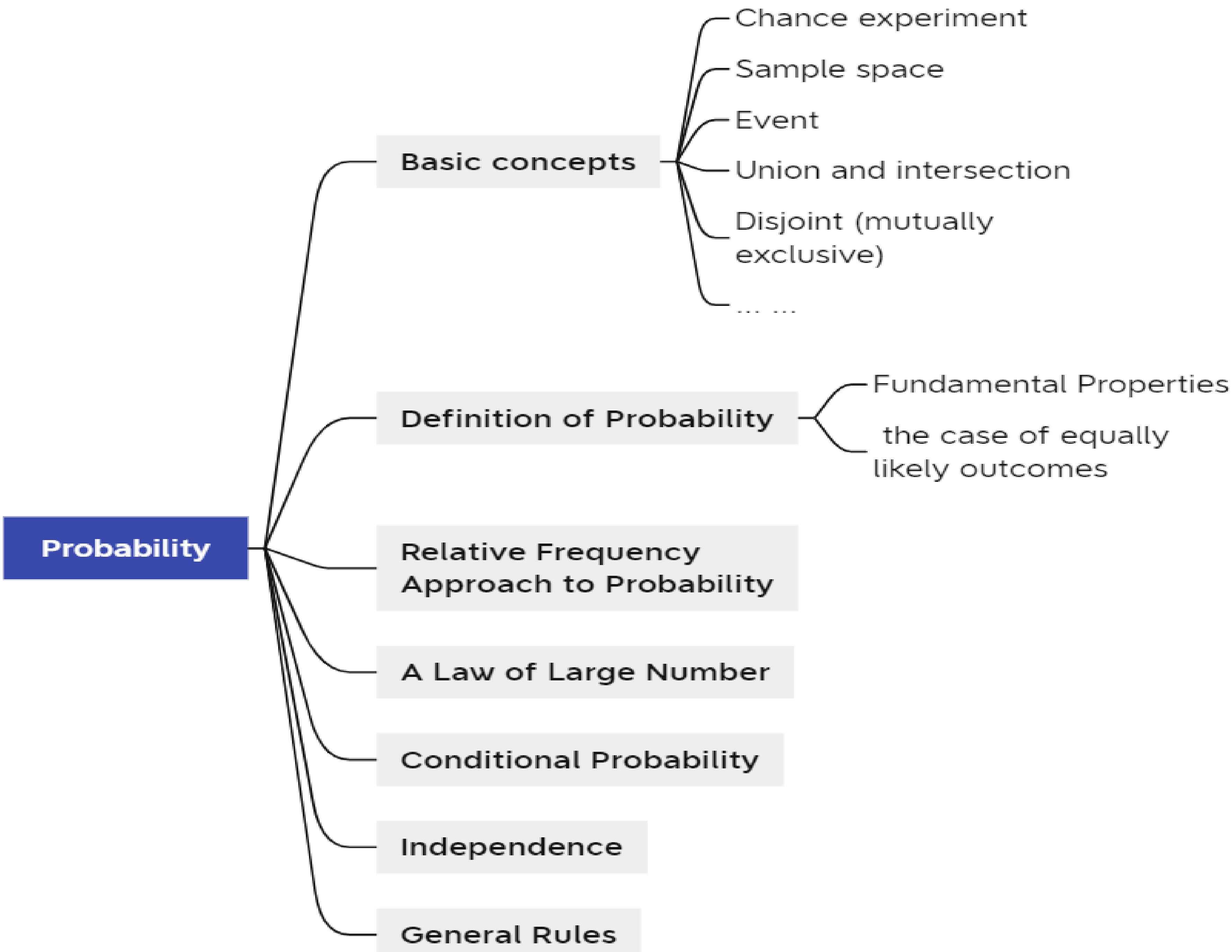

Probability
Basic concepts
Chance experiment
Sample space
Event
Union and intersection
Disjoint (mutually exclusive)
... ...
Definition of Probability
Fundamental Properties
the case of equally likely outcomes
Relative Frequency Approach to Probability
A Law of Large Number
Conditional Probability
Independence
General Rules

Unit 4 Probability

Lecture 1

We make decision based on

uncertainty

every day!

Basic Concept

- **Outcome: "element"**

  e.g. when a single coin is tossed, there are two possible outcomes:

  Heads and Tails.

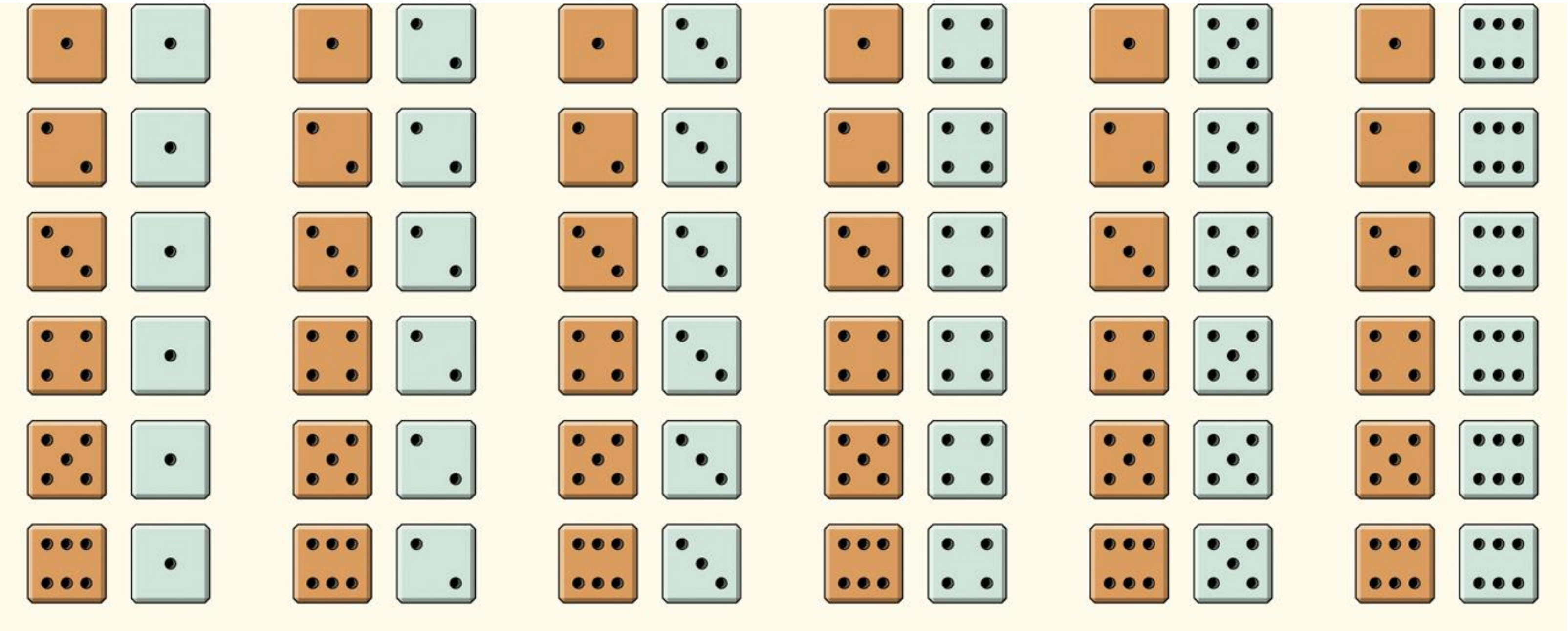
Consider rolling both a red die and a green die.

How many possible outcomes in all?

Basic Concept

Consider rolling both a red die and a green die. How many possible outcomes in all?

36 possible outcomes



Since the dice are **fair**, **each outcome is equally likely**.

Each outcome has probability **1/36.**

Basic Concept

- **Outcome: "element"**

e.g. when a single coin is tossed, there are two possible outcomes: Heads and Tails.

Consider rolling both a red die and a green die.

How many possible outcomes in all?

36 possible outcomes

Do we know in advance what the result of a particular roll will be?

No, we do not know!

A chance experiment

Basic Concept

- **Outcome: "element"**
- **Chance experiment:**

  A chance experiment is any situation where there is uncertainty about which of two or more possible outcomes will result.

- **Sample space: "set", "a collection of elements"**

  A set of all possible outcomes of an experiments.

- **Event: "set"** An event is any collection of outcomes from the sample space of a chance experiment.
  - **Simple event:** An event consisting of exactly one outcome.

Practice:

Rolling a six-sided die (fair dice with 6 faces)

Outcomes ?

Sample space ?

Event that the number of face is even ?

Simple event ?

Rolling a six-sided die (fair dice with 6 faces)

Outcomes:

$$1,2,\ldots,6$$

Sample space :

$$\{1,2,3,4,5,6\}$$

Event that the number of face is even :

$$\{\{2\},\{4\},\{6\}\}$$

Simple event:

$$\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\}\}$$

Basic Concept

➢ Complement, Union, Intersection

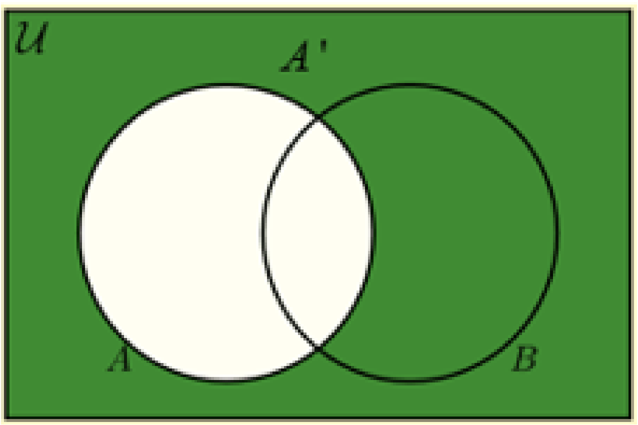
DEFINITION

Let A and B denote two events.

1. The event ***not A*** consists of all experimental outcomes that are not in event A. *Not A* is sometimes called the *complement* of A and is usually denoted by A^C, A', or $\overline{A}$.
2. The event ***A or B*** consists of all experimental outcomes that are in at least one of the two events, that is, in A or in B or in both of these. *A or B* is called the *union* of the two events and is denoted by $A \cup B$.
3. The event ***A and B*** consists of all experimental outcomes that are in both of the events A and B. *A and B* is called the *intersection* of the two events and is denoted by $A \cap B$.

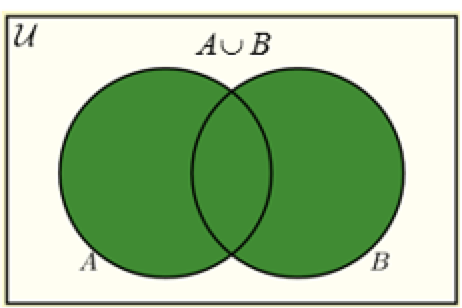
Basic Concept

- **Venn Diagram:** In a **Venn diagram**, the collection of all possible outcomes is typically shown as the interior of a rectangle. Other events are then identified by specified regions inside this rectangle.



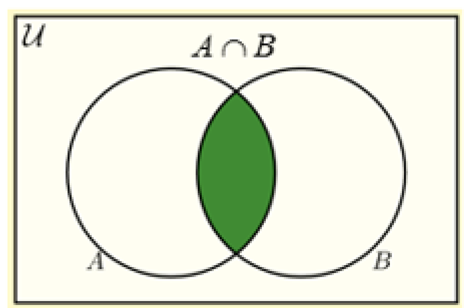
A complement

Elements that don't belong to *A*.



A union *B*

Elements that belong to either *A* or *B* or both.



A intersect *B*

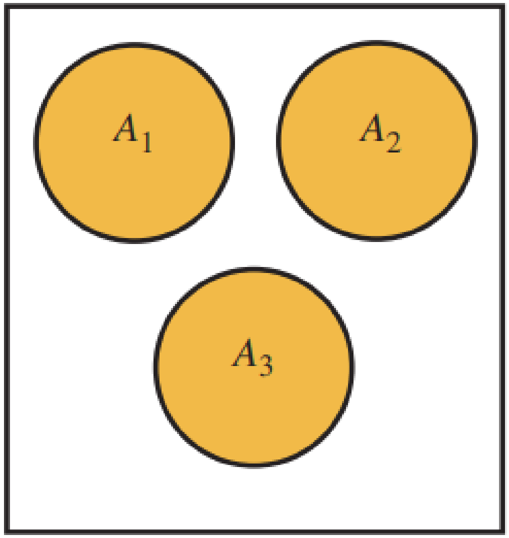
Elements that belong to both *A* and *B*.

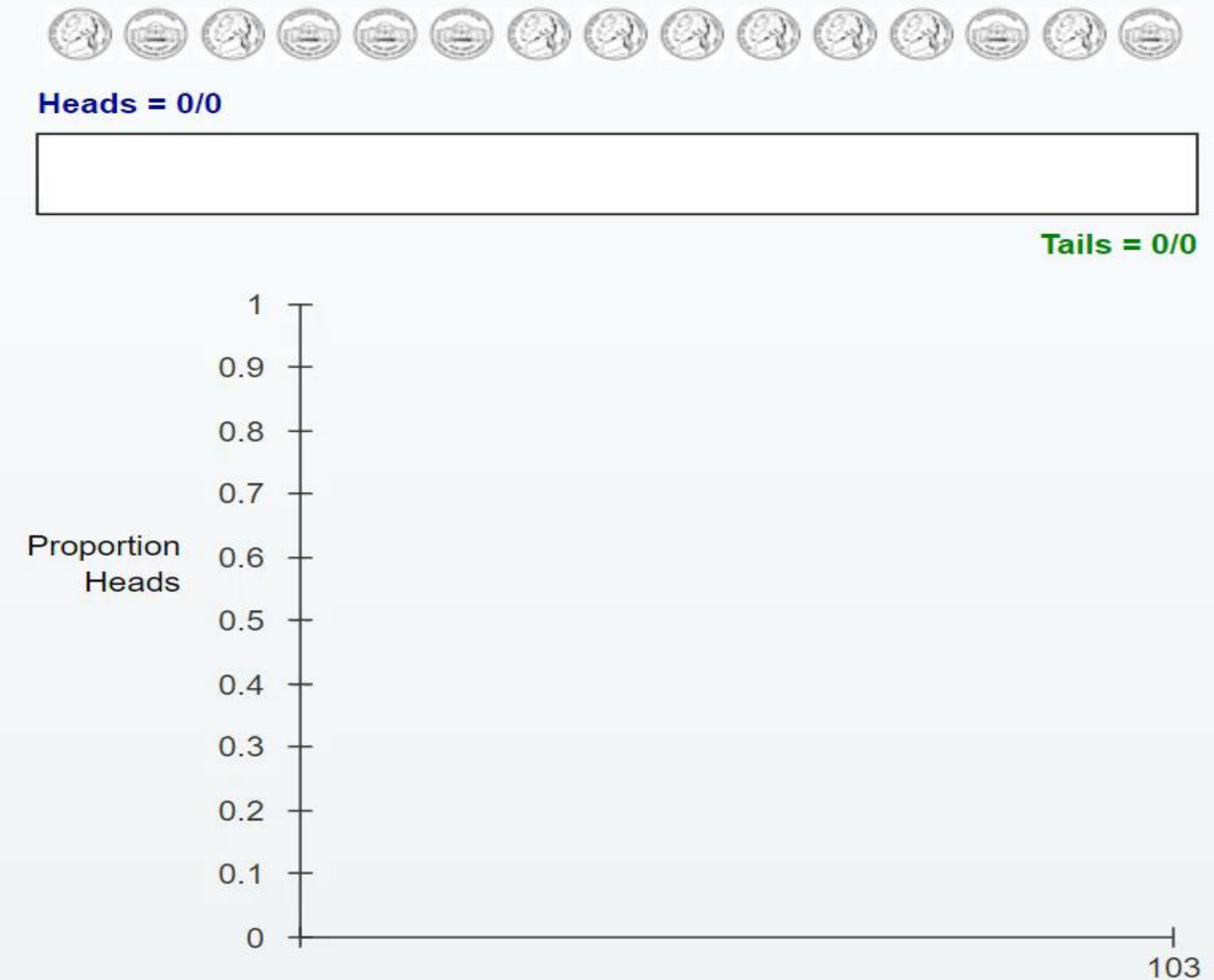
Basic Concept

- **Disjoint (mutually exclusive)**

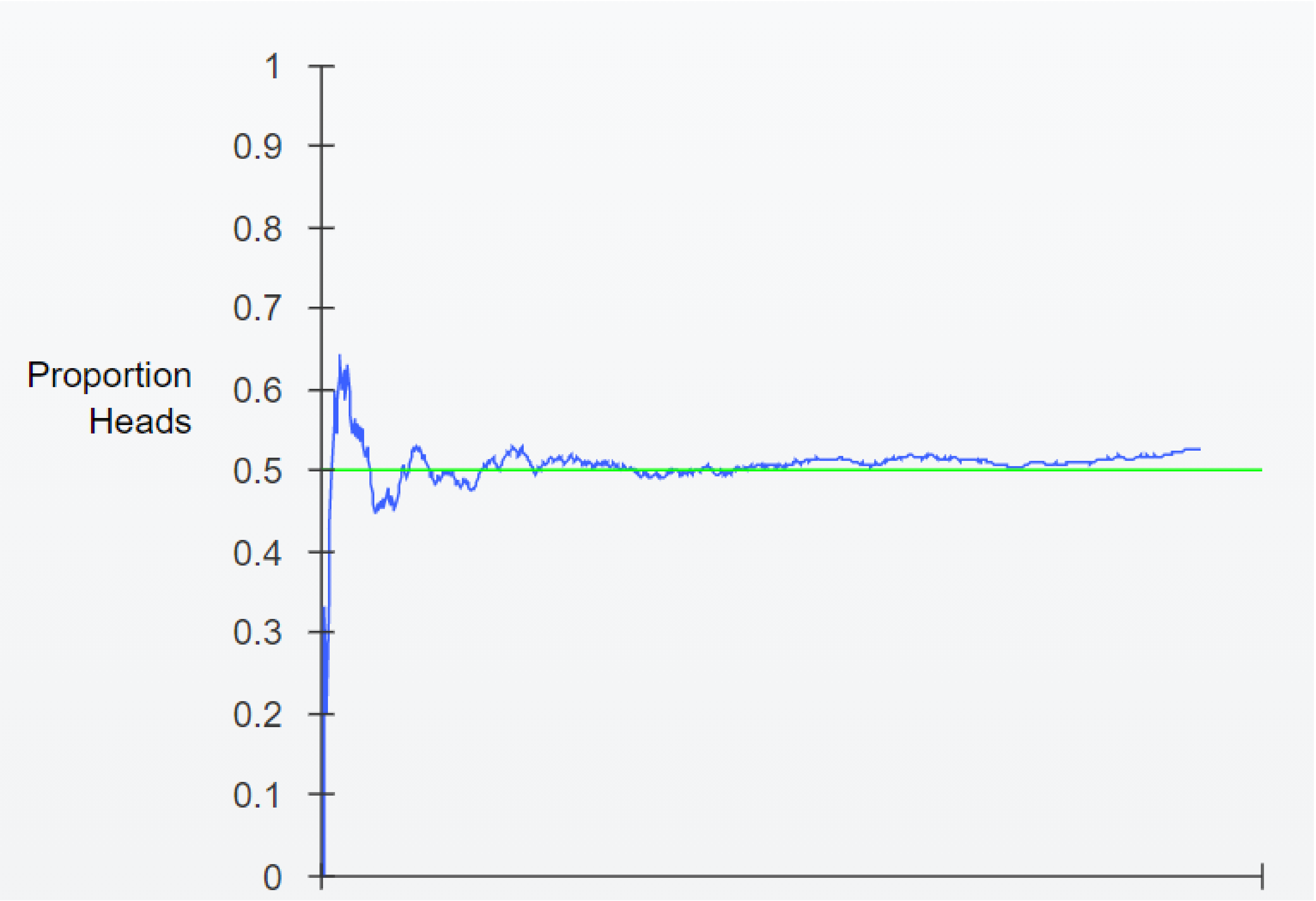
DEFINITION

Two events that have no common outcomes are said to be **disjoint** or **mutually exclusive.**




Heads = 0/0
Tails = 0/0
Proportion Heads
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
103

http:


1
0.9
0.8
0.7
Proportion
Heads
0.6
0.5
0.4
0.3
0.2
0.1
0

Probability

Definition:

The **probability** of any outcome of a chance process is a number between 0 (never occurs) and 1 (always occurs) that describes the proportion of times the outcome would occur in a very long series of repetitions.

Relative Frequency Approach To Probability

The **probability of an event E,** denoted by $P(E)$**,** is defined to be the value approached by the relative frequency of occurrence of E in a very long series of trials of a chance experiment. Thus, if the number of trials is quite large,

$$P(E) \approx \frac{\text{number of times } E \text{ occurs}}{\text{number of trials}}$$

Simple definition of probability

$$\text{Probability} = \frac{\text{Number of outcomes in an event}}{\text{Total number of possible outcomes}}$$

In the dice-rolling example, suppose we define event A as "sum is 5".

What is the value of P(A)?

- Only used when all outcomes are **equally likely**.

In the dice-rolling example, suppose we define event A as "sum is 5."

There are 4 outcomes that result in a sum of 5.

Since each outcome has probability 1/36, P(A) = 4/36=1/9.

Suppose event B is defined as "sum is not 5." What is P(B)?

$$P(B) = 1 - 1/9$$

$$= 8/9$$

Chance behavior is unpredictable in the short run, but has a regular and predictable pattern in the long run.

The **law of large numbers** says that if we observe more and more repetitions of any chance process, the proportion of times (relative frequency) that a specific outcome occurs approaches a single value (probability).

Law of large numbers:

After many **trials**, the relative frequency of outcomes will approach their **probability**.

You have a fair coin. You flip it 10 times.
In which situation is getting tails (T) on your next flip more likely?

<u>Situation 1:</u> HTTHHTHTHT

<u>Situation 2:</u> HTTHHHHHHH

It's **equally** likely!

The probability doesn't change because of previous results.

Practice

There are 8 red marbles and 12 blue marbles in a jar. What is the probability of selecting a red marble from the jar?

R: Event of selecting a red marble from the jar

P(R): Probability of Event R occurring

$$\text{Probability} = \frac{\text{Number of outcomes in an event}}{\text{Total number of possible outcomes}}$$

$$P(\mathbf{R}) = \frac{\text{Number of outcomes in event } R}{\text{Total number of possible outcomes}}$$

$$P(R) = \frac{8}{8+12} = \frac{8}{20} = 40.0\%$$

Properties of Probabilities

$$\text{Probability} = \frac{\text{Number of outcomes in an event}}{\text{Total number of possible outcomes}}$$

Properties:

1. Probabilities are always between **0 – 1** (0% - 100%).

8 red marbles in a jar and 12 blue marbles in a jar.

$$1.\ P(R) = \frac{8}{8+12} = \frac{8}{20} = .400$$ ✓

Properties of Probabilities

$$\text{Probability} = \frac{\text{Number of outcomes in an event}}{\text{Total number of possible outcomes}}$$

Properties:

1. Probabilities are always between **0 – 1** (0% - 100%).
2. If S is the sample space for an experiment, P(S)= 1

8 red marbles in a jar and 12 blue marbles in a jar.

1. $P(R) = \frac{8}{8+12} = \frac{8}{20} = .400$
2. $P(B) = \frac{12}{20} = .600$

   $P(R)+P(B) = 1$

Properties of Probabilities

$$\text{Probability} = \frac{\text{Number of outcomes in an event}}{\text{Total number of possible outcomes}}$$

Properties:

1. Probabilities are always between **0 – 1** (0% - 100%).
2. If S is the sample space for an experiment, $P(S) = 1$
3. Complement rule:

   the probability of event A **not** happening, $P(A^C)$,

   is equal to $1 - P(A)$.

8 red marbles in a jar and 12 blue marbles in a jar.

1. $P(R) = \frac{8}{8+12} = \frac{8}{20} = .400$ ✓

2. $P(R)+P(B) = 1$ ✓

3. $P(R^C) = 1 - P(R)$??????

$P(B) = 1 - P(R)$

$P(B) = 1 - 0.400 = 0.600$ ✓

Properties of Probabilities

Properties:

1. Probabilities are always between **0 – 1** (0% - 100%).
2. If S is the sample space for an experiment, P(S)= 1
3. Complement rule:
   the probability of event A **not** happening, $P(A^C)$,
   is equal to 1 – P(A).

4. If two events E and F are disjoint, then P(E or F) =

$$P(E \cup F) = P(E)+P(F)$$

Not disjoint:

$$P(E \text{ or } F)= P(E)+P(F)-P(E \cap F)$$

The Addition Rule for Disjoint (Mutually Exclusive) Events

Let E and F be two disjoint events. One of the basic properties (axioms) of probability is

$$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F)$$

This property of probability is known as the addition rule for disjoint events. More generally, if events $E_1, E_2, \ldots, E_k$ are disjoint, then

$$P(E_1 \text{ or } E_2 \text{ or } \ldots \text{ or } E_k) = P(E_1 \cup E_2 \cup \cdots \cup E_k) = P(E_1) + P(E_2) + \cdots + P(E_k)$$

In words, the probability that any of these k disjoint events occurs is the sum of the probabilities of the individual events.

Example: Distance Learning

Distance-learning courses are rapidly gaining popularity among college students. Randomly select an undergraduate student who is taking distance-learning courses for credit and record the student's age. Here is the probability model:

Age group (yr):	18 to 23	24 to 29	30 to 39	40 or over
Probability:	0.57	0.17	0.14	0.12

(a) Show that this is a legitimate probability model.

(b) Find the probability that the chosen student is not in the traditional college age group (18 to 23 years).

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(a) Show that this is a legitimate probability model.

- each probability is between 0 and 1
- the sum of all probabilities is 1:

$0.57+0.17+0.14+0.12=1$

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(b) Find the probability that the chosen student is not in the traditional college age group (18 to 23 years).

P(not 18 to 23 years old) = 1 – P(18 to 23 years old)

= 1-0.57

= 0.43