




# Workshop 6 – Deep Learning - BackProp

Advanced Analytics and Applications [AAA]

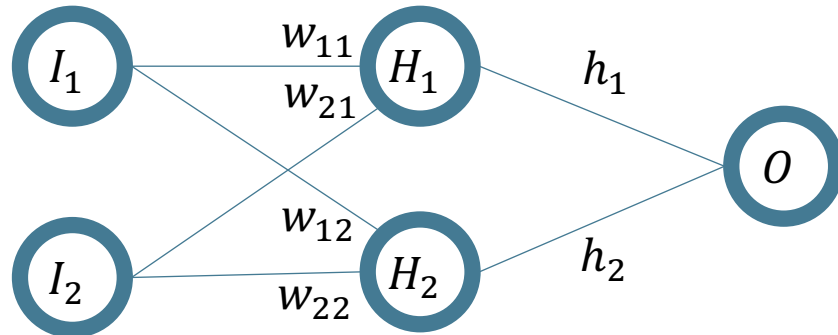


Calculation



Programming

## Question 2: Calculation BackProp



Assume we have the following neural network setup.

- Learning rate  $\eta = 0.05$
- Linear activation function
- One Data Point:

$$I_1 = 2 \quad I_2 = 3 \quad O = 1$$

- Random Initial weights:

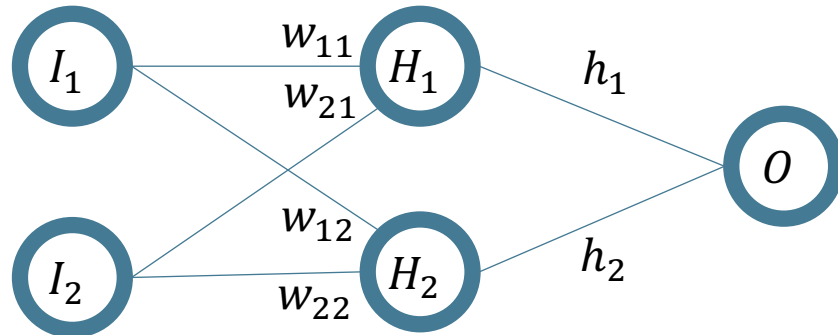
$$w_{11} = 0.11 \quad h_1 = 0.14$$

$$w_{21} = 0.21 \quad h_2 = 0.15$$

$$w_{12} = 0.12$$

$$w_{22} = 0.08$$

## Question 2: Calculation BackProp



$$\hat{O} = [2 \quad 3] * \begin{bmatrix} 0.11 & 0.12 \\ 0.21 & 0.08 \end{bmatrix} * \begin{bmatrix} 0.14 \\ 0.15 \end{bmatrix} = 0.191$$

Calculate the prediction using this configuration.

- Learning rate  $\eta = 0.05$
- Linear activation function
- One Data Point:

$$I_1 = 2 \quad I_2 = 3 \quad O = 1$$

- Random Initial weights:

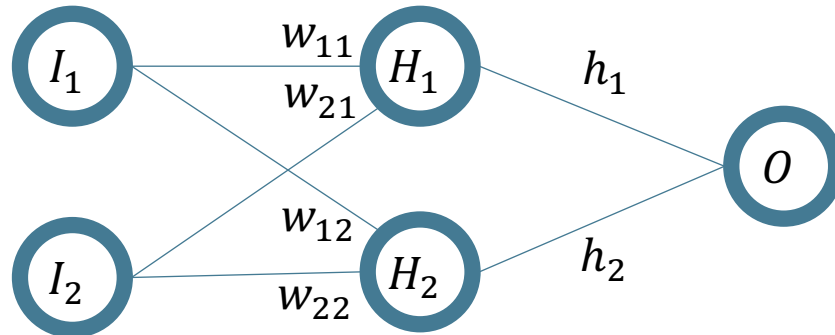
$$w_{11} = 0.11 \quad h_1 = 0.14$$

$$w_{21} = 0.21 \quad h_2 = 0.15$$

$$w_{12} = 0.12$$

$$w_{22} = 0.08$$

## Question 2: Calculation BackProp



$$L = \frac{1}{2}(0.191-1)^2 = 0.327$$

Calculate the error using a MSE-like loss function (it is not MSE)

- Learning rate  $\eta = 0.05$
- Linear activation function
- One Data Point:

$$I_1 = 2 \quad I_2 = 3 \quad O = 1$$

- Random Initial weights:

$$w_{11} = 0.11 \quad h_1 = 0.14$$

$$w_{21} = 0.21 \quad h_2 = 0.15$$

$$w_{12} = 0.12$$

$$w_{22} = 0.08$$

## Question 2: Calculation BackProp

$$\begin{aligned}
 \frac{\partial L}{\partial h_1} &= \frac{\partial L}{\partial \hat{O}} * \frac{\partial \hat{O}}{\partial h_1} \\
 &= \frac{\partial(\frac{1}{2}(\hat{O} - O)^2)}{\partial \hat{O}} * \frac{\partial[(I_1 * w_{11} + I_2 w_{21})h_1 + (I_1 * w_{12} + I_2 w_{22})h_2]}{\partial h_1} \\
 &= \frac{\partial((\hat{O} - O))}{\partial \hat{O}} * 2 * \frac{1}{2}(\hat{O} - O) * (I_1 * w_{11} + I_2 w_{21}) \\
 &= (\hat{O} - O) * (I_1 * w_{11} + I_2 w_{21})
 \end{aligned}$$

Similarly for  $\frac{\partial L}{\partial h_2} = (\hat{O} - O) * (I_1 * w_{12} + I_2 w_{22})$

Calculate Derivative of Loss with respect to  $h_1$

- Learning rate  $\eta = 0.05$
- Linear activation function
- One Data Point:

$$I_1 = 2 \quad I_2 = 3 \quad O = 1$$

- Random Initial weights:

$$w_{11} = 0.11 \quad h_1 = 0.14$$

$$w_{21} = 0.21 \quad h_2 = 0.15$$

$$w_{12} = 0.12$$

$$w_{22} = 0.08$$

## Question 2: Calculation BackProp

$$\begin{aligned}
 \frac{\partial L}{\partial w_{11}} &= \frac{\partial L}{\partial \hat{O}} * \frac{\partial \hat{O}}{\partial H_1} * \frac{\partial H_1}{\partial w_{11}} \\
 &= \frac{\partial(\frac{1}{2}(\hat{O} - O)^2)}{\partial \hat{O}} * \frac{\partial[(H_1)h_1 + (H_2)h_2]}{\partial H_1} * \frac{\partial H_1}{\partial w_{11}} \\
 &= \frac{\partial((\hat{O} - O))}{\partial \hat{O}} * 2 * \frac{1}{2}(\hat{O} - O) * (h_1) * (I_1) \\
 &= (\hat{O} - O) * (h_1) * (I_1)
 \end{aligned}$$

$(I_1 * w_{11} + I_2 w_{21})$

Similarly for  $\frac{\partial L}{\partial w_{12}} = (\hat{O} - O) * (h_2) * (I_1)$

$$\frac{\partial L}{\partial w_{21}} = (\hat{O} - O) * (h_1) * (I_2)$$

$$\frac{\partial L}{\partial w_{22}} = (\hat{O} - O) * (h_2) * (I_2)$$

## Calculate Derivative of Loss with respect to $w_{11}$

- Learning rate  $\eta = 0.05$
- Linear activation function
- One Data Point:

$$I_1 = 2 \quad I_2 = 3 \quad O = 1$$

- Random Initial weights:

$$w_{11} = 0.11 \quad h_1 = 0.14$$

$$w_{21} = 0.21 \quad h_2 = 0.15$$

$$w_{12} = 0.12$$

$$w_{22} = 0.08$$



## Question 2: Calculation BackProp

$$w_{11}^{k+1} = w_{11}^k - \eta(\hat{O} - O) * (h_1) * (I_1)$$

$$w_{12}^{k+1} = w_{12}^k - \eta(\hat{O} - O) * (h_2) * (I_1)$$

$$w_{21}^{k+1} = w_{21}^k - \eta(\hat{O} - O) * (h_1) * (I_2)$$

$$w_{22}^{k+1} = w_{22}^k - \eta(\hat{O} - O) * (h_2) * (I_2)$$

$$h_1^{k+1} = h_1^k - \eta(\hat{O} - O) * (I_1 * w_{11} + I_2 w_{21})$$

$$h_2^{k+1} = h_2^k - \eta(\hat{O} - O) * (I_1 * w_{12} + I_2 w_{22})$$

## Updating weights using derivatives

- Learning rate  $\eta = 0.05$
- Linear activation function
- One Data Point:

$$I_1 = 2 \quad I_2 = 3 \quad O = 1$$

- Random Initial weights:

$$w_{11} = 0.11 \quad h_1 = 0.14$$

$$w_{21} = 0.21 \quad h_2 = 0.15$$

$$w_{12} = 0.12$$

$$w_{22} = 0.08$$



## Question 2: Calculation BackProp

$$\begin{aligned} w_{11}^1 &= w_{11}^0 - \eta(\hat{O} - O) * (h_1) * (I_1) \\ &= 0.11 - 0.05(0.191 - 1) * 0.14 * 2 \\ &\cong 0.12 \end{aligned}$$

$$w_{12}^1 = 0.13$$

$$w_{21}^1 = 0.23$$

$$w_{22}^1 = 0.10$$

$$h_1^1 = 0.17$$

$$h_2^1 = 0.17$$

$$\text{Results in updated prediction: } \hat{O} = [2 \quad 3] * \begin{bmatrix} 0.12 & 0.13 \\ 0.23 & 0.10 \end{bmatrix} * \begin{bmatrix} 0.17 \\ 0.17 \end{bmatrix} = 0.26$$

We got closer to the real 0 – that's how backprop works in a nutshell 😊

## Updating weights using derivatives with real values

- Learning rate  $\eta = 0.05$
- Linear activation function
- One Data Point:

$$I_1 = 2 \quad I_2 = 3 \quad O = 1$$

- Random Initial weights:

$$w_{11} = 0.11 \quad h_1 = 0.14$$

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Calculation

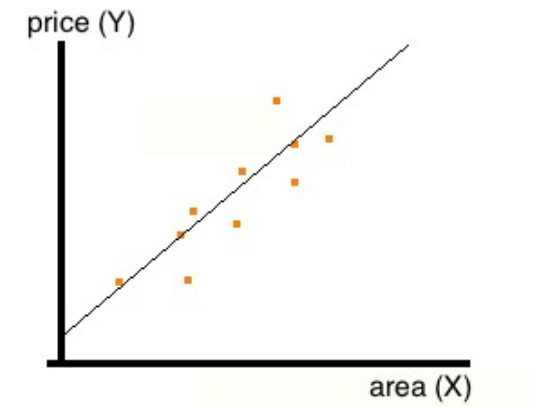


Programming

## 1 Classifying newswires



## 2 Regression with Keras



## 1 Classifying newswires



## 2 Regression with Keras



# Classifying newswires with keras

## The dataset: Reuters-21578



# Classifying newswires with keras

## Fact sheet: Reuters-21578



11228 newswires  
Over 46 topics



# Classifying newswires with keras

## Fact sheet: Reuters-21578



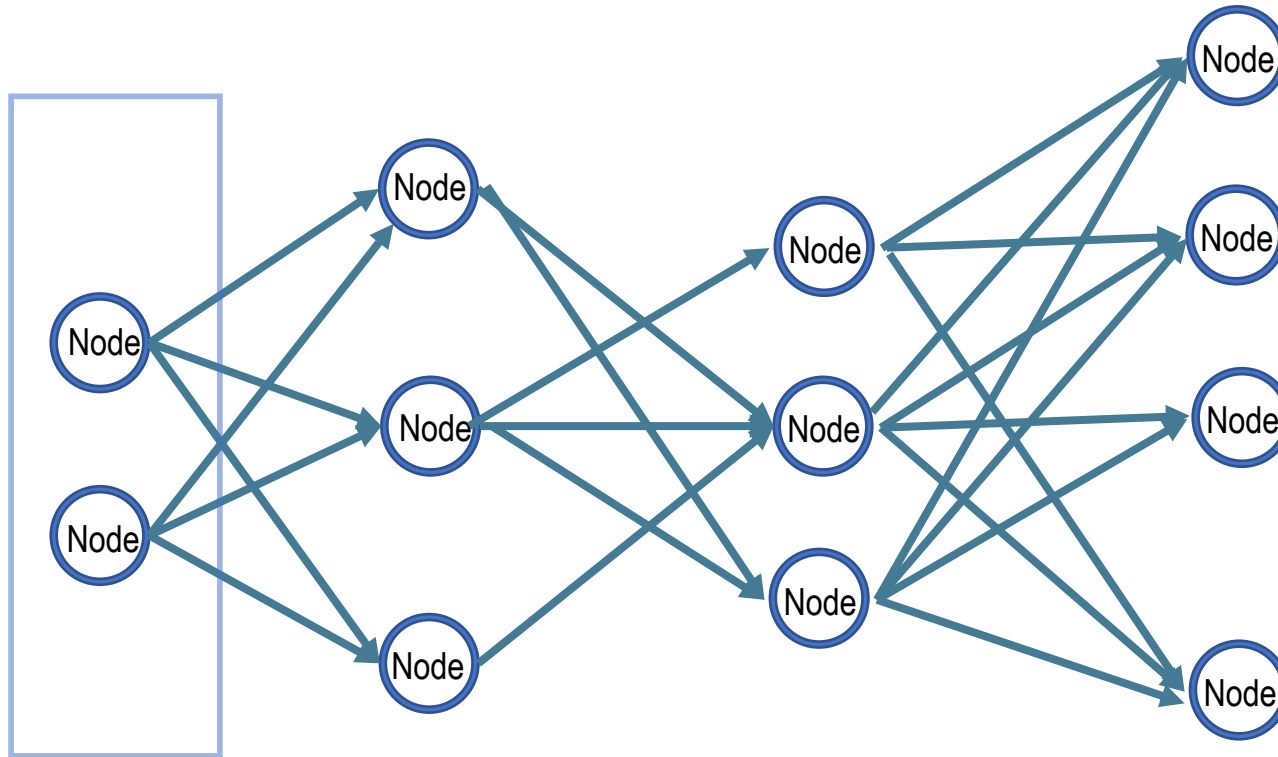
1. Toy dataset in **keras**.
2. Each newswire is encoded as a list of **word indexes**.
3. For convenience, words are indexed by overall **frequency** in the dataset, so that **for instance** the integer **"3"** encodes the 3rd most frequent word in the data.



# Our objective: Train a deep learning network that can classify unseen documents

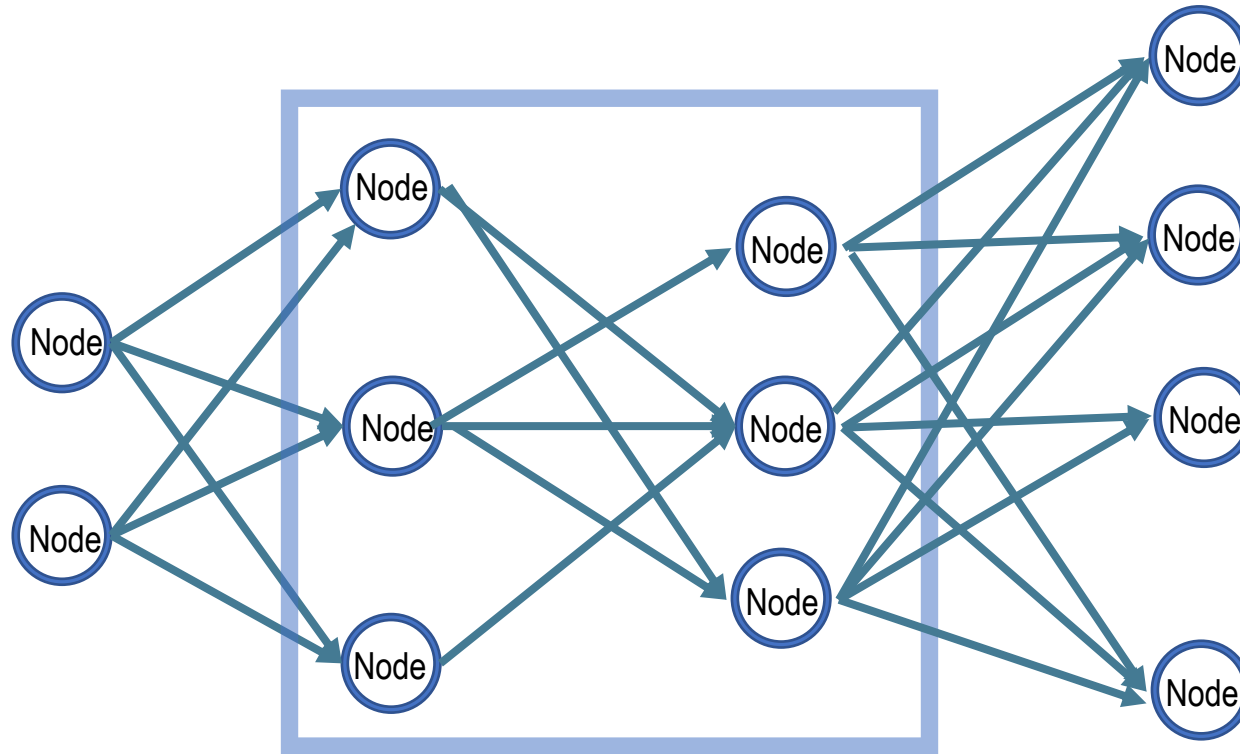


# Our objective: Train a deep learning network that can classify unseen documents



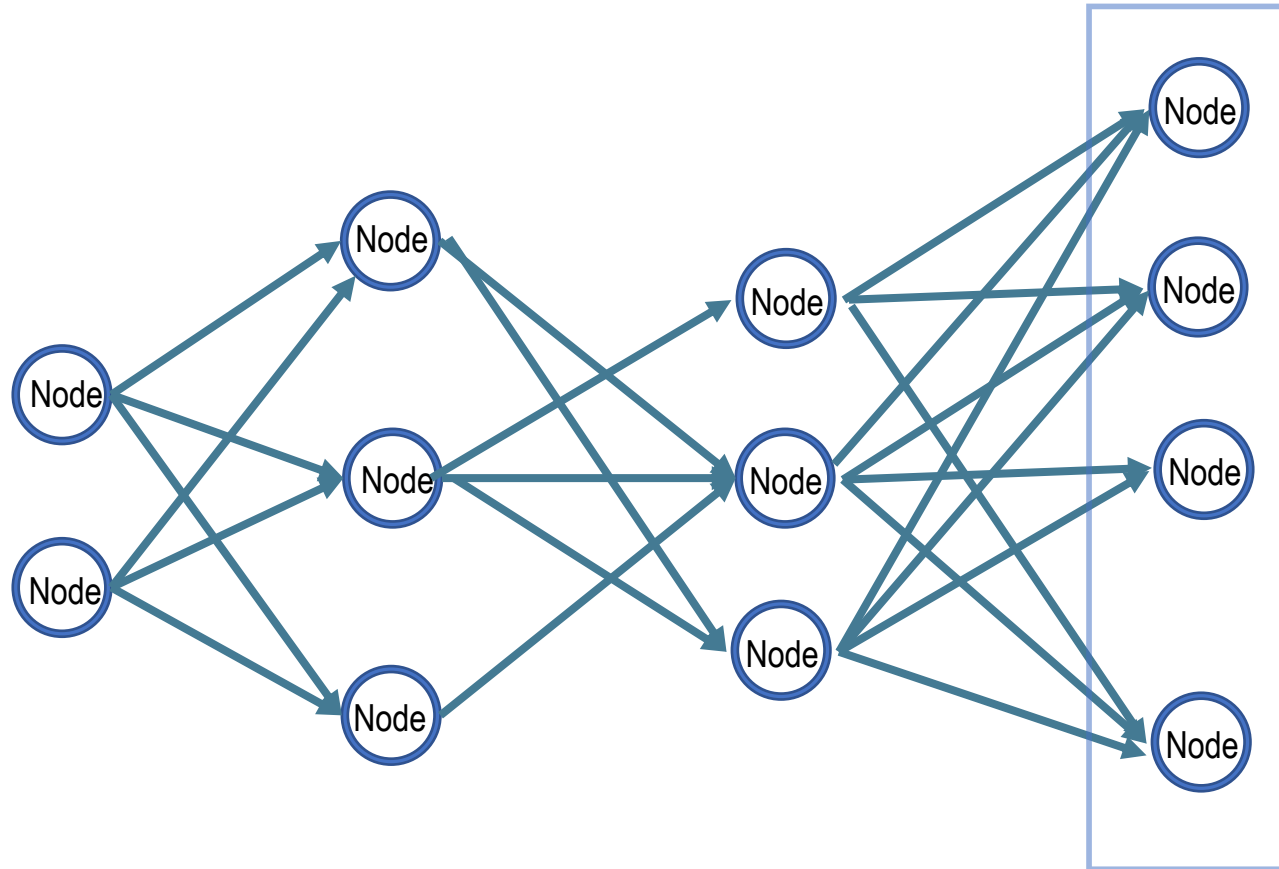
Input Layer with 10000 Input Nodes

# Our objective: Train a deep learning network that can classify unseen documents



64 Nodes, 2 Hidden Layers,  
Activation Function: **ReLU**

# Our objective: Train a deep learning network that can classify unseen documents

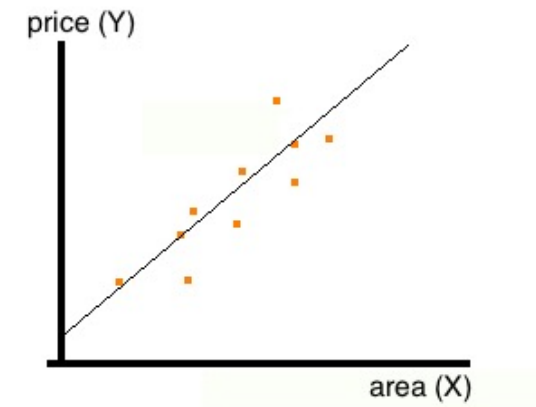


46 Nodes, 1 Output Layer,  
Activation Function: **SoftMAX**

## 1 Classifying newswires



## 2 Regression with Keras



# The boston housing price data set



Price of House?



# The boston housing price data set



1. Sample contains 404 training and 102 test samples.
2. 13 attributes (independent variables) – i.e., attributes of the houses at different location
3. The target value are the median values of the houses at a location (in k \$)



# The boston housing price data set

## Variables in order:

CRIM	per capita crime rate by town
ZN	proportion of residential land zoned for lots over 25,000 sq.ft.
INDUS	proportion of non-retail business acres per town
CHAS	Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
NOX	nitric oxides concentration (parts per 10 million)
RM	average number of rooms per dwelling
AGE	proportion of owner-occupied units built prior to 1940
DIS	weighted distances to five Boston employment centres
RAD	index of accessibility to radial highways
TAX	full-value property-tax rate per \$10,000
PTRATIO	pupil-teacher ratio by town
B	$1000(B_k - 0.63)^2$ where $B_k$ is the proportion of blacks by town
LSTAT	% lower status of the population
MEDV	Median value of owner-occupied homes in \$1000's

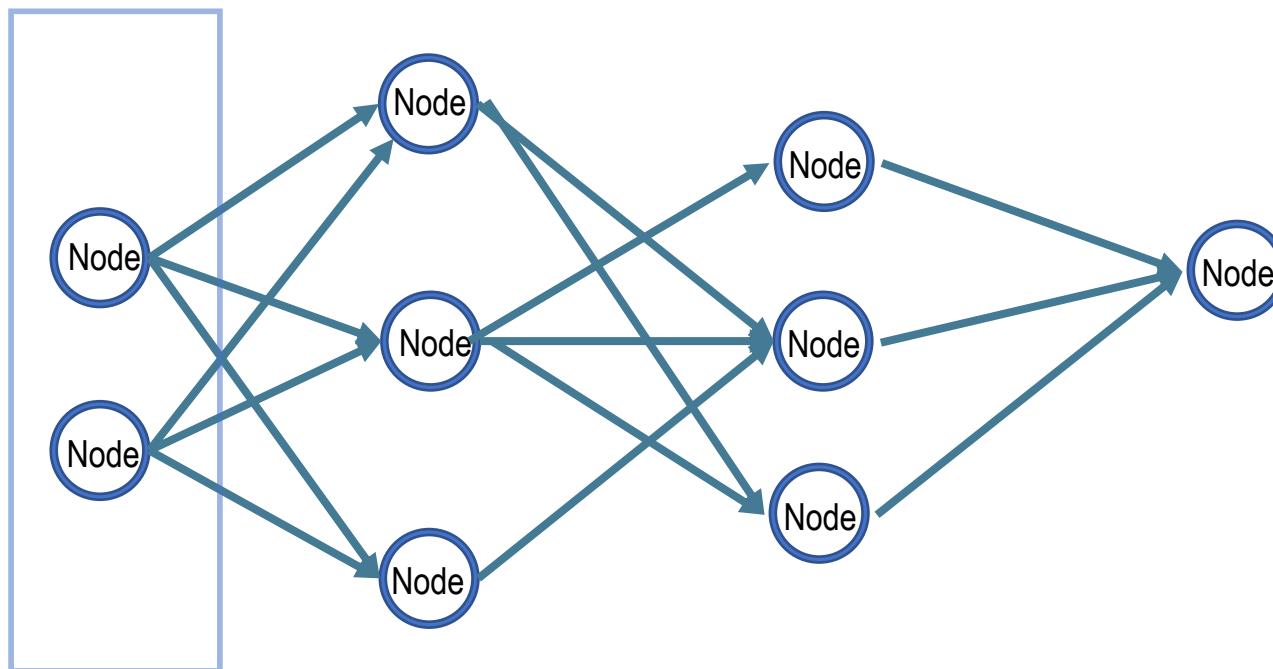
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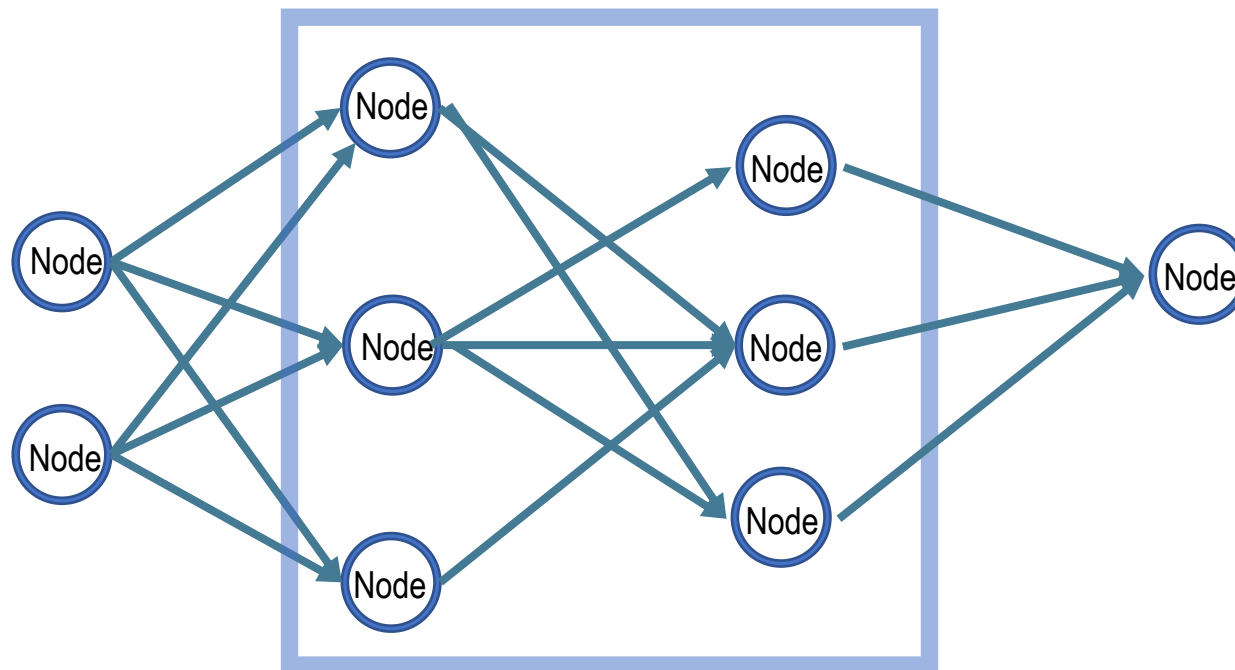
The attributes have different scales and ranges.  
We need to normalize this before training the network.

# Our objective: Train a deep learning network that can predict house prices



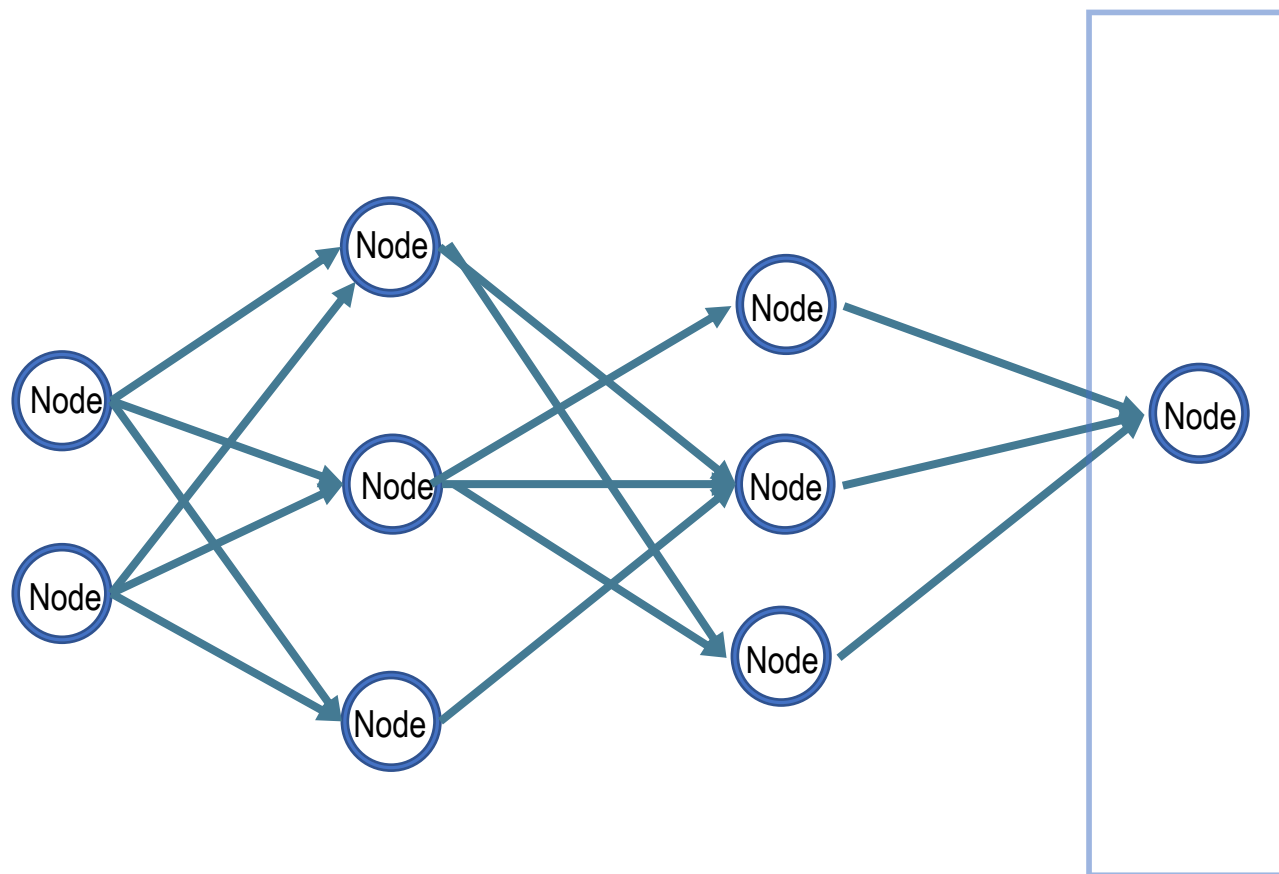
Input Layer with 13 Input Nodes

# Our objective: Train a deep learning network that can predict house prices



2 Hidden Layers, 64 Nodes in each layer  
Activation Function: **ReLU**

# Our objective: Train a deep learning network that can classify unseen documents



**Loss: MSE**

**Metric: MAE**

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2$$

$$\text{MAE} = \frac{1}{n} \sum_{j=1}^n |y_j - \hat{y}_j|$$

*1 Output Node, Real Value Output*  
*Activation Function: **Linear***

# Contact



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