



Caltech



FOURIER NEURAL OPERATORS FOR FAST WEATHER MODELING

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CHALLENGE: DRAMATIC RISE IN EXTREME WEATHER ACROSS THE GLOBE

CLIMATE SCIENCE REQUIRES MILLION-X SPEEDUPS

Computational constraints limit model resolution

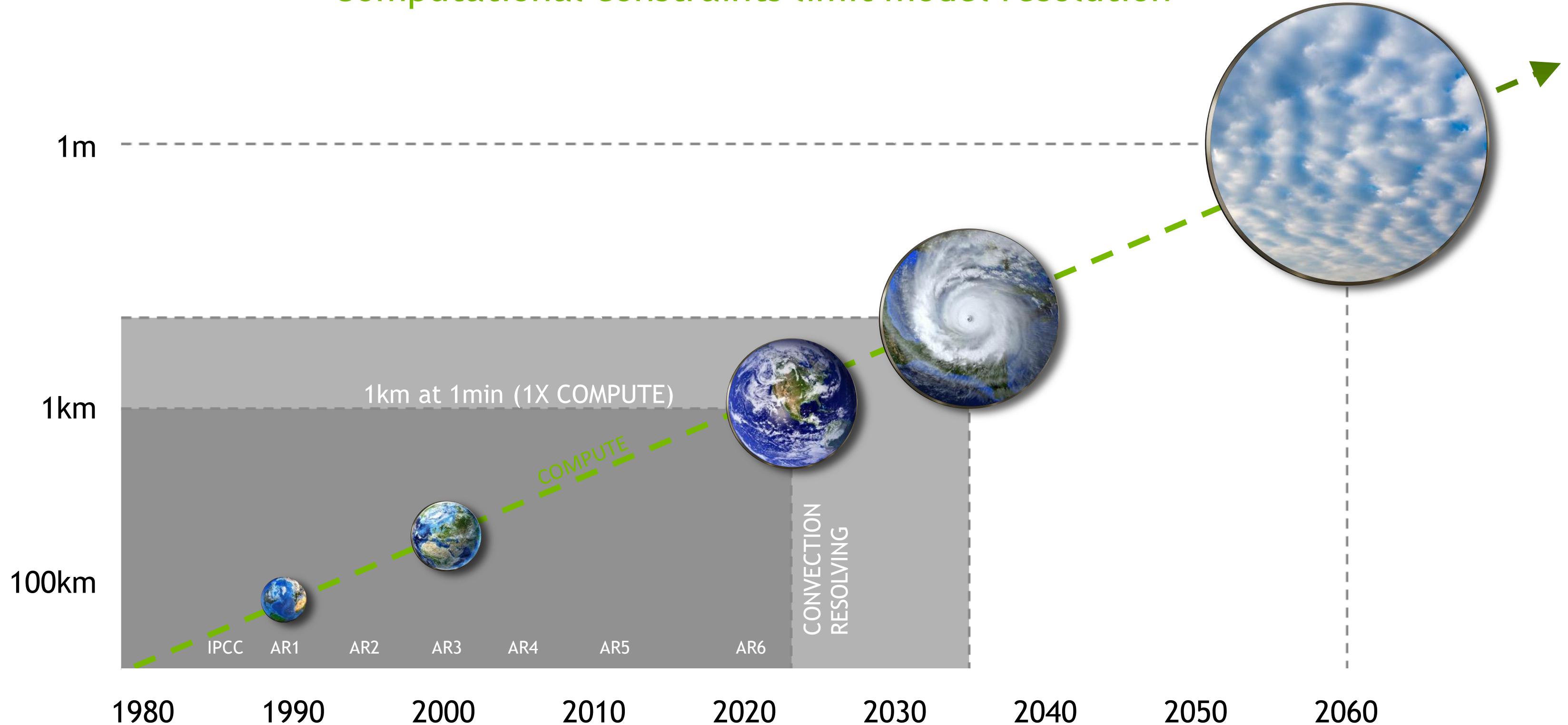
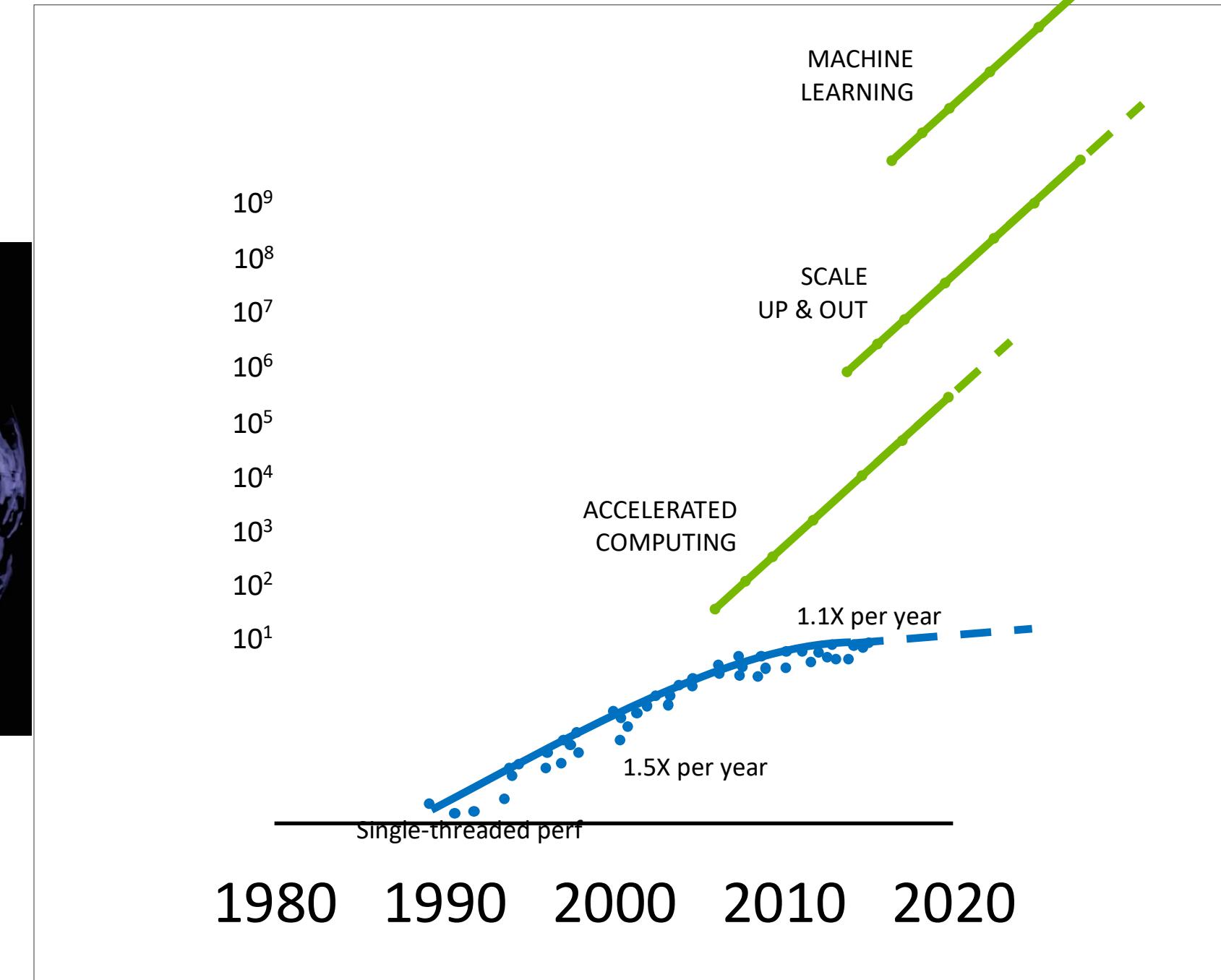
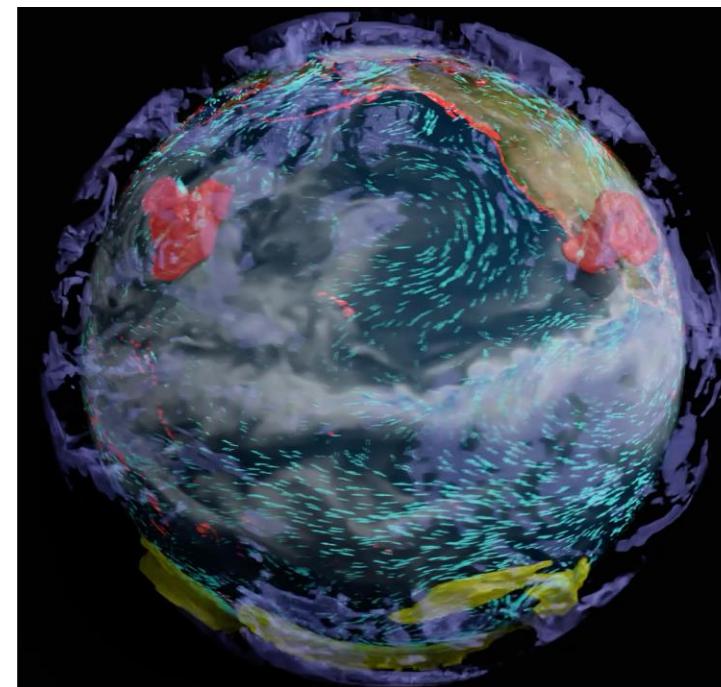
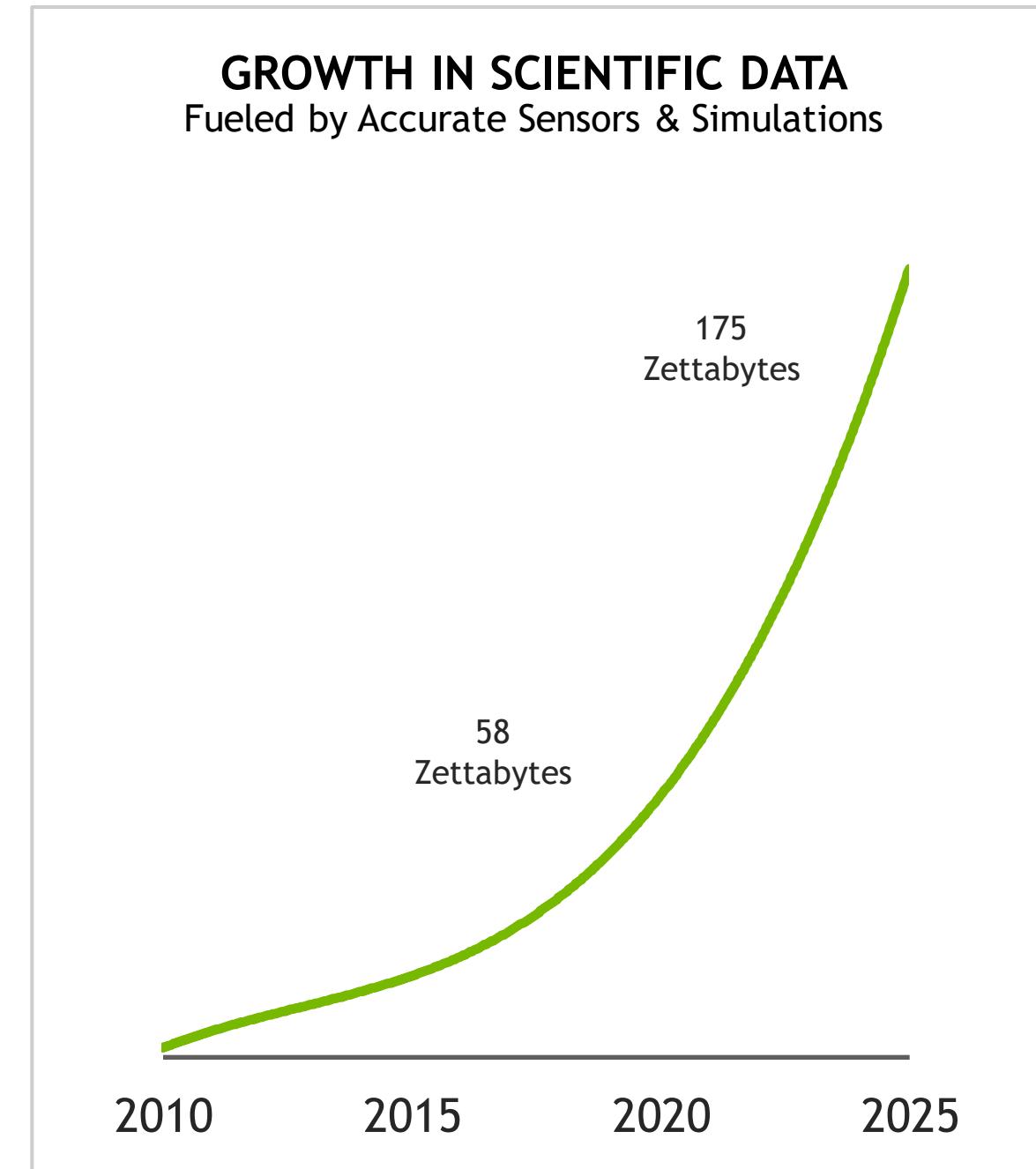
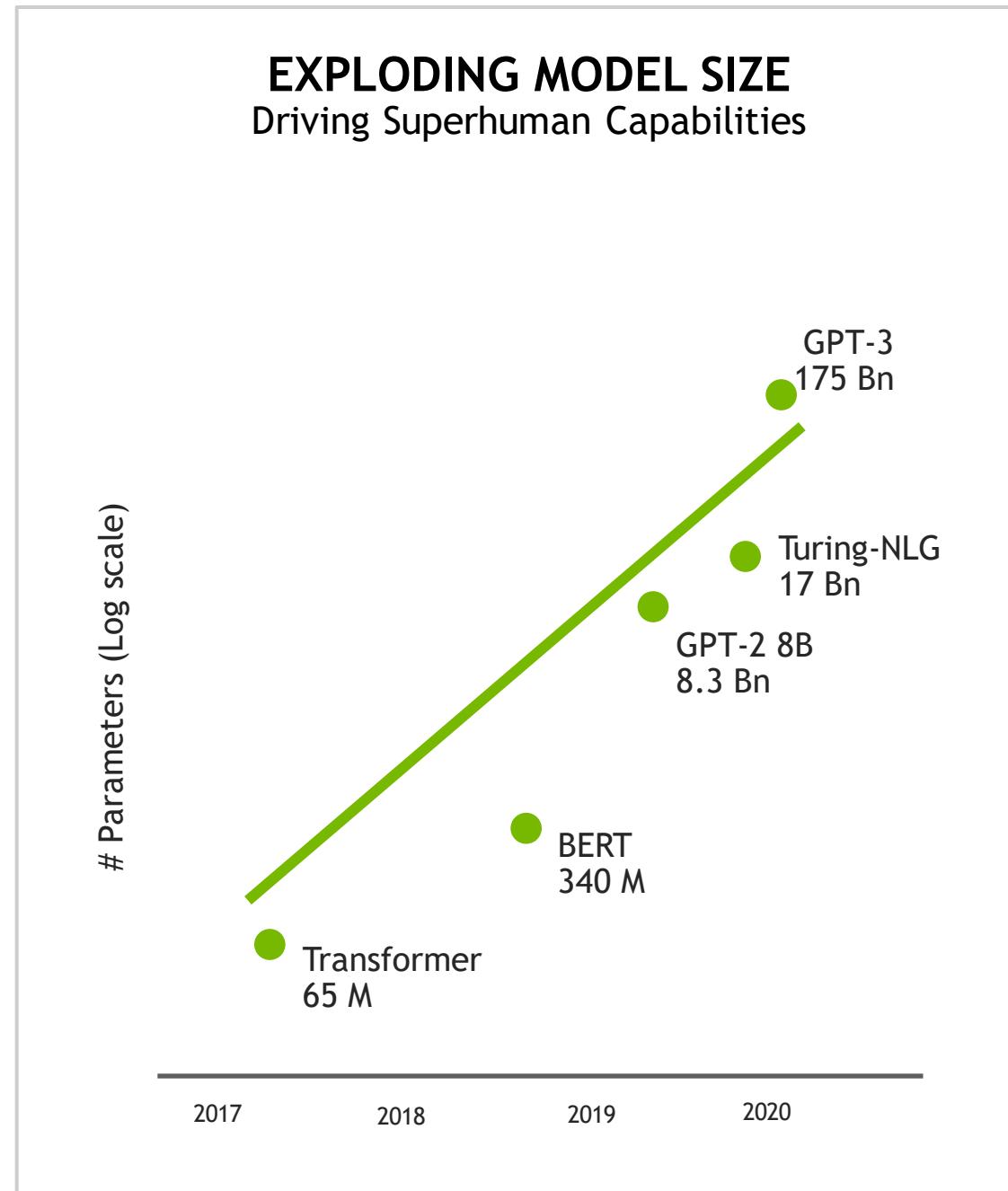


Figure adapted from: Schneider, T., Teixeira, J., Bretherton, C. et al. Climate goals and computing the future of clouds. *Nature Climate Change* 7, 3–5 (2017). <https://doi.org/10.1038/nclimate3190>

MILLION-X LEAP IN SCIENTIFIC COMPUTING



EXPLODING DATA AND MODEL SIZE

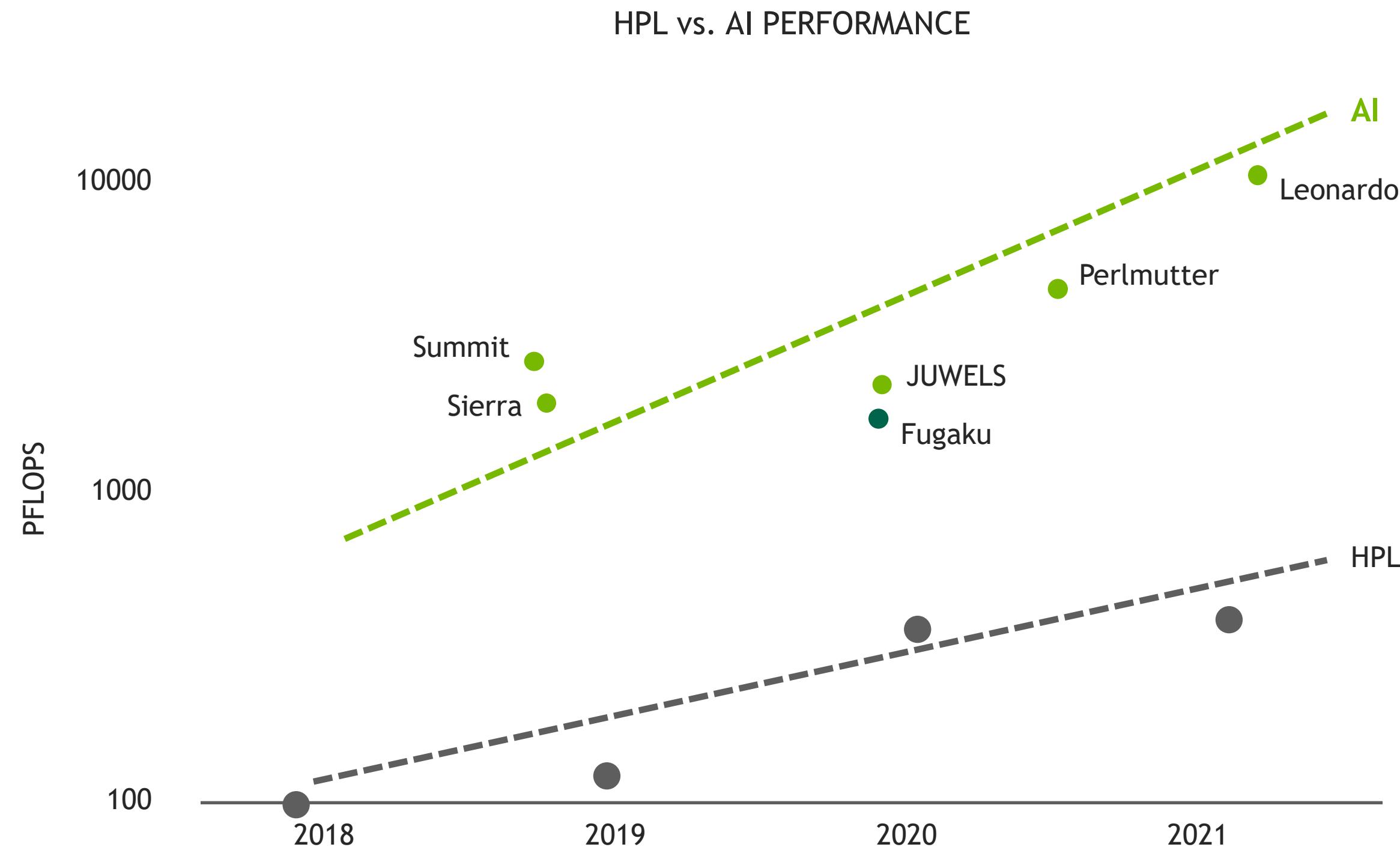


Source for Big Data Growth chart: IDC – The Digitization of the World (May, 2020)



THE ERA OF EXASCALE AI SUPERCOMPUTING

AI is the New Growth Driver for Modern HPC

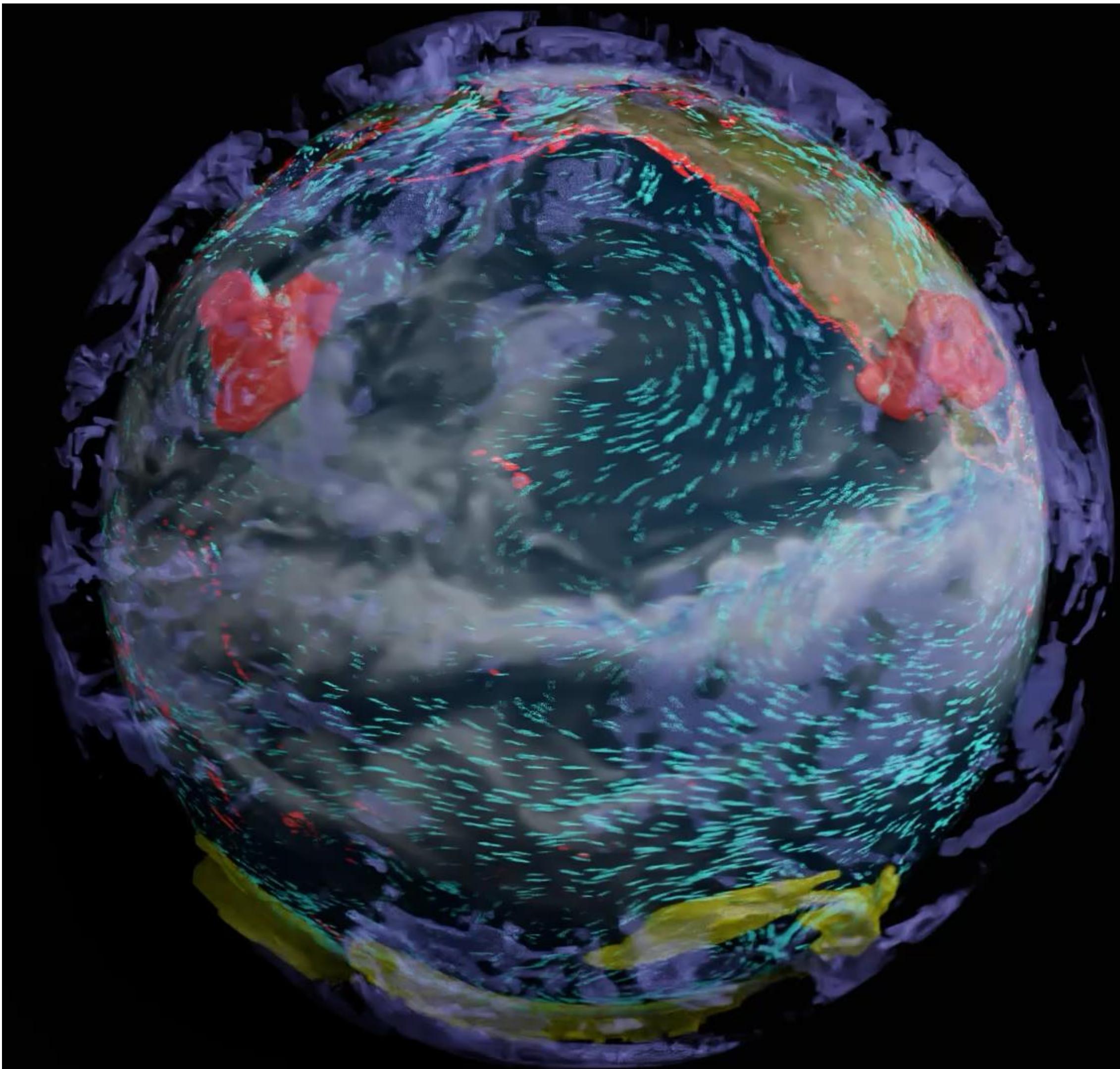


HPL: Based on #1 system in June Top500
AI: Peak system FP16 FLOPS

ANNOUNCING EARTH-2 SUPERCOMPUTER

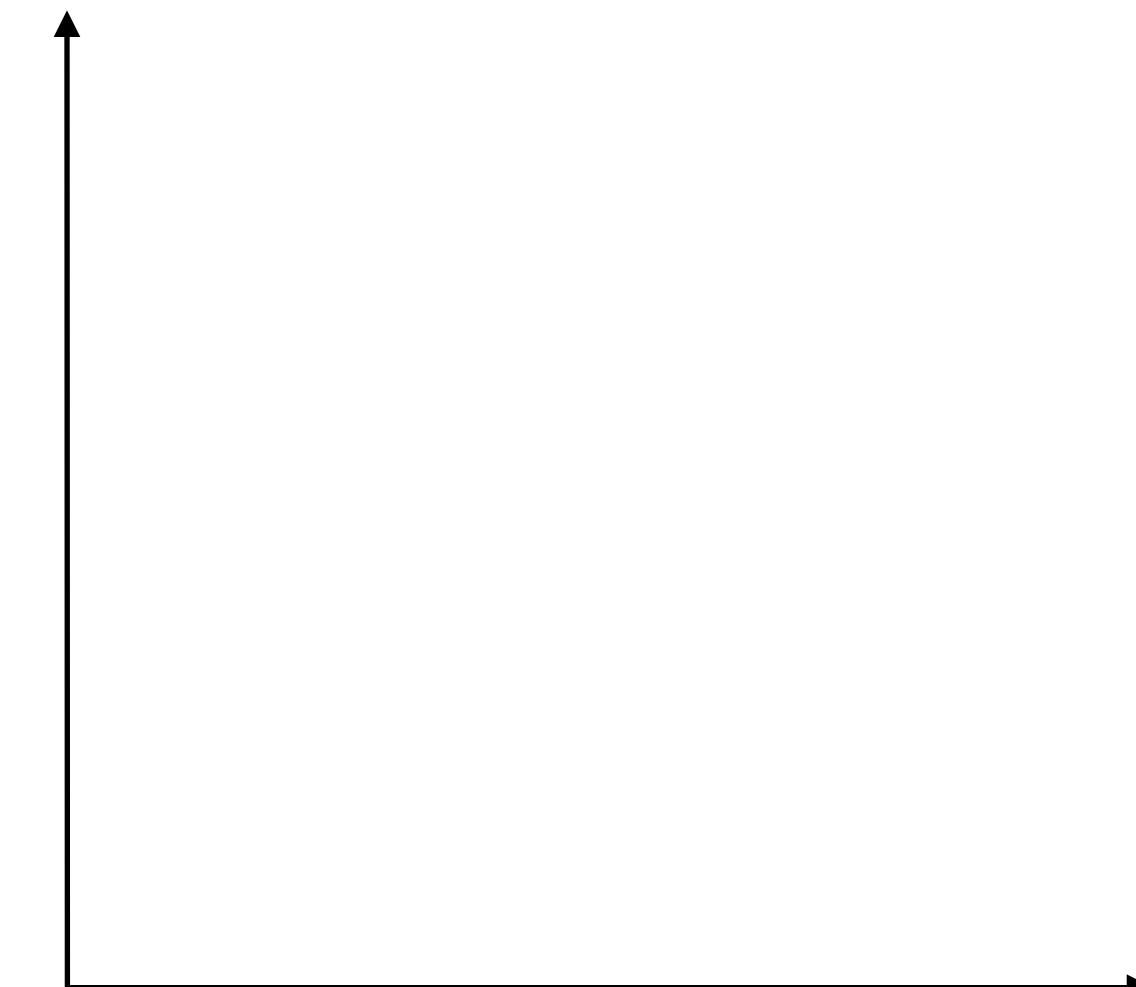
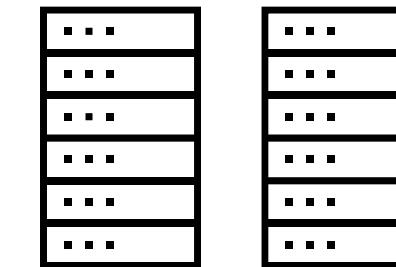
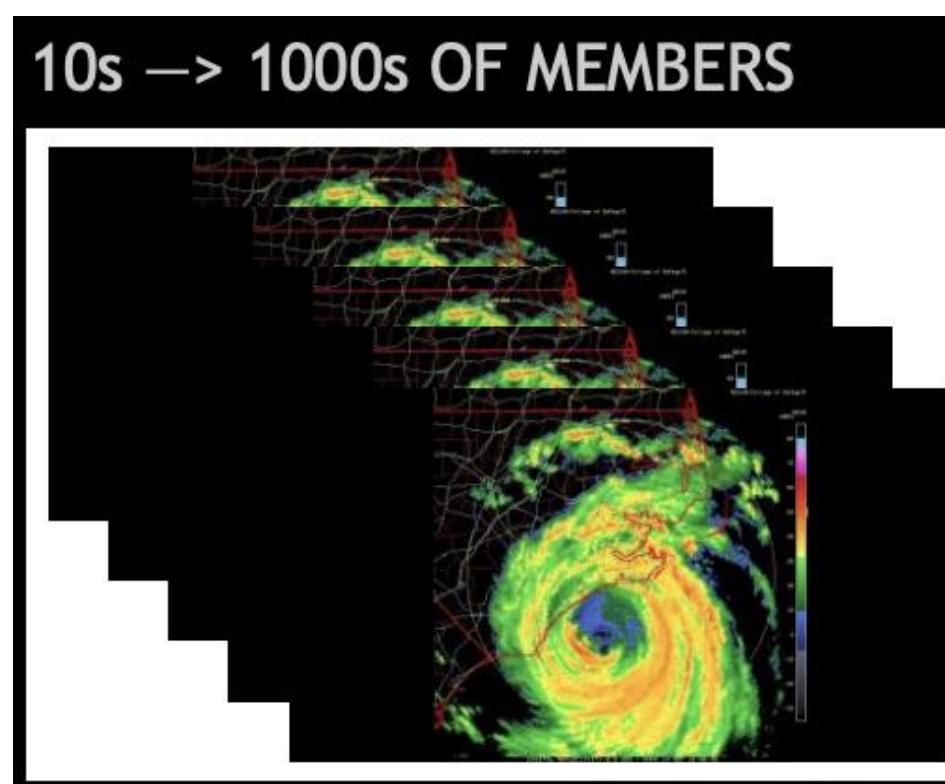
Enabling Million-X Development
in Climate Modeling

- Pledge to dedicate engineering and AI resources at NVIDIA
- Fine-scale weather and climate modeling
- Uncertainty calibration for extreme weather risks
- Simulation is the answer

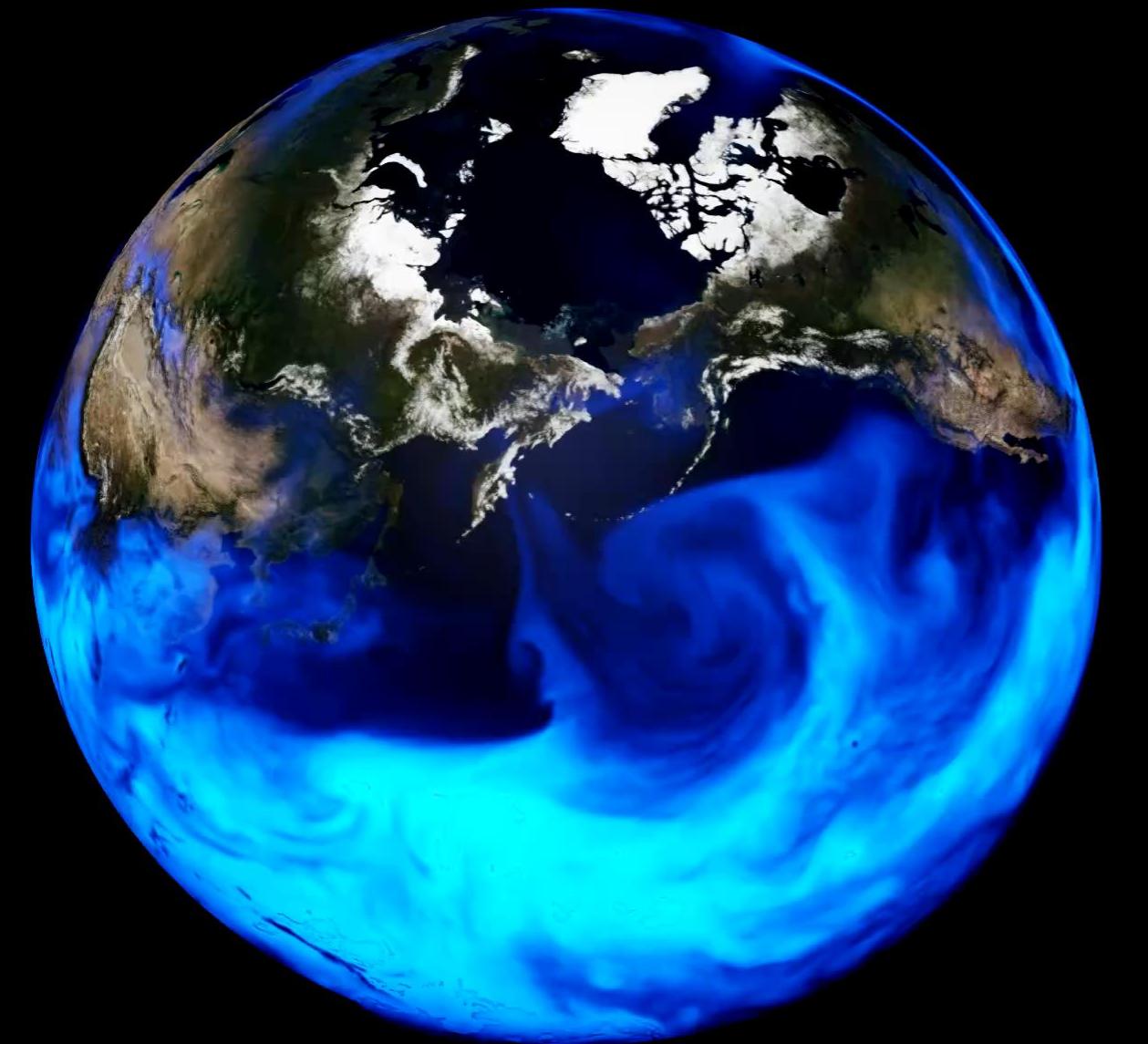


CLIMATE SCIENCE REQUIRES MILLION-X SPEEDUPS

Computational constraints limit the size of ensembles and how many scenarios can be explored



Ground Truth



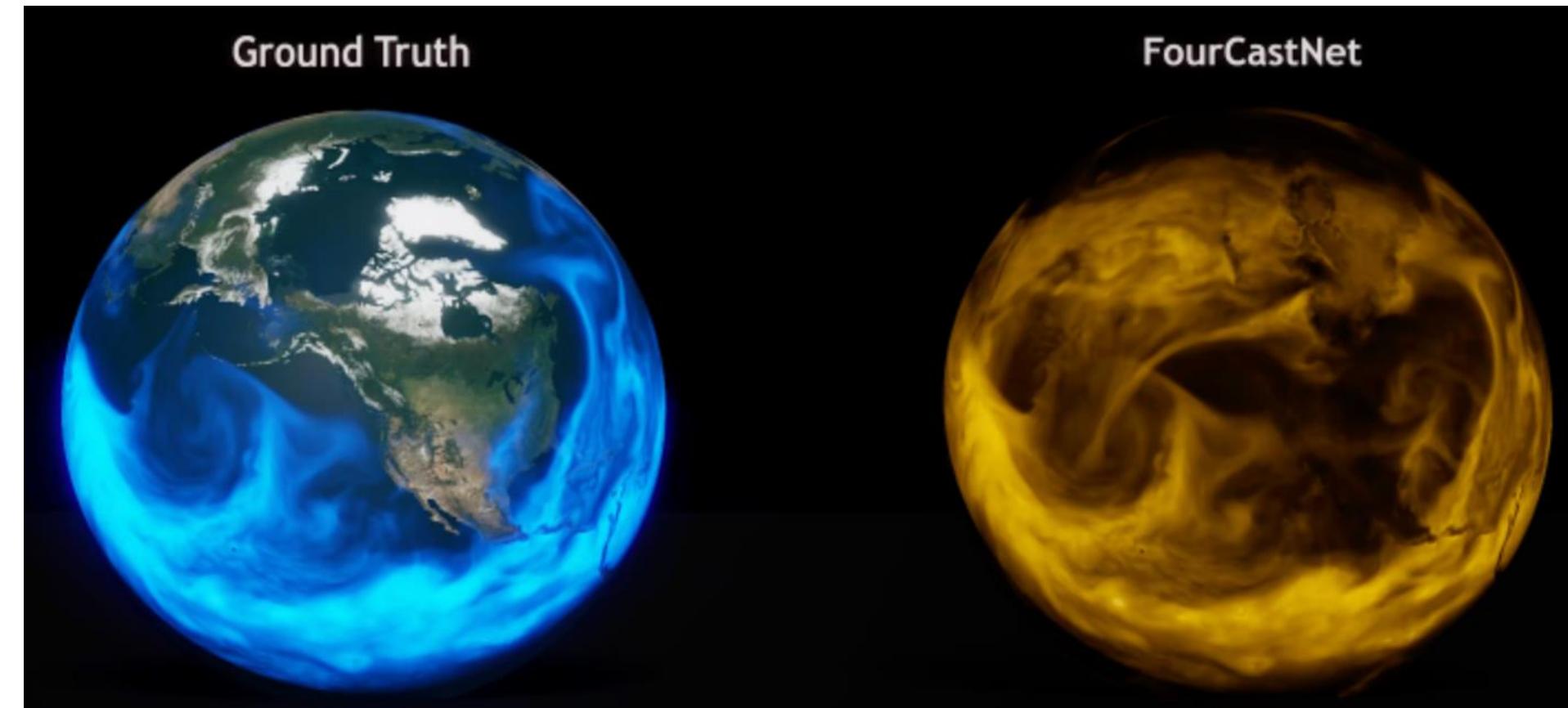
FourCastNet



FINE-SCALE WEATHER PREDICTION

FourCastNet uses FNO to obtain unprecedented speed and accuracy

- FourCastNet is a data-driven model trained on 10 TB of weather data
- Uses FNO with transformer backbone
- Unparalleled accuracy of surface winds and precipitation up to one week.
- 8x higher resolution than any other AI model for weather forecasting.
- 1000-member ensemble in a fraction of a second.
- 4 to 5 orders of magnitude speedup over NWP.
- 25000X smaller energy footprint.

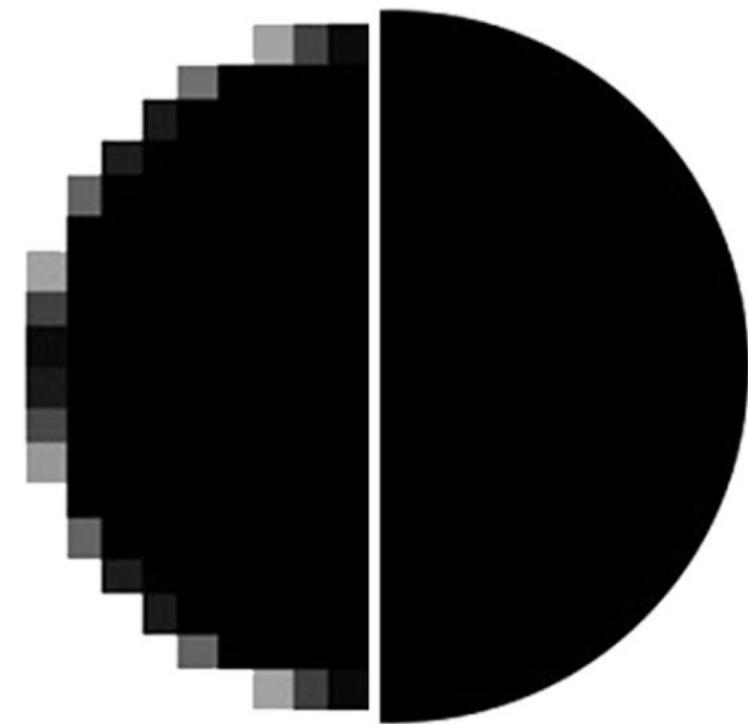
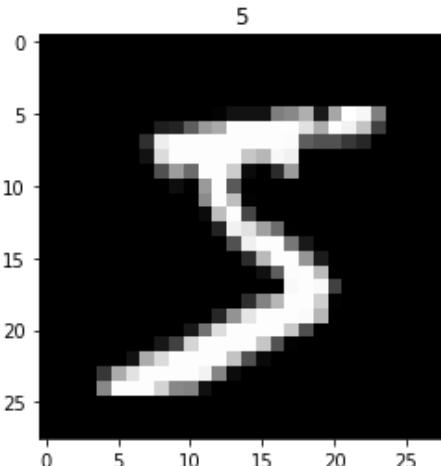
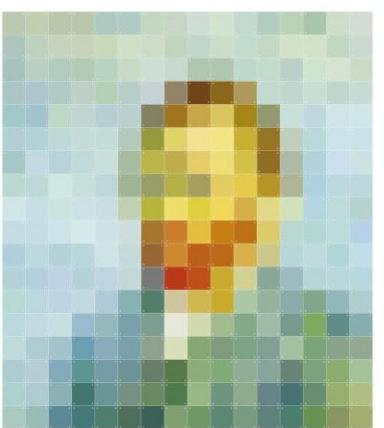


GRID-FREE LEARNING FOR CONTINUOUS PHENOMENA

Learning operators between continuous inputs and outputs

Neural Network

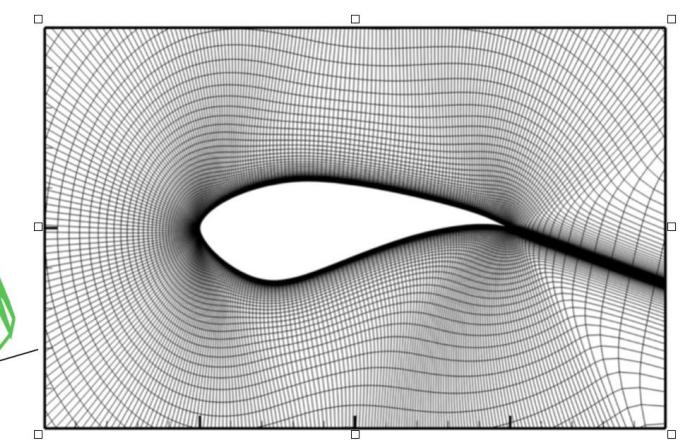
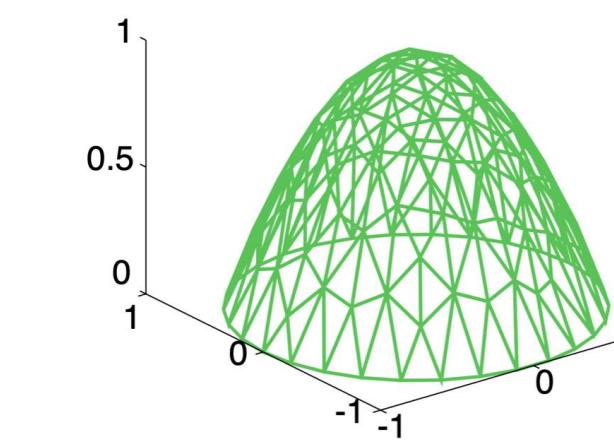
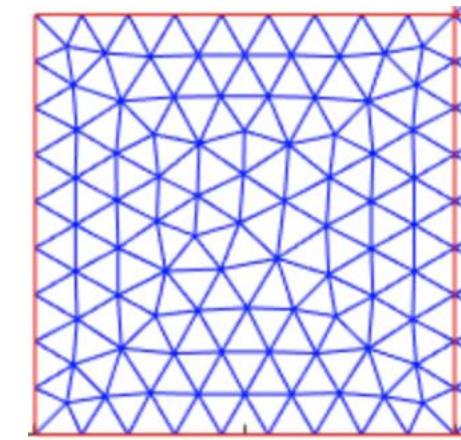
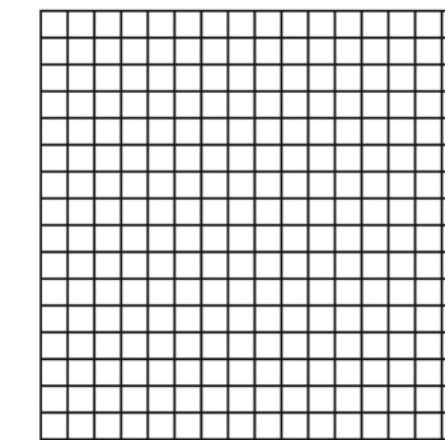
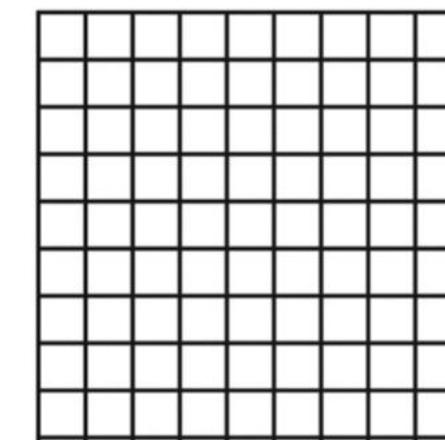
Function: Mapping in finite dimensions.



Discretized input

Neural Operator

Operator: Mapping in infinite dimensions
(function space).

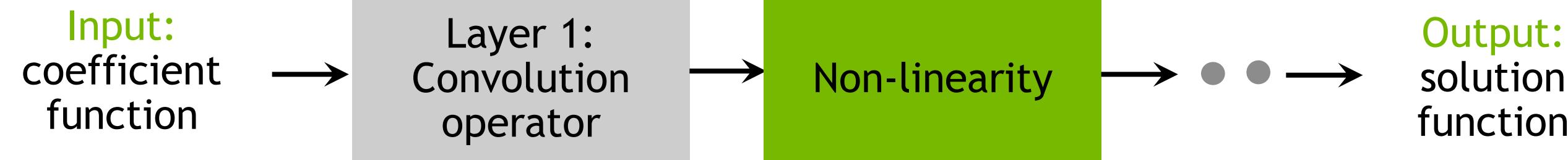
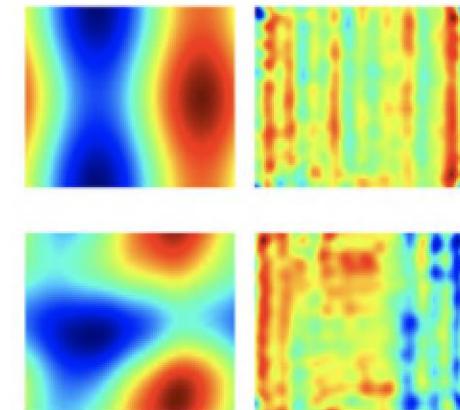
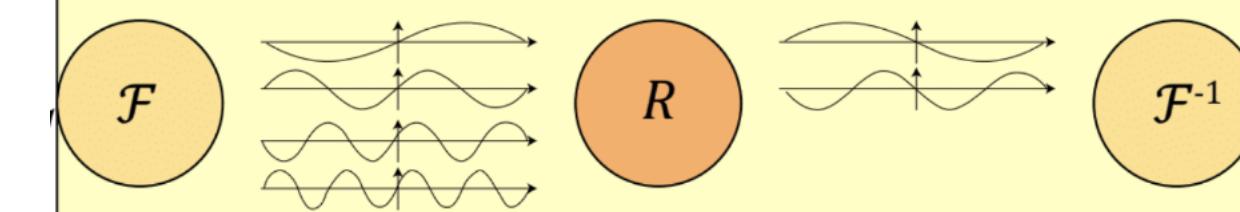


Continuous function

FNO: FOURIER NEURAL OPERATOR

New Framework for Learning PDEs

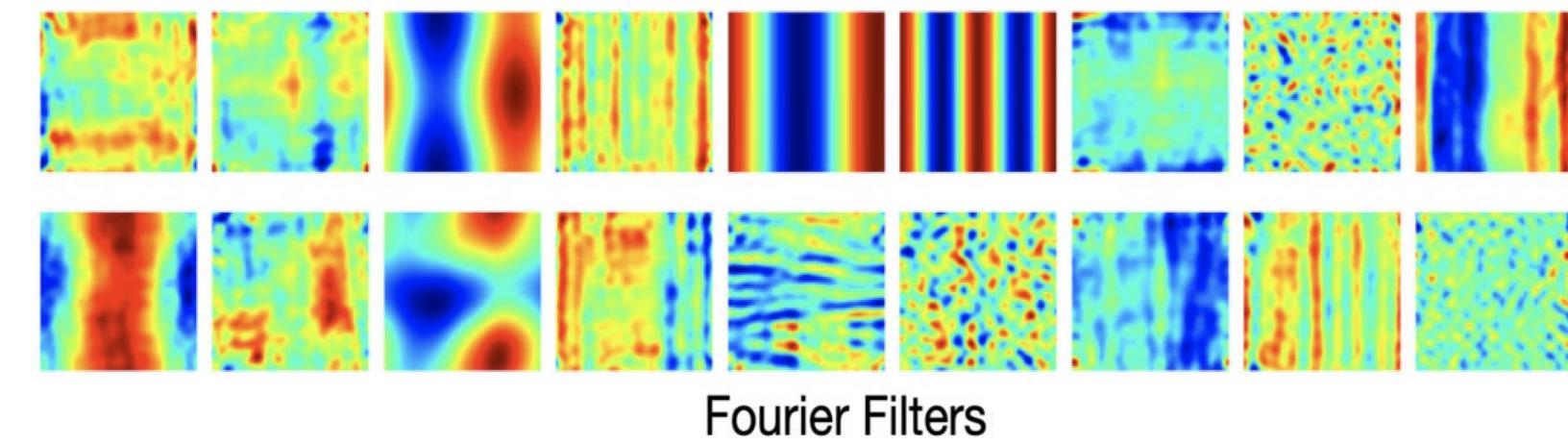
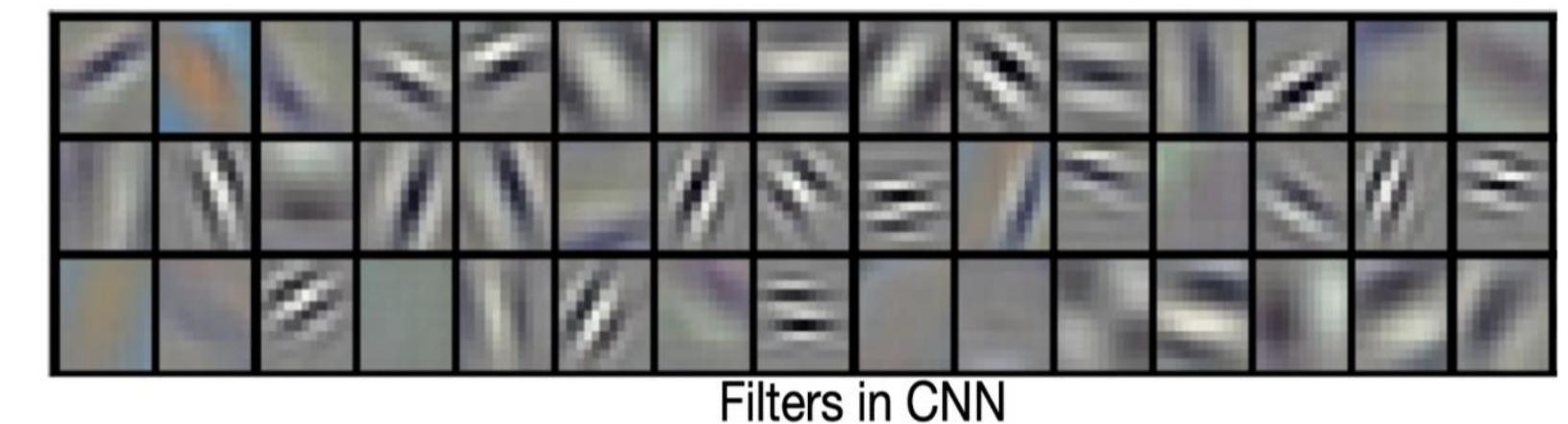
- Use Fourier Transform to implement kernel integration
- Reduce to learning weights in frequency domain



Fourier Neural Operator can approximate any operator: solutions of non-linear PDEs

FOURIER TRANSFORM FOR GLOBAL CONVOLUTION

$$\begin{aligned} Kv_l &= \int_D \kappa(x, y, a(x), a(y)) v_l(y) dy \\ &\approx \int_D \kappa(x - y) v_l(y) dy \\ &= \mathcal{F}^{-1}((\mathcal{F}\kappa) \cdot (\mathcal{F}v_l)) \\ &:= \mathcal{F}^{-1}(R \cdot (\mathcal{F}v_l)) \end{aligned}$$



Learn R in Fourier Domain

NEURAL OPERATOR

Learning in infinite dimensions

Composing linear integral operator with non-linearity.

Input:
Initial and
boundary
conditions



Linear
operator

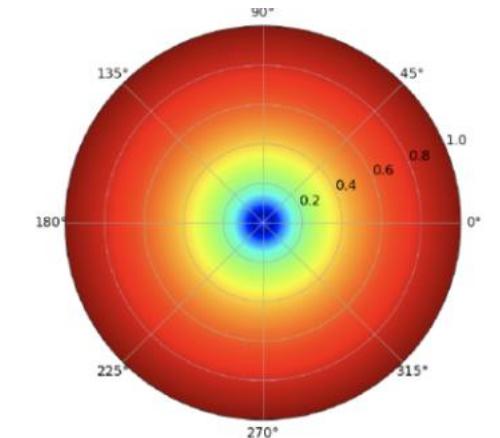


Non-linearity

Output:
Solution
function

Integral Operator

$$\int K(x, y)v(y)dy$$



$K(x, y)$

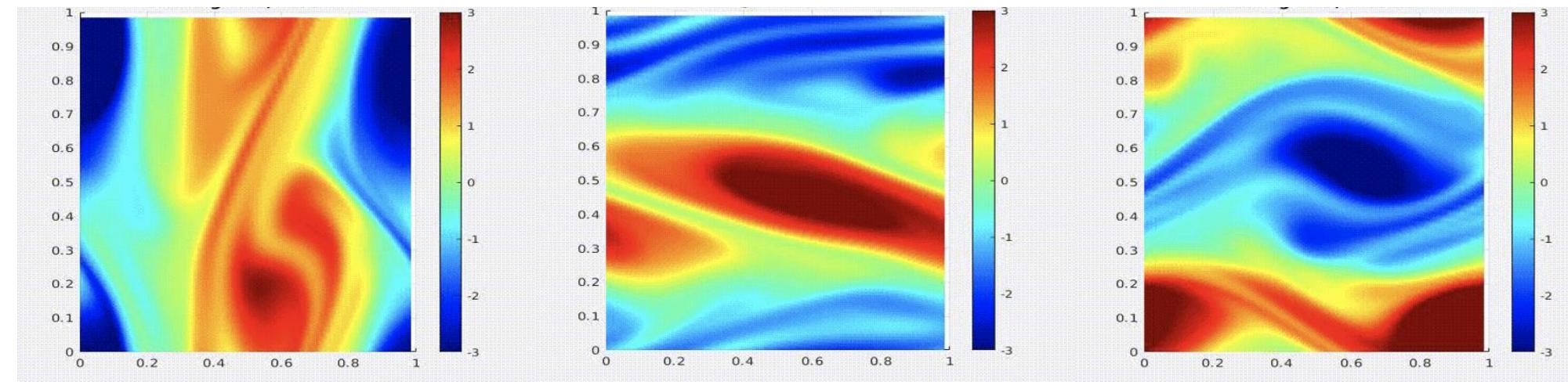
Kernel of integral operator

Neural Operator can approximate any continuous operator

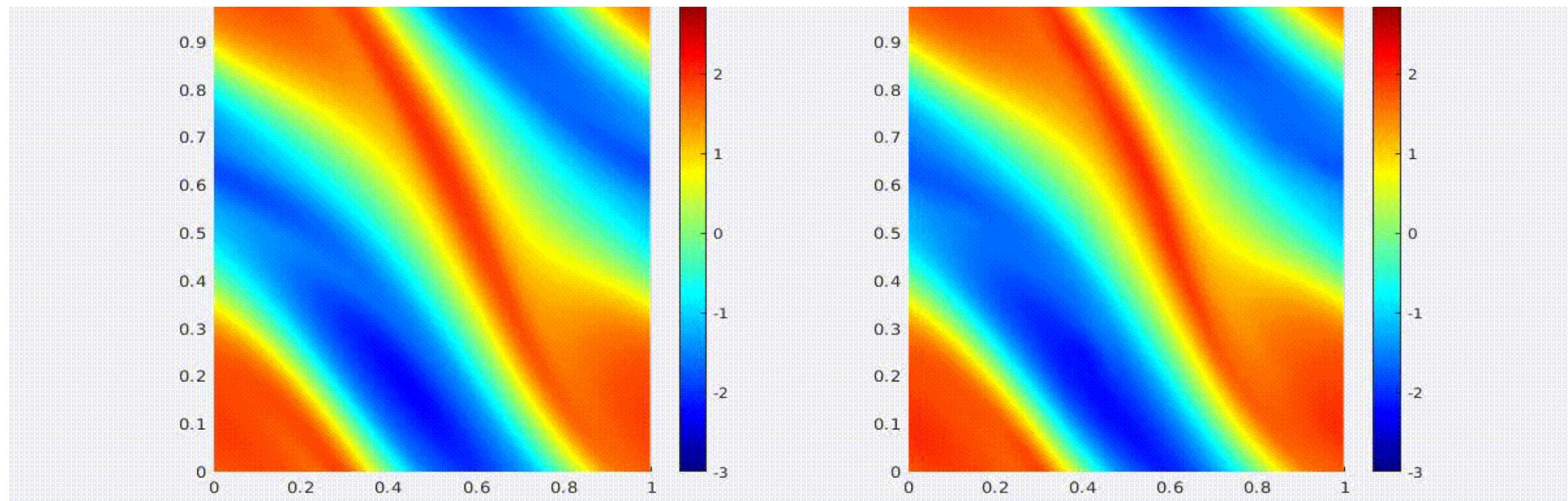
FIRST ML METHOD TO SOLVE FLUID FLOW

Solution in turbulent regime. 1000x faster than traditional solvers

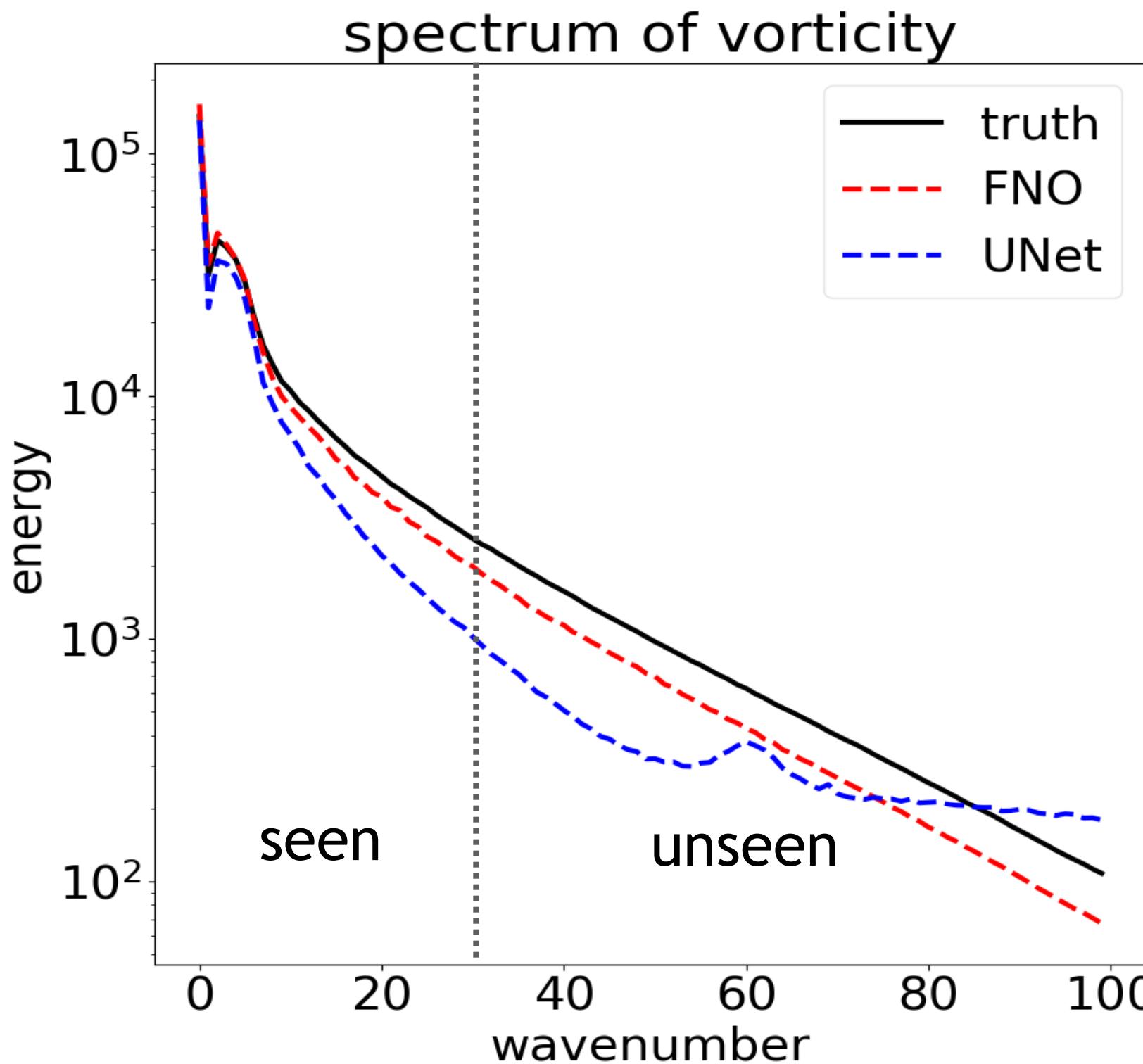
Train using 64*64*20 data (coarse)



Directly evaluate on 256*256*80 (zero-shot super resolution)



FNO CAPTURES THE ENERGY SPECTRUM



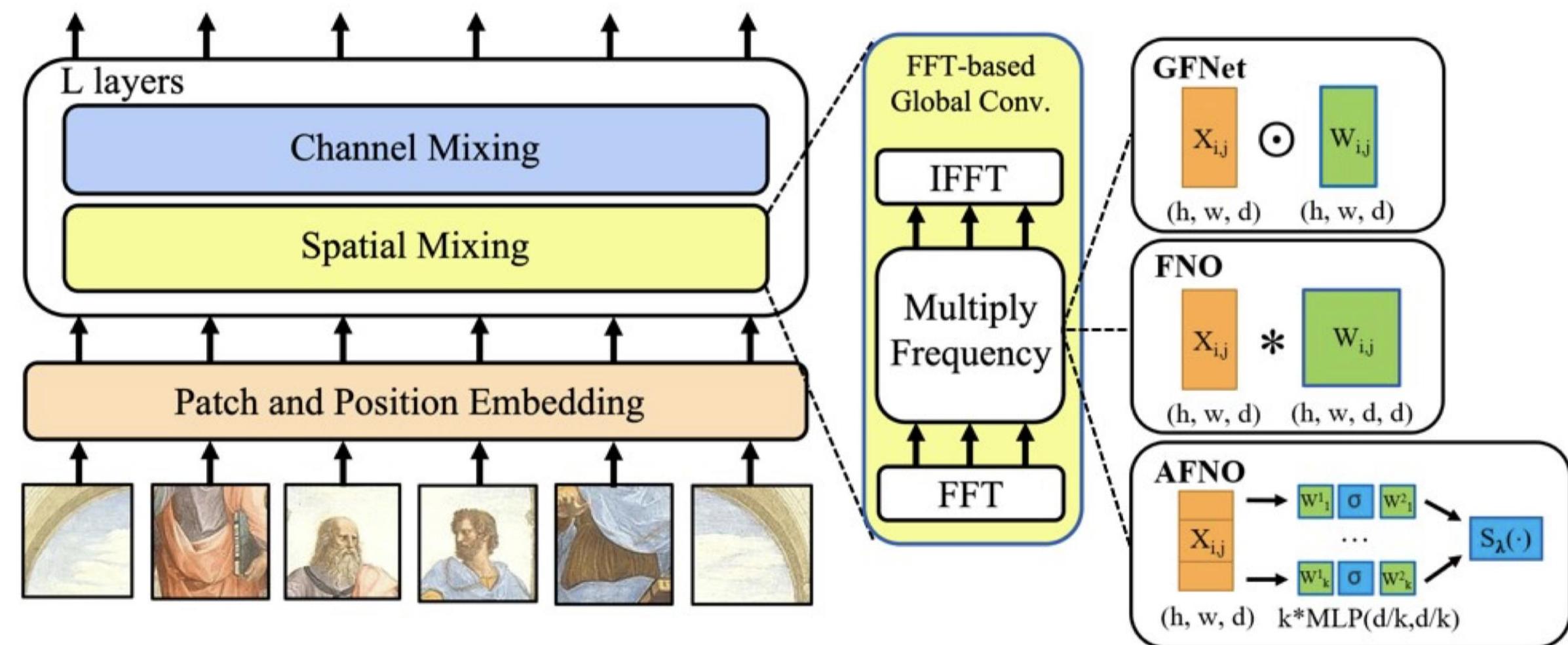
Zero shot super-resolution: train on 64x64, test on 256x256.
Extrapolate to unseen wavenumbers (32->128).

TRANSFORMERS ARE SPECIAL CASES OF NEURAL OPERATOR

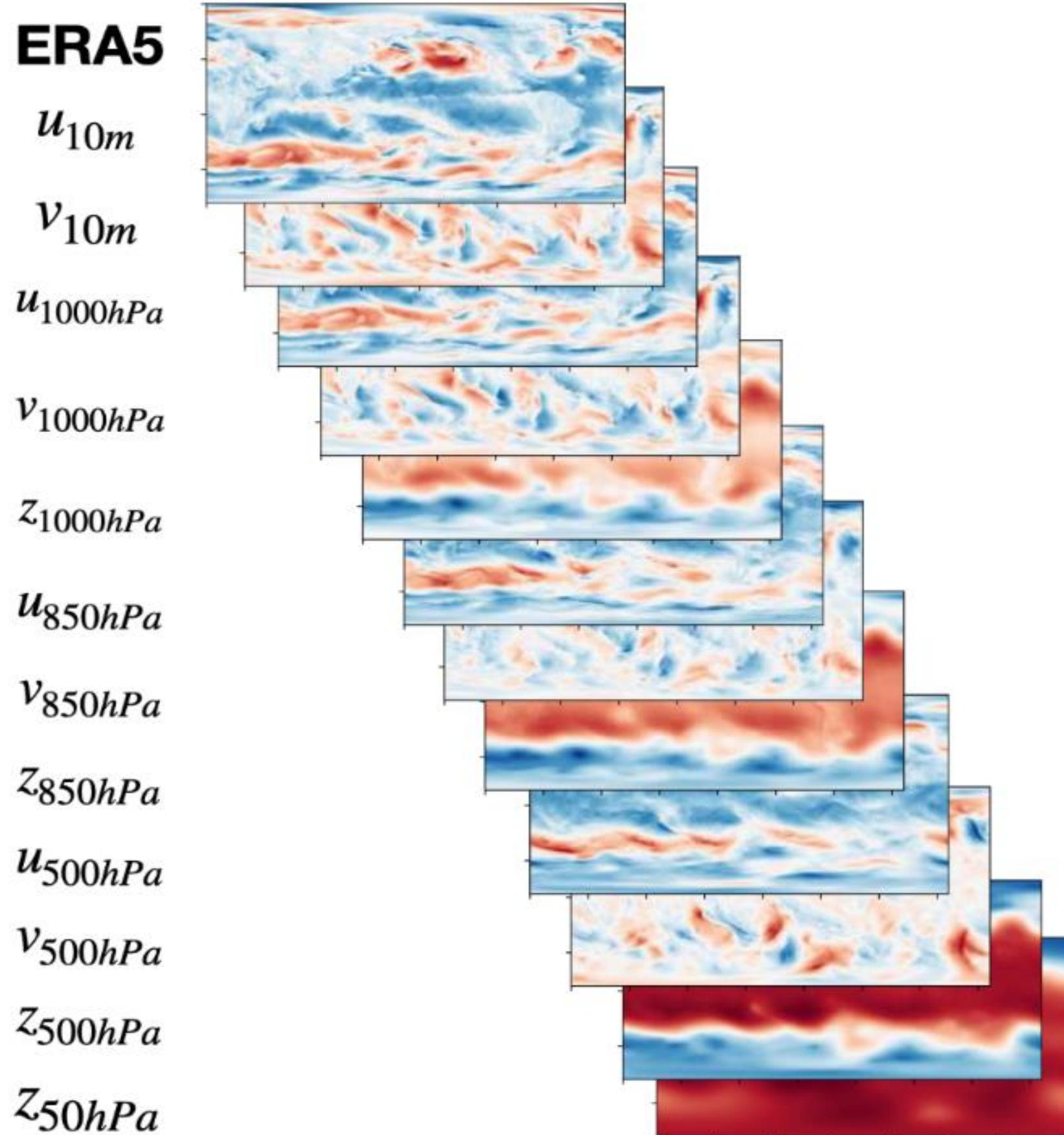
Adaptive Fourier neural operator for efficiency

Neural operator is a continuous generalization of Transformers.

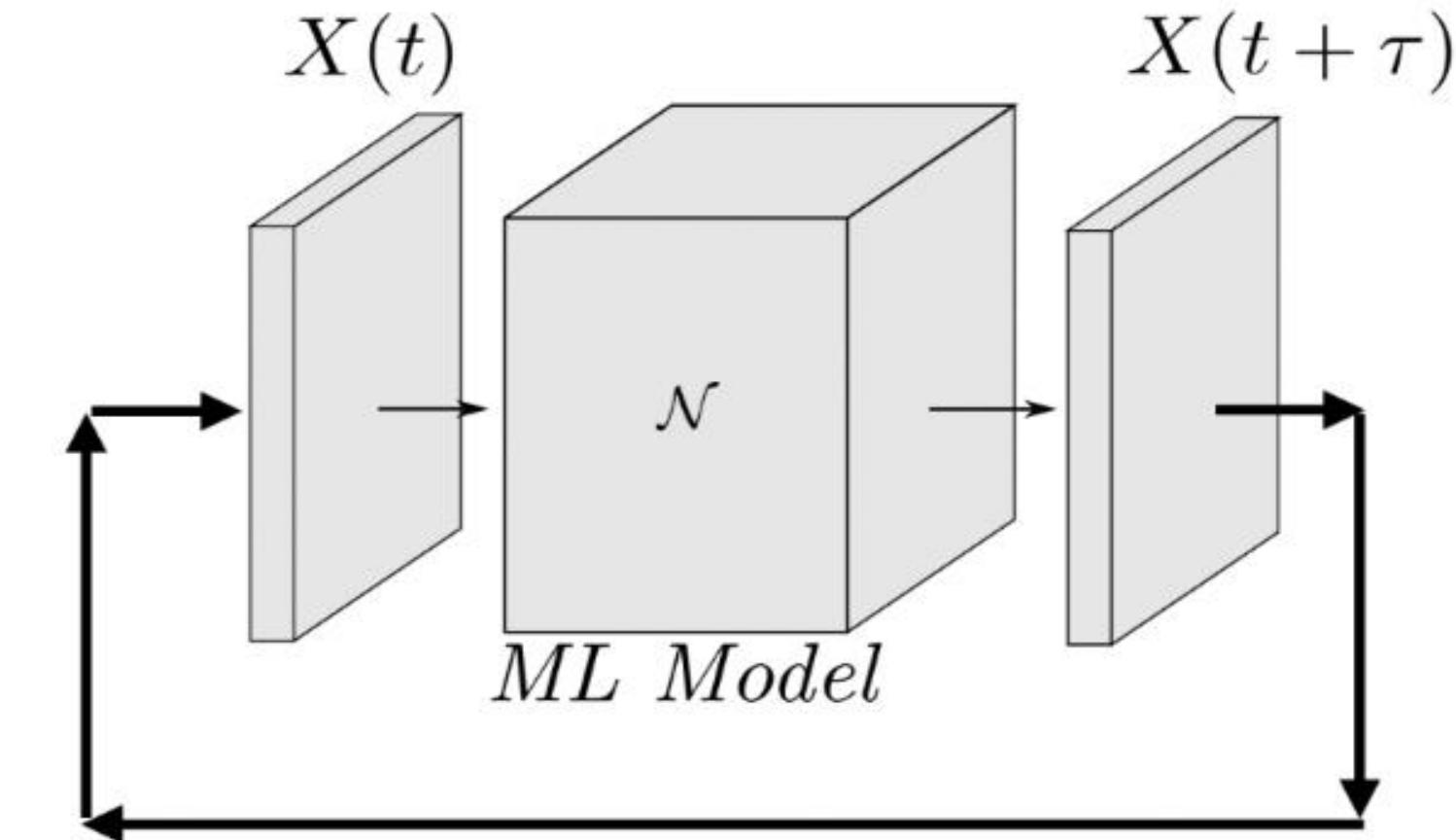
- Attention: kernel-integration.
- Query-key: low-rank kernel
- Use Fourier transform in kernel for efficiency.



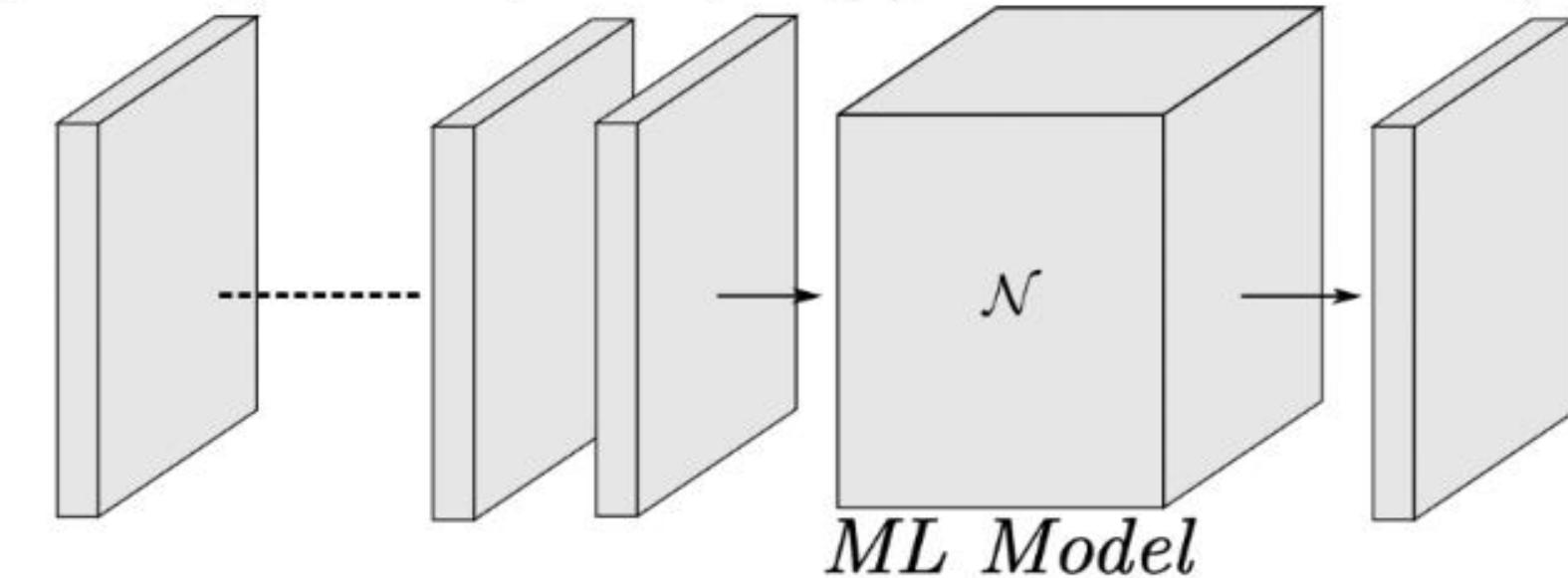
FOURCASTNET (FOURIER FORECASTING NETWORK)



Input: $[24 \times 720 \times 1440]$

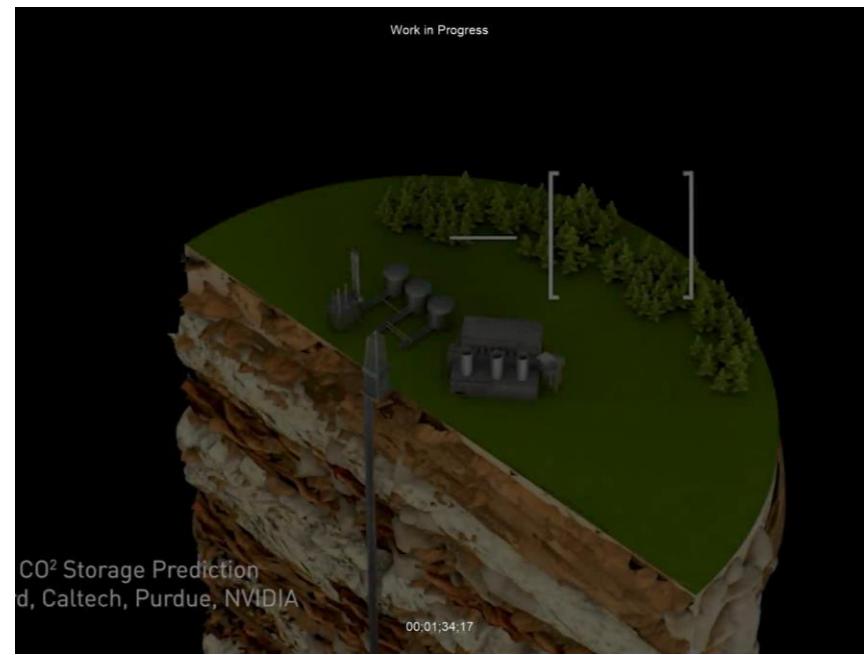


$X(t - k\tau)$



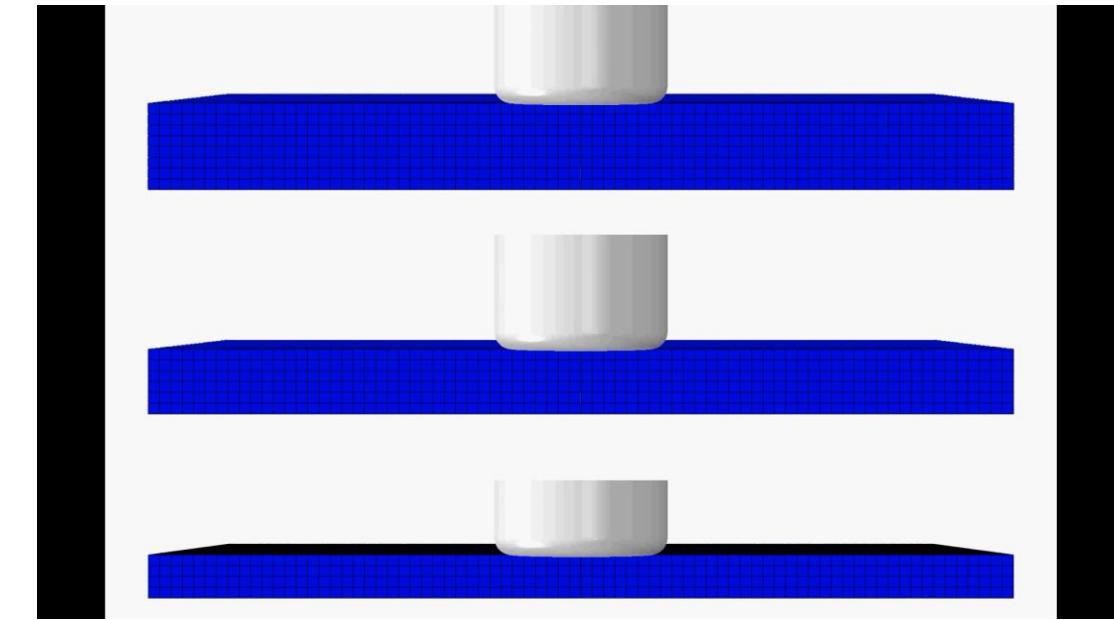
MANY OTHER APPLICATIONS FOR FNO

CO₂ Sequestration



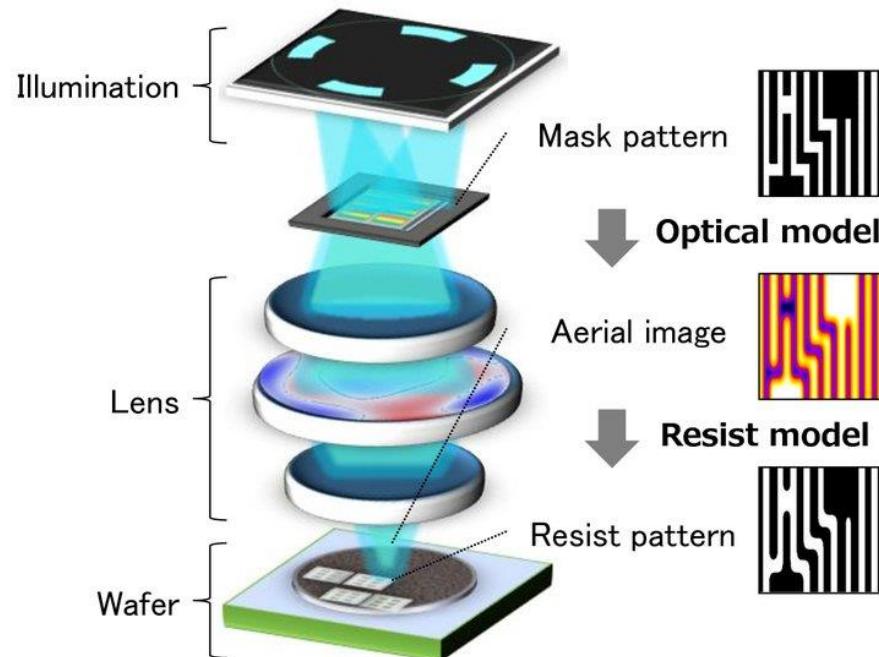
FNO is 10^4 x faster

Stress Prediction in Materials



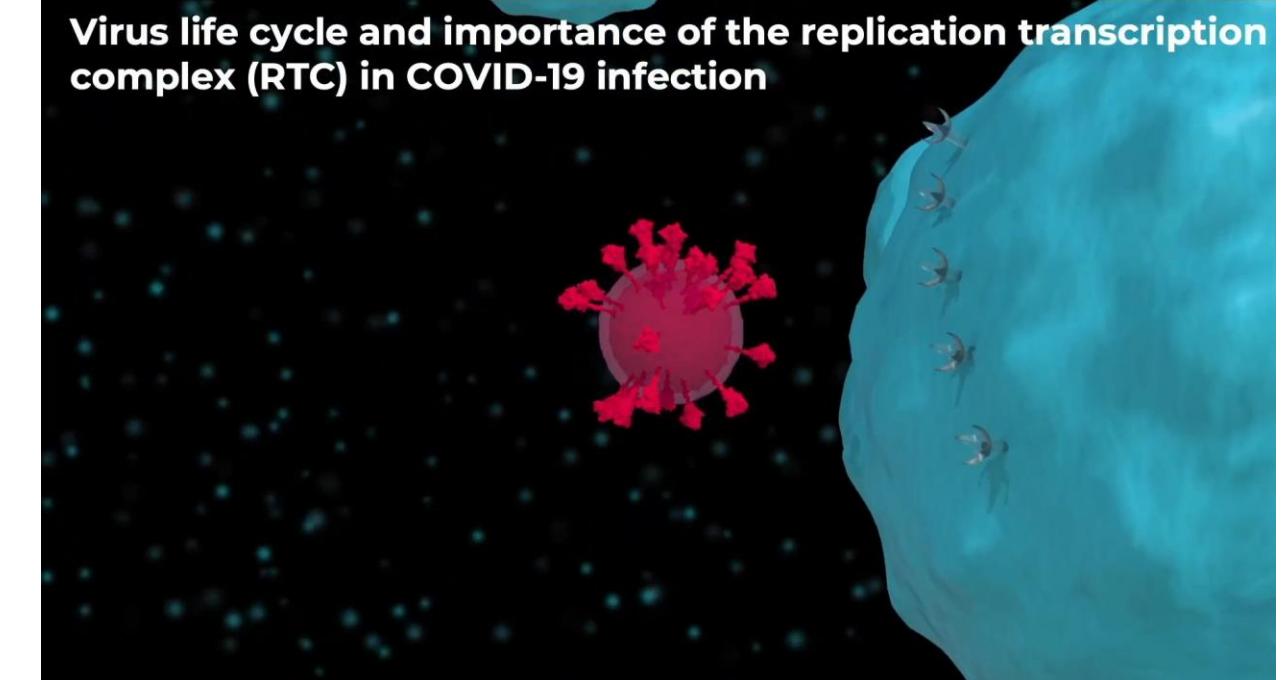
PCA-neural operator is 10^5 x faster

Computational Lithography



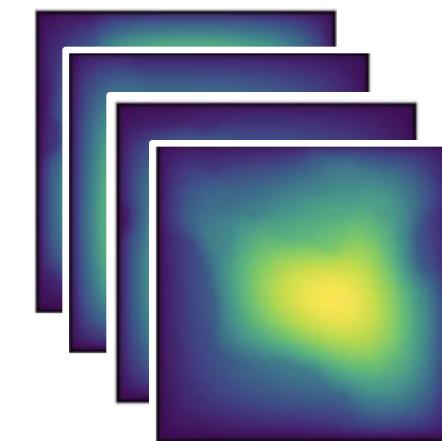
For first time,
metal layer
contour
simulation at
 $1nm^2$ /pixel
resolution with
any tile size.

Molecular dynamics



PINO: PHYSICS-INFORMED NEURAL OPERATOR

Data loss: compare prediction
and ground-truth solution



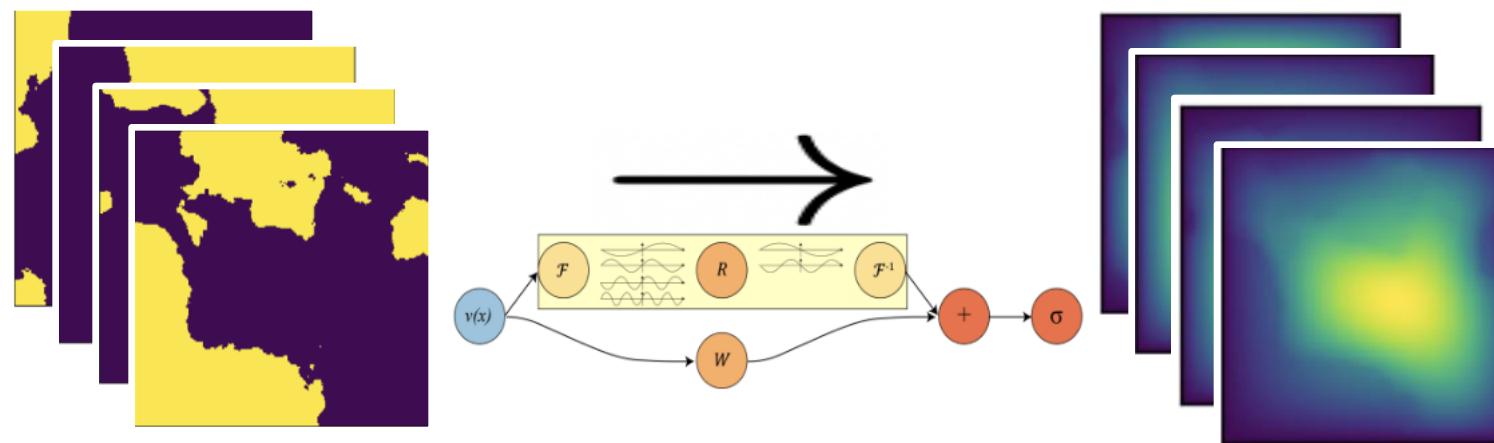
Equation loss: plug prediction
in PDE and compute residual

$$\begin{aligned} -\nabla \cdot (a(x)\nabla u(x)) &= f(x), & x \in D \\ u(x) &= 0, & x \in \partial D \end{aligned}$$

PINO: PHYSICS-INFORMED NEURAL OPERATOR

1. Pre-train:

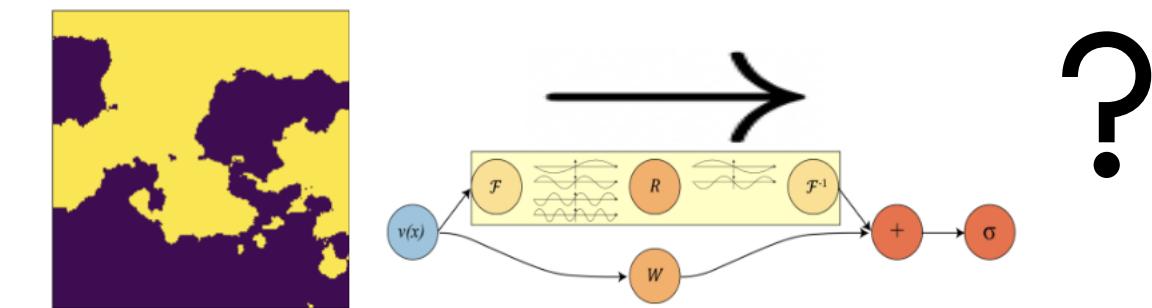
Learn a solution operator



Equation loss
Data loss (if available)

2. Test-time optimize:

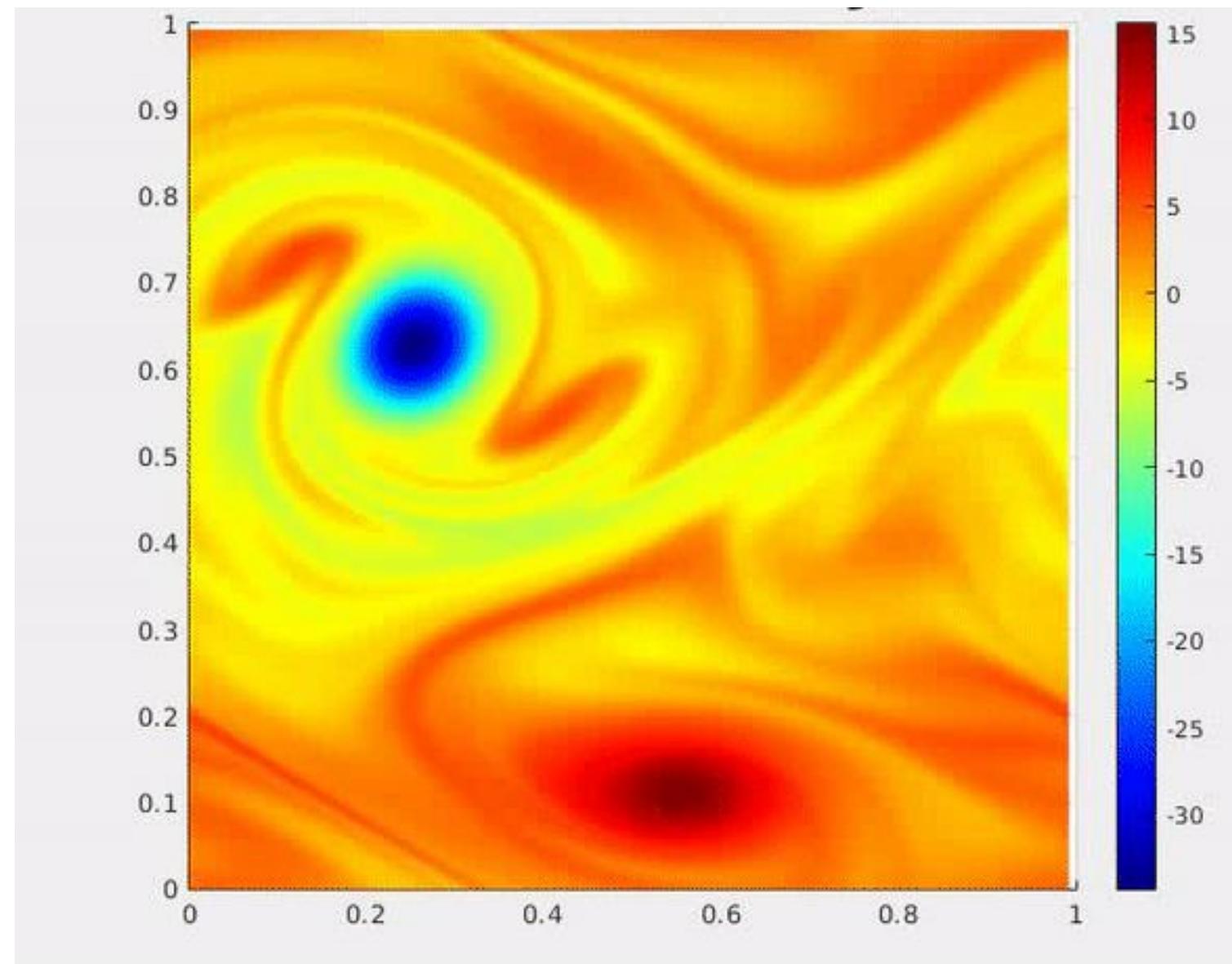
Fine-tune learned operator to solve for a specific instance



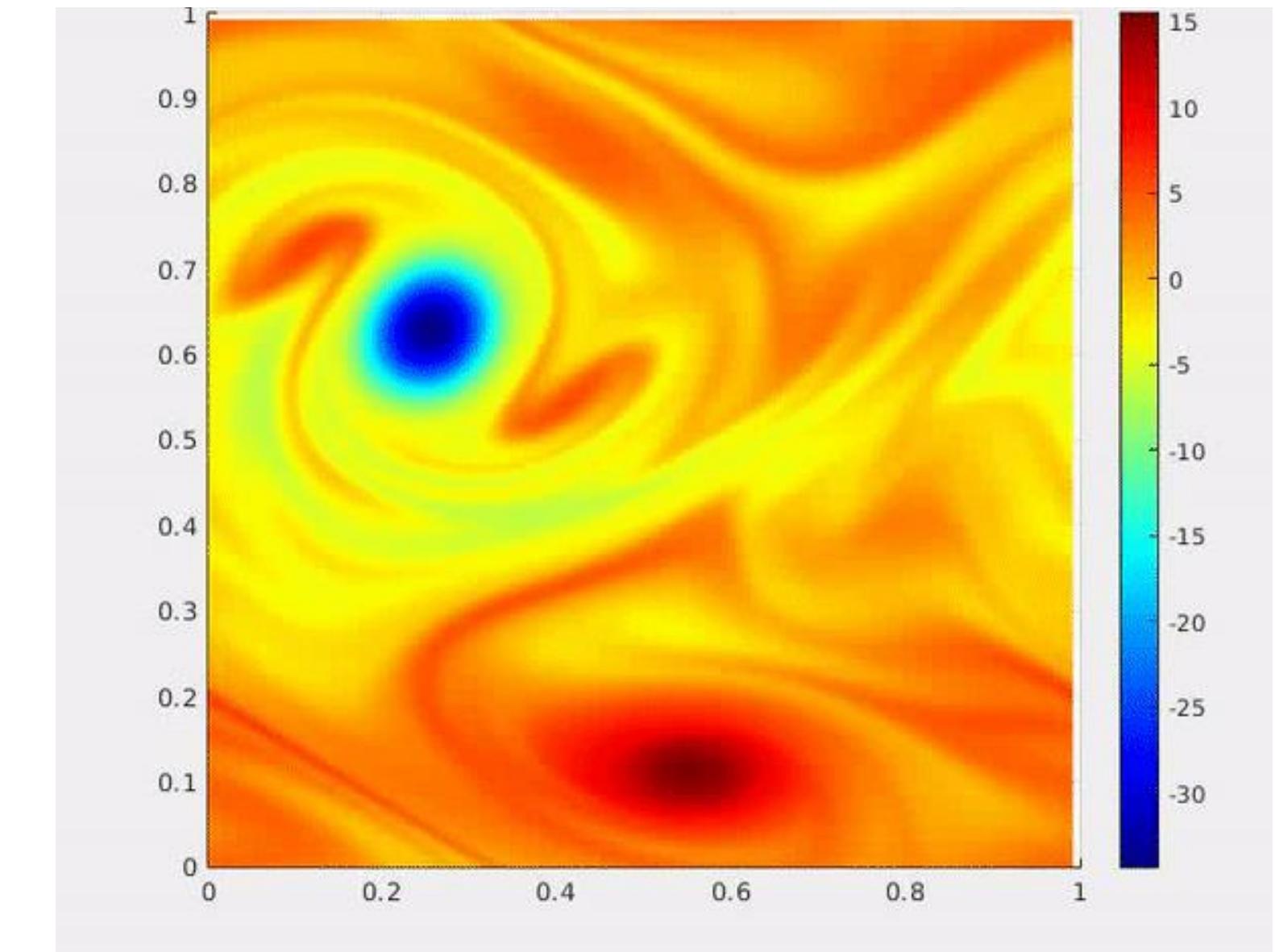
Equation loss
Operator loss (to prevent forgetting)

PINO: PHYSICS-INFORMED NEURAL OPERATOR

True vorticity

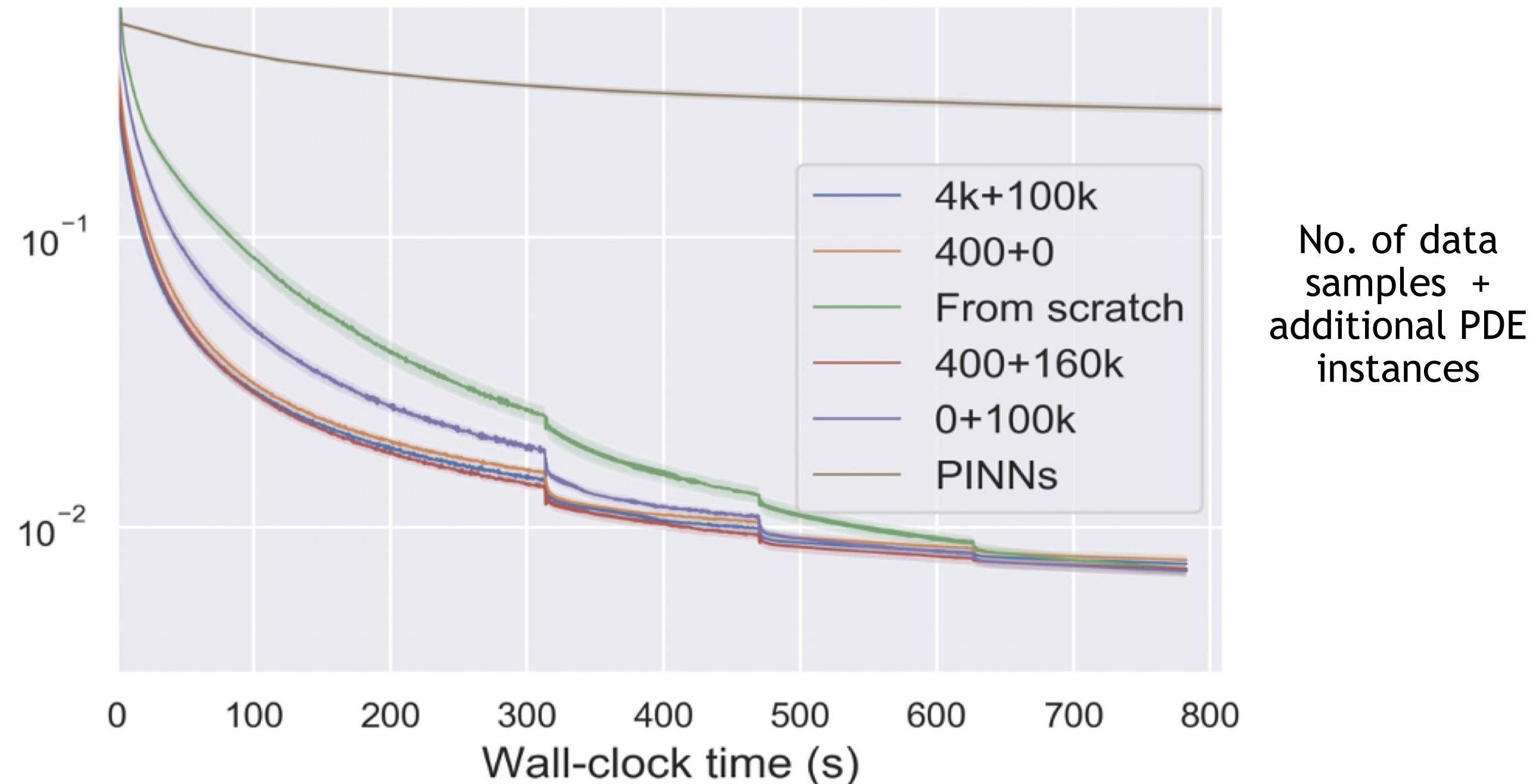


Predicted vorticity



- PINO gets 2% error on Re500 [$2\pi \times 2\pi \times 1s$]
- Easily generalize from one Reynolds number to another

PINO: PHYSICS-INFORMED NEURAL OPERATOR



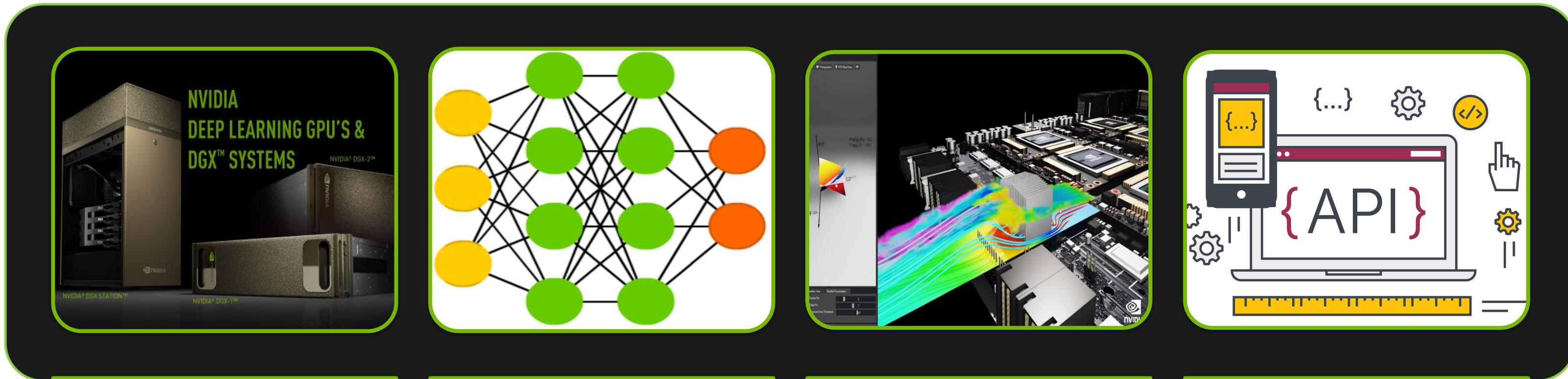
PINO vs PINN

- Learned operator ansatz has better optimization landscape
- Optimize function-wise vs optimize pointwise

Physics-informed neural networks --
M.Raissi, P.Perdikaris, G.E.Karniadakis

NVIDIA MODULUS

AI-accelerated Physics Simulation Platform



Solve larger problems faster
with XLA, AMP and TF32 support, and Multi-GPU, Multi-Node implementation

Several advanced networks model Multiple Physics in **Forward, Inverse and Data Assimilation simulations** with accuracy & convergence

Parameterized system representation to solve **multiple scenarios simultaneously**

APIs for implementing new Physics, Geometry, and Domains and detailed **User Guide examples**

CONCLUSION

- AI4science is the future of science
- Principled algorithms for zero-shot generalization
- Operator learning extends neural networks to learning in infinite dimensional spaces
- Orders of magnitude speedup while maintaining accuracy

