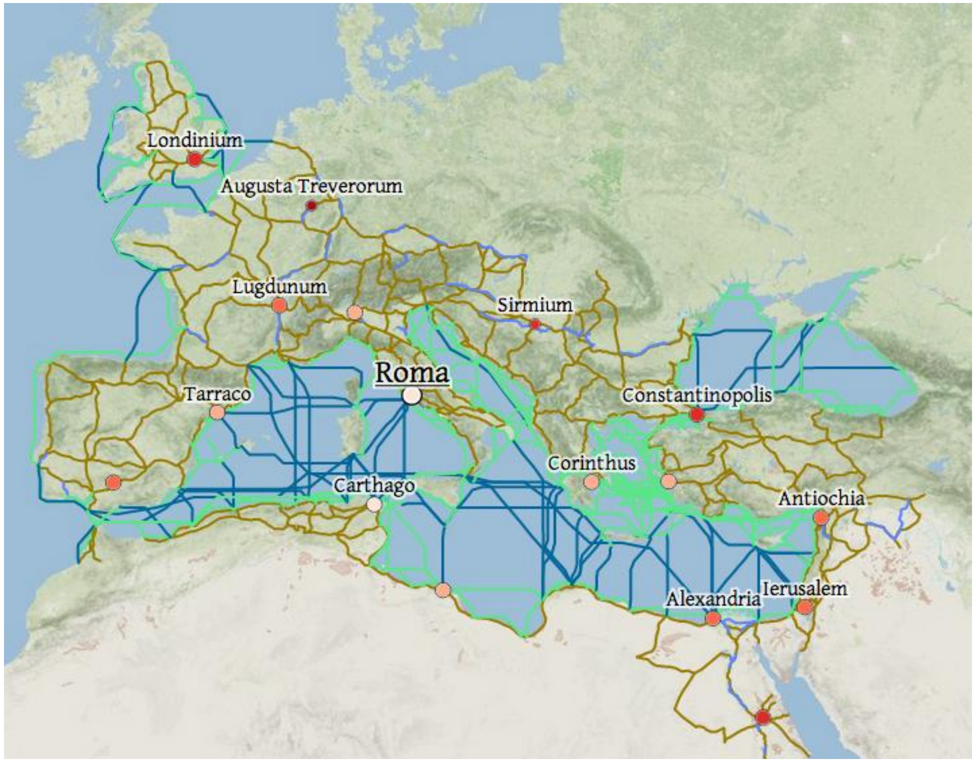
Imperial College London

Modelling Spatial Exchange. I

Ray Rivers (ICL)



Modelling Interaction in Landscape Archaeology, Kiel, August 2018

Prologue:

This lecture is about modelling exchange and mobility in early society

The models that we have available are 'easy' to implement/code

Problems lie elsewhere!

Main issues:

- ❑ When is modelling appropriate?
- ❑ When it is, how do we construct and choose models?

Most of this talk will address these issues rather than the details of the models themselves!

Talk Structure:

I shall argue that spatial exchange networks provide a framework to address these questions

Overview: Limitations of modelling

Part I: Exchange Networks

Part II. Model generics

Part III: Summary

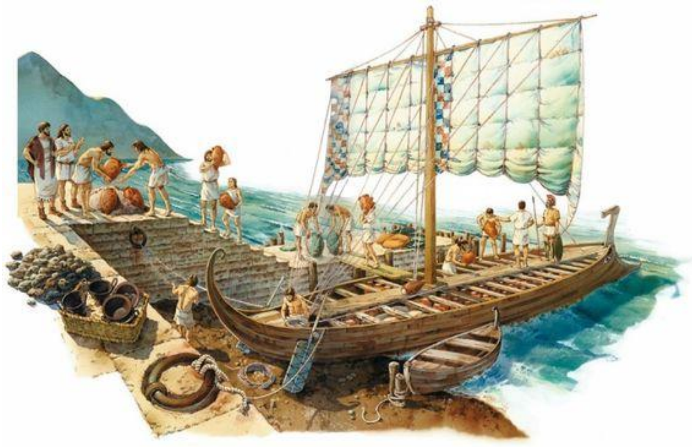
Overview:

Broad Question:

Can we understand the archaeological record in terms of patterns of social interaction, particularly for pre- and proto-historical societies?

A traditional historical approach looks for dramatic narrative – typically war and conquest

Misleading approach:



Fortunately, for most of the people most of the time drama passes by

Just as economic models are poor at describing crises our analysis is only applicable when 'history is idling'.

We are modelling 'ordinary' times

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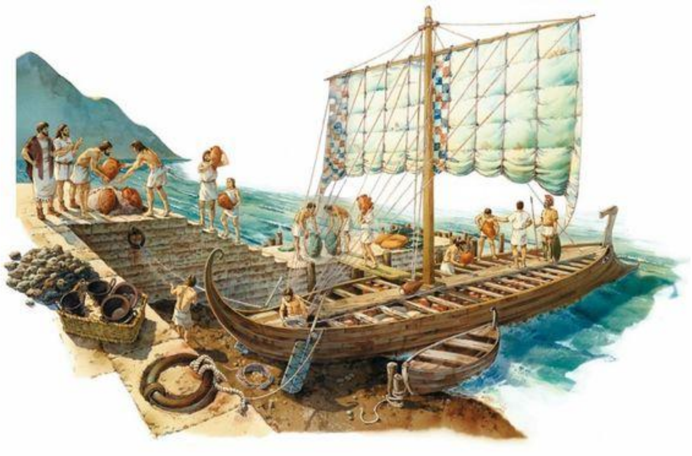

Fortunately, for most of the people most of the time drama passes by

Just as economic models are poor at describing crises our analysis is only applicable when 'history is idling'.

Making a virtue of the mundane

We are modelling 'ordinary' times

Outcomes often the 'obvious'!

Overview: 'Ordinary' times

Applications: Large-scale 'cultures' functioning for long periods of time:

Examples that follow in Part II are taken from

- Assyrian mercantile networks (EBA)
- Cycladic culture (EBA)
- Minoan culture (MBA)
- Mycenaean culture (MBA/LBA)
- Archaic Greece (IA)
- Phoenician maritime networks (IA)

Mediterranean 'Cultures'

Overview: ‘Ordinary’ times

Applications: Large-scale ‘cultures’ functioning for long periods of time:

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Is this Systems Theory/ Processualism 2.0?

Overview: Processualism Yes and No!

Musical metaphor:

- Modelling provides the basic harmony
- ‘Sociopolitics’ – whose details we cannot model - provides the melodic line

- the chaconne (passacaglia) consists of a melody over a harmonic ground - a series of chords.

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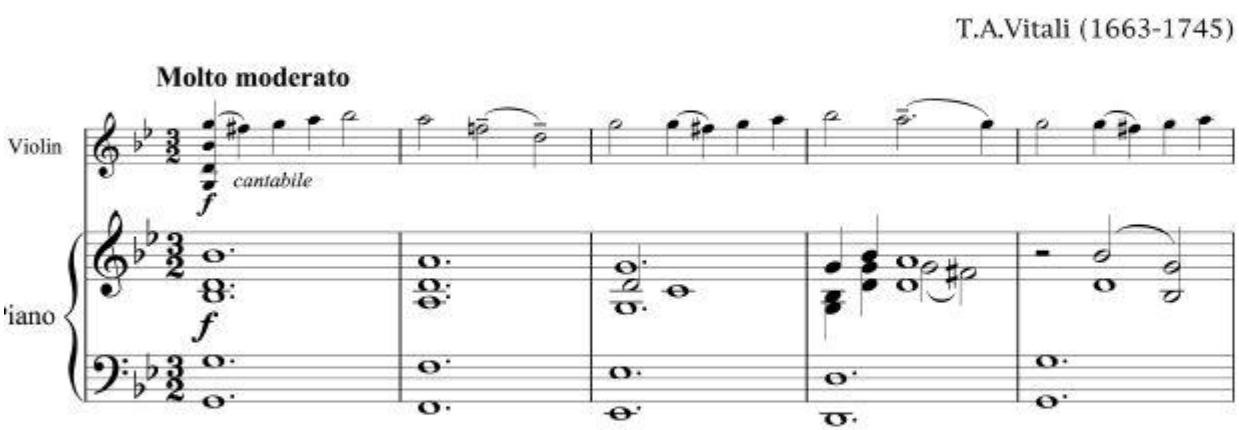

'culture':

- singular events

- mundane/routine daily living events!

Overview: Processualism Yes and No!

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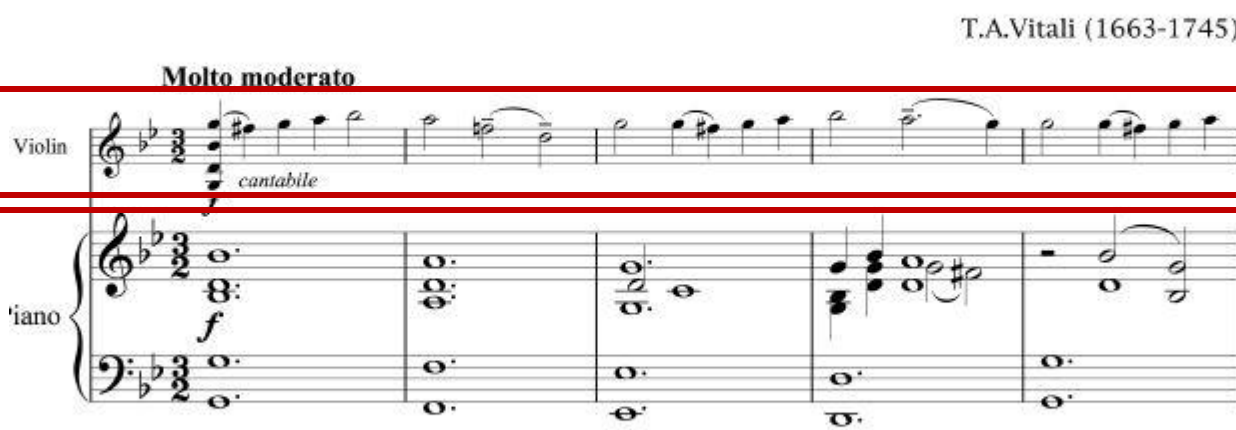

'culture':

Unretrodictable

'Retrodictable'
modelling

NO systemic logic for 'melody'!

Overview: Processualism Yes and No!

Musical metaphor:

- Modelling provides the basic harmony
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Somewhat fancifully!

T.A.Vitali (1663-1745)

‘culture’:

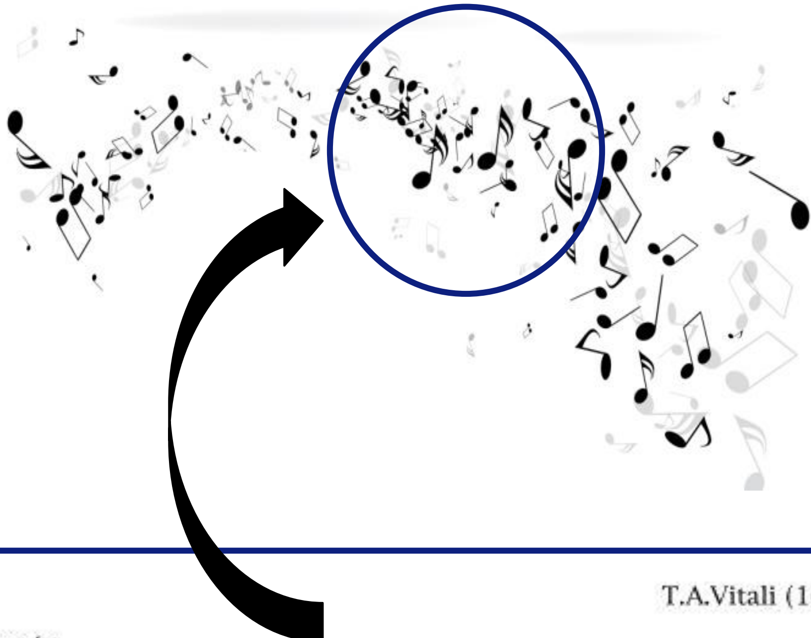
Post-processual analysis

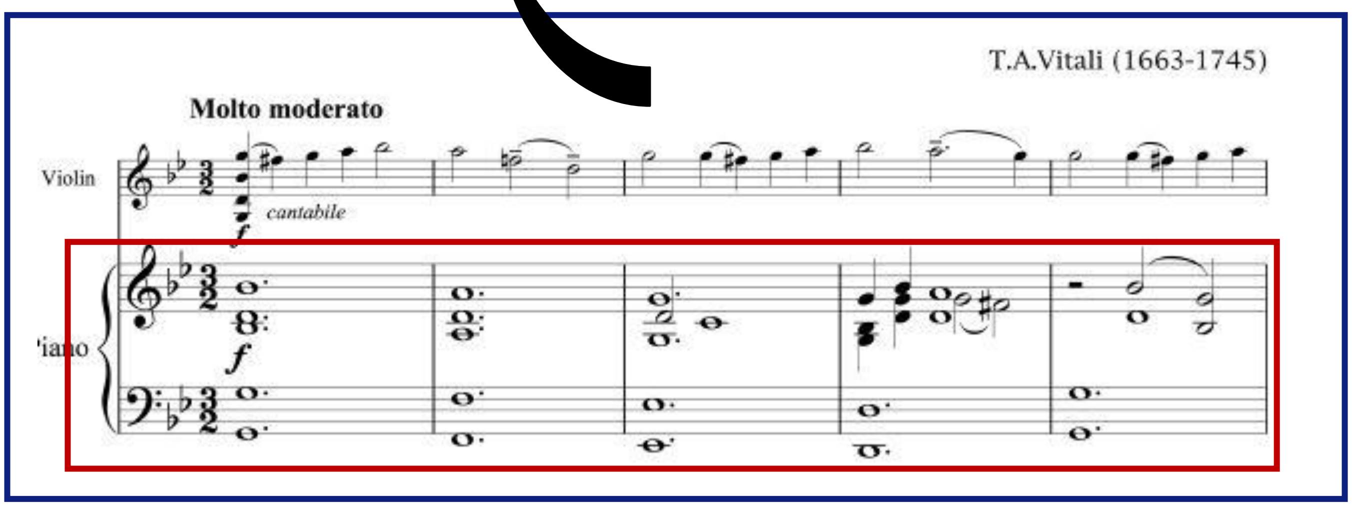
‘Processualism 2.0

Stress: We need both melody and harmony for a description of the society in question!

Overview: Processualism Yes and No!

Problem with data: Incomplete data is for the whole piece, not just the harmony!





Trying to match the harmony with data in ignorance of the melody is very difficult!

Part I: Exchange Networks

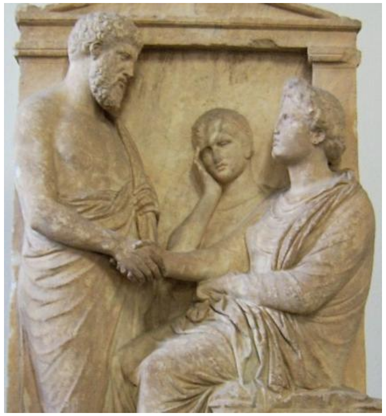
1. Networks

2. Links (exchange)

3. Nodes (sites)

4. Data

I. Exchange Networks



Question: How do ‘cultures’ sustain themselves?

Requires social interaction on a large scale!

Specifically:

Long-distance ‘exchange’ between relevant groups
as a mechanism/agent for social/cultural cohesion

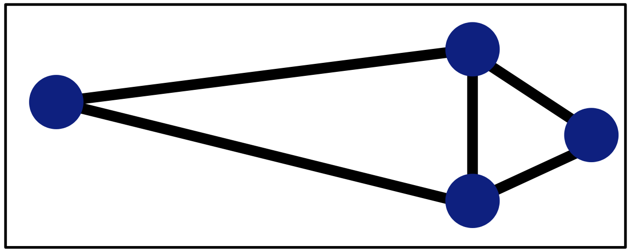
Organised through Spatial Networks:

Spatial networks systematise interactions by linking

- ❑ Spatial geography (space) to
- ❑ Social geography (place)

I. Exchange Networks

A network is a *graph,* built from



- Nodes (vertices)
- Links (edges) between nodes

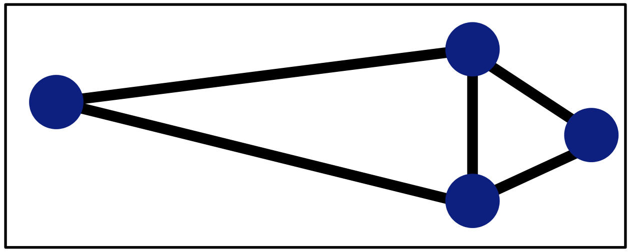
  - a link between two nodes means that they are connected in some way

I. Exchange Networks

A network is a *graph,* built from

➢ Nodes (vertices)

➢ Links (edges) between nodes



- a link between two nodes means that they are connected in some way

(Spatial) Exchange Networks

Constructed from

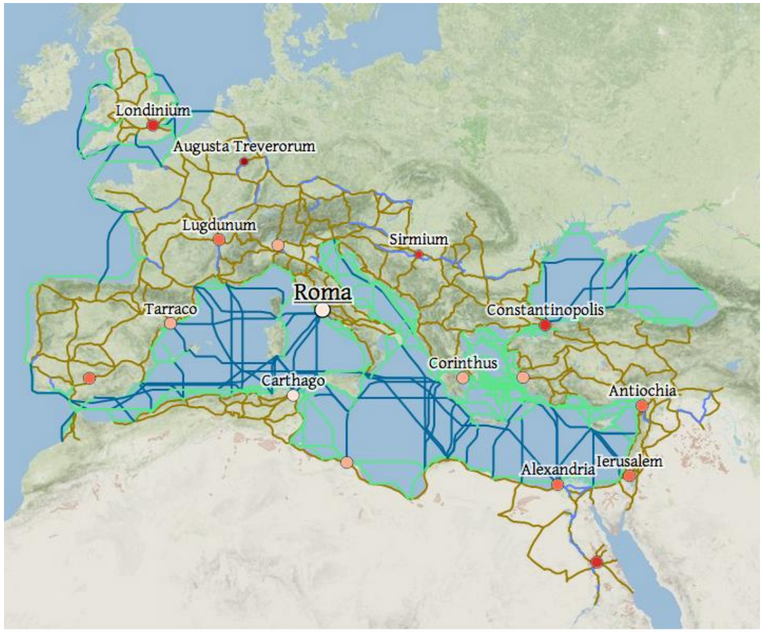
➢ nodes, comprising social groups

- henceforth termed 'sites'

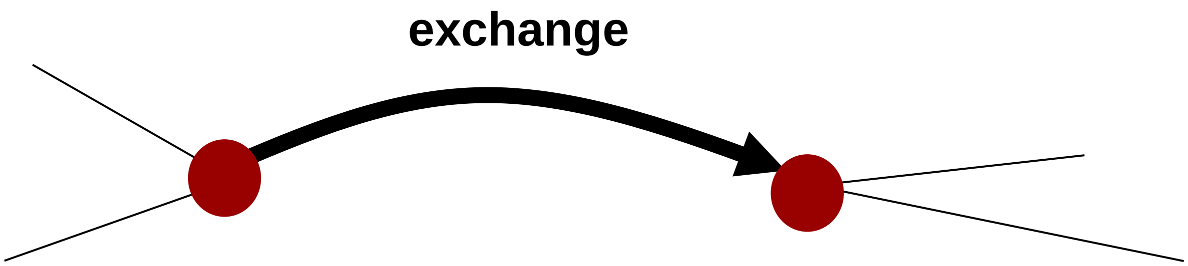
whose

➢ links describe the exchange between them

- 'exchange' and 'link' synonymous



2. What is being exchanged?



Material goods: ‘trade’, barter, storage, elite goods,..

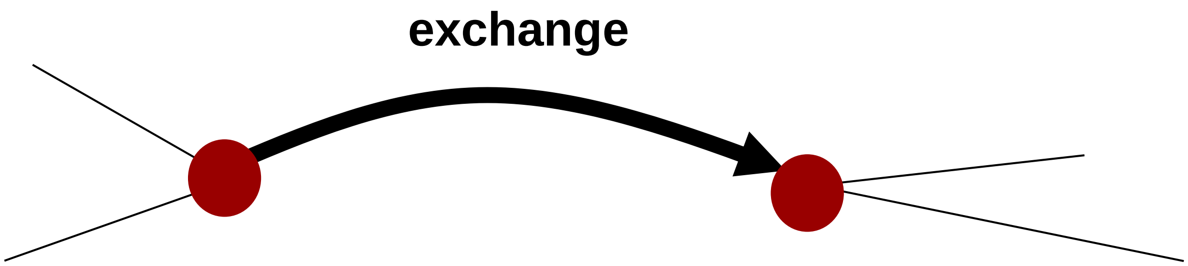
People: traders, artesans, transhumance, migration, slaves, hostages, exogamy,..

‘Culture’: technology (e.g. potters wheel), religious cults (e.g. Zeus Hypsistos)

Restrict ourselves to the most obvious material exchange!

In general, it is this that survives in the pre-historic record in large enough quantities to permit modelling!

3. Sites: Who is exchanging?



Exchange ultimately takes place between individuals,
but can’t usually work at the level of individual exchange
- need to aggregate individuals to larger units

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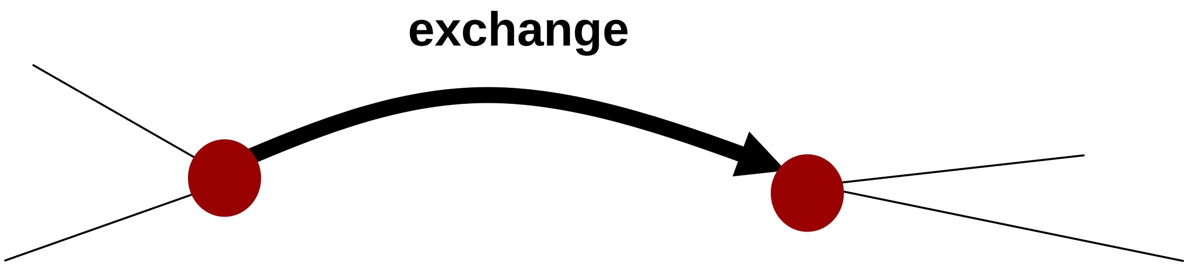

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This is 'coarse-graining'

- introduces a scale δ characteristic of site
  e.g. site size, site population

Perhaps thought of as zooming out in Google Earth!

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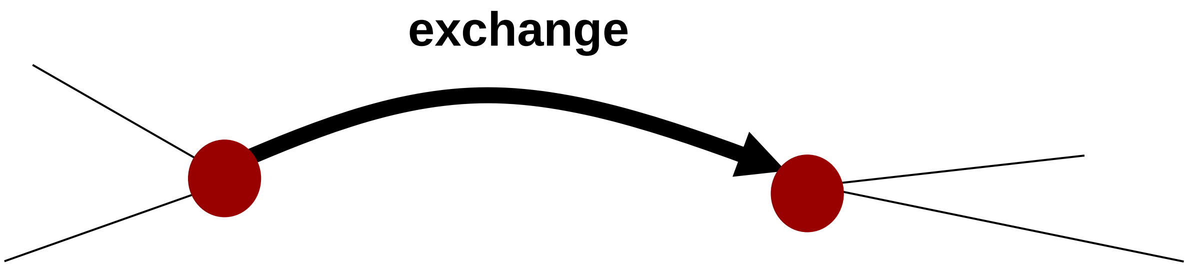

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Perhaps thought of as zooming out in Google Earth!

Micro-level
- the Trader

Meso-level; the Town

Macro-level: the Island

4. Data: Sites

Sites are coarse-grained:

Characterised in particular by

- Position/size/carrying capacity/population

  - at scale δ

- levels of activity as indicated by artefact assemblages and type of assemblage

  - organised at scale δ

4. Data: Sites

Qualitatively:

Data is statistically poor:

- Excavations necessarily partial
  - unknown/missing sites!
- Nonetheless, often a prodigious amount of excavated material
- Documented artefacts only a tiny fraction
- Most artefacts locally produced, exchange a small fraction of that small fraction
- Exchange difficult to identify by source
- Possibly not quantified

Lots of data! NOT Big Data

Small/Dirty Data!

4. Data: Sites

Qualitatively:

Data is very limited in nature:

Largely restricted to artefacts (e.g. grave goods)

- Even within the context of basic activity, much is biodegradable (fabrics, wood, agricultural produce) and hence invisible

- Equally true of human mobility (e.g. slaves) ~ 50% of population in Greek city states

Lots of data! NOT Big Data

Small/Dirty Data!

4. Data: Exchange

Exchange is crucial but ‘uncommon’ *in the record* both for material goods and people:

Qualitatively:

- Akrotiri: only ~10% of artefacts imported
- Mediterranean ~ 5% (local) human mobility

Distance matters:

Most movement of things and people is short-range – hardly counts as ‘exchange’

Most locally produced!

4. Data: Exchange

Quantitatively:

Exchange data is statistically poor:

- Absolute numbers of both total and exchange artefacts not reliable

- The best we can usually ask concerns relative frequencies
  e.g. what are the likely sources for ‘imported’ artefacts and the likely destinations for ‘exported’ artefacts ?

- More generally, only make qualitative statements about site significance (e.g. urbanisation) and flows

Lots of data! NOT Big Data

Small/Dirty Data!

Henceforth restrict ourselves to movement of objects ('trade') rather than people

Any questions/comments !

Part II: Model Generics

1. Model generics and model cycle:

2. Model inputs: distance scales d (geography) and D (technology)

3. Coarse-graining d

4. Coarse-graining D and exchange E

5. Modelling for coarse-graining exchange E

Multi-level/scale modelling:

Requires coarse-graining/
block analysis/pixillation/mode
truncation of inputs/outputs/model

1. Model generics:

- ❑ **Data modelling**

  - derived from patterned observations in data

whereas

- ❑ **Theory modelling**

  - derived from theories about processes that produce these patterned observations – ‘agency’

1. Model generics:

- **Data modelling**

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  - ‘bricks and mortar’

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  - ‘architecture’

1. Model generics:

- **Data modelling**

  - derived from patterned observations in data
  - ‘bricks and mortar’
  - induction

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  - deduction

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First choice for understanding **‘Big’** data

whereas

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Default choice for understanding **‘Dirty’** data

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  - 'architecture'
  - deduction

Default choice for understanding **'Dirty'** data

Restrict ourselves to Theory Modelling!

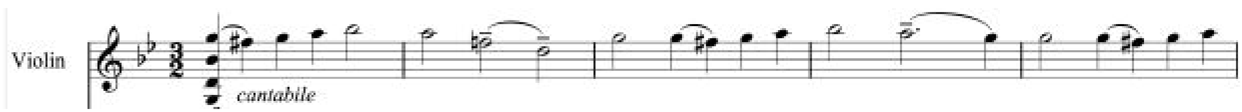
1. Model generics:

❑ Theory Modelling for Small Data:

- Designed to help our understanding of how the ‘real world’ works rather than demonstrate what happens in detailed reality.

- Paints with a very broad brush – just the harmonies

- No ‘melody’



Expect clashes with historians for whom the melody is crucial!

1. Model cycle:

Think of model M as a black box (code)!

The type of model reflects the ‘agency’ that we attribute to the functioning of the network

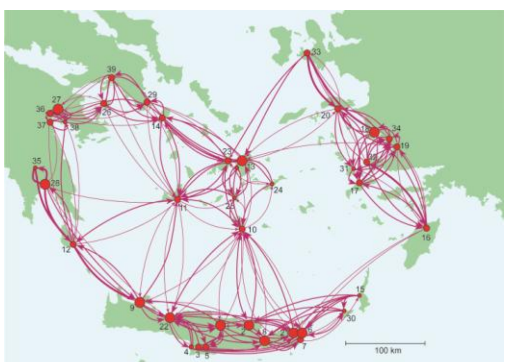
How to choose the model will be discussed later!

1. Model cycle:

Think of model M as a black box (code)!

Inputs:

Outputs:



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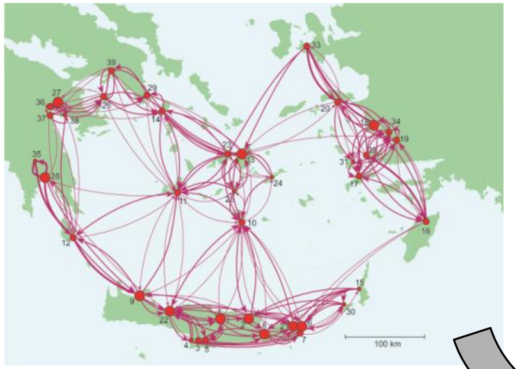

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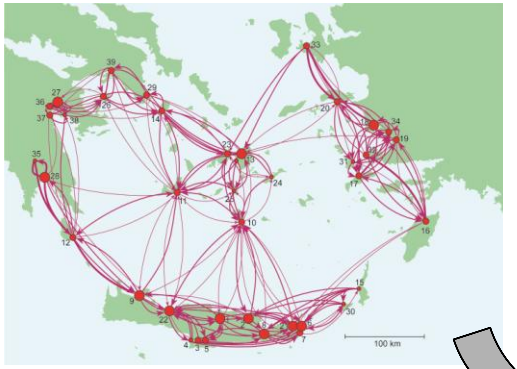

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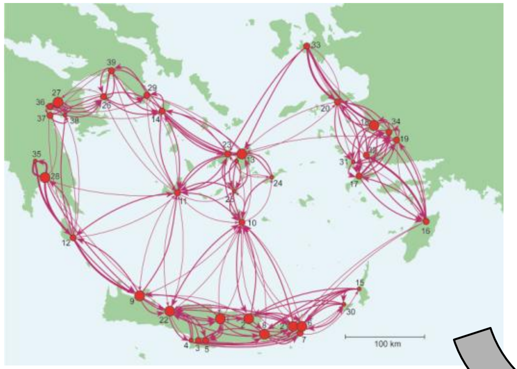
Inputs:

Serious matching problem!

Data:

Outputs:



1. Model cycle:

Inputs:

Control parameters (G):

‘Geography’ – at least as
many as there are sites

Initial settings (back of the box)
- fixed!

Largely model independent

Calibration parameters (Λ):

These implement the ‘agency’ that
the modelling encodes

Dials (front of the box) – few in number

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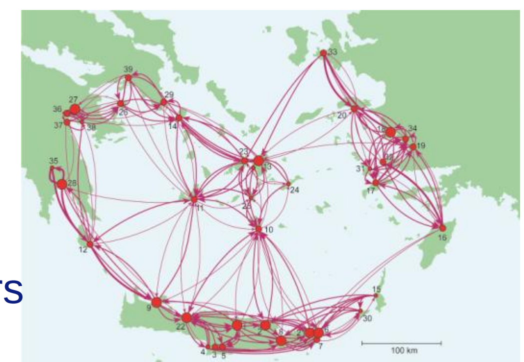
Outputs:

- Exchange parameters E
  - at least as many as there are links
- Perhaps further site parameters



Together written as E = M(G, Λ)

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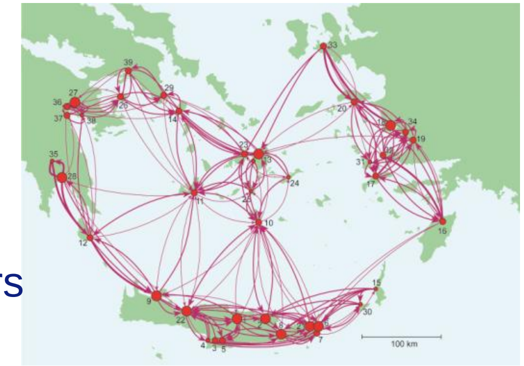


Together written as E = M(G, Λ)

Derived Outputs:

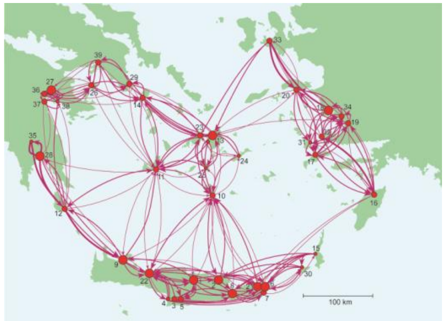
Network analysis:

- Site significance (rank)
- Centrality
- Betweenness
- etc.

Often the primary questions - but NOT here!

1. Model cycle:

Outputs:



Data:

Huge number of detailed outputs $M(G, \Lambda)$ controlled by very few parameters Λ

Large amount of poor, incomplete, aggregated data Δ - but enough to perhaps be in disagreement with few-parameter fitting

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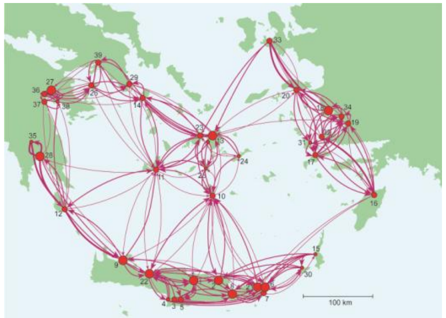
Outputs:



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Modus operandi:

Try to coarse-grain (block-analyse) both to get the best composite ‘image’
- exercise in pixillation (geographic) or suitable modes (structures)

e.g. no point having many detailed outputs incapable of being tested!

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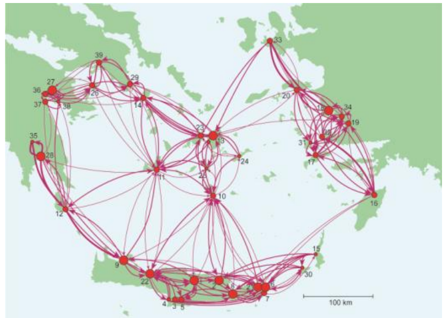
Outputs:



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Modus operandi:

Key example:
Characteristic distances

Try to coarse-grain (block-analyse) both to get the best composite ‘image’
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e.g. no point having many detailed outputs incapable of being tested!

2. Model inputs: Distance scales

Many sites A,B,..with many separations d_{AB} and many methods to get from A to B

Can distil TWO characteristic qualitative scales:

1. Geography: Distance scale ***d*** for the network:

   - typical intersite separation $\boldsymbol{d \approx \langle d_{AB} \rangle}$

2. Technology: Distance scale ***D*** for travel/exchange:

   e.g. ‘effective’ distance you can travel on a single journey

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Example: pizza delivery

d – typical separation of customers
D – distance scooter can go on one filling

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3. d/D is a measure of the ease of establishing vigorous network

   d/D > 1; 'difficult' to establish network

   d/D < 1; 'easy' to establish network

2. Example: Bronze Age E. Mediterranean

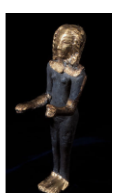
Bronze Age sees the development of maritime exchange from local inter-island paddling to long-distance mercantile cargo movements

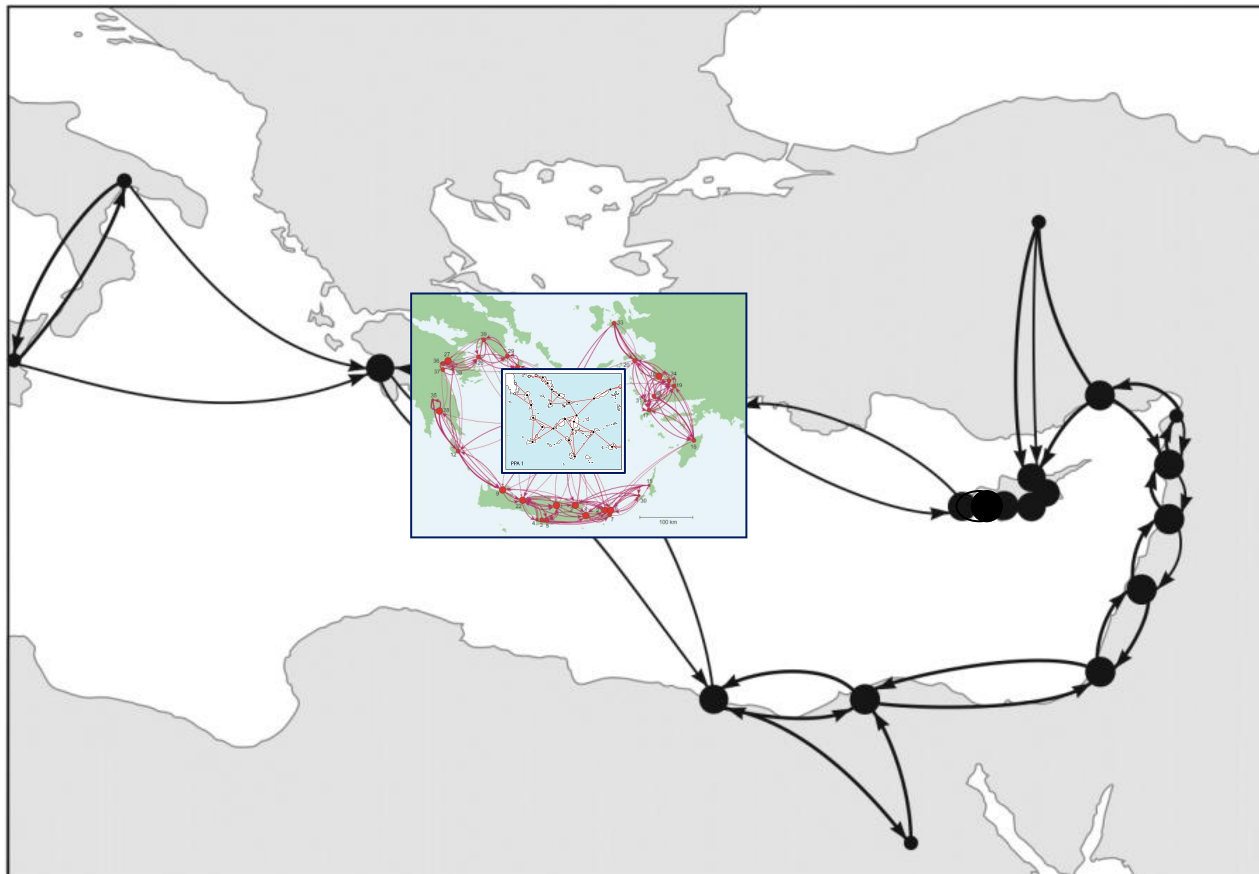
Geography: distance scale **d** needed to connect sites with relative ease.

- EBA Cyclades
  d ≈ 50km

- MBA S. Aegean
  d ≈ 100km

- LBA E. Mediterranean
  d ≈ 400km





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Technology: distance scale **D** for exchange

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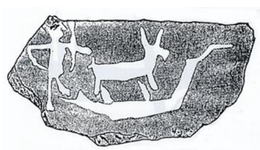
  d ≈ 50km
  D ≈ 20km



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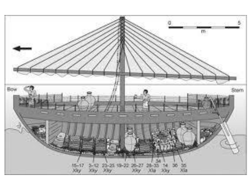

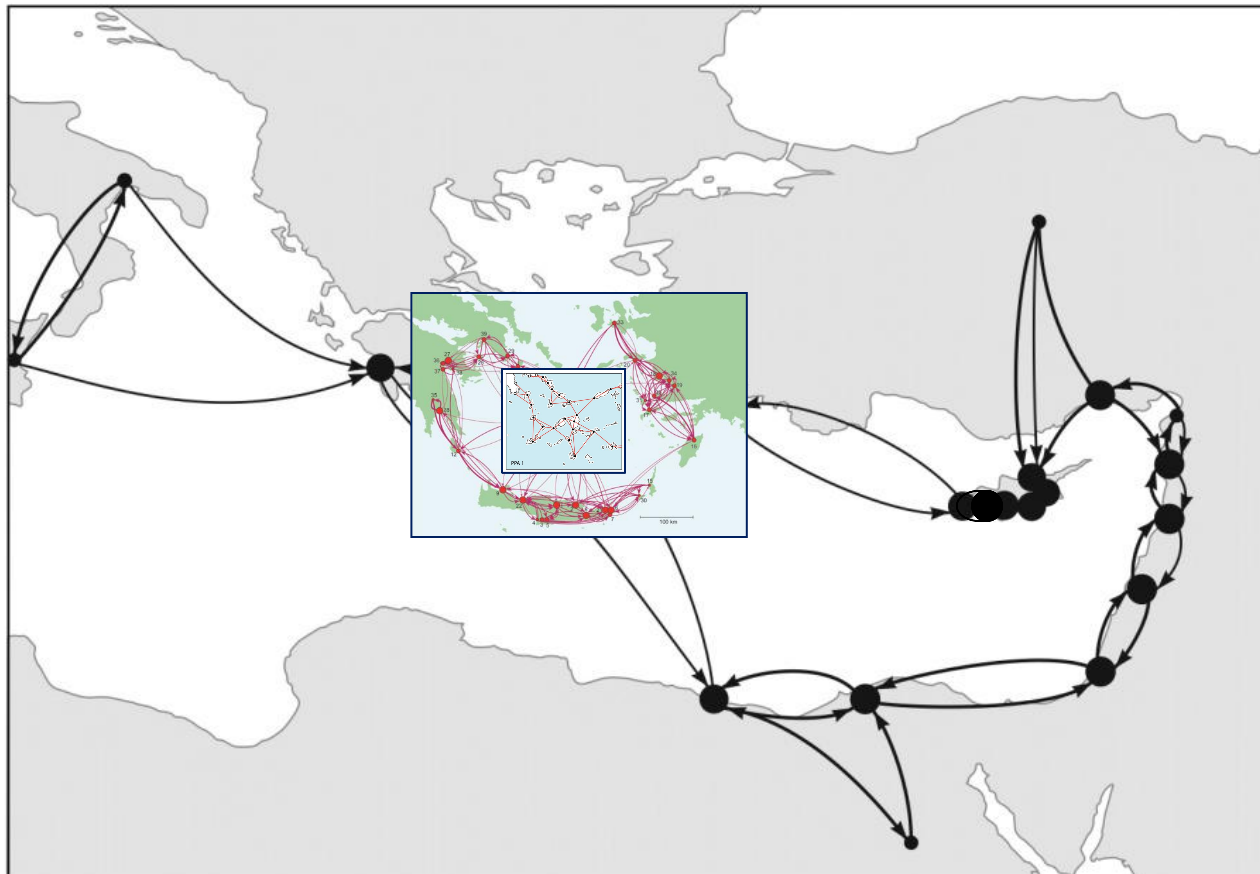

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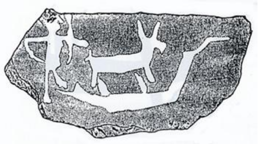
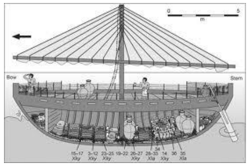
Ease of setting up vigorous networks:

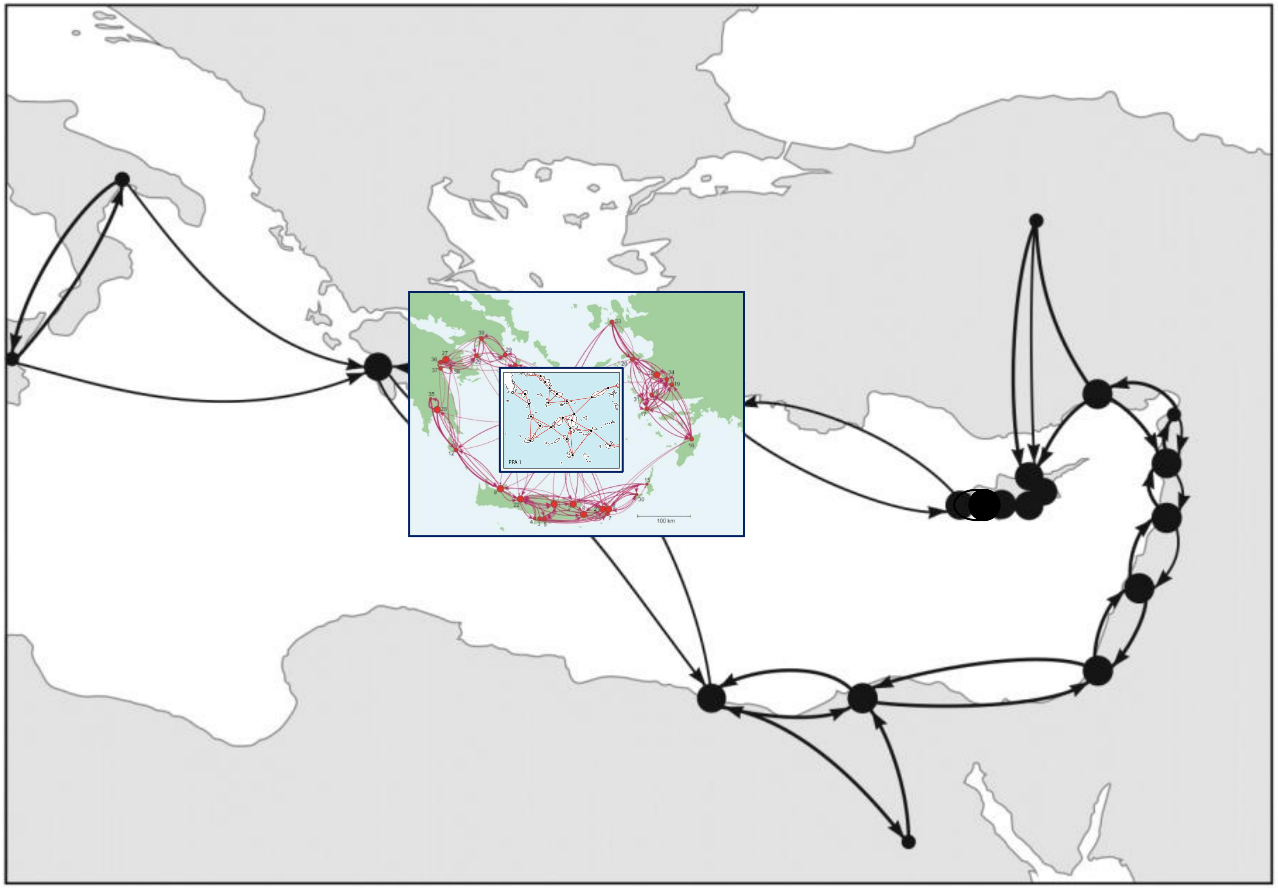
- EBA Cyclades
  difficult



- MBA S. Aegean
  transitional

- LBA E. Mediterranean
  easy





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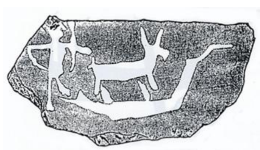
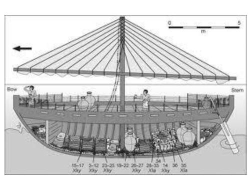
- EBA Cyclades
  difficult

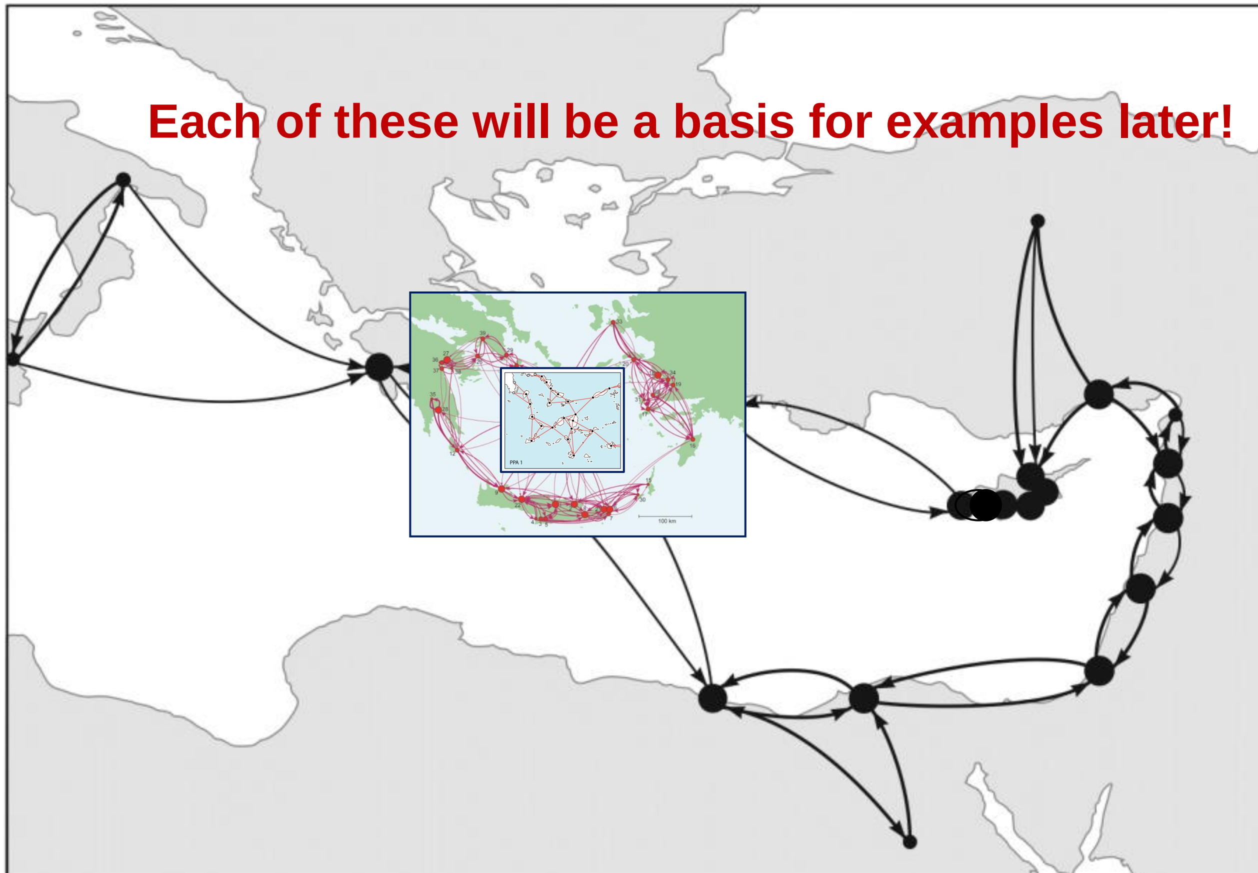


- MBA S. Aegean
  transitional

- LBA E. Mediterranean
  easy



Each of these will be a basis for examples later!



3. Distance scales: Coarse-graining d

The example shows that there are several composite distances d :

1. Site coarse-graining:

The different d’s quoted for the BA are not intrinsic to the geography (the Mediterranean is the Mediterranean) but depends on the *site* coarse-graining or clustering

3. Distance scales: Coarse-graining d

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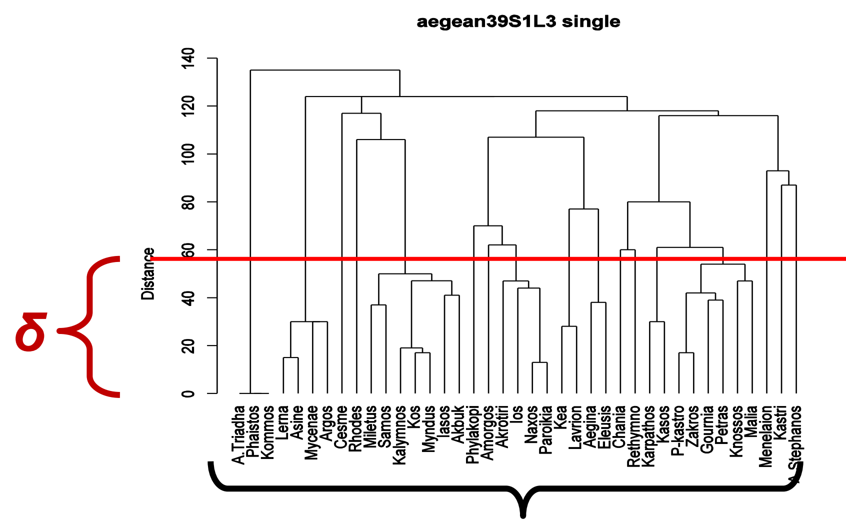
Understood with distance dendograms

Coarse-graining/clustering distance scale δ



$$\boxed{d = d(\delta)}$$

d – block separation at level

3. Distance scales: Coarse-graining d

Further composite distances d :

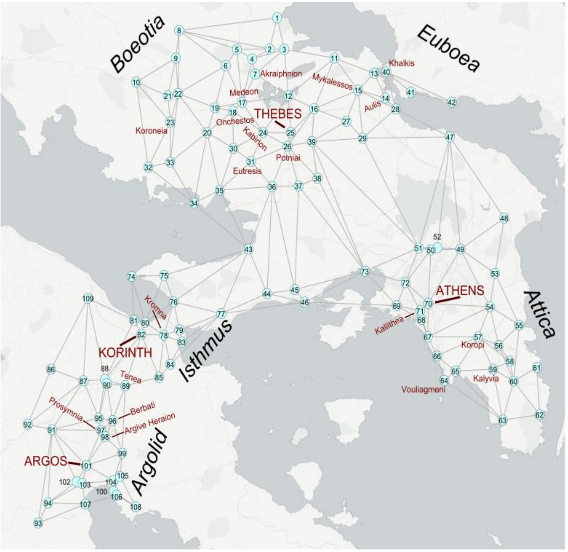
At sea the choice of route is relatively straightforward

Not the case on land where not all sites are connected to one another simply

2. Path redundancy
   - paths are no longer simply geographical

- Is there a route to go from A to B directly?
  - L-space

- Do we go from A to B via intermediate sites C, D,..?
  - P-space



Understood with *path* coarse-graining *P*

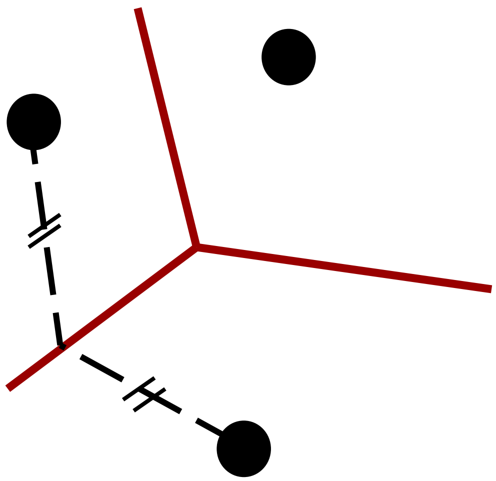
Very non-unique!

3. Distance scales: Coarse-graining d

Simplest P-space coarse-graining implemented with
Voronoi tesselation and Delaunay triangulation

Step I. Voronoi tesselation (Thiessen polygons):

- ❑ Partition space into ‘zones of influence, one zone for each site.
- ❑ Zone boundaries equidistant from adjacent sites

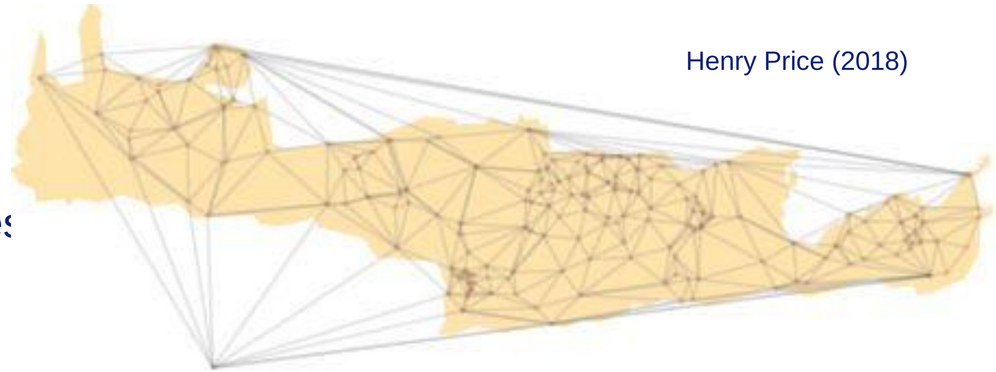


3. Distance scales: Coarse-graining d

Step II. Delaunay triangulation:

- ❑ Link only those sites in contiguous zones

- ❑ Paths only intersect at sites
  - no uninhabited cross-roads!
  - less appropriate for sea-travel!

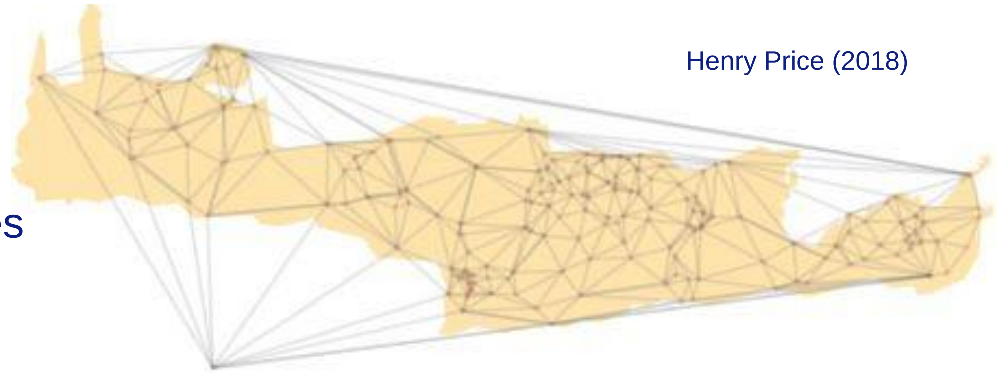


Henry Price (2018)

C14 BCE Mycenaean Crete:

3. Distance scales: Coarse-graining d

Step II. Delaunay triangulation:



C14 BCE Mycenaean Crete:

- ❑ Link only those sites in contiguous zones
- ❑ Paths only intersect at sites
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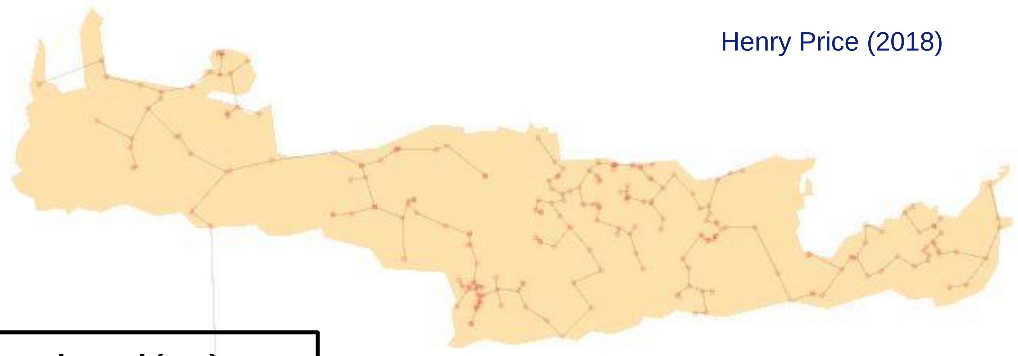
Further thinning out of links



Minimal spanning tree

- ❑ Gabriel graphs
- ❑ Minimal spanning trees (non-unique)
- ❑ d depends on coarse-graining $\boxed{d = d(P)}$

3. Distance scales: Effective d

Geographical separation is a poor guide to how we feel distance!
e.g. mountains in C. Crete

- ❑ Distances by sea/land, plain/mountain differ
  Depends on how exchange is implemented (technology)!

Two related notions:

- ➢ Effective geographical distances

  Upwind/downwind, plain/hill, ..., distances can usually can be converted into effective geographic distances on a basis of travel 'cost'/energy expended/time taken by introducing relative frictional coefficients f.

- ➢ Least cost paths

  In terms of effective distances paths are as 'short' as possible (geodesics) subject to caveats of Delaunay/Gabriel paths, etc.

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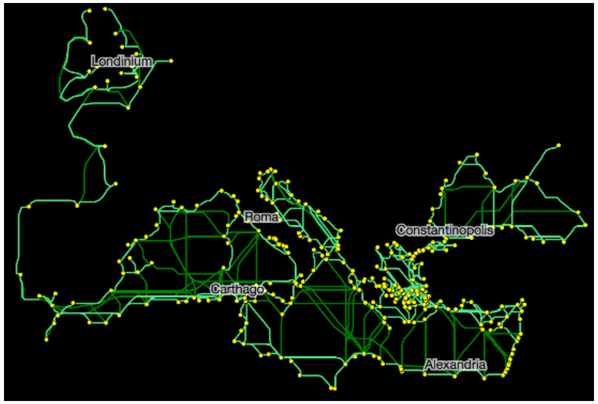


Roman ‘cost surface; (with wind)
ORBIS (http://orbis.stanford.edu)

- ➢ Least cost paths

  In terms of effective distances paths are as ‘short’ as possible (geodesics) subject to caveats of Delaunay/Gabriel paths, etc.

3. Distance scales: Effective d

Outcome: $\boxed{d = d(\delta, P, f)}$ where P and f are related

- ❑ Crucial to check sensitivity of results to variations in choice of δ, P, f
- ❑ Do not have to have d_{BA} equal to d_{AB}
- ➢ d_{BA} *not* equal to d_{AB} is crucial for sea and riverine exchange!

For simplicity we shall ignore unless specifically stated!

3. Distance scales: Effective D

Distance scale D:

Much more difficult to anticipate with same accuracy:

D also interpreted as the inverse ‘cost’ or effort for exchange per unit distance.
The higher the cost or the harder the exchange, the smaller is D.

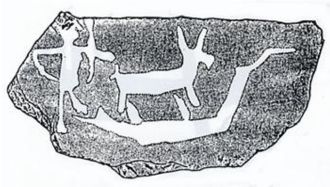
Use this interpretation henceforth:

4. Distance scales: Coarse-graining D

In fact, there are several distances D:

- Different categories of exchange E (e.g. slaves, oil) may require different D

- Different modes of exchange T (e.g. canoe, sail) for same category require different D



We can, in principle, introduce different D in models for different types of exchange E and different modes of exchange T

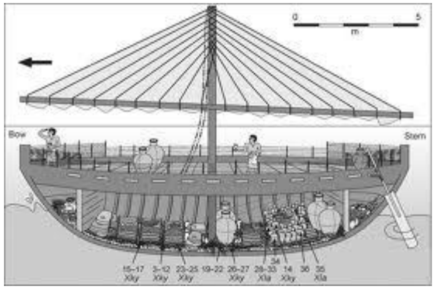
4. Distance scales: Coarse-graining D

Most extreme assumptions – Null model

- ❑ Assume that different categories of exchange E from A to B can be conflated with a single 'effective' number T_{AB}

- ➢ when T_{AB} is relatively large there is strong exchange from A to B

- ➢ when T_{AB} is relatively small there is weak exchange from A to B



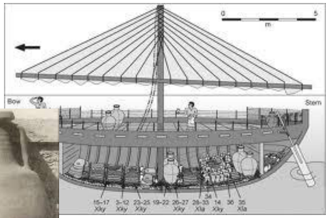
- ❑ Assume that different modes of transportation T can be conflated with a single distance scale D

  e.g. *ad valoram* 'costing'

Often label exchange in network diagrams so that the thickness of links reflects T_{AB}

5. Coarse-graining Exchange: Constant Elasticity of Exchange (CES)

There are economic models permitting aggregation of different exchanges (goods!) to give a single coarse-grained D and E



Most simply:

- Introduce CES utility functions $U_i(\sigma)$ which measure the welfare or benefits of individuals due to composite exchange (Anderson and van Wincoop 2003)

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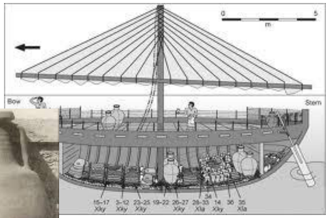

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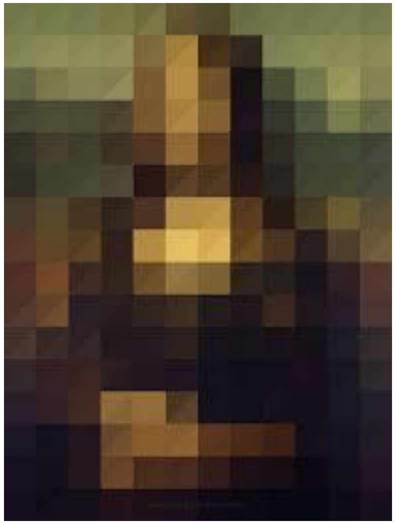
Elasticity of Exchange $\sigma > 1$

- measures the willingness to make substitutions in cargoes when some types of exchange become more difficult/'costly' to effect.

Most simple outcome: Effective distance scale $D_{eff} = D_{ad\ val} /(\sigma - 1)$

5. Coarse-graining Errors

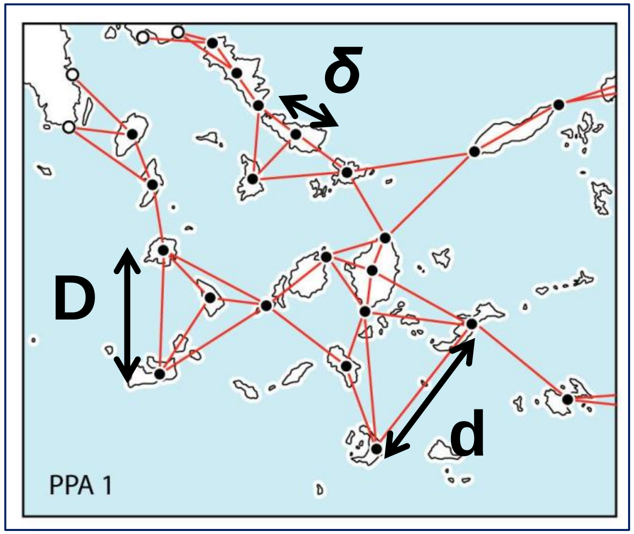


Coarse-graining leads to a loss of information:

This should be distinguished from other forms of uncertainty

Best seen by example: EBA Cyclades by canoe

- Typical intra-island coarse-graining - δ
- If we are coarse-graining at inter-island level, then **d** is typical inter-island separation
- EBA: D < d for canoe travel



Anticipated error in E:

$\varepsilon \sim (\delta/d)^2$ and/or $(\delta/D)^2$ (Which D?)

For EBA Cyclades this is Large: > 10%

III. Summary:

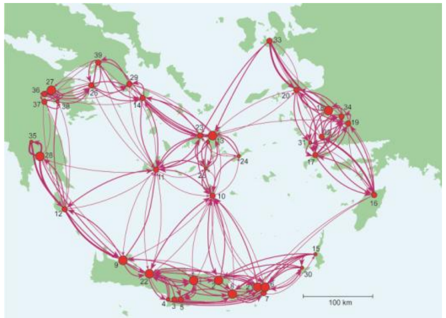
- We are dealing with multiscale societies with poor data!
- Our approach is to try to coarse-grain consistently to a single effective scale
- Don’t expect too much. You are not a historian!

“The purpose of a good model is to formulate simple concepts and hypotheses concerning them, and to demonstrate that, despite their simplicity, they give approximate accounts of otherwise complex behaviour of phenomena. If a model ‘works’ ... then it shows that the assumptions and hypotheses built into the model contribute to an explanation of the phenomena”

- Alan Wilson 1981

III. Summary: Review

Outputs:



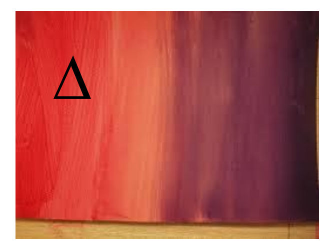
Data:

Huge number of detailed outputs E controlled by very few parameters Λ

Large amount of poor, incomplete, aggregated data Δ

If coarse-grained data Δ matches theoretical output E at some scales (some Λ) then:

- model gives insight into ‘agency’ as to why the network ‘works’
- parallels social narratives that are often part of the data modelling
- enables us to fill in gaps in Δ



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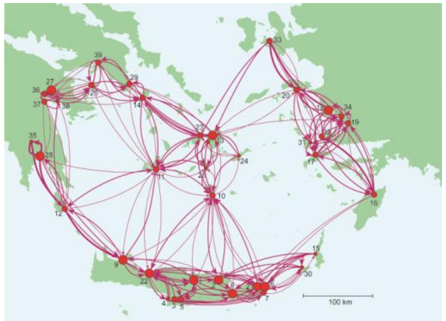


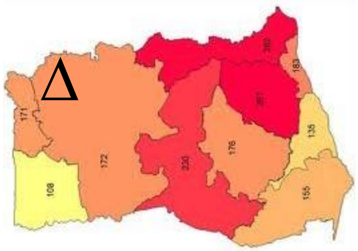
Data:

Huge number of detailed outputs E controlled by very few parameters Λ

Large amount of poor, incomplete, aggregated data Δ

If coarse-grained data Δ does not match theoretical output E at any scales (any Λ) then:

- either have the ‘wrong’ model
- have the ‘correct’ model but the unpredictable ‘melody’ is dominant over the ‘harmony’



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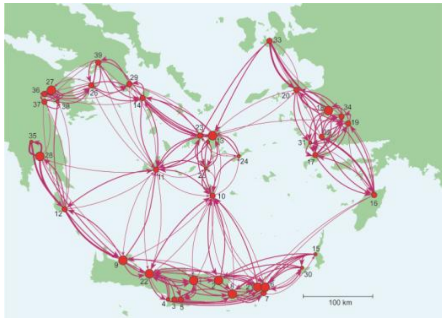

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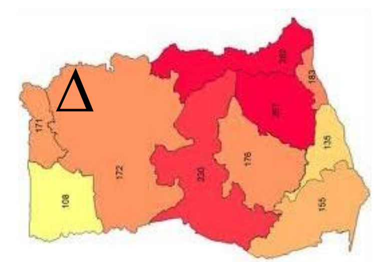


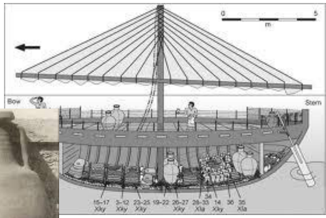
Either way we need to know what models are available!

We shall discuss how to choose and use models in Part II

Any questions/comments !

Notes I: Constant Elasticity of Exchange (CES)

There are economic models permitting aggregation of different exchanges (goods!) to give a single coarse-grained D and E



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- Then maximise benefits subject to constraints on resources.

Elasticity of Exchange $\sigma > 1$

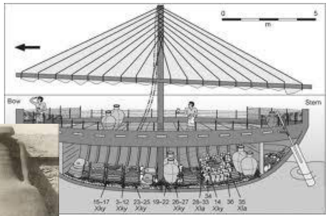
- measures the willingness to make substitutions in cargoes when some types of exchange become more difficult/'costly' to effect.

$\sigma > 1$: substitution favourable ($\sigma < 1$: different exchanges complementary)

- Most simple outcome:

  A single aggregated exchange T_{AB} with effective distance scale $D_{eff} = D_{ad\ val} / (\sigma - 1)$

Notes I: Constant Elasticity of Exchange (CES)



There is an economic model permitting such a procedure:

Constant Elasticity of Exchange (CES):

In economics, the utility function measures the welfare or satisfaction of a consumer as a function of consumption of real goods such as food, clothing and composite goods rather than nominal goods measured in nominal terms. Utility function is widely used in the rational choice theory to analyze human behavior.

CES utility function. The same **CES functional** form arises as a **utility function** in consumer theory. For **example**, if there exist types of consumption goods , then aggregate consumption could be defined using the **CES** aggregator: Here again, the coefficients are share parameters, and is the elasticity of substitution.

is given by

$$u(x_1, x_2) = \frac{x_1^\delta}{\delta} + \frac{x_2^\delta}{\delta}$$

where $\delta \neq 0$ is a fixed constant, which may be positive or negative. Also, when $\delta = 0$ we define

$$u(x_1, x_2) = x_1 x_2$$

As the parameter δ varies, the associated indifference curves have a wide variety of shapes representing different degrees of complementarity or substitutability of goods. Note that the cases $\delta = 1, \delta = 0$ are examples we have seen before: perfect substitutes and Cobb-Douglas, respectively.

(a) For the case $\delta =$ [illegible], draw the three indifference curves through [illegible].

(b) Calculate the MRS as a function of δ, x_1, x_2.

(c) For a fixed x_1 and x_2, what happens to the MRS as $\delta \to -\infty$? (The answer will depend on x_1 and x_2; how?) Using this answer

Notes I: Constant Elasticity of Exchange (CES)

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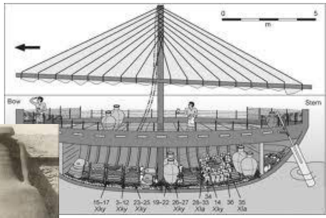
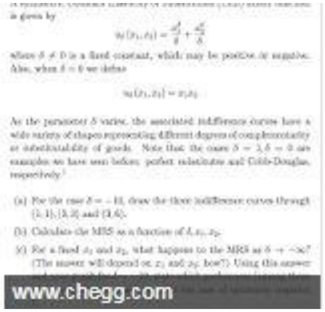


Constant Elasticity of Exchange (CES):

CES utility function. The same **CES functional** form arises as a **utility function** in consumer theory. For **example**, if there exist types of consumption goods , then aggregate consumption could be defined using the **CES** aggregator: Here again, the coefficients are share parameters, and is the elasticity of substitution.



CES utility function [edit]

The same CES functional form arises as a utility function in consumer theory. For example, if there exist n types of consumption goods x_i, then aggregate consumption X could be defined using the CES aggregator:

$$X = \left[\sum_{i=1}^{n} a_i^{\frac{1}{s}} x_i^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}}.$$

Here again, the coefficients a_i are share parameters, and s is the elasticity of substitution. Therefore, the consumption goods x_i are perfect substitutes when s approaches infinity and perfect complements when s approaches zero. The CES aggregator is also sometimes called the *Armington aggregator*, which was discussed by Armington (1969).[7]

CES utility functions are a special case of homothetic preferences.

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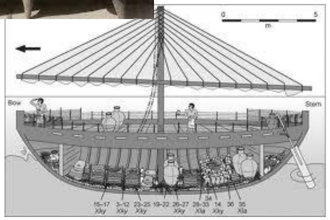

Constant Elasticity of Exchange (CES):

Simply, it's the elasticity of any input substitution, or swap. It could be capital and labour substitution, for example. Or maybe product A for product B. You are simply questioning what will happen if, say, the price of A goes up: just how good a substitute is B? If it's a really good substitute it may go up faster than A goes down, or the opposite may happen. Or it may move in 1:1 proportion. That's elasticity of substitution.

When the elasticity of substitution is less than 1, relative demand for an input variable falls, but by proportionally less than the relative rise in its price. That indicates that the items compared are gross complements. You are thus less likely to swap or substitute products (even though it costs you more).

OTOH when the elasticity of substitution is greater than 1 (which is what you are asking, of course) the reduction in relative input quantity exceeds the increase in relative price, indicating that they are gross substitutes. Which is to say that the larger the magnitude of the elasticity of substitution, the more likely the item is to substitute and you swap over to the other product more easily.