1. Using a left Riemann sum with three subintervals [0,1], [1,2], [2,3], what is the approximation of $\int_0^3 (3-x)(x+1)\,dx$?

x	1	3	5	8	10
$f(x)$	7	12	16	23	17

2. The function f is continuous on the closed interval $[1,10]$ and has values as shown in the table above. Using a right Riemann sum with four subintervals $[1,3]$, $[3,5]$, $[5,8]$, $[8,10]$, what is the approximation of $\int_1^{10} f(x)\,dx$?

(A) 96 (B) 116 (C) 132 (D) 159

3. The function f is continuous on the closed interval [0,12] and has values as shown in the table below. Use a midpoint Riemann sum with 4 subintervals of equal length to approximate the area that lies under f and above the x-axis from x=0 to x=12.

x	0	1.5	3	4.5	6	7.5	9	10.5	12
$f(x)$	1	1.45	2.8	5.05	8.2	12.25	17.2	23.05	29.8

4. Which of the following integrals is equal to $\lim_{n\to\infty}\sum_{i=1}^{n}(-1+\frac{3i}{n})^2\frac{3}{n}$?

(A) $\int_{-1}^{2} x^2\,dx$

(B) $\int_{-1}^{0} x^2\,dx$

(C) $\int_{-1}^{2} (-1+x)^2\,dx$

(D) $\int_{-1}^{0} (-1+\frac{x}{3})^2\,dx$

5. The expression $\frac{1}{30}\left[\sqrt{\frac{1}{30}}+\sqrt{\frac{2}{30}}+\sqrt{\frac{3}{30}}+\dots+\sqrt{\frac{30}{30}}\right]$ is a Riemann sum approximation for

(A) $\int_0^1 \sqrt{x}\,dx$

(B) $\frac{1}{30}\int_0^1 \sqrt{x}\,dx$

(C) $\frac{1}{30}\int_0^{30} \sqrt{x}\,dx$

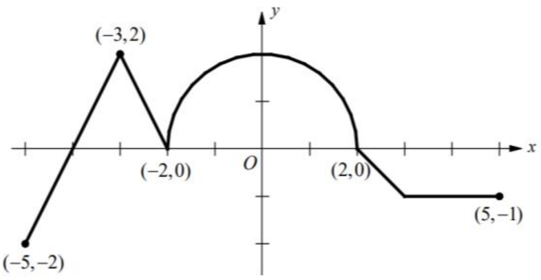
(D) $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

7. If n is a positive integer, then $\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\cdots+\left(\frac{n}{n}\right)^2\right]$ can be expressed as

(A) $\int_0^1 \frac{1}{x}\,dx$ (B) $\int_0^1 \frac{1}{x^2}\,dx$ (C) $\int_0^1 x^2\,dx$ (D) $\frac{1}{2}\int_0^1 x^2\,dx$

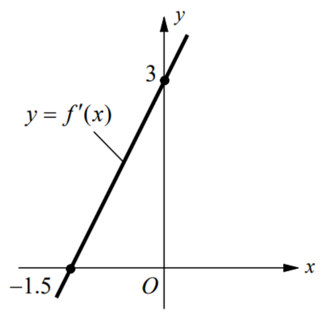
8. If n is a positive integer, then $\lim_{n\to\infty} \frac{2}{n}\left[\sqrt{\frac{2}{n}}+\sqrt{\frac{4}{n}}+\cdots+\sqrt{\frac{2n}{n}}\right]$ can be expressed as

(A) $\int_0^1 \sqrt{x}\,dx$ (B) $\int_0^2 \sqrt{x}\,dx$ (C) $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ (D) $\int_0^2 \frac{1}{\sqrt{x}}\,dx$



9. The graph of $y = f(x)$ consists of four line segments and a semicircle as shown in the figure above. Evaluate each definite integral by using geometric formulas.

(a) $\int_{-5}^{-2} f(x)\,dx$ (b) $\int_{-2}^{2} f(x)\,dx$ (c) $\int_{2}^{5} f(x)\,dx$ (d) $\int_{-5}^{5} |f(x)|\,dx$

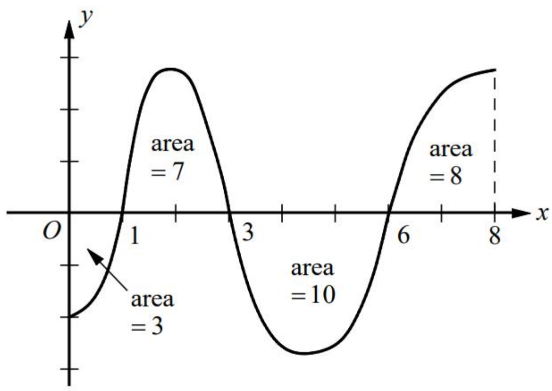


10. The graph of f', the derivative of f, is the line shown in the figure above. If $f(3)=11$, then $f(-3)=$

11. If $f(x)=\sqrt{x^4-3x+4}$ and g is the antiderivative of f, such that $g(3)=7$, then $g(0)=$

(A) -2.966 (B) -1.472 (C) -0.745 (D) 1.086

12.



The figure above shows the graph of f', the derivative of a differentiable function f, on the closed interval $0 \le x \le 8$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. Given $f(6)=9$, find each of the following.

(a) $f(0)$ (b) $f(1)$ (c) $f(3)$ (d) $f(8)$

13. If $\int_a^b f(x)\,dx = 2a - 5b$, then $\int_a^b [f(x) - 2]\,dx =$

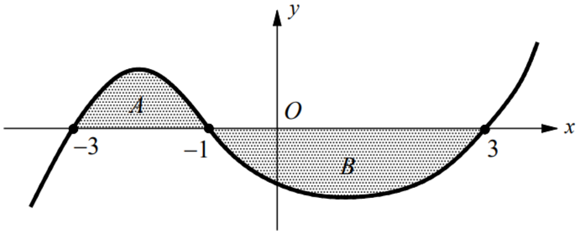
(A) $-7b$ (B) $-3b$ (C) $4a - 7b$ (D) $4a - 3b$

14. If $\int_1^6 f(x)\,dx = \frac{15}{2}$ and $\int_6^4 f(x)\,dx = 5$, then $\int_1^4 f(x)\,dx =$

(A) $\frac{5}{2}$ (B) $\frac{9}{2}$ (C) $\frac{19}{2}$ (D) $\frac{25}{2}$

15. If $\int_{-2}^6 f(x)\,dx = 10$ and $\int_2^6 f(x)\,dx = 3$, then $\int_2^6 f(4-x)\,dx =$

(A) 3 (B) 6 (C) 7 (D) 10



16. The graph of $y = f(x)$ is shown in the figure above. If A and B are positive numbers that represent the areas of the shaded regions, what is the value of $\int_{-3}^3 f(x)\,dx - 2\int_{-1}^3 f(x)\,dx$, in terms of A and B?

(A) $-A - B$ (B) $A + B$ (C) $A - 2B$ (D) $A - B$

17. Let f and g be continuous functions with the following properties.

(1) $g(x) = f(x) - n$ where n is a constant.

(2) $\int_0^4 f(x)\,dx - \int_4^6 g(x)\,dx = 1$

(3) $\int_4^6 f(x)\,dx = 5n - 1$

(a) Find $\int_0^4 f(x)\,dx$ in terms of n.

(b) Find $\int_0^6 g(x)\,dx$ in terms of n.

(c) Find the value of k if $\int_0^2 f(2x)\,dx = kn$.

1. If $\frac{dy}{dx} = 3x^2 - 1$, and if $y = -1$ when $x = 1$, then $y =$

(A) $x^3 - x + 1$

(B) $x^3 - x - 1$

(C) $-x^3 + x - 1$

(D) $-x^3 + 1$

2. The closed interval $[a,b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \ldots, x_n$ where $0 < a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{\sqrt{x_i}} \Delta x$?

(A) $\frac{1}{\sqrt{b}} - \frac{1}{\sqrt{a}}$

(B) $\frac{(\sqrt{b} - \sqrt{a})}{2}$

(C) $2(\sqrt{b} - \sqrt{a})$

(D) $\sqrt{b} - \sqrt{a}$

3. A curve has a slope of $-x+2$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(2,1)$?

(A) $\frac{1}{2}x^2 - 2x - 4$

(B) $2x^2 + x - 8$

(C) $-\frac{1}{2}x^2 + 2x - 1$

(D) $x^2 - 2x + 1$

4. $\int (x^2 - 2)\sqrt{x}\, dx =$

(A) $\frac{2}{5}x^2\sqrt{x} - \frac{2}{3}x\sqrt{x} + C$

(B) $\frac{2}{5}x^2\sqrt{x} - \frac{4}{3}x\sqrt{x} + C$

(C) $\frac{2}{7}x^3\sqrt{x} - \frac{4}{3}x\sqrt{x} + C$

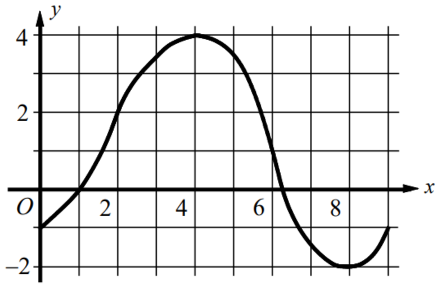
(D) $\frac{2}{7}x^3\sqrt{x} - \frac{2}{3}x^2\sqrt{x} + C$

1. $\frac{d}{dx}\int_1^{x^2} \sqrt{3+t^2}\, dt =$

(A) $\sqrt{3+x^2}$ (B) $\sqrt{3+x^4}$ (C) $2x\sqrt{3+x^4}$ (D) $2\sqrt{3+x^2}$

3. If $F(x)=\int_0^{\sqrt{x}} \cos(t^2)\, dt$, then $F'(4)=$

(A) $\cos 2$ (B) $\frac{\cos 4}{4}$ (C) $\frac{\cos 4}{\sqrt{2}}$ (D) $\sqrt{2}\cos 4$



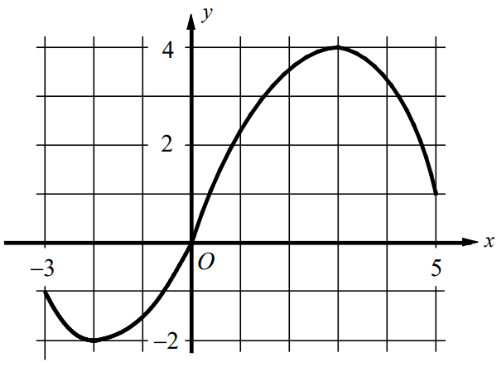
graph of g

5. The graph of the function g, shown in the figure above, has horizontal tangents at $x=4$ and $x=8$. If $f(x)=\int_0^{\sqrt{x}} g(t)\, dt$, what is the value of $f'(4)$?

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{3}{2}$

6. If $F(x)=\int_0^{x^2} \frac{\sqrt{t^2+3}}{2t}\, dt$, then $F''(1)=$

(A) -1 (B) 0 (C) 1 (D) $\frac{3}{2}$ (E) $\frac{8}{5}$

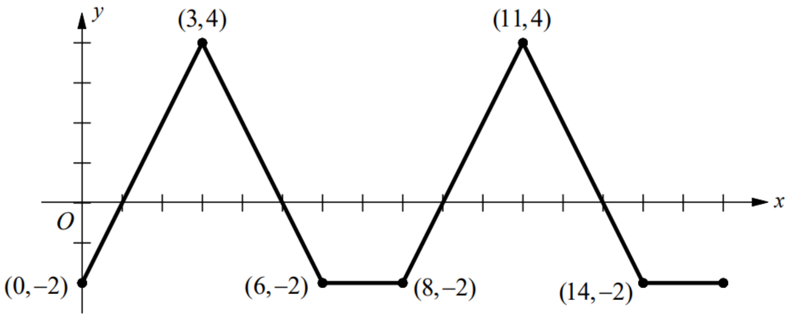


graph of f

7. The graph of a function f, whose domain is the closed interval $[-3,5]$, is shown above. Let g be the function given by $g(x)=\int_{-3}^{2x-1} f(t)\, dt$.

(a) Find the domain of g.

(b) Find $g'(3)$.



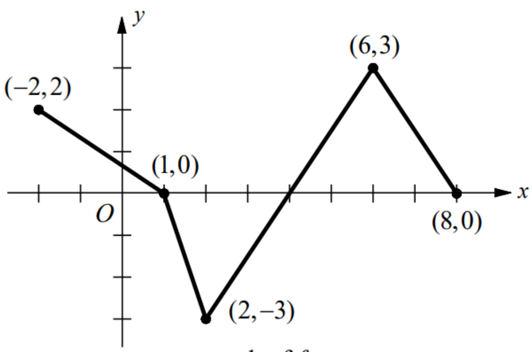
graph of f

The graph above shows two periods of f. The function f is defined for all real numbers x and is periodic with a period of 8. Let h be the function given by $h(x) = \int_0^x f(t)\,dt$.

(a) Find $h(8)$, $h'(6)$, and $h''(4)$.

(b) Find the values of x at which h has its minimum and maximum on the closed interval $[0,8]$. Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 35$.



graph of f

8. The graph of f, consisting of four line segments, is shown in the figure above. Let g be the function given by $g(x) = \int_{-2}^x f(t)\,dt$.

   (a) Find $g'(1)$.

   (b) Find the x-coordinate for each point of inflection of the graph of g on the interval $-2 < x < 8$.

   (c) Find the average rate of change of g on the interval $2 \le x \le 8$.

   (d) For how many values of c, where $2 < c < 8$, is $g'(c)$ equal to the average rate found in part (c)? Explain your reasoning.

1. Use the trapezoidal rule to approximate the integral $\int_{1}^{3} \sqrt{1+x^2}\, dx$ with four subintervals.

2. If three equal subdivisions on $\left[\frac{\pi}{2}, \pi\right]$ are used, what is the trapezoidal approximation of $\int_{\pi/2}^{\pi} \sin x\, dx$?

(A) $\frac{\pi}{12}(\sin\frac{2\pi}{3}+\sin\frac{5\pi}{6}+\sin\pi)$

(B) $\frac{\pi}{12}(\sin\frac{\pi}{2}+\sin\frac{2\pi}{3}+\sin\frac{5\pi}{6}+\sin\pi)$

(C) $\frac{\pi}{12}(\sin\frac{\pi}{2}+2\sin\frac{2\pi}{3}+2\sin\frac{5\pi}{6}+\sin\pi)$

(D) $\frac{\pi}{6}(\sin\frac{\pi}{2}+\sin\frac{2\pi}{3}+\sin\frac{5\pi}{6}+\sin\pi)$

3. If three equal subdivisions on $[0,6]$ are used, what is the trapezoidal approximation of $\int_{0}^{6} \ln(x+1)\, dx$?

(A) $\frac{1}{3}(\ln 1+\ln 9+\ln 25+\ln 7)$

(B) $\frac{1}{2}(\ln 1+\ln 9+\ln 25+\ln 7)$

(C) $\ln 1+\ln 3+\ln 5+\ln 7$

(D) $\ln 1+\ln 9+\ln 25+\ln 7$

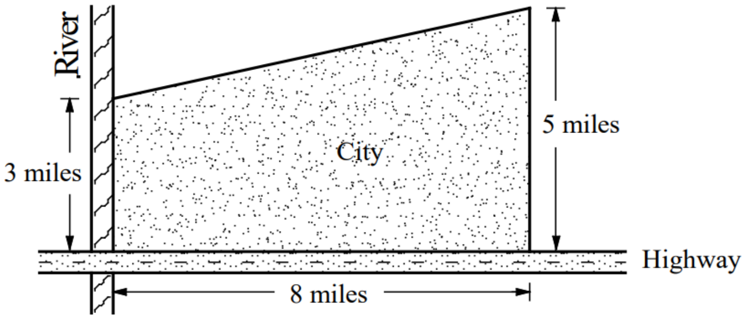
4.

x	1	3	5	9	12
$f(x)$	4	10	14	11	7

A function f is continuous on the closed interval $[1,12]$ and has values that are given in the table above. Using subintervals $[1,3]$, $[3,5]$, $[5,9]$, and $[9,12]$, what is the trapezoidal approximation of $\int_{1}^{12} f(x)\, dx$?

(A) 97 (B) 115 (C) 128 (D) 136

The region shown in the figure above represents the boundary of a city that is bordered by a river and a highway. The population density of the city at a distance of x miles from the river is modeled by $D(x) = \dfrac{6}{\sqrt{x+16}}$, where $D(x)$ is measured in thousands of people per square mile. According to the model, which of the following expressions gives the total population, in thousands of the city?



(A) $\int_0^8 (4)\left(\dfrac{6}{\sqrt{x+16}}\right) dx$

(B) $\int_0^8 (4x)\left(\dfrac{6}{\sqrt{x+16}}\right) dx$

(C) $\int_0^8 \left(\dfrac{1}{4}x\right)\left(\dfrac{6}{\sqrt{x+16}}\right) dx$

(D) $\int_0^8 \left(\dfrac{1}{4}x+3\right)\left(\dfrac{6}{\sqrt{x+16}}\right) dx$

The population density of a circular region is given by $f(r) = 10 - 3\sqrt{r}$ people per square mile, where r is the distance from the center of the city, in miles. Which of the following expressions gives the number of people who live within a 3 mile radius from the center of the city?

(A) $\pi\int_0^3 r^2(10-3\sqrt{r})\, dr$

(B) $\pi\int_0^3 (r+3)^2(10-3\sqrt{r})\, dr$

(C) $2\pi\int_0^3 (r+3)(10-3\sqrt{r})\, dr$

(D) $2\pi\int_0^3 r(10-3\sqrt{r})\, dr$