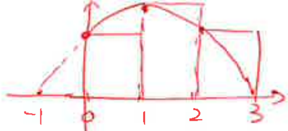


1. Using a left Riemann sum with three subintervals [0,1], [1,2], [2,3], what is the approximation of $\int_0^3 (3-x)(x+1)\,dx$?

$1\times f(0) + 1\times f(1) + 1\times f(2)$
$= 3+4+3 = 10$

x	1	3	5	8	10
$f(x)$	7	12	16	23	17

D 2. The function f is continuous on the closed interval $[1,10]$ and has values as shown in the table above. Using a right Riemann sum with four subintervals $[1,3]$, $[3,5]$, $[5,8]$, $[8,10]$, what is the approximation of $\int_1^{10} f(x)\,dx$? $\approx 2\times 12 + 2\times 16 + 3\times 23 + 2\times 17 = 159$

(A) 96 (B) 116 (C) 132 (D) 159

3. The function f is continuous on the closed interval [0,12] and has values as shown in the table below. Use a midpoint Riemann sum with 4 subintervals of equal length to approximate the area that lies under f and above the x-axis from x=0 to x=12.

x	0	1.5	3	4.5	6	7.5	9	10.5	12
$f(x)$	1	1.45	2.8	5.05	8.2	12.25	17.2	23.05	29.8

$I \approx 3\times[f(1.5)+f(4.5)+f(7.5)+f(10.5)]$
$= 125.4$

A 4. Which of the following integrals is equal to $\lim_{n\to\infty}\sum_{i=1}^{n}(-1+\frac{3i}{n})^2\frac{3}{n}$? $\frac{3}{n} = \frac{b-a}{n}$ $\therefore b-a=3$

(A) $\int_{-1}^{2} x^2\,dx$

(B) $\int_{-1}^{0} x^2\,dx$

(C) $\int_{-1}^{2} (-1+x)^2\,dx$

(D) $\int_{-1}^{0} (-1+\frac{x}{3})^2\,dx$

lower Limit $= -1$ $\Rightarrow$ $b=2$, $a=-1$

$c_i = -1+\Delta x\, i$
$= -1+\frac{3}{n} i$
$f(c_i) = c_i^2$

-1, $-1+\Delta x$, 2

A 5. The expression $\frac{1}{30}\left[\sqrt{\frac{1}{30}}+\sqrt{\frac{2}{30}}+\sqrt{\frac{3}{30}}+\dots+\sqrt{\frac{30}{30}}\right]$ is a Riemann sum approximation for

30 terms
$\therefore n=30$

(A) $\int_0^1 \sqrt{x}\,dx$ $\Delta x = \frac{b-a}{30}$

(B) $\frac{1}{30}\int_0^1 \sqrt{x}\,dx$

(C) $\frac{1}{30}\int_0^{30} \sqrt{x}\,dx$

(D) $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

$c_i = \frac{i}{30}$ $\therefore f(c_i) = \sqrt{c_i}$ $\therefore f(x) = \sqrt{x}$

0, $\frac{1}{30}$, $\frac{30}{30} = 1$

$x: b-a=1$
$\therefore a=0$ $b=1$

$I = \int_0^1 \sqrt{x}\,dx$

C 7. If n is a positive integer, then $\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\dots+\left(\frac{n}{n}\right)^2\right]$ can be expressed as

Δx

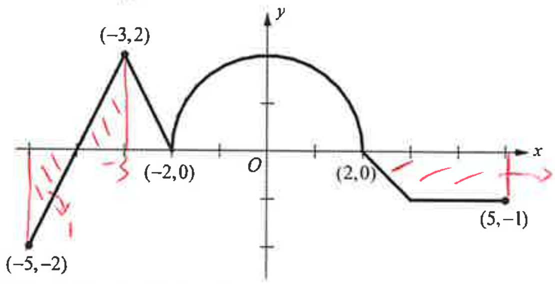
(A) $\int_0^1 \frac{1}{x}\,dx$ (B) $\int_0^1 \frac{1}{x^2}\,dx$ (C) $\int_0^1 x^2\,dx$ (D) $\frac{1}{2}\int_0^1 x^2\,dx$

$c_i = \frac{i}{n}$
$f(c_i) = c_i^2$
$\therefore f(x) = x^2$

$b-a=2$ $\quad$ n terms $\quad$ $c_i = \frac{2i}{n}$ $\quad$ $f(c_i) = \sqrt{c_i}$

B 8. If n is a positive integer, then $\lim\limits_{n\to\infty} \frac{2}{n}\left[\sqrt{\frac{2}{n}} + \sqrt{\frac{4}{n}} + \cdots + \sqrt{\frac{2n}{n}}\right]$ can be expressed as $\quad$ $\therefore f(x) = \sqrt{x}$

(A) $\int_0^1 \sqrt{x}\,dx$ $\qquad$ (B) $\int_0^2 \sqrt{x}\,dx$ $\qquad$ (C) $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ $\qquad$ (D) $\int_0^2 \frac{1}{\sqrt{x}}\,dx$ $\qquad$ $0 \quad \frac{2}{n} \quad \cdots \quad \frac{2n}{n} = 2$



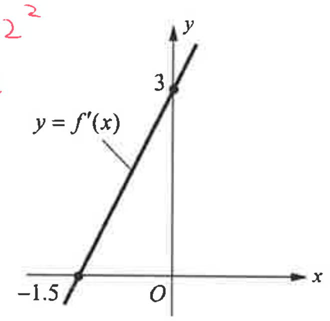
9. The graph of $y = f(x)$ consists of four line segments and a semicircle as shown in the figure above. Evaluate each definite integral by using geometric formulas.

(a) $\int_{-5}^{-2} f(x)\,dx$ $\qquad$ (b) $\int_{-2}^{2} f(x)\,dx$ $\qquad$ (c) $\int_{2}^{5} f(x)\,dx$ $\qquad$ (d) $\int_{-5}^{5} |f(x)|\,dx$ $= 1+1+1+2\pi+2.5$

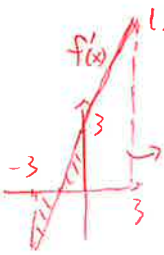
$= \int_{-3}^{-2} f(x)\,dx$ $\qquad$ $= S_{\text{semi-circle}}$ $\qquad$ $= -2.5$ $\qquad$ $= 5.5 + 2\pi$

$= 1 \times 2 \times \frac{1}{2}$ $\qquad$ $= \frac{1}{2}\cdot\pi\, 2^2$

$= 1$ $\qquad$ $= 2\pi$



$\Rightarrow$



$\rightarrow \frac{(3+9)\times 3}{2} = 18$

FTC
$\Downarrow$

10. The graph of f', the derivative of f, is the line shown in the figure above. If $f(3) = 11$, then $f(-3) =$

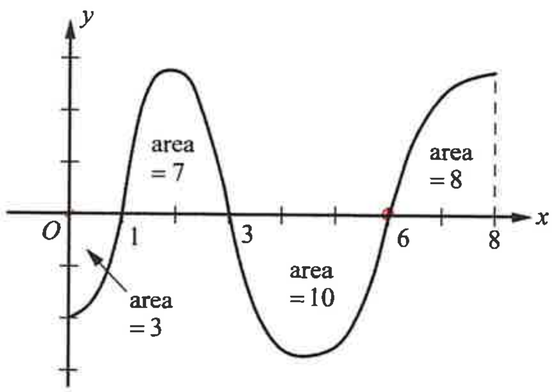
$f(3) - f(-3) = \int_{-3}^{3} f'(x)\,dx = 18$ $\qquad$ $\therefore f(-3) = -7$

A 11. If $f(x) = \sqrt{x^4 - 3x + 4}$ and g is the antiderivative of f, such that $g(3) = 7$, then $g(0) =$

(Calculator)

(A) -2.966 $\qquad$ (B) -1.472 $\qquad$ (C) -0.745 $\qquad$ (D) 1.086 $\qquad$ $g(3) - g(0) = \int_0^3 \sqrt{x^4 - 3x + 4}\,dx$

12.



The figure above shows the graph of f', the derivative of a differentiable function f, on the closed interval $0 \le x \le 8$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. Given $f(6) = 9$, find each of the following.

(a) $f(0)$ $\qquad$ (b) $f(1)$ $\qquad$ (c) $f(3)$ $\qquad$ (d) $f(8)$

$f(6) - f(0) = -3 + 7 - 10$ $\qquad$ $f(1) - f(0) = -3$ $\qquad$ $f(6) - f(3) = -10$ $\qquad$ $f(8) - f(6) = 8$

$\therefore f(0) = 15$ $\qquad$ $\therefore f(1) = 12$ $\qquad$ $\therefore f(3) = 19$ $\qquad$ $\therefore f(8) = 17$

C 13. If $\int_a^b f(x)\,dx = 2a - 5b$, then $\int_a^b [f(x) - 2]\,dx =$ $\int_a^b f(x)\,dx - 2(b-a)$

(A) $-7b$ (B) $-3b$ (C) $4a - 7b$ (D) $4a - 3b$

D 14. If $\int_1^6 f(x)\,dx = \frac{15}{2}$ and $\int_6^4 f(x)\,dx = 5$, then $\int_1^4 f(x)\,dx =$ $\int_1^6 f(x)dx - \int_4^6 f(x)dx$

$= -\int_4^6 f(x)\,dx$

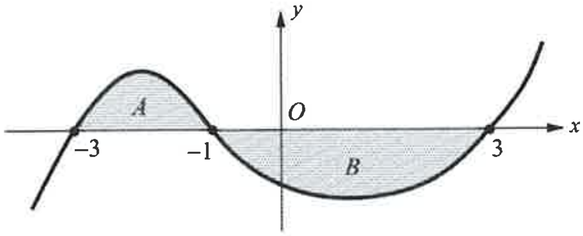
(A) $\frac{5}{2}$ (B) $\frac{9}{2}$ (C) $\frac{19}{2}$ (D) $\frac{25}{2}$

(*) Substitution

C 15. If $\int_{-2}^6 f(x)\,dx = 10$ and $\int_2^6 f(x)\,dx = 3$, then $\int_2^6 f(4-x)\,dx =$ $-\int_2^{-2} f(x)\,dx = \int_{-2}^{2} f(x)\,dx$

(A) 3 (B) 6 (C) 7 (D) 10 $= \int_{-2}^{6} f(x)\,dx +$

$\int_6^2 f(x)\,dx$



B 16. The graph of $y = f(x)$ is shown in the figure above. If A and B are positive numbers that represent the areas of the shaded regions, what is the value of $\underbrace{\int_{-3}^3 f(x)\,dx}_{A-B} - 2\underbrace{\int_{-1}^3 f(x)\,dx}_{-B}$, in terms of A and B?

(A) $-A - B$ (B) $A + B$ (C) $A - 2B$ (D) $A - B$

17. Let f and g be continuous functions with the following properties.

(1) $g(x) = f(x) - n$ where n is a constant.

(2) $\int_0^4 f(x)\,dx - \int_4^6 g(x)\,dx = 1$ (a) $\Rightarrow \int_0^4 f(x)dx - \int_4^6 f(x) - n\,dx = \int_0^4 f(x)dx - \int_4^6 f(x)\,dx + 2n = 1$

(3) $\int_4^6 f(x)\,dx = 5n - 1$ $\Rightarrow$ $\int_0^4 f(x)dx = 1 - 2n + (5n-1) = 3n$

(a) Find $\int_0^4 f(x)\,dx$ in terms of n. $\therefore \int_0^6 f(x)\,dx = \int_0^4 f(x)\,dx + \int_4^6 f(x)\,dx = 3n + 5n - 1 = 8n-1$

(b) Find $\int_0^6 g(x)\,dx$ in terms of n. (b) $\int_0^6 f(x) - n\,dx = \int_0^6 f(x)\,dx - 6n = 8n - 1 - 6n = 2n - 1$

(*) Substitution

(c) Find the value of k if $\int_0^2 f(2x)\,dx = kn$.

(C) $u = 2x$ $\therefore I = \int_0^4 f(u)\cdot\frac{1}{2}du$

$du = 2dx$

$x = 0 \rightarrow u = 0$ $= \frac{1}{2}\int_0^4 f(x)\,dx = kn$

$x = 2 \rightarrow u = 4$ $= \frac{3}{2}n$

$\therefore k = \frac{3}{2}$

Antiderivatives ⇓

B 1. If $\frac{dy}{dx} = 3x^2 - 1$, and if $y = -1$ when $x = 1$, then $y =$

passes $(1, -1)$

(A) $x^3 - x + 1$

(B) $(x^3 - x - 1)' = 3x^2 - 1$

(C) $(-x^3 + x - 1)' \neq 3x^2 - 1$

(D) $-x^3 + 1$

C 2. The closed interval $[a, b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \ldots, x_n$ where $0 < a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{\sqrt{x_i}} \Delta x$?

$f(x) = \frac{1}{\sqrt{x}}$

$\int_a^b x^{-\frac{1}{2}} dx$

$= \left[2x^{\frac{1}{2}}\right]_a^b$

(A) $\frac{1}{\sqrt{b}} - \frac{1}{\sqrt{a}}$

(B) $\frac{(\sqrt{b} - \sqrt{a})}{2}$

(C) $2(\sqrt{b} - \sqrt{a})$

(D) $\sqrt{b} - \sqrt{a}$

C 3. A curve has a slope of $-x + 2$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(2, 1)$?

slope $= f'(x) = -x + 2$

$\therefore f(x) = -\frac{1}{2}x^2 + 2x + C$

(A) $\frac{1}{2}x^2 - 2x - 4$

(B) $2x^2 + x - 8$

(C) $-\frac{1}{2}x^2 + 2x - 1$

(D) $x^2 - 2x + 1$

C 4. $\int (x^2 - 2)\sqrt{x}\, dx =$ $\int x^{\frac{5}{2}} - 2x^{\frac{1}{2}} dx = \frac{2}{7}x^{\frac{7}{2}} - \frac{4}{3}x^{\frac{3}{2}} + C$

(A) $\frac{2}{5}x^2\sqrt{x} - \frac{2}{3}x\sqrt{x} + C$

(B) $\frac{2}{5}x^2\sqrt{x} - \frac{4}{3}x\sqrt{x} + C$

(C) $\frac{2}{7}x^3\sqrt{x} - \frac{4}{3}x\sqrt{x} + C$

(D) $\frac{2}{7}x^3\sqrt{x} - \frac{2}{3}x^2\sqrt{x} + C$

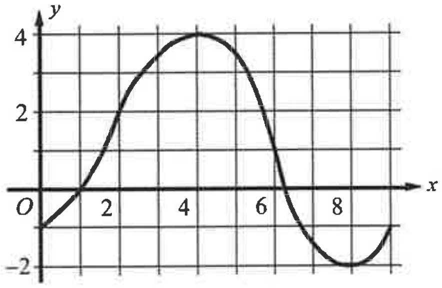
[Fundamental Theorem of Calculus II] ⇓

C 1. $\frac{d}{dx}\int_1^{x^2}\sqrt{3+t^2}\,dt =$ $\sqrt{3+(x^2)^2}\cdot 2x$

(A) $\sqrt{3+x^2}$ (B) $\sqrt{3+x^4}$ (C) $2x\sqrt{3+x^4}$ (D) $2\sqrt{3+x^2}$

B 3. If $F(x)=\int_0^{\sqrt{x}}\cos(t^2)\,dt$, then $F'(4)=$ $F'(x)=\cos((\sqrt{x})^2)\cdot\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}\cos x\cdot x^{-\frac{1}{2}}$

(A) $\cos 2$ (B) $\frac{\cos 4}{4}$ (C) $\frac{\cos 4}{\sqrt{2}}$ (D) $\sqrt{2}\cos 4$



graph of g

B 5. The graph of the function g, shown in the figure above, has horizontal tangents at $x=4$ and $x=8$. If $f(x)=\int_0^{\sqrt{x}} g(t)\,dt$, what is the value of $f'(4)$? $f'(x)=g(\sqrt{x})\cdot\frac{1}{2}x^{-\frac{1}{2}}$

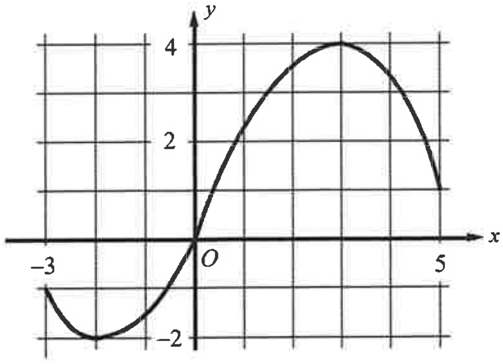
(A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{3}{2}$

A 6. If $F(x)=\int_0^{x^2}\frac{\sqrt{t^2+3}}{2t}\,dt$, then $F''(1)=$ $F'(x)=\frac{\sqrt{x^4+3}}{2x^2}\cdot 2x = \frac{\sqrt{x^4+3}}{x} = (x^2+3x^{-2})^{\frac{1}{2}}$ $(x>0)$

(A) −1 (B) 0 (C) 1 (D) $\frac{3}{2}$ (E) $\frac{8}{5}$

$F''(x)=\frac{1}{2}(x^2+3x^{-2})^{-\frac{1}{2}}(2x+(-6)x^{-3})$

$F''(1)=\frac{1}{2}\cdot\frac{1}{2}\cdot(-4)=-1$



graph of f

7. The graph of a function f, whose domain is the closed interval $[-3,5]$, is shown above. Let g be the function given by $g(x)=\int_{-3}^{2x-1} f(t)\,dt$.

(a) Find the domain of g. (a) $-3\le 2x-1\le 5$

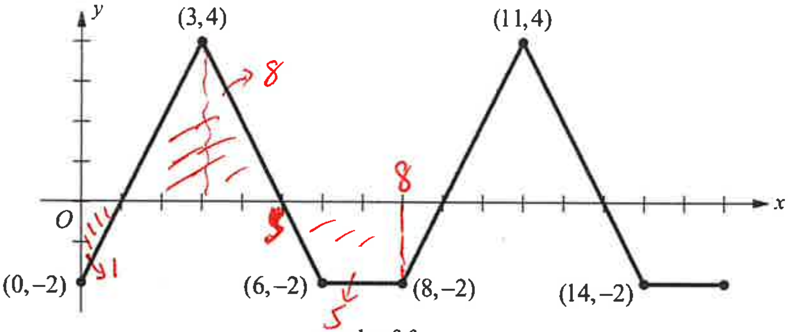
$\therefore -1\le x\le 3$

(b) Find $g'(3)$.

$\therefore$ Domain $[-1,3]$

(b) $g'(x)=f(2x-1)\cdot 2$

$g'(3)=2\cdot f(5)=2$



graph of f

The graph above shows two periods of f. The function f is defined for all real numbers x and is periodic with a period of 8. Let h be the function given by $h(x)=\int_0^x f(t)\,dt$.

(a) $h(8) = -1+8-5 = 2$ $\quad h'(x) = f(x)$

(a) Find $h(8)$, $h'(6)$, and $h''(4)$. $\quad h'(6) = f(6) = -2 \quad h''(x) = f'(x)$

$h''(4) = f'(4) = -2$

(b) Find the values of x at which h has its minimum and maximum on the closed interval $[0,8]$. Justify your answer.

(b) $h'(x) = f(x)$

x	0	$(0,1)$	1	$(1,5)$	5	$(5,8)$	8
$h'(x)$		$-$	0	$+$	0	$-$	
$h(x)$	max	↘	min	↗	max	↘	min

(c) Write an equation for the line tangent to the graph of h at $x = 35$.

(c) $h'(35) = f(35) = f(32+3) = f(3) = 4$

$h(35) = 4\int_0^8 f(t)\,dt + \int_0^3 f(t)\,dt$

$= 8 + 3 = 11$

$\therefore$ passes $(35, 11)$ with slope $= 4$

$\therefore y - 11 = 4(x-35)$

$h(0) = 0$

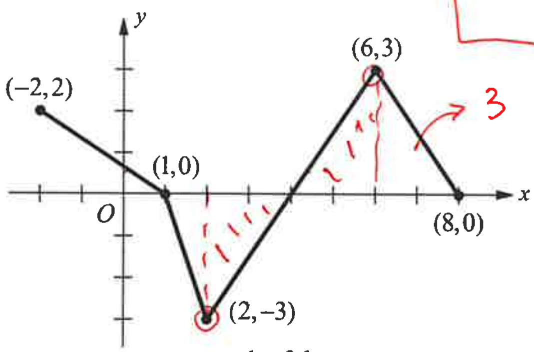
$h(5) = 0 - 1 + 8 > 0$

$\therefore$ At $x = 5$ h attains its max.

$h(1) = -1$

$h(8) = 2$

$\therefore$ h attains its min at $x = 1$



graph of f

8. The graph of f, consisting of four line segments, is shown in the figure above. Let g be the function given by $g(x)=\int_{-2}^x f(t)\,dt$.

(a) Find $g'(1)$ (a) $g'(x) = f(x) \quad \therefore g'(1) = f(1) = 0$

(b) $g''(x) = f'(x)$

$g''(x)$ changes its sign at $x = 2$ and $x = 6$

(b) Find the x-coordinate for each point of inflection of the graph of g on the interval $-2 < x < 8$.

(c) Find the average rate of change of g on the interval $2 \le x \le 8$. (c) $\text{AROC} = \dfrac{g(8)-g(2)}{8-2} = \dfrac{\int_2^8 f(t)\,dt}{6} = \dfrac{1}{2}$

(d) For how many values of c, where $\boxed{2 < c < 8}$, is $g'(c)$ equal to the average rate found in part (c)? Explain your reasoning.

(d) $g'(x) = f(x) = \frac{1}{2}$

Two values of c for $2 < c < 8$.