

changed. This is normal for new scientific directions and technologies. The reader should not be surprised if in other sources they meet different formulations, terms. A detailed description of the functionality of CAS and CMS can be found in [6], the current state, an overview of the systems on the market can be clarified in [7], a brief outline (basic features) for beginners can be found in [8].

IV. COMPUTER ALGEBRA TOOLS

Regarding the classification of CAS. A fairly complete list with the functionality of symbolic computation systems and the platforms they operate on can be found in [7]. Classification attributes of CAS are: functional purpose, type of architecture, means of implementation, fields of application, integral quality assessments.

A. *Basic functionality of CAS*

AS allow computer-assisted implementation of analytical and numerical methods for solving problems, presenting the results in mathematical notation, providing graphic visualization, design of the results, and preparation for publication. Using CAS and a computer, it is possible to perform in analytical form the following computations:

- simplifying expressions or reducing to the standard form;
- substituting symbolic and numeric values into expressions;
- extraction of common factors and divisors;
- exponentiation of products and powers, factorization;
- decomposition into simple fractions;
- finding limits of functions and sequences;
- operations with series;
- differentiation in full and partial derivatives;
- finding undetermined and definite integrals;
- continuity analysis of functions;
- finding extremes of functions and their asymptotes;
- operations with vectors;
- matrix operations;
- finding solutions to linear and nonlinear equations;
- symbolic solution of optimization problems;
- algebraic solution of differential equations;
- integral transformations;
- direct and inverse fast Fourier transform;
- interpolation, extrapolation and approximation;
- statistical computations;

- machine theorem proving.

If a problem has an accurate analytical solution, the user of CAS can get this solution in explicit form (of course, we are talking about problems for which the algorithm of solution construction is known).

- integer arithmetic for large numbers;
- calculation of fundamental constants with arbitrary precision;
- support for number theory functions;
- editing mathematical expressions in twodimensional form;
- graphing of analytically defined functions;
- function graphing using table values;
- plotting function graphs in two or three dimensions;
- animation of the plots of various types;
- use of special-purpose extension packages;
- programming in the built-in language;
- automatic formal verification;
- program synthesis.

CAS in the modern implementation are not only applicable for the study of various mathematical and scientific and technical problems using built-in and additional functions, but also contain all the components of programming languages – de facto are problem-oriented high-level programming languages.

Mathematica and Maple are the leaders of CAS -

B. *Noncommercial general-purpose CAS*

A distinctive feature of the current state of IT is that commercial software products in many cases can be fully or partially replaced by non-commercial software, analogues with open source – free software. This includes software products which, with or without modification, have no restrictions on use, copying or transferring to other users, whether for a fee or for free. The following is a reference to software released under the GPL.

Maxima. Maxima is a free, full-featured computer algebra system, a descendant of Macsyma, which was developed as part of the Artificial Intelligence Project at the Massachusetts Institute of Technology from 1968 to 1982 (development stages and leaders of the development teams of the main sections listed in [9]). Experts note that Maxima, unlike Mathematica and Maple, is mainly oriented toward applied mathematical calculations. In this connection, the system lacks or reduces sections related to theoretical methods, such as number theory, group theory, algebraic fields, and mathematical logic. At the same time, numbers in mathematical expressions in the system are assumed to be real by default. This allows to get analytical solutions for many computations encountered in applied problems (such as algebraic transformations and simplifications, integration, solution of differential equations), for which solutions do not exist in the complex domain. Maxima itself is a console

program; it “draws” all mathematical formulas with regular text characters. This has some advantages. For example, you can use Maxima itself as a kernel to build various graphical special interfaces on top of it. There are several examples to date. We can recommend the following textbook as a basic introduction to Maxima CAS, available in electronic form [10].

Axiom. Axiom is a free computer algebra system [11]. It consists of an interpreter environment, a compiler, and a library describing a strict, mathematically correct type hierarchy. Its development was begun in 1971 by a group of IBM researchers, led by Richard Dimick Jenks. The original name of the system was Scratchpad. Originally the project was seen as a research platform for developing new ideas in computational mathematics. It was sold to the Numerical Algorithms Group (NAG) in the 1990s, named Axiom, and became a commercial product, but was not a commercial success and was withdrawn from the market in October 2001. NAG made Axiom free software and opened the source code under a modified BSD license. Development of the system continues, with releases of new versions [11]. In 2007, Axiom had two open-source forks: OpenAxiom and FriCAS.

OpenAxiom [12] released version 1.4.2 in April 2013. The main changes implemented in this version relate to the work of the compiler. The aforementioned system for preparing and editing documents with GNU TeXmacs mathematical notation can be used as an OpenAxiom interface.

Another branch of Axiom that is being actively developed is FriCAS[13], version 1.3.8 (version 22/06/2022) is currently in use. FriCAS favourably differs from other general-purpose CAS by the developed type hierarchy corresponding to real mathematical structures. Axiom and the named branches are inferior to Maxima at this stage in the pace of development. It is better for beginners to focus on Maxima.

The above information about Maxima, Axiom CAS is specifically given because their codes are open, can be used in the Ecosystem OSTIS. C. *Proprietary* CMS, CAS

MATLAB programming system m (short for Matrix Laboratory) was developed by The MathWorks, Inc. It is one of the oldest, thoroughly developed and time-tested systems for automating mathematical calculations, built on an extended representation and application of matrix operations. Nowadays the system has gone far beyond specialized matrix and has become one of the most powerful universal integrated CMSs. MATLAB includes tools for developing complex programs with an advanced graphical interface, is an effective environment for conducting research, creating models, solving natural science and engineering problems [14]. The system has become de-facto one of the world standards in the field of modern mathematical and scientific and technical software. First of all, CMS is focused on numerical calculations, with matrix algebra being particularly prominent. The effectiveness of the system is primarily due to its orientation to work with multidimensional arrays, large and sparse matrices with software emulation of parallel calculations and

simplified tools for setting cycles. Recent versions of the system support 64-bit microprocessors and multicore microprocessors such as Intel Core 2 Duo and Quad. System functionality is provided by a rich command library and its own programming language. MATLAB is the largest of all PC-oriented systems due to large number of extension packages it comes with. Its file size exceeds 3 GB. MATLAB works on most modern operating systems, including Linux, macOS, Solaris (support for Solaris is discontinued since R2010b) and Windows. There are many publications describing the system and its components – in Russian [15] can be noted.

Versions history of MATLAB can be traced in [16]. Focusing only on notable items in terms of AI, machine learning, and data mining for versions after 2012 (code R2012* means the 2012 version):

- MATLAB 8.2 R2013b – added table data type, Java runtime updated to version 7;
- MATLAB 8.4 R2014b – added improved user toolbar, new functions and packages, such as py (to use Python), web page counter, histograms, TCP client, and others;
- MATLAB 8.6 R2015b – new runtime mechanism (LXE) and new classes, such as graphs and orgraphs, have been added to handle graphs;
- MATLAB 9.1 R2016b – official MATLAB engine for JAVA, new encoding and decoding functions for JSON, new "string" data type added; algorithms for handling non-memory data, including algorithms for dimensionality reduction, descriptive statistics, kmeans clustering, linear regression, logistic regression and discriminant analysis; Bayesian optimization to automatically adjust machine learning algorithm parameters, component neighborhood analysis (NCA) to select machine learning model functions;
- MATLAB 9.5 R2018b – implemented graph axis interaction, which provides efficient data analysis with panning, zooming; added functions: removing outliers in an array, table or schedule; setting a local environment about each element in the input data;
- MATLAB 9.6 R2019a – Added Functions to specify the location of a missing value, detect outliers using percentiles; Implemented improvements for artificial intelligence and analytics;
- MATLAB 9.7 R2019b – includes updates on artificial intelligence (new features allow users to train advanced network architectures using custom learning cycles, automatic differentiation, common weights, and custom loss functions; users can create generative adversarial GANs, Siamese networks,

variational autocoders, and attention networks; Deep Learning Toolbox can also now export to ONNX format networks that combine CNN and LSTM layers, and networks that include 3D CNN layers);

- MATLAB 9.11 R2021b – added: a set of tools for statistics and machine learning (signal and image analysis, preprocessing and parameter extraction using wavelet methods and interactive applications for artificial intelligence models), k-means clustering in real problems;
- MATLAB 9.13 R2022b includes updates on artificial intelligence, a set of system identification tools – create nonlinear state-space models based on deep learning using neural ordinary differential equations (ODEs); machine learning and deep learning techniques can also represent nonlinear dynamics in nonlinear ARX and Hammerstein-Wiener models.

ATLAB is a commercial system; there are noncommercial versions of its type that are compatible in basic language constructs, but not compatible in library functions. For example, Scilab, Maxima, Euler Math Toolbox and Octave.

MATLAB includes a command interpreter, graphical shell, editor-debugger, profiler, compiler, symbolic kernel Maple for analytical calculations, mathematical libraries and Toolboxes libraries, designed to work with special classes of tasks.

MATLAB language. MATLAB system is both an operating environment and a programming language. MATLAB language is a high-level interpreted programming language. Programs written in MATLAB are of two types: functions and scripts. MATLAB programs, both console and GUI, can be compiled using MATLAB Compiler component into independent executable applications or dynamic libraries. MATLAB Builder programs extend the capabilities of MATLAB Compiler and allow you to create independent Java, .NET or Excel components.

Basic extension packages. A feature of MATLAB CMS is the ability to create special toolboxes. MathWorks supplies more than 80 toolboxes that are used in many areas. In recent releases, the company classifies them into three families – MATLAB, SIMULINK and Polyspace [14], as well as partner products.

Maple. Mathematica and Maple are the leaders of CAS, and they are often compared. This seems counterproductive. Each of the systems named has its own characteristics, and they have their own strengths and weaknesses; constantly competing with each other, they are evolving and improving. Most CAS users have experienced several other systems before choosing their primary system. The exchange of opinions, the analysis of publications, and presentations at conferences and seminars allow us to state that each system has its own adherents, and it is useless to convince specialists who

have been using CAS for quite a long time that a system other than the one they prefer is somehow better than the others. In most cases, the main factor in using a particular CAS is the user's habit. However, many note that having mastered any of the systems, it is easy to work with others.

In terms of completeness and interface solutions, Mathematica and Maple tools for symbolic and numerical computation are perfect, not the lack of any functions or tools, but the skills of the users. It is impossible to give a complete overview of the capabilities of Maple, as well as Mathematica. It is unlikely that any of the authors of even specialized publications with a book focus on a particular class of problems can present all of the tools of the named CAS from the spectrum they cover. This material can only be regarded as an introduction to the capabilities of the system, mentioning classes of problems of interest to undergraduates, graduate students, postgraduates, researchers, and programmers. Again, the functionality of Mathematica and Maple in almost everything related to mathematics, applied mathematics, and computer science is not only sufficient, but also redundant. Since the main list of features of CAS has already been given above, and in Maple they are implemented, here we will note what in a number of sources is either omitted or called by other terms.

There are many books devoted to the Maple system around the world, a list of which can be found on the developer's section [17]. Publications in Russian can be tracked by [18], [19]. Despite its fundamental nature and focus on the most serious mathematical calculations, Maple-class systems are needed by a fairly wide category of users: university students and teachers, engineers, graduate students, researchers, and even students of mathematics classes of general education and special schools. Maple is a typical integrated software system [19]–[21].

The main Maple document is the Worksheet, which is similar to editing in a text editor. Text can be formatted at the paragraph level, with different styles, or as symbols. The content of the document can be structured into sections, sub-sections, etc., all the way down to cells.

Like most CAS, the Maple interface combines text and command processor functions. Since version 8, Maplets have been added to the system to support visually oriented dialog.

Maple (like Mathematica) integrates three languages: communication language, implementation, and programming.

The Maple kernel and all of its components improve from version to version. Many of the functions built into the system, like the kernel functions, can be used without any declaration, others need to be declared. There are several problem-oriented packages (packages) that cover many branches of classical and modern mathematics. The total number of functions in Maple, including those built