

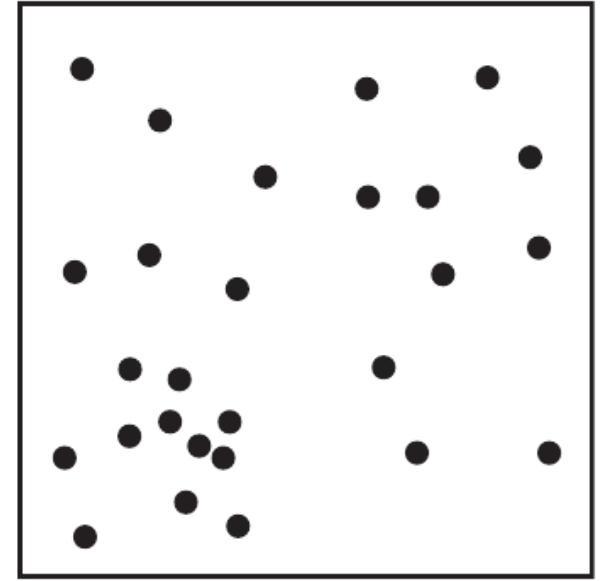
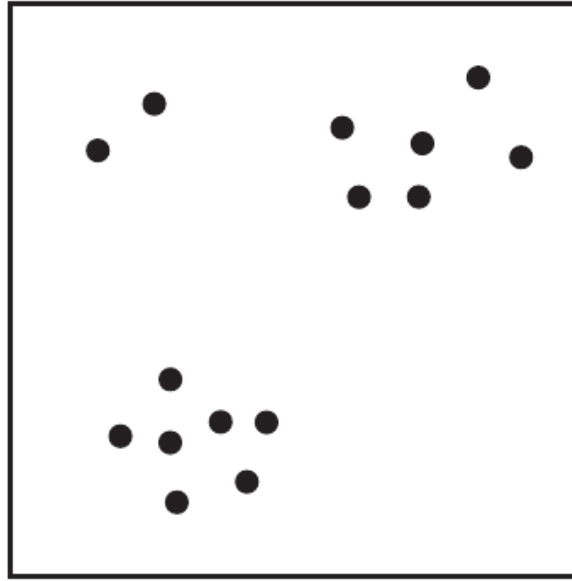
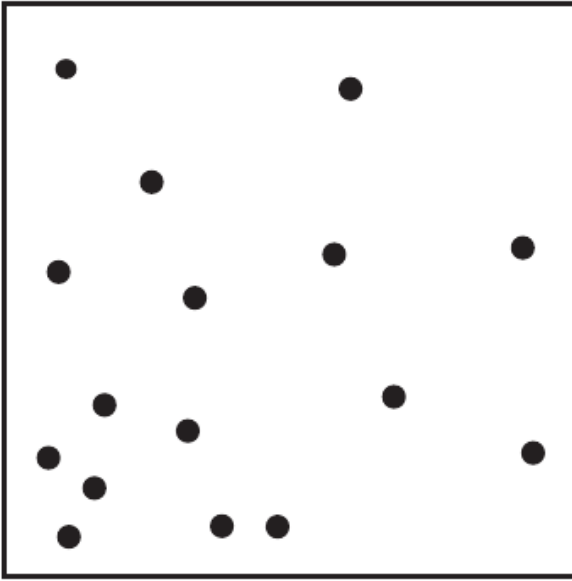
V/S Modellieren in der Landschaftsarchäologie

Freie Universität Berlin
M.Sc. Landschaftsarchäologie

Dr. Daniel Knitter

Point Pattern

What caused these patterns? First- or Second order effects?



Point pattern analysis aims to distinguish between both effects
– helping you to understand the genuine process creating the pattern –

Point Pattern

Simple measures:

- mean center
- standard distance
- intensity of a pattern

$$\bar{\mathbf{s}} = (\mu_x, \mu_y) = \left(\frac{\sum_{i=1}^n x_i}{n}, \frac{\sum_{i=1}^n y_i}{n} \right)$$

$$d = \sqrt{\frac{\sum_{i=1}^n (x_i - \mu_x)^2 + (y_i - \mu_y)^2}{n}}$$

$$\hat{\lambda} = \frac{n}{a} = \frac{\#(S \in A)}{a}$$

Number of
events in the
pattern found
in study region

Area of study region

Point Pattern – first order effects

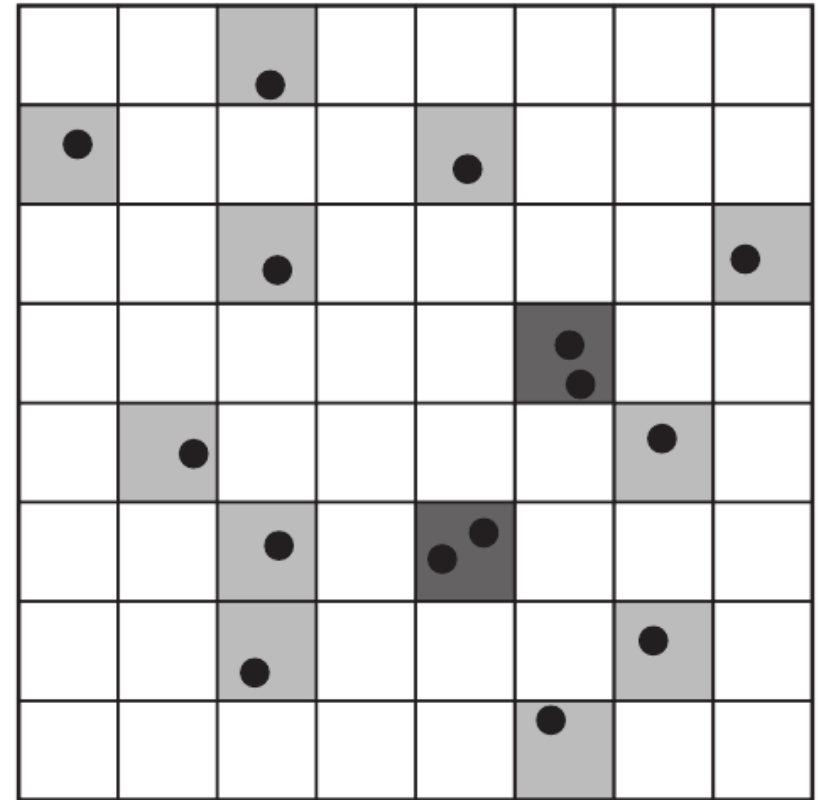
Intensity – more advanced

Independent Random Process (IRP)

= Complete Spatial Randomness (CSR)

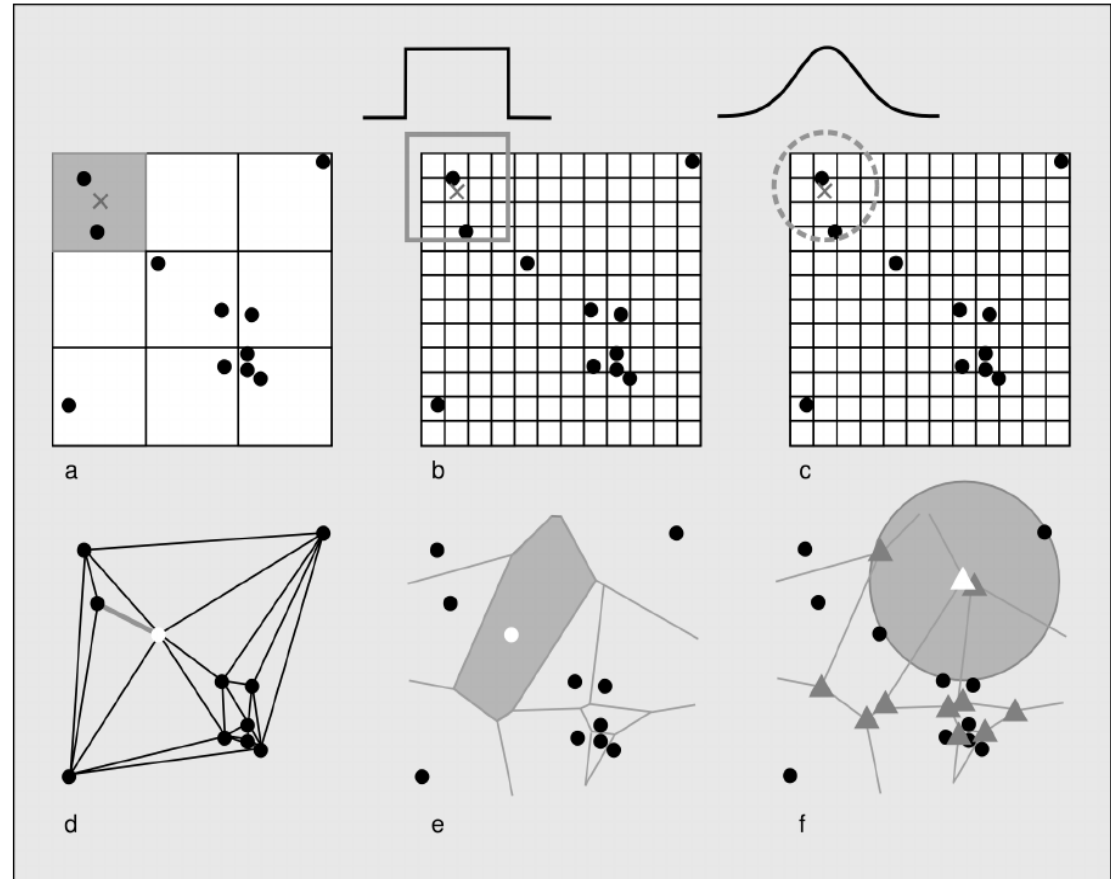
- any event has equal probability to be located a quadrant
- the occurrence of points is independent of the Positioning of other events

$$P(\text{event in a quadrant}) = 1/64$$



Point Pattern – first order effects

Intensity – possibilities to calculate



Point Pattern – first order effects

Intensity – more advanced

Independent Random Process (IRP)

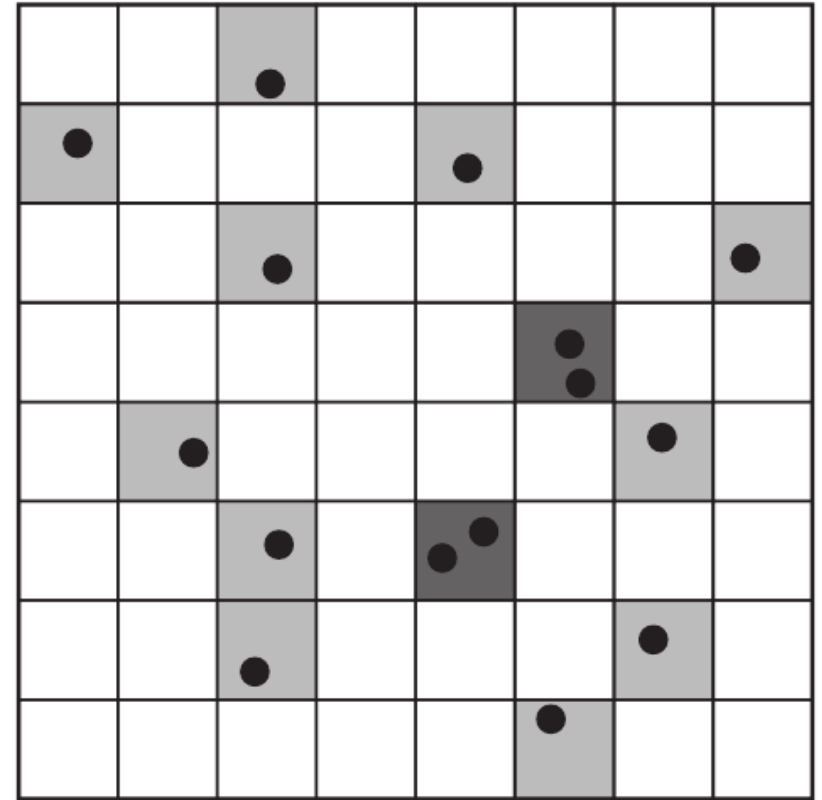
= Complete Spatial Randomness (CSR)

Probability of k events in a quadrant is calculated with Poisson distribution

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

intensity

Base of natural logarithm (2.78...)



Point Pattern – first order effects

Intensity – more advanced
Kernel density estimation (KDE)

Edge correction

kernel location

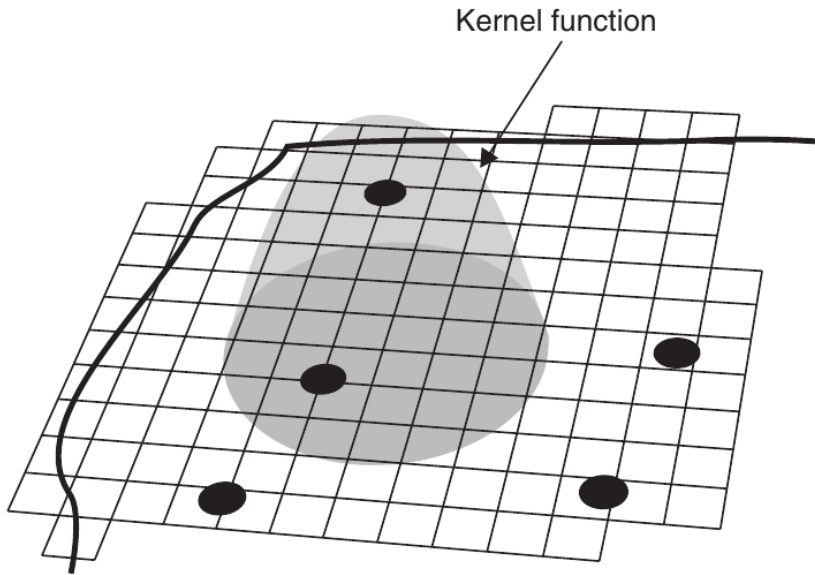
$$\hat{\lambda}_p = e(p) \sum_i k(x_i - p)$$

(Gaussian) Kernel function

mean

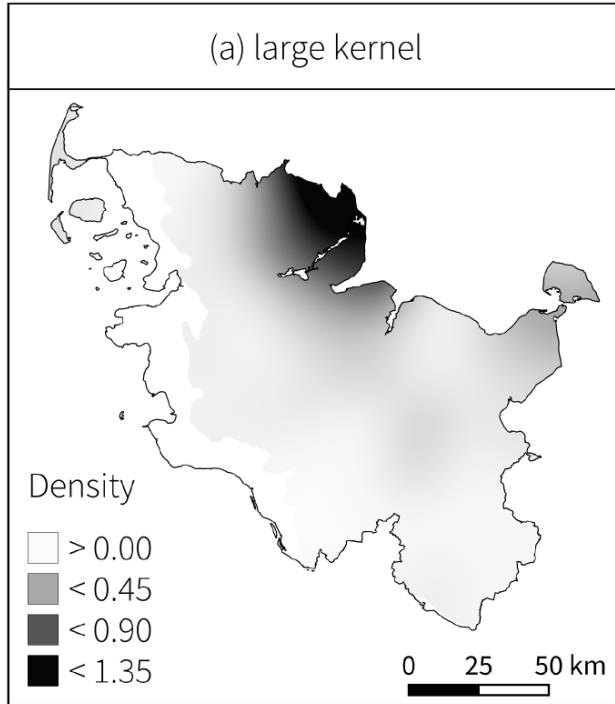
$$\frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Standard deviation



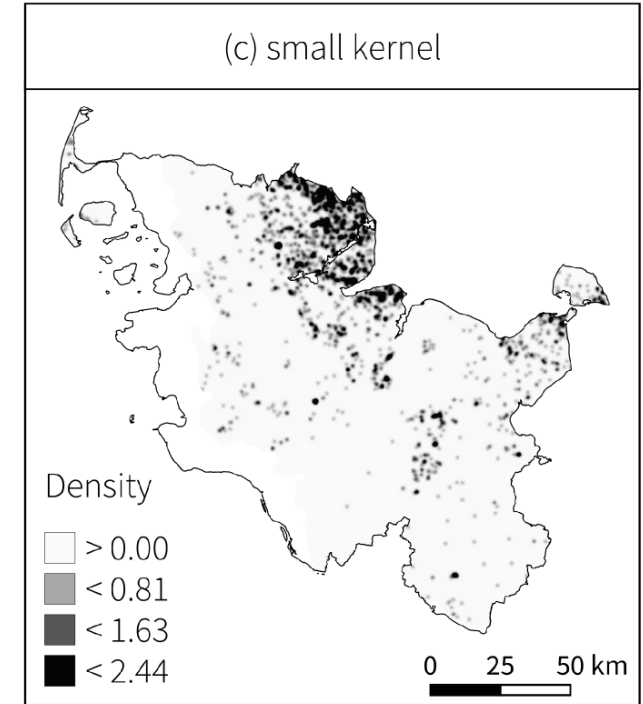
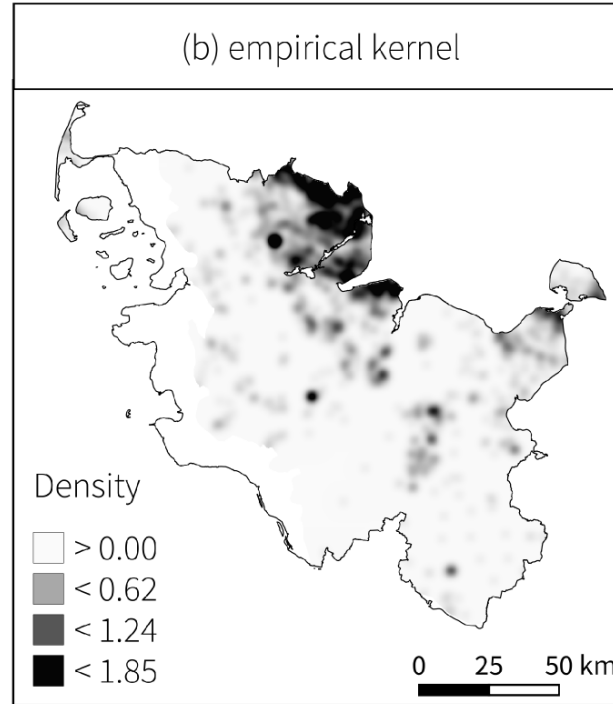
Point Pattern – first order effects

Sigma large



General trend

Sigma small



Local trend

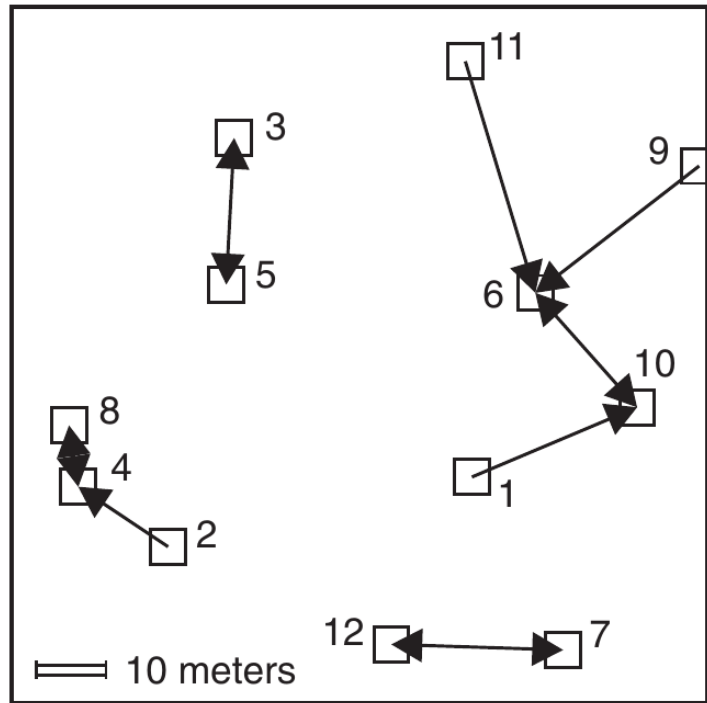
Point Pattern – second order effects

Point patterns described by distance measures

- nearest-neighbor distance → euclidean distance

mean nearest-neighbor distance

$$\bar{d}_{\min} = \frac{\sum_{i=1}^n d_{\min}(\mathbf{s}_i)}{n}$$



Are the points clustered or dispersed?

Use Clark and Evan's R statistic of nearest neighbor distances

$$R = \bar{d}_{\min} / \frac{1}{(2\sqrt{\lambda})}$$

more clustered < 1 > more dispersed

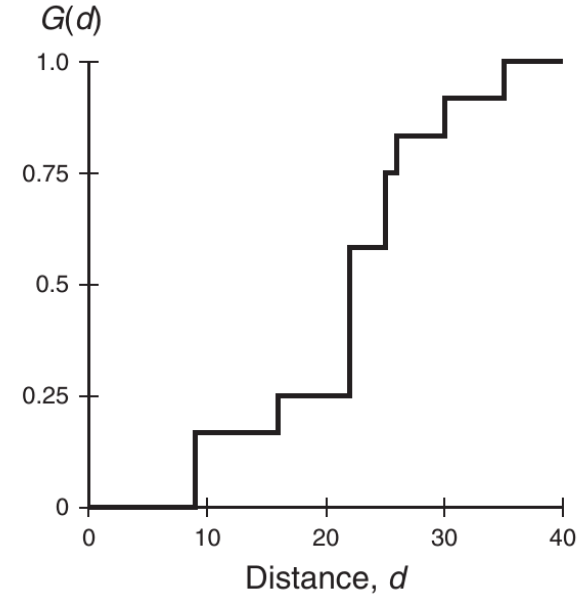
Point Pattern – second order effects

Point patterns described by distance measures

→ cumulative frequency distribution of the nearest-neighbor distances = $G(d)$

$$G(d) = \frac{\#(d_{\min}(\mathbf{s}_i) < d)}{n}$$

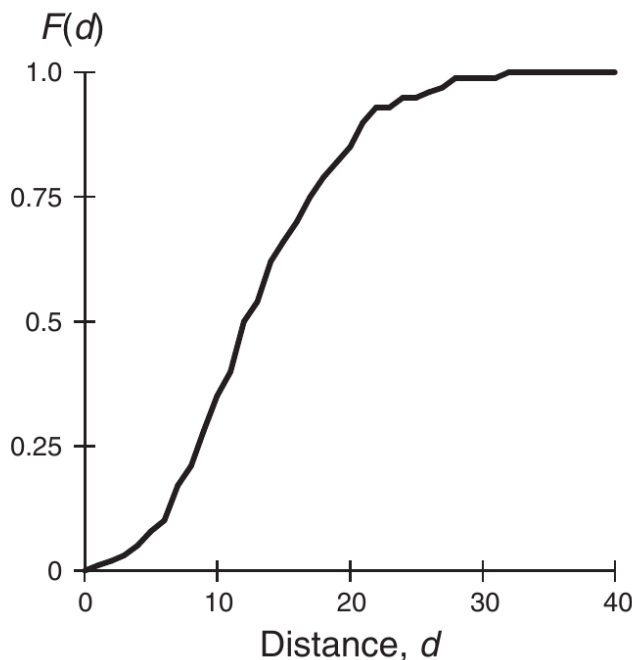
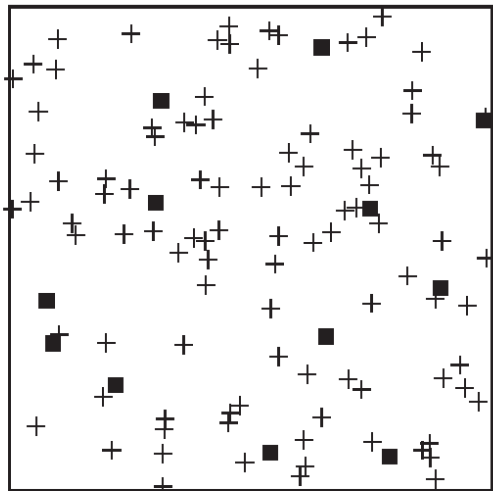
→ $G(d)$ tells us what fraction of all n-n distances is less than d



Point Pattern – second order effects

Point patterns described by distance measures

→ cumulative frequency distribution of the nearest-neighbor distances of *arbitrary events* to known events = $F(d)$



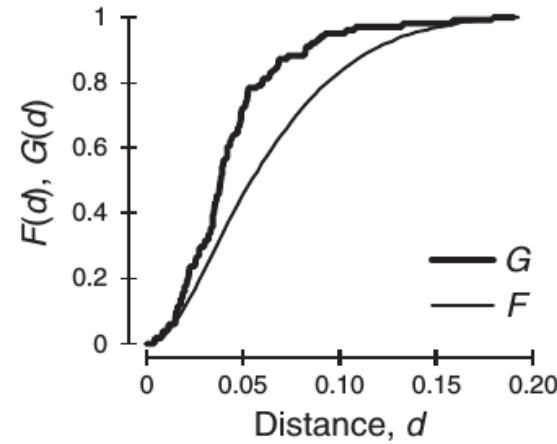
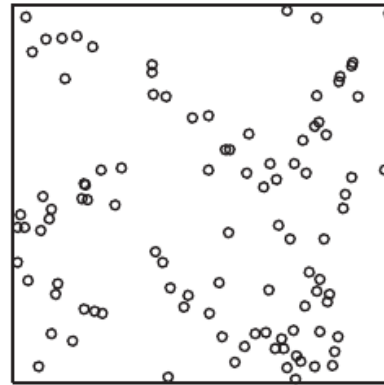
minimum distance from
random point p_i to an event

$$F(d) = \frac{\#[d_{\min}(\mathbf{p}_i, S) < d]}{m}$$

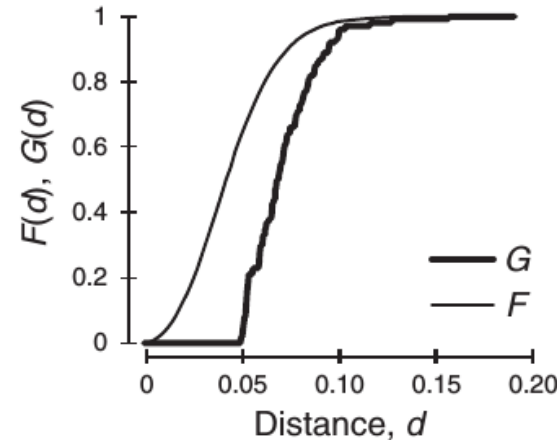
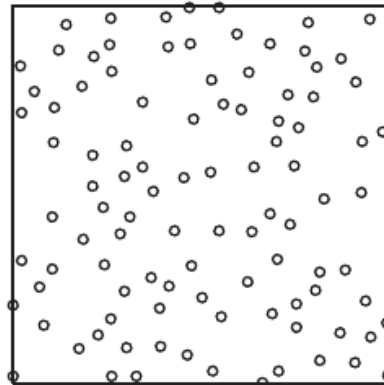
set of randomly selected
locations

Point Pattern – second order effects

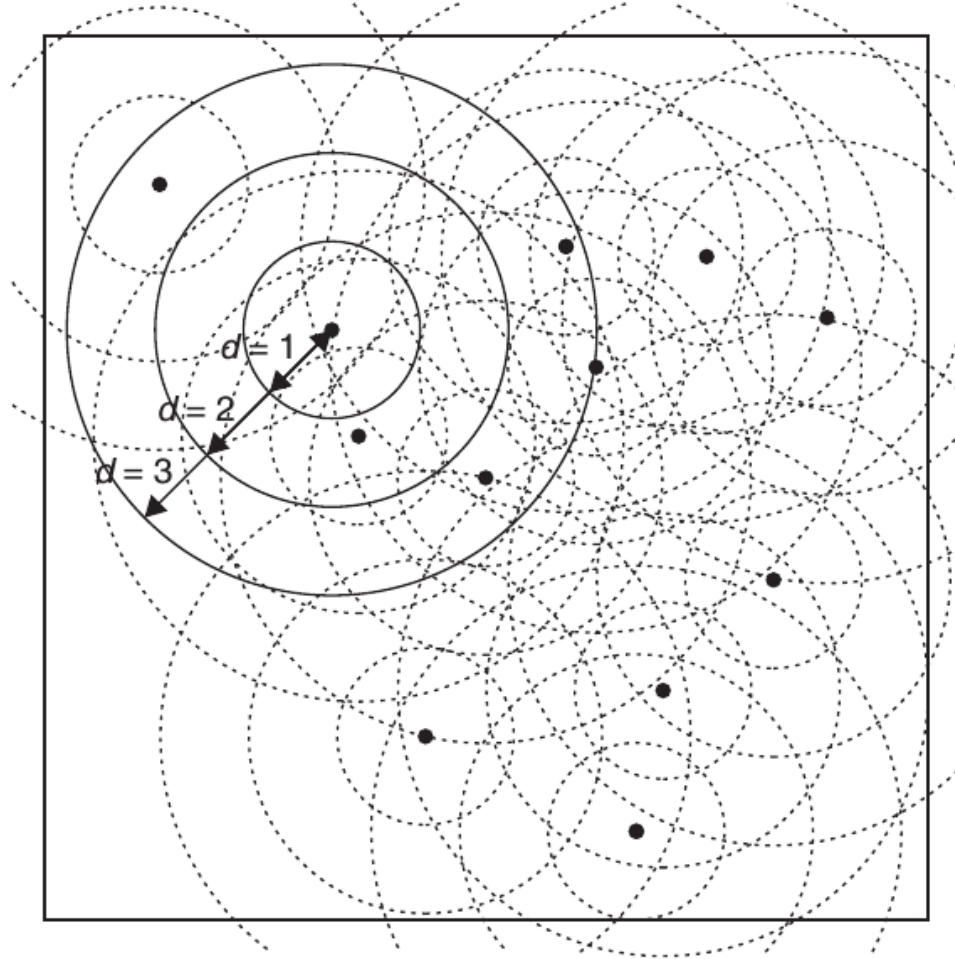
Clustered



Evenly spaced



Point Pattern – second order effects



To get rid of the nearest-neighbor limitation:
use the K Function

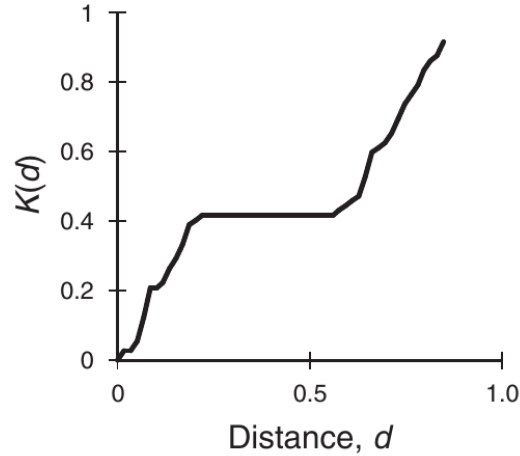
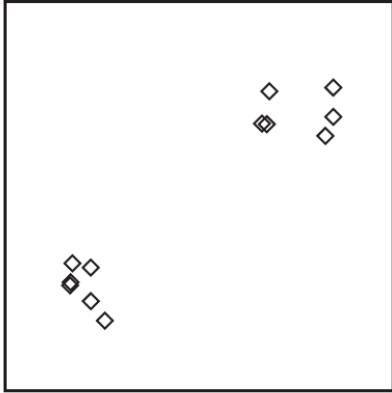
Events in circle radius d centered at s

$$K(d) = \frac{\sum_{i=1}^n \# [S \in C(\mathbf{s}_i, d)]}{n\lambda}$$

Event density in the study area

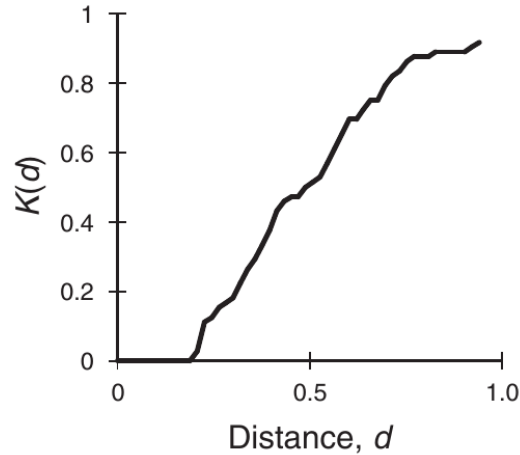
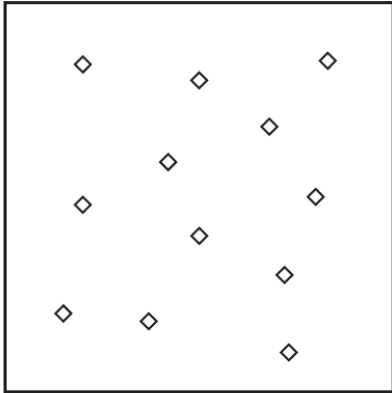
Point Pattern – second order effects

Clustered



To get rid of the nearest-neighbor limitation:
use the K Function

Evenly spaced



Events in circle radius d centered at s

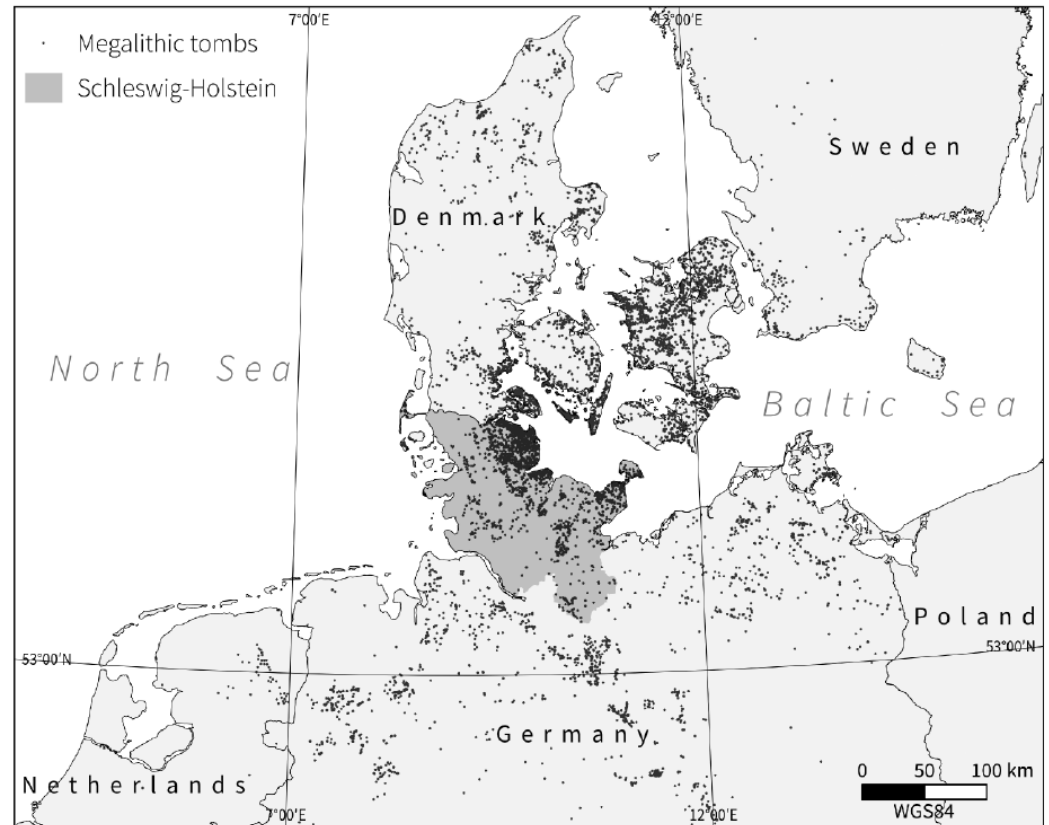
$$K(d) = \frac{\sum_{i=1}^n \#[S \in C(\mathbf{s}_i, d)]}{n\lambda}$$

Event density in the study area

Point Pattern – second order effects

Shows the point pattern clustering/dispersion; does it deviate from CSR?

→ Simulation approach based on Monte Carlo simulations



Point Pattern – second order effects

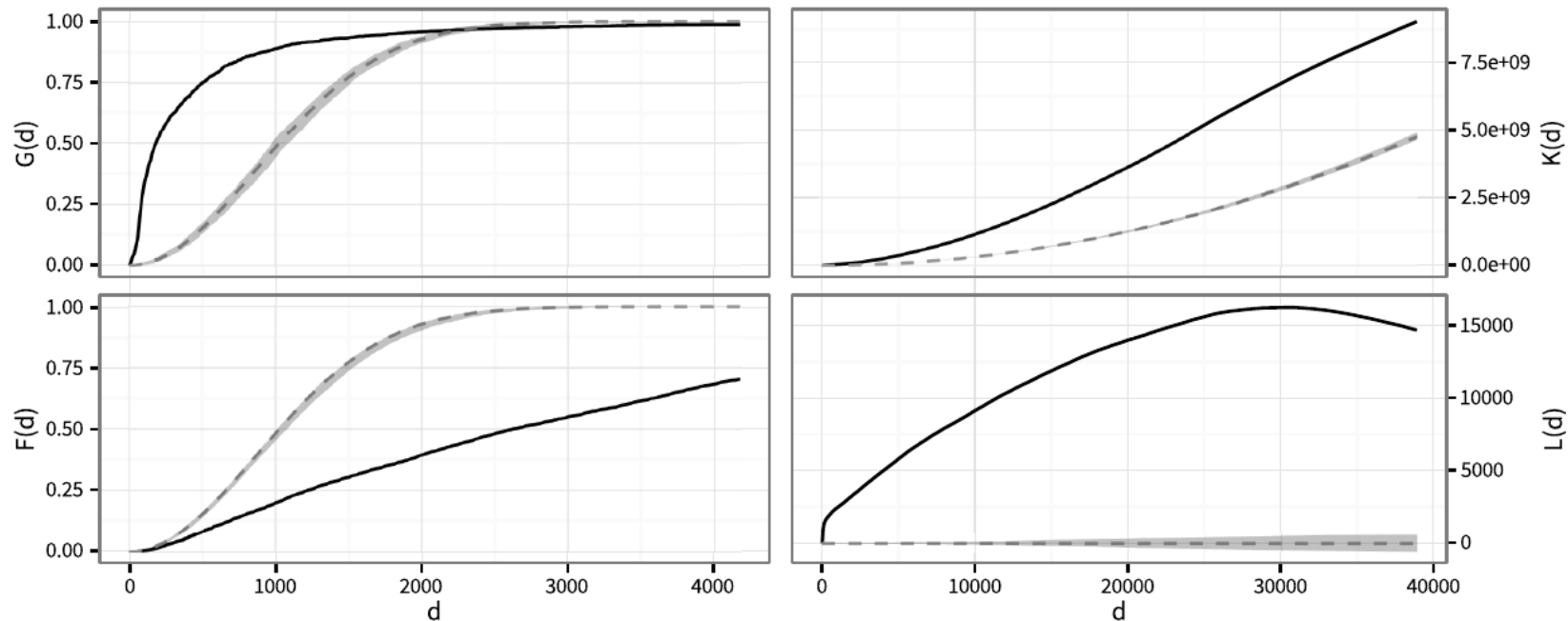
Shows the point pattern clustering/dispersion; does it deviate from CSR?

→ Simulation approach based on
Monte Carlo simulations

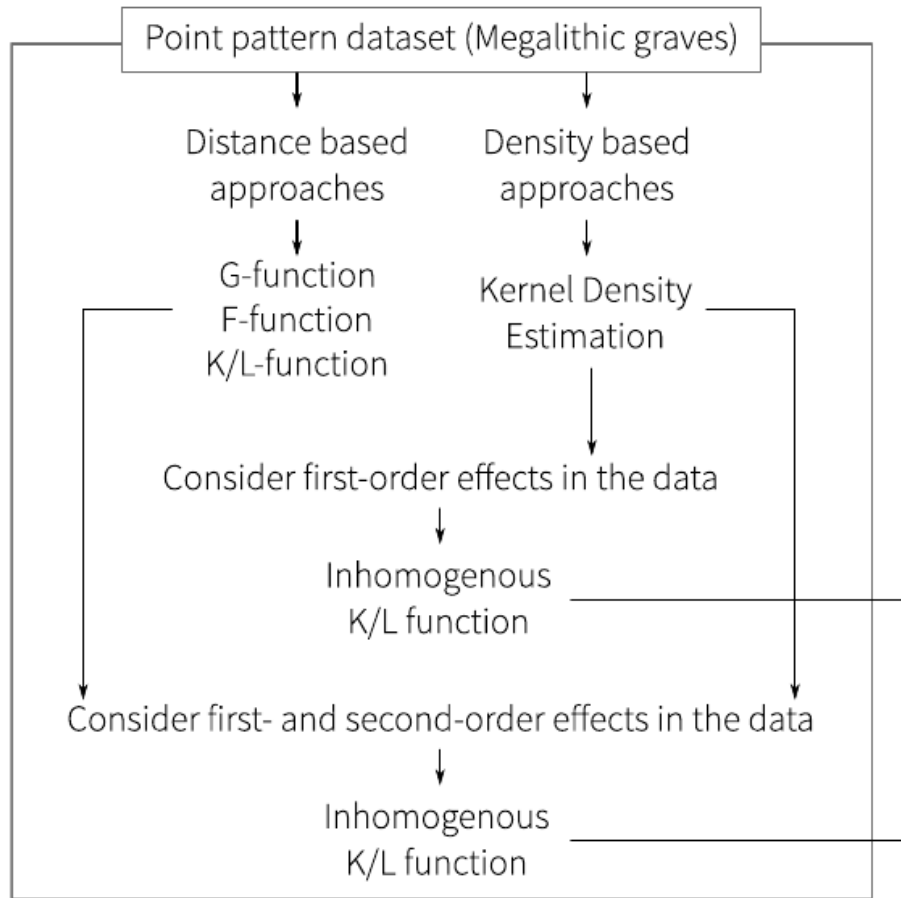
— Empirical distribution — Theoretical distribution

$$L(d) = \sqrt{\frac{K(d)}{\pi}}$$

Just a
square root
transformation
of $K(d)$



Point Pattern – a look ahead



Point Pattern – a look ahead

