

[The inference relation is a reflexive, transitive, monotone binary relation on sets of premises (judgments (propositions), logical formulas). The properties of the inference relation are the rules of inference by Gentzen.]

sequent

⇒ *note**:

[A sequent is a tuple (of an implicative form) between a conjunctive set of logical formulas (conjunction) and a disjunctive set of logical formulas (disjunction). An example of a sequent is an expression (judgement) like: $A1 \wedge A2 \wedge \dots \wedge An \Rightarrow C1 \vee C2 \vee \dots \vee Cm$.]

antecedent'

⇒ *first domain**:

sequent

⇒ *second domain**:

conjunction

consequent'

⇒ *first domain**:

sequent

⇒ *second domain**:

disjunction

Inference relation on sequents

⇒ *note**:

[The inference relation on sequents satisfies the rules of inference of the sequent calculus.]

metastructure

⇒ *note**:

[A metastructure is a structure whose fullconnectively represented element is another structure.]

modal operator

⇒ *note**:

[A modal operator is a logical connectives that links a logical formula to a structure (and sometimes other elements) in a metastructure. An example of a modal operator is the knowledge operator: Δ .]

modal inference rule

⇒ *note**:

[A modal inference rule the modal operator binding of a formula is true (has a true interpretation) in a structure if and only if the formula is true (has a true interpretation) in the structure that precedes it. An example of an inference rule is the knowledge operator: $\Gamma \cup \{\alpha\} \vdash \Gamma \cup \{\Delta\alpha\}$.]

relation of becoming of structures

⇒ *note**:

[The relation of becoming of structures is a binary relation on the set of structures having a nonempty common support. The roles in the tuples of the relation of becoming are the role relations of the previous structure (preceding structure) and the subsequent structure.]

thinking sequence

:= [fate of thinking]

:= [thought]

⇒ *note**:

[A sequence of thinking is a sequence of sc-sets of propositions (logical formulas).]

⇒ *subdividing**:

{• *irrational thinking sequence*
• *irrational thinking sequence*
}

sequence of rational thinking of classical logic

:= [fate of rational thinking of classical logic]

⊂ *rational thinking sequence*

⇒ *note**:

[A sequence of rational thinking is a sequence (given by the relation of becoming of structures) of (classically) satisfiable sc-sets (sc-subsets or sc-supersets) of propositions.]

sequence of classical rational deductive thinking

⊂ *sequence of rational thinking of classical logic*

⊃ *sequence of rational deductive thinking of classical logic on finite sc-sets*

⇒ *note**:

[A sequence of classical rational deductive thinking is a sequence of rational thinking, the sequence (of becoming) of satisfiable sc-sets of propositions deductively logically following (according to classical rules) one after another.]

sequence of classical rational deductive cognition

:= [will]

⇒ *note**:

[A sequence of classical rational deductive cognition is a sequence (given by the relation of becoming of structures) of non-contradictory scsupersets of propositions logically following from one to another.]

Non-classical logics [27], [29]–[31], [33], [34] consider (1) non-classical inference whose inference relation having unusual properties [27], [29]–[31], following which, it is possible or impossible to deduce results which is deduced in classical logic, as well as (2) other scales of truth constants for logical formulas, their interpretations, and [33], [34] values that are different from false and 111 (ground) truth. The event-like nature of event (distensible) sets [3],

[4], used by the languages of the unified knowledge representation model, allows naturally not only representing transitions between deducible sets of facts under the conditions of the open world assumption (hypothesis) but also the non-monotonic modifiable reasoning corresponding to them under the conditions of the closed world assumption (hypothesis). To implement non-monotone inference, special relations of non-monotone inference are introduced. Modifiable reasoning considers four kinds of predicates:

$$\begin{aligned} P1i &= P1j \\ P2i &\subseteq P2j \\ P3i &\supseteq P3j \\ P4i &? P4j \end{aligned}$$

An efficient solution of the problem is possible by adding additional restrictions on the class of considered formulas and functions (Horn clauses, monotone predicates). The transition to fuzzy logic allows reducing a discrete problem to a continuous problem (embed it in a continuous model which is close to quantor elimination methods). The used formalism of event (distensible) sets [3], [4] allows representing fuzzy logic expressions in a natural way. Let us consider some definitions of fuzzy connectives that link and express causality and exclusivity in Lukasiewicz's fuzzy logic [34].

$$\begin{aligned} \varphi \dot{\rightarrow} \psi & \\ 1 &\stackrel{\text{def}}{=} \tilde{0} \dot{\rightarrow} \tilde{0} \\ \sim \varphi &\stackrel{\text{def}}{=} \varphi \dot{\rightarrow} \tilde{0} \\ \varphi \dot{\wedge} \psi &\stackrel{\text{def}}{=} (\varphi \dot{\rightarrow} \psi) \dot{\rightarrow} \psi \\ \varphi \dot{\vee} \psi &\stackrel{\text{def}}{=} \sim (\sim \varphi \dot{\wedge} \sim \psi) \\ \varphi \dot{\oplus} \psi &\stackrel{\text{def}}{=} (\sim \varphi) \dot{\rightarrow} \psi \\ \varphi \dot{\otimes} \psi &\stackrel{\text{def}}{=} \sim (\sim \varphi \dot{\oplus} \sim \psi) \\ \varphi \dot{\ominus} \psi &\stackrel{\text{def}}{=} \sim (\varphi \dot{\rightarrow} \psi) \\ \varphi \dot{\leftrightarrow} \psi &\stackrel{\text{def}}{=} (\varphi \dot{\rightarrow} \psi) \vee (\psi \dot{\rightarrow} \varphi) \end{aligned}$$

To implement the inference in other non-classical logics, including those dealing with subject domains in which non-deterministic sets and structures are considered, additional concepts are also considered.

nonmonotonic inference on finite sc-set of premise

\Rightarrow *note**

[Nonmonotonic inference on a finite sc-set of premises is a relation between (finite) sc-sets of true logical statements (premises). If there is no embedding of the structure of an atomic logical formula in the relational structure (sc-subset of the subject area) of the sc-set of true (consistent) premises and the negation of this atomic formula is true with respect to them, then there is an sc-set with a relational structure belonging to it, including all elements of the

previously mentioned relational structure and the constants of this atomic formula, to which all premises of the previously mentioned sc-set of true (consistent) premises and the mentioned atomic logical formula belong.]

inferencing set

\Rightarrow *note**:

[An inferencing set is an event sc-set, the (temporary) belonging of logical formulas to which is established in the order of formation of the process of deriving these logical formulas.]

fuzzy truth*

\Rightarrow *note**:

[Fuzzy truth connects a finite sc-set with temporal belongings on a finite set of finite phenomena of belonging to a statement. On the membership phenomena, a finite sc-subset of the sc-relation of becoming (immediately before, immediately after) is given, which defines the structure of the corresponding sc-subsets. This structure is a directed tree. Fuzzy truth is a binary relation between the (fuzzy) membership of a link of a formal theory statement and a finite sc-set and a real number from 0.0 to 1.0. The fuzzy truth of the negation of the statement is equal to the difference 1.0 and the fuzzy truth of the statement belonging to the negation. The fuzzy truth of the conjunction of propositions does not exceed the minimum of the fuzzy truth of the elements of this conjunction and is not lower than the (boundary or drastic) product of the fuzzy truth of the same elements of the conjunction. The fuzzy truth of a disjunction does not exceed the (boundary or drastically) sum of the fuzzy truth of the elements of this conjunction and not less than the maximum fuzzy truth of the same elements of the conjunction. The fuzzy truth of atomic propositions is equal to the arithmetic mean of the isomorphic embedding of the proposition structure in each of the sc-subsets of the finite sc-set that are included in the structure given by the becoming sc-relation.

constructively true proposition*

\Rightarrow *note**:

[A constructively true proposition* is a subset of a true proposition*. True atomic logical formulas or their true interpretations are constructively true if and only if they have an isomorphic embedding in the domain where all elements of the embedding 11 are full-connectively represented. A conjunction of

constructively true logical formulas (or having corresponding constructively true interpretations) is constructively true (or has a constructively true corresponding interpretation). The disjunction of at least one constructively true logical formula (or having a corresponding complete constructively true interpretation) is constructively true (or has a constructively true corresponding interpretation). The negation of a false logical formula (or having a false corresponding interpretation) is constructively true (or has a constructively true interpretation). If all the logical formulas in the disjunction are false (have corresponding false interpretations), then the disjunction is also false (has a corresponding false interpretation). The negation of a false logical formula (or having a false corresponding interpretation) is constructively true (or has a constructively true interpretation). An implication with a false premise (or having a corresponding false interpretation) is constructively true (or has a constructively true interpretation). An implication with a constructively true consequence (or having a corresponding constructively true interpretation) is constructively true (or has a constructively true interpretation). A constructively true implication (or having a constructively true interpretation) with a constructively true premise (or having a corresponding constructively true interpretation) has a constructively true consequence (or having a corresponding constructively true interpretation). A constructively true implication (or having a constructively true interpretation) with a false consequence (or having a corresponding false interpretation) has a false premise (or having a corresponding false interpretation). The existence of variable values for a logical formula is constructively true (or has a corresponding constructive true interpretation) if the universality of variable values for that logical formula is constructively true (or has a corresponding constructive true interpretation). If a logical formula has only constructively true corresponding interpretations, then the universality of the values of the variable for this logical formula is constructively true (or has a corresponding constructively true interpretation).]

right proposition*

⇒ *note**:

[A right proposition is a proposition that is true or uncorrupted.]

uncorrupted proposition*

⇒ *note**

[An uncorrupted proposition is a proposition whose truth or falsity (untruth) does not lead to a contradiction.]

The approach based on the semantic space [1], [8], in addition to considering useful metric properties, allows imposing additional requirements on the topological properties of the corresponding logics, when all substructures of the relational structure of the subject domain, on which the logical formulas are interpreted, are meaningfully closed.

IV. CONCLUSION

The author thanks the scientific staff of the Department of Intelligent Information Technologies of the Belarusian State University of Informatics and Radioelectronics for their help in the work and valuable comments.

REFERENCES

- [1] V. Golenkov, N. Gulyakina, and D. Shunkevich, *Standart otkrytoi tekhnologii ontologicheskogo proektirovaniya, proizvodstva i ekspluatatsii semanticheski sovmestimyykh gibridnykh intellektual'nykh komp'yuternykh sistem* [Open technology standard for ontological design, production and operation of semantically compatible hybrid intelligent computer systems], V. Golenkov, Ed. Minsk: Bestprint, 2021, (In Russ.).
- [2] V. V. Golenkov and N. A. Gulyakina, "Next-generation intelligent computer systems and technology of complex support of their life cycle," *Otkrytye semanticheskie tekhnologii proektirovaniya intellektual'nykh sistem* [Open semantic technologies for intelligent systems], pp. 27–40, 2022.
- [3] V. P. Ivashenko, "Modeli i algoritmy integratsii znaniy na osnove odnorodnykh semanticheskikh setei (diss. na soiskanie stepeni kand. tekhn. nauk: 05.13.17) [models and algorithms for knowledge integration based on homogeneous semantic networks (diss. for the degree of candidate of technical sciences: 05.13.17)]," dis. ... kand. tekhn. nauk : 05.13.17, Belarus. gos. un-t informatiki i radioelektroniki [Belarusian State University for Informatics and Radioelectronics], Minsk, 2014, (In Russ.).
- [4] V. Ivashenko, "General-purpose semantic representation language and semantic space," *Otkrytye semanticheskie tekhnologii proektirovaniya intellektual'nykh sistem* [Open semantic technologies for intelligent systems], pp. 41–64, 2022.
- [5] —, "String processing model for knowledge-driven systems," *Doklady BGUIR*, vol. 18, pp. 33–40, 10 2020.
- [6] V. Golenkov, V. Golovko, N. Gulyakina, and V. Krasnoproshin, "The standardization of intelligent computer systems as a key challenge of the current stage of development of artificial intelligence technologies," pp. 73–88, 2020.
- [7] W. Lowe, "Towards a theory of semantic space," 01 2001, pp. 576–581.
- [8] Y. I. Manin and M. Marcolli, "Semantic spaces," *Mathematics in Computer Science*, vol. 10, pp. 459–477, 2016.
- [9] I. Mel'cuk, *Language: from Meaning to Text*, D. Beck, Ed. Moscow Boston: Academic Studies Press, 04 2016.
- [10] Y. M. A. Kostrikin, *Linear Algebra and Geometry*. Gordon and Breach Science Publishers, 1997.
- [11] D. Bohm and B. Hiley, *The Undivided Universe: An Ontological Interpretation of Quantum Theory*. London: Routledge, 1993.