Maintaining hydrostatic stability in an ice sheet coupled s-coordinate ocean model

Stefan Jendersie[1]
Dan Lowry[2],
Nick Golledge[1],

[1] Antarctic Research Centre, Victoria University of Wellington, New Zealand
[2] GNS Science, Lower Hutt, New Zealand

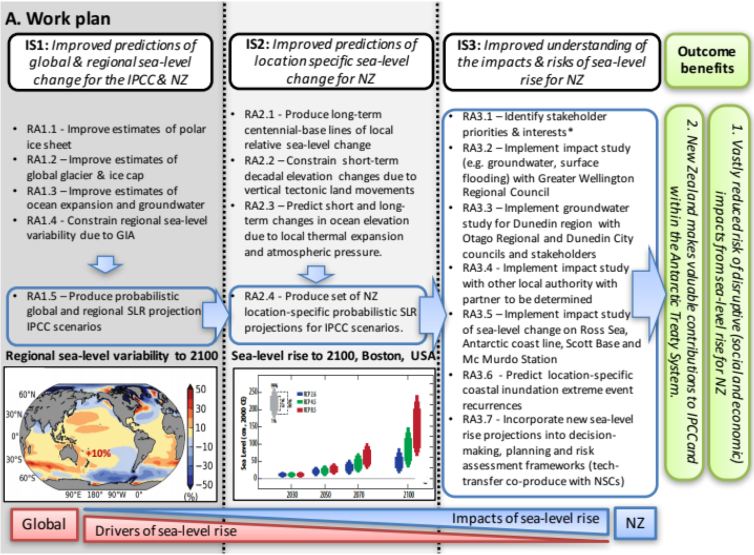
Antarctic Science Platform
National Modelling Hub

VICTORIA UNIVERSITY WELLINGTON
TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI

MINISTRY OF BUSINESS, INNOVATION & EMPLOYMENT
HĪKINA WHAKATUTUKI

NZ SEA RISE
TE TAI PARI O AOTEAROA

1 | NZ SeaRise Programme



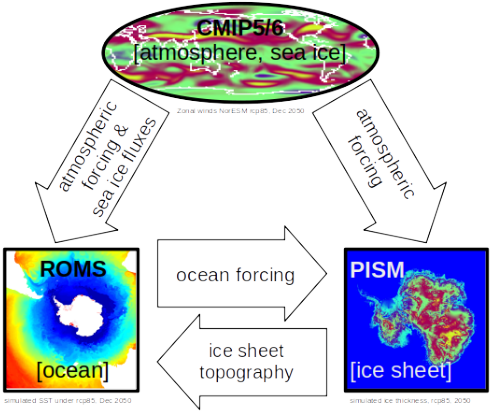
2 | Antarctic Ice Loss Prediction to 2300 - Coupling PISM to ROMS

Parallel Ice Sheet Model

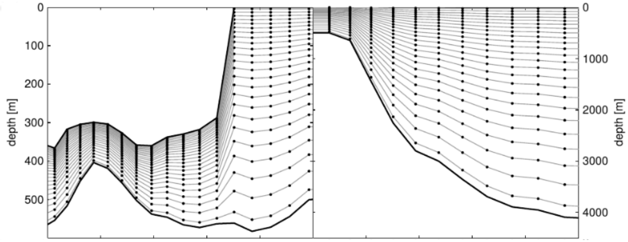
- open source, parallel, simulations at high resolution
- extensible atmospheric/ocean coupling
- shallow, hybrid, and higher-order stress balances
- marine ice sheet physics
- subglacial hydrology and till model

Regional Ocean Modeling System

- hydrostatic, primitive equation, Boussinesq ocean circulation model
- designed primarily for coastal applications
- terrain-following vertical coordinates allow for greater vertical resolution in shallow water and regions with complex bathymetry
- ice ocean boundary layer is better resolved than in z-coordinates



3 | ROMS Terrain following Vertical Coordinate System



Jendersie et al. (2016)

- vertical model layers vary in thickness
- resolution can be pushed towards certain parts of the water column e.g. surface or bottom
- ice ocean boundary layer is better resolved than in z-coordinates

3 | ROMS Terrain following Vertical Coordinate System

- stretching can be a simple function like

$$
\begin{aligned}
z(\sigma, h)_{i,j,k} &= \sigma_k h_{i,j} \\
\sigma_k &= \frac{2k-1}{2N}
\end{aligned}
$$

with z, the depth position of a grid point; (i, j) and (k) its horizontal and vertical numerical index; the ocean depth $h_{i,j}$; and N the number of vertical layers

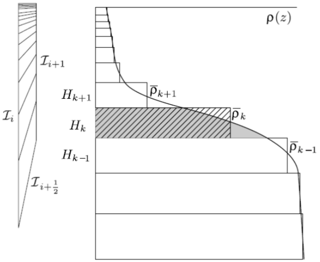
- usually computed from a convoluted transformation functional like

$$
\begin{aligned}
z(h, S, \zeta)_{i,j,k} &= S(h, \sigma, C)_{i,j,k} + \zeta(t)_{i,j} \left[1 + \frac{S(h, \sigma, C)_{i,j,k}}{h_{i,j}}\right] \\
S(h, \sigma, C) &= h_c \sigma + \left[h_{i,j} - h_c\right] C(\sigma)_k \\
C(\sigma) &= (1 - \theta_B) \frac{\sinh(\theta_S \sigma)}{\sinh \theta_S} + \theta_B \left[\frac{tanh[\theta_S(\sigma + \frac{1}{1})]}{2 \tanh(\frac{1}{2}\theta_S)} - \frac{1}{2}\right]
\end{aligned}
$$

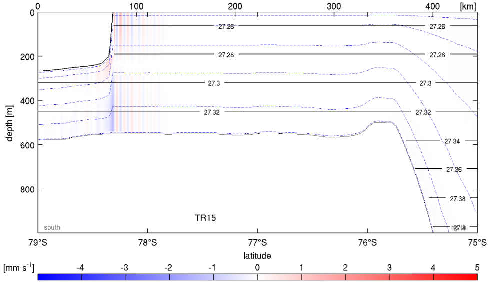
with θ_S & θ_B resoluton coefficients for surface and bottom

4 | PGE - Pressure Gradient Error

- with steep step changes in topography terrain following coordinates are prone to numerical errors
- piece wise reconstruction of the density profile at every baroclinic time step
- the local difference in vertical resoluton (evaluated horizontally) causes differences in the density integration
- spurious pressure gradients hence advection



Shchepetkin and McWilliams (2005)



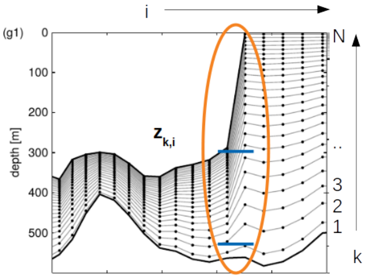
Jendersie et al. (2016)

5 | The Hydrostatic Stability Number - rx_1

$$rx_0 = \frac{|h_i - h_{i,j}|}{h_i + h_{i,j}}$$

$$rx_1 = \frac{\left|z_{i,k} - z_{i+1,k} + z_{i,k-1} - z_{i+1,k-1}\right|}{(z_{i,k} + z_{i+1,k} - z_{i,k-1} - z_{i+1,k-1})}$$

- rx are defined at half way between grid points
- rx_0 is a simple measure of depth change with respect to the total depth
- rx_1 expresses the ratio of step change between numerical neighbours z_i and z_{i+1} and thickness of the associated grid cells
- if both neighbouring cells share a level $rx_1 \approx 1$
- different values for stable rx1 in the literature, generally recommended $rx_1 < 6$
- rx_1 provides a metric to assess the stability of a given modelling bathymetry and the chosen grid

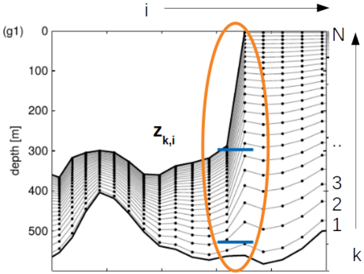


Jendersie et al. (2016)

5 | The Hydrostatic Stability Number - rx_1

$$
\begin{aligned}
rx_0 &= \frac{|h_i - h_{i,j}|}{h_i + h_{i,j}} \\
rx_1 &= \frac{\left|z_{i,k} - z_{i+1,k} + z_{i,k-1} - z_{i+1,k-1}\right|}{(z_{i,k} + z_{i+1,k} - z_{i,k-1} - z_{i+1,k-1})}
\end{aligned}
$$

- mostly used methods of mitigation is selective smoothing of bathymetric surfaces; Shapiro filter, Laplacian filter, Martinho & Batteen scheme etc
- selective application easily conducted for rx_0 but not rx_1
- rx_0 useful for ocean bathymetry but not for ice shelves, especially near grounding lines
- treat local problem of steepness between $z_{i,k}$ and $z_{i+1,k}$
- needs a method that treats both surfaces in conjunction and actually optimizes rx_1
- method needs to be aware of the vertical stretching and its parameters



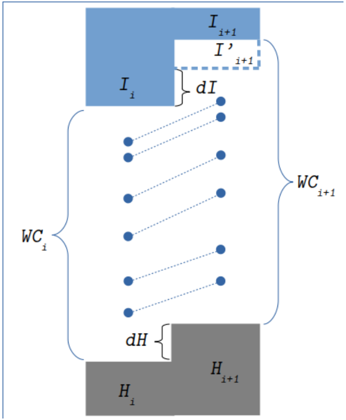
Jendersie et al. (2016)

6 | 4D Lookup Table for rx_1

- the simple approach - change I_{i+1}
- where

$$rx > rx_{max}$$
modify
$$I_{i+1} \rightarrow I'_{i+1}$$
that satisfies
$$rx \approx rx_{max}$$

- thre are either *no* or *two* solutions for I'_{i+1}
- *no* solution
  $|H_i - H_{i+1}|$ is big $\rightarrow rx$ is violated at the bottom;
  no I'_{i+1} will satisfy rx_{max}
- *two* solutions
  $I'_{i+1} > I_i$ & $I'_{i+1} < I_i$ (non symmetrical!)
- implicit assumptions $\theta_B \geq 1$ & $\theta_S \geq 1$

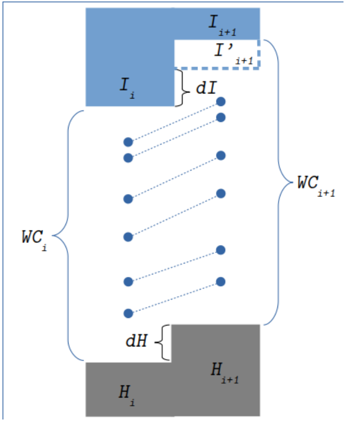


6 | 4D Lookup Table for rx_1

- is there an analytical solution for $I'_{i+1}(rx_{max}, H_i, H_{i+1}, I_i, S)$?
- find I'_{i+1} from a 4D look up table RX
- with the variable transformations

$$
\begin{aligned}
WC_i &= I_i - H_i \\
dI_{i,i+1} &= \frac{I_i - I'_{i+1}}{WC_i} \\
dH_{i,i+1} &= \frac{H_i - H_{i+1}}{WC_i}
\end{aligned}
$$

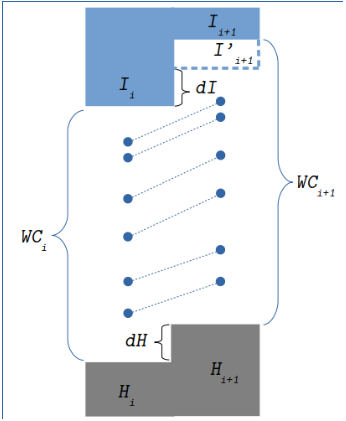
- rx_1 is now a function of dH, dI, WC_i, I_i, S



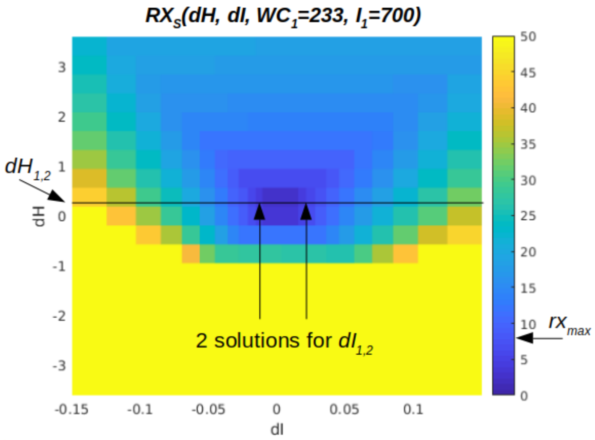
6 | 4D Lookup Table for rx_1

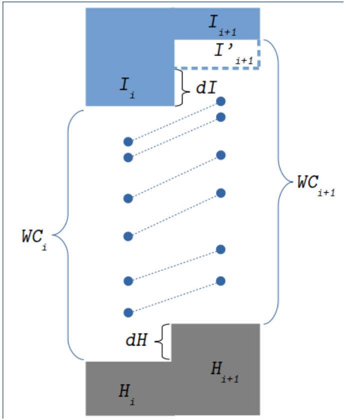
- for a given S compute rx_1 over ranges of
  $I_i\{0 \rightarrow 2500\}[m]$
  $WC_i\{0 \rightarrow 2500\}[m]$
  $dI\{-2 \rightarrow 2\}$
  $dH\{-2 \rightarrow 2\}$
- yields lookup table $\mathrm{RX}_S(dH, dI, WC_i, I_i)$ for a given coordinate stretching $S(\sigma, C, N, \theta_S, \theta_B)$
- finding dI by querying RX_S

$$\mathrm{RX}_S(dH, WC_i, I_i)|_{rx_{max}} \rightarrow dI$$

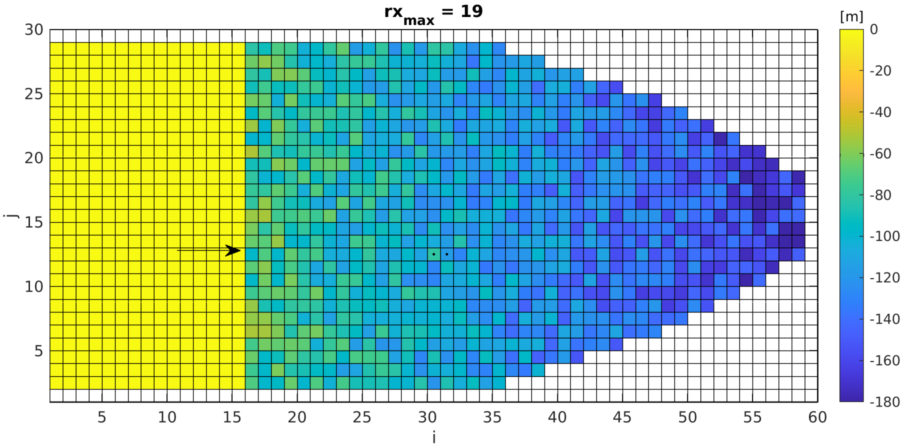


6 | 4D Lookup Table for rx_1

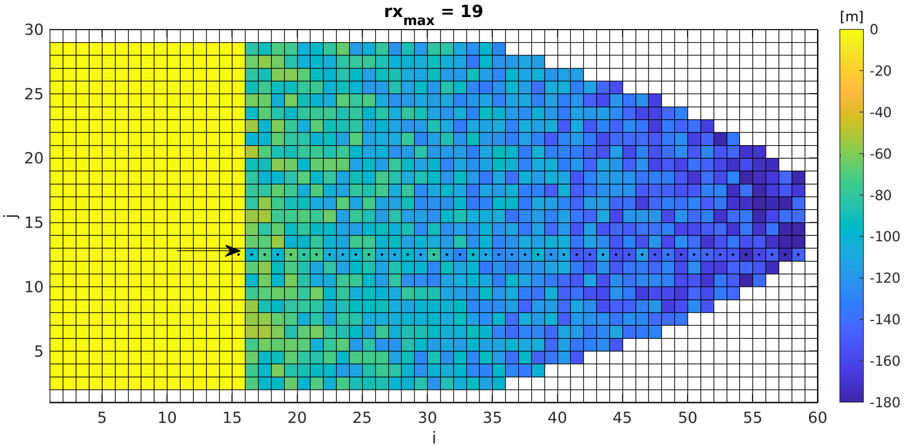




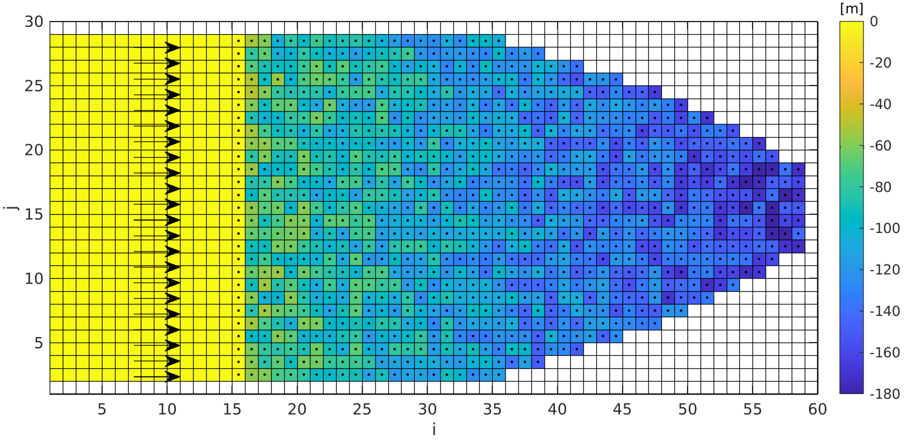
7 | Progressively Rotating 1d Forward Correction of rx_1



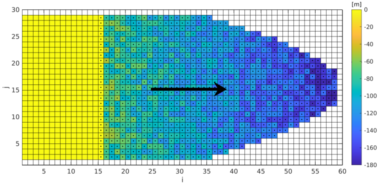
7 | Progressively Rotating 1d Forward Correction of rx_1



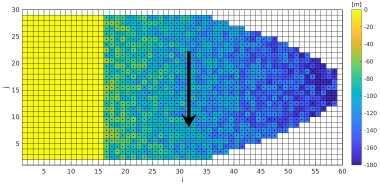
7 | Progressively Rotating 1d Forward Correction of rx_1



7 | Progressively Rotating 1d Forward Correction of rx_1

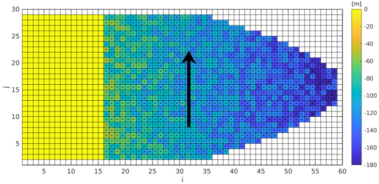


check rx_1 $\Longrightarrow$

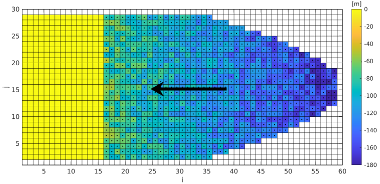


check rx_1 $\Uparrow$

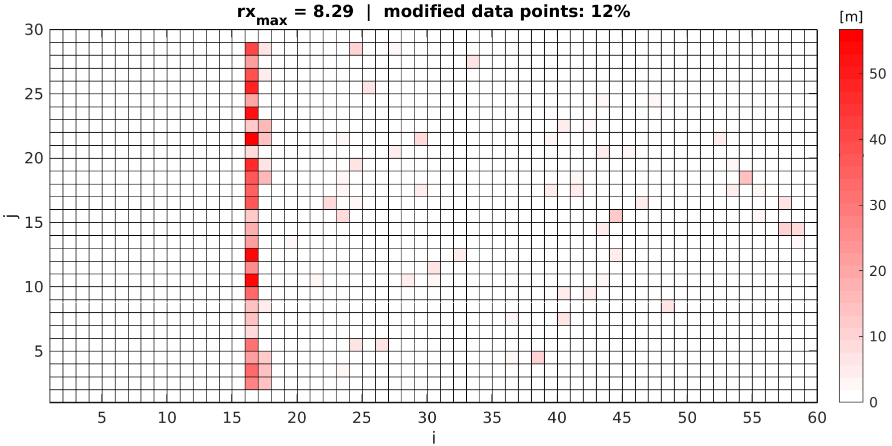
check rx_1 $\Downarrow$



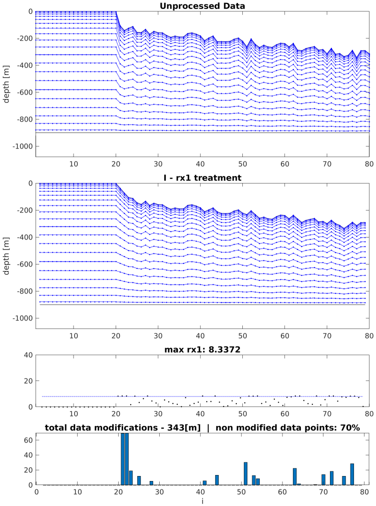
check rx_1 $\Longleftarrow$

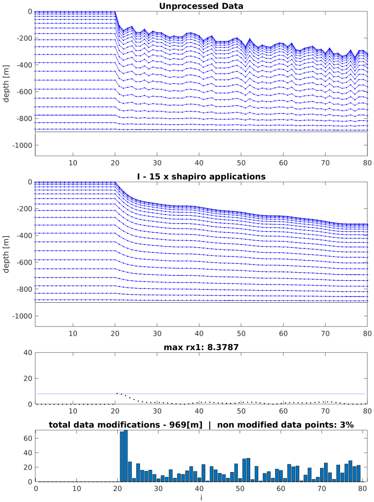


7 | Progressively Rotating 1d Forward Correction of rx_1



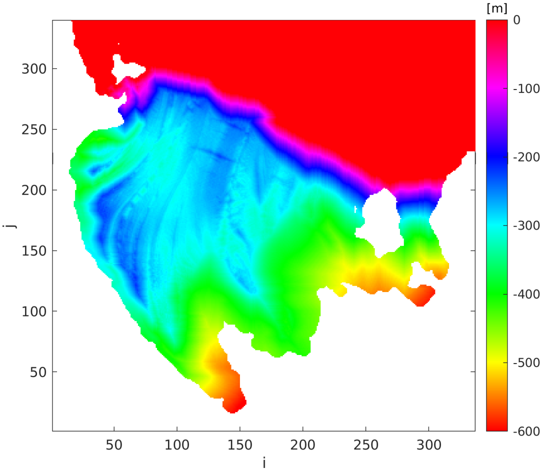
7 | Progressively Rotating 1d Forward Correction of rx_1





8 | Results - Ice Draft

- Ross Ice Shelf
- 3km horizontal grid spacing, $rx_1 \approx 4$
- western and central base topography is largely unmodified
- suture zones and fractures are still represented
- modifications at the ice shelf front and towards the deep grounding zones



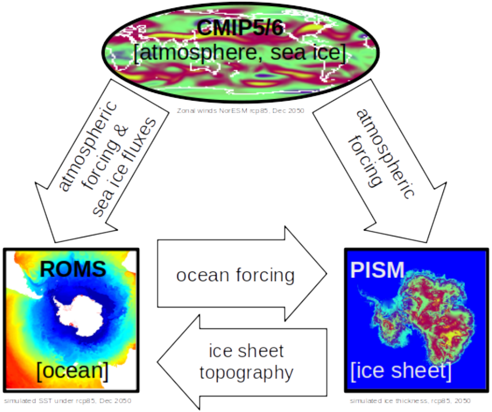
8 | Results - Ocean - Ice Sheet Coupled Model

- **coupler** - 2000 lines (bash script)
- rx_1 **treatment** - 7000 lines (matlab)

- **ROMS → PISM Coupling Point [freq=1yr]**
- melt rate, ice base temperature (annual averages)
  - regridding
  - unit conversion
  - conserve fresh water flux per individual ice shelf

- **PISM → ROMS Coupling Point [freq=5yr]**
- ice shelf draft, dynamic ice & ocean masks (snap shots)
  - regridding
  - remove ocean narrows and tight corners
  - rx_1 treatment for each individual ice shelf

- conserved quantities: fresh water flux
- not conserved: ice thickness

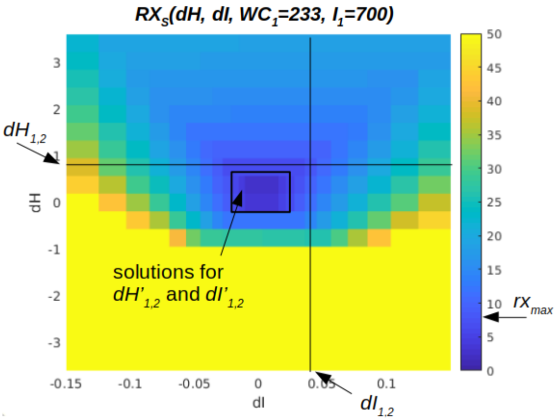


9 | Summary & Outlook

- rx_1 correction method works reliable (converges)
- minimum modification of data points
- ensures baroclinic ocean time stepping near the theoretical limit (dt=20min @ 8km)

- expand method to correct both surfaces in conjunction
- implement in a suitable coupler framework

...

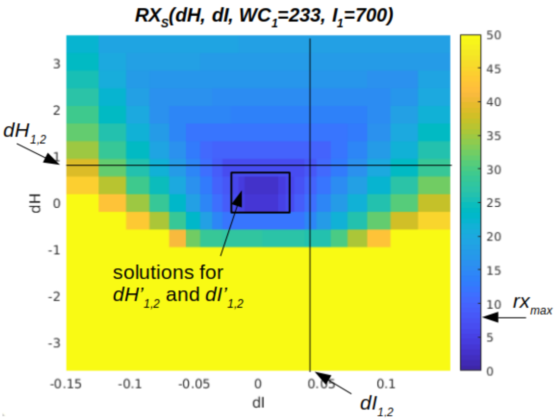


9 | Summary & Outlook

- rx_1 correction method works reliable (converges)
- minimum modification of data points
- ensures baroclinic ocean time stepping near the theoretical limit (dt=20min @ 8km)

- expand method to correct both surfaces in conjunction
- implement in a suitable coupler framework

...



THANK YOU!