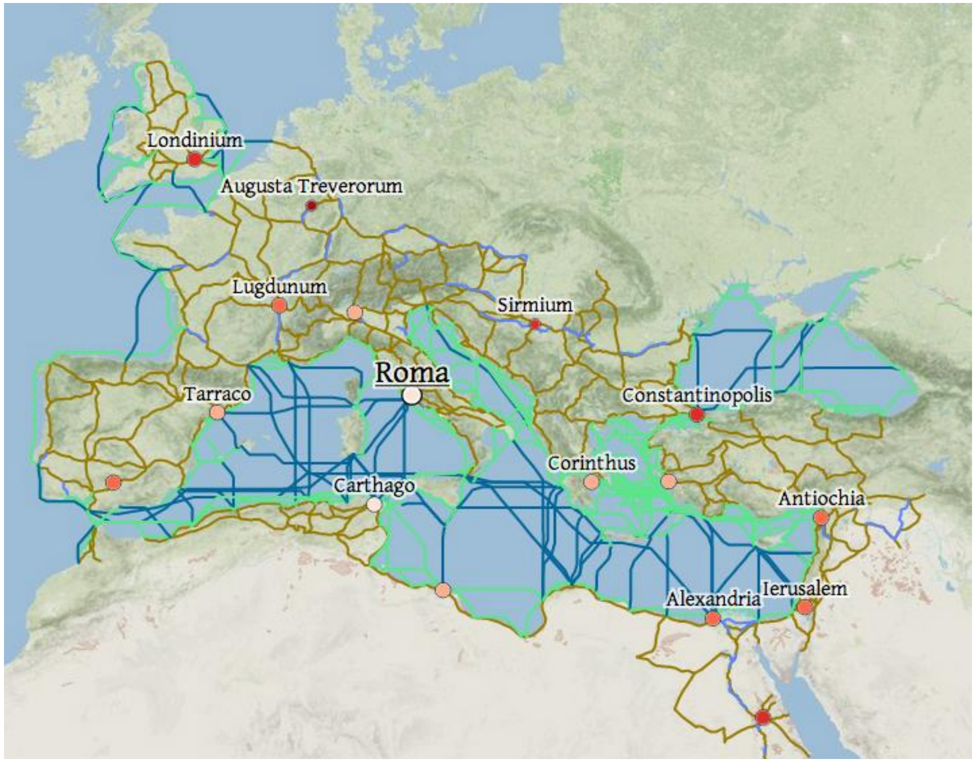
Imperial College London

Modelling Spatial Exchange. II

Ray Rivers (ICL)



Modelling Interaction in Landscape Archaeology, Kiel, August 2018

Talk Structure:

The issue now is how to choose and use models:

Part III. Model choice and applications

Division of networks into

1. ‘the easiest’
2. ‘the most likely’
3. ‘the best’

Part IV: Brief conclusions

Examples taken from

- Assyrian mercantile networks (EBA)
- Cycladic culture (EBA)
- Minoan culture (MBA)
- Mycenaean culture (MBA/LBA)
- Archaic Greece (IA)
- Phoenician maritime networks (IA)

III. Model Choice and Applications

Several approaches:

Agent Based Modelling:

nodes as agents following basic rules

Network Optimisation:

Networks that exist do so because they do something well!

Dynamical Equations:

often adopted from biological systems

Increasing Detail

Coarse Graining Increasing

III. Model Choice and Applications

Not as distinct as they look!

Several approaches:

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nodes as agents following basic rules

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Dynamical Equations:
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Coarse Graining Increasing

Increasing Detail

III. Model Choice and Applications

Not as distinct as they look!

Several approaches:

Restrict ourselves to:

Network Optimisation:

Networks that exist do so because they do something well!

III. Model Choice and Applications

Three main genres of models of ‘optimal’ networks:

Are our networks:

1. **‘Local’ networks/’the easiest’ networks:**
   i.e. we do the ‘easiest’ thing; exploit local connections ‘first’ and expand from there

2. **‘The most likely’ networks:**
   i.e. we live in the most likely of all possible worlds commensurate with our understanding

3. **The ‘best’ networks:**
   i.e. we live in the ‘best’ of all possible worlds with some trade-off between costs and benefits

III. Model Choice and Applications

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   ‘Intervening Opportunity’ modelling (IOM)

2. **‘The most likely’ networks:**
   i.e. we live in the most likely of all possible worlds commensurate with our understanding

   Max(imum)Ent(ropy) modelling: e.g. ‘gravity’ modelling

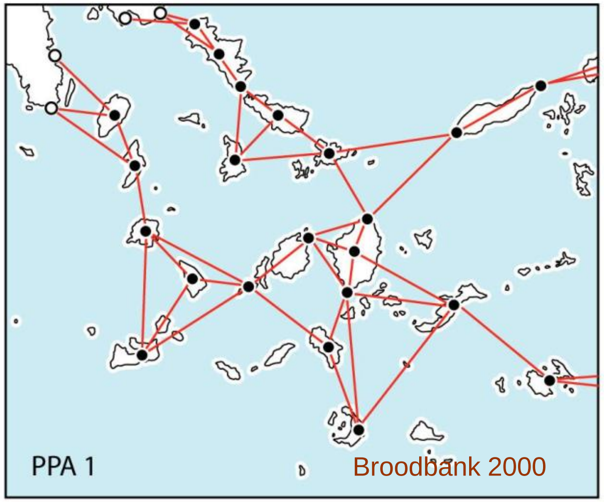
3. **The ‘best’ networks:**
   i.e. we live in the ‘best’ of all possible worlds with some trade-off between costs and benefits

   Cost-benefit modelling

1. ‘Local’ networks:

Local agency: Communities try to satisfy their needs locally

.



EBA Cyclades (3000 -2000 BCE)

- Large-scale picture follows from putting these local patterns together
- Pre-supposes no socio-political organisation with a long reach
- relatively non-hierarchical society!

1. ‘Local’ networks: Maximum ‘distance’ model (MDM)

Local agency: ‘So far and no further’

- Any site can only sustain useful exchange with sites within an effective distance D determined by the cost or effort available for exchange.

- Do not discriminate between different categories of ‘exchange’.
  - links unweighted!

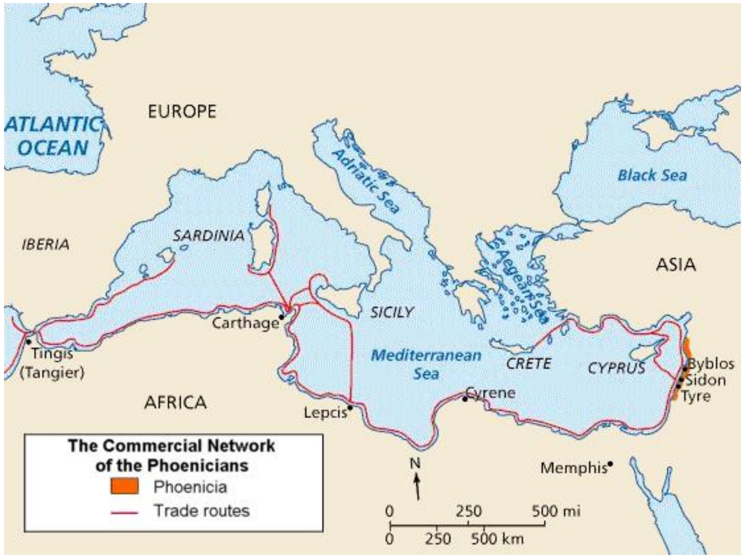
Link such sites!

That is,

- $T_{AB} = T_{BA} = 1$ if $d_{AB} = d_{BA} < D$
- $T_{AB} = T_{BA} = 0$ if $d_{AB} = d_{BA} > D$
- Links unweighted and non-directional

1. Example I: MDM for Iron Age Phoenician Mediterranean.

Model for exchange between coastal sites
when long-range sea travel is possible.



For a site A define

I. $N_A(D)$ = # of coastal sites that can be reached

from A by a sea trip of effective length D

II. $L_A(D)$ = length of coastline that can be reached

from A by a sea trip of effective length D

- proxy for market access with D ~ 550km

Of Mice and Merchants: Trade and Growth in the Iron Age

J D Bakker, S Maurer, J-S Pischke and F Rauch, July 2018

CEP (LSE) Discussion Paper No. 1558

Iron Age Phoenician network

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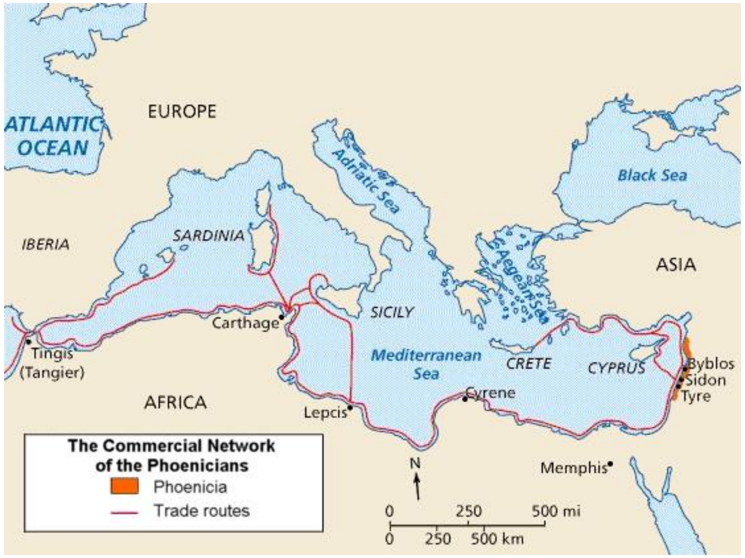

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Then the urbanisation measure U_A for A has the form

I. $U_A(D) = C_A N_A(D)^b$

or

II. $U_A(D) = C_A L_A(D)^b$

C_A ; climactic variation across Med – control parameter

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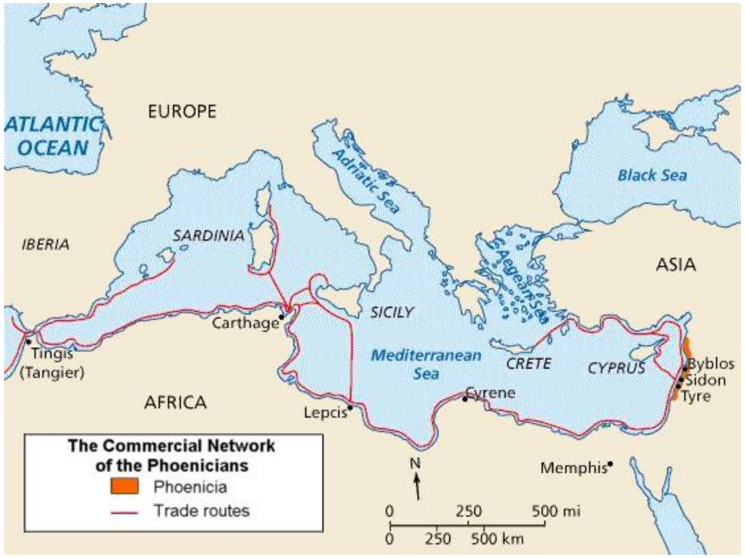

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Iron Age Phoenician network

Ln $U_A(D)$ ~ 'GDP' of A ~ population of A – Pleiades data set

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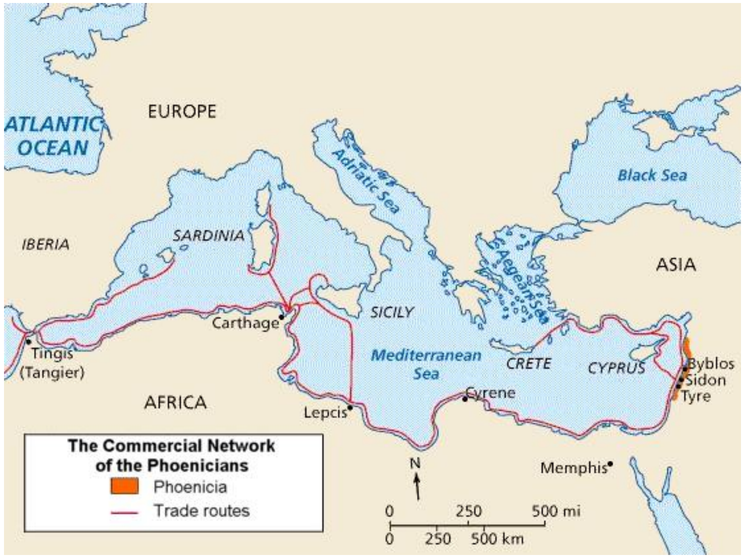

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$\boxed{\mathit{Ln}\ U_A(D) \sim \text{'GDP' of A} \sim \text{population of A} – \text{Pleiades data set}}$

- exchange data (artefacts) NOT used!
- only site data (I)
- unweighted links (II)
- no derived network analysis!

1. Example II: Mycenaean trade/exchange 1400-1200 BCE (LMII – LMIII)

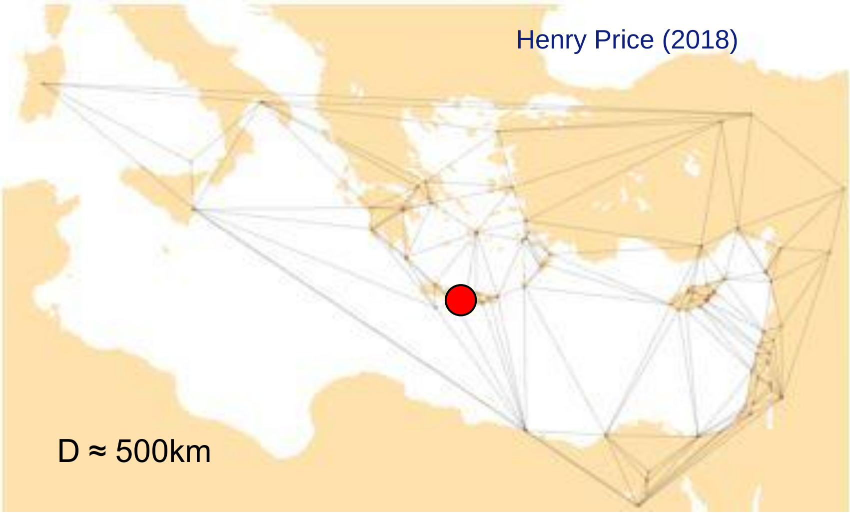
Example of a 'good' data set:

Paula Gheorghiade, U. of Toronto,
PhD thesis (in preparation)

- Imports/local production at ONLY 5 important Cretan sites
- 8000 catalogued items (< 1% of excavated material!)
- 1750 imported items (can be within Crete) – largely at 2 sites!
- imports from 40 regional sites (constituted by > 200 local sites)
  sometimes imports identified to specific sites, sometimes to regions

1. Example II: MDM for Mycenaean trade/exchange

Model for exchange between coastal sites
when long-range sea travel is possible.



Mycenaean E. Mediterranean 1400-1200 BCE

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- proxy for market access with D ~ 500km

Then the urbanisation measure U_A for A has the form

I: $U_A(D) = C_A N_A(D)^b$

or

II: $U_A(D) = C'_A L_A(D)^b$

C_A ; climactic variation across Med - control parameter

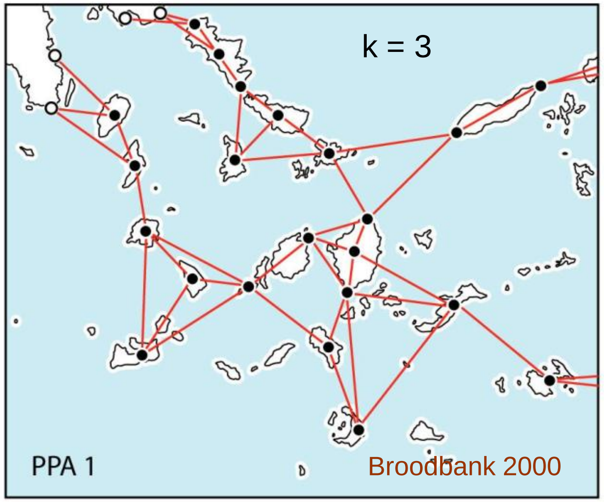
b ; calibration parameter

Use as null models prior to incorporating regional analysis using detailed exchange data

Not difficult to implement but haven't completed

1. 'Local' networks: Proximal Point Analysis (PPA)

Local agency: 'So many and no more'



EBA Cyclades (3000 -2000 BCE)

- Rather than distance, we are interested in rank, how near a neighbour we are. If B is the rth nearest neighbour to A it has rank $R_A(B) = r$.

PPA:

- Any site A can only sustain useful connections/exchange with a limited no. **k** of ranked sites.
- Exchange conflated and unweighted

That is,

- $T_{AB} = 1$ if $R_A(B) \leq k$
- $T_{AB} = 0$ otherwise
- In general $T_{AB} \neq T_{BA}$ since $R_A(B) \neq R_B(A)$ - directed networks!
- In practice often drop directional arrows – as above!

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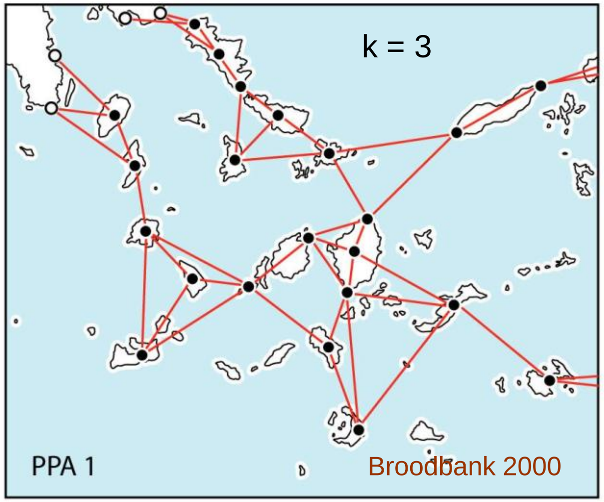

EBA Cyclades (3000 -2000 BCE)

Question:

What determines a site’s ‘importance’?

Some sites with very poor resources show high levels of activity

Some sites with good resources show little evidence

Can network ‘centrality’ overcome poor resources?

Q. Which are the most ‘important’ sites?

A. Provisional answer: Those nodes with highest order – no. of connections

1. Example III: Proximal Point Analysis (PPA) for EBA Cyclades

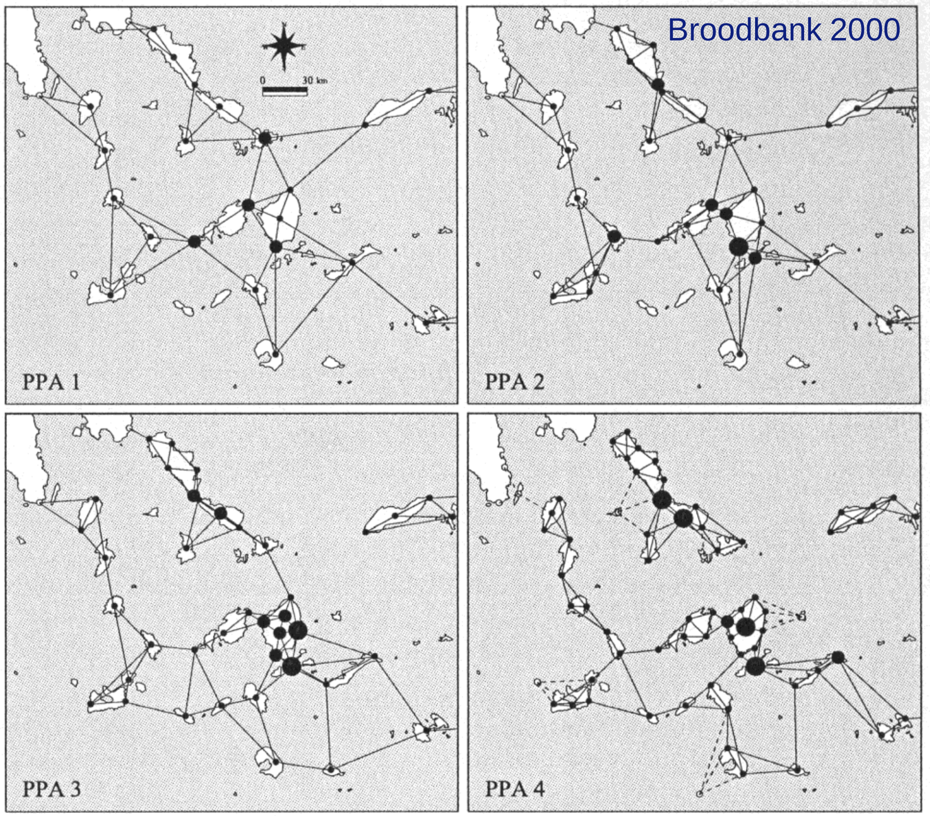

Fig. 75 Nodes of intense communication in the Cyclades as modelled by PPAs 1–4 (five and six linkages only).

The higher the order the more
'important' the site - larger the blob

PPA

- Dependent on position and number of sites – villages (50 – 150) inhabitants
  In this case with the passage of time creating more sites!

- Dependent on k (k = 3)

1. Example III: Proximal Point Analysis (PPA) for EBA Cyclades

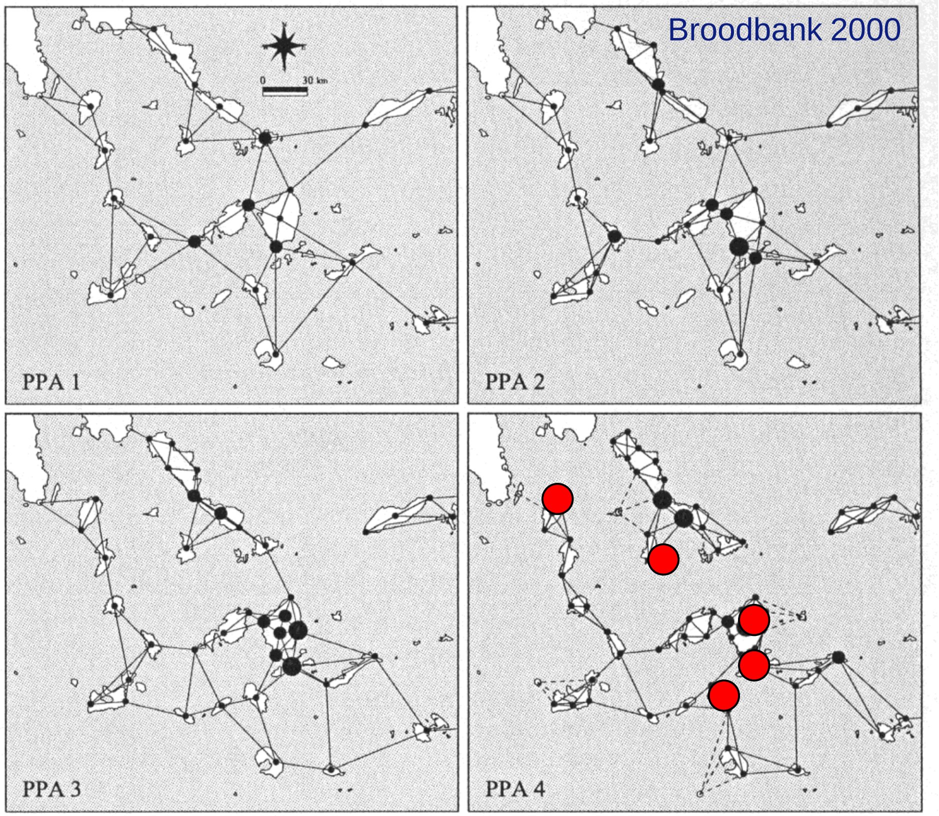

Fig. 75 Nodes of intense communication in the Cyclades as modelled by PPAs 1–4 (five and six linkages only).

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PPA

- Dependent on position and number of sites – villages (50 – 150) inhabitants
  In this case with the passage of time creating more sites!

- Dependent on k (k = 3)

Data: ●

- Agreement so-so!
- Resources (obsidian) not taken into account.
  Should not always conflate exchange!

1. ‘Local’ Networks: Intervening Opportunity Network Models (IOM)

IOM have their origins in urban commuting models/sales networks

- Stouffer and Schneider

Local agency: Opportunism

"The number of persons going a given distance is directly proportional to the number of opportunities at that distance and inversely proportional to the number of intervening opportunities.“ Stouffer

Extended to exchange we would argue that:

- the likelihood that a transaction (exchange) will take place from a source A increases with the number of accessible potential targets
- the likelihood that a transaction (exchange) will take place from A to B falls off inversely with the number of intermediate opportunities C at a shorter (more accessible) distance from A than B – rank more relevant than distance!

1. ‘Local’ Networks: Intervening Opportunity Network Models (IOM)

Generics:

Exchange from i to k:

$T_{ik}\ \alpha\ S_i P(1|\ i,k) = S_i P(1|\ S_i, S_k, S_{ik})$

- S_i - population of site i
- S_{ik} - population /resources between sites i and j as measured by ‘effective’ distance without including the resources of i and k themselves

  e.g. for ‘flat’ distances the population/resources in a circle radius d_{ik} centred on i

1. ‘Local’ Networks: Intervening Opportunity Network Models (IOM)

Many models/variations: Weighted/directional links

- disagree with the Phoenician MDM!

Simple observation: for a linear distribution of sites along a coast, say,

Typically,

$$T_{AB} = \exp(-b\, R_A(B))$$

That is: $T_{AB} = 1, x, x^2, x^3, \ldots x^k \ldots\ldots$(rank k) - Smoothed out PPA!

x is the calibration parameter – weighted networks

Most IOMs much more sophisticated than this!

1. ‘Local’ Networks (IOM): Radiation Model

Extreme model: Control parameters - site positions and sizes!

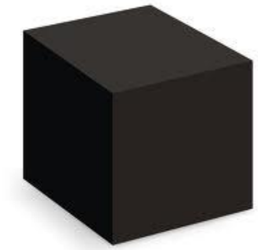
- no calibration parameters!

Simini Barabasi ‘radiation’ model (simplest form)

- S_i = *population of i*
- S_{ij} = *population/resources between i and j as measured by ‘effective’ distance without including resources of i and j themselves*

  - # of intervening opportunities

$$T_{ij} = \frac{S_i S_j}{(S_i + S_{ij})(S_i + S_j + S_{ij})}$$



Simini, F., González, M.C., Maritan, A. and Barabási, A.-L. (2012). `A universal model for mobility and migration patterns`, Nature, 484, 10856

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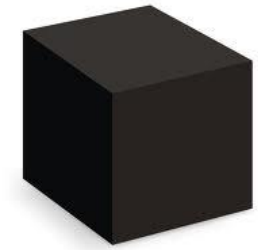


$$T_{ij} = \frac{S_i S_j}{(S_i + S_{ij})(S_i + S_j + S_{ij})}$$

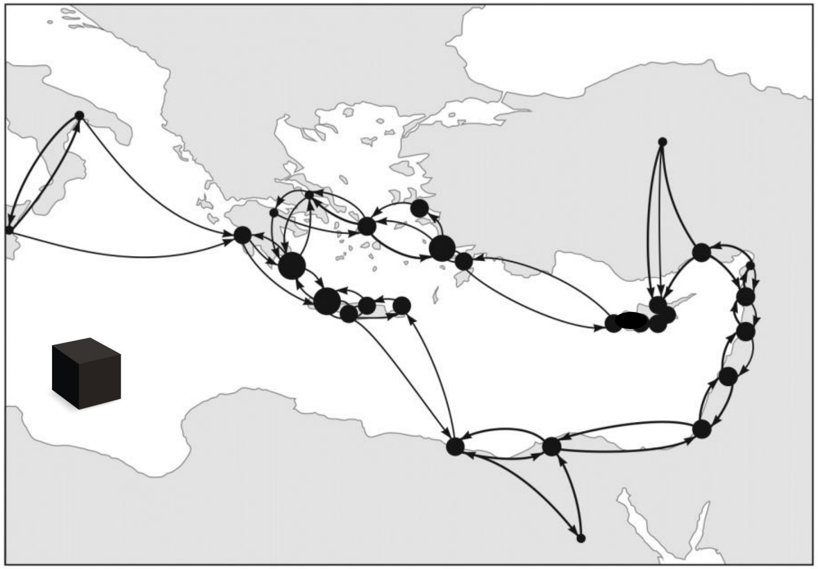
Works poorly – generalised in ad-hoc ways to include calibration parameters – but a good null model

Simini, F., González, M.C., Maritan, A. and Barabási, A.-L. (2012). `A universal model for mobility and migration patterns`, Nature, 484, 10856

1. Example IV: Mycenaean trade/exchange

Extreme model: Control parameters - site positions and sizes!

- no calibration parameters!



Rivers, RJ., Evans, TS., and Knappett, C., in “Maritime Networks: Spatial Structures and Time Dynamics”, ed. C. Ducruet, New York, Routledge, Taylor & Francis Group (2015)

MDM and Radiation Models provide two very different null models

Radiation model is plausible first step and MDM not yet performed

Any questions/comments !

2. ‘Most likely’ Networks: Basic Idea!

Poor data: Have to rely heavily on guesswork!

Best guess!

“All other things being equal, I would expect .... to have happened”

Q. How do we quantify this statement?

Old problem - recently revived

2. ‘Most likely’ Networks:

J Bernoulli
1654

Bayes
1702

Laplace
1749

Principle of insufficient reason (Laplace)

List all the ‘worlds’ which are compatible with your knowledge/ignorance. Each is equally likely. Otherwise you are withholding information!

The most typical of these is the way in which the system is most likely to have behaved.

PIR: Making best use of limited information we have about the system.

2. ‘Most likely’ Networks:

J Bernoulli 1654

Bayes 1702

Laplace 1749

Boltzmann 1844

Maynard Keynes 1883

Jaynes 1922

Wilson 1939

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PIR: Making best use of limited information we have about the system.

Principle of maximum entropy (Jaynes)

Entropy: # of questions with which you need to interrogate the system to have complete knowledge.

PIR ≈ Principle of Maximum Entropy (MaxEnt).

Most likely state of the system is the one with maximum entropy given our limited knowledge.

AKA: Principle of Maximum Ignorance

- epistemic modesty (Jaynes)

2. ‘Most likely’ Networks: Implementing MaxEnt

Multi-step process: Occam’s razor

Minimal knowledge: Null model

Input:

- Exchange takes place but it is (collectively) limited in scope

  - stuff happens!

- Exchange costs/takes effort but (collectively) only so many resources available

  - it costs!

2. 'Most likely' Networks: Implementing MaxEnt

Multi-step process: Occam's razor

Minimal knowledge: Null model

Input:

- Exchange takes place but it is (collectively) limited in scope

  \- stuff happens!

- Exchange costs/takes effort but (collectively) only so many resources available

  \- it costs!

Output:

- Exchange falls off with distance in a way that reflects how cost/effort increases with distance!

2. 'Most likely' Networks: Implementing MaxEnt

Multi-step process: Occam's razor

Example: Suppose

- T_{ij} = # of amphorae transported from i to j (i,j = 1,2,3,...N)
- Cost/effort of transporting a single amphora from i to j is c_{ij}

Further suppose

- TOTAL activity/level of exchange

$$\sum_{ij} T_{ij} = T$$

- TOTAL effort/cost in achieving that activity

$$\sum_{j} c_{ij} T_{ij} = C$$

2. 'Most likely' Networks: Implementing MaxEnt

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Microstates:

In principle enumerate each state of the network commensurate with these constraints and look for the most typical.

Corresponds to maximising the entropy

$$S = - \sum_{ij} T_{ij} (\ln T_{ij} - 1)$$

subject to these constraints

2. ‘Most likely’ Networks: Implementing MaxEnt

Outcome: Gravity model with Boltzmann distribution

Exchange from site i to site j falls off exponentially with the unit ‘cost’ from i to j.

$$\boxed{T_{ij} \;\; \alpha \;\; \exp(-\beta\, c_{ij})}$$

Swap T and C for β and coefficient of proportionality (now given)!

No assumptions about motives of individuals!

Deterrence function:

$$f_{ij} = \exp(-\beta\, c_{ij})$$

is termed the ’*deterrence function*’ for exchange - impedence to flow of ‘goods’

2. 'Most likely' Networks: Deterrence Functions

Knowledge of c_{ij} poor, apart from increasing with distance d_{ij} !

Assume

- *ad valoram* costing
- c_{ij} only depends on the separation d_{ij}

Then $f_{ij} = f(d_{ij}/D)$ defines a function $f(x)$ universal for the network

D is a characteristic distance scale necessary to make
the argument of f dimensionless

'Gravity' choice (mimicking Newton)

$$f(d/D) = a\,(D/d)^2$$

unrealistic for small d (as is any simple inverse power law p>0)

$$f(d/D) = a\,(D/d)^p$$

- nonetheless, still used occasionally!

2. ‘Most likely’ Networks: Deterrence Functions

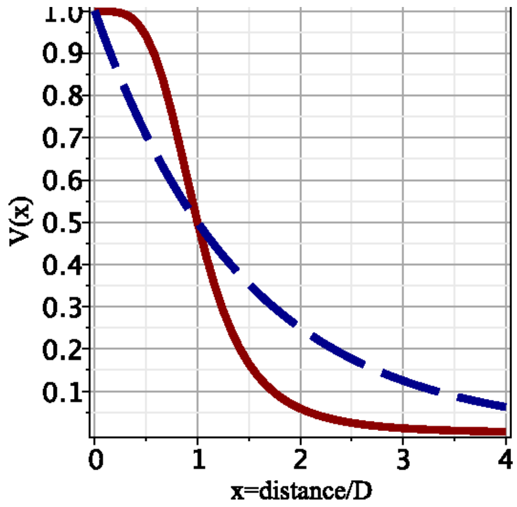
Two obvious choices:

- Equal effort/’cost’ for equal distance/time in exchange/transport of artefacts

$$T_{ij} \;\; \alpha \;\; \exp(- d_{ij} / D)$$

Blue dotted line

‘Deterrence’ function f(x):



- Maximum distance model (MDM; so far and no further)

$$T_{ij} = 1, \quad d < D$$
$$T_{ij} = 0, \quad d > D$$

Difference between red and blue – model discrepancy/inadequacy

- Smoothed out MDM Red line

Appropriate for sea travel with embarkation and disembarkation costs

2. ‘Most likely’ Networks: Simple Gravity Model (SGM)

Coarsegrain:

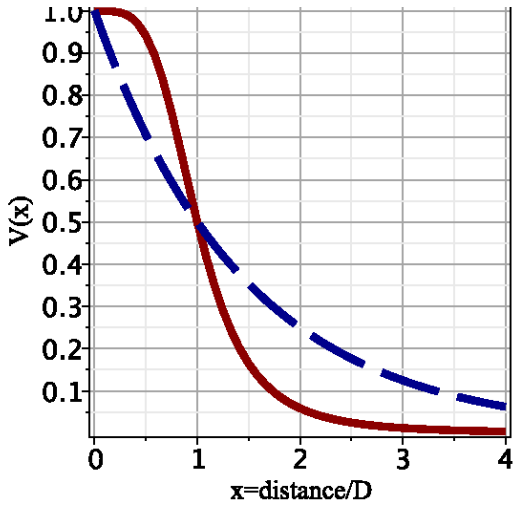
- Assume S_i sources for exchange on site i
- Assume T_j targets for exchange on site j

Then, in appropriate units

$$\boxed{T_{ij} = S_i\, T_j\, f(d_{ij}/D)}$$

Simple Gravity Model (SGM)

‘Deterrence’ function f(x):



- Typically set $S_i = T_i$ as a measure of population of site i

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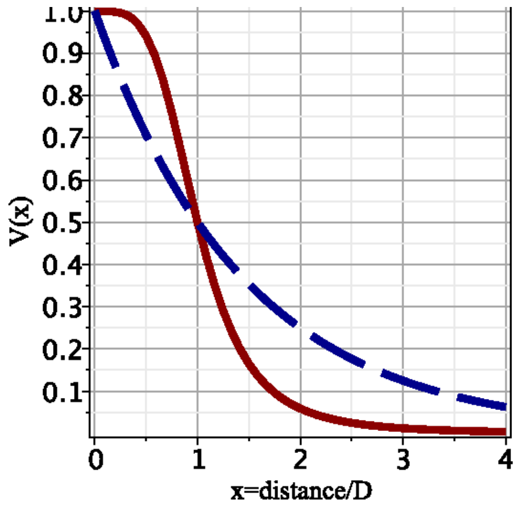


- Typically set $S_i = T_i$ as a measure of population of site i

Note: The SGM is a pseudo-network

Removing a link leaves the other links unchanged

- the whole is just the sum of the parts

2. ‘Most likely’ Networks: Output Constrained Gravity Model (OCGM)

Proposal: Networking can enhance benefits of intermittent favourable wind

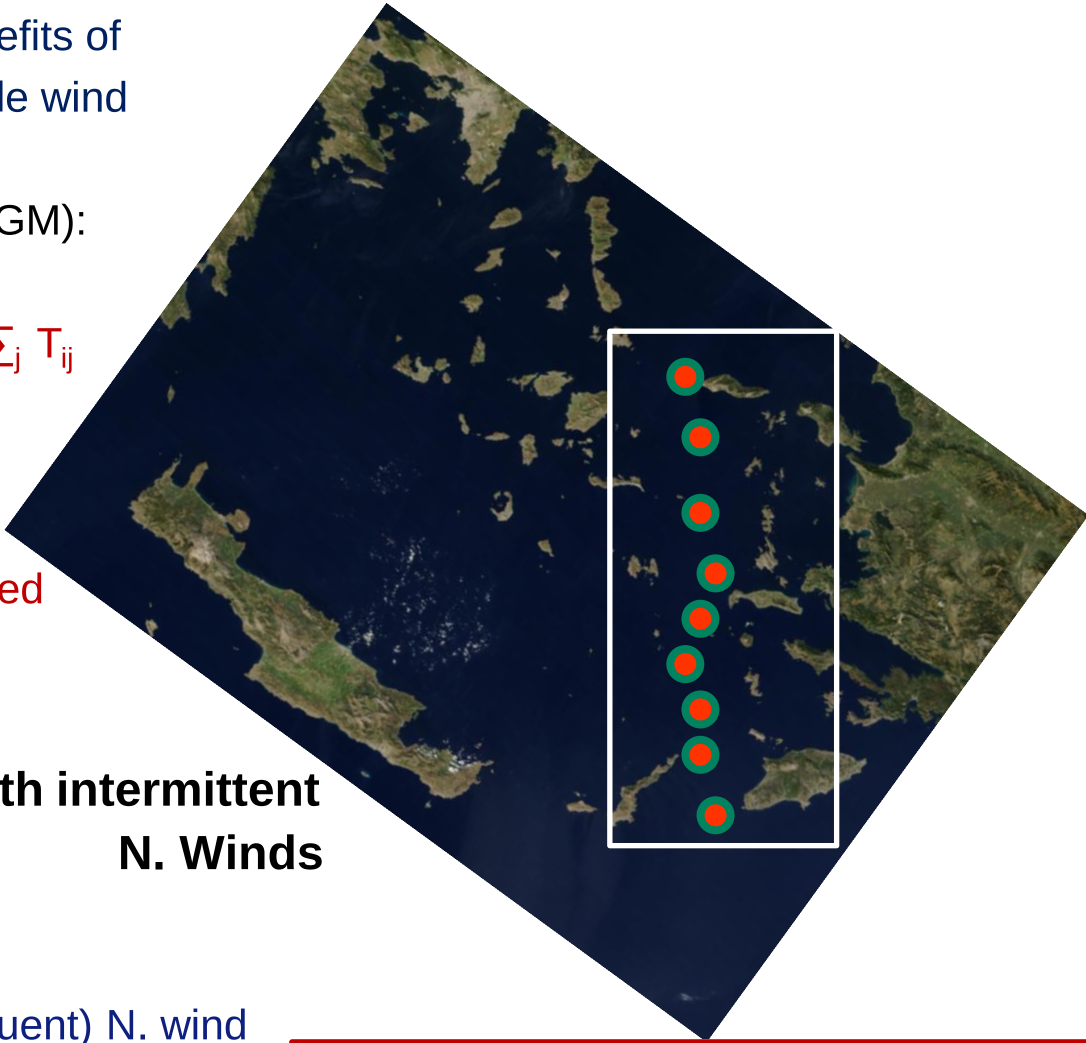
Simplest extension of Gravity Model (OCGM):

We follow PPA in limiting outflows $O_i = \sum_j T_{ij}$

Then $\boxed{T_{ij} = A_i\, O_i\, T_j\, f(d_{ij}/D)}$

where $A_i^{-1} = \sum_j T_j\, f(d_{ij}/D)$ - truly networked

It is the ‘most likely’ network!



Example V. Linear archipelago with intermittent N. Winds

Different friction coefficients for:

- Upwind and downwind travel in (frequent) N. wind
- Upwind and downwind travel in (rare) S. Wind

RJR, TSE, & CK, “Winds and Maritime Linkages in Ancient Greece” in “Advances in Shipping Data Analysis and Modelling. ...”, ed. C. Ducruet, Routledge (2017)

2. 'Most likely' Networks: Doubly Constrained Gravity Model

Doubly Constrained Gravity Model (DCGM):

We limit *both* the outflows $O_i = \sum_j T_{ij}$ and the inflows $I_k = \sum_j T_{jk}$

Then

$$\boxed{T_{ik} = A_i B_k O_i I_k f(d_{ik}/D)}$$

where $A_i^{-1} = \sum_k B_k I_k f(d_{ik}/D)$ and $B_k^{-1} = \sum_i A_i I_i f(d_{ik}/D)$

Mercantile transactions: Typically take $O_i = I_i = T_i \alpha S_i$ (detailed balance)
where S_i is a measure of population ('GDP')

If $d_{ij} = d_{ji}$ then $\boxed{T_{ik} = T_{ki} = \alpha_i f(d_{ik}/D) \alpha_k}$ where $T_i = \alpha_i \sum_j f(d_{ik}/D) \alpha_k$

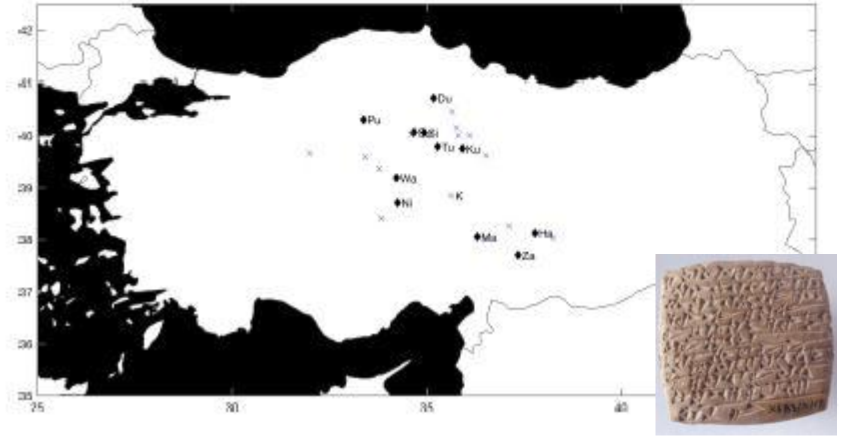
Note: It is the MaxEnt 'most likely' network - ratios follow easily!

2. Example VI: DCGM for Assyrian Trading Networks (Texts!)

Assyrian merchants (mainly 1895-1865 BCE)

- 24000 tablets
- 12000 deciphered
- 400 merchant itineraries involving exchange

- 31 sites (15 unknown)



Aim: Identify missing sites from ratios of exchange

Trade, Merchants and lost cities of the Bronze Age, Barjamovic et al. NBER working paper 23992, http://www.nber.org/papers/w23992

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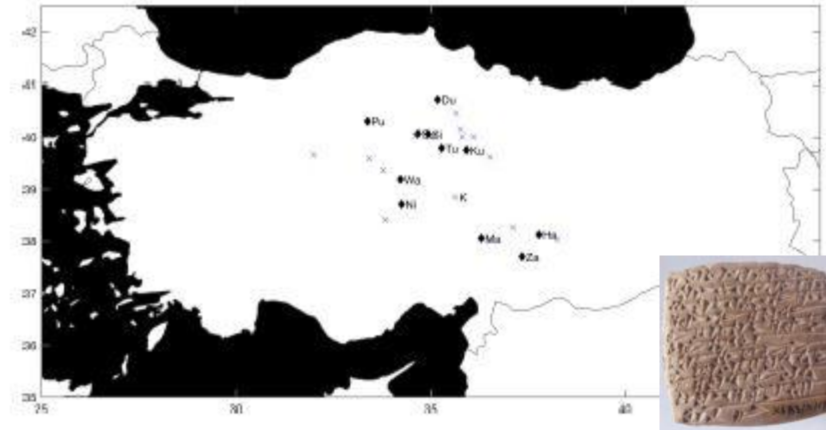

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Assume a DCGM:

Equivalent to adopting a generalisation of *ad valorem* no arbitrage CES that enables us to conflate exchange to a single variable and impose a single distance scale D (Eaton and Kortum 2002)

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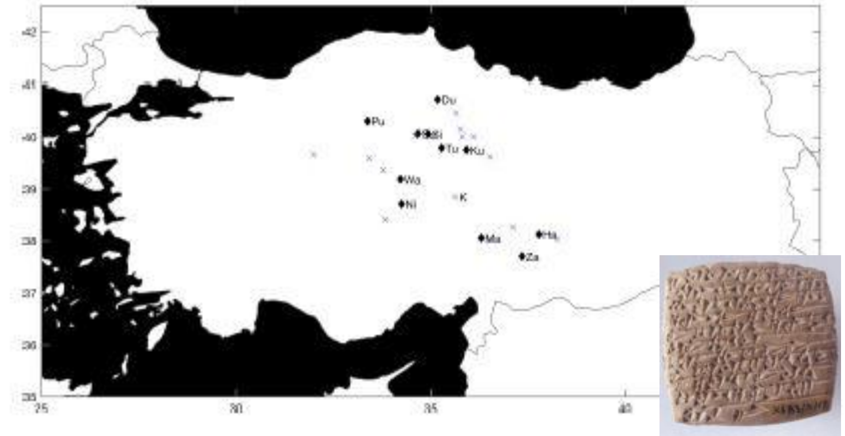

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Data just about good enough to suggest identity of missing sites!

Triangulation via $f(d_{ik}/D)$!

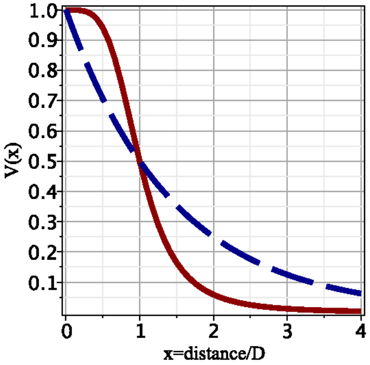
2. ‘Most likely’ Networks: The Wilson ‘Retail’ Model

Designed to describe the dominance of supermarkets and shopping centres and the collapse of High Street shops!

It’s historical counterpart is that of city state formation – the emergence of the polis as a result of

- Urbanisation: Emergence of dominant settlements
- Synoikism: Surrendering of local sovereignty to a wider community

‘Deterrence’ function f(x):



Two calibration parameters:

- distance scale D
- ‘attractiveness Y’

  - benefit of concentrated resources

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Example VII: 9th C BCE Greek city states (Rihll and Wilson, 1987)

Thebes, Athens, Carthage as the Tesco, Aldi, Carrefour of geometric/archaic Greece!

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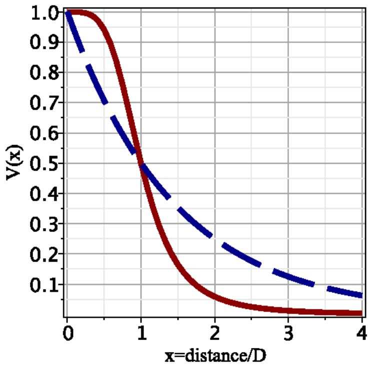

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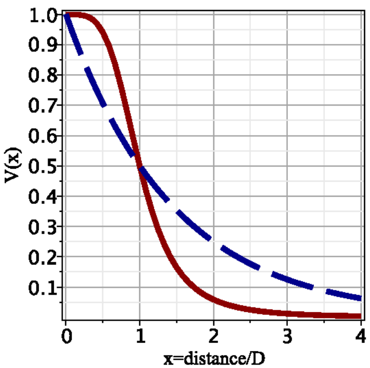
Several subsequent applications:
e.g.
- Bronze Age Khabur triangle (Davies et al., JAS 2014)
- La Tène W. Europe (Filet 2017), ...

‘Deterrence’ function f(x):



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2. ‘Most likely’ Networks: The Wilson ‘Retail’ Model

Constrained entropy model: entropy a ‘superconcept’

- As before, constrain outflows $O_i = \sum_j T_{ij}$
- Inflows $I_k = \sum_j T_{jk}$ unconstrained but constrain inflow entropy

$$S(I) = - \sum_j I_j (\ln I_j - 1)$$

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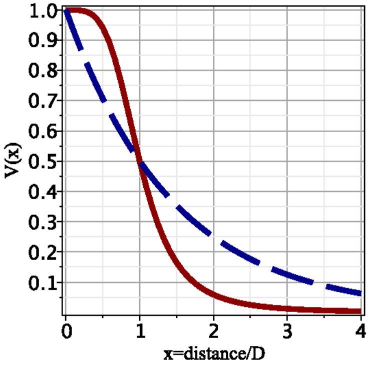

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$$S(I) = - \sum_j I_j (\ln I_j - 1)$$

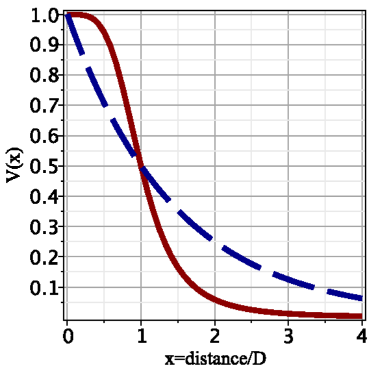
Then

$$\boxed{T_{ij} = A_i O_i (I_j)^{\Upsilon} f(d_{ij}/D)}$$

where A_i, I_j determined by self-consistent equations

$$(A_i)^{-1} = \sum_j (I_j)^{\Upsilon} f(d_{ij}/D) \quad \text{and} \quad I_j = \sum_I A_i O_i (I_j)^{\Upsilon} f(d_{ij}/D)$$

'Deterrence' function f(x):



Two calibration parameters:

- distance scale D
- 'attractiveness Υ

- benefit of concentrated resources

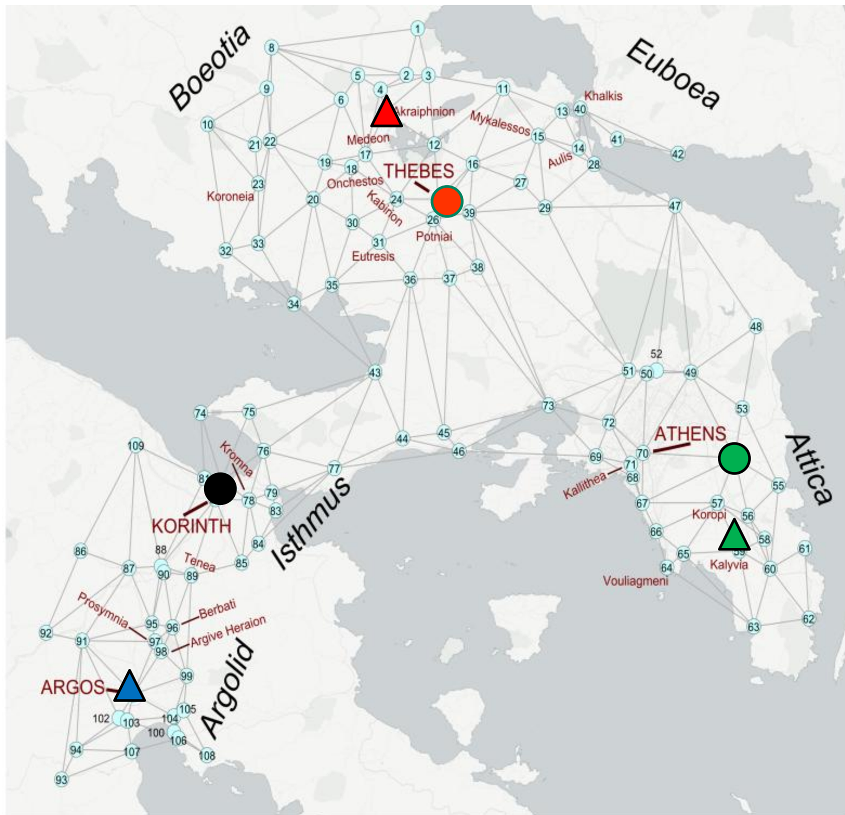
2. Example VII: Greece in 9th and 8th C BCE

Outputs:

Rihll & Wilson (1987, 1991); Rivers & Evans (2015, 2017)

As Υ increases above unity there is a transition in which a few important sites grow at the expense of smaller ones with a handful of second-tier cities.

Identifiable regional structure appears



In particular:

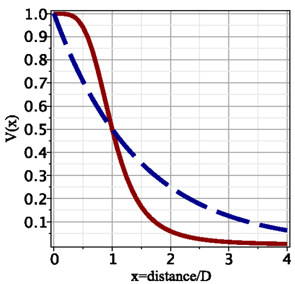
- Thebes
- Corinth
- Athens
- ......
- Argos
- Akraiphnion
- Kalyvia
- .....

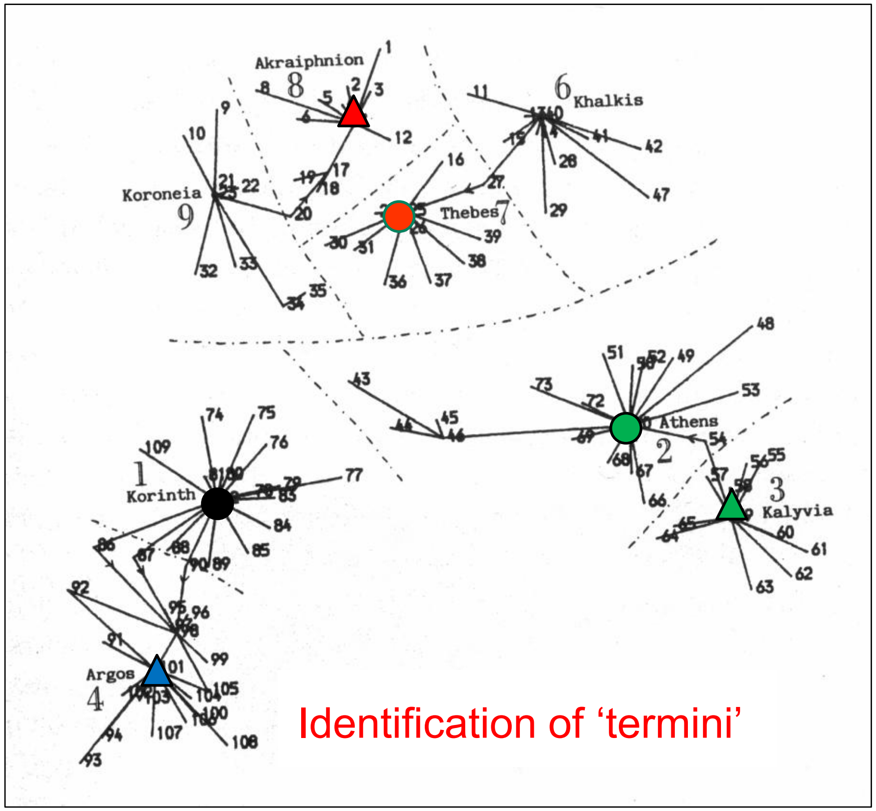
2. Example VII: Greece in 9th and 8th C BCE

Outputs:

Rihll & Wilson (1987), *Histoire & Mesure* 2: 5-32.

R&W use exponential decay for f(x) – blue line





Identification of 'termini'

In particular:

- Thebes
- Corinth
- Athens
- ......
- Argos
- Akraiphnion
- Kalyvia
- .....

In the RW analysis these were the key sites of that period!

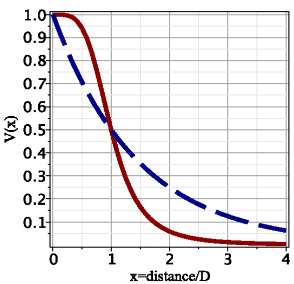
Data very poor – nothing more than historical knowledge of site importance!

2. Example VII: Greece in 9th and 8th C BCE

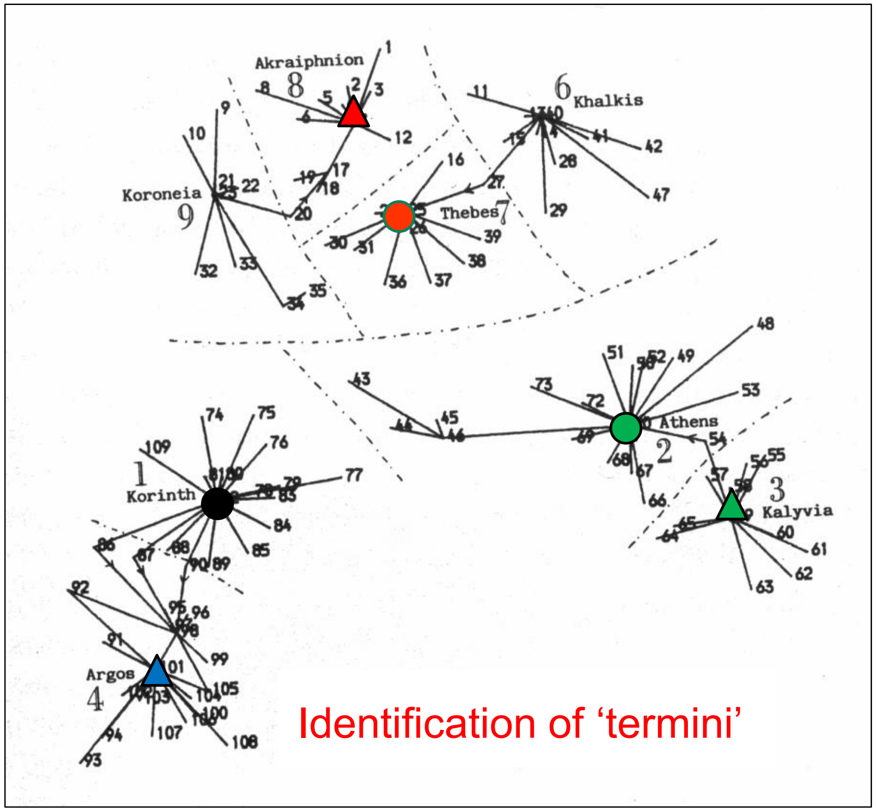
Outputs:

Rivers & Evans (2017) Frontiers in Digital Humanities
https://www.frontiersin.org/articles/10.3389/fdigh.2017.00008.

R&E use power law decay for f(x) – red line plus different distance coarse-grainings d(δ, P, f) - contingency!



In general key sites are robust – one exception – Thebes – which comes and goes!



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- Thebes
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- Kalyvia
- .....

Harmony is correct – have missed the melody; something important happens at Thebes!

2. Understanding the Wilson ‘Retail’ Model

Why ‘retail’ model?

Equivalent to assuming:

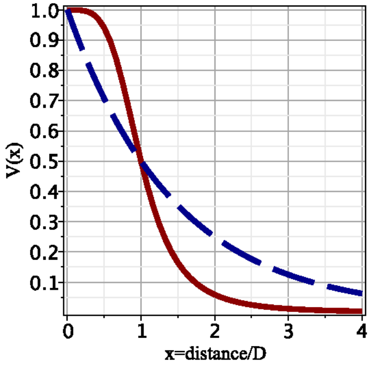
i. Interaction between two places is proportional to the size of the origin zone and the importance and distance from the origin zone of all the other sites which compete as destination zones

ii. the ‘importance’ of a place is proportional to the interaction it attracts from other places

- quantified in terms of footfall and retail space

i. ‘inflow’ of a site ~ site ‘importance’

‘Deterrence’ function f(x):



Two calibration parameters:

- distance scale D
- ‘attractiveness Y’

- benefit of concentrated resources

2. Epistemic - Ontic duality

- On the one hand we have seen how *epistemic* MaxEnt models can have an *ontic* effective agent-based model (ABM) realisation

DCGM MaxEnt $\longleftrightarrow$ Ricardian no arbitrage *ad valorem* CES Wiebull...

RW MaxEnt $\longleftrightarrow$ Retail management, ABM, Lokta-Volterra equations, ...

- Converse equally applicable:

IOM (Stouffer/Schneider) $\longleftrightarrow$ Constrained MaxEnt

2. Epistemic - Ontic duality

Mathematics identical but interpretation different in two ways:

- ABM approach with its emphasis on contemporary economic practice looks as if we are trying to impose the present on the past

  From an epistemic perspective we are interrogating our own knowledge without imputing specific activity to any agents

- MaxEnt has no obvious dynamical content unlike ABM, which is intrinsically dynamical

- From MaxEnt viewpoint 'history' is an attempt to maintain 'good' functionality at all times as circumstances change

- From ABM viewpoint 'history' is an attempt to achieve 'good' functionality from a non-optimal beginning

Any questions/comments !

3. The ‘best’ Networks: Cost-Benefit Models

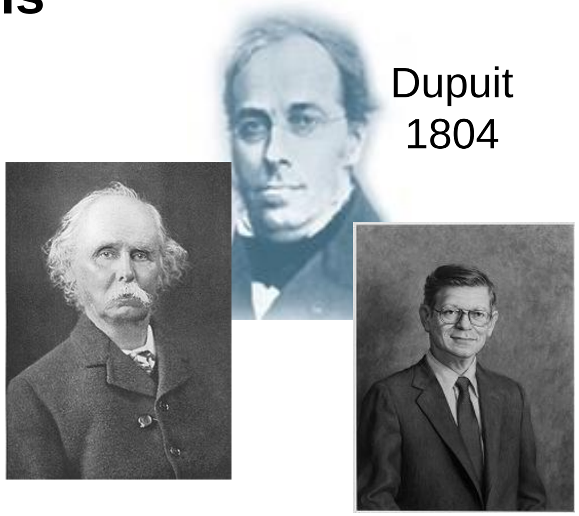
Leibniz
1646

Maupertuis (Pangloss?)
1698

Lagrange
1736

Dupuit
1804

Marshall
1842



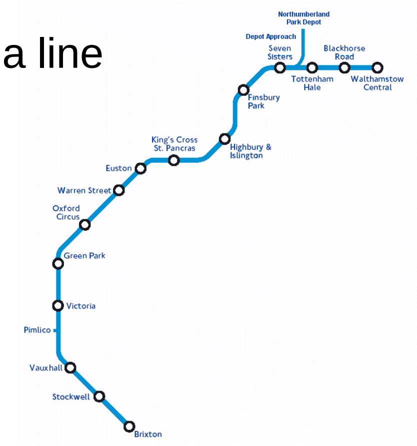
Eckstein 1927

Principle of sufficient reason (Leibniz)

Cost-benefit analysis

We live in the ‘best’ of all possible worlds!

Victoria line



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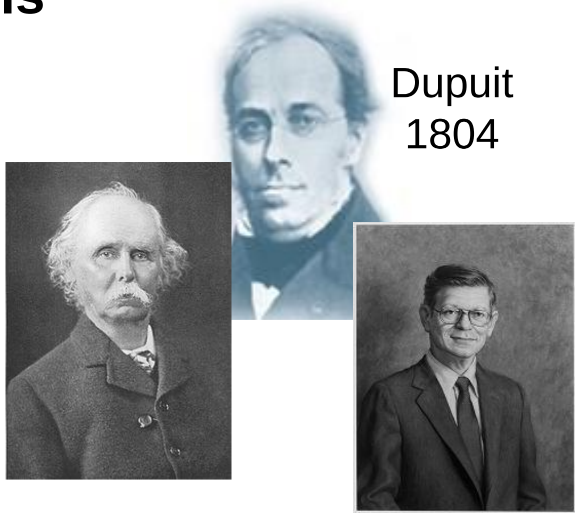

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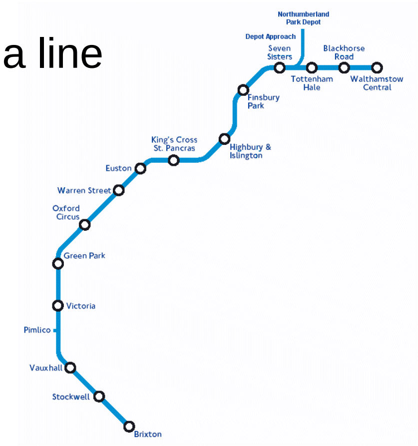

Living in the ‘most likely’ possible world and the ‘best’ possible world are not as orthogonal as they look!

Can be dual realisations (NOT alternative truths!)

3. The ‘best’ Networks: Cost-Benefit Models

Agency: The network tries to become ‘efficient’ by

- expanding benefits **B** of exchange, exploiting local resources
- reducing cost/effort **C** of sustaining the network

Realised through a social potential $\mathbf{H = C(T_{ij}) - B(T_{ij})}$

The more efficient the network the smaller H!

A network ‘landscape’:

- each position is a network
- H is the elevation

the lower the elevation the ‘better’ the network!

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A network 'landscape':

- each position is a network
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the lower the elevation the 'better' the network!

Look for the 'best'

– be satisfied with the 'good'

- 'Satisficing' strategy
- Bounded rationality

3. The ‘best’ Networks: Model Construction

- Some generalities but ultimately *bespoke*
- More like a construction kit than a black box!

Aim:

Only use models in an environment that imposes structure

e.g. ‘Goldilocks’ scenario:

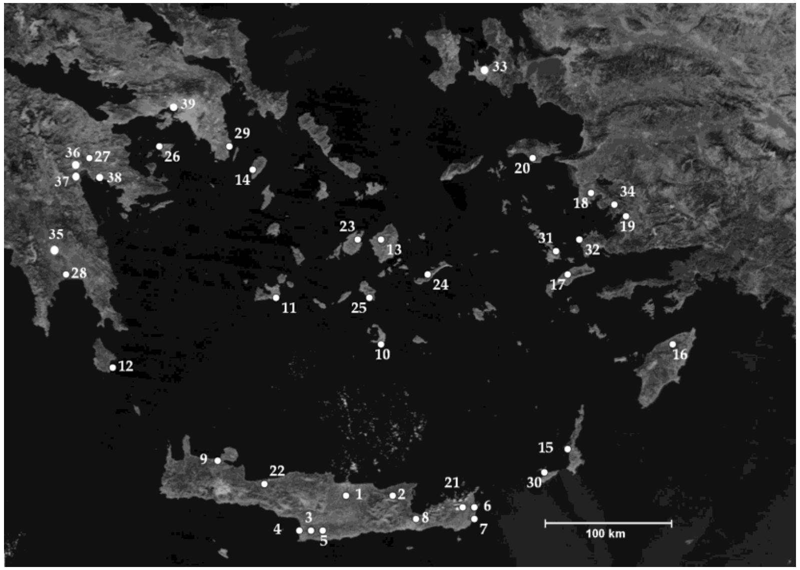
Treading the tightrope between ‘boom’ and ‘bust’

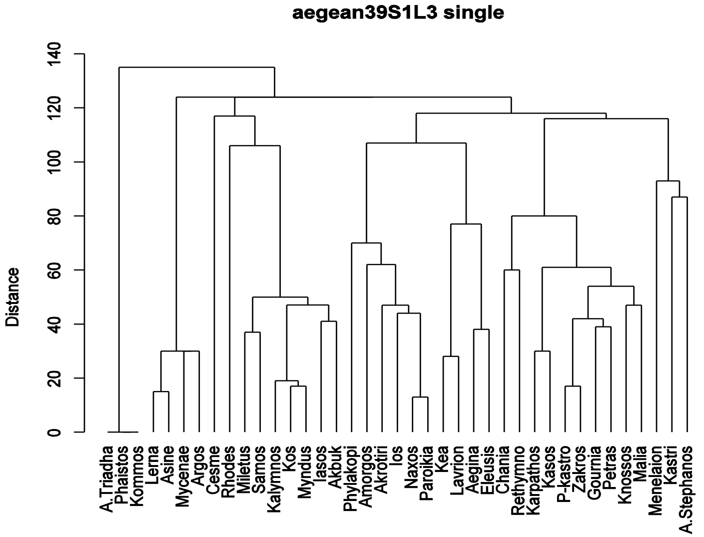
Our model is ‘ariadne’

3. Example VIII: Cost-benefit analysis for MBA Aegean

Minoanisation: 1800 – 1450 BCE

- prodigious amount of excavated material
- documented artefacts not organised for exchange analysis
- many examples, but not quantified





Lots of data! NOT Big Data

3. Example VIII: Cost-benefit analysis for MBA Aegean

Minoanisation: 1800 – 1450 BCE

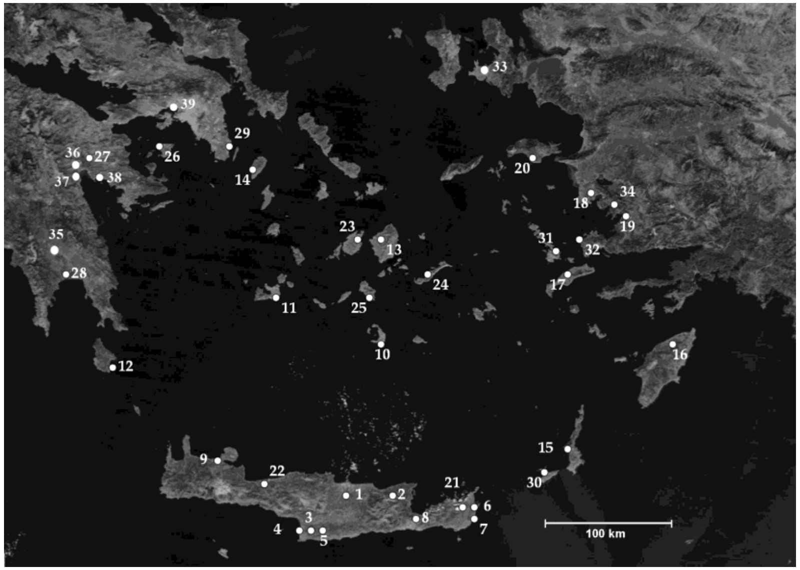
We expect to see signals of

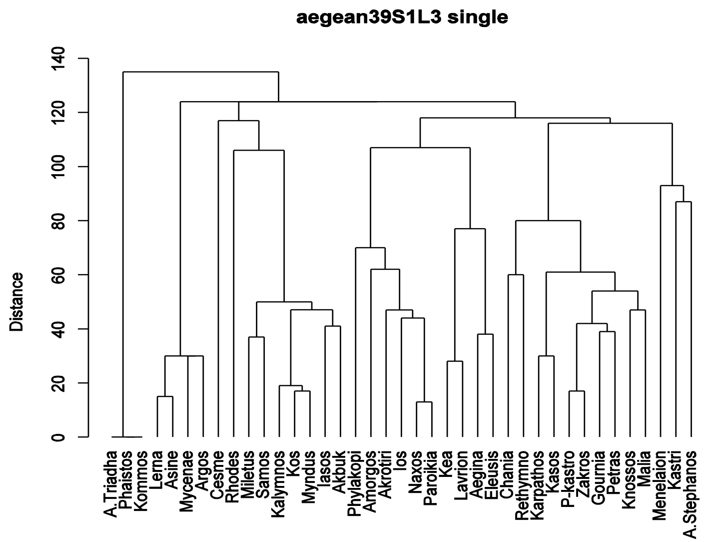
- An ability of the network to thrive for a distance scale in f(x) of 120 -120 km and not for smaller distance scales

- A demonstration of the importance of city states in N. Crete

- A rearrangement of exchange patterns after the eruption of Thera in accord with the record

- An understanding of robustness and stability





3. Example VIII: Cost-benefit analysis for MBA Aegean (*ariadne*)

$$H =$$

$$-\kappa \sum_i S_i v_i (1 - v_i)$$

- benefits of local resources

$$-\lambda \sum_{i,j} V(d_{ij} / D).S_i v_i .e_{ij} .S_j v_j$$

- benefits of exchange

$$+ j \sum_i S_i v_i$$

- costs of sustaining population

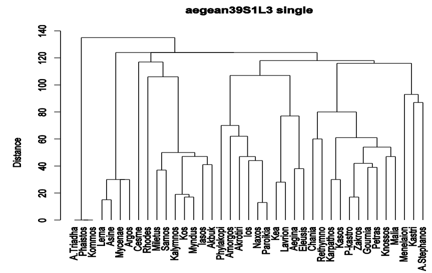
$$+ \mu \sum_{i,j} S_i v_i e_{ij}$$

- costs of sustaining network

Knappett, Evans & Rivers, 2008. Modelling maritime interaction in the Aegean Bronze Age, Antiquity 82

- S_i - carrying capacity of site i
- v_i - fraction of resources of i that are exploited
- e_{ij} - relative benefits of unit exchange from i to j
- S_i v_i - population of i

3. Example VIII: Cost-benefit analysis for MBA Aegean (*ariadne*)



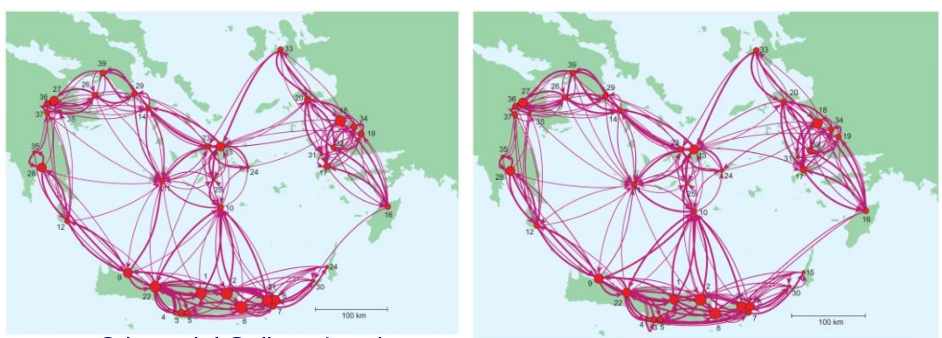
$$H =$$
$$-\kappa \sum_i S_i v_i (1 - v_i)$$
$$-\lambda \sum_{i,j} V(d_{ij}/D).S_i v_i.e_{ij}.S_j v_j$$
$$+ j \sum_i S_i v_i$$
$$+ \mu \sum_{i,j} S_i v_i e_{ij}$$

Model aims for ‘best’

Settles for the ‘good’

It works!

‘Goldilocks’ scenario

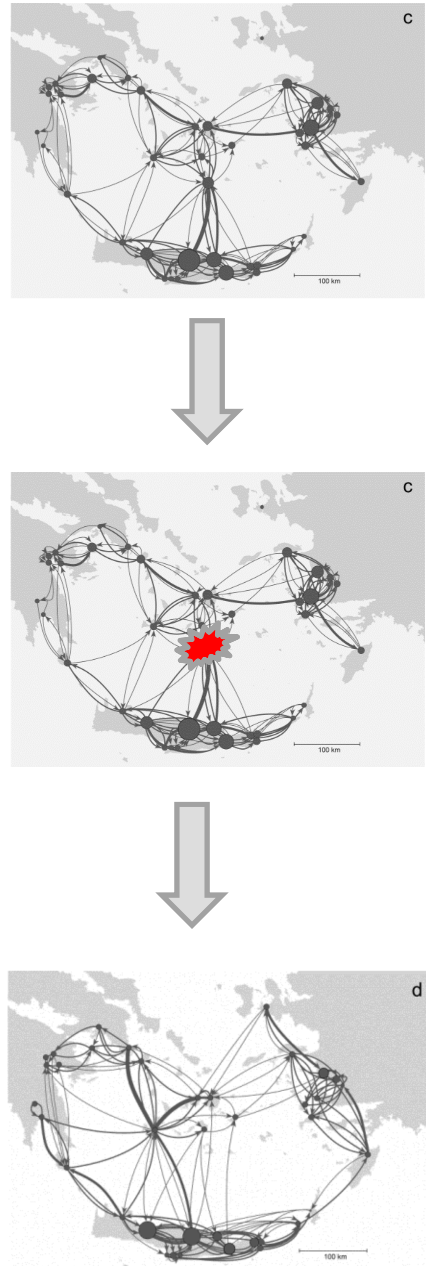


Stochastic optimisation

- contingency!

3. Example VIII: Eruption of Thera (robustness)

$$
\begin{aligned}
H = \\
&-\kappa \sum_{i} S_i v_i (1 - v_i) \\
&-\lambda \sum_{i,j} V(d_{ij} / D).S_i v_i .e_{ij} .S_j v_j \\
&+ j \sum_{i} S_i v_i \\
&+ \mu \sum_{i,j} S_i v_i e_{ij}
\end{aligned}
$$



“the evidence points to, if anything, an increase in Minoan trading activity in LM IB, particularly in our excavations at Ayia Irini, Keos (14) where we literally had thousands of LM IB vases imported from outside” (Pichler 1980)

3. Example IX: Late post-eruption behaviour (network instability)

‘Burning of the Palaces’

$$
\begin{aligned}
H = &\\
&-\kappa \sum_i S_i v_i (1 - v_i)\\
&-\lambda \sum_{i,j} V(d_{ij}/D).S_i v_i.e_{ij}.S_j v_j\\
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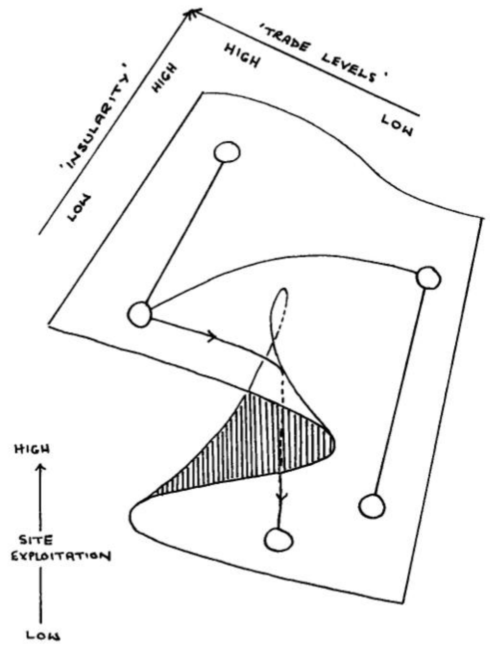
&+ \mu \sum_{i,j} S_i v_i e_{ij}
\end{aligned}
$$



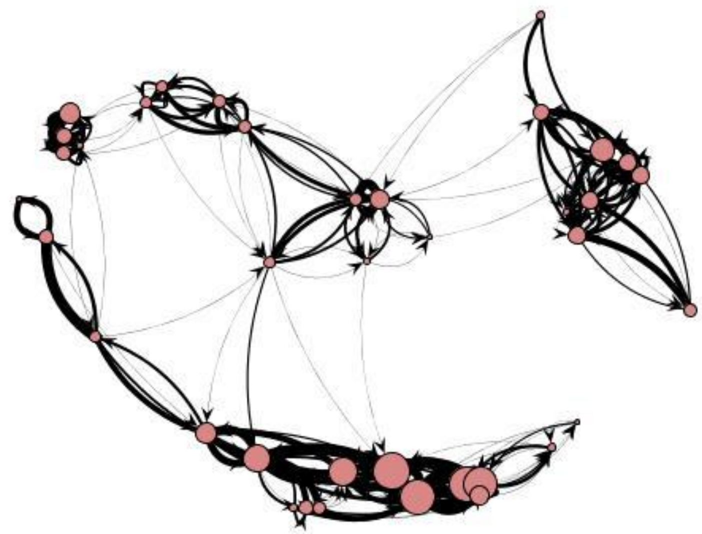
Basic catastrophe theory

“ a centralised economy which may be working under some adversity which might be increased population … people coming in from Thera … What I think you would expect to see is not a gradual decline, but an increasing intensity in the various subsystems of the culture system, including an increasing level of trade, until the system breaks down altogether. *There is a parallel here with a stock exchange collapse”*

Renfrew (1980)

3. Example IX: Late post-eruption behaviour (network instability)

Increasing costs of sustaining network (e.g. piracy)



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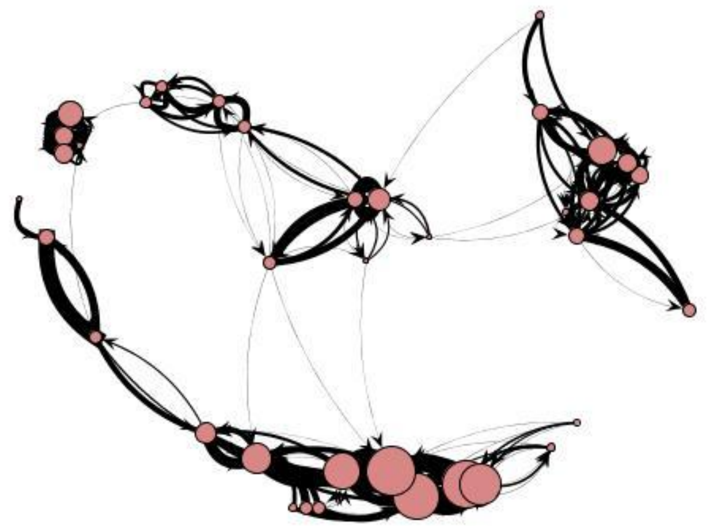

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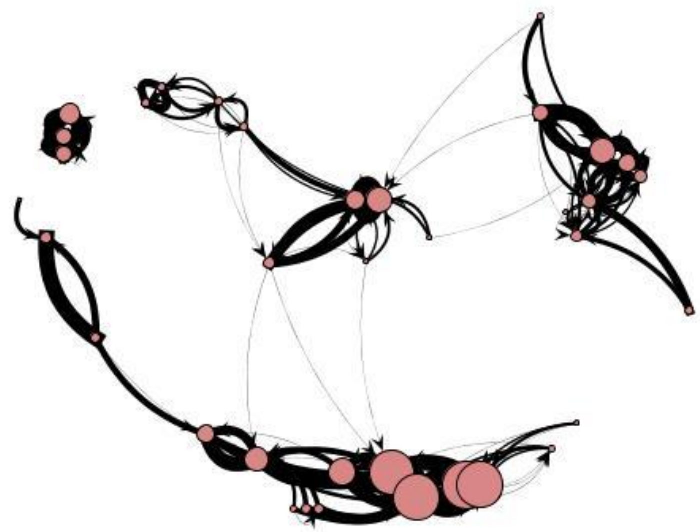

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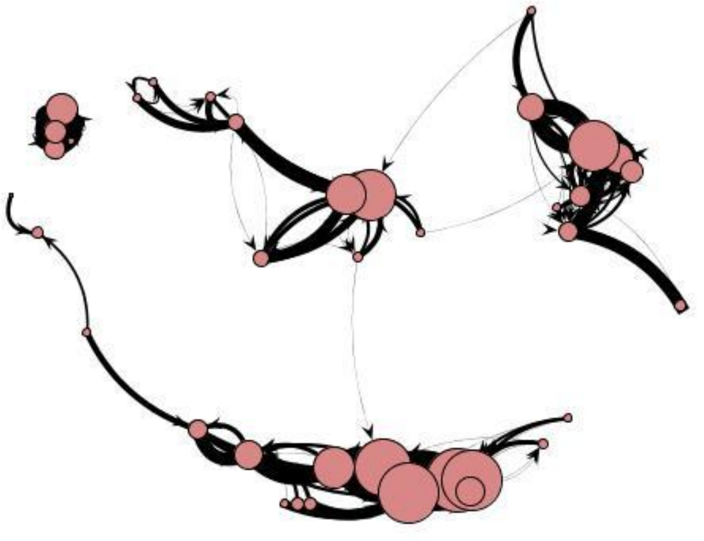

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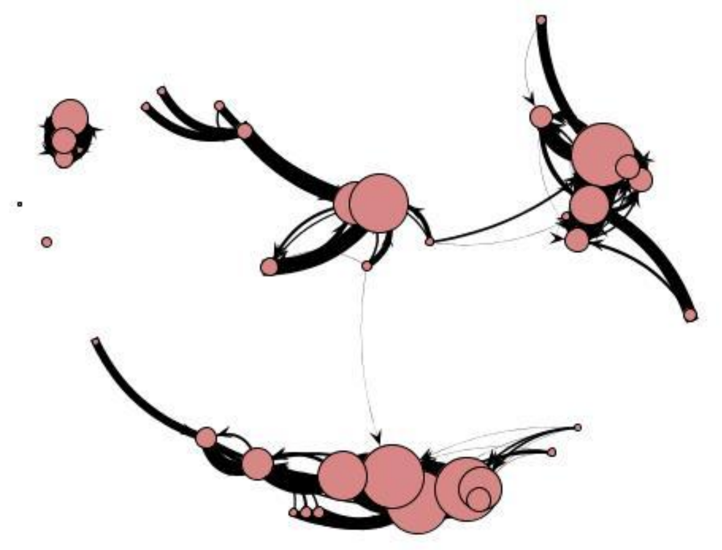

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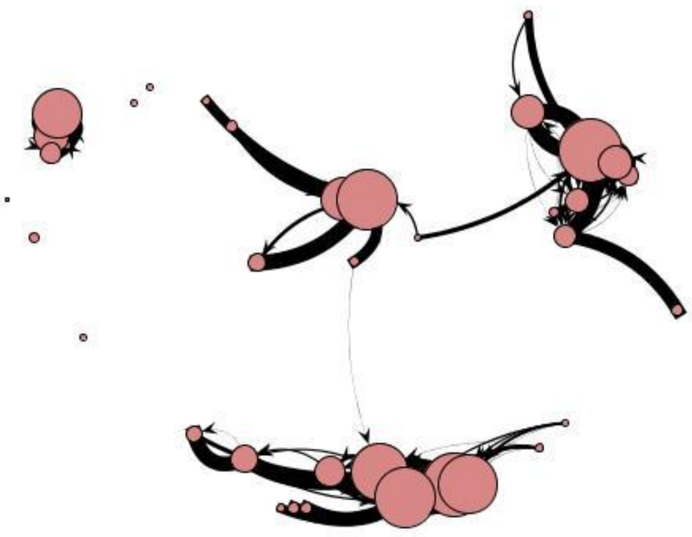
3. Example IX: Late post-eruption behaviour (network instability)

Increasing costs of sustaining network (e.g. piracy)

- Network puts all its eggs in fewer baskets
- Loses its weak links and becomes unstable
- Contingency now plays an important role in how collapse occurs



Not inevitable:

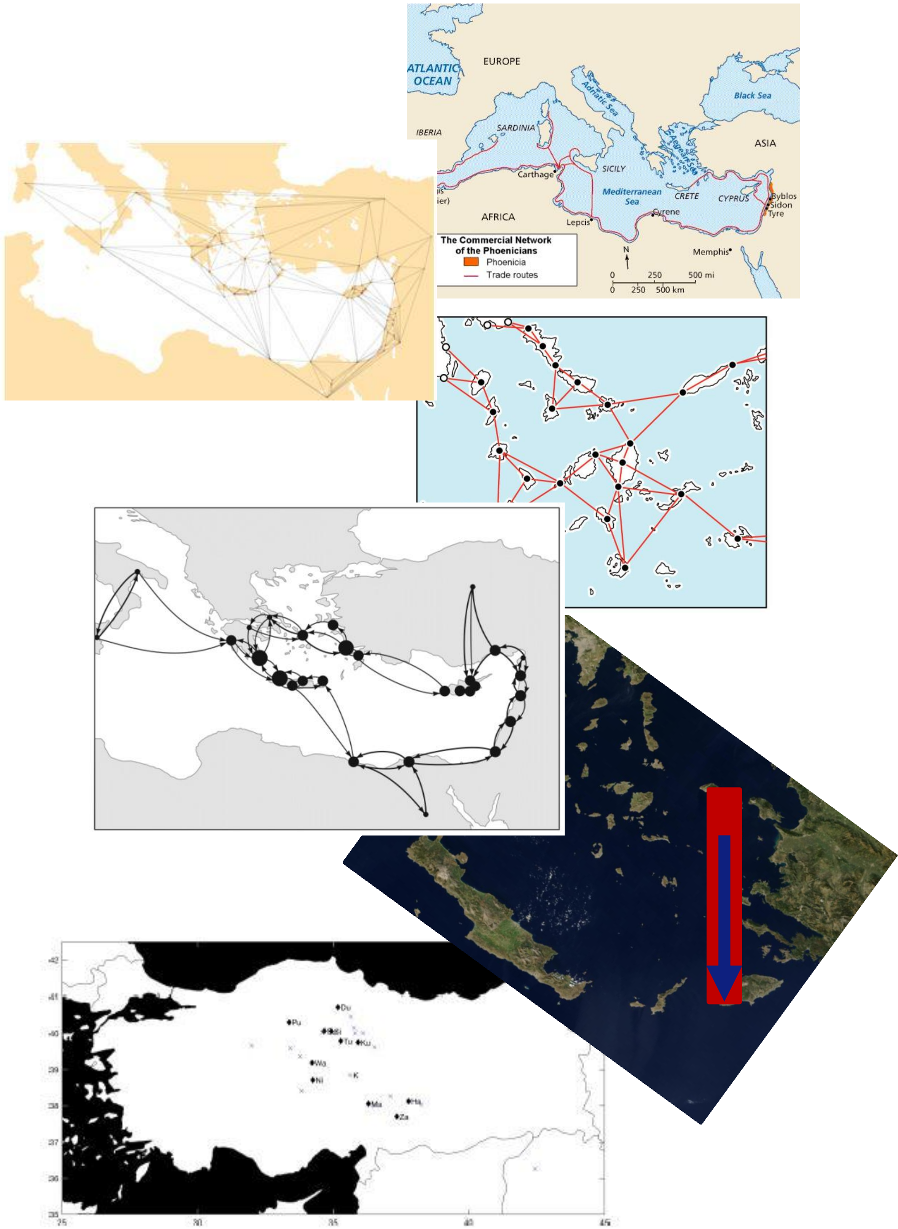
Could be

- External invasion
- Natural causes (earthquake)

aegean38S1L3a, -j -2.00, -m 3.00, -k 1.00, -l 4.00, -dl 110, -ds 5.00, -bt 3.44e+07, -bs 1.20, -a 4.00, -

IV. Summary: ‘Successful’ applications

- Iron Age Phoenician Mediterranean (MDM)
  Bakker et al. 2018
- LBA Mycenaean E. Mediterranian (MDM+)
  Gheorghiade, Rivers et al., 2018
- EBA Cyclades (PPA)
  Broodbank 2005
- BA E. Mediterranean (Radiation IOM)
  Rivers et al. 2015
- Aegean wind (OCGM)
  Rivers et al. 2017
- BA Assyrian trade networks (DCGM)
  Barjamovic et al. 2018



IV. Summary: ‘Successful’ applications

- Iron Age Greece (RW retail model)
  Rihll & Wilson 1987, 1991
  Rivers & Evans 2014,2017

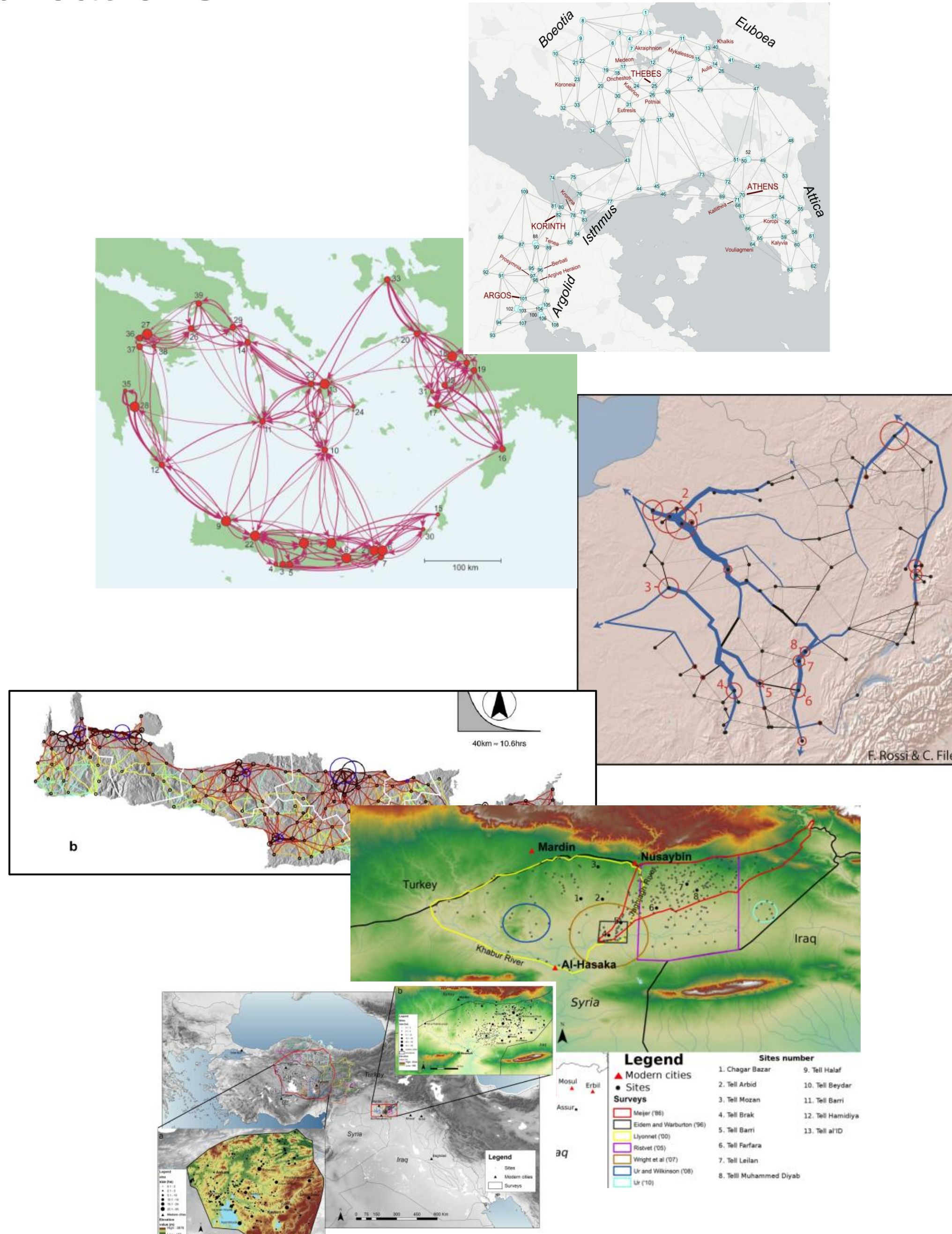
- MBA Aegean (CB *ariadne* model)
  Rivers et al., 2006, 2008, 2009, 2011

- Iron Age Gaul (RW model)
  Rossi & Filet 2017

- BA Crete (RW model)
  Bevan et al. 2013, 2016

- Iron Age Syria (RW model)
  Davies et al. 2013

- BA Anatolia (RW model)
  Palmisano et al. 2015



IV. Conclusions:

- ❑ Be as simple as you can!

- ➢ Simple: Take the model with the least necessary assumptions as possible
  Null Model (intentionally oversimplified) e.g. geographical distances!

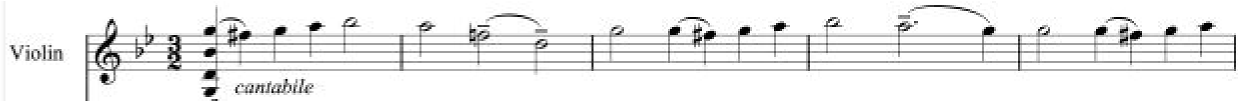
- ➢ Robustness: Test the model until it breaks
  - some form of sensitivity analysis is obligatory!

- ❑ Increase the sophistication and repeat!

- ➢ Don’t be more sophisticated than the data – coarse-grain both models and data

- ❑ Don’t expect too much. You are not a historian!

- ➢ You will never get the melodic line:



- ➢ But it can still work in its way!

:

“The purpose of a good model is to formulate simple concepts and hypotheses concerning them, and to demonstrate that, despite their simplicity, they give approximate accounts of otherwise complex behaviour of phenomena. If a model ‘works’ ... then it shows that the assumptions and hypotheses built into the model contribute to an explanation of the phenomena”

- Alan Wilson 1981

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Carl Knappett, Henry Price, Paula Gheorghiade and Clara Filet

Thank You!

“A hypothesis is important if it "explains" much by little.... To be important, therefore, a hypothesis must be descriptively false in its assumptions, ... Truly important and significant hypotheses will be found to have "assumptions" that are wildly inaccurate descriptive representations of reality, and, in general, the more significant the theory, the more unrealistic the assumptions.”

- Milton Friedman 1953

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Notes I: Intervening Opportunity Network Models (IOM)

Schneider:

- Total travel time from a site is minimized, subject to the condition that every destination point has a stated probability of being accepted if it is considered.
- The probability of a destination being accepted, if it is considered, is a constant, independent of the order in which destinations are considered.
- It is an output-constrained model in that the site outflows $O_i = \sum_j T_{ij}$ are assumed given

- generalisation of PPA

Output:

$P(1 | S_i, S_k, S_{ik}) = \exp(- a S_{ik})[1 – \exp(- a S_k)]$

- calibration parameter a

I don't know of any historical context to which it has been applied!