Integration

✓ We know how to calculate velocity v(t) from position function s(t). This helped us to understand the idea of the derivative or rate of change of a function. $s'(t) = v(t)$

Now we consider the reverse problem: given v(t), find s(t)

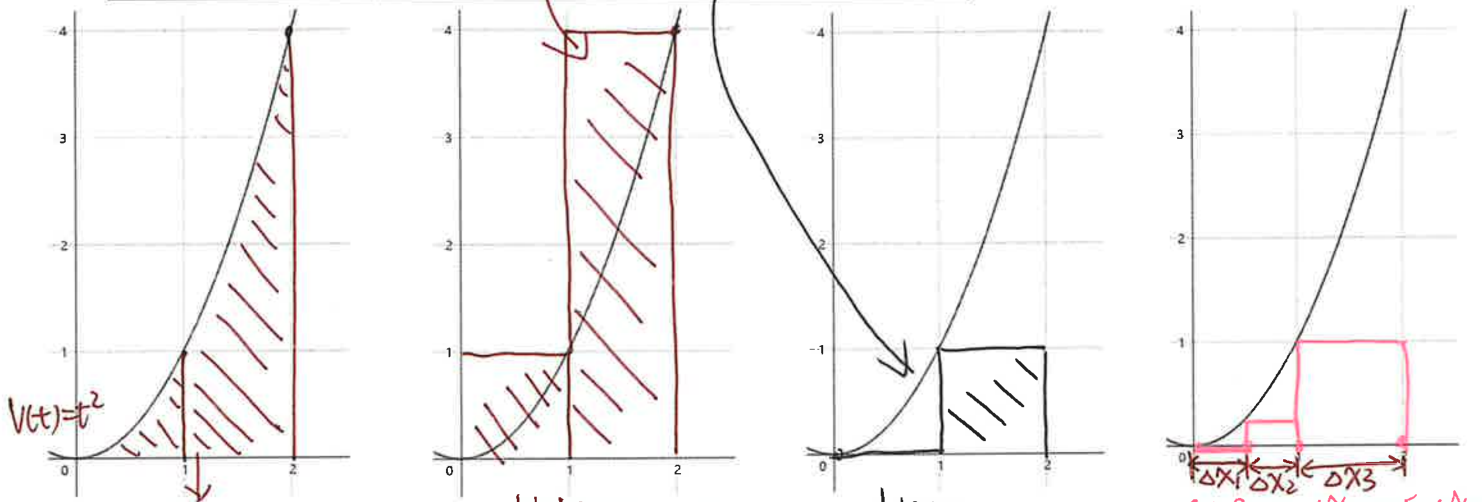
Given velocity, how do we calculate the distance the car has traveled?

This will give us the idea of **definite integration**.

➢ **Approximating the area under a curve using rectangles**

Consider the function of $v(t) = t^2$ for $t \in [0,2]$. Use the rectangle(s) to estimate the area under the curve.

Time Interval	0~1	1~2	total distance in the first two seconds:
Velocity is at most	1	4	$[s(2) - s(0)] =$ $1\times1+4\times1=5$ ft
Velocity is at least	0	1	$[s(2) - s(0)] =$ $1\times1=1$ ft



estimate: total distance
↓
estimate: Area under the curve on [0,2]

distance: at most 5 ft

distance: at least 1 ft

Δ: $a=0 < x_1=0.5 < x_2=1 < x_3=2$

$c_1=0$ $c_2=0.5$ $c_3=1$

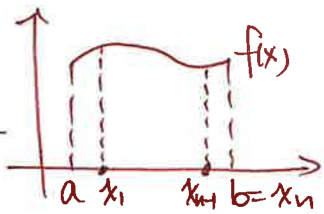
c_i: left point of the ith interval.

➢ **Def. Riemann Sum**

Let f be a continuous function defined on the closed interval $[a,b]$, and let Δ be a partition of $[a,b]$ given by $a = x_0 < x_1 < \cdots < x_n = b$, where Δx_i is the width of the ith interval. If c_i is any point in the ith interval, ⇒ 选取作为长方形的高. then the sum $\sum_{i=1}^{n} f(c_i)\Delta x_i = f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + \cdots + f(c_n)\Delta x_n$ is called a **Riemann Sum** for f on the interval $[a,b]$.

Riemann Sum = $f(c_1)\cdot\Delta x_1 + f(c_2)\cdot\Delta x_2 + f(c_3)\cdot\Delta x_3$

● If every subinterval is of equal width, then $\Delta x = \frac{b-a}{n}$



➢ **Left, Right, and Midpoint Riemann Sum Approximation**

ith interval: $a+(i-1)\Delta x$ $a+i\cdot\Delta x$

If c_i is the left endpoint of each subinterval then $\sum_{i=1}^{n} f(c_i)\Delta x$ is called a Left Riemann Sum. $c_i = a+(i-1)\Delta x$

If c_i is the right endpoint of each subinterval then $\sum_{i=1}^{n} f(c_i)\Delta x$ is called a Right Riemann Sum. $c_i = a+i\cdot\Delta x$

If c_i is the midpoint of each subinterval then $\sum_{i=1}^{n} f(c_i)\Delta x$ is called a Midpoint Riemann Sum. $c_i = a+(i-\frac{1}{2})\Delta x$

● The general form of Riemann sum is $\sum_{i=1}^{n} f(c_i)\Delta x = \sum_{i=1}^{n} f(c_i)\cdot\frac{b-a}{n}$, where $c_i = a+\Delta x(i-\lambda)$

$\lambda \in [0,1]$

(equal width)

Q1. Approximate the area of the region bounded by the graph of $f(x) = -x^2 + x + 2$, the x-axis, and the vertical lines x = 0 and x = 2

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	2	$\frac{9}{4}$	2	$\frac{5}{4}$	0

(1) by using a left Riemann sum with four subintervals

$0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2$

$\frac{1}{2}\left(f(0)+f(\tfrac{1}{2})+f(1)+f(\tfrac{3}{2})\right) = 1.875$

(2) by using a right Riemann sum with four subintervals

$\frac{1}{2}\left(f(\tfrac{1}{2})+f(1)+f(\tfrac{3}{2})+f(2)\right) = 1.375$

(3) by using a midpoint Riemann sum with four subintervals

$\frac{1}{4} \quad \frac{3}{4} \quad \frac{5}{4} \quad \frac{7}{4}$

$0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2$

$\frac{1}{2}\left[f(\tfrac{1}{4})+f(\tfrac{3}{4})+f(\tfrac{5}{4})+f(\tfrac{7}{4})\right]$

t (hours)	0	2	4	5	6	9	12
$P'(t)$ people/hour	41	30	54	26	21	44	11

Q2. Tiffani posts a picture of her posing with Sir Isaac from the pop group Sir Isaac and the Newtones on her Instagram at 9 AM. The rate that her followers view her picture is modeled with selected values shown in the table above where $t = 0$ represents 9 AM and the rate is measured in people per hour. Use the data in the table above to approximate the following.

(1) Use a Right Riemann Sum with 3 subintervals to approximate the area between P′(t) and the t-axis from t = 0 to t = 5. Include units of measure with your answer.

$\Delta x_1 \quad \Delta x_2 \quad \Delta x_3$

$0 \quad 2 \quad 4 \quad 5$

$2\times f(2) + 2\times f(4) + 1\times f(5) = 194$ people

(2) Use a Left Riemann Sum with 4 subintervals to approximate the area between P′(t) and the t-axis from t = 4 to t = 12. Include units of measure with your answer.

$\Delta x_1 \quad \Delta x_2 \quad \Delta x_3 \quad \Delta x_4$

$4 \quad 5 \quad 6 \quad 9 \quad 12$

$1\times f(4) + 1\times f(5) + 3\times f(6) + 3\times f(9) = 275$ people

(3) Use a Midpoint Riemann Sum with 3 subintervals to approximate the area between P′(t) and the t-axis from t = 0 to t = 12. Include units of measure with your answer.

$\Delta x_1 = 4 \quad \Delta x_2 = 2 \quad \Delta x_3 = 6$

$0 \quad 2 \quad 4 \quad 5 \quad 6 \quad 9 \quad 12$

$4\times f(2) + 2\times f(5) + 6\times f(9) = 436$ people

Riemann Sum = sum of areas of rectangles

$n \uparrow$. error $\downarrow$

$n \to \infty$ Riemann Sum $\to$ actual area

➢ **Def. Definite Integrals** ⇒ The limit of Riemann Sum can be defined as Definite Integral.

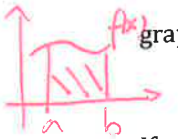
If f is a continuous function defined for $a \le x \le b$, then the definite integral of f from a to b is

a, b: limits of integration

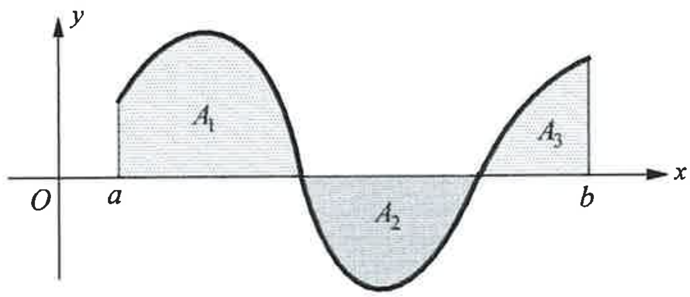
$$\int_a^b \underbrace{f(x)}_{\text{integrand}}\, dx = \lim_{n\to\infty} \boxed{\sum_{i=1}^{n}} f(c_i) \cdot \boxed{\Delta x}$$

$\int$ dx

If $y = f(x)$ is continuous and nonnegative over a closed interval $[a, b]$ then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by $Area = \int_a^b f(x)\, dx$



If $y = f(x)$ takes on both positive and negative values over a closed interval $[a, b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is obtained by adding the absolute value of the definite integral over each subinterval where $f(x)$ does not change sign.



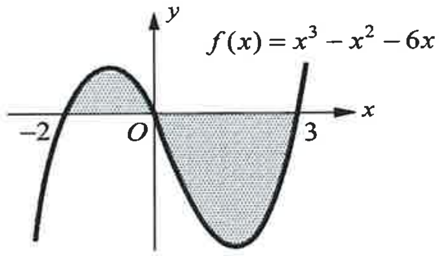
The definite integral of $f(x)$ over $[a, b]$ is $\int_a^b f(x)\, dx =$ $A_1 - A_2 + A_3$

The total area between the curve and the x-axis over $[a, b]$ is $\int_a^b |f(x)|\, dx =$ $A_1 + A_2 + A_3$

Q3. The figure shows the graph of $f(x) = x^3 - x^2 - 6x$.

(a) Find the definite integral of $f(x)$ on $[-2,3]$ using calculator. $-\frac{125}{12}$

(b) Find the area between the graph of $f(x)$ and the x-axis on $[-2,3]$.



(b) $\int_{-2}^{0} f(x)\,dx + \left[-\int_0^3 f(x)\,dx\right] = \int_{-2}^{3} |f(x)|\, dx = 21.083$

(right point)

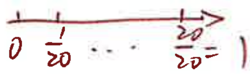
$c_i = \frac{i}{n} = \frac{i}{20}$ $\quad f(c_i) = c_i^2 \quad \therefore f(x) = x^2$

Q4. The expression $\frac{1}{20}\Big[\underbrace{\big(\overset{c_1}{\tfrac{1}{20}}\big)^2 + \big(\overset{c_2}{\tfrac{2}{20}}\big)^2 + \cdots + \big(\overset{c_{20}}{\tfrac{20}{20}}\big)^2}_{20 \text{ terms} \Rightarrow n=20}\Big]$ is a Riemann sum approximation for $\int_0^1 x^2\,dx$

Subintervals are of equal width

$\Delta x = \frac{b-a}{n} = \frac{b-a}{20} = \frac{1}{20}$

$\therefore b - a = 1$



Integration

$c_1 \quad c_2 \quad c_{20} \qquad c_i = \frac{i}{10} \qquad f(c_i) = c_i \qquad \therefore f(x) = x$

Q5. The expression $\frac{1}{10}\left[\frac{1}{10}+\frac{2}{10}+\cdots+\frac{20}{10}\right]$ is a Riemann sum approximation for $\int_0^2 x\,dx$

$\underbrace{\frac{1}{10}+\frac{2}{10}+\cdots+\frac{20}{10}}_{20 \text{ terms}} \Downarrow n=20$

$\Delta x = \frac{1}{10}$

$\Downarrow$

$\frac{b-a}{n} = \frac{b-a}{20} = \frac{1}{10}$

$0 \quad \frac{1}{10} \quad \cdots \quad \frac{20}{10} = 2$

$\therefore b-a=2$

D **Q6.** Which of the following limits is equal to $\int_1^3 x^3\,dx$

$\Delta x = \frac{3-1}{n} = \frac{2}{n}$

(A) $\lim_{n\to\infty}\sum_{i=1}^{n}(1+\frac{i}{n})^3\frac{1}{n}$ (C) $\lim_{n\to\infty}\sum_{i=1}^{n}(1+\frac{2i}{n})^3\frac{1}{n}$

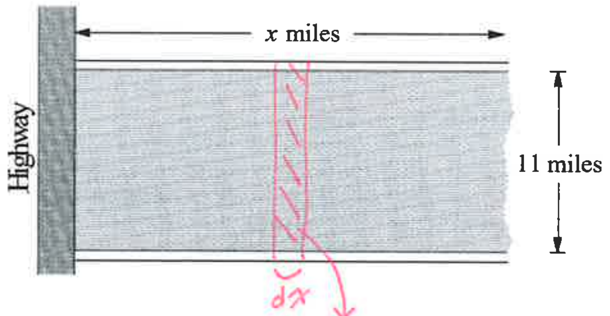
(B) $\lim_{n\to\infty}\sum_{i=1}^{n}(1+\frac{i}{n})^3\frac{2}{n}$ (D) $\lim_{n\to\infty}\sum_{i=1}^{n}(1+\frac{2i}{n})^3\frac{2}{n}$

$1 \quad 1+\Delta x$

right point of the ith interval :

$1+\underbrace{\frac{2}{n}}_{\Delta x} i = 1+\frac{2i}{n}$

Q7. (*)(Calculator) A rectangular region located beside a highway and between two straight roads 11 miles apart are shown in the figure below. The population density of the region at a distance x miles from the highway is given by $D(x) = 15x\sqrt{x} - 3x^2$, where $0 \le x \le 25$. How many people live between 16 to 25 miles from the highway?



population = $D(x) \cdot 11\,dx$

$\therefore$ Total population = $\int_{16}^{25} 11\,D(x)\,dx$

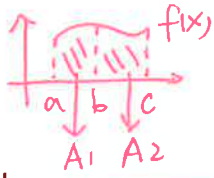
- **Properties of definite integral**

1. $\int_a^a f(x)\,dx =$ $\underline{0}$

$\int_a^b f(x)dx + \int_b^a f(x)dx = \int_a^a f(x)dx = 0$

2. $\int_a^b f(x)\,dx =$ $\underline{-}$ $\int_b^a f(x)\,dx$ ⇒ Upper Limit might be smaller than the lower limit.

3. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx =$ $\underline{\int_a^c f(x)\,dx}$



4. $\int_a^b f(x) \pm g(x)\,dx =$ $\underline{\int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx}$

Riemann Sum:

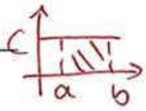
$\sum_{i=1}^{n}[f(c_i) \pm g(c_i)]\,\Delta x$

$= \sum_{i=1}^{n} f(c_i)\,\Delta x \pm \sum_{i=1}^{n} g(c_i)\,\Delta x$

$\downarrow \int_a^b f(x)\,dx \qquad \downarrow \int_a^b g(x)\,dx$

5. $\int_a^b cf(x)\,dx =$ $\underline{c\int_a^b f(x)\,dx}$ $\Leftarrow \sum_{i=1}^{n} c\,f(c_i)\,\Delta x = c\sum_{i=1}^{n} f(c_i)\,\Delta x$ $\downarrow \int_a^b f(x)\,dx$

6. $\int_a^b c\,dx =$ $\underline{c(b-a)}$



(4.5) ⇒ linear combination

$\int k_1 f + k_2 g\,dx$

$= k_1 \int f\,dx + k_2 \int g\,dx$

求导: $(k_1 f + k_2 g)' = k_1 f' + k_2 g'$

7. If f is even, then $\int_{-a}^{a} f(x)\,dx =$ $\underline{2}$ $\int_0^a f(x)\,dx$

If f is odd, then $\int_{-a}^{a} f(x)\,dx =$ $\underline{0}$

8. If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx$ $\underline{\geq 0}$

If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx$ $\underline{\geq}$ $\int_a^b g(x)\,dx$

If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then $\underline{m(b-a)} \leq \int_a^b f(x)\,dx \leq \underline{M(b-a)}$

Q1. Suppose $\int_0^{1\cdot 25} \cos(x^2)\,dx = 0.98$ and $\int_0^1 \cos(x^2)\,dx = 0.90$. What are the values of the following integrals?

(a) $\int_1^{1\cdot 25} \cos(x^2)\,dx = \int_0^{1.25} \cos(x^2)\,dx - \int_0^1 \cos(x^2)\,dx = 0.08$

(b) $\int_{-1}^{1} \cos(x^2)\,dx = 2\int_0^1 \cos(x^2)\,dx = 1.8$

(c) $\int_{1.25}^{-1} \cos(x^2)\,dx = -\int_{-1}^{1.25} \cos(x^2)\,dx = -1.88$

Q2. Suppose that $\int_{-3}^{4} f(x)\,dx = 5$, $\int_{-3}^{4} g(x)\,dx = -4$, and $\int_{-3}^{1} f(x)\,dx = 2$.

Find (a) $\int_{-3}^{4} [2f(x) - 3g(x)]\,dx$ (b) $\int_{1}^{4} f(x)\,dx$ (c) $\int_{-3}^{4} [g(x)+2]\,dx$.

(a) $I = 2\int_{-3}^{4} f(x)dx - 3\int_{-3}^{4} g(x)\,dx = 22$

(b) $I = \int_{-3}^{4} f(x)\,dx - \int_{-3}^{1} f(x)\,dx = 3$

(c) $I = \int_{-3}^{4} g(x)\,dx + 2\times[4-(-3)] = 10$

Q3. Let f and g be continuous on the interval $[1,5]$. Given $\int_1^3 f(x)\,dx = -3$, $\int_1^5 f(x)\,dx = 7$, and $\int_1^5 g(x)\,dx = 9$, find the following definite integrals.

(a) $\int_3^5 f(x)\,dx$ $= \int_1^5 f(x)\,dx - \int_1^3 f(x)\,dx = 10$

(b) $\int_1^3 [f(x)+3]\,dx$ $= \int_1^3 f(x)\,dx + 3\times(3-1) = 3$

(c) $\int_5^1 2g(x)\,dx$ $= 2\cdot\left(-\int_1^5 g(x)\,dx\right) = -18$

(d) $\int_5^5 g(x)\,dx + \int_5^3 f(x)\,dx$ $= 0 + (-1)\int_3^5 f(x)\,dx = -10$

(e) (*) $\int_{-1}^{3} f(x+2)\,dx$ $= \int_1^5 f(x)\,dx = 7$

➢ **Def. Antiderivative** ~ "Inverse Function"

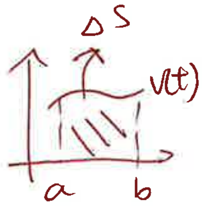
A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x on I.

If F is an antiderivative of f on I, then $F(x)+C$ represents the most general antiderivative of f on I.

$\because (\text{constant})' = 0$

➢ **The Fundamental Theorem of Calculus (FTC)**

Let f be continuous on $[a, b]$ then $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x)$ is an antiderivative of f.



Understanding: $f \sim v(t)$ $\quad F \sim s(t)$ $\quad s'(t) = v(t)$

$\int_a^b f(x)dx \sim \int_a^b v(t)\,dt$ = Area under the velocity curve

= 发生的位移变化 = final position − initial position $= S(b) - S(a)$

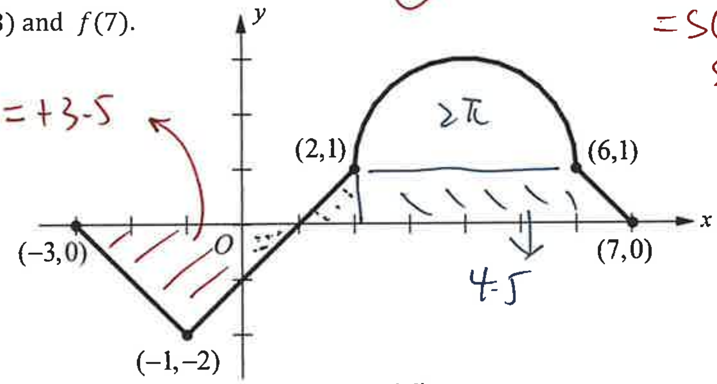
Q1. Let f be a function defined on the closed interval [-3,7] with $f(2) = 3$. The graph of f' consists of three line segments and a semicircle, as shown below. Find $f(-3)$ and $f(7)$.

$f(-3) - f(2) = \int_2^{-3} f'(x)dx = -\int_{-3}^{2} f'(x)dx = +3.5$

$\therefore f(-3) = 6.5$

$f(7) - f(2) = \int_2^7 f'(x)dx = 4.5 + 2\pi$

$\therefore f(7) = 7.5 + 2\pi$



graph of f'

Q2. (Calculator) If f is the antiderivative of $\dfrac{\sqrt{x}}{1+x^3}$ such that $f(1) = 2$, then $f(3) =$

$f'(x)$

$\int_1^3 \frac{\sqrt{x}}{1+x^3}dx = f(3) - f(1) = 0.397$

$\therefore f(3) = 2.397$

Q3. (Calculator) If $f'(x) = \cos(x^2 - 1)$ and $f(-1) = 1.5$, then $f(5) =$ 3.024

$\int_{-1}^{5} f'(x)\,dx = f(5) - f(-1)$

Q4. If f is a continuous function and $F'(x) = f(x)$ for all real numbers x, then $\int_2^{10} f(\frac{1}{2}x)\,dx =$

C

(A) $\frac{1}{2}[F(5) - F(1)]$

(B) $\frac{1}{2}[F(10) - F(2)]$

(C) $2[F(5) - F(1)]$

(D) $2[F(10) - F(2)]$

$(F(\frac{1}{2}x))' = F'(\frac{1}{2}x)\cdot\frac{1}{2}$

$= f(\frac{1}{2}x)\frac{1}{2}$

$\therefore \underbrace{(2F(\frac{1}{2}x))'}_{\Downarrow\ \text{Antiderivative}} = f(\frac{1}{2}x)$

$\therefore$ FTC: $I = 2F(\frac{1}{2}\cdot 10) - 2F(\frac{1}{2}\cdot 2) = 2[F(5) - F(1)]$

The limits (upper & lower) are not definite

不定积分

Integration

➢ Indefinite Integral

The set of all antiderivatives of f is the **indefinite integral** of f with respect to x denoted by $\int f(x)\,dx$.

$$\int f(x)\,dx = F(x) + C \iff F'(x) = f(x)$$

⇒ How to find F(x)?
⇒ whose derivative is f(x)?

➢ Indefinite Integrals

$\int k\,dx = kx + C$ $\quad \int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ $\quad \int e^x\,dx = e^x + C$

$\int \sin x\,dx = -\cos x + C$ $\quad \int \cos x\,dx = \sin x + C$ $\quad \int \sec^2 x\,dx = \tan x + C$ $\quad \int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$ $\quad \int \csc x \cot x\,dx = -\csc x + C$

$\int kf(x)\,dx = k\int f(x)dx$ $\quad \int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

➢ Integral of Natural Logarithmic Function

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ $\quad (x>0)$

For $x \neq 0$, $\int \frac{1}{x}dx = \ln|x| + C$

$\ln|x| = \begin{cases} \ln x, & x>0 \\ \ln(-x), & x<0 \end{cases}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(\ln(-x)) = \frac{1}{-x}\cdot(-1) = \frac{1}{x}$

$\therefore \frac{d}{dx}(\ln|x|) = \frac{1}{x}$ for $x \neq 0$.

$\therefore \int \frac{1}{x}dx = \ln|x| + C$

Q1. $\int_{\frac{\pi}{2}}^{x} \cos t\,dt = ?$ $\quad F(x) - F(\frac{\pi}{2}) = \sin x - \sin\frac{\pi}{2} = \sin x - 1$

$F(t) = \sin t$

Q2. Find an antiderivative for each of the following functions.

a. $f(x) = 3x^2$ $\quad F(x) = x^3 + C$

b. $g(x) = \cos x + 3$ $\quad G(x) = \sin x + 3x + C$

Q3. Find the antiderivative of $x^3 - 3x + 2$.

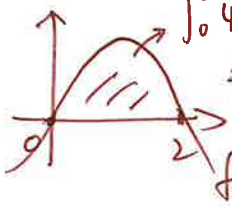
$\frac{1}{4}x^4 - \frac{3}{2}x^2 + 2x + C$

Q4. Find the general indefinite integral $\int \sqrt{x} - \sec x \tan x\ dx$

$= \int x^{\frac12}\,dx - \int \sec x \tan x\,dx = \frac{2}{3}x^{\frac32} - \sec x + C$

Q5. The area of the region in the first quadrant enclosed by $f(x) = 4x - x^3$ and the x-axis is

(A) $\frac{11}{4}$ $\quad$ (B) $\frac{7}{2}$ $\quad$ (C) 4 $\quad$ (D) $\frac{11}{2}$



$\int_0^2 4x - x^3\,dx = \left[2x^2 - \frac{1}{4}x^4\right]_0^2 = 4$

Q6. Find $\int_1^e \frac{x^2+3}{x}dx = \int_1^e x + \frac{3}{x}\,dx = \left[\frac12 x^2 + 3\ln|x|\right]_1^e = \left(\frac12 e^2 + 3\right) - \frac12 = \frac12 e^2 + \frac52$

Q7.(*) $\int_0^5 \sqrt{25 - x^2}\,dx = \frac{25}{4}\pi$

$y = \sqrt{25 - x^2}$

$y^2 + x^2 = 25$

$\frac14 *$ Area of a circle $= \frac14 \cdot \pi \cdot 25$

Integration

Accordng to the FTC.

➢ **Fundamental Theorem:**

$= " F(x) - F(a) "$ $\quad \frac{d}{dx}[F(x) - F(a)] = F'(x) - 0 = f(x)$

- Let f be continuous on $[a, b]$ then $F(x) = \int_a^x f(t)\,dt$ is continuous on $[a, b]$ and differentiable on (a, b), and $F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt =$ $f(x)$

- If $F(x) = \int_a^{u(x)} f(t)\,dt$, then $F'(x) = \frac{d}{dx}\int_a^{u(x)} f(t)\,dt =$ $f(u(x)) \cdot u'(x)$

$= " F(u(x)) - F(a) "$ $\quad \frac{d}{dx}\left(F(u(x))\right) = F'(u(x)) \cdot u'(x) = f(u(x)) \cdot u'(x)$

Q1. If $F(x) = \int_1^x \frac{1}{1+u^3}\,du$, then $F'(x) =$ $\frac{1}{1+x^3}$

Q2. If $F(x) = \int_1^{x^2+1} \sqrt{t}\,dt$, then $F'(x) =$ $\sqrt{x^2+1} \cdot 2x$

$\frac{2}{3} t^{\frac{3}{2}}$

$\frac{d}{dx}\left(\left[\frac{2}{3} t^{\frac{3}{2}}\right]_1^{x^2+1}\right) = \frac{d}{dx}\left(\frac{2}{3}(x^2+1)^{\frac{3}{2}} - \text{constant}\right) = \frac{2}{3} \cdot \frac{3}{2}(x^2+1)^{\frac{1}{2}} \cdot (x^2+1)'$

$= (x^2+1)^{\frac{1}{2}} \cdot 2x$

Q3. For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, if $F(x) = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}$, then $F'(x) =$ $\frac{1}{\sqrt{1-\sin^2 x}} \cdot \cos x$

$= 1$

$x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\therefore \cos x > 0$

C

Q4. Let f be the function given by $f(x) = \int_0^x \cos(t^2+2)\,dt$ for $0 \le x \le \pi$. On which of the following intervals is f increasing?

$f'(x) > 0$ $\quad f'(x) = \cos(x^2+2) > 0$

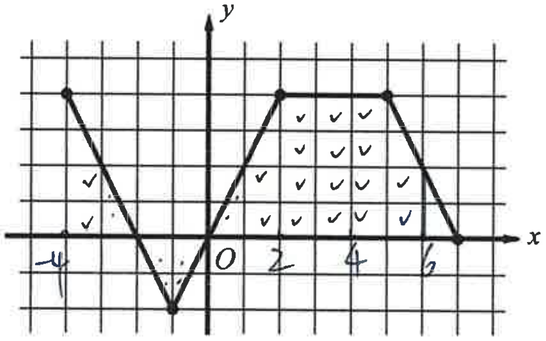
(A) $0 \le x \le \frac{\pi}{2}$

(B) $0 \le x \le 1.647$

(C) $1.647 \le x \le 2.419$

(D) $\frac{\pi}{2} \le x \le \pi$

Q5. The graph of the function f shown below consists of four line segments. If g is the function defined by $g(x)=\int_{-4}^{x} f(t)\,dt$, find the value of $g(6)$, $g'(6)$, and $g''(6)$.



$g(6)=\int_{-4}^{6} f(t)\,dt$ = Area.

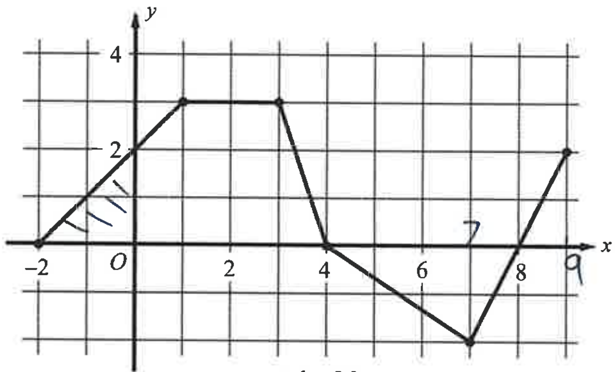
$= 21.5$

$g'(6)$: $g'(x)=f(x)$

$\therefore g'(6)=f(6)=2$

$g''(x) = f'(x)$

$g''(6) = f'(6) = -2$



graph of f

Q6. Let g be the function given by $g(x)=\int_{-2}^{x} f(t)\,dt$. The graph of the function f, shown above, consists of five line segments.

(a) Find $g(0)$, $g'(0)$ and $g''(0)$.

(b) For what values of x, in the open interval $(-2,9)$, is the graph of g concave up?

(c) For what values of x, in the open interval $(-2,9)$, is g increasing?

(a)

$g(0)=\int_{-2}^{0} f(t)\,dt = 2$

$g'(x) = f(x) \Rightarrow g'(0)=f(0)=2$

$g''(x)=f'(x)$

$\therefore g''(0)=f'(0)=1$

(b) $g'(x)=f(x)$ $g''(x)=f'(x)$

concave up: $g''(x)>0 \Rightarrow x\in(-2,1),\ (7,9)$

(c) $g'(x)=f(x)>0$

$\Rightarrow x\in(-2,4),\ (8,9)$