FairIRT_Adult_race_new

October 14, 2024

[1]:
```
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.colors as mcolors

from scipy import stats
from matplotlib import rcParams

from irt import Beta3
import matplotlib.pyplot as plt
```

[2]:
```
pij = pd.read_csv('./Audlt_STS_race.csv')
pij.set_index(pij.columns[0], inplace=True)

random_seed = 42
pij = pij.sample(n=1000, random_state=random_seed)
```

[3]:
```
array = pij.values.flatten()

transformed_data, best_lambda = stats.boxcox(array)
transformed_array = transformed_data.reshape(pij.shape)

res = pd.DataFrame(transformed_array, index=pij.index, columns=pij.columns)

array = res.values

min_val = np.min(array)
max_val = np.max(array)
normalized_array = (array - min_val) / (max_val - min_val)

normalized_df = pd.DataFrame(normalized_array, index=pij.index, columns=pij.
 ↪columns)
```

[4]:
```
def ICC_function(abilities, difficulties, discriminations):
    a = ((1-abilities)/ abilities)
    b = (difficulties / (1-difficulties))
    c = a*b
    d = c**discriminations
```

```
    return (1 / (d+1))
```

[5]:
```
b4 = Beta3(
        learning_rate=100,
        epochs=10000,
        n_respondents=normalized_df.shape[1],
        n_items=normalized_df.shape[0],
        n_workers=-1,
        random_seed=1,
    )
b4.fit(normalized_df.values)
```

```
100%|      | 10000/10000 [00:16<00:00, 599.94it/s]
```

[5]:
```
<irt.Beta3 at 0x342d85be0>
```

[6]:
```
new_pij = pd.DataFrame(index=range(1000), columns=range(24))

for i in range(1000):
    for j in range(14):
        alpha = (b4.abilities[j] / b4.difficulties[i]) ** b4.discriminations[i]
        beta_val = ((1 - b4.abilities[j]) / (1 - b4.difficulties[i])) ** b4.
 ↪discriminations[i]
        new_pij.iloc[i, j] = (alpha)/(alpha+beta_val)
```

[7]:
```
def plot_discriminations_difficulties(discriminations, difficulties,␣
 ↪normalized_df, font_size=10, font_ann_size=5, base_point_size=500):
    rcParams['font.family'] = 'serif'
    rcParams['font.serif'] = ['Times New Roman']

    sns.set_style('whitegrid')
    fig, ax = plt.subplots(figsize=(3, 3))
    ax.grid(color='black', linestyle='--', linewidth=0.5)

    ax.spines['bottom'].set_color('black')
    ax.spines['left'].set_color('black')
    ax.spines['right'].set_color('black')
    ax.spines['top'].set_color('black')
    ax.xaxis.label.set_color('black')
    ax.yaxis.label.set_color('black')
    ax.tick_params(axis='x', colors='black')
    ax.tick_params(axis='y', colors='black')

    point_sizes = base_point_size * (1 - np.abs(difficulties - 0.5) * 2)

    colors = []
    for disc, diff in zip(discriminations, difficulties):
        base_color = '#482878' if disc < 0 else '#35b779'
```

```
        intensity = 1
        color = mcolors.to_rgba(base_color, intensity)
        colors.append(color)

    scatter = ax.scatter(discriminations, difficulties, s=point_sizes, c=colors)

    plt.xlabel(r'Discrimination', fontsize=font_size, family='Times New Roman',␣
 ↪color='black')
    plt.ylabel(r'Difficulty', fontsize=font_size, family='Times New Roman',␣
 ↪color='black')

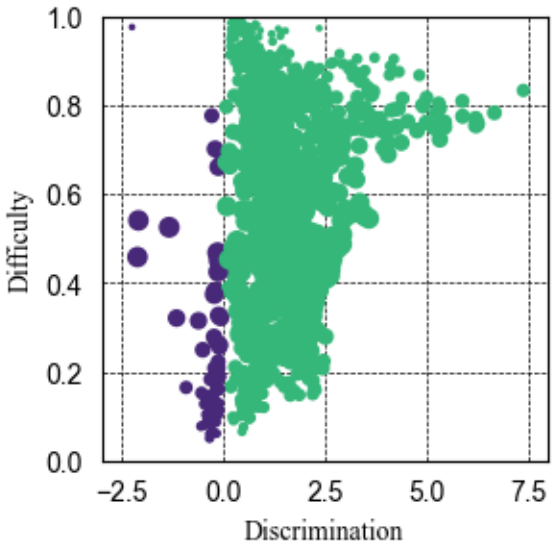
    ax.set_ylim(0, 1)

    if discriminations.min() < 0:
        x_min = int(discriminations.min()) - 1
    else:
        x_min = int(discriminations.min())

    if discriminations.max() > 0:
        x_max = int(discriminations.max()) + 1
    else:
        x_max = int(discriminations.max())

    ax.set_xlim(x_min, x_max)
    plt.savefig("Figure6a.pdf", format="pdf", bbox_inches='tight')
    plt.show()

plot_discriminations_difficulties(b4.discriminations, b4.difficulties, new_pij,␣
 ↪font_size=10, font_ann_size=8, base_point_size=50)
```



[8]:
```
fairness_model = normalized_df.apply(np.mean,axis=0).to_numpy()
```

[9]:
```
def create_abilities_fairness_table(name, abilities, fairness_model):
    df = pd.DataFrame({
        'Model': name,
        'Ability': abilities,
        'STS': fairness_model
    })

    return df

df = create_abilities_fairness_table(pij.columns, b4.abilities, fairness_model)

df
```

[9]:

	Model	Ability	STS
0	GBM_2	0.838897	0.709320
1	GBM_5	0.827377	0.690295
2	GBM_3	0.838161	0.706577
3	GBM_4	0.824131	0.666666
4	GBM_grid_4	0.805147	0.650855
5	GBM_grid_2	0.809815	0.679544
6	GBM_grid_3	0.808127	0.669700
7	GBM_1	0.841759	0.702578
8	GBM_grid_1	0.786608	0.636572
9	GBM_grid_5	0.823302	0.672017
10	XRT_1	0.891550	0.746396
11	DRF_1	0.750537	0.595940
12	DL_1	0.718731	0.613512
13	GLM_1	0.473743	0.385080

[10]:
```
def f(theta,delta_j,a_j):
    term1 = (delta_j / (1 - delta_j)) ** a_j
    term2 = (theta / (1 - theta)) ** (-a_j - 1)
    numerator = a_j * term1 * term2
    denominator = (1 + term1 * (theta / (1 - theta)) ** -a_j) ** 2
    return numerator / denominator * (1 / (1 - theta) ** 2)
```

[11]:
```
abilities = np.linspace(0.001, 0.999, 1000)
```

[12]:
```
min_ability = np.min(b4.abilities)
max_ability = np.max(b4.abilities)

plt.figure(figsize=(3, 3))
plt.rcParams["font.family"] = "Times New Roman"
```

```
sns.set_style('whitegrid')
fig, ax = plt.subplots(figsize=(3, 3))
ax.grid(color='black', linestyle='--', linewidth=0.5)

ax.spines['bottom'].set_color('black')
ax.spines['left'].set_color('black')
ax.spines['right'].set_color('black')
ax.spines['top'].set_color('black')
ax.xaxis.label.set_color('black')
ax.yaxis.label.set_color('black')
ax.tick_params(axis='x', colors='black')
ax.tick_params(axis='y', colors='black')

markers = ['o', 's', 'D', '^', 'v', 'P']
num_markers = len(markers)
colors = ['red', 'blue', 'green', 'orange', 'purple', 'brown']
num_colors = len(colors)
linestyles = ['-', '--', '-.', ':', (0, (3, 1, 1, 1)),
              (0, (5, 1)), (0, (5, 10)), (0, (1, 1)),
              (0, (3, 5, 1, 5)), (0, (3, 10, 1, 10))]
num_linestyles = len(linestyles)
i = 0
j = 0
id = 0
list = [0.25,0.11,0.02,0.76,0.86]

added_labels = set()

for index in [837,797,447,765,698]:
    total_f_theta = 0

    for i in range(b4.abilities.shape[0]):
        f_theta = abs(f(b4.abilities[i], b4.difficulties[index], b4.
 ↪discriminations[index]))
        total_f_theta += f_theta

    linestyle = linestyles[j % num_linestyles]
    fairness_2 = ICC_function(abilities, b4.difficulties[index], b4.
 ↪discriminations[index])
    if b4.discriminations[index]>0:
        plt.plot(abilities, fairness_2, label=f'{index+1}␣
 ↪(FI={round(total_f_theta, 2)})',color='#35b779', linestyle=linestyle)
    else:
        plt.plot(abilities, fairness_2, label=f'{index+1}␣
 ↪(FI={round(total_f_theta, 2)})',color='#482878', linestyle=linestyle)
    j += 1
    id += 1
```

```
plt.fill_betweenx(np.arange(-0.05, 1.15, 0.1), min_ability, max_ability,␣
 ↪color='gray', alpha=0.2, label='Model Range')

plt.xlim(-0.05, 1.05)
plt.ylim(-0.05, 1.05)


plt.gca().set_aspect('equal', adjustable='box')

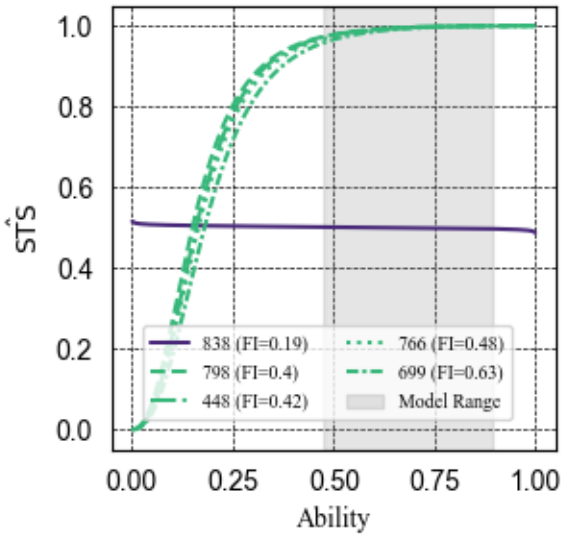
legend = plt.legend(loc='upper left', fontsize=7, bbox_to_anchor=(0.05, 0.3),␣
 ↪ncol=2)

for text in legend.get_texts():
    text.set_fontname('Times New Roman')
    text.set_color('black')

plt.xlabel(r'Ability', fontsize=10, family='Times New Roman')
plt.ylabel(r'$\hat{\mathrm{STS}}$', fontsize=10, family='Times New Roman')

plt.grid(True)
plt.savefig("Figure6b.pdf", format="pdf", bbox_inches='tight')
plt.show()
```

```
<Figure size 300x300 with 0 Axes>
```



[13]:
```
from irt_special import Beta3
```

[14]:
```
def ICC_function_special(abilities, difficulties):
    a = ((1-abilities)/ abilities)
    b = (difficulties / (1-difficulties))
    c = a*b
    d = c
    return (1 / (d+1))
```

[15]:
```
b4_special = Beta3(
        learning_rate=100,
        epochs=10000,
        n_respondents=normalized_df.shape[1],
        n_items=normalized_df.shape[0],
        n_workers=-1,
        random_seed=1,
    )
b4_special.fit(normalized_df.values)
```

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```

[15]:
```
<irt_special.Beta3 at 0x343fb4d30>
```

[16]:
```
new_pij = pd.DataFrame(index=range(1000), columns=range(14))

for i in range(1000):
    for j in range(14):
        alpha = (b4_special.abilities[j] / b4_special.difficulties[i])
        beta_val = ((1 - b4_special.abilities[j]) / (1 - b4_special.
 ↪difficulties[i]))
        new_pij.iloc[i, j] = (alpha)/(alpha+beta_val)
```

[18]:
```
def generate_table(b4_special, new_pij, index):
    delta_j_values = b4_special.difficulties[index]
    Delta_j_values = delta_j_values / (1 - delta_j_values)
    log_Delta_j_values = np.log(Delta_j_values)

    print(f"log_Delta_j_values (for difficulty at index {index}):␣
 ↪{round(log_Delta_j_values, 3)}")

    results = []

    for i in range(b4_special.abilities.shape[0]):
        theta_i_values = b4_special.abilities[i]
        Theta_i_values = (1 - theta_i_values) / theta_i_values
        log_Telta_j_values = np.log(Theta_i_values)
        res = np.log(1 - new_pij[i][index]) - np.log(new_pij[i][index])

        results.append({
            "log_Telta_j_values": round(log_Telta_j_values, 3),
```

```
            "res": round(res, 3)
        })

    df = pd.DataFrame(results)


    return df

generate_table(b4_special, new_pij, 837)
```

```
log_Delta_j_values (for difficulty at index 837): 1.489
```

[18]:

	log_Telta_j_values	res
0	-1.823	-0.334
1	-1.721	-0.232
2	-1.832	-0.343
3	-1.604	-0.115
4	-1.443	0.046
5	-1.621	-0.132
6	-1.574	-0.085
7	-1.812	-0.323
8	-1.363	0.126
9	-1.579	-0.090
10	-2.171	-0.682
11	-1.076	0.413
12	-1.135	0.354
13	0.391	1.880

[17]:

```
```