot(-1)(x^2-a)^{-2}(2x) \qquad f'(1) = -\frac{2a}{(1-a)^2} = \frac{1}{2}$$

$$\therefore a=-1$$

D 17. If $y=x^2\cdot f(x)$, then $y''=$

(A) $x^2f''(x)+xf'(x)+2f(x)$

(B) $x^2f''(x)+xf'(x)+f(x)$

(C) $x^2f''(x)+2xf'(x)+f(x)$

(D) $x^2f''(x)+4xf'(x)+2f(x)$

18. Let $h(x)=x\cdot f(x)\cdot g(x)$. Find $h'(1)$, if $f(1)=-2$, $g(1)=3$, $f'(1)=1$, and $g'(1)=\frac{1}{2}$.

$$h'(x) = f\cdot g + x f' g + x f g'$$

$$h'(1) = f(1)g(1) + f'(1)g(1) + f(1)g'(1)$$

$$= (-6) + 3 + (-2)\cdot\frac{1}{2} = -4$$

19. Let $g(x)=\dfrac{x}{\sqrt{x}-1}$. Find $g''(4)$.

$$g'(x)=\frac{(\sqrt{x}-1)-\frac{1}{2}x^{-\frac{1}{2}}x}{(\sqrt{x}-1)^2} = \frac{\sqrt{x}-1-\frac{1}{2}\sqrt{x}}{x+1-2\sqrt{x}} = \frac{\frac{1}{2}\sqrt{x}-1}{x-2\sqrt{x}+1}$$

$$g''(x)=\frac{\frac{1}{4}x^{-\frac{1}{2}}(x-2\sqrt{x}+1)-(1-x^{-\frac{1}{2}})\cdot(\frac{1}{2}x^{\frac{1}{2}}-1)}{(x-2\sqrt{x}+1)^2}$$

$$g''(4)=\frac{\frac{1}{4}\cdot\frac{1}{2}-(1-\frac{1}{2})(\frac{1}{2}\cdot 2-1)}{1} = \frac{1}{8}+\frac{1}{2}\cdot 0 = \frac{1}{8}$$

20. If $f(x) = (x^2 - 3x)^{\frac{3}{2}}$, then $f'(4) =$

$f'(x) = \frac{3}{2}(x^2-3x)^{\frac{1}{2}} \cdot (2x-3)$

$f'(4) = 15$

21.

A

If $f(x) = (3-\sqrt{x})^{-1}$, then $f''(4) =$

$f'(x) = -(3-\sqrt{x})^{-2}\left(-\frac{1}{2}x^{-\frac{1}{2}}\right) = \frac{1}{2}\cdot(3-\sqrt{x})^{-2}x^{-\frac{1}{2}}$

$f''(x) = \frac{1}{2}(-2)(3-\sqrt{x})^{-3}\left(-\frac{1}{2}x^{-\frac{1}{2}}\right)\cdot x^{-\frac{1}{2}} + \frac{1}{2}(3-\sqrt{x})^{-2}\cdot\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$

$= \frac{1}{2}(3-\sqrt{x})^{-3}x^{-1} - \frac{(3-\sqrt{x})^{-2}x^{-\frac{3}{2}}}{4}$

$f''(4) = \frac{3}{32}$

(A) $\frac{3}{32}$ (B) $\frac{3}{16}$ (C) $\frac{3}{4}$ (D) $\frac{9}{4}$

22.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	1	−1
2	−2	1	−1	3
3	1	4	2	3
4	5	2	1	−2

The table above gives values of f, f', g, and g' at selected values of x.

(1) Find $h'(1)$, if $h(x) = f(g(x))$.

$h'(x) = f'(g(x))\cdot g'(x)$

$h'(1) = f'(g(1))\cdot g'(1) = f'(2)\cdot(-1) = 1$

(2) Find $h'(2)$, if $h(x) = x f(x^2)$.

$h'(x) = f(x^2) + x f'(x^2)\cdot 2x$

$h'(2) = f(4) + 8f'(4) = 5 + 8\times 1 = 13$

(3) Find $h'(3)$, if $h(x) = \frac{f(x)}{\sqrt{g(x)}}$.

$h'(x) = \frac{f'(x)g(x) - \frac{1}{2}g(x)^{-\frac{1}{2}}g'(x)f(x)}{g(x)}$

$h'(3) = \frac{2\times 2 - \frac{1}{2}\times\frac{1}{\sqrt{4}}\cdot 3\times 1}{4} = \frac{13}{16}$

(4) Find $h'(2)$, if $h(x) = [f(2x)]^2$

$h'(x) = 2f(2x)\cdot f'(2x)\cdot 2 = 4f(2x)f'(2x)$

$h'(2) = 4f(4)f'(4) = 20$

(5) Find $h'(1)$, if $h(x) = (x^9 + f(x))^{-2}$

$h'(x) = (-2)(x^9+f(x))^{-3}\cdot(9x^8+f'(x))$

$h'(1) = -2\cdot\frac{1}{64}\cdot 10$

$= -\frac{5}{16}$

23. Let $f(x)=xe^x$ and $f^{(n)}(x)$ be the nth derivative of f with respect to x. If $f^{(10)}(x)=(x+n)\,e^x$, what is the value of n?

$n=10$

$f'(x)= e^x + xe^x$

$f''(x)= e^x + e^x + xe^x$

$\vdots$

$f^{(n)}(x)= ne^x + xe^x$

B 24. If $y=x^x$, then $y'=$

$\ln y = \ln x^x = x\ln x \Rightarrow \frac{1}{y}y' = x\cdot\frac{1}{x}+\ln x$

$y'=(1+\ln x)\cdot x^x$

(A) $x^x \ln x$ (B) $x^x(1+\ln x)$ (C) $x^x(x+\ln x)$ (D) $\dfrac{x^x \ln x}{x}$

D 25. If $y=e^{\sqrt{x^2+1}}$, then $y'= e^{\sqrt{x^2+1}}$

$\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x$

$= e^{\sqrt{x^2+1}}(x^2+1)^{-\frac{1}{2}}x$

(A) $\sqrt{x^2+1}\,e^{\sqrt{x^2+1}}$

(B) $2x\sqrt{x^2+1}\,e^{\sqrt{x^2+1}}$

(C) $\dfrac{e^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$

(D) $\dfrac{xe^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$

B 26. If $y=x^{\ln\sqrt{x}}$, then $y'=$

$\ln y = \ln\sqrt{x}\cdot\ln x = \frac{1}{2}\cdot(\ln x)^2$

$\frac{1}{y}y' = \frac{1}{2}\cdot 2\ln x\cdot\frac{1}{x} = \ln x\cdot\frac{1}{x}$

$y'=\frac{\ln x}{x}\cdot x^{\ln\sqrt{x}}$

(A) $\dfrac{x^{\ln\sqrt{x}}\ln x}{2x}$

(B) $\dfrac{x^{\ln\sqrt{x}}\ln x}{x}$

(C) $\dfrac{2x^{\ln\sqrt{x}}\ln x}{x}$

(D) $\dfrac{x^{\ln\sqrt{x}}(1+\ln x)}{x}$

A 27. If $3xy+x^2-2y^2=2$, then the value of $\frac{dy}{dx}$ at the point $(1,1)$ is

$3y+3xy'+2x-4yy'=0$

(A) 5 (B) $\frac{7}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{7}{2}$

$y'=\frac{3y+2x}{4y-3x}$ $y'(1,1)=\frac{5}{1}=5$

C 28. If $3x^4-x^2-y^2=0$, then the value of $\frac{dy}{dx}$ at the point $(1,\sqrt{2})$ is

$12x^3-2x-2yy'=0$

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{3\sqrt{2}}{2}$ (C) $\frac{5\sqrt{2}}{2}$ (D) $\frac{7\sqrt{2}}{2}$

$y'=\frac{12x^3-2x}{2y}$

$y'(1,\sqrt{2})=\frac{12-2}{2\sqrt{2}}=\frac{5}{2}\sqrt{2}$

B

29.

If $x^2y+2xy^2=5x$, then $\frac{dy}{dx}=$

$2xy+x^2y'+2y^2+2x\cdot 2yy'=5$

(A) $\frac{5-4xy-4y}{x^2+4xy}$

(B) $\frac{5-2xy-2y^2}{x^2+4xy}$

(C) $\frac{5-2xy-y^2}{x^2+2xy}$

(D) $\frac{5-xy-2y}{x^2-2xy}$

$y'=\frac{3x^2+y^2}{6y-2xy}$

$\Uparrow$

$6yy'-3x^2-(y^2+x\,2yy')=0$

D

30.

An equation of the line tangent to the graph of $3y^2-x^3-xy^2=7$ at the point $(1,2)$ is

slope $=y'(1,2)=\frac{3+4}{12-4}=\frac{7}{8}$

(A) $y=\frac{3}{4}x-\frac{3}{8}$ (B) $y=\frac{3}{4}x+\frac{1}{2}$ (C) $y=-\frac{7}{8}x+\frac{3}{2}$ (D) $y=\frac{7}{8}x+\frac{9}{8}$

B 31.

An equation of the line normal to the graph of $2x^2+3y^2=5$ at the point $(1,1)$ is

$4x+6yy'=0$ $y'=\frac{-2}{3}$ $\therefore$ slope of normal line $=\frac{3}{2}$

(A) $y=\frac{3}{2}x+1$ (B) $y=\frac{3}{2}x-\frac{1}{2}$ (C) $y=-\frac{2}{3}x+\frac{5}{3}$ (D) $y=-\frac{2}{3}x+\frac{3}{2}$

32. Consider the curve given by $x^3-xy+y^2=3$.

(a) Find $\frac{dy}{dx}$. $\frac{y-3x^2}{2y-x}$

(b) $x=1\Rightarrow 1-y+y^2=3\Rightarrow y=2$ or -1

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

slope $=-\frac{1}{3}$ or $\frac{4}{3}$

$\therefore \begin{cases} m=-\frac{1}{3} \\ (x,y)=(1,2)\end{cases}$ OR $\begin{cases} m=\frac{4}{3} \\ (x,y)=(1,-1)\end{cases}$

$\Rightarrow y=-\frac{1}{3}x+\frac{7}{3}$ OR $y=\frac{4}{3}x-\frac{7}{3}$

(c) Find the x-coordinate of each point on the curve where the tangent line is horizontal.

(c) $y-3x^2=0$ $x^3-xy+y^2=3$ Sub into $y=3x^2$

Calculator: $x=-0.709$ $x=0.822$

33. Consider the curve $x^2+y^2-xy=7$.

(a) Find $\frac{dy}{dx}=\frac{y-2x}{2y-x}$

(b) Find all points on the curve whose x-coordinate is 2, and write an equation for the tangent line at each of these points.

$4+y^2-2y=7\Rightarrow y=-1$ OR 3

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(C) $\begin{cases} 2y=x \\ x^2+y^2-xy=7\end{cases}$

$\Rightarrow x=\pm\frac{2}{3}\sqrt{21}$

$\Rightarrow \begin{cases} y=-1 \\ \frac{dy}{dx}=\frac{5}{4}\end{cases}$ $\begin{cases} y=3 \\ \frac{dy}{dx}=-\frac{1}{4}\end{cases}$

$\therefore y+1=\frac{5}{4}(x-2)$

OR $y-3=-\frac{1}{4}(x-2)$

P 34. If $y=(\sin x)^{1/x}$, then $y'=$

(A) $(\sin x)^{\frac{1}{x}}\left[\frac{\ln(\sin x)}{x}\right]$

(B) $(\sin x)^{\frac{1}{x}}\left[\frac{x-\ln(\sin x)}{x^2}\right]$

(C) $(\sin x)^{\frac{1}{x}}\left[\frac{x\sin x-\ln(\sin x)}{x^2}\right]$

(D) $(\sin x)^{\frac{1}{x}}\left[\frac{x\cot x-\ln(\sin x)}{x^2}\right]$

$\ln y = \frac{1}{x}\ln\sin x$

$\frac{1}{y}y' = -\frac{1}{x^2}\ln(\sin x)+\frac{1}{x}\frac{\cos x}{\sin x}$

$= \frac{-\ln(\sin x)+x\cot x}{x^2}$

$y' = \text{———}\cdot(\sin x)^{\frac{1}{x}}$

35. If $f(x)=e^{\tan x}$, then $f'\left(\frac{\pi}{4}\right)=$

$f'(x)=e^{\tan x}\sec^2 x$ $\quad f'(\frac{\pi}{4})=e^1\cdot(\sqrt{2})^2=2e$

36. If $f(x)=\ln(\cos x)$, then $f'(x)=$

$f'(x)=\frac{1}{\cos x}\cdot(-\sin x)=-\tan x$

C 37. If $f(x)=\ln[\sec(\ln x)]$, then $f'(e)=$

$f'(x)=\frac{1}{\sec(\ln x)}\cdot\sec(\ln x)\tan(\ln x)\cdot\frac{1}{x}$

$f'(e)=\frac{1}{\sec 1}\cdot\sec 1\cdot\tan 1\cdot\frac{1}{e}$

$=\tan 1\cdot\frac{1}{e}$

(A) $\frac{\cos 1}{e}$ (B) $\frac{\sin 1}{e}$ (C) $\frac{\tan 1}{e}$ (D) $\frac{\cot 1}{e}$

B 38. $\lim_{h\to 0}\frac{\cos(\frac{\pi}{3}+h)-\frac{1}{2}}{h}=f'(\frac{\pi}{3})$ $\quad f(x)=\cos x$ $\quad f'(x)=-\sin x$

(A) $-\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

C 39. $\lim_{h\to 0}\frac{\sin 2(x+h)-\sin 2x}{h}=\lim_{h\to 0}\frac{\sin(2(x+h))-\sin 2(x)}{h}=f'(x)$

$f(x)=\sin 2x$

$f'(x)=2\cos 2x$.

(A) $2\sin 2x$ (B) $-2\sin 2x$ (C) $2\cos 2x$ (D) $-2\cos 2x$

D 40. If $f(x)=\sin(\cos 2x)$, then $f'(\frac{\pi}{4})=$

$f'(x)=\cos(\cos 2x)\cdot(-\sin 2x)\cdot 2$

$f'(\frac{\pi}{4})=\cos 0\cdot(-2)=-2$

(A) 0 (B) -1 (C) 1 (D) -2

A 41. If $y=a\sin x+b\cos x$, then $y+y''=$

$y'=a\cos x-b\sin x$

(A) 0 (B) $2a\sin x$ (C) $2b\cos x$ (D) $-2a\sin x$

$y''=-a\sin x-b\cos x$

C 42. $\frac{d}{dx}\sec^2(\sqrt{x}) =$

(A) $\frac{2\sec(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

(B) $\frac{2\sec^2(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

(C) $\frac{\sec^2(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

(D) $\frac{\sec(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

B 43. $\frac{d}{dx}\left[x^2\cos 2x\right] =$

(A) $-2x\sin 2x$

(B) $2x(-x\sin 2x + \cos 2x)$

(C) $2x(x\sin 2x - \cos 2x)$

(D) $2x(x\sin 2x - \cos 2x)$

44. If $f(\theta) = \cos\pi - \frac{1}{2\cos\theta} + \frac{1}{3\tan\theta}$, then $f'(\frac{\pi}{6}) =$

$f'(\theta) = 0 - \frac{1}{2}\cdot(-1)(\cos\theta)^{-2}\cdot(-\sin\theta) + \frac{1}{3}\cdot(-1)\underline{(\tan\theta)^{-2}\cdot\sec^2\theta} = \frac{1}{\sin^2\theta}$

$f'(\frac{\pi}{6}) = -\frac{1}{2}\,\frac{\frac{1}{2}}{\frac{3}{4}} + (-\frac{1}{3})\cdot 4 = -\frac{5}{3}$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	$-1/2$	$3/2$	4	$\sqrt{2}$
$\pi/4$	-2	1	2	3

45. The table above gives values of f, f', g, and g' at selected values of x.

Find $h'(\frac{\pi}{4})$, if $h(x) = f(x)\cdot g(\tan x)$.

$h'(x) = f'(x)g(\tan x) + f(x)g'(\tan x)\sec^2 x$

$h'(\frac{\pi}{4}) = f'(\frac{\pi}{4})g(1) + f(\frac{\pi}{4})g'(1)\cdot 2$

$= 3 + (-2)\cdot\sqrt{2}\cdot 2$

$= 3 - 4\sqrt{2}$

D 46. If $xy + \tan(xy) = \pi$, then $\frac{dy}{dx} =$

(A) $-y\sec^2(xy)$ (B) $-y\cos^2(xy)$ (C) $-x\sec^2(xy)$ (D) $-\frac{y}{x}$

47.

Find the value of the constants a and b for which the function

$f(x)=\begin{cases}\sin x, & x<\pi \\ ax+b, & x\geq\pi\end{cases}$ is differentiable at $x=\pi$. $\Rightarrow$ ctns at $x=\pi$

$\Downarrow$

$f'(x)=\begin{cases}\cos x, & x<\pi \\ a, & x>\pi\end{cases}$

$f'(\pi^-)=f'(\pi^+)$

$\therefore \cos\pi = a$

$\therefore a=-1$

$\lim_{x\to\pi^-} f(x) = \lim_{x\to\pi^+} f(x) = f(\pi)$

$\sin\pi = a\pi+b = a\pi+b$

$\therefore a\pi+b=0$

$\Downarrow$

$\because a=-1$

$\therefore b=\pi$

C 48.

An equation of the line normal to the graph of $y=\tan x$, at the point $(\frac{\pi}{6},\frac{1}{\sqrt{3}})$ is

(A) $y-\frac{1}{\sqrt{3}}=-\frac{1}{4}(x-\frac{\pi}{6})$

(B) $y-\frac{1}{\sqrt{3}}=\frac{1}{4}(x-\frac{\pi}{6})$

(C) $y-\frac{1}{\sqrt{3}}=-\frac{3}{4}(x-\frac{\pi}{6})$

(D) $y-\frac{1}{\sqrt{3}}=\frac{3}{4}(x-\frac{\pi}{6})$

$y'=\sec^2 x$

$y'(\frac{\pi}{6})=\frac{4}{3}$ = slope of tangent line

$\therefore$ slope of normal $=-\frac{3}{4}$

C 49.

If $x+\sin y=y+3$, then $\frac{d^2y}{dx^2}=$

(A) $\frac{-\sin y}{(1-\cos y)^2}$ (B) $\frac{-\sin y}{(1+\cos y)^2}$ (C) $\frac{-\sin y}{(1-\cos y)^3}$ (D) $\frac{-\sin y}{(1+\cos y)^3}$

$1+\cos y\cdot y'=y' \Rightarrow y'=\frac{1}{1-\cos y}$

$\frac{d}{dx}(\cos y\cdot y')=\frac{d}{dx}(y')$

$-\sin y\cdot y'\cdot y'+\cos y\cdot y''=y''$

$y''=\frac{\sin y\cdot(y')^2}{\cos y-1}=\frac{\sin y\cdot\frac{1}{(1-\cos y)^2}}{\cos y-1}$

$=\frac{-\sin y}{(1-\cos y)^3}$

50.

C

Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(3)=4$ and $f'(4)=\frac{3}{2}$, then $g'(3)=$

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

B

51.

If $f(-3)=2$ and $f'(-3)=\frac{3}{4}$, then $(f^{-1})'(2)=$

(A) $\frac{1}{2}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{4}$

52.

A

If $f(x)=x^3-x+2$, then $(f^{-1})'(2)=$

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 4 (D) 6

53.

P

If $f(x)=\sin x$, then $(f^{-1})'(\frac{\sqrt{3}}{2})=$

(A) $\frac{1}{2}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) 2

54.

D

If $f(x)=1+\ln x$, then $(f^{-1})'(2)=$

(A) $-\frac{1}{e}$ (B) $\frac{1}{e}$ (C) $-e$ (D) e

55.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
−1	3	−2	2	6
0	−2	−1	0	−3
1	0	1	−1	2
2	−1	4	3	−1

The functions f and g are differentiable for all real numbers. The table above gives the values of the functions and their first derivatives at selected values of x.

(a) If f^{-1} is the inverse function of f, write an equation for the line tangent to the graph of $y=f^{-1}(x)$ at $x=-1$.

(a) $(f^{-1})'(-1)=\frac{1}{4}$ passes $(-1, f^{-1}(-1)) = (-1, 2)$ $\Rightarrow$ $y-2=\frac{1}{4}(x+1)$

(b) Let h be the function given by $h(x)=f(g(x))$. Find $h(1)$ and $h'(1)$.

(b) $h(1)=f(g(1))=3$

$h'(x)=f'(g(x))g'(x)$

$h'(1)=-4$

(c) Find $(h^{-1})'(3)$, if h^{-1} is the inverse function of h.

(c) $(h^{-1})'(3)=\dfrac{1}{h'(h^{-1}(3))}$

$=\dfrac{1}{h'(1)}=-\frac{1}{4}$

56. $\frac{d}{dx}(\arcsin x^2) =$ $\frac{2x}{\sqrt{1-x^4}}$

57. If $f(x) = \arctan(e^{-x})$, *then* $f'(-1) =$ $\frac{-e}{1+e^2}$

58. If $f(x) = \arctan(\sin x)$, *then* $f'\left(\frac{\pi}{3}\right) =$ $\frac{2}{7}$

59. If $f(x) = \cos(\sin^{-1} x)$, *then* $f'(x) =$ $\frac{-x}{\sqrt{1-x^2}}$

60. Let f be the function given by $f(x) = x^{\tan^{-1} x}$.

(a) Find $f'(x) =$ $x^{\tan^{-1}x}\left[\frac{\ln x}{1+x^2} + \frac{\tan^{-1}x}{x}\right]$

(b) Write an equation for the line tangent to the graph of f at $x = 1$.

$f'(1) = \frac{\pi}{4}$

passes $(1, f(1)) = (1, 1)$

$y - 1 = \frac{\pi}{4}(x-1)$

C

61. Some values of differentiable function f are shown in the table below.
What is the approximation value of $f'(3.5)$?

x	3.0	3.3	3.8	4.2	4.9
$f(x)$	21.8	26.1	32.5	38.2	48.7

(A) 8 (B) 10 (C) 13 (D) 16

62.

t Month	1	2	3	4	5	6
$F(t)$ Temperature	−8	0	25	50	72	88

The normal daily maximum temperature F for a certain city is shown in the table above.

(a) Use data in the table to find the average rate of change in temperature from $t = 1$ to $t = 6$. (a) $\frac{F(6)-F(1)}{6-1} = 19.2\ °F/\text{mon}$

(b) Use data in the table to estimate the rate of change in maximum temperature at $t = 4$.

(b) $F'(4) \approx \frac{F(5)-F(3)}{2} = 23.5\ °F/\text{mon}$

(c) The rate at which the maximum temperature changes for $1 \le t \le 6$ is modeled by $F(t) = 40 - 52\sin(\frac{\pi t}{6} - 5)$ degrees per minute. Find $F'(4)$ using the given model.

(c) $F'(t) = -52\cos\left(\frac{\pi}{6}t - 5\right) \cdot \left(\frac{\pi}{6}\right)$

$F'(4) = 26.472\ °F/\text{mon}$