Quantum Club Research Team Interview Problem Report

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Question

The superposition principal states that a state function can be expressed as a linear combination of it's normalized eigenstates. When a measurement is performed, the statefunction collapses to one of these eigenstates.

The measurement performed above is a quantum measurement, however, humans can only observe classical information. How would you "see/observe" the result of the above collapse?

The Schrodinger Wave Equation

Every quantum particle must satisfy the following equation -

$$i\hbar\frac{\partial\Psi}{\partial t} = V\Psi - \frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}$$

Here Ψ is a function of the position and time of the particle called the **wave function**.

What does this truly mean? One possible meaning is found in the **Born's Statistical Interpretation**

$$\int_a^b |\Psi(x,t)|^2\,dx = Probability\ of\ finding\ the\ particle\ between\ a\ and\ b\ at\ time\ t$$

What is superposition? Why and where was the concept introduced?

What is a quantum state?

A quantum state is nothing but a mathematical entity that provides a probability distribution for the outcomes of each possible measurement on a system.

When one says that a single quantum system is to be "found" in a certain state, or "has" this state, it simply means that if we measure a physical quantity of that quantum system then we will get a particular value from a set of “allowed” values. Which of the values we get is chosen randomly with an associated probability from the aforementioned probability distribution.

How is the outcome of a measurement chosen? What values do physical properties such as position, momentum etc. have before measurement?

Nobody knows. This is the big mystery of quantum mechanics. There are various “interpretations” being researched to find the answer to this question. The most popular one is the **Copenhagen interpretation**.

Copenhagen Interpretation

> A quantum system before measurement has no definite values for any physical property. It is the act of measurement that forces the system into a particular value from a set of values. Each value has a probability of being the one in which the system is forced into. Immediate subsequent measurements will however not choose from the set of allowed values, but rather, the outcome of the first measurement will be repeated no matter how many subsequent measurements are made.

The aforementioned set of “allowed” values that a measurement outcome is chosen from is called the *eigenspectrum* of the physical observable being measured. The values are each an *eigenvalue* of the observable. Post measurement, the state of the system is referred to as an *eigenstate*.

Superposition

The superposition principal states that a state can be expressed as a linear combination of it's normalized eigenstates.

This is a representation of the statement that the system has no definite values. The linear combination represents the fact that if the Ψ_1 or Ψ_2 are solutions to the wave equation then the linear combination $\Psi = c_1\Psi_1 + c_2\Psi_2$ is also a solution to the wave equation.

Hence If Ψ_1 and Ψ_2 are considered to be eigenstates then any state Ψ can indeed be written as a linear combination of the eigenstates, each of which correspond to a specific measurement outcome. Here c_1 and c_2 can be considered to be the probability amplitudes i.e. the square root of the probabilities of the corresponding outcome. The normalization is necessary to ensure that the probabilities of measurement outcomes add upto 1.

Measurement

According to the copenhagen interpretation, when a system in the state $\Psi_{pre-measurement} = c_1\Psi_1 + c_2\Psi_2 + ... + c_n\Psi_n$ is measured then one of the outcomes is chosen. However, when subsequent measurements are done then only the same outcome is there. This means that the post measurement outcome can be expressed as $\Psi_{post-measurement} = 0\Psi_1 + 0\Psi_2 + ... + 1\Psi_i + ... + 0\Psi_n$ where Ψ_i is the outcome of the first measurement.

Clearly the probability distribution has changed. This is referred to as the *collapse* of the wave function.

It is said that the wave function has collapsed to one of the eigenstates.

Mathematical framework for measurement

Let us use *dirac notation* for the remainder of this document.

A state can be represented as

$$|\Psi_{pre-measurement}\rangle = c_1 |\Psi_1\rangle + c_2 |\Psi_2\rangle + ... + c_n |\Psi_n\rangle$$

Measurements can be described as a collection $\{M_m\}$ of measurement operators that

- satisfy the completeness equation $\sum M_m = I$.
- are hermitian
- are positive

The index m refers to all measurement outcomes that may occur. If state of system is $|\psi\rangle$ before measurement, then after measurement it will be

$$|\Psi_{post-measurement}\rangle = \frac{M_m |\psi\rangle}{\sqrt{p(m)}}$$

where $p(m) = \langle\psi| M_m^\dagger M_m |\psi\rangle$ is the probability of outcome m.

POVM Measurements

A POVM is a set of positive hermitian operators $\{E_m\}$such that $\sum E_m = I$. Probability of outcome m is measured by $\langle\psi| E_m |\psi\rangle$.

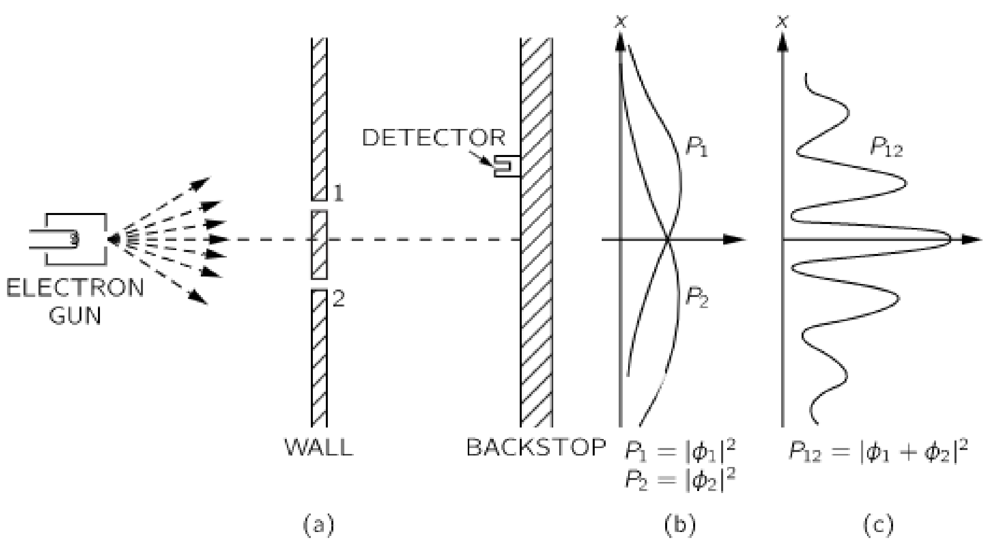
Projective Measurements

A projective measurement is a POVM measurement where E_m is a projective operator. This means that E_m is a POVM operator with following additonal conditions

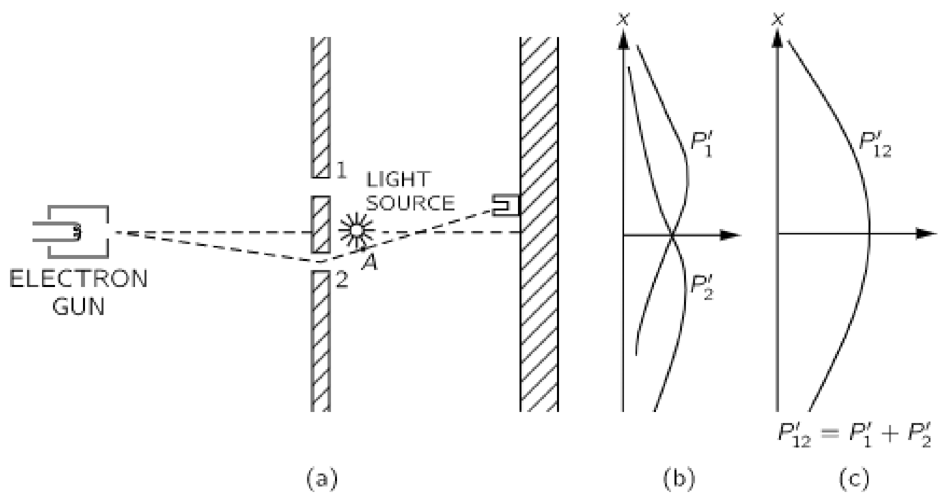
- it is idempotent, i.e. $E_m^2 = E_m$
- are pairwise orthogonal, i.e. $E_i E_j = \delta_{ij}$ where δ_{ij} is the Kronecker delta.

Experimental Example

Lets consider the electron double slit experiment. We make an electron gun which emits electrons. All the electrons which come out of the gun will have (nearly) the same energy. In front of the gun is a wall (just a thin metal plate) with two holes in it. Beyond the wall is another plate which will serve as a "backstop." In front of the backstop we place a movable detector. Before it reaches the detector the electron will not have a definite position, as it is in a linear superposition of states. First state will refer to the condition of the electron coming out of the first hole and the second state will refer to the condition of the electron coming out of the second hole. The detector will force the collapse of the electron's wave function and the electron will fall somewhere on the backstop with probability according to the given graphs. The probability distribution in graph b is the distribution if either of the holes are covered. Graph c is the distribution if neither are covered. We can clearly see the superposition.



Now what happens if we place a light source near the wall? It will disturb the electron thus counting as a measurement. The wave function will collapse before it reaches the backstop and the electron will now behave classically, as if it came out of either one of the holes.



The probability distribution in graph b is the distribution if either of the holes are covered. Graph c is the distribution if neither are covered. We can clearly see that the electron is now behaving classically. **This shows the collapse of the wave function upon measurement.**