

Principle of superposition:

The superposition principle states that, for all linear systems, the net response caused by two or more stimuli is the sum of responses that would have been caused by each stimulus individually.

The classical vector superposition:

The classical vector superposition states that, the resultant of two or more vectors /waves is addition of results as they were acted individually. A common application of classical superposition is finding the total magnitude and direction of quantities such as force, electric field, magnetic field, etc. For example, to calculate the total electric force F_{total} on a charge q_2 produced by other charges q_1 and q_3 , one would sum the forces produced by each individual charge:

$F_{\text{total}} = F_{12} + F_{32}$. The challenge here is that forces are vectors, so vector addition is needed, as shown in Fig. 1.1.

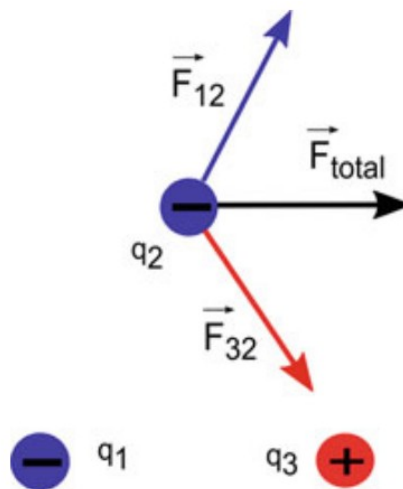


Fig. 1.1 A classical superposition is used to calculate the total electric force on a charge q_2 due to charges q_1 and q_3 .

Very often usage of principle of superposition is in interference of waves. Two pulses on a string which pass through each other will interfere following the principle of superposition as shown Fig. 1.2. Noise-canceling headphones use superposition by creating sound waves with the same magnitude as the incoming sound wave but completely out of phase, thereby canceling the sound wave. This destructive interference is illustrated in the second figure of Fig. 1.2.

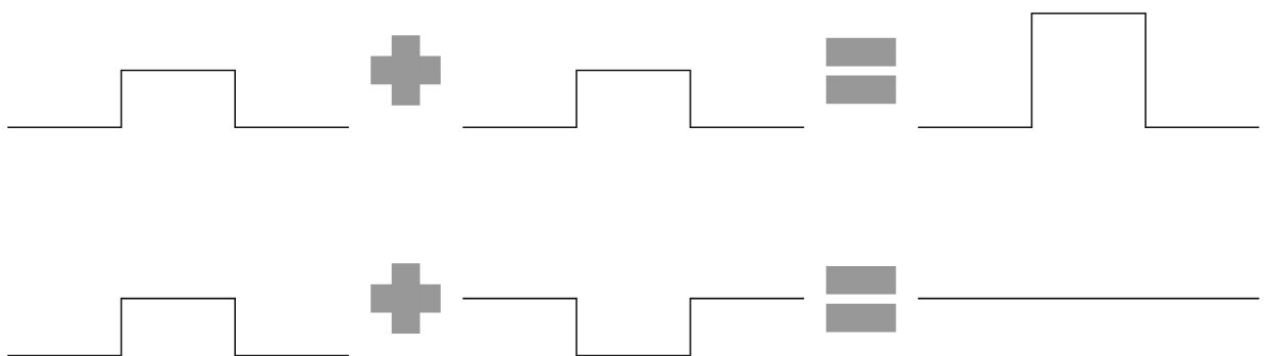


Fig. 1.2 constructive and destructive interference due to the classical superposition principle.

Quantum superposition:

Quantum superposition is a phenomenon associated with quantum systems. Quantum systems include small objects such as nuclei, electrons, elementary particles, and photons, for which the wave-particle duality and other non-classical effects are observed. The superposition principle is the idea that a system is in all possible states at the same time, until it is measured. After measurement it then falls to one of the basis states that form the superposition, thus destroying the original configuration. The superposition principle explains the “quantum weirdness” observed with many experiments. A classic example of this is the double-slit experiment. Here, two slits in a barrier allow for the passage of (for example) electrons. The result of this experiment is an interference pattern not predicted by classical mechanics.

One aspect of quantum superposition can be explained using a coin analogy. A coin has a 50/50 probability of landing as either heads or tails, as shown in Fig. 1.3.

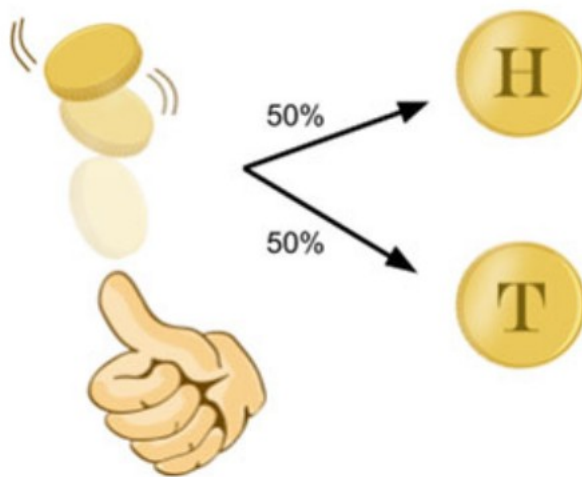


Fig. 1.3 A tossed coin has a 50% chance of landing on heads or tails.

We can say that the coin is in a superposition of both heads and tails. When it lands, it has a definite state, either heads or tails. Generally, the word “state” means any particular way that a system can possibly be described. For example, the coin can be either heads, or tails, or a combination of heads or tails while flipped in the air. While the coin is being flipped it is in a state of superposition. When we observe the coin, we are making a measurement which destroys the superposition.

At any given time, a system can be described as being in a particular state. The state is related to its quantized values. For example, a tossed coin is either in a heads state or a tails state. An electron orbiting a hydrogen atom could be in the ground state or an excited state. A quantum system is special because it can be in a superposition of these definite states, i.e., both heads and tails simultaneously. The outcome of a measurement is to observe some definite state with a given probability.

In Schrödinger’s famous thought experiment, Schrödinger’s cat is placed in a closed box with a single atom that has some probability of emitting deadly radiation at any time. Since radioactive nuclear decay is a spontaneous process, it is impossible to predict for certain when the nucleus decays. Therefore, you do not know whether the cat is alive or dead unless you open and look in the

box. It can be said that the cat is both alive AND dead with some non-zero probability. That is, the cat is in a quantum superposition state until you open the box and measure its state. Upon measurement, the cat is obviously either alive OR dead and the superposition has collapsed to a definite, non-superposition state.

$$|\text{cat}\rangle = \alpha \left| \text{cat alive} \right\rangle + \beta \left| \text{cat dead} \right\rangle$$


Fig. 1.4 The state of Schrödinger's cat expressed as superposition of alive and dead.

The superposition principle states that a statefunction (Ψ) can be expanded as a linear combination of the normalized eigenstates (ϕ_n) of a particular operator that constitute a basis of the space occupied by Ψ . For the discrete case:

$$|\Psi\rangle = \sum_{n=1}^{\infty} b_n |\phi_n\rangle$$

where the coefficients b_n are, in general, functions of time and are given by:

$$b_n = \langle \phi_n | \Psi \rangle$$

which is the projection of Ψ onto the eigenvector ϕ_n . This representation is similar to how a 3-dimensional vector is a superposition of its projections onto the basis vectors i, j, k (which correspond to the eigenstates ϕ_n). Unlike the "normal" vector, however, measurement of an observable will destroy the original statefunction.

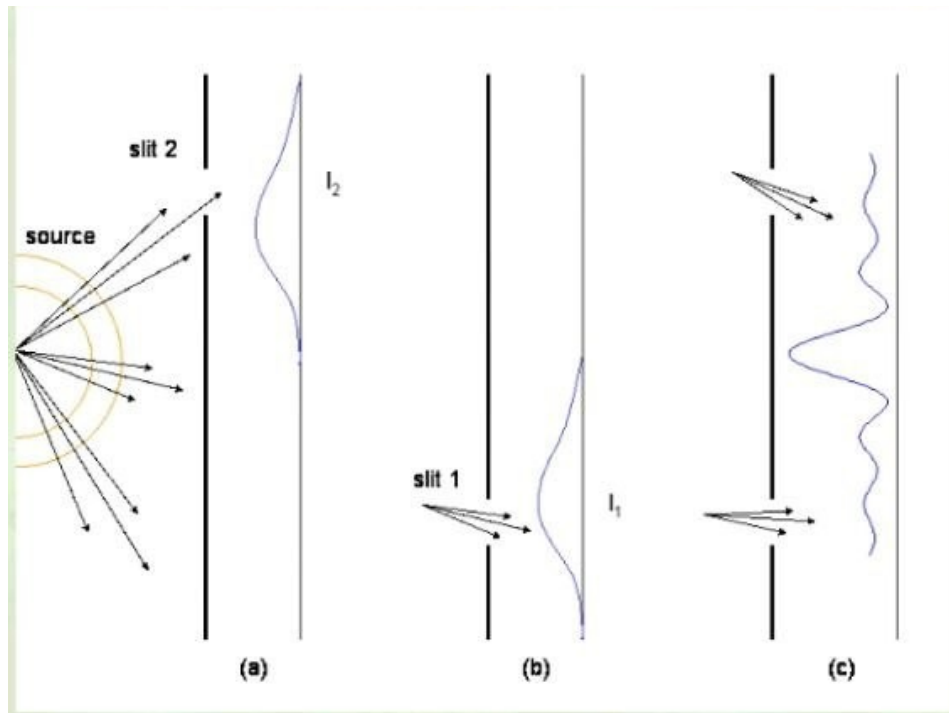
Because the coefficients of expansion represent probabilities of what measurement will obtain, the superposition principle allows calculation of:

1. The expectation (average) value of an observable.
2. The probability that measurement of an observable will give a particular value.

The Double-Slit Experiment :

For this experiment, a beam of light is aimed at a barrier with two vertical slits. The light passes through the slits and the resulting pattern is recorded on a photographic plate. When one slit is covered, the pattern is what would be expected: a single line of light, aligned with whichever slit is open.

Intuitively, one would expect that if both slits are open, the pattern of light will reflect two lines of light aligned with the slits. In fact, what happens is that the photographic plate separates into multiple lines of lightness and darkness in varying degrees.



Photons are fired at the screen. Here, some kind of detector is behind the screen and records the impact (intensity) of the photons. First slit one is closed, then slit two. The results are illustrated in (a) and (b) respectively. The intensity for the electron passing through the first slot is I_1 and the intensity for the electron passing through the second is I_2 . The result with both slits open is shown in (c), where an interference pattern is observed. Classical physics would predict an intensity that is merely the addition of the two individual intensities, or

$$I = I_1 + I_2$$

This does not account for the interference pattern, however.

To deal with this pattern we consider the wavefunction (i.e., the quantum mechanical solution), Y , for which the intensity is given by,

$$I = |Y|^2.$$

So for the first slot only being opened, $I_1 = |Y_1|^2$ and for the second only, $I_2 = |Y_2|^2$.

The resultant wavefunction for both slots being open is,

$$Y = Y_1 + Y_2$$

The superposition principle gives the resultant wavefunction for both slots being opened. Until a measurement is made, the system is "in" all possible states. Here, the possible states are the electron going through slot one (Y_1) and the electron going through slot two (Y_2).

The corresponding intensity is,

$$I = |Y_1 + Y_2|^2$$

When the intensity I is expanded there results,

$$I = |Y_1 + Y_2|^2 = I_1 + I_2 + 2 \operatorname{Re}(Y_1^* Y_2)$$

The $2 \operatorname{Re}(Y_1^* \diamond Y_2)$ term is called the "interference term." This results in the oscillation pattern in (c).

The superposition of states thus explains the quantum interference pattern. When both slits are open, the description of the system is the superposition of the states when each slot is opened individually (i.e., $Y = Y_1 + Y_2$) and it is just this superposition that accounts for the interference. This is true until one tries to determine which path is taken by a photon, after which the state of the system collapses. The classical interpretation of particles bombarding a detector fails to adequately describe the situation.

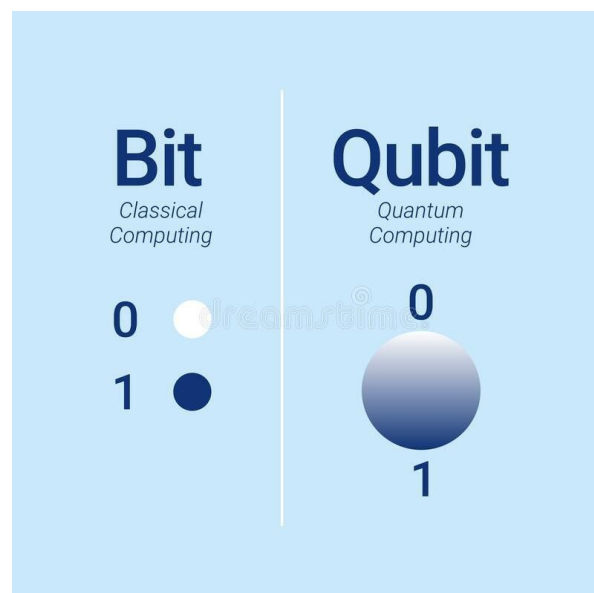
What is being illustrated by this result is that interference is taking place between the waves going through the slits, in what, seemingly, should be two non-crossing trajectories. Each photon not only goes through both slits; it simultaneously takes every possible trajectory en route to the photographic plate.

In order to see how this might possibly occur, other experiments have focused on tracking the paths of individual photons. Surprisingly, the measurement in some way disrupts the photons' trajectories and somehow, the results of the experiment become what would be predicted by classical physics: two bright lines on the photographic plate, each aligned with the slits in the barrier. This has led scientists to conclude that superposition cannot be directly observed; one can only observe the resulting consequence, interference.

In computing, the concept of superposition has important implications for the way information will be processed and stored in the future. For example, today's classical computers process information in bits of one or zero, similar to a light switch being turned on or off. The quantum supercomputers of tomorrow, however, will process information as qubits -- one, zero or a superposition of the two states.

Exponential speedups in computation power:

One of the most studied applications of quantum superposition is the possible speedup in computation. The concept of qubit can be explained by saying that it can be both 1 and 0 at same time, seems like a absurd case to make. This is the reason why even researchers working on Quantum Technology are not able to fully visualize it's power and capabilities. Consider that a quantum particle is going through a maze. A quantum particle have the unique property of being at places at the same time, due to the painciple of superposition. So, when a quantum prarticle encounters various paths to take within the maze, it can decide to take all of those paths at the same time using superposition. This process closely resembles the paradigm of parallel computing. Due to quantum superposition, the quantum particle is able to navigate the maze in exponentially less time than the classical bit.



Conclusion:

The very idea of reality and the way things work becomes totally different and interesting when we deal with Quantum Superposition. As Bohr said, “If you aren’t confused by quantum mechanics, you haven’t understood it”. This Quantum Superposition is always weird by stating that, “a particle is in all possible states”. There are numerous applications of Quantum Superposition, Researchers are working on creating full fledged quantum computers to answer problems which are just too hard for classical computers today. Many fields such as cryptography, machine learning, information security may soon undergo a drastic change due to the advent of the Quantum computing capabilities. World of science and technology is getting weirder and harder to explain day by day.
