

Continuous-Time Spline Visual-Inertial Odometry, Jacobians

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I. STATE

We include the state from the main paper for reference.

$$\begin{aligned} \text{state} &= [f_x, f_y, c_x, c_y, s, r, p, \mathbf{kf}_1, \mathbf{kf}_2, \dots, \mathbf{kf}_N] \\ \mathbf{kf}_j &= [\mathbf{s}_{dso}, \mathbf{b}_j^a, \mathbf{b}_j^g, \mathbf{l}_j^r, \mathbf{q}_j^p, \mathbf{q}_j^r, \mathbf{c}_j^p, \mathbf{c}_j^r] \\ \mathbf{s}_{dso} &= [{}^w\mathbf{p}_{c_j}, {}^w\boldsymbol{\varphi}_{c_j}, a_j, b_j, d_{j1}, d_{j2}, \dots, d_{jM}] \end{aligned}$$

II. ERROR STATE JACOBIANS

A. Acceleration

We synthesize acceleration from spline using the following equations (Eq. 14 & 15 in the main paper):

$$\begin{aligned} {}^i\mathbf{a}(t) &= {}^i\mathbf{R} \cdot {}^w\mathbf{R}(t)^T \cdot [s \cdot (2\mathbf{q}_j^p + 6t\mathbf{c}_j^p) + \mathbf{g}] + \mathbf{b}_j^a \quad (1) \\ \mathbf{g} &= 9.8 \cdot [-\sin(p)\cos(r), \sin(r), -\cos(p)\cos(r)]^T \end{aligned}$$

We define the acceleration error to the accelerator measurement ${}^i\tilde{\mathbf{a}}(t)$ (Eq. 22 in the main paper):

$$\mathbf{e}_{acc} = {}^i\mathbf{a}(t) - {}^i\tilde{\mathbf{a}}(t)$$

The Jacobians with respect to the global variables in the state are:

$$\begin{aligned} \frac{\partial \mathbf{e}_{acc}}{\partial s} &= {}^i\mathbf{R} \cdot {}^w\mathbf{R}(t)^T \cdot (2\mathbf{q}_j^p + 6t\mathbf{c}_j^p) \\ \frac{\partial \mathbf{e}_{acc}}{\partial r} &= {}^i\mathbf{R} \cdot {}^w\mathbf{R}(t)^T \cdot 9.8 \cdot \begin{bmatrix} \sin(p)\sin(r) \\ \cos(r) \\ \cos(p)\sin(r) \end{bmatrix} \\ \frac{\partial \mathbf{e}_{acc}}{\partial p} &= {}^i\mathbf{R} \cdot {}^w\mathbf{R}(t)^T \cdot 9.8 \cdot \begin{bmatrix} -\cos(p)\cos(r) \\ 0 \\ \sin(p)\cos(r) \end{bmatrix} \end{aligned}$$

The Jacobian with respect to the acceleration bias \mathbf{b}_j^a is:

$$\frac{\partial \mathbf{e}_{acc}}{\partial \mathbf{b}_j^a} = \mathbf{I}_{3 \times 3}$$

The Jacobians with respect to the position spline coefficients are:

$$\begin{aligned} \frac{\partial \mathbf{e}_{acc}}{\partial \mathbf{q}_j^p} &= {}^i\mathbf{R} \cdot {}^w\mathbf{R}(t)^T \cdot s \cdot 2 \\ \frac{\partial \mathbf{e}_{acc}}{\partial \mathbf{c}_j^p} &= {}^i\mathbf{R} \cdot {}^w\mathbf{R}(t)^T \cdot s \cdot 6t \end{aligned}$$

We expand ${}^w\mathbf{R}(t)$ in Eq. 1:

$$\begin{aligned} {}^w\mathbf{R}(t) &= {}^w\mathbf{R} \cdot \exp((t\mathbf{l}_j^r + t^2\mathbf{q}_j^r + t^3\mathbf{c}_j^r)^\wedge) \\ &= \exp({}^w\boldsymbol{\varphi}_{c_j}^\wedge) \cdot \exp((t\mathbf{l}_j^r + t^2\mathbf{q}_j^r + t^3\mathbf{c}_j^r)^\wedge) \end{aligned}$$

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The Jacobian with respect to the keyframe orientation (left multiplication) is:

$$\frac{\partial \mathbf{e}_{acc}}{\partial {}^w\boldsymbol{\varphi}_{c_j}} = {}^i\mathbf{R} \cdot {}^w\mathbf{R}(t)^T \cdot [s \cdot (2\mathbf{q}_j^p + 6t\mathbf{c}_j^p) + \mathbf{g}]^\wedge$$

The Jacobians with respect to the rotation spline coefficients are:

$$\begin{aligned} \frac{\partial \mathbf{e}_{acc}}{\partial \mathbf{l}_j^r} &= {}^i\mathbf{R} \cdot \{{}^w\mathbf{R}(t)^T \cdot [s \cdot (2\mathbf{q}_j^p + 6t\mathbf{c}_j^p) + \mathbf{g}]\}^\wedge \cdot t \\ \frac{\partial \mathbf{e}_{acc}}{\partial \mathbf{q}_j^r} &= {}^i\mathbf{R} \cdot \{{}^w\mathbf{R}(t)^T \cdot [s \cdot (2\mathbf{q}_j^p + 6t\mathbf{c}_j^p) + \mathbf{g}]\}^\wedge \cdot t^2 \\ \frac{\partial \mathbf{e}_{acc}}{\partial \mathbf{c}_j^r} &= {}^i\mathbf{R} \cdot \{{}^w\mathbf{R}(t)^T \cdot [s \cdot (2\mathbf{q}_j^p + 6t\mathbf{c}_j^p) + \mathbf{g}]\}^\wedge \cdot t^3 \end{aligned}$$

B. Angular Velocity

We synthesize angular velocity from spline using the following equation (Eq. 16 in the main paper):

$${}^i\boldsymbol{\omega}(t) = {}^i\mathbf{R} \cdot (\mathbf{l}_j^r + 2t\mathbf{q}_j^r + 3t^2\mathbf{c}_j^r) + \mathbf{b}_j^g$$

We define the angular velocity error to the gyroscope measurement ${}^i\tilde{\boldsymbol{\omega}}(t)$ (Eq. 22 in the main paper):

$$\mathbf{e}_{gyro} = {}^i\boldsymbol{\omega}(t) - {}^i\tilde{\boldsymbol{\omega}}(t)$$

The Jacobian with respect to the gyroscope bias \mathbf{b}_j^g is:

$$\frac{\partial \mathbf{e}_{gyro}}{\partial \mathbf{b}_j^g} = \mathbf{I}_{3 \times 3}$$

The Jacobians with respect to the rotation spline coefficients are:

$$\begin{aligned} \frac{\partial \mathbf{e}_{gyro}}{\partial \mathbf{l}_j^r} &= {}^i\mathbf{R} \\ \frac{\partial \mathbf{e}_{gyro}}{\partial \mathbf{q}_j^r} &= {}^i\mathbf{R} \cdot 2t \\ \frac{\partial \mathbf{e}_{gyro}}{\partial \mathbf{c}_j^r} &= {}^i\mathbf{R} \cdot 3t^2 \end{aligned}$$

III. CONSTRAINT JACOBIANS

A. Velocity Constraint

The velocity continuity constraint (Eq. 19) is:

$$\begin{aligned} \mathbf{c}_v(j) &= ({}^w\mathbf{p}_{c_{j-1}} - {}^w\mathbf{p}_{c_j})/t_{j-1} - t_{j-1}\mathbf{q}_j^p - t_{j-1}^2\mathbf{c}_j^p \\ &\quad - ({}^w\mathbf{p}_{c_j} - {}^w\mathbf{p}_{c_{j+1}})/t_j - t_j\mathbf{q}_{j+1}^p - 2t_j^2\mathbf{c}_{j+1}^p = \mathbf{0} \end{aligned}$$

The Jacobians with respect to the keyframe positions are:

$$\begin{aligned}\frac{\partial \mathbf{c}_v(j)}{\partial {}^w \mathbf{p}_{c_{j-1}}} &= \frac{1}{t_{j-1}} \cdot \mathbf{I}_{3 \times 3} \\ \frac{\partial \mathbf{c}_v(j)}{\partial {}^w \mathbf{p}_{c_j}} &= -\left(\frac{1}{t_{j-1}} + \frac{1}{t_j}\right) \cdot \mathbf{I}_{3 \times 3} \\ \frac{\partial \mathbf{c}_v(j)}{\partial {}^w \mathbf{p}_{c_{j+1}}} &= \frac{1}{t_j} \cdot \mathbf{I}_{3 \times 3}\end{aligned}$$

The Jacobians with respect to the position spline coefficients are:

$$\begin{aligned}\frac{\partial \mathbf{c}_v(j)}{\partial \mathbf{q}_j^p} &= -t_{j-1} \cdot \mathbf{I}_{3 \times 3} \\ \frac{\partial \mathbf{c}_v(j)}{\partial \mathbf{c}_j^p} &= -t_{j-1}^2 \cdot \mathbf{I}_{3 \times 3} \\ \frac{\partial \mathbf{c}_v(j)}{\partial \mathbf{q}_{j+1}^p} &= -t_j \cdot \mathbf{I}_{3 \times 3} \\ \frac{\partial \mathbf{c}_v(j)}{\partial \mathbf{c}_{j+1}^p} &= -2t_j^2 \cdot \mathbf{I}_{3 \times 3}\end{aligned}$$

B. Rotation Constraint

The rotation continuity constraint (Eq. 17) is:

$$\mathbf{c}_r(j) = {}^{w}_{c_{j-1}} \mathbf{R}^T \cdot {}^w_{c_j} \mathbf{R} \cdot \exp((t_{j-1} \mathbf{l}_j^r + t_{j-1}^2 \mathbf{q}_j^r + t_{j-1}^3 \mathbf{c}_j^r)^\wedge) = \mathbf{I}$$

The Jacobians with respect to the rotation spline coefficients are:

$$\begin{aligned}\frac{\partial \mathbf{c}_r(j)}{\partial \mathbf{l}_j^r} &= {}^{w}_{c_{j-1}} \mathbf{R}^T \cdot {}^w_{c_j} \mathbf{R} \cdot \exp(t_{j-1} \mathbf{l}_j^r + t_{j-1}^2 \mathbf{q}_j^r + t_{j-1}^3 \mathbf{c}_j^r) \cdot t_{j-1} \\ &= t_{j-1} \cdot \mathbf{I}_{3 \times 3} \\ \frac{\partial \mathbf{c}_r(j)}{\partial \mathbf{q}_j^r} &= {}^{w}_{c_{j-1}} \mathbf{R}^T \cdot {}^w_{c_j} \mathbf{R} \cdot \exp(t_{j-1} \mathbf{l}_j^r + t_{j-1}^2 \mathbf{q}_j^r + t_{j-1}^3 \mathbf{c}_j^r) \cdot t_{j-1}^2 \\ &= t_{j-1}^2 \cdot \mathbf{I}_{3 \times 3} \\ \frac{\partial \mathbf{c}_r(j)}{\partial \mathbf{c}_j^r} &= {}^{w}_{c_{j-1}} \mathbf{R}^T \cdot {}^w_{c_j} \mathbf{R} \cdot \exp(t_{j-1} \mathbf{l}_j^r + t_{j-1}^2 \mathbf{q}_j^r + t_{j-1}^3 \mathbf{c}_j^r) \cdot t_{j-1}^3 \\ &= t_{j-1}^3 \cdot \mathbf{I}_{3 \times 3}\end{aligned}$$

The Jacobians with respect to the keyframe orientations are:

$$\begin{aligned}\frac{\partial \mathbf{c}_r(j)}{\partial {}^w \boldsymbol{\varphi}_{c_j}} &= {}^{w}_{c_{j-1}} \mathbf{R}^T \\ \frac{\partial \mathbf{c}_r(j)}{\partial {}^w \boldsymbol{\varphi}_{c_{j-1}}} &= -{}^{w}_{c_{j-1}} \mathbf{R}^T\end{aligned}$$