➢ Separable Differential Equation

1. The solution to the differential equation $\frac{dy}{dx} = \frac{3x^2}{2y}$, where $y(3) = 4$, is

2. If $\frac{dy}{dx} = \frac{x + \sec^2 x}{y}$ and $y(0) = 2$, then $y =$

3. What is the value of $m + b$, if $y = mx + b$ is a solution to the differential equation $\frac{dy}{dx} = \frac{1}{4}x - y + 1$?

Exponential Growth or Decay

1. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles every four hours, in how many hours will the number of bacteria triple?

(A) $\ln(\frac{27}{2})$ (B) $\ln(\frac{81}{2})$ (C) $\frac{4\ln 2}{\ln 3}$ (D) $\frac{4\ln 3}{\ln 2}$

2. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 15 years what is the value of k?

(A) 0.035 (B) 0.046 (C) 0 .069 (D) 0.078

3. A baby weighs 6 pounds at birth and 9 pounds three months later. If the weight of baby increasing at a rate proportional to its weight, then how much will the baby weigh when she is 6 months old?

(A) 11.9 (B) 12.8 (C) 13.5 (D) 14.6

4. Temperature F changes according to the differential equation $\frac{dF}{dt} = kF$, where k is a constant and t is measured in minutes. If at time $t = 0$, $F = 180$ and at time $t = 16$, $F = 120$, what is the value of k?

(A) -0.025 (B) -0.032 (C) -0.045 (D) -0.058

- **Logistic**

1. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = 3P - 0.0006P^2$, where the initial population is $P(0) = 1000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 1000 (B) 2000 (C) 3000 (D) 5000

2. A healthy population $P(t)$ of animals satisfies the logistic differential equation $\frac{dP}{dt} = 5P(1 - \frac{P}{240})$, where the initial population is $P(0) = 150$ and t is the time in years. For what value of P is the population growing the fastest?

(A) 48 (B) 60 (C) 120 (D) 240

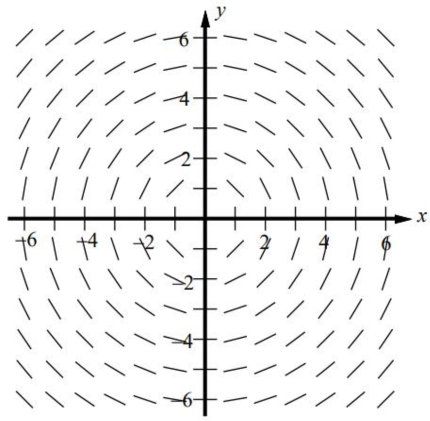
3. A population is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{150}\right)$, where the initial population is $P(0) = 800$ and t is the time in years. What is the slope of the graph of P at the point of inflection?

(A) 5 (B) 7.5 (C) 10 (D) 12.5

4. A certain rumor spreads in a small town at the rate $\frac{dy}{dt} = y(1 - 3y)$, where y is the fraction of the population that has heard the rumor at any time t. What fraction of the population has heard the rumor when it is spreading the fastest?

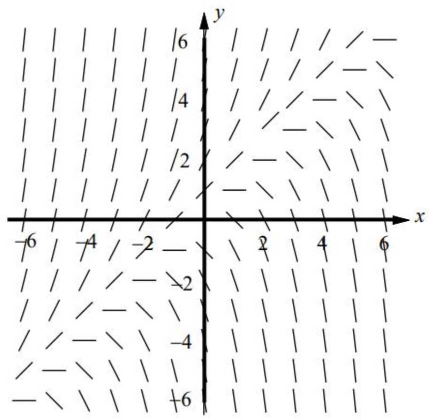
(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$

➢ Slope Field



1. Shown above is a slope field for which of the following differential equations?

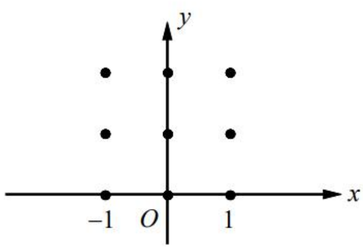
(A) $\frac{dy}{dx} = \frac{x}{y}$ (B) $\frac{dy}{dx} = -\frac{x}{y}$ (C) $\frac{dy}{dx} = \frac{x^2}{y}$ (D) $\frac{dy}{dx} = -\frac{x^2}{y}$



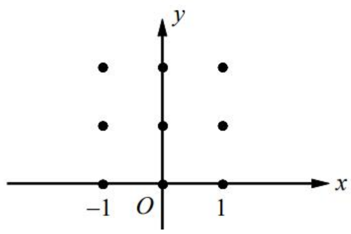
2. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = x + y$ (B) $\frac{dy}{dx} = x - y$ (C) $\frac{dy}{dx} = -x + y$ (D) $\frac{dy}{dx} = x^2 - y$

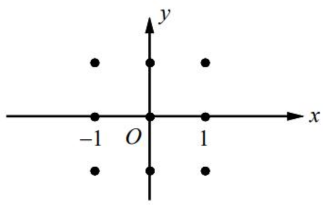
3. On the axis provided, sketch a slope field for the differential equation $\frac{dy}{dx} = y - x^2$.



4. On the axis provided, sketch a slope field for the differential equation $\frac{dy}{dx} = x^2 + y^2$.



5. On the axis provided, sketch a slope field for the differential equation $\frac{dy}{dx} = (x+1)(y-2)$.



➢ Euler's Method

1. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 1 + 2x - y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?

(A) 2.5 (B) 2.75 (C) 3.25 (D) 3.75

2. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - xy$ with the initial condition $f(0.5) = 0$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 0.5$ with a step size of 0.5?

(A) 0.825 (B) 0.906 (C) 1.064 (D) 1.178

3. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = \arctan(xy)$ with the initial condition $f(0) = 1$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 0$ with a step size of 1?

(A) $\frac{\pi}{2}$ (B) $1 + \frac{\pi}{4}$ (C) $1 + \frac{\pi}{2}$ (D) π

x	-1	-0.6	-0.2	0.2	0.6
$f'(x)$	1	2	-0.5	-1.5	1.2

4. The table above gives selected values for the derivative of a function f on the interval $-1 \le x \le 0.6$. If $f(-1) = 1.5$ and Euler's method is used to approximate $f(0.6)$ with step size of 0.8, what is the resulting approximation?

(A) 1.9 (B) 2.1 (C) 2.3 (D) 2.5

$x_0 = 0$	$f(x_0) = 1$
$x_1 = 0.5$	$f(x_1) \approx 1.5$
$x_2 = 1$	$f(x_2) \approx 3$

5. Consider the differential equation $\dfrac{dy}{dx} = kx + y - 2x^2$, where k is a constant. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. Euler's method, starting at $x = 0$ with step size of 0.5, is used to approximate $f(1)$. Steps from this approximation are shown in the table above. What is the value of k?

(A) 2.5 (B) 3 (C) 3.5 (D) 4

6. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{1}{2}x - y - \dfrac{1}{2}$.

(a) Find $\dfrac{d^2y}{dx^2}$ in terms of x and y.

(b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(0, -\dfrac{1}{2})$. Does the graph of f have relative minimum, a relative maximum, or neither at the point $(0, -\dfrac{1}{2})$? Justify your answer.

(c) Let $y = g(x)$ be another solution to the given differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 0.5, gives the approximation $g(1) \approx 1$. Find the value of k.