Adult Mortality in the Metropolis of London 1100–1850: a Bayesian View Based on Osteological Data

Supplement: Code structure, data source and processing

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Prerequisites

The calculations were made in R using R-Studio. The structure of the code is essentially based on the structure of the text. The raw code is in the file order_of_code.R. The file extended with Markdown is order_of_code-doc.RMD and the file order_of_code-doc.html is generated from it.

Note: The base path for rmd files is the folder in which they are located, not the r-project. Consequently, order_of_code.R and order_of_code-doc.RMD are both located in the root folder of the project.

Depending on the hardware, the subsequent code can run for several hours or even a few days.

Install "Just Another Gibbs Sampler" (JAGS) (Plummer, 2003) if you want to run the Bayesian analyses anew. Version 4.3 - as used here - can be downloaded in pre-compiled form for a number of OS here: $\frac{\text{https:}}{\text{sourceforge.net/projects/mcmc-jags/}}.$ The manual can be found here: $\frac{\text{https:}}{\text{people.stat.sc.edu/hansont/stat740/jags_user_manual.pdf}}$

The code makes extensive use of the function **source** to call external code. Thus, the main part of the code remains slim, well structured and readable.

Install required packages, set some options and link the sources for the helper functions.

Remark: Depending on your R version the package osmplotr may be installed from github using devtools::install_github ("ropensci/osmplotr").

```
options(dplyr.summarise.inform = FALSE)

source("./functions/gomp_MLE.R")
source("./functions/gomp_MLE_adapted.R")
source("./functions/gomp_MLE_interval.R")
source("./functions/gomp_anthr_age.R")
source("./functions/gomp_anthr_age_r.R")
source("./functions/gomp_known_age.R")
source("./functions/gomp_known_age_r.R")
source("./functions/helper_functions.R")
source("./functions/lt_MC.R")
source("./functions/lt_MC_Gomp.R")
RNGkind("L'Ecuyer-CMRG") # conservative random number generator to avoid periodicity
```

Important for saving time: Decide to run extensive code anew (app. 6 h +). In addition, you can set the folder for preprocessed files.

```
runCodeNew <- FALSE
#runCodeNew <- TRUE

# Ask for credentials of the Human Mortality Database if the code runs anew
if (runCodeNew){
    HMD_username <- readline(prompt = "Enter username: ")
    HMD_password <- readline(prompt="Enter password: ")
    credentials <- c(HMD_username, HMD_password)
}

# Specify filename prefix for saved files and create a folder if needed:
saveFileDir = "preprocessed_files"
if (saveFileDir %in% list.files(getwd())) {
    # Dir exists
}else{
    dir.create(file.path(".", saveFileDir), showWarnings = FALSE )
}</pre>
```

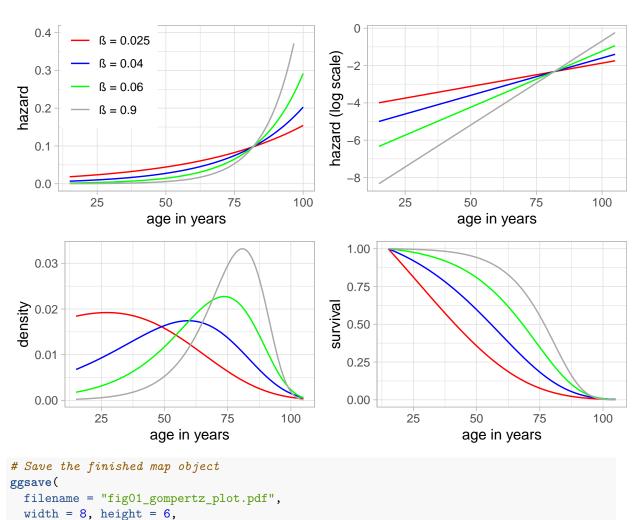
NUT.I.

Chapter 01 Introduction

Figure 1: Exemplary life table curves generated by Gompertz functions with different values for the β parameter.

```
# beta model values
beta1 <- 0.025
beta2 <- 0.04
beta3 <- 0.06
beta4 <- 0.09
# hgompertz(x, shape, rate):
# x = age, shape = beta value, rate = derived from Sasaki & Kondo 2016 fig. 1, 2
# rate values according Sasaki & Kondo 2016 fig. 1, line 6, 30
Sab < -2.624
Sbb <- 0.0393
Ma < -7.119
Mb < -0.0718
M1 <- Sab * (beta1 - Mb) / Sbb + Ma
M2 <- Sab * (beta2 - Mb) / Sbb + Ma
M3 <- Sab * (beta3 - Mb) / Sbb + Ma
M4 <- Sab * (beta4 - Mb) / Sbb + Ma
gridExtra::grid.arrange (
  ggplot() + xlim(15, 100) + ylim(0, 0.4) +
    geom_function(fun = function(x) flexsurv::hgompertz(x - 15, 0.025, exp(M1)),
                  aes(col = "\u03B2 = 0.025")) +
    geom_function(fun = function(x) flexsurv::hgompertz(x - 15, 0.04, exp(M2)),
                  aes(col = "\u03B2 = 0.04")) +
    geom_function(fun = function(x) flexsurv::hgompertz(x - 15, 0.06, exp(M3)),
                  aes(col = "\u03B2 = 0.06")) +
    geom_function(fun = function(x) flexsurv::hgompertz(x - 15, 0.09, exp(M4)),
                  aes(col = "\u03B2 = 0.9")) +
    ylab("hazard") + xlab("age in years") +
    theme_light() +
    scale_colour_manual(values = c("red","blue","green", "dark grey")) +
    theme(legend.position = c(0.2, 0.7), legend.title = element_blank()),
  ggplot() + xlim(15, 105) +
    geom_function(fun = function(x) log(flexsurv::hgompertz(x - 15, 0.025, exp(M1))),
                  colour = "red") +
    geom_function(fun = function(x) log(flexsurv::hgompertz(x - 15, 0.04, exp(M2))),
                  colour= "blue") +
    geom_function(fun = function(x) log(flexsurv::hgompertz(x - 15, 0.06, exp(M3))),
                  colour= "green") +
    geom_function(fun = function(x) log(flexsurv::hgompertz(x - 15, 0.09, exp(M4))),
                  colour= "dark grey") +
    xlab("age in years") + ylab("hazard (log scale)") +
    theme_light(),
  ggplot() + xlim(15, 105) +
    geom_function(fun = function(x) flexsurv::dgompertz(x - 15, 0.025, exp(M1)),
                  colour = "red") +
    geom_function(fun = function(x) flexsurv::dgompertz(x - 15, 0.04, exp(M2)),
                  colour= "blue") +
    geom_function(fun = function(x) flexsurv::dgompertz(x - 15, 0.06, exp(M3)),
                  colour= "green") +
```

```
geom_function(fun = function(x) flexsurv::dgompertz(x - 15, 0.09, exp(M4)),
                  colour= "dark grey") +
    xlab("age in years") + ylab("density") +
    theme_light(),
  # gomp_lx() s. functions\helper_functions.R
  ggplot() + xlim(15, 105) + ylim(0, 1) +
    geom_function(fun = function(x) gomp_lx(x - 15, exp(M1), 0.025),
                  colour = "red") +
    geom_function(fun = function(x) gomp_lx(x - 15, exp(M2), 0.04),
                  colour = "blue") +
    geom_function(fun = function(x) gomp_lx(x - 15, exp(M3), 0.06),
                  colour = "green") +
    geom_function(fun = function(x) gomp_lx(x - 15, exp(M4), 0.09),
                  colour = "dark grey") +
    ylab("survival") + xlab("age in years") +
    theme_light(),
  ncol = 2
) -> gompertz_plot
```



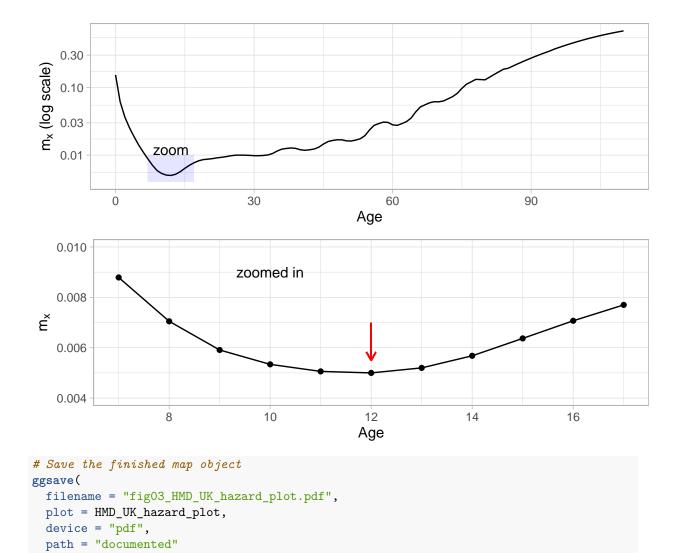
plot = gompertz_plot,
device = cairo_pdf,
path = "documented"

)

Chapter 02 Materials and methods

Figure 2: Hazard curve (mx) for HMD UK data of the year 1841.

```
# Login needed to retrieve data from the Human Mortality Database
# https://mortality.org/
if (runCodeNew){
 login <- askYesNo(paste("Login for Human Mortality Database needed.",</pre>
                          "Do you want to proceed?", sep = "\n"),
                    default = FALSE)
  # qet dx
 if (login){
    HMD_UK_result_1_year <- HMDHFDplus::readHMDweb("GBRTENW", "bltper_1x1",</pre>
                                                    credentials[1],
                                                    credentials[2])
    # saves results in Rda-object
   save(HMD_UK_result_1_year, file = file.path(".", saveFileDir,
                                                 "HMD_UK_result_1_year.Rda") )
} else {load(file.path(".", saveFileDir, "HMD_UK_result_1_year.Rda") )
 }
gridExtra::grid.arrange(
 ggplot(HMD_UK_result_1_year[which(HMD_UK_result_1_year$Year == 1841),]) +
   geom\_line(aes(x = Age, y = mx)) +
   scale_y_continuous(trans='log10') + labs(y = expression(m[x] * " (log scale)")) +
    annotate("rect", xmin = 7, xmax = 17, ymin = 0.004, ymax = 0.01,
             alpha = .1,fill = "blue") +
   annotate (geom = "text", x = 12, y = 0.012, label = "zoom") +
   theme_light(),
  ggplot(HMD_UK_result_1_year[which(HMD_UK_result_1_year$Year == 1841),],
         aes(x = Age, y = mx)) + scale_x_continuous(breaks=seq(8,16,2),
                                                    limits=c(7, 17)) +
    geom_line() + geom_point() +
   ylim(0.004, 0.01) +
   labs(y = expression(m[x]))+
   annotate (geom = "text", x = 10, y = 0.009, label = "zoomed in") +
   geom_segment(aes(x = 12, y = 0.007, xend = 12, yend = 0.0055),
                 arrow = arrow(length = unit(0.25, "cm")), colour = "red") +
   theme_light(),
 ncol = 1
) -> HMD UK hazard plot
```

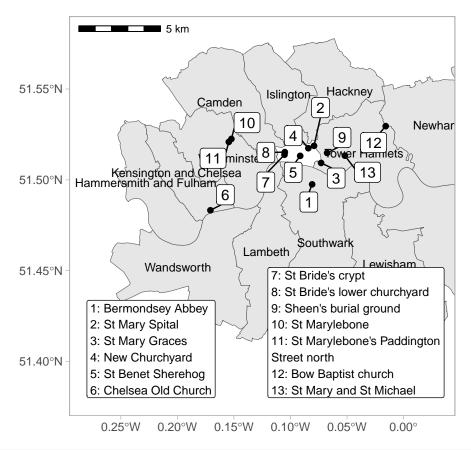


Saving 6.5 x 4.5 in image

Chapter 03 Data

Figure 3: Major cemeteries in Greater London 1100–1850 used in the present study.

```
# Get the coordinates of sites to be plotted
sites_data <- rbind.data.frame(</pre>
    c("1", "Bermondsey Abbey", 51.4975, -0.080833),
    c("2", "St Mary Spital", 51.518716, -0.079161),
    c("3", "St Mary Graces", 51.509289, -0.072916),
    c("4", "New Churchyard", 51.517403, -0.084216),
    c("5", "St Benet Sherehog", 51.513194, -0.091389),
    c("6", "Chelsea Old Church", 51.483222, -0.170795),
    c("7","St Bride's crypt",51.513802,-0.105292),
    c("8", "St Bride's lower churchyard", 51.515253, -0.104973),
    c("9", "Sheen's burial ground", 51.51480, -0.06760),
    c("10", "St Marylebone", 51.5225, -0.152222),
    c("11",paste("St Marylebone's Paddington", "Street north", sep="\n"),
      51.520869, -0.154515),
    c("12", "Bow Baptist church", 51.529540, -0.01580),
    c("13", "St Mary and St Michael", 51.51330, -0.05190)
colnames(sites data) <-c("nr", "name", "lat", "lon")</pre>
sites_data$lat<-as.numeric(sites_data$lat)</pre>
sites_data$lon<-as.numeric(sites_data$lon)</pre>
dat_sites <- st_as_sf(sites_data,</pre>
                      coords = c("lon", "lat"),
                       crs = 4326)
# Build a bounding box by the coordinates + 10% of the extent as frame
bbox <- matrix(</pre>
 c(
    min(sites_data$lon) - (0.1*(max(sites_data$lon) - min(sites_data$lon))),
    min(sites_data$lat) - (0.1*(max(sites_data$lat) - min(sites_data$lat))),
    max(sites_data$lon) + (0.1*(max(sites_data$lon) - min(sites_data$lon))),
    max(sites_data$lat) + (0.1*(max(sites_data$lat) - min(sites_data$lat)))
 ),
 byrow = FALSE, nrow = 2, ncol = 2,
 dimnames = list(c('x','y'),c('min','max'))
)
# Querry the osm data
# If the bbox has been changed please run the statement to get the new osm data.
if (!exists ("q admin8") | (runCodeNew == TRUE)) {
 q_admin8 <- bbox %>% opq() %>%
    add_osm_feature(key = "boundary", value = "administrative") %>%
    add_osm_feature(key = "admin_level", value = "8") %>%
    osmdata_sf()
 }
# Build the map
London_map <- ggplot() +</pre>
 geom_sf(data = q_admin8$osm_multipolygons, fill=rgb(0.9,0.9,0.9)) +
 geom_sf_text(data = q_admin8$osm_multipolygons, aes(label=sub('.*of ','',name)),
               size=3) +
 geom sf(data = dat sites,aes(), shape = 16, colour = "black", size = 2) +
 ggrepel::geom_label_repel(data = dat_sites, aes(label = nr, geometry = geometry),
```



```
# Save the finished map object
ggsave(
  filename = "fig04_london_map.pdf",
  plot = London_map,
  device = "pdf",
    path = "documented"
)
```

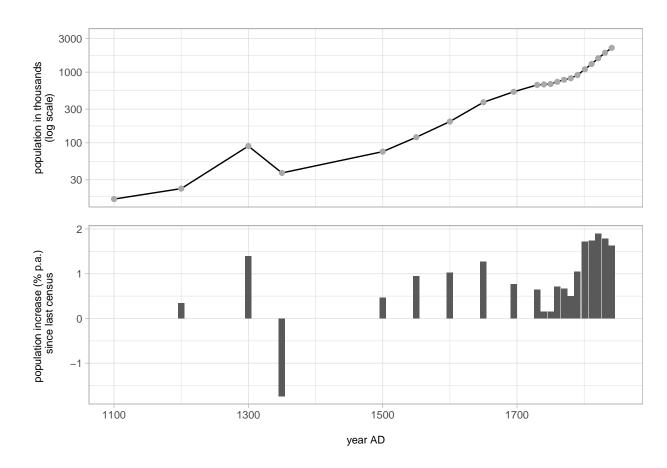
Saving 6.5×4.5 in image

Table 1: Major cemeteries of London, without and with (r) compensation of population growth. beta – Gompertz beta parameter; M – modal age; ex20 – life expectancy at age 20; ex25 – life expectancy at age 25. Ranges computed with credible HDIs of 95%.

Map no.	Name	Period	Excavation year	Social character
1	Bermondsey Abbey	1089-1538	1984-1995	monks
2	St Mary Spital	1120 - 1538	1991 - 2007	commoners
NA		period 14: 1120–1200		
NA		period 15: 1200–1250		
NA		period 16: 1250–1400		
NA		period 17: 1400–1539		
3	St Mary Graces	1350 - 1540	1986 – 1988	commoners
4	New Churchyard	1569 - 1739	2011 – 2015	low status
5	St Benet Sherehog	1670 - 1740	1994 – 1996	higher status
6	Chelsea Old church	1712 – 1842	2000	mixed
7	St Brides crypt	1740-1853		high
8	St Brides lower churchyard	1770 - 1849		low
9	Sheens burial ground	1763 - 1853	2006 – 2007	low
10	St Marylebone	1767 - 1859	1992; 2003	high status
11	St Marylebone Paddington Street north	1772 - 1853	2012 – 2013	high status
12	Bow Baptist church	1816-1853	2006; 2008	villagers
13	St Mary and St Michaels burial ground	1843 - 1854	2004 – 2005	low, Irish immigrants

Figure 4: Population development of London, compiled from Finlay & Shearer (1986), 39 table 1; Landers (1993), 41; 179 table 5.7; Weinreb, Hibbert, Keay, & Keay (2008), 655–657.

```
source("./chapter_03_data/London_population.R")
grid::grid.newpage()
grid::grid.draw(rbind(london_pop1, london_pop2))
```



Footnote 6: Re-calculation of population increase rates of London from Razzell & Spence (2007).

 $Calculated \ in \ ./chapter_03_data/London_population.R$

```
knitr::kable(razz_df)%>%
kableExtra::kable_styling(latex_options = "HOLD_position")
```

date	population	rate.per.year
1520	55000	NA
1600	200000	0.016
1650	400000	0.014
1700	575000	0.007
1750	675000	0.003
1801	960000	0.007
1851	2685000	0.021

Chapter 04 Results

Simulations

Figure 5: Comparing observed and estimated Gompertz β -values by different algorithms with known age-at-death (n = 1,000).

```
source("./chapter_supplement/simulations_run.R")
gridExtra::grid.arrange(grobs = plot_list_shapes, ncol = 3)
```

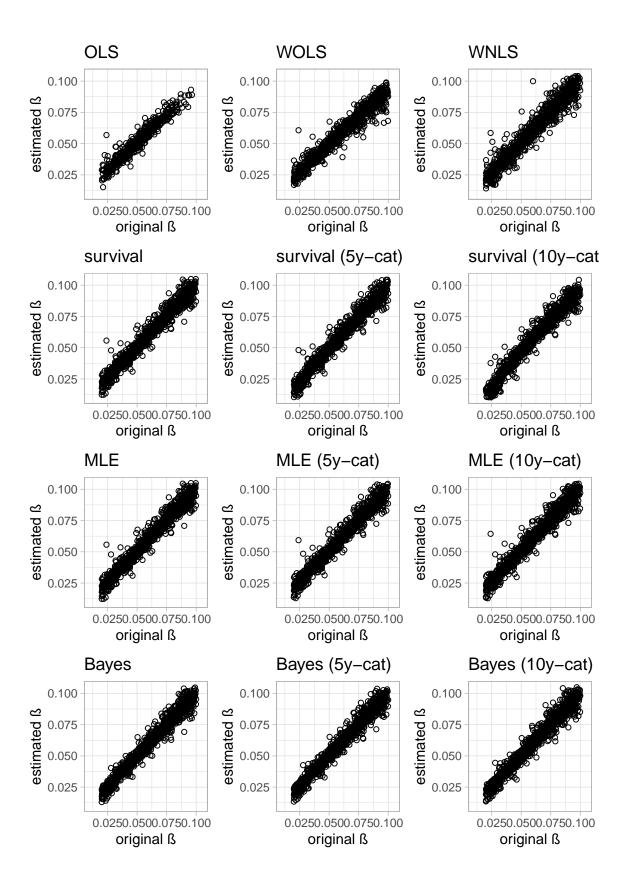


Figure 6: Difference between observed and estimated Gompertz β -values by different algorithms with known age-at-death (n = 1,000).

source("./chapter_supplement/simulations_run.R")
gridExtra::grid.arrange(grobs = plot_list_diff, ncol = 3)

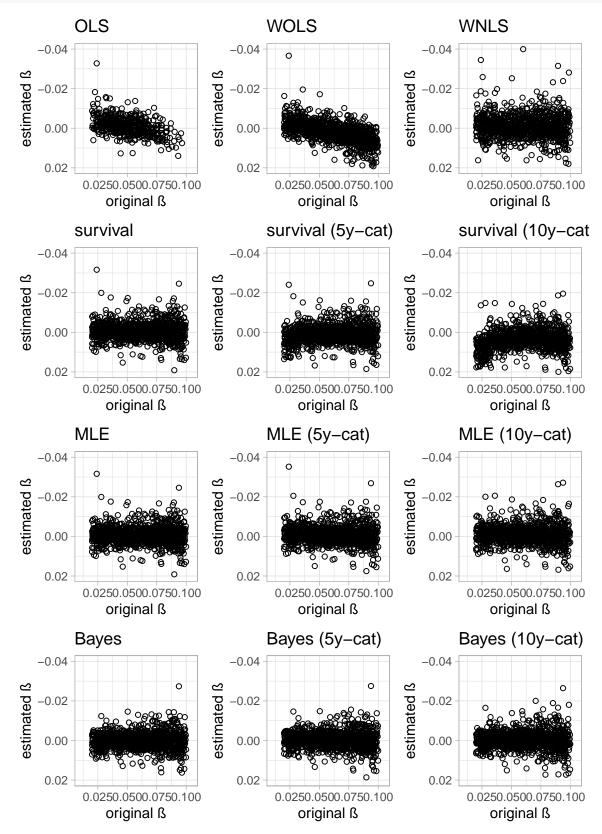


Table 2: Root mean square errors (RMSE) for different formulas for fitting known age-at-death, in ascending order.

```
# table of RMSEs
kable(rmse_result[order(rmse_result$RMSE) ,]) %>%
kableExtra::kable_styling(latex_options = "HOLD_position")
```

	method	RMSE	NAs
11	Bayes (5y-cat)	0.0045054	0
10	Bayes	0.0045342	0
7	MLE	0.0047464	0
4	survival	0.0047465	0
12	Bayes (10y-cat)	0.0048206	0
8	MLE (5y-cat)	0.0048251	0
5	survival (5y-cat)	0.0048980	0
1	OLS	0.0049008	545
9	MLE (10y-cat)	0.0051441	0
2	WOLS	0.0060401	0
3	WNLS	0.0061083	14
6	survival (10y-cat)	0.0067095	0

Figure 7: Comparison of methods to estimate the original Gompertz β from simulated data with anthropological age categories (n = 1,000).

```
source("./chapter_supplement/simulations_run.R")
gridExtra::grid.arrange(grobs = plot_list_estim_shapes, ncol = 2)
```

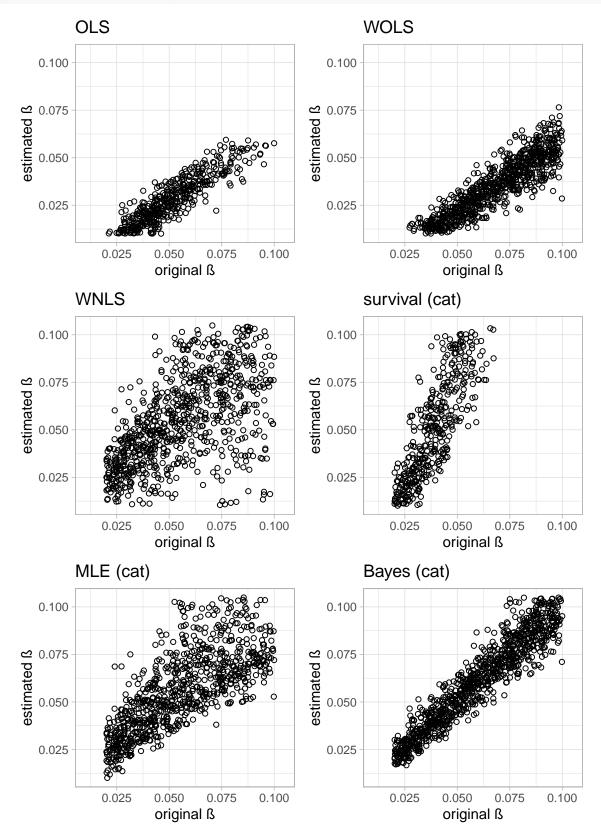


Table 3: Root mean square errors in different methods to estimate the original Gompertz β from simulated data with anthropological age categories, in ascending order.

```
# table of RMSEs
kable(rmse_estim_result[order(rmse_estim_result$RMSE) ,]) %>%
kableExtra::kable_styling(latex_options = "HOLD_position")
```

	method	RMSE	NAs
6	Bayes (cat)	$\begin{array}{c} 0.0099026 \\ 0.0202114 \end{array}$	0
7	MLE_wo_OL		71
$\begin{array}{c} 1 \\ 3 \\ 2 \end{array}$	OLS WNLS WOLS	$\begin{array}{c} 0.0243214 \\ 0.0293842 \\ 0.0328083 \end{array}$	442 73 2
5	MLE (cat)	$0.0761568 \\ 0.4731442$	0
4	survival (cat)		0

Written sources

Preprocessing of data used in figure 9: Estimated modal ages.

Basic statistics

The data is referenced and aggregated in "./chapter_04_results/historical_lifetables.R". In this file, all records from individual preprocessing files located in "./liftables_preprocessed/" are sourced. The corresponding data files are stored in "./data/".

 $\label{london_1728_1840.R} London_1728_1840.R, Mortality_bills_1728_1840.txt, Source: Roberts \& Cox (2003), 304 Table 6.5; > 100 years and < 1 year collapsed$

Table 5: London Mortality bills 1728-1840.

parameter	ranges
beta	0.0324-0.0419
M	43.4-54.8

Table 6: London Mortality bills 1728-1840, corrected for population growth.

parameter	ranges
beta_r	0.0327-0.0501
M_r	45.1-64.4
r	0.002-0.019

London 1841 raw all.R, London 1841 raw.txt, Source: Graham (1842), 19 table q.

Table 7: Census data for London from 1841.

parameter	modes	HDI.ranges
beta M	$0.0547 \\ 60.4164$	0.0510-0.0585 58.9-61.7

English_Mortality.R, wrigley_et_al_1997_england_1640-1809.txt, Source: Wrigley, Oeppen, Davies, & Schofield (1997), 290 table 6.19

Table 8: English mortality data.

parameter	ranges
beta	0.0438-0.0608
M	52.2-67.4

$HMD_UK_ranges.R$

The data from the Human Mortality Database (https://mortality.org/) were retrieved with a personal account using the R package HMDHFDplus. Therefore, we only provide the processed data here.

```
kable(HMD_UK_ranges, caption = "Human Mortality Database UK.") %>%
kableExtra::kable_styling(latex_options = "HOLD_position")
```

Table 9: Human Mortality Database UK.

parameter	ranges
beta	0.05-0.0654
M	64.2 - 70.2

English_Peers.R, russell.txt, Source: La Poutré & Janssen (2021), table 2

```
kable(peers_ranges, caption = "English Peers") %>%
kableExtra::kable_styling(latex_options = "HOLD_position")
```

Table 10: English Peers

parameter	modes	HDI.ranges
beta	0.0613	0.0559 - 0.0660
M	58.1758	56.4 - 59.8
e20	33.4148	NA
e25	29.4926	NA

Medieval_England.R, Christ_church_monks.txt, Source: Hatcher, Piper, & Stone (2006), 28 table 2

```
kable(monks_ranges, caption = "Christ Church monks") %>%
kableExtra::kable_styling(latex_options = "HOLD_position")
```

Table 11: Christ Church monks

parameter	modes	HDI.ranges
beta	0.0461	0.0398 - 0.0523
M	52.7659	48.9 - 56.0
e20	31.0948	NA
e25	27.7530	NA

Extended statistics

Table 12: London Mortality bills 1728-1840.

year	parameter	PSRF Point est.	PSRF Upper C.I.	Mean	Median	Mode	ESS	MC
X1728	alpha	1.000108	1.000395	0.0146970	0.0146465	0.0145984	16365.6	0.0000
X1728	beta	1.000120	1.000451	0.0349080	0.0349052	0.0347825	16828.8	0.0000
X1728	\mathbf{M}	1.000140	1.000453	44.6140062	44.8625026	45.0618057	16114.3	0.0233
X1730	alpha	1.000025	1.000130	0.0156646	0.0156056	0.0154789	18337.2	0.0000
X1730	beta	1.000039	1.000168	0.0325426	0.0325361	0.0324144	18117.0	0.0000
X1730	M	1.000030	1.000142	42.2093093	42.5573739	43.4233691	18020.2	0.0277
X1740	alpha	1.000112	1.000403	0.0153878	0.0153418	0.0152749	17990.7	0.0000
X1740	beta	1.000101	1.000385	0.0337646	0.0337606	0.0335736	17926.6	0.0000
X1740	M	1.000157	1.000484	43.0932517	43.3416150	44.0450446	17518.4	0.0233
X1750	alpha	1.000308	1.001170	0.0151942	0.0151449	0.0150989	18259.5	0.0000
X1750	beta	1.000351	1.001292	0.0342717	0.0342730	0.0342242	18121.9	0.0000
X1750	\mathbf{M}	1.000317	1.001174	43.5548106	43.8136623	44.5458564	17813.4	0.0230
X1760	alpha	1.000338	1.000801	0.0145711	0.0145269	0.0143509	17449.6	0.0000
X1760	beta	1.000241	1.000520	0.0350287	0.0350196	0.0349168	17085.2	0.0000
X1760	M	1.000363	1.000866	44.8827829	45.1095653	45.6086874	17106.4	0.0221
X1770	alpha	1.000175	1.000588	0.0143221	0.0142774	0.0142718	17407.1	0.0000
X1770	beta	1.000110	1.000381	0.0355969	0.0356004	0.0358743	17291.4	0.0000
X1770	\mathbf{M}	1.000178	1.000591	45.4300281	45.6462299	45.7955369	17214.1	0.0214
X1780	alpha	1.000269	1.000935	0.0136238	0.0135783	0.0135358	16804.1	0.0000
X1780	beta	1.000239	1.000906	0.0367220	0.0367253	0.0367228	16729.5	0.0000
X1780	M	1.000314	1.001006	46.8874156	47.0760558	47.5932925	16847.5	0.0195
X1790	alpha	1.000213	1.000318	0.0126517	0.0126141	0.0126627	16987.5	0.0000
X1790	beta	1.000153	1.000189	0.0385170	0.0385078	0.0385633	16735.7	0.0000
X1790	\mathbf{M}	1.000247	1.000390	48.8232330	48.9686652	49.3336238	17313.3	0.0172
X1800	alpha	1.000066	1.000135	0.0116898	0.0116448	0.0115679	15913.4	0.0000
X1800	beta	1.000108	1.000229	0.0399437	0.0399437	0.0398114	16280.5	0.0000
X1800	\mathbf{M}	1.000048	1.000103	50.7070955	50.8409969	51.0720614	16902.5	0.0156
X1810	alpha	1.000554	1.001977	0.0115040	0.0114672	0.0113409	15693.3	0.0000
X1810	beta	1.000705	1.002539	0.0384284	0.0384204	0.0384387	15514.8	0.0000
X1810	M	1.000514	1.001824	51.3269015	51.4533360	51.7564282	16513.8	0.0160
X1820	alpha	1.000033	1.000085	0.0104903	0.0104573	0.0103834	15105.4	0.0000
X1820	beta	1.000043	1.000117	0.0403661	0.0403518	0.0400258	14941.4	0.0000
X1820	${ m M}$	1.000026	1.000068	53.3477816	53.4500376	53.7200147	16242.5	0.0145
X1830	alpha	1.000262	1.000742	0.0098565	0.0098250	0.0097444	14287.5	0.0000
X1830	beta	1.000328	1.000824	0.0420331	0.0420192	0.0419423	14811.9	0.0000
X1830	M	1.000204	1.000597	54.4845443	54.5690888	54.8316675	16013.8	0.0134

X1840	alpha	1.000065	1.000175	0.0103000	0.0102638	0.0101621	14542.4	0.0000
X1840	beta	1.000062	1.000189	0.0409226	0.0409164	0.0407728	14161.3	0.0000
X1840	${ m M}$	1.000053	1.000143	53.6798657	53.7773975	53.7555529	15909.8	0.0144

Table 13: London Mortality bills 1728-1840, corrected for population growth.

year	parameter	PSRF Point est.	PSRF Upper C.I.	Mean	Median	Mode	ESS	MC
X1728	alpha	1.000493	1.001271	0.0120388	0.0119639	0.0117705	13219.2	0.0000
X1728	beta	1.000293	1.000781	0.0373256	0.0373497	0.0374499	14626.6	0.0000
X1728	M	1.000607	1.001494	50.2038587	50.4495217	50.9806000	13687.7	0.0264
X1728	rate	1.000147	1.000459	0.0066686	0.0066694	0.0067571	27227.8	0.0000
X1730	alpha	1.000269	1.000770	0.0148858	0.0147951	0.0145680	14694.8	0.0000
X1730	beta	1.000150	1.000455	0.0324320	0.0324651	0.0327068	16554.5	0.0000
X1730	M	1.000349	1.000914	43.6813384	44.1622266	45.0521696	14720.2	0.0375
X1730 X1740	rate	1.000125	$1.000492 \\ 1.000720$	0.0017597 0.0146610	0.0017647	0.0017560	31387.5 12838.5	0.0000 0.0000
X1740 X1740	alpha	1.000329	1.001036	0.0140010 0.0338783	0.0145783	$0.0142673 \\ 0.0340453$	14559.4	0.0000
	beta	1.000371			0.0339015			
X1740	M	1.000313	1.000633	44.5110399	44.8625313	45.6855802	12927.7	0.0334
X1740	rate	1.000079	1.000225	0.0015451	0.0015464	0.0015893	25927.1	0.0000
X1750	alpha	1.000091	1.000301	0.0126650	0.0125920	0.0124096	13381.2	0.0000
X1750	beta	1.000158	1.000520	0.0363874	0.0364000	0.0364127	15055.2	0.0000
X1750	M	1.000109	1.000338	48.8679712	49.1288156	49.6582623	13657.6	0.0280
X1750	rate	1.000002	1.000003	0.0061501	0.0061570	0.0061586	28511.9	0.0000
X1760	alpha	1.000046	1.000138	0.0118787	0.0118090	0.0117666	13167.5	0.0000
X1760	beta	1.000017	1.000101	0.0375028	0.0375223	0.0376118	15087.9	0.0000
X1760	M	1.000075	1.000168	50.5533524	50.7836185	51.6251396	13738.2	0.0259
X1760	rate	1.000014	1.000079	0.0068523	0.0068519	0.0068244	26661.9	0.0000
X1770	alpha	1.000380	1.001203	0.0121911	0.0121201	0.0120472	13162.6	0.0000
X1770	beta	1.000576	1.001801	0.0375407	0.0375624	0.0378417	14253.2	0.0000
X1770	M	1.000298	1.000964	49.8528284	50.0817560	50.3580496	13551.1	0.0264
X1770 X1770	rate	1.000236	1.000304	0.0053016	0.0052956	0.0052041	27499.5	0.0000
X1770 X1780	alpha	1.001089	1.003872	0.0100912	0.0100242	0.0098793	13153.1	0.0000
X1780	beta	1.000765	1.002770	0.0407636	0.0407668	0.0406275	14796.3	0.0000
X1780	M	1.001100	1.003857	54.2249121	54.3794171	54.9416590	14389.5	0.0204
X1780	rate	1.000574	1.002146	0.0098681	0.0098605	0.0099363	27001.6	0.0000
X1790	alpha	1.001213	1.004101	0.0075877	0.0075309	0.0073853	13394.6	0.0000
X1790	beta	1.000912	1.003290	0.0458236	0.0458462	0.0458421	14781.8	0.0000
X1790	M	1.001011	1.003327	59.2913458	59.3690803	59.3913393	16477.9	0.0152
X1790	rate	1.000444	1.001551	0.0163690	0.0163788	0.0165687	24985.7	0.0000
X1800	alpha	1.000387	1.001251	0.0067681	0.0067205	0.0066553	12952.0	0.0000
X1800	beta	1.000332	1.001184	0.0477523	0.0477522	0.0478150	14065.9	0.0000
X1800	M	1.000285	1.000812	60.9817744	61.0368872	61.0870784	17235.5	0.0136
X1800	rate	1.000109	1.000405	0.0172752	0.0172792	0.0173804	24208.7	0.0000
X1810	alpha	1.000103	1.000403	0.0112132	0.0112152 0.0062051	0.0060517	12219.0	0.0000
X1810	beta	1.000202	1.000974	0.0466685	0.0466843	0.0466222	13819.9	0.0000
X1810 X1810	M	1.000300	1.000314	63.1437477	63.1965586	63.1840070	16815.0	0.0000
X1810 X1810	rate	1.000099	1.000312	0.0186911	0.0186887	0.0186977	21565.1	0.0141 0.0000
X1820	alpha	1.000215	1.000279	0.0057095	0.0056649	0.0055172	11737.7	0.0000

X1820	beta	1.000270	1.000463	0.0488446	0.0488583	0.0489581	12730.0	0.0000
X1820	M	1.000058	1.000058	64.0351053	64.0783090	64.1736064	17185.5	0.0129
X1820	rate	1.000055	1.000145	0.0183072	0.0183033	0.0183194	21399.3	0.0000
X1830	alpha	1.000851	1.002718	0.0056217	0.0055807	0.0055025	11196.7	0.0000
X1830	beta	1.000811	1.002726	0.0500179	0.0500137	0.0501418	12016.8	0.0000
X1830	M	1.000547	1.001650	63.7874764	63.8285439	63.9588124	16675.9	0.0124
X1830	rate	1.000415	1.001353	0.0169139	0.0169158	0.0170653	21735.5	0.0000
X1840	alpha	1.001912	1.006772	0.0055618	0.0055194	0.0053917	11400.5	0.0000
X1840	beta	1.001683	1.006126	0.0496698	0.0496734	0.0495466	12494.4	0.0000
X1840	M	1.001165	1.004232	64.1728612	64.2122002	64.4409872	16778.0	0.0127
X1840	rate	1.000770	1.002796	0.0184375	0.0184351	0.0184647	20471.4	0.0000

Table 14: Census data for London from 1841.

	PSRF Point est.	PSRF Upper C.I.	Mean	Median	Mode	ESS	MCSE	HDImass
a	1.000227	1.000481	0.0045780	0.0045654	0.0045126	15366.4	0.0000030	0.95
b	1.000173	1.000361	0.0547652	0.0547581	0.0546507	15261.7	0.0000155	0.95
M	1.000098	1.000288	60.3512664	60.3640713	60.4164328	26323.3	0.0044223	0.95

```
kable(eng_mort_result, caption = "English mortality data.") %>%
kableExtra::kable_styling(latex_options = c("HOLD_position", "scale_down"))
```

Table 15: English mortality data.

year	parameter	PSRF Point est.	PSRF Upper C.I.	Mean	Median	Mode	ESS	МС
X1640	alpha	1.000116	1.000213	0.0109640	0.0109440	0.0108724	20086.8	0.0000
X1640	beta	1.000128	1.000269	0.0473794	0.0473646	0.0471615	19457.0	0.0000
X1640	M	1.000098	1.000168	55.8790864	55.9257645	56.0762781	22766.9	0.0074
X1650	alpha	1.000196	1.000699	0.0086495	0.0086305	0.0086386	17477.0	0.0000
X1650	beta	1.000238	1.000818	0.0534298	0.0534191	0.0532174	17097.8	0.0000
X1650	M	1.000122	1.000456	59.0892102	59.1180884	59.1222939	22875.6	0.0058
X1660	alpha	1.000538	1.001899	0.0091323	0.0091133	0.0090836	17736.5	0.0000
X1660	beta	1.000442	1.001588	0.0507107	0.0507012	0.0506517	17263.0	0.0000
X1660	M	1.000484	1.001711	58.8100849	58.8427992	58.8427140	22595.3	0.0063
X1670	alpha	1.000280	1.000678	0.0109726	0.0109471	0.0109033	20090.3	0.0000
X1670	beta	1.000338	1.000812	0.0444972	0.0445019	0.0446316	19399.7	0.0000
X1670	\mathbf{M}	1.000211	1.000529	56.4443366	56.5061422	56.5992418	22544.4	0.0083
X1680	alpha	1.000560	1.001531	0.0134499	0.0134251	0.0133309	22339.5	0.0000
X1680	beta	1.000479	1.001399	0.0435646	0.0435605	0.0437501	21337.1	0.0000
X1680	M	1.000597	1.001611	51.9412993	52.0117614	52.1884363	23152.8	0.0091
X1690	alpha	1.000203	1.000424	0.0100036	0.0099815	0.0099278	19068.4	0.0000
X1690	beta	1.000217	1.000487	0.0457255	0.0457144	0.0457837	18458.1	0.0000
X1690	\mathbf{M}	1.000156	1.000323	58.2265512	58.2748183	58.3311613	22552.3	0.0076
X1700	alpha	1.000247	1.000813	0.0097904	0.0097696	0.0097670	19048.9	0.0000
X1700	beta	1.000228	1.000792	0.0469162	0.0469091	0.0468449	18524.9	0.0000
X1700	M	1.000210	1.000697	58.3940436	58.4388608	58.4010030	22362.5	0.0073
X1710	alpha	1.000629	1.002202	0.0076344	0.0076175	0.0075883	16583.9	0.0000
X1710	beta	1.000810	1.002798	0.0547320	0.0547184	0.0547561	16149.3	0.0000
X1710	M	1.000354	1.001230	61.0059909	61.0278089	61.0396872	23904.2	0.0053

X1720	alpha	1.000438	1.001584	0.0080028	0.0079824	0.0079301	17173.5	0.0000
X1720	beta	1.000453	1.001645	0.0560063	0.0560001	0.0558861	16622.0	0.0000
X1720	M	1.000297	1.001094	59.7548666	59.7785061	59.8227855	23615.5	0.0051
X1730	alpha	1.000128	1.000485	0.0067962	0.0067781	0.0067500	15125.2	0.0000
X1730	beta	1.000132	1.000481	0.0561362	0.0561233	0.0561525	14537.5	0.0000
X1730	M	1.000086	1.000325	62.6345173	62.6558894	62.6742840	23395.6	0.0050
X1740	alpha	1.000185	1.000678	0.0067863	0.0067674	0.0067507	15383.6	0.0000
X1740	beta	1.000260	1.000953	0.0566936	0.0566911	0.0568096	15043.4	0.0000
X1740	M	1.000063	1.000245	62.4655115	62.4849711	62.4345863	22893.7	0.0051
X1750	alpha	1.000389	1.001407	0.0053793	0.0053645	0.0053635	13451.8	0.0000
X1750	beta	1.000443	1.001627	0.0595510	0.0595305	0.0594825	12966.5	0.0000
X1750	M	1.000167	1.000607	65.4071998	65.4213962	65.4642394	24607.7	0.0044
X1760	alpha	1.000180	1.000387	0.0080565	0.0080378	0.0079359	17802.8	0.0000
X1760	beta	1.000187	1.000346	0.0488550	0.0488417	0.0488743	16840.4	0.0000
X1760	M	1.000119	1.000328	61.9016665	61.9336352	62.0050035	23618.5	0.0063
X1770	alpha	1.000469	1.001234	0.0067654	0.0067476	0.0066997	14881.8	0.0000
X1770	beta	1.000501	1.001341	0.0538935	0.0538804	0.0536262	14373.8	0.0000
X1770	M	1.000270	1.000709	63.5277738	63.5489910	63.4921572	22585.6	0.0054
X1780	alpha	1.000347	1.001235	0.0066384	0.0066225	0.0066110	15713.1	0.0000
X1780	beta	1.000349	1.001303	0.0570824	0.0570677	0.0571475	15127.7	0.0000
X1780	M	1.000195	1.000710	62.7166530	62.7352113	62.7235429	24154.0	0.0048
X1790	alpha	1.000180	1.000376	0.0058127	0.0057951	0.0057825	13899.0	0.0000
X1790	beta	1.000229	1.000478	0.0580897	0.0580826	0.0579337	13447.1	0.0000
X1790	M	1.000055	1.000174	64.6577738	64.6756380	64.7246120	23430.8	0.0047
X1800	alpha	1.000697	1.002167	0.0046142	0.0045977	0.0045567	12391.5	0.0000
X1800	beta	1.000709	1.002222	0.0608363	0.0608357	0.0608310	11940.3	0.0000
X1800	M	1.000239	1.000822	67.4325112	67.4451148	67.4486965	26907.8	0.0039

kable(HMD_UK_result, caption = "Human Mortality Database UK.") %>%
kableExtra::kable_styling(latex_options = c("HOLD_position","scale_down"))

Table 16: Human Mortality Database UK.

year	parameter	PSRF Point est.	PSRF Upper C.I.	Mean	Median	Mode	ESS	MC
X1841	alpha	1.000163	1.000417	0.0036316	0.0036148	0.0035704	13786.9	0.0000
X1841	beta	1.000156	1.000468	0.0519888	0.0519880	0.0522191	13557.1	0.0000
X1841	\mathbf{M}	1.000083	1.000148	66.2542881	66.2748935	66.3111134	25227.7	0.0056
X1845	alpha	1.000079	1.000297	0.0042938	0.0042780	0.0042669	14758.1	0.0000
X1845	beta	1.000067	1.000247	0.0498996	0.0498859	0.0500069	14606.7	0.0000
X1845	M	1.000065	1.000255	64.2120906	64.2353737	64.2258466	24443.1	0.0062
X1850	alpha	1.000973	1.003549	0.0037558	0.0037391	0.0036783	13963.2	0.0000
X1850	beta	1.000868	1.003159	0.0517607	0.0517548	0.0515669	13902.2	0.0000
X1850	\mathbf{M}	1.000670	1.002447	65.7445767	65.7656894	65.8470383	24788.4	0.0057
X1855	alpha	1.000156	1.000325	0.0034310	0.0034165	0.0034148	13071.2	0.0000
X1855	beta	1.000111	1.000252	0.0533405	0.0533264	0.0532089	12901.9	0.0000
X1855	\mathbf{M}	1.000055	1.000176	66.5070750	66.5214941	66.5377049	24496.9	0.0055
X1860	alpha	1.000103	1.000169	0.0034785	0.0034615	0.0034062	13432.3	0.0000
X1860	beta	1.000118	1.000200	0.0532621	0.0532640	0.0532106	13350.6	0.0000
X1860	M	1.000016	1.000032	66.2956647	66.3134949	66.3500169	24949.1	0.0054
X1865	alpha	1.000093	1.000333	0.0035991	0.0035837	0.0035330	13810.1	0.0000
X1865	beta	1.000108	1.000353	0.0530530	0.0530491	0.0530859	13658.1	0.0000
X1865	M	1.000045	1.000177	65.7780212	65.7953467	65.8802464	25507.4	0.0054
X1870	alpha	1.000077	1.000191	0.0035027	0.0034908	0.0034765	13760.3	0.0000

X1870	beta	1.000110	1.000242	0.0536849	0.0536587	0.0535650	13616.9	0.0000
X1870	M	1.000013	1.000070	65.9071237	65.9205478	65.8577523	25746.8	0.0052
X1875	alpha	1.000092	1.000343	0.0030605	0.0030478	0.0030211	12815.0	0.0000
X1875	beta	1.000131	1.000471	0.0568401	0.0568279	0.0568980	12685.7	0.0000
X1875	M	1.000031	1.000141	66.4668857	66.4785272	66.5086127	26537.0	0.0047
X1880	alpha	1.000119	1.000291	0.0027376	0.0027242	0.0026867	12292.1	0.0000
X1880	beta	1.000171	1.000388	0.0580794	0.0580802	0.0580583	12275.9	0.0000
X1880	M	1.000016	1.000046	67.6639684	67.6752899	67.6594434	26763.5	0.0045
X1885	alpha	1.000329	1.000699	0.0024297	0.0024169	0.0023954	11472.9	0.0000
X1885	beta	1.000279	1.000617	0.0605593	0.0605577	0.0605172	11375.3	0.0000
X1885	M	1.000124	1.000309	68.1749533	68.1842585	68.1928238	26834.9	0.0042
X1890	alpha	1.000121	1.000451	0.0024118	0.0023997	0.0023802	11867.7	0.0000
X1890	beta	1.000146	1.000531	0.0614665	0.0614588	0.0613662	11669.1	0.0000
X1890	M	1.000045	1.000148	67.7519542	67.7613179	67.8037905	28081.6	0.0040
X1895	alpha	1.000328	1.001210	0.0019825	0.0019735	0.0019634	10586.6	0.0000
X1895	beta	1.000267	1.000994	0.0637922	0.0637712	0.0636859	10594.5	0.0000
X1895	M	1.000224	1.000819	69.4924393	69.4994274	69.5249671	28489.7	0.0038
X1900	alpha	1.000051	1.000194	0.0017882	0.0017794	0.0017686	10113.1	0.0000
X1900	beta	1.000061	1.000247	0.0653147	0.0653019	0.0653664	10036.7	0.0000
X1900	M	1.000007	1.000020	70.1656919	70.1713832	70.1615274	28126.0	0.0036

```
kable(peers_result, caption = "English Peers.") %>%
kableExtra::kable_styling(latex_options = c("HOLD_position", "scale_down"))
```

Table 17: English Peers.

	PSRF Point est.	PSRF Upper C.I.	Mean	Median	Mode	ESS	MCSE	HDImass
a	1.000356	1.001061	0.0044095	0.0043879	0.0043263	11315.3	0.0000044	0.95
b	1.000307	1.000854	0.0609452	0.0609518	0.0612976	11121.9	0.0000244	0.95
\mathbf{M}	1.000196	1.000696	58.1500750	58.1648975	58.1757927	20660.4	0.0059675	0.95

```
kable(monks_result, caption = "Christ Church monks.") %>%
  kableExtra::kable_styling(latex_options = c("HOLD_position","scale_down"))
```

Table 18: Christ Church monks.

start	end	parameter	PSRF Point est.	PSRF Upper C.I.	Mean	Median	Mode	ESS
1395	1505	alpha	1.000569	1.001661	0.0102993	0.0102563	0.0100488	13991.9 (
1395	1505	beta	1.000498	1.001436	0.0459467	0.0459287	0.0461060	13623.6 (
1395	1505	\mathbf{M}	1.000504	1.001504	52.5281106	52.6296486	52.7658924	15808.5 (

London cemeteries

The data is mainly hard coded in the file ./chapter_04_results/Wellcome_DB.R.

Only St. Bride's crypt is excluded but available from the Museum of London upon request. For general information: https://www.museumoflondon.org.uk go for: Collections > Archaeology at the Museum of London > Wellcome Osteological Research Database > St. Bride's Church Fleet Street. If runCodeNew == TRUE the file ./lifetables_processing/stbrides_crypt.R will ask for the location of the retrieved dataset (Excel sheet) and process the data. In any other case pre-processed data will be loaded.

```
source("./lifetables_processing/stbrides_crypt.R")
source("./chapter_04_results/Wellcome_DB.R")
```

kable(wellcome_result) %>%

kableExtra::kable_styling(latex_options = c("HOLD_position","scale_down"))

cemetery	start	end	parameter	PSRF Point est.	PSRF Upper C.I.	Me
Bermondsey Abbey	1089	1538	alpha	1.000095	1.000352	0.01198
Bermondsey Abbey	1089	1538	beta	1.000115	1.000319	0.0410
Bermondsey Abbey	1089	1538	M	1.000071	1.000266	41.8366
St. Mary Graces	1350	1540	alpha	1.000196	1.000732	0.0197
St. Mary Graces	1350	1540	beta	1.000244	1.000894	0.0346
St. Mary Graces	1350	1540	\mathbf{M}	1.000288	1.000951	27.6620
St. Mary Hospital, 1120-1200	1120	1200	alpha	1.000021	1.000058	0.02493
St. Mary Hospital, 1120-1200	1120	1200	beta	1.000010	1.000041	0.0363
St. Mary Hospital, 1120-1200	1120	1200	M	1.000014	1.000048	21.90008
St. Mary Hospital, 1200-1250	1200	1250	alpha	1.000142	1.000479	0.0299
St. Mary Hospital, 1200-1250	1200	1250	beta	1.000145	1.000390	0.03569
St. Mary Hospital, 1200-1250	1200	1250	\mathbf{M}	1.000138	1.000388	16.4726
St. Mary Hospital, 1250-1400	1250	1400	alpha	1.000058	1.000230	0.01833
St. Mary Hospital, 1250-1400	1250	1400	beta	1.000039	1.000160	0.0580'
St. Mary Hospital, 1250-1400	1250	1400	M	1.000056	1.000226	31.84030
St. Mary Hospital, 1400-1539	1400	1539	alpha	1.000184	1.000413	0.02624
St. Mary Hospital, 1400-1539	1400	1539	beta	1.000202	1.000384	0.03740
St. Mary Hospital, 1400-1539	1400	1539	\mathbf{M}	1.000255	1.000509	21.15625
New Churchyard	1569	1739	alpha	1.000100	1.000375	0.02574
New Churchyard	1569	1739	beta	1.000143	1.000447	0.03654
New Churchyard	1569	1739	\mathbf{M}	1.000167	1.000489	21.2258
St. Benet Sherehog	1670	1740	alpha	1.000103	1.000388	0.01593
St. Benet Sherehog	1670	1740	beta	1.000166	1.000541	0.03548
St. Benet Sherehog	1670	1740	\mathbf{M}	1.000123	1.000414	34.06949
Chelsea Old church	1712	1842	alpha	1.000092	1.000284	0.00839
Chelsea Old church	1712	1842	beta	1.000107	1.000363	0.04224
Chelsea Old church	1712	1842	${ m M}$	1.000066	1.000214	50.24129
St. Marylebone	1742	1817	alpha	1.000026	1.000095	0.01259
St. Marylebone	1742	1817	beta	1.000017	1.000034	0.04209
St. Marylebone	1742	1817	M	1.000036	1.000085	40.5179
St. Marylebone Paddington Street north	1772	1853	alpha	1.000045	1.000166	0.00990
St. Marylebone Paddington Street north	1772	1853	beta	1.000055	1.000206	0.04886
St. Marylebone Paddington Street north	1772	1853	\mathbf{M}	1.000039	1.000116	44.6690
St. Bride's lower churchyard	1770	1849	alpha	1.000035	1.000073	0.0061'
St. Bride's lower churchyard	1770	1849	beta	1.000155	1.000339	0.0510'
St. Bride's lower churchyard	1770	1849	\mathbf{M}	1.000130	1.000460	53.43680
Sheen's burial ground	1763	1854	alpha	1.000091	1.000320	0.01296
Sheen's burial ground	1763	1854	beta	1.000171	1.000495	0.03539
Sheen's burial ground	1763	1854	${ m M}$	1.000120	1.000420	39.99564
Bow Baptist Church	1816	1854	alpha	1.000037	1.000121	0.0177'
Bow Baptist Church	1816	1854	beta	1.000013	1.000063	0.03440
Bow Baptist Church	1816	1854	\mathbf{M}	1.000031	1.000063	30.6580
St. Mary and St. Michael	1843	1853	alpha	1.000523	1.001826	0.01864
St. Mary and St. Michael	1843	1853	beta	1.000441	1.001467	0.0402
St. Mary and St. Michael	1843	1853	M	1.000616	1.001944	30.8426
St. Bride's crypt (known age)	1740	1853	alpha	1.001301	1.004385	0.0048
St. Bride's crypt (known age)	1740	1853	beta	1.001527	1.004999	0.0492
St. Bride's crypt (known age)	1740	1853	\mathbf{M}	1.000820	1.002819	59.11689
St. Bride's crypt (estimates)	1740	1853	alpha	1.000667	1.001933	0.0050

St. Bride's crypt (estimates)	1740	1853	beta	1.000585	1.001723	0.04623
St. Bride's crypt (estimates)	1740	1853	M	1.000256	1.000830	60.03387

kable(wellcome_result_r, caption = "London cemeteries data, corrected for population growth.") %>%
kableExtra::kable_styling(latex_options = c("HOLD_position", "scale_down"))

Table 20: London cemeteries data, corrected for

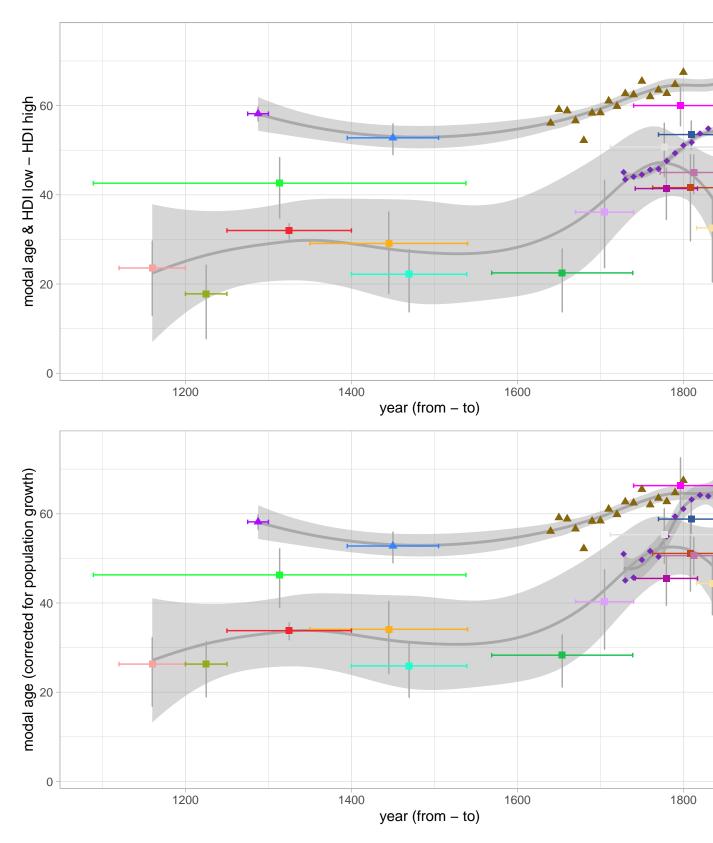
cemetery	start	end	parameter	PSRF Point est.	PSRF Upper C.I.	Me
Bermondsey Abbey	1089	1538	alpha	1.0004384	1.0015608	0.01005
Bermondsey Abbey	1089	1538	beta	1.0003921	1.0013937	0.04337
Bermondsey Abbey	1089	1538	\mathbf{M}	1.0004266	1.0014866	45.68224
Bermondsey Abbey	1089	1538	rate	1.0000854	1.0002686	0.00564
St. Mary Graces	1350	1540	alpha	1.0000932	1.0003161	0.01704
St. Mary Graces	1350	1540	beta	1.0001339	1.0004049	0.03715
St. Mary Graces	1350	1540	\mathbf{M}	1.0001704	1.0003953	32.62808
St. Mary Graces	1350	1540	rate	1.0000468	1.0001575	0.00501
St. Mary Hospital, 1120-1200	1120	1200	alpha	1.0000497	1.0001475	0.02288
St. Mary Hospital, 1120-1200	1120	1200	beta	1.0000265	1.0000918	0.03798
St. Mary Hospital, 1120-1200	1120	1200	M	1.0000797	1.0001963	24.99273
St. Mary Hospital, 1120-1200	1120	1200	rate	0.9999984	1.0000119	0.00284
St. Mary Hospital, 1200-1250	1200	1250	alpha	1.0000343	1.0001359	0.02311
St. Mary Hospital, 1200-1250	1200	1250	beta	1.0000008	1.0000202	0.03978
St. Mary Hospital, 1200-1250	1200	1250	M	1.0000240	1.0001056	25.40988
St. Mary Hospital, 1200-1250	1200	1250	rate	1.0000671	1.0002342	0.01276
St. Mary Hospital, 1250-1400	1250	1400	alpha	1.0000605	1.0000968	0.01623
St. Mary Hospital, 1250-1400	1250	1400	beta	1.0000277	1.0000352	0.06103
St. Mary Hospital, 1250-1400	1250	1400	${ m M}$	1.0000627	1.0001135	33.69499
St. Mary Hospital, 1250-1400	1250	1400	rate	1.0000509	1.0000976	0.00487
St. Mary Hospital, 1400-1539	1400	1539	alpha	1.0000461	1.0001787	0.02323
St. Mary Hospital, 1400-1539	1400	1539	beta	1.0000550	1.0001895	0.03989
St. Mary Hospital, 1400-1539	1400	1539	\mathbf{M}	1.0000630	1.0002150	25.31239
St. Mary Hospital, 1400-1539	1400	1539	rate	1.0000106	1.0000576	0.00474
New Churchyard	1569	1739	alpha	1.0000443	1.0000878	0.02147
New Churchyard	1569	1739	beta	1.0001137	1.0002395	0.03990
New Churchyard	1569	1739	\mathbf{M}	1.0001096	1.0002228	27.31358
New Churchyard	1569	1739	rate	1.0000195	1.0000586	0.00756
St. Benet Sherehog	1670	1740	alpha	1.0001384	1.0005211	0.01351
St. Benet Sherehog	1670	1740	beta	1.0002094	1.0006563	0.03777
St. Benet Sherehog	1670	1740	M	1.0001248	1.0004113	38.90225
St. Benet Sherehog	1670	1740	rate	1.0000249	1.0001064	0.00542
Chelsea Old church	1712	1842	alpha	1.0001480	1.0002573	0.00647
Chelsea Old church	1712	1842	beta	1.0002599	1.0005036	0.04538
Chelsea Old church	1712	1842	M	1.0000299	1.0000809	55.07137
Chelsea Old church	1712	1842	rate	1.0000055	1.0000176	0.00759
St. Marylebone	1742	1817	alpha	1.0002096	1.0006937	0.01014
St. Marylebone	1742	1817	beta	1.0001959	1.0007254	0.04512
St. Marylebone	1742	1817	M	1.0001775	1.0005507	45.06300
St. Marylebone	1742	1817	rate	1.0000264	1.0001090	0.00746
St. Marylebone Paddington Street north	1772	1853	alpha	1.0000658	1.0001793	0.00677
St. Marylebone Paddington Street north	1772	1853	beta	1.0002920	1.0009451	0.05405
St. Marylebone Paddington Street north	1772	1853	\mathbf{M}	1.0000059	1.0000339	50.56973
St. Marylebone Paddington Street north	1772	1853	rate	1.0000299	1.0001073	0.01344

St. Bride's lower churchyard	1770	1849	alpha	1.0004322	1.0014789	0.00388
St. Bride's lower churchyard	1770	1849	beta	1.0010900	1.0037600	0.05765
St. Bride's lower churchyard	1770	1849	M	1.0001688	1.0005250	59.00471
St. Bride's lower churchyard	1770	1849	rate	1.0000394	1.0001612	0.01325
Sheen's burial ground	1763	1854	alpha	1.0000293	1.0000680	0.00848
Sheen's burial ground	1763	1854	beta	1.0000180	1.0000412	0.04066
Sheen's burial ground	1763	1854	M	1.0000358	1.0000988	50.56928
Sheen's burial ground	1763	1854	rate	0.9999938	0.9999965	0.01314
Bow Baptist Church	1816	1854	alpha	1.0001794	1.0006411	0.01103
Bow Baptist Church	1816	1854	beta	1.0000875	1.0003385	0.04078
Bow Baptist Church	1816	1854	M	1.0001823	1.0006454	43.96280
Bow Baptist Church	1816	1854	rate	1.0000471	1.0001859	0.01745
St. Mary and St. Michael	1843	1853	alpha	1.0000453	1.0000833	0.01227
St. Mary and St. Michael	1843	1853	beta	1.0001569	1.0004401	0.04595
St. Mary and St. Michael	1843	1853	M	1.0000225	1.0000500	40.68548
St. Mary and St. Michael	1843	1853	rate	1.0000285	1.0000843	0.01764
St. Bride's crypt (known age)	1740	1853	alpha	1.0004443	1.0013905	0.00326
St. Bride's crypt (known age)	1740	1853	beta	1.0004021	1.0012201	0.05486
St. Bride's crypt (known age)	1740	1853	${ m M}$	1.0002073	1.0007301	63.60241
St. Bride's crypt (known age)	1740	1853	rate	0.9999985	1.0000287	0.01001
St. Bride's crypt (estimates)	1740	1853	alpha	1.0004308	1.0015894	0.00344
St. Bride's crypt (estimates)	1740	1853	beta	1.0008845	1.0032184	0.04914
St. Bride's crypt (estimates)	1740	1853	${ m M}$	1.0000909	1.0001930	66.51165
St. Bride's crypt (estimates)	1740	1853	rate	1.0000276	1.0001275	0.00999

Figure 6: Estimated modal ages from written sources and osteological data compared, upper panel: without population growth correction, lower panel: with population growth correction. Horizontal bars indicate the time span the data point covers. Vertical bars indicate 95% HDI for credible ranges and are only displayed for small n, i.e. English Peers, Christ Church monks and osteological data.

```
# get symbols & colors from palette alphabet (max n = 26), alt. glasbey (32), polychrome(36)
plotcolors<-palette.colors(palette = 'alphabet')</pre>
plotsymbols < -c(17, 18, 15)
# MOLA Welcome data without correction of population growth
english_wellcome <- rbind(english_mortality_prep, wellcome_prep)</pre>
# slight modifications
english_wellcome <- english_wellcome %>%
 mutate(data = factor(data, levels = unique(data))) %>%
 mutate(source = gsub('written', 'England & Wales written', source)) %>%
 mutate(source = gsub('osteological', 'London osteological', source)) %>%
 mutate(source = ifelse(data=="London 1728-1840", "London written", source)) %>%
 mutate(source = factor(source, levels = c('England & Wales written', 'London written',
                         'London osteological'))) %>%
 mutate(start = as.numeric(start)) %>%
 mutate(end = as.numeric(end)) %>%
 mutate(year = ifelse(is.na(year), (start + end)/2, substr(year, 2,5))) %>%
 mutate(year = as.numeric(year))
ggplot(english_wellcome, aes(x = year, y = M, colour = data, shape = source) ) +
 ylab("modal age & HDI low - HDI high") +
 xlab("year (from - to)") + ylim(2, 75) + theme_light() +
  scale_color_manual(values=unname(plotcolors)) +
```

```
scale_shape_manual(values=plotsymbols) +
  geom_smooth(color = "dark grey", method = 'loess', formula = 'y ~ x') +
  geom_errorbar(aes(ymin = HDIlow, ymax= HDIhigh), width=0, colour = "dark grey") +
  geom_errorbarh(aes(xmax = start, xmin = end, height = 1)) +
  geom_point(size= 2 ) +
  guides(size = "none",colour=guide_legend(ncol=1)) +
  scale_x_continuous (breaks = seq(1200, 1800, by = 200)) +
  theme(legend.position="none") -> english_wellcome_plot
# MOLA Welcome data with correction of population growth (_r)
english_wellcome_r <- rbind(english_mortality_prep_r, wellcome_prep_r)</pre>
# slight modifications
english wellcome r <- english wellcome r %>%
  mutate(data = factor(data, levels = unique(data))) %>%
  mutate(source = gsub('written', 'England & Wales written', source)) %>%
  mutate(source = gsub('osteological', 'London osteological', source)) %>%
  mutate(source = ifelse(data=="London 1728-1840", "London written", source)) %>%
  mutate(source = factor(source, levels = c('England & Wales written', 'London written',
                          'London osteological'))) %>%
  mutate(start = as.numeric(start)) %>%
  mutate(end = as.numeric(end)) %>%
  mutate(year = ifelse(is.na(year), (start + end)/2, substr(year, 2,5))) %>%
  mutate(year = as.numeric(year))
ggplot(english_wellcome_r, aes(x = year, y = M, colour = data, shape = source) ) +
  ylab("modal age (corrected for population growth)") +
  xlab("year (from - to)") + ylim(2, 75) + theme_light() +
  scale_color_manual(values=unname(plotcolors)) +
  scale shape manual(values=plotsymbols) +
  geom_smooth(color = "dark grey", method = 'loess', formula = 'y ~ x') +
  geom_errorbar(aes(ymin = HDIlow, ymax= HDIhigh), width=0, colour = "dark grey") +
  geom_errorbarh(aes(xmax = start, xmin = end, height = 1)) +
  geom_point(size= 2 )+
  guides(size = "none",colour=guide_legend(ncol=1)) +
  scale_x_continuous (breaks = seq(1200, 1800, by = 200)) -> english_wellcome_plot_r
# get the legend and remove it afterwards
ewp_legend <- get_legend(english_wellcome_plot_r)</pre>
english_wellcome_plot_r <- english_wellcome_plot_r + theme(legend.position="none")
# build the image
grid::grid.newpage()
ewp<-plot_grid(english_wellcome_plot, english_wellcome_plot_r, ncol=1)</pre>
modal ages plot \leftarrow plot grid(ewp, ewp legend, ncol = 2, rel widths = c(.75, .25))
# Save the finished map object
ggsave(
  filename = "fig06 modal ages plot.pdf",
 width = 11, height = 8.5,
 plot = modal_ages_plot,
  device = "pdf",
  path = "documented"
plot(modal_ages_plot)
```



The following data overview is build during pre-processing in ./chapter_04_results/Wellcome_DB.R and saved to a textfile (sep = \t t).

Table 3: Major cemeteries of London, without and with (r) compensation of population growth. beta – Gompertz beta parameter; M – modal age; ex20 – life expectancy at age 20; ex25 – life expectancy at age 25. Ranges computed with credible HDIs of 95%.

```
kable(wellcome_overview_all) %>%
kableExtra::kable_styling(latex_options = c("HOLD_position", "scale_down"))
```

cemetery	beta	beta range	M	M_range	ex20	ex25	r beta	r beta
Bermondsey Abbey	0.0414	0.0319-0.0504	42.6	34.7-48.4	25.5	22.7	0.0439	0.0337-
Bow Baptist Church	0.0345	0.026-0.0429	32.6	20.4-39.4	22.7	20.4	0.0410	0.0318-
Chelsea Old church	0.0422	0.0328-0.0518	50.7	44-56.1	30.4	27.3	0.0458	0.0348-
New Churchyard	0.0365	0.0299-0.0432	22.5	13.7-27.9	17.0	15.0	0.0398	0.0331-
Sheen's burial ground	0.0350	0.0263-0.0447	41.6	29.6-49.1	27.3	24.6	0.0407	0.0305-
St. Benet Sherehog	0.0353	0.0265-0.0446	36.1	23.6-43.3	23.7	21.2	0.0378	0.0283-
St. Bride's crypt (estimates)	0.0455	0.0377-0.0547	60.0	55.4-64.7	36.8	33.2	0.0494	0.0384-
St. Bride's crypt (known age)	0.0491	0.0423-0.0565	59.4	55.5-62.5	35.7	32.0	0.0547	0.0477-
St. Bride's lower churchyard	0.0511	0.0424-0.0599	53.5	50.2-56.6	31.0	27.5	0.0577	0.0483-
St. Mary Graces	0.0349	0.0267 0.0427	29.1	17.8 - 36.2	20.8	18.6	0.0372	0.0288-
St. Mary Hospital, 1120-1200	0.0366	0.0289-0.0437	23.6	12.9-29.7	17.4	15.4	0.0382	0.0304-
St. Mary Hospital, 1200-1250	0.0355	0.0289-0.0424	17.8	7.7 - 24.3	15.7	13.8	0.0399	0.0328-
St. Mary Hospital, 1250-1400	0.0577	0.0529 - 0.0634	32.0	30-33.6	16.0	13.4	0.0608	0.0556-
St. Mary Hospital, 1400-1539	0.0374	0.0307 - 0.0443	22.2	13.7 - 27.8	16.7	14.7	0.0400	0.0329-
St. Mary and St. Michael	0.0405	0.0323 - 0.0482	31.7	24.2 - 37	20.0	17.6	0.0460	0.0378-
St. Marylebone	0.0423	0.0331-0.0508	41.4	34.4-46.1	24.5	21.7	0.0452	0.0358-
St. Marylebone Paddington Street north	0.0486	0.0399 0.0578	45.0	40.2-49	25.4	22.4	0.0538	0.0448-

Data are hard coded in the code. Sources: Miles, Powers, Wroe-Brown, & Walker (2008), 97–103 table 32 (St Marylebone); Henderson, Walker, & Miles (2015), 81 (St Marylebone north of Paddington street)

Table 22: St Marylebone, corrected with population growth rate of 2.75 per-cent.

parameter	modes	HDI.ranges
Marylebone beta	0.0527	0.0433-0.0619
Marylebone M	54.4880	49.6 - 59.3
Marylebone north beta	0.0593	0.0499 0.0685
Marylebone north M	55.3931	51.4-59.9

The following plot is build in ./lifetables_processing/stbrides_crypt.R within the if-statement on runCodeNew (s. data limitations above).

Figure 8: St. Bride's Crypt. Density of actual ages and Bayesian model of Gompertz distribution of actual ages and osteological estimates (without correction for population growth).

```
plot(stbrides_crypt_plot)
```

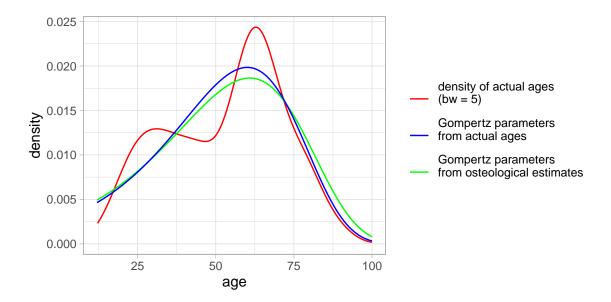
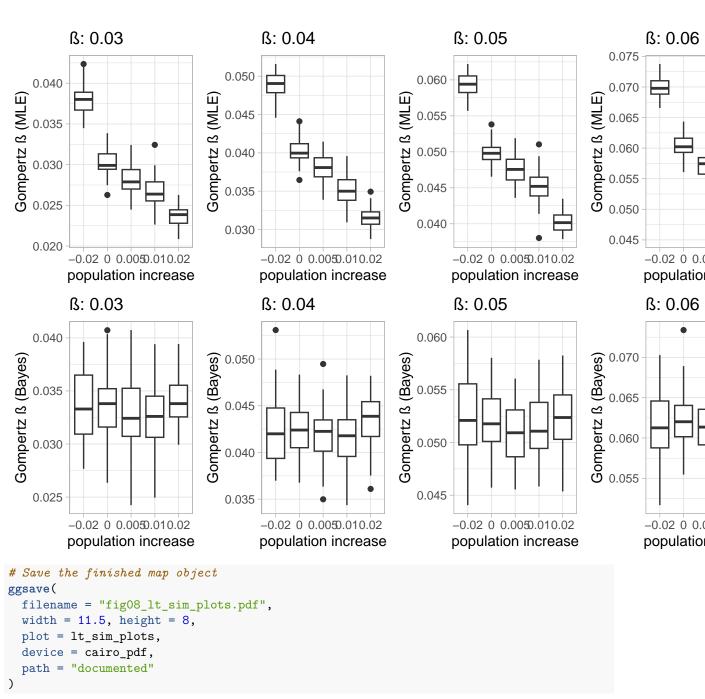


Figure 9: Simulation of population increase with known age-at-death and Maximum Likelihood Estimation (MLE) (top four) and osteological estimates, Bayesian model and including rate of increase (bottom four).

```
if (runCodeNew){
  set.seed(3673)
  lt_sim_list <- list()</pre>
  for(k in 1:4) {
    lt_sim \leftarrow lt.MC.Gomp(pop_start = c(10000, 1000, 500, 200, 100),
                          pop_inc = c(-0.02, 0, 0.005, 0.01, 0.02),
                          years = 200,
                          obs_start = 150,
                          obs_end = 200,
                          beta = (k + 2)/100,
                          bayes = TRUE)
    lt sim list[[k]] <- lt sim</pre>
  }
  # saves results in Rda-object
  save(lt_sim_list, file = file.path(".", saveFileDir, "lt_sim_list.Rda") )
load(file.path(".", saveFileDir, "lt_sim_list.Rda") )
lt_sim_plot_list <- list()</pre>
for (i in 1:4) {
lt_sim_plot_list[[i]] <- ggplot(lt_sim_list[[i]],</pre>
                                 aes(y = surv_Gompertz_shape,
                                     x = as.factor(pop inc))) +
  geom boxplot() +
  ggtitle(paste0("\u03B2: ", (i + 2)/100)) +
  ylab("Gompertz \u03B2 (MLE)") + xlab("population increase") +
  theme(plot.margin = unit(c(0,0.5,0.5,0), "cm"),
        plot.title = element_text(size = 12),
        axis.title = element_text(size = 10),
        axis.text = element_text(size = 8)) + theme_light()
                               ggplot(lt_sim_list[[i]],
lt_sim_plot_list[[i + 4]] <-</pre>
                                        aes(y = bayes_gomp_b,
                                            x = as.factor(pop_inc)) +
 geom_boxplot()
```

Original Gompertz



Supporting information

The chapter 'Supporting information' provides details about the London cemeteries included in the study, the Gompertz parameters of the Coale & Demeny life tables, and the simulations and their results.

The Coale & Demeny life tables

Calculation of the lowest β -value for any of the Coale & Demeny life tables (Coale & Demeny (1983)) which is 0.0391 (the female table "West", level 1).

```
source("./chapter_supplement/coale_demeny_life_tables_gompertz.R")
min(gompertz_df$Gompertz_shape)
```

[1] 0.03913138

Simulations

Figure 10: Bayesian model of simulated data with anthropological age categories and random error in "age estimation." Difference of estimated to original Gompertz β in relation to original β (left) and sample size (right).

```
#source("./chapter_supplement/simulations_run.R")
gridExtra::grid.arrange(grobs = plot_list_bayes_diff, ncol = 2)
```

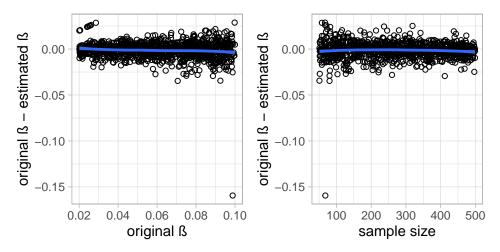


Table 4. Bayesian model with simulated data-set to compare the impact of thinning and additional steps. n=500, Gompertz $\beta=0.05.$

```
source("./chapter_supplement/bayes_complete.R") # can take a few minutes
kable (bayes_complete) %>%
  kableExtra::kable_styling(latex_options = c("HOLD_position","scale_down"))
```

Warning in styling_latex_scale(out, table_info, "down"): Longtable cannot be
resized.

mode	thinning	steps	parameter	PSRF Point est.	PSRF Upper C.I.	Mean	Median	
known_age	1	10000	a	1.003210	1.006108	0.0028582	0.0028441	0.0
known_age	1	10000	b	1.001893	1.003920	0.0480363	0.0480265	0.0
known_age	1	10000	\mathbf{M}	1.001259	1.003216	73.8375371	73.8501409	73.6
known_age	20	100000	a	1.000011	1.000109	0.0028495	0.0028327	0.0
known_age	20	100000	b	1.000005	1.000090	0.0480993	0.0480962	0.0
known_age	e 20	100000	M	1.000000	1.000078	73.8543153	73.8768307	73.9
estimation	1	10000	a	1.007384	1.023678	0.0028312	0.0028026	0.0
estimation	1	10000	b	1.011691	1.038693	0.0510970	0.0508290	0.0
estimation	1	10000	${ m M}$	1.005548	1.017117	71.9242579	71.9119671	72.0
estimation	20	100000	a	1.000247	1.000941	0.0027769	0.0027509	0.0
estimation	20	100000	b	1.000354	1.001342	0.0516959	0.0514776	0.0
estimation	20	100000	M	1.000091	1.000334	71.8362699	71.7754410	71.6

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