PERTEMUAN 6
TURUNAN FUNGSI NON-ALJABAR

Informatika

Universitas Ahmad Dahlan

REVIEW

- $\sin x$
- $\cos x$
- $\tan x = \sin x \,/\, \cos x$
- csc (cosecan) $x = 1/\sin x$
- $\sec$ (secan) $x = 1/\cos x$
- $\cot$ (cotangent)$x = 1/\tan x$
- $\sin^2 x + \cos^2 x = 1$
- $\tan^2 x + 1 = \sec^2 x$
- $\cot^2 x + 1 = \csc^2 x$

$$\sin(-x) = -\sin x$$
$$\csc(-x) = -\csc x$$
$$\cos(-x) = \cos x$$
$$\sec(-x) = \sec x$$
$$\tan(-x) = -\tan x$$
$$\cot(-x) = -\cot x$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\tan(x \pm y) = \frac{(\tan x \pm \tan y)}{(1 \mp \tan x \tan y)}$$

TURUNAN FUNGSI TRIGONOMETRI

- $\frac{d}{dx}\sin x = \cos x$
- $\frac{d}{dx}\cos x = -\sin x$
- $\frac{d}{dx}\tan x = sec^2 x$
- $\frac{d}{dx}\cot x = -\csc^2 x$
- $\frac{d}{dx}\sec x = \sec x \tan x$
- $\frac{d}{dx} csc\, x = -\csc x \cot x$

- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$
- $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}(csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

CONTOH

Hitung turunan pertama dari $y = x^2 \sin x$

Jawab :

- $\frac{dy}{dx} = x^2 \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x^2$
- $\frac{dy}{dx} = x^2 \cos x + \sin x \; 2x$

LATIHAN

Hitung turunan pertama dari

- $f(x) = \frac{\sec x}{1+\tan x}$
- $f(x) = 3x^2 - 2\cos x$
- $f(x) = \sin x + \frac{1}{2}\cot x$
- $f(x) = \sqrt{x}\sin x$
- $f(x) = 2\csc x + 5\cos x$

FUNGSI EKSPONENSIAL

Jika $b > 0$ maka domain b^x adalah semua bilangan real x

Range adalah $(0, +\infty)$ jika $b \neq 1$ dan b^x differentiable

Aturan pangkat :

- $b^p b^q = b^{p+q}$
- $(b^p)^q = b^{pq}$
- $\frac{b^p}{b^q} = b^{p-q}$

FUNGSI LOGARITMA

$\log_b x$ / $b_{log}\ x$ adalah bilangan yang unik

$(b_{\log x} = c \rightarrow b^c = x)$

- $b^{\log_b x} = x$
- $\log_b(b^x) = x$
- $\log_b x$ dan b^x adalah invers

Domain $\log_b x = range\ b^x = (0, +\infty)$

Range $\log_b x = domain\ b^x = (-\infty, +\infty)$

HUKUM LOGARITMA

$\log_b x$ hanya terdefinisi untuk $x > 0$

Hukum – Hukum logaritma

- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b ac = \log_b a + \log_b c$
- $\log_b a/c = \log_b a - \log_b c$
- $\log_b a^r = r \log_b a$
- $\log_b \frac{1}{c} = -\log_b c$

HUKUM LOGARITMA

- $\log_{10} x = \log x$
- $\log_e x = \ln x$

Bilangan e

- $e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x \cong 2.71828 \ldots$
- $\ln 1 = 0$
- $\ln e = 1$
- $\ln e^x = x = e^{\ln x}$

TURUNAN FUNGSI EKSPONENSIAL

- $e^x = \exp(x)$
- $\ln x = \log_e x$
- $\frac{d}{dx} e^x = e^x$
- $a^x = e^{\ln a^x} = e^{x \ln a}$
- $\frac{d}{dx} a^x = (\ln a)\, a^x$

CONTOH

Hitung turunan pertama dari $f(x) = e^x - x$

- $f'(x) = \frac{d}{dx}e^x - \frac{d}{dx}x$
- $f'(x) = e^x - 1$

Hitung turunan pertama dari $f(x) = x^{\sqrt{x}}$

- $f(x) = x^{\sqrt{x}} = e^{\sqrt{x}\ln x}$

Misalkan $u = \sqrt{x}\ln x$ dan $y = e^u$

- $f'(x) = \frac{dy}{du}\frac{du}{dx} = e^u\left(\sqrt{x}\frac{1}{x} + \ln x\frac{1}{2\sqrt{x}}\right) = e^{\sqrt{x}\ln x}\left(\sqrt{x}\frac{1}{x} + \ln x\frac{1}{2\sqrt{x}}\right)$
- $f'(x) = x^{\sqrt{x}}\left(\frac{2+\ln x}{2\sqrt{x}}\right)$

LATIHAN

Hitung turunan pertama dari

- $f(x) = \sqrt{x} - 2e^{x}$
- $f(x) = 5e^{x} + 3x$
- $f(x) = e^{x+1} + 1$
- $f(x) = 5x^{2}/2e^{x}$
- $f(x) = x^{3} - 2e^{x}$

TURUNAN FUNGSI LOGARITMA

- $\frac{d}{dx} \ln x = \frac{1}{x}$
- $\log_a x = \frac{\ln x}{\ln a}$
- $\frac{d}{dx} \log_a x = \frac{\log_a x}{\ln a} = \frac{1}{x \ln a}$

CONTOH

Hitung turunan pertama dari $\mathrm{f(x)} = \ln(\sin x)$

Dimisalkan $u = \sin x$ dan $y = \ln u$

- $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = \dfrac{d}{du}\ln u \dfrac{d}{dx}\sin x$
- $\dfrac{dy}{dx} = \dfrac{1}{u}\cos x = \dfrac{1}{\sin x}\cos x = \cot x$

LATIHAN

Hitung turunan pertama dari

- $f(x) = \sqrt{\ln x}$
- $f(x) = \log_{10}(2 + \sin x)$
- $f(x) = \ln\left(\frac{x+1}{\sqrt{x-2}}\right)$
- $f(x) = \log_5(xe^x)$
- $f(x) = \ln(x^3 + 1)$