

Using Task Features for Zero-Shot Knowledge Transfer in Lifelong Learning

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Abstract

Knowledge transfer between tasks can improve the performance of learned models, but requires an accurate estimate of the inter-task relationships to identify the relevant knowledge to transfer. These inter-task relationships are typically estimated based on training data for each task, which is inefficient in lifelong learning settings where the goal is to learn each consecutive task rapidly from as little data as possible. To reduce this burden, we develop a lifelong reinforcement learning method based on coupled dictionary learning that incorporates high-level task descriptors to model the inter-task relationships. We show that using task descriptors improves the performance of the learned task policies, providing both theoretical justification for the benefit and empirical demonstration of the improvement across a variety of dynamical control problems. Given only the descriptor for a new task, the lifelong learner is also able to accurately predict the task policy through zero-shot learning using the coupled dictionary, eliminating the need to pause to gather training data before addressing the task.

1 Introduction

Transfer and multi-task learning (MTL) methods reduce the amount of experience needed to train individual task models by reusing knowledge from other related tasks. These techniques typically select the relevant knowledge to transfer by modeling inter-task relationships, based on training data for each task [Baxter, 2000; Ando & Zhang, 2005; Bickel *et al.*, 2009; Maurer *et al.*, 2013]. However, this process requires sufficient training data for each task to identify these relationships before knowledge transfer can succeed.

Consider instead the human ability to rapidly bootstrap a model for a new task, given *only a high-level task description*, before obtaining experience on the actual task. For example, viewing only the image on the box of a new Ikea chair, we can immediately identify previous related assembly tasks and begin formulating a plan to assemble the chair. In the same manner, an experienced inverted pole balancing agent may be able to predict the controller for a new pole given its mass and length, prior to interacting with the physical system.

Inspired by this idea, we explore the use of high-level task descriptions to improve knowledge transfer between multiple machine learning tasks. We focus on lifelong learning scenarios [Thrun, 1996; Ruvoletto & Eaton, 2013], in which multiple tasks arrive consecutively and the goal is to rapidly learn each new task by building upon previous knowledge. Although we focus on reinforcement learning (RL) tasks in this paper, our approach extends easily to regression and classification.

Our algorithm, Task Descriptors for Lifelong Learning (TaDeLL), encodes the task descriptions as feature vectors that identify each task, treating these descriptors as side information in addition to training data on the individual tasks. This idea of using task features for knowledge transfer has been explored previously by Bonilla *et al.* [2007] in an offline batch MTL setting, and more recently by Sinapov *et al.* [2015] in a computationally expensive method for estimating transfer relationships between pairs of tasks. In comparison, our approach operates online over consecutive tasks and is much more computationally efficient.

We use *coupled dictionary learning* to model the inter-task relationships between both the task descriptions and the individual task policies in lifelong learning. The coupled dictionary enforces the notion that tasks with similar descriptions should have similar policies, but still allows dictionary elements the freedom to accurately represent the different task policies. We connect the coupled dictionary to the concept of mutual coherence in sparse coding, providing theoretical justification for why the task descriptors improve performance, and verify this improvement empirically.

In addition to improving the task policies, we show that the task descriptors enable the learner to accurately predict the policies for unseen tasks given only their description—this process of learning without data is known as *zero-shot learning*. This capability is particularly important in the online setting of lifelong learning, enabling the system to accurately predict policies for new tasks through transfer, without requiring it to pause to gather training data on each task.

2 Related Work

Batch MTL [Caruana, 1997] methods often model the relationships between tasks to determine the knowledge to transfer [Baxter, 2000; Ando & Zhang, 2005; Bickel *et al.*, 2009; Maurer *et al.*, 2013]. These techniques include modeling

a task distance metric [Ben-David *et al.*, 2007], using correlations to determine when transfer is appropriate [Wang *et al.*, 2014], or building models based on nearest neighbors [Parameswaran & Weinberger, 2010]. More recently, MTL has been extended to a lifelong learning setting, in which tasks arrive consecutively for regression and classification [Ruvolo & Eaton, 2013] and for reinforcement learning [Bou Ammar *et al.*, 2014; 2015]. However, all of these methods require training data for each task in order to assess their relationships and determine the knowledge to transfer.

Instead of relying solely on the tasks’ training data, several works have explored the use of high level task descriptors to model the inter-task relationships in MTL and transfer learning settings. Task descriptors have been used in combination with neural networks [Bakker & Heskes, 2003] to define a task-specific prior or to control the gating network between individual task clusters. Bonilla *et al.* [2007] explore similar techniques for multi-task kernel machines, using task features in combination with the data for a gating network over individual task experts or to augment the original task training data. These papers focus on multi-task classification and regression in batch settings where the system has access to the data and features for all tasks, in contrast to our study of task descriptors for lifelong learning over consecutive RL tasks.

In the work most similar to ours, Sinapov *et al.* [2015] use task descriptors to estimate the transferability between each pair of tasks for transfer learning. Given the descriptor for a new task, they identify the source task with the highest predicted transferability, and use that source task as a warm start in RL. Though effective, their approach is very computationally expensive, since they estimate the transferability for every task pair through repeated simulation. Their evaluation is also limited to a transfer learning setting, and they do not consider the effects of transfer over consecutive tasks or updates to the transferability model, as we do in the lifelong setting.

Our work is also related to the Simple Zero-Shot Learning (Simple ZSL) method by Romera-Paredes and Tor [2015], which learns a multi-class linear model, and factorizes the linear model parameters, assuming the descriptors are coefficients over a latent basis to reconstruct the models. Our approach assumes a more flexible relationship: that both the model parameters and task descriptors can be reconstructed from separate latent bases that are coupled together through their coefficients. In comparison to our lifelong learning approach, Simple ZSL operates in an offline multi-class setting.

3 Background

3.1 Reinforcement Learning

A reinforcement learning (RL) agent must select sequential actions in an environment to maximize its expected return. An RL task is typically formulated as a Markov Decision Process (MDP) $\langle \mathcal{X}, \mathcal{A}, P, R, \gamma \rangle$, where $\mathcal{X} \subseteq \mathbb{R}^d$ is the set of states, and \mathcal{A} is the set of actions that the agent may execute, $P : \mathcal{X} \times \mathcal{A} \times \mathcal{X} \rightarrow [0, 1]$ is the state transition probability describing the systems dynamics, $R : \mathcal{X} \times \mathcal{A} \times \mathcal{X} \rightarrow \mathbb{R}$ is the reward function, and $\gamma \in [0, 1)$ is the discount assigned to rewards over time. At time step h , the agent is in state $x_h \in \mathcal{X}$ and chooses an action $a \in \mathcal{A}$ ac-

cording to policy $\pi : \mathcal{X} \times \mathcal{A} \mapsto [0, 1]$, which is represented as a function defined by a vector of control parameters $\theta \in \mathbb{R}^n$. The agent then receives reward r_h according to R and transitions to state x_{h+1} according to P . This sequence of states, actions, and rewards is given as a trajectory $\tau = \{(x_1, a_1, r_1), \dots, (x_H, a_H, r_H)\}$ over a horizon H . The goal of RL is to find the optimal policy π^* with parameters θ^* that maximizes the expected reward. However, learning an individual task still requires numerous trajectories, motivating the use of transfer to reduce the number of interactions with the environment.

Policy Gradient (PG) methods [Sutton *et al.*, 1999], which we employ as our base learner, are a class of RL algorithms that are effective for solving high dimensional problems with continuous state and action spaces, such as robotic control [Peters & Schaal, 2008]. The goal of PG is to optimize the expected average return: $\mathcal{J}(\theta) = E \left[\frac{1}{H} \sum_{h=1}^H r_h \right] = \int_{\mathbb{T}} p_{\theta}(\tau) \mathfrak{R}(\tau) d\tau$, where \mathbb{T} is the set of all possible trajectories, the average reward on trajectory τ is given by $\mathfrak{R}(\tau) = \frac{1}{H} \sum_{h=1}^H r_h$, and $p_{\theta}(\tau) = P_0(x_1) \prod_{h=1}^H p(x_{h+1} | x_h, a_h) \pi(a_h | x_h)$ is the probability of τ under an initial state distribution $P_0 : \mathcal{X} \mapsto [0, 1]$. Most PG methods (e.g., episodic REINFORCE [Williams, 1992], PoWER [Kober & Peters, 2009], and Natural Actor Critic [Peters & Schaal, 2008]) optimize the policy by maximizing a lower bound on $\mathcal{J}(\theta)$, comparing trajectories generated by π_{θ} against those generated by a new candidate policy $\pi_{\tilde{\theta}}$. For details, see Kober & Peters [2009].

3.2 Lifelong Machine Learning

In a lifelong learning setting [Thrun, 1996; Ruvolo & Eaton, 2013], the learner faces multiple, consecutive tasks and must rapidly learn each new task by building upon its previous experience. The learner may encounter a previous task at any time, and so must optimize performance across all tasks seen so far. A priori, the agent does not know the total number of tasks T_{\max} , the task distribution, or the task order.

At time t , the lifelong learner encounters task $\mathcal{Z}^{(t)}$. In this paper, each task $\mathcal{Z}^{(t)}$ is specified by an MDP $\langle \mathcal{X}^{(t)}, \mathcal{A}^{(t)}, P^{(t)}, R^{(t)}, \gamma^{(t)} \rangle$, but the lifelong learning setting and our approach can equivalently handle classification or regression tasks. The agent will learn each task consecutively, acquiring training data (i.e., trajectories) in each task before advancing to the next. The agent’s goal is to learn the optimal policies $\{\pi_{\theta^{(1)}}, \dots, \pi_{\theta^{(T)}}\}$ with corresponding parameters $\{\theta^{(1)}, \dots, \theta^{(T)}\}$, where T is the number of unique tasks seen so far ($1 \leq T \leq T_{\max}$). Ideally, knowledge learned from previous tasks $\{\mathcal{Z}^{(1)}, \dots, \mathcal{Z}^{(T-1)}\}$ should accelerate training and improve performance on each new task $\mathcal{Z}^{(T)}$. Also, the lifelong learner should scale effectively to large numbers of tasks, learning each new task rapidly from minimal data.

The Efficient Lifelong Learning Algorithm (ELLA) [Ruvolo & Eaton, 2013] and PG-ELLA [Bou Ammar *et al.*, 2014] were developed to operate in this lifelong learning setting for classification/regression and RL tasks, respectively. Both approaches assume the parameters for each task model can be factorized using a shared knowledge base L , facilitating

transfer between tasks. Specifically, the model parameters for task t are given by $\theta^{(t)} = \mathbf{L}\mathbf{s}^{(t)}$, where $\mathbf{L} \in \mathbb{R}^{d \times k}$ is the shared basis over the model space, and $\mathbf{s}^{(t)} \in \mathbb{R}^k$ are the sparse coefficients over the basis. This factorization has been effective for transfer in both lifelong and multi-task learning [Kumar & Daumé, 2012; Maurer *et al.*, 2013].

Under this assumption, the MTL objective for PG is:

$$\min_{\mathbf{L}, \mathbf{S}} \frac{1}{T} \sum_t \left[-\mathcal{J}(\theta^{(t)}) + \mu \|\mathbf{s}^{(t)}\|_1 \right] + \lambda \|\mathbf{L}\|_F^2, \quad (1)$$

where $\mathbf{S} = [\mathbf{s}^{(1)} \dots \mathbf{s}^{(T)}]$ is the matrix of sparse vectors, $\mathcal{J}(\theta^{(t)})$ is the PG objective for task t , $\|\cdot\|_F$ is the Frobenius norm, the L_1 norm is used to approximate the true vector sparsity of $\mathbf{s}^{(t)}$, and μ and λ are tunable regularization parameters. To solve this objective in a lifelong learning setting, Bou Ammar *et al.* [2014] approximate the multi-task objective by first substituting in the lower-bound to the PG objective, then taking a second-order Taylor expansion of the objective around an estimate $\alpha^{(t)}$ of the single-task policy parameters for each task t , and updating only the coefficients $\mathbf{s}^{(t)}$ for the current task at each time step. This process reduces the MTL objective to the problem of sparse coding the single-task policies in the shared basis \mathbf{L} , and enables \mathbf{S} and \mathbf{L} to be solved efficiently by the following online update rules that constitute PG-ELLA [Bou Ammar *et al.*, 2014]:

$$\mathbf{s}^{(t)} \leftarrow \arg \min_{\mathbf{s}} \|\alpha^{(t)} - \mathbf{L}\mathbf{s}\|_{\Gamma^{(t)}}^2 + \mu \|\mathbf{s}\|_1 \quad (2)$$

$$\mathbf{A} \leftarrow \mathbf{A} + (\mathbf{s}^{(t)} \mathbf{s}^{(t)\top}) \quad (3)$$

$$\mathbf{b} \leftarrow \mathbf{b} + \text{vec} \left(\mathbf{s}^{(t)\top} \otimes \left(\alpha^{(t)\top} \Gamma^{(t)} \right) \right) \quad (4)$$

$$\mathbf{L} \leftarrow \text{mat} \left(\left(\frac{1}{T} \mathbf{A} + \lambda \mathbf{I}_{k \times d, k \times d} \right)^{-1} \frac{1}{T} \mathbf{b} \right), \quad (5)$$

where $\|\mathbf{v}\|_{\mathbf{A}}^2 = \mathbf{v}^\top \mathbf{A} \mathbf{v}$, $\Gamma^{(t)}$ is the Hessian of the PG lower bound on $\mathcal{J}(\alpha^{(t)})$, and \mathbf{A} , \mathbf{b} are initialized to zero matrices.

Though effective for lifelong learning, this approach requires significant training data to estimate the policy for each new task before the learner can solve it. We eliminate this restriction by incorporating task descriptors into lifelong learning, enabling zero-shot transfer to new tasks.

4 Task Descriptors

While most MTL and lifelong learning methods use task training data to model inter-task relationships, high level descriptions can describe task differences. For example, in multi-task medical domains, patients are often grouped into tasks by demographic data and disease presentation [Oyen & Lane, 2012]. In control problems, the dynamical system parameters (e.g., the spring, mass, and damper constants in a spring-mass-damper system) describe the task. Descriptors can also be derived from external sources, such as Wikipedia [Huang *et al.*, 2012; Pennington *et al.*, 2014]. Such task descriptors have been used extensively for zero-shot learning [Palatucci *et al.*, 2009; Socher *et al.*, 2013].

Formally, we assume that each task $\mathcal{Z}^{(t)}$ has an associated descriptor $\mathbf{m}^{(t)}$ that is given to the learner upon first presentation of the task. The learner has no knowledge of future

tasks, or the distribution of task descriptors. The descriptor is represented by a feature vector $\phi(\mathbf{m}^{(t)}) \in \mathbb{R}^{d_m}$, where $\phi(\cdot)$ performs feature extraction and (possibly) a non-linear basis transformation on the features. We make no assumptions on the uniqueness of $\phi(\mathbf{m}^{(t)})$, although in general tasks will have different descriptors.¹ In addition, each task also has associated training data $\mathbf{X}^{(t)}$ to learn the model; in the case of RL tasks, the data consists of trajectories that are dynamically acquired by the agent through experience in the environment.

5 Lifelong Learning with Task Descriptors

We incorporate task descriptors into lifelong learning via sparse coding with a coupled dictionary, enabling the descriptors and learned policies to augment each other. Although we focus on RL tasks, our method can easily be adapted to classification or regression, as described in the Appendix.²

5.1 Coupled Dictionary Optimization

As described previously, many multi-task and lifelong learning approaches have found success with factorizing the policy parameters $\theta^{(t)} = \mathbf{L}\mathbf{s}^{(t)}$. In effect, each column of the shared basis \mathbf{L} serves as a reusable policy component representing a cohesive chunk of knowledge. In lifelong learning, the basis \mathbf{L} is refined over time as the system learns more tasks. The coefficient vectors $\mathbf{S} = [\mathbf{s}^{(1)} \dots \mathbf{s}^{(T)}]$ encode the task policies in this shared basis, providing an embedding of the tasks based on how their policies share knowledge.

We make a similar assumption about the task descriptors—that the descriptor features $\phi(\mathbf{m}^{(t)})$ can be linearly factorized³ using a latent basis $\mathbf{D} \in \mathbb{R}^{d_m \times k}$ over the descriptor space. This basis captures relationships among the descriptors, with coefficients that similarly embed tasks based on commonalities in their descriptions. From a co-view perspective [Yu *et al.*, 2014], both the policies and descriptors provide information about the task, and so each can augment the learning of the other. Each underlying task is common to both views, and so we seek to find task embeddings that are consistent for *both* the policies and their corresponding task descriptors. We can enforce this by coupling the two bases \mathbf{L} and \mathbf{D} , sharing the same coefficient vectors \mathbf{S} to reconstruct both the policies and descriptors. Therefore, for task $\mathcal{Z}^{(t)}$,

$$\theta^{(t)} = \mathbf{L}\mathbf{s}^{(t)} \quad \phi(\mathbf{m}^{(t)}) = \mathbf{D}\mathbf{s}^{(t)}. \quad (6)$$

To optimize the coupled bases \mathbf{L} and \mathbf{D} during the lifelong learning process, we employ techniques for coupled dictionary optimization from the sparse coding literature [Yang *et al.*, 2010], which optimizes the dictionaries for multiple feature spaces that share a joint sparse representation. This notion of coupled dictionary learning has led to high performance algorithms for image super-resolution [Yang *et al.*,

¹This raises the question of what descriptive features to use, and how task performance will change if some descriptive features are unknown. We explore these issues empirically in the Appendix.

²The Appendix is available online at <https://goo.gl/HibKQ3>

³This is potentially non-linear w.r.t $\mathbf{m}^{(t)}$, since ϕ can be non-linear.

Algorithm 1 TaDeLL (k, λ, μ)

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1:  $T \leftarrow 0$ 
2:  $\mathbf{L} \leftarrow \text{RandomMatrix}_{d,k}$ ,  $\mathbf{D} \leftarrow \text{RandomMatrix}_{m,k}$ 
3: while some task  $(\mathcal{Z}^{(t)}, \phi(\mathbf{m}^{(t)}))$  is available do
4:   if isNewTask( $\mathcal{Z}^{(t)}$ ) then
5:      $T \leftarrow T + 1$ 
6:      $\mathbb{T}^{(t)} \leftarrow \text{sampleRandomTrajectories}(\mathcal{Z}^{(t)})$ 
7:   else
8:      $\mathbb{T}^{(t)} \leftarrow \text{sampleTrajectories}(\mathcal{Z}^{(t)}, \pi_{\alpha^{(t)}})$ 
9:   end if
10:  Compute  $\alpha^{(t)}$  and  $\Gamma^{(t)}$  from  $\mathbb{T}^{(t)}$ 
11:   $\mathbf{s}^{(t)} \leftarrow \arg \min_{\mathbf{s}} \|\beta^{(t)} - \mathbf{K}\mathbf{s}\|_{\mathbf{A}^{(t)}}^2 + \mu \|\mathbf{s}\|_1$ 
12:   $\mathbf{L} \leftarrow \text{updateL}(\mathbf{L}, \mathbf{s}^{(t)}, \alpha^{(t)}, \Gamma^{(t)})$ 
13:   $\mathbf{D} \leftarrow \text{updateD}(\mathbf{D}, \mathbf{s}^{(t)}, \phi(\mathbf{m}^{(t)}), \gamma \mathbf{I}_{d_m, d_m})$ 
14:  for  $t \in \{1, \dots, T\}$  do:  $\theta^{(t)} \leftarrow \mathbf{L}\mathbf{s}^{(t)}$ 
15: end while

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2010], allowing the reconstruction of high-res images from low-res samples, and for multi-modal retrieval [Zhuang *et al.*, 2013], cross-domain image recognition [Huang & Wang, 2013], and cross-domain retrieval [Yu *et al.*, 2014].

Given the factorization in Eq. (6), we can re-formulate the multi-task objective (Eq. 1) for the coupled dictionaries as

$$\min_{\mathbf{L}, \mathbf{D}, \mathbf{S}} \frac{1}{T} \sum_t \left[-\mathcal{J}(\theta^{(t)}) + \gamma \left\| \phi(\mathbf{m}^{(t)}) - \mathbf{D}\mathbf{s}^{(t)} \right\|_2^2 + \mu \left\| \mathbf{s}^{(t)} \right\|_1 \right] + \lambda (\|\mathbf{L}\|_F^2 + \|\mathbf{D}\|_F^2), \quad (7)$$

where γ balances the policy’s fit to the task descriptor’s fit.

To solve Eq. 7 online, we approximate $\mathcal{J}(\cdot)$ by a second-order Taylor expansion around $\alpha^{(t)}$, the minimizer for the PG lower bound of $\mathcal{J}(\cdot)$ (i.e., $\pi_{\alpha^{(t)}}$ is the single-task policy for $\mathcal{Z}^{(t)}$ based on the observed trajectories), following Bou Ammar *et al.* [2014]. This simplifies Eq. (7) to

$$\min_{\mathbf{L}, \mathbf{D}, \mathbf{S}} \frac{1}{T} \sum_t \left[\left\| \alpha^{(t)} - \mathbf{L}\mathbf{s}^{(t)} \right\|_{\Gamma^{(t)}}^2 + \gamma \left\| \phi(\mathbf{m}^{(t)}) - \mathbf{D}\mathbf{s}^{(t)} \right\|_2^2 + \mu \left\| \mathbf{s}^{(t)} \right\|_1 \right] + \lambda (\|\mathbf{L}\|_F^2 + \|\mathbf{D}\|_F^2). \quad (8)$$

We can merge the first two terms inside the sum by choosing:

$$\beta^{(t)} = \begin{bmatrix} \alpha^{(t)} \\ \phi(\mathbf{m}^{(t)}) \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{L} \\ \mathbf{D} \end{bmatrix}, \quad \mathbf{A}^{(t)} = \begin{bmatrix} \Gamma^{(t)} & \mathbf{0} \\ \mathbf{0} & \gamma \mathbf{I}_{d_m, d_m} \end{bmatrix},$$

where $\mathbf{0}$ and \mathbf{I} are the zero and $d_m \times d_m$ identity matrices, respectively, allowing us to re-write Eq. (8) concisely as

$$\min_{\mathbf{K}, \mathbf{S}} \frac{1}{T} \sum_t \left[\left\| \beta^{(t)} - \mathbf{K}\mathbf{s}^{(t)} \right\|_{\mathbf{A}^{(t)}}^2 + \mu \left\| \mathbf{s}^{(t)} \right\|_1 \right] + \lambda \|\mathbf{K}\|_F^2. \quad (9)$$

This objective can now be solved efficiently online, as a series of per-task update rules given in Algorithm 1. \mathbf{L} and \mathbf{D} are updated independently using Equations 3–5, following a recursive construction based on an eigenvalue decomposition.

The complete implementation of our approach is available online at [URL_SUPPRESSED_FOR_BLIND_REVIEW](#).

Algorithm 2 Zero-Shot Transfer to a New Task $\mathcal{Z}^{(t_{new})}$

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1: Inputs: task descriptor  $\mathbf{m}^{(t_{new})}$ , learned bases  $\mathbf{L}$  and  $\mathbf{D}$ 
2:  $\tilde{\mathbf{s}}^{(t_{new})} \leftarrow \arg \min_{\mathbf{s}} \left\| \phi(\mathbf{m}^{(t_{new})}) - \mathbf{D}\mathbf{s} \right\|_2^2 + \mu \|\mathbf{s}\|_1$ 
3:  $\tilde{\theta}^{(t_{new})} \leftarrow \mathbf{L}\tilde{\mathbf{s}}^{(t_{new})}$ 
4: Return:  $\pi_{\tilde{\theta}^{(t_{new})}}$ 

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5.2 Zero-Shot Transfer Learning

In a lifelong setting, when faced with a new task, the agent’s goal is to learn an effective policy for that task as quickly as possible. At this stage, previous multi-task and lifelong learners incurred a delay before they could produce a decent policy, since they needed to acquire data from the new task in order to identify related knowledge and train the new policy.

Incorporating task descriptors enables our approach to predict a policy for the new task immediately, given *only* the descriptor. This ability to perform zero-shot learning is provided by the use of coupled dictionary learning, which allows us to observe a data instance in one feature space (i.e., the task descriptor), and then recover its underlying latent signal in the other feature spaces (i.e., the policy parameters) using the dictionaries and sparse coding [Yang *et al.*, 2010].

Given only the descriptor $\mathbf{m}^{(t_{new})}$ for a new task $\mathcal{Z}^{(t_{new})}$, we can estimate the embedding of the task in the latent descriptor space via LASSO on the learned dictionary \mathbf{D} :

$$\tilde{\mathbf{s}}^{(t_{new})} \leftarrow \arg \min_{\mathbf{s}} \left\| \phi(\mathbf{m}^{(t)}) - \mathbf{D}\mathbf{s} \right\|_2^2 + \mu \|\mathbf{s}\|_1. \quad (10)$$

Since the estimate given by $\mathbf{s}^{(t_{new})}$ also serves as the coefficients over the latent policy space \mathbf{L} , we can immediately predict a policy for the new task as: $\tilde{\theta}^{(t_{new})} = \mathbf{L}\tilde{\mathbf{s}}^{(t_{new})}$. This zero-shot transfer learning procedure is given as Algorithm 2.

5.3 Theoretical Analysis

This section discusses why incorporating task descriptors through coupled dictionaries can improve performance of the learned policies and enable zero-shot transfer to new tasks. In the Appendix², we also prove the convergence of TaDeLL. A full sample complexity analysis is beyond the scope of this paper, and, indeed, remains an open problem for zero-shot learning [Romera-Paredes & Torr, 2015].

To analyze the policy improvement, since the policy parameters are factored as $\theta^{(t)} = \mathbf{L}\mathbf{s}^{(t)}$, we proceed by showing that incorporating the descriptors through coupled dictionaries can improve both \mathbf{L} and \mathbf{S} . In this analysis, we employ the concept of *mutual coherence*, which has been studied extensively in the sparse recovery literature. Mutual coherence measures the similarity of a dictionary’s elements as $M(\mathbf{Q}) = \max_{1 \leq i \neq j \leq n} \left(\frac{|\mathbf{q}_i^\top \mathbf{q}_j|}{\|\mathbf{q}_i\|_2 \|\mathbf{q}_j\|_2} \right)$, where \mathbf{q}_i is the i^{th} column of a dictionary $\mathbf{Q} \in \mathbb{R}^{d \times k}$. If $M(\mathbf{Q}) = 0$, then \mathbf{Q} is an invertible orthogonal matrix and so sparse recovery can be solved directly by inversion; $M(\mathbf{Q}) = 1$ implies that \mathbf{Q} is not full rank and a poor dictionary. Intuitively, low mutual coherence indicates that the dictionary’s columns are considerably different, and thus such a “good” dictionary can represent many different policies, potentially yielding more knowledge transfer. This intuition is shown in the following theorem:

Theorem 5.1. [Donoho et al., 2006] Suppose we have noisy observations $\hat{\theta}$ of the linear system $\theta = Qs$, such that $\|\hat{\theta} - \theta\|_2 \leq \epsilon$. Let s^* be a solution to the system, and let $K = \|s\|_0$. If $K < 0.5(1 + M(Q)^{-1})$, then s^* is the unique sparsest solution of the system. Moreover, if s^+ is the LASSO solution for the system from the noisy observations, then: $\|s^* - s^+\|_2^2 \leq \frac{4\epsilon^2}{1 - M(Q)(4K-1)}$.

Therefore, an L with low mutual coherence would lead to more stable solutions of the $\theta^{(t)}$'s against inaccurate single-task estimates of the policies (the $\alpha^{(t)}$'s). We next show that our approach likely lowers the mutual coherence of L .

TaDeLL alters the problem from training L to training the coupled dictionaries L and D (contained in K). Let $s^{*(t)}$ be the solution to Eq. 1 for task t , which is unique under sparse recovery theory, so $\|s^{*(t)}\|_0$ remains unchanged for all tasks. Theorem 5.1 implies that, if $M(K) < M(L)$, coupled dictionary learning can help with a more accurate recovery of the $s^{(t)}$'s. To show this, we note that Eq. 7 can also be derived as a result of an MAP estimate from a Bayesian perspective, enforcing a Laplacian distribution on the $s^{(t)}$'s and assuming L to be a Gaussian matrix with elements drawn i.i.d: $l_{ij} \sim \mathcal{N}(0, \sigma^2)$. When considering such a random matrix $Q \in \mathbb{R}^{d \times k}$, Donoho & Huo [2001] proved that asymptotically $M(Q) \propto \sqrt{\frac{\log(dk)}{d}}$ as $d \rightarrow \infty$. Using this as an estimate for $M(L)$ and $M(K)$, since d is finite and incorporating task descriptors increases d , most likely $M(K) < M(L)$, implying that TaDeLL learns a superior dictionary. Moreover, if $M(D) \leq M(L)$, the theorem implies we can use D alone to recover the task policies through zero-shot learning.

To show that task features can also improve the sparse recovery, we rely on the following theorem about LASSO:

Theorem 5.2. [Negahban et al., 2009] Let s^* be a unique solution to the system $\theta = Qs$ with $\|s\|_0 = K$ and $Q \in \mathbb{R}^{d \times k}$. If s^+ is the LASSO solution for the system from noisy observations, then with high probability: $\|s^* - s^+\|_2 \leq c' \sqrt{K \frac{\log k}{d}}$, where the constant $c' \in \mathbb{R}^+$ depends on properties of the linear system and observations.

This theorem shows that the error reconstruction for LASSO is proportional to $\frac{1}{\sqrt{d}}$. When we incorporate the descriptor through $\beta^{(t)}$, the RHS denominator increases from d to $(d + d_m)$ while K and k remain constant, yielding a tighter fit. Therefore, task descriptors can improve learned dictionary quality and sparse recovery accuracy. To ensure an equivalently tight fit for $s^{(t)}$ using either policies or descriptors, Theorem 5.2 suggests it should be that $d_m \geq d$ to ensure that zero-shot learning yields similarly tight estimates of $s^{(t)}$.

Computational Complexity Each update begins with one PG step to update $\alpha^{(t)}$ and $\Gamma^{(t)}$ at a cost of $O(\xi(d, n_t))$, where $\xi(\cdot)$ depends on the base PG learner and n_t is the number of trajectories obtained for task $\mathcal{Z}^{(t)}$. The cost of updating L and $s^{(t)}$ alone is $O(k^2 d^3)$ [Ruvolo & Eaton, 2013], and so the cost of updating K through coupled dictionary learning is $O(k^2(d + d_m)^3)$. This yields an overall per-update cost of $O(k^2(d + d_m)^3 + \xi(d, n_t))$, which is independent of T .

6 Experiments

We evaluated our method on learning control policies for three benchmark systems and an application to quadrotors.

6.1 Benchmark Dynamical Systems

Spring Mass Damper (SM) The SM system is described by three parameters: the spring constant, mass, and damping constant. The system's state is given by the position and velocity of the mass. The controller applies a force to the mass, attempting to stabilize it to a given position.

Cart Pole (CP) The CP system involves balancing an inverted pendulum by applying a force to the cart. The system is characterized by the cart and pole masses, pole length, and a damping parameter. The states are the position and velocity of the cart and the angle and rotational velocity of the pole.

Bicycle (BK) This system focuses on keeping a bicycle balanced upright as it rolls along a horizontal plane at constant velocity. The system is characterized by the bicycle mass, x - and z -coordinates of the center of mass, and parameters relating to the shape of the bike (the wheelbase, trail, and head angle). The state is the bike's tilt and its derivative; the actions are the torque applied to the handlebar and its derivative.

6.2 Methodology

In each domain we generated 40 tasks, each with different dynamics, by varying the system parameters. The reward for each task was taken to be the distance between the current state and the goal. For lifelong learning, tasks were encountered consecutively with repetition, and learning proceeded until each task had been seen at least once. We used the same random task order between methods to ensure fair comparison. The learners sampled trajectories of 100 steps, and the learning session during each task presentation was limited to 30 iterations. For MTL, all tasks were presented simultaneously. We used Natural Actor Critic [Peters & Schaal, 2008] as the base learner for the benchmark systems and episodic REINFORCE [Williams, 1992] for quadrotor control. We chose k and the regularization parameters independently for each domain to optimize the combined performance of all methods on 20 held-out tasks, and set $\gamma = \text{mean}(\text{diag}(\gamma^{(t)}))$ to balance the fit to the descriptors and the policies. We measured learning curves based on the final policies for each of the 40 tasks, averaging results over seven trials. The system parameters for each task were used as the task descriptor features $\phi(m)$; we also tried several non-linear transformations as $\phi(\cdot)$, but found the linear features worked well.

6.3 Results on Benchmark Systems

Figure 1 compares our TaDeLL approach for lifelong learning with task descriptors to 1.) PG-ELLA [Bou Ammar et al., 2014], which does not use task features, 2.) GO-MTL [Kumar & Daumé, 2012], the MTL optimization of Eq. 1, and 3.) single-task learning using PG. For comparison, we also performed an offline MTL optimization of Eq. 7 via alternating optimization, and plot the results as TaDeMTL. The shaded regions on the plots denote standard error bars.

We see that task descriptors improves lifelong learning on every system, even driving performance to a level that is unachievable from training the policies from experience alone

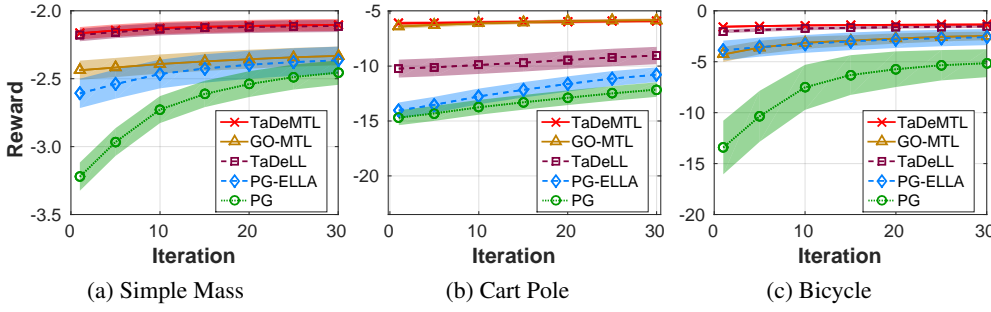


Figure 1: Performance of multi-task (solid lines), lifelong (dashed), and single-task learning (dotted) on benchmark dynamical systems.

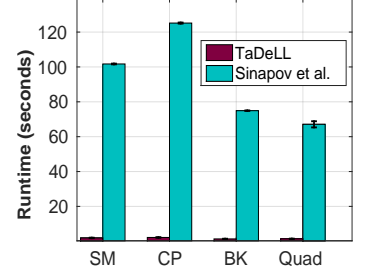


Figure 2: Runtime comparison.

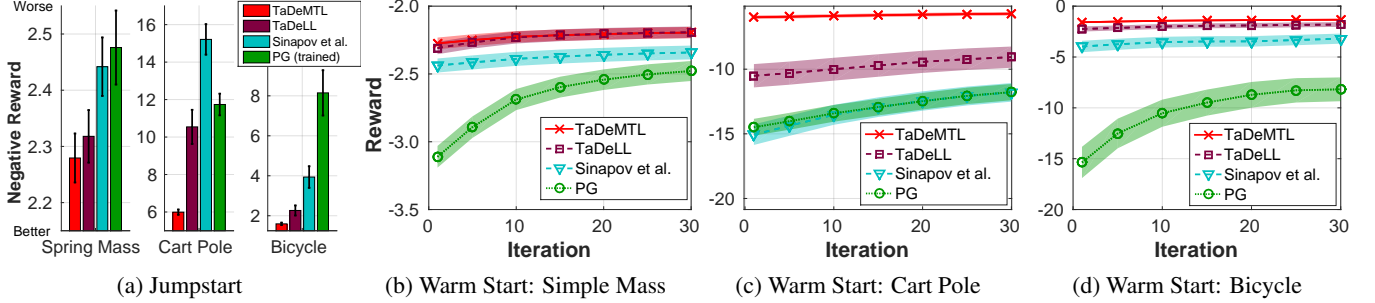


Figure 3: Zero-shot transfer to new tasks. Figure (a) shows the initial “jumpstart” improvement on each task domain; Figures (b)–(d) depict the result of using zero-shot policies as warm start initializations for PG in order to further improve results.

via GO-MTL in the SM and BK domains. The difference between TaDeLL and TaDeMTL is also negligible for all domains except CP (which had very diverse tasks), demonstrating the effectiveness of our online optimization.

Figure 3 shows that task descriptors are effective for zero-shot transfer to new tasks. We generated an additional 40 tasks for each domain to measure zero-shot performance, averaging results over these new tasks. Figure 3a shows that our approach improves the initial performance (i.e., the “jumpstart” [Taylor & Stone, 2009]) on new tasks, outperforming Sinapov et al. [2015]’s method and single-task PG, which was allowed to train on the task. We attribute the especially poor performance of Sinapov et al. on CP to the fact that the CP policies differ substantially; in domains where the source policies are vastly different from the target policies, Sinapov et al.’s algorithm does not have an appropriate source to transfer. Their approach is also much more computationally expensive (quadratic in the number of tasks) than our approach (linear in the number of tasks), as shown in Figure 2; details of the runtime experiments are included in the Appendix². Figures 3b–3d shows that the zero-shot policies can be used effectively as a warm start initialization for a PG learner, which is then allowed to improve the policy.

6.4 Application to Quadrotor Control

We also applied our approach to the more challenging domain of quadrotor control, focusing on zero-shot transfer to new stability tasks. To ensure realistic dynamics, we use the model of Bouabdallah and Siegwart [2005], which has been verified on physical systems. The quadrotors are character-

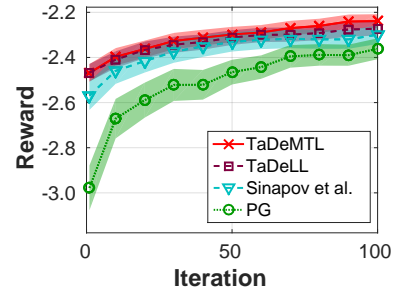


Figure 4: Warm start learning on quadrotor control.

ized by three inertial constants and the arm length, with their state consisting of roll/pitch/yaw and their derivatives.

Figure 4 shows the results of our application, demonstrating that TaDeLL can predict a controller for new quadrotors through zero-shot learning that has equivalent accuracy to PG, which had to train on the system. As with the benchmarks, TaDeLL is effective for warm start learning with PG.

7 Conclusion

We proposed a coupled dictionary method for incorporating task descriptors into lifelong learning. Using task descriptors not only improves the performance of the learned policies, but enables us to predict policies for new tasks without seeing any training data through zero-shot transfer. Experiments show that our method outperforms other approaches on a variety of dynamical system control problems, and requires substantially less computational time than similar methods.

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Online Appendix to “Using Task Features for Zero-Shot Knowledge Transfer in Lifelong Learning”

Submitted for blind review to IJCAI 2016

Note to reviewers: If the main paper is accepted for publication, this Appendix will be either included as supplemental conference material, or posted online to a URL described in the main paper.

A Appendix

This appendix provides further details on the approach, theory, and experiments described in the main paper.

A.1 Extension to Classification and Regression

Although we derived our approach for RL tasks, TaDeLL can easily be adapted to classification or regression by the following modifications to Algorithm 1:

1. Instead of sampling trajectories from the environment, each time step TaDeLL would observe a labeled training set $(\mathbf{X}^{(t)}, \mathbf{y}^{(t)})$ for task $\mathcal{Z}^{(t)}$, where $\mathbf{X}^{(t)} \subseteq \mathbb{R}^{n_t \times d}$. For classification tasks, $\mathbf{y}^{(t)} \in \{+1, -1\}^{n_t}$, and for regression tasks, $\mathbf{y}^{(t)} \in \mathbb{R}^{n_t}$.
2. Set $\alpha^{(t)}$ to be the parameters of a single-task model trained via classification or regression (e.g., logistic or linear regression) on that data set, as described by Ruvolo and Eaton [2013].
3. Set $\Gamma^{(t)}$ to be the Hessian of the corresponding loss function around the single-task solution $\alpha^{(t)}$ (available in [Ruvolo & Eaton, 2013]), and
4. Using the reconstructed $\theta^{(t)}$ as the model parameters for the corresponding classification or regression model for that task.

This modification procedure would support any parametric classification or regression base learner, such as logistic or linear regression, where the loss function is second-order differentiable in order to compute the Hessian.

A.2 Theoretical Convergence of TaDeLL

In this section, we prove theoretically the convergence of TaDeLL, showing that the learned dictionaries become increasingly stable as it learns more tasks. **We build upon key theoretical results from Bou Ammar et al. [2014] and Ruvolo & Eaton [2013], demonstrating that these results apply to coupled dictionary learning with task descriptors, and use them to prove convergence.**

Let $\hat{g}_T(\mathbf{L})$ represent the sparse coded approximation to the MTL objective, which can be defined as:

$$\hat{g}_T(\mathbf{L}) = \frac{1}{T} \sum_{t=1}^T \|\alpha^{(t)} - \mathbf{L} \mathbf{s}^{(t)}\|_{\Gamma^{(t)}}^2 + \mu \|\mathbf{s}^{(t)}\|_1 + \lambda \|\mathbf{L}\|_F^2.$$

This equation can be viewed as the cost for \mathbf{L} when the sparse coefficients are kept constant. Let \mathbf{L}_T be the version of the dictionary \mathbf{L} obtained after observing T tasks. Given these definitions, we consider the following theorem:

Theorem A.1. [Ruvolo & Eaton, 2013]

1. The trained dictionary \mathbf{L} is stabilized over learning with rate: $\mathbf{L}_T - \mathbf{L}_{T-1} = O(\frac{1}{T})$
2. $\hat{g}_T(\mathbf{L}_T)$ converges almost surely.
3. $\hat{g}_T(\mathbf{L}_T) - \hat{g}_T(\mathbf{L}_{T-1})$ converges almost surely to zero.


This theorem requires two conditions:

1. The tuples $\Gamma^{(t)}, \alpha^{(t)}$ are drawn i.i.d from a distribution with compact support to bound the norms of \mathbf{L} and $\mathbf{s}^{(t)}$.
2. For all t , let \mathbf{L}_κ be the subset of the dictionary \mathbf{L}_t , where only columns corresponding to non-zero element of $\mathbf{s}^{(t)}$ are included. Then, all eigenvalues of the matrix $\mathbf{L}_\kappa^T \Gamma^{(t)} \mathbf{L}_\kappa$ need to be strictly positive.

Bou Ammar et al. [2014] show that both of these conditions are met for the lifelong learning framework given in Eqs. 2–5. When we incorporate the task descriptors into this framework, we alter $\alpha^{(t)} \rightarrow \beta^{(t)}$, $\mathbf{L} \rightarrow \mathbf{K}$, and $\Gamma^{(t)} \rightarrow \mathbf{A}^{(t)}$. Note both $\beta^{(t)}$ and $\mathbf{A}^{(t)}$ are formed by adding deterministic entries and thus can be considered to be drawn i.i.d (because $\Gamma^{(t)}$ and $\alpha^{(t)}$ are assumed to be drawn i.i.d). Therefore, incorporating task descriptors does not violate Condition 1.

To show that Condition 2 holds, if we analogously form \mathbf{K}_κ , then eigenvalues of \mathbf{K}_κ are strictly positive because its eigenvalues are either eigenvalues of \mathbf{L} (which are strictly positive according to [Bou Ammar et al., 2014]) or the regularizing parameter γ by definition. Thus, both conditions are met and convergence follows directly from Theorem A.1.

A.3 Parameter Analysis

 **Eric:** This section still needs cleaning.

We examine whether or not it is sufficient to use a subset of task features when learning our coupled dictionary. This addresses issues of how performance will be affected if all the parameters are not known and only partial information is available. In figure 5 we look at how the number and selection of parameters affects the performance of learning. Specifically we use the jumpstart measure for spring mass damper systems, using all possible combinations of mass M , damping constant D , and spring constant K for the task features. Our results show that more features is better.

To analyze the dependency of our approach on the number of tasks being used to train the dictionary, we vary the

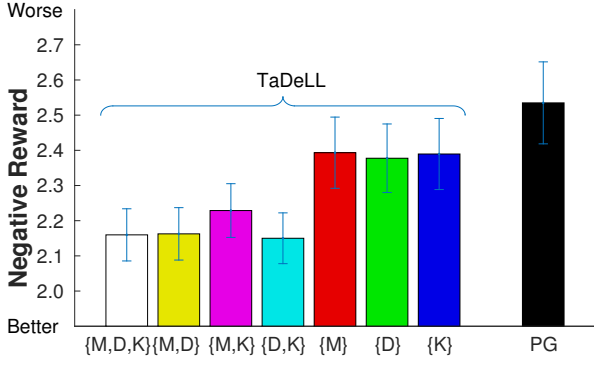


Figure 5: Performance varying the number of task features.

number of tasks and look at the quality of the zero-shot performance. While in the case of even a few tasks both the multi-task and lifelong learning approaches improve performance of standard policy gradients, however when it comes to building a dictionary that is able to predict a good model, more tasks is better. As we would expect, Figure 6 shows that performance improves with the number of tasks used to train the dictionaries.

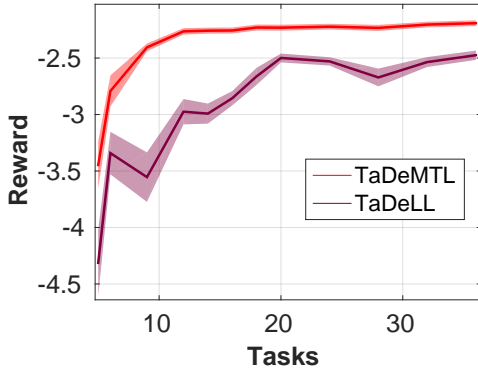


Figure 6: Zero-shot performance as a function of the number of tasks used to train the dictionary. As more tasks are used, the performance of the zero-shot transfer improves.

To improve our understanding of the task relationships in each domain, Figure 7 plots the first two parameters in the learned policy. We see that the learned cart pole policies are much more diverse than the learned policies for the other tasks.

A.4 Computational Efficiency

This section provides details on the runtime experiments, which were included as Figure 2 in the main paper.

We compared the average per-task runtime of our approach to that of Sinapov et al. [2015], the most closely related method to our approach. Since Sinapov et al.’s method requires training transferability predictors between all pairs of tasks, its total runtime grows quadratically with the number of tasks. In comparison, our online algorithm is highly efficient. As shown in Section 5.3, the per-update cost of TaDeLL is

$O(k^2(d+m)^3 + \xi(d, n_t))$. Note that this per-update cost is independent of the number of tasks T , giving TaDeLL a total runtime that scales linearly in the number of tasks.

Figure 2 shows the per-task runtime for each algorithm based on a set of 40 tasks, as evaluated on an Intel Core I7-4700HQ CPU. TaDeLL samples tasks randomly with replacement and terminates once every task has been seen. For the method of Sinapov et al., we used 10 PG iterations for calculating the warm start benefit, ensuring fair comparison between the methods. These results show a substantial reduction in computational time for TaDeLL: two orders of magnitude over the 40 tasks.

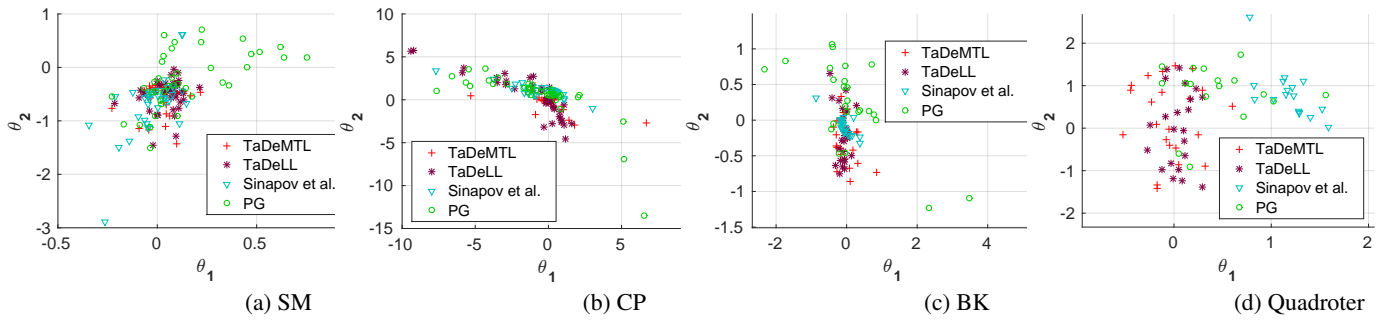


Figure 7: Learned Policies. We visualize the first two parameters of the policies learned for each task, revealing that the policies for cart pole are more diverse than for the other two problems. This also shows that the different methods often learn very different policies.