HIERARCHICAL MODELING OF HETEROGENEOUS NETWORKS FOR ANIMAL PRODUCTION SYSTEMS

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Joint work with Kessinee Chitakasempornkul, Guilherme J. Rosa and Abigail Jager

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Multiple-Trait Models

$$\begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \\ y_{5i} \\ y_{6i} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{x}_{1i}^{'}\boldsymbol{\beta}_1^{*} \\ \mathbf{x}_{2i}^{'}\boldsymbol{\beta}_2^{*} \\ \mathbf{x}_{3i}^{'}\boldsymbol{\beta}_3^{*} \\ \mathbf{x}_{4i}^{'}\boldsymbol{\beta}_4^{*} \\ \mathbf{x}_{5i}^{'}\boldsymbol{\beta}_5^{*} \\ \mathbf{x}_{6i}^{'}\boldsymbol{\beta}_6^{*} \end{bmatrix}}_{\text{Trt structure}} + \underbrace{\begin{bmatrix} \mathbf{z}_{i}^{'}\mathbf{b}_1^{*} \\ \mathbf{z}_{i}^{'}\mathbf{b}_2^{*} \\ \mathbf{z}_{i}^{'}\mathbf{b}_3^{*} \\ \mathbf{z}_{i}^{'}\mathbf{b}_4^{*} \\ \mathbf{z}_{i}^{'}\mathbf{b}_5^{*} \\ \mathbf{z}_{i}^{'}\mathbf{b}_6^{*} \end{bmatrix}}_{\text{Design structure}} + \begin{bmatrix} e_{1i}^{*} \\ e_{2i}^{*} \\ e_{3i}^{*} \\ e_{4i}^{*} \\ e_{5i}^{*} \\ e_{6i}^{*} \end{bmatrix}$$

$$\mathbf{b}^{*} \sim N\left(\mathbf{0}, \mathbf{B}^{*} \otimes \mathbf{I}_q\right)$$

$$\mathbf{e}_i^{*} = \left\{e_{ji}^{*}\right\}_{j=1}^{6} \sim N\left(\mathbf{0}, \mathbf{R}^{*}\right) \quad i = 1, \ldots, n$$

Sorensen and Gianola 2004

Structural Equation Mixed Models

$$
\begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \\ y_{5i} \\ y_{6i} \end{bmatrix}
=
\underbrace{\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_{21} & 0 & 0 & 0 & 0 & 0 \\
\lambda_{31} & \lambda_{32} & 0 & 0 & 0 & 0 \\
\lambda_{41} & \lambda_{42} & \lambda_{43} & 0 & 0 & 0 \\
\lambda_{51} & \lambda_{52} & \lambda_{53} & \lambda_{54} & 0 & 0 \\
\lambda_{61} & \lambda_{62} & \lambda_{63} & \lambda_{64} & \lambda_{65} & 0
\end{bmatrix}
\begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \\ y_{5i} \\ y_{6i} \end{bmatrix}}_{\text{Structural coefficients}}
+
\underbrace{\begin{bmatrix} \mathbf{x}'_{1i}\boldsymbol{\beta}_1^* \\ \mathbf{x}'_{2i}\boldsymbol{\beta}_2^* \\ \mathbf{x}'_{3i}\boldsymbol{\beta}_3^* \\ \mathbf{x}'_{4i}\boldsymbol{\beta}_4^* \\ \mathbf{x}'_{5i}\boldsymbol{\beta}_5^* \\ \mathbf{x}'_{6i}\boldsymbol{\beta}_6^* \end{bmatrix}}_{\text{Trt structure}}
+
\underbrace{\begin{bmatrix} \mathbf{z}'_i\mathbf{b}_1^* \\ \mathbf{z}'_i\mathbf{b}_2^* \\ \mathbf{z}'_i\mathbf{b}_3^* \\ \mathbf{z}'_i\mathbf{b}_4^* \\ \mathbf{z}'_i\mathbf{b}_5^* \\ \mathbf{z}'_i\mathbf{b}_6^* \end{bmatrix}}_{\text{Design structure}}
+
\begin{bmatrix} e_{1i}^* \\ e_{2i}^* \\ e_{3i}^* \\ e_{4i}^* \\ e_{5i}^* \\ e_{6i}^* \end{bmatrix}
$$

$$\mathbf{b}^* \sim N\left(\mathbf{0}, \mathbf{B}^* \otimes \mathbf{I}_q\right)$$

$$\mathbf{e}_i^* = \left\{e_{ji}^*\right\}_{j=1}^{6} \sim N\left(\mathbf{0}, \mathbf{R}^*\right) \quad i = 1, \ldots, n$$

Sorensen and Gianola 2004

Equivalence: SEM and MTM

$$\mathbf{y}_i = \boldsymbol{\Lambda}\mathbf{y}_i + \mathbf{X}_i\boldsymbol{\beta}^* + \mathbf{Z}_i\mathbf{u}^* + \mathbf{e}_i^* \quad \text{SEM}$$

$$(\mathbf{I}-\boldsymbol{\Lambda})\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta}^* + \mathbf{Z}_i\mathbf{u}^* + \mathbf{e}_i^*$$

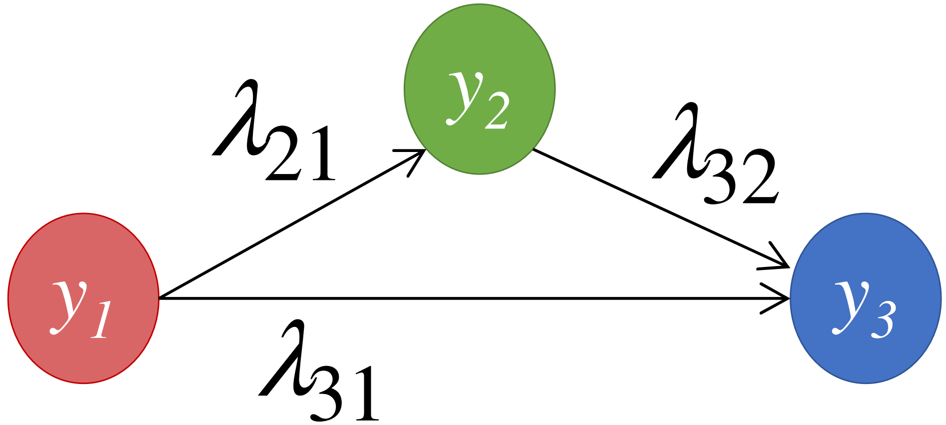
$$\mathbf{y}_i = (\mathbf{I}-\boldsymbol{\Lambda})^{-1}\mathbf{X}_i\boldsymbol{\beta}^* + (\mathbf{I}-\boldsymbol{\Lambda})^{-1}\mathbf{Z}_i\mathbf{u}^* + (\mathbf{I}-\boldsymbol{\Lambda})^{-1}\mathbf{e}_i^*$$

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{u} + \mathbf{e}_i \quad \text{MTM}$$

MTM		SEM
$\mathbf{X}_i\boldsymbol{\beta}$	$=$	$(\mathbf{I}-\boldsymbol{\Lambda})^{-1}\mathbf{X}_i\boldsymbol{\beta}^*$
$\mathbf{Z}_i\mathbf{b}$	$=$	$(\mathbf{I}-\boldsymbol{\Lambda})^{-1}\mathbf{Z}_i\mathbf{b}^*$
$\mathbf{B}$	$=$	$(\mathbf{I}-\boldsymbol{\Lambda})^{-1}\mathbf{B}^*(\mathbf{I}-\boldsymbol{\Lambda})^{-1'}$
$\mathbf{R}$	$=$	$(\mathbf{I}-\boldsymbol{\Lambda})^{-1}\mathbf{R}^*(\mathbf{I}-\boldsymbol{\Lambda})^{-1'}$

Adapted from Rosa et al. 2011

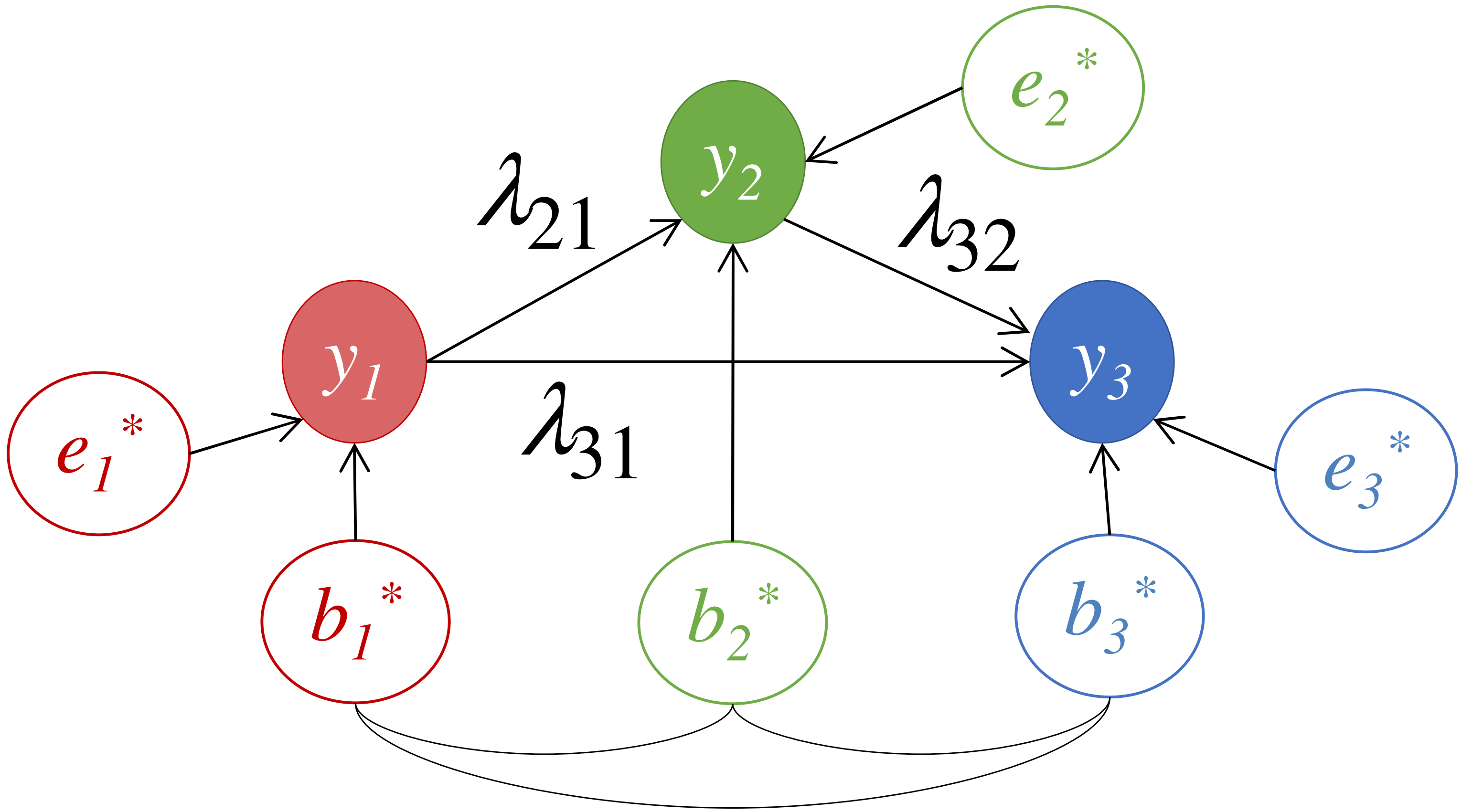
SEM as Graphical Models



$$\begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \\ y_{5i} \\ y_{6i} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 & 0 & 0 & 0 \\ \lambda_{31} & \lambda_{32} & 0 & 0 & 0 & 0 \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & 0 & 0 & 0 \\ \lambda_{51} & \lambda_{52} & \lambda_{53} & \lambda_{54} & 0 & 0 \\ \lambda_{61} & \lambda_{62} & \lambda_{63} & \lambda_{64} & \lambda_{65} & 0 \end{bmatrix} \begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \\ y_{5i} \\ y_{6i} \end{bmatrix} + \begin{bmatrix} \mathbf{x}_{1i}'\boldsymbol{\beta}_1^* \\ \mathbf{x}_{2i}'\boldsymbol{\beta}_2^* \\ \mathbf{x}_{3i}'\boldsymbol{\beta}_3^* \\ \mathbf{x}_{4i}'\boldsymbol{\beta}_4^* \\ \mathbf{x}_{5i}'\boldsymbol{\beta}_5^* \\ \mathbf{x}_{6i}'\boldsymbol{\beta}_6^* \end{bmatrix} + \begin{bmatrix} \mathbf{z}_i'\mathbf{b}_1^* \\ \mathbf{z}_i'\mathbf{b}_2^* \\ \mathbf{z}_i'\mathbf{b}_3^* \\ \mathbf{z}_i'\mathbf{b}_4^* \\ \mathbf{z}_i'\mathbf{b}_5^* \\ \mathbf{z}_i'\mathbf{b}_6^* \end{bmatrix} + \begin{bmatrix} e_{1i}^* \\ e_{2i}^* \\ e_{3i}^* \\ e_{4i}^* \\ e_{5i}^* \\ e_{6i}^* \end{bmatrix}$$

Pearl 2009

SEM as Graphical Models



$$\begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \\ y_{5i} \\ y_{6i} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 & 0 & 0 & 0 \\ \lambda_{31} & \lambda_{32} & 0 & 0 & 0 & 0 \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & 0 & 0 & 0 \\ \lambda_{51} & \lambda_{52} & \lambda_{53} & \lambda_{54} & 0 & 0 \\ \lambda_{61} & \lambda_{62} & \lambda_{63} & \lambda_{64} & \lambda_{65} & 0 \end{bmatrix} \begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \\ y_{5i} \\ y_{6i} \end{bmatrix} + \begin{bmatrix} \mathbf{x}_{1i}'\boldsymbol{\beta}_1^* \\ \mathbf{x}_{2i}'\boldsymbol{\beta}_2^* \\ \mathbf{x}_{3i}'\boldsymbol{\beta}_3^* \\ \mathbf{x}_{4i}'\boldsymbol{\beta}_4^* \\ \mathbf{x}_{5i}'\boldsymbol{\beta}_5^* \\ \mathbf{x}_{6i}'\boldsymbol{\beta}_6^* \end{bmatrix} + \begin{bmatrix} \mathbf{z}_i'\mathbf{b}_1^* \\ \mathbf{z}_i'\mathbf{b}_2^* \\ \mathbf{z}_i'\mathbf{b}_3^* \\ \mathbf{z}_i'\mathbf{b}_4^* \\ \mathbf{z}_i'\mathbf{b}_5^* \\ \mathbf{z}_i'\mathbf{b}_6^* \end{bmatrix} + \begin{bmatrix} e_{1i}^* \\ e_{2i}^* \\ e_{3i}^* \\ e_{4i}^* \\ e_{5i}^* \\ e_{6i}^* \end{bmatrix}$$

Pearl 2009

Structural Equation Models: Assumptions

$$\lambda_{jj',i} = \lambda_{jj'} \text{ for } \forall i$$

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Investigating causal biological relationships between reproductive performance traits in high-performing gilts and sows[1]

Kessinee Chitakasempornkul, Mariana B Meneget, Guilherme J M Rosa, Fernando B Lopes, Abigail Jager, Márcio A D Gonçalves, Steve S Dritz, Mike D Tokach, Robert D Goodband, Nora M Bello

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Objectives

To develop methodological extensions to hierarchical SEM that explicitly accommodate heterogeneous structural coefficients

Infer on heterogeneous functional networks in complex systems

- Simulations
- Data application in a designed experiment in swine reproduction

Structural Equation Models: Extensions

$$\lambda_{jj',i} = \underbrace{\boxed{\boldsymbol{x}'_{jj',i}\boldsymbol{\delta}_{jj'}}}_{\text{Trt structure}} + \underbrace{\boxed{\boldsymbol{z}'_{jj',i}\boldsymbol{v}_{jj'}}}_{\text{Design structure}}$$

$$\boldsymbol{v}_{jj'} \sim N(\boldsymbol{0}, \sigma^2_{jj'}\boldsymbol{I}_q)$$

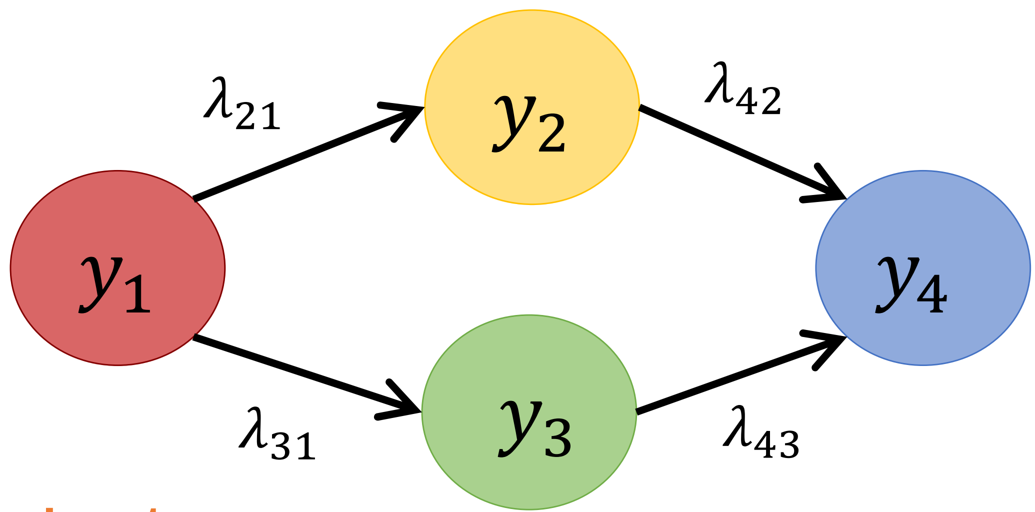
$j = 1, \ldots, J$ outcomes

$j' < j$

$i = 1, \ldots, n$ subjects

Simulation Study:
Data Generation

n = 2,000
q = 100
j = 1,2,3,4 outcomes, $j' < j$
i = 1, … , n subjects



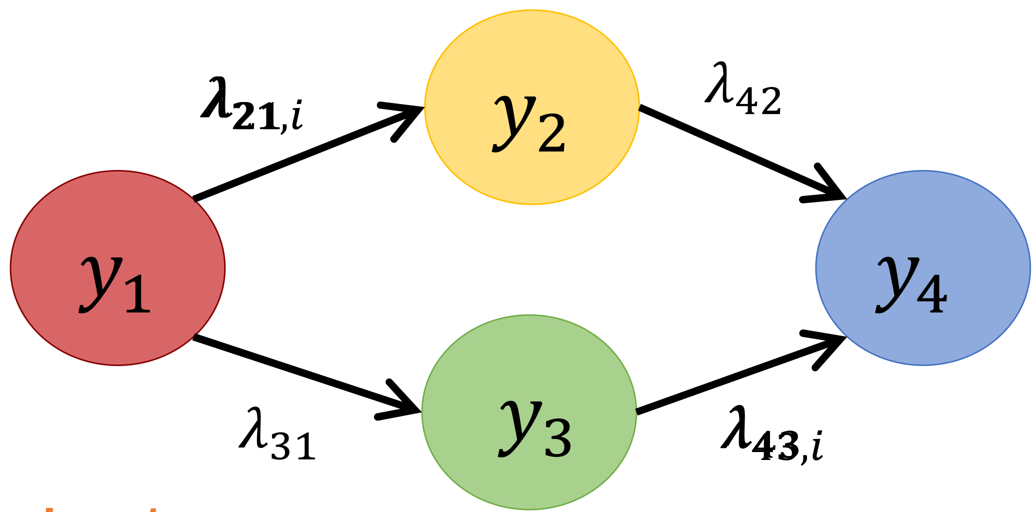
Scenario 1

$\lambda_{jj',i} = \lambda_{jj'}$ for $\forall i$

10 data replicates per scenario

Simulation Study:
Data Generation

n = 2,000
q = 100
j = 1,2,3,4 outcomes, $j' < j$
i = 1, … , n subjects



Scenario 1

$\lambda_{jj',i} = \lambda_{jj'}$ for $\forall i$

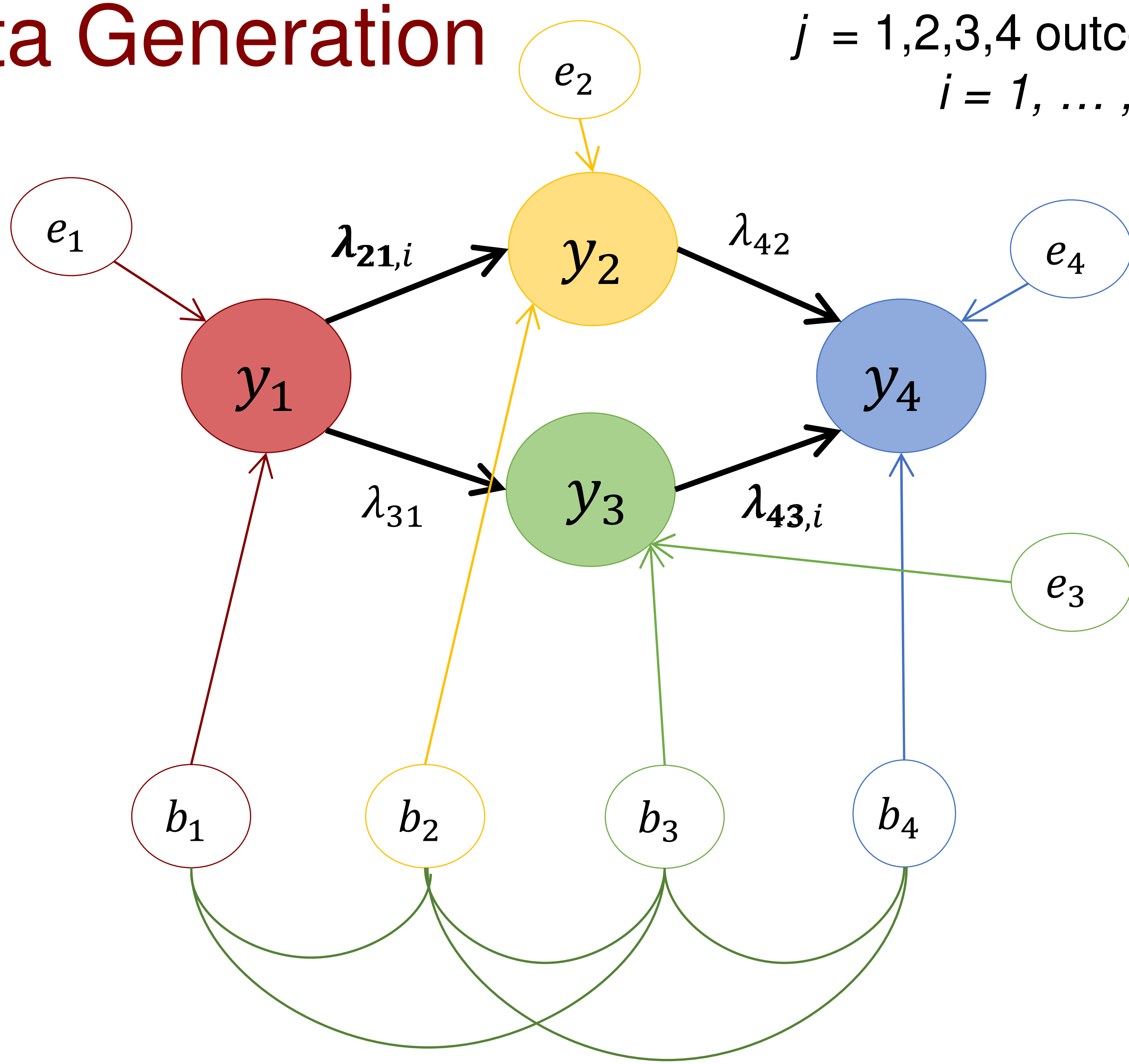
Scenario 2

$$\lambda_{21,i} = \boldsymbol{x}'_{21,i}\boldsymbol{\delta}_{21} + \boldsymbol{z}'_{21,i}\boldsymbol{v}_{21}$$
$$\boldsymbol{v}_{21} \sim N\left(\mathbf{0}, \sigma^2_{\boldsymbol{v}_{21}}\boldsymbol{I}_q\right)$$
$$\lambda_{43,i} = \boldsymbol{x}'_{43,i}\boldsymbol{\delta}_{43} + \boldsymbol{z}'_{43,i}\boldsymbol{v}_{43}$$
$$\boldsymbol{v}_{43} \sim N\left(\mathbf{0}, \sigma^2_{\boldsymbol{v}_{43}}\boldsymbol{I}_q\right)$$

10 data replicates per scenario

Simulation Study:
Data Generation

n = 2,000
q = 100
j = 1,2,3,4 outcomes, $j' < j$
i = 1, … , n subjects



10 data replicates per scenario

Simulation Study: Alternative Models

$$\boldsymbol{y} = \boldsymbol{\Lambda}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{Z}\boldsymbol{b} + \boldsymbol{e}$$

Model 0: Multiple-trait model ↔ Fully recursive SEM

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 & 0 \\ \lambda_{31} & \lambda_{32} & 0 & 0 \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & 0 \end{bmatrix}$$

Model 1: SEM with homogeneous $\lambda_{jj'}$

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 & 0 \\ \lambda_{31} & 0 & 0 & 0 \\ 0 & \lambda_{42} & \lambda_{43} & 0 \end{bmatrix}$$

Model 2*: SEM with heterogeneous $\lambda_{21,i}$ and $\lambda_{43,i}$

$$\lambda_{21,i} = \boldsymbol{x}'_{21,i}\boldsymbol{\delta}_{21} + \boldsymbol{z}'_{21,i}\boldsymbol{v}_{21} \qquad \boldsymbol{v}_{21} \sim N(\boldsymbol{0}, \sigma^2_{v_{21}}\boldsymbol{I}_q)$$

$$\lambda_{43,i} = \boldsymbol{x}'_{43,i}\boldsymbol{\delta}_{43} + \boldsymbol{z}'_{43,i}\boldsymbol{v}_{43} \qquad \boldsymbol{v}_{43} \sim N(\boldsymbol{0}, \sigma^2_{v_{43}}\boldsymbol{I}_q)$$

Model 2: SEM with all $\lambda_{jj',i}$ heterogeneous

$$\lambda_{jj',i} = \boldsymbol{x}'_{jj',i}\boldsymbol{\delta}_{jj'} + \boldsymbol{z}'_{jj',i}\boldsymbol{v}_{jj'} \qquad \boldsymbol{v}_{jj'} \sim N(\boldsymbol{0}, \sigma^2_{v_{jj'}}\boldsymbol{I}_q)$$

Hierarchical Bayesian Framework

$$\underbrace{p(\boldsymbol{\beta},\boldsymbol{b},\boldsymbol{\lambda}_{jj'},\boldsymbol{R},\boldsymbol{B}|\boldsymbol{y})}_{\text{Posterior}} \propto \underbrace{p(\boldsymbol{y}|\boldsymbol{\beta},\boldsymbol{b},\boldsymbol{\lambda}_{jj'},\boldsymbol{R})}_{\text{Data Likelihood}} \underbrace{p(\boldsymbol{\beta})p(\boldsymbol{\lambda}_{jj'}|\boldsymbol{\delta}_{jj'},\boldsymbol{v}_{jj'})p(\boldsymbol{\delta}_{jj'})}_{\text{Prior Specifications}}$$

$$p\left(\boldsymbol{v}_{jj'}|\sigma^2_{\boldsymbol{v}_{jj'}}\right)p(\sigma^2_{\boldsymbol{v}_{jj'}})\,p(\boldsymbol{b}|\boldsymbol{B})p(\boldsymbol{B})p(\boldsymbol{R})$$

Priors

$$p(\boldsymbol{\beta}) \propto 1 \qquad p\left(\sigma^2_{e_j}\right) = \chi^{-2}(v_{e_j}, s^2_{e_j})$$

$$p(\boldsymbol{b}|\boldsymbol{B}) = N(\boldsymbol{0}, \boldsymbol{B} \otimes \boldsymbol{I}_q) \qquad p(\boldsymbol{B}) = IW(\boldsymbol{v}_{\boldsymbol{B}_0}, \boldsymbol{V}_{\boldsymbol{B}_0})$$

Models with homogeneous $\lambda_{jj'}$	**Models with heterogeneous $\lambda_{jj',i}$**
$p(\boldsymbol{\lambda}_{jj'}) \propto 1$	$p(\boldsymbol{\delta}_{jj'}) \propto 1$ $p\left(\boldsymbol{v}_{jj'}\|\sigma^2_{\boldsymbol{v}_{jj'}}\right) = N(\boldsymbol{0}, \sigma^2_{\boldsymbol{v}_{jj'}}\boldsymbol{I}_q)$ $p\left(\sigma^2_{\boldsymbol{v}_{jj'}}\right) = \chi^{-2}(v_{\boldsymbol{v}_{jj'}}, s^2_{\boldsymbol{v}_{jj'}})$

- Markov Chain Monte Carlo
- 270K iterations after 60K burn-in with thinning 2
- Convergence diagnostics

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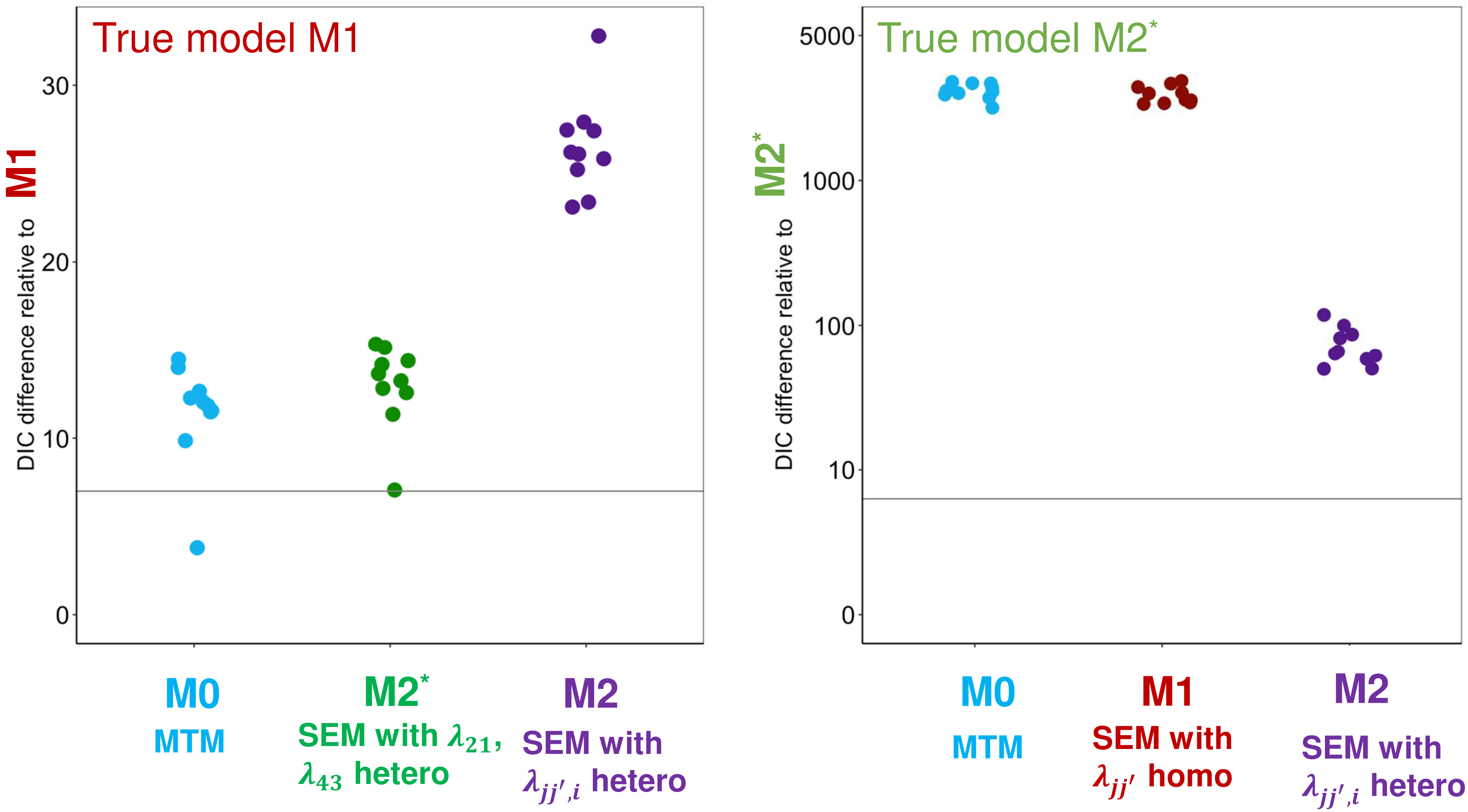
Simulation Study:
Model Fit

i = 1, …, n subjects
j = 1,2,3,4 outcomes
j' < j

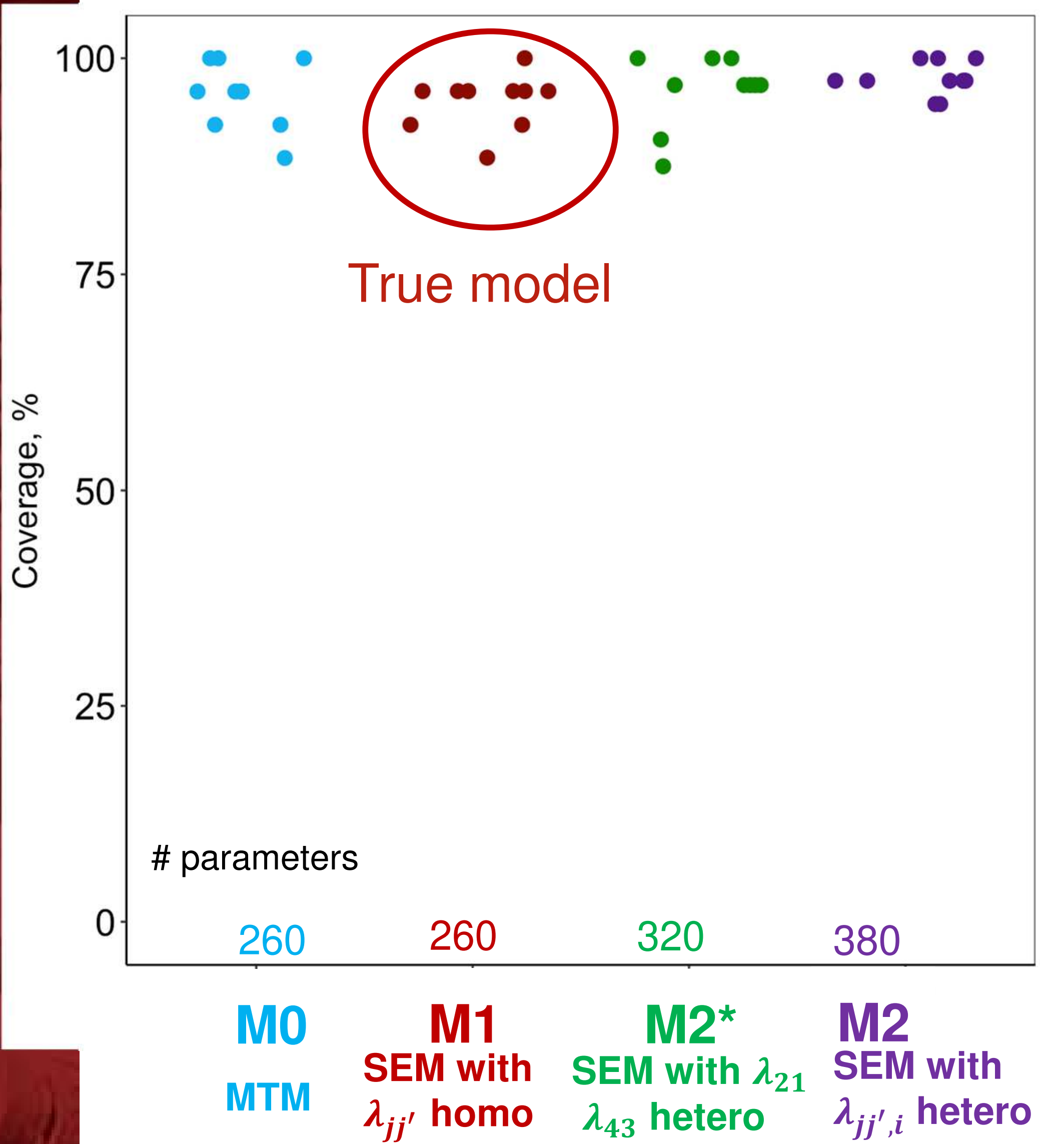
Scenario 1: Homogeneous $\lambda_{jj'}$

Scenario 2: Heterogeneous $\lambda_{21,i}$, $\lambda_{43,i}$

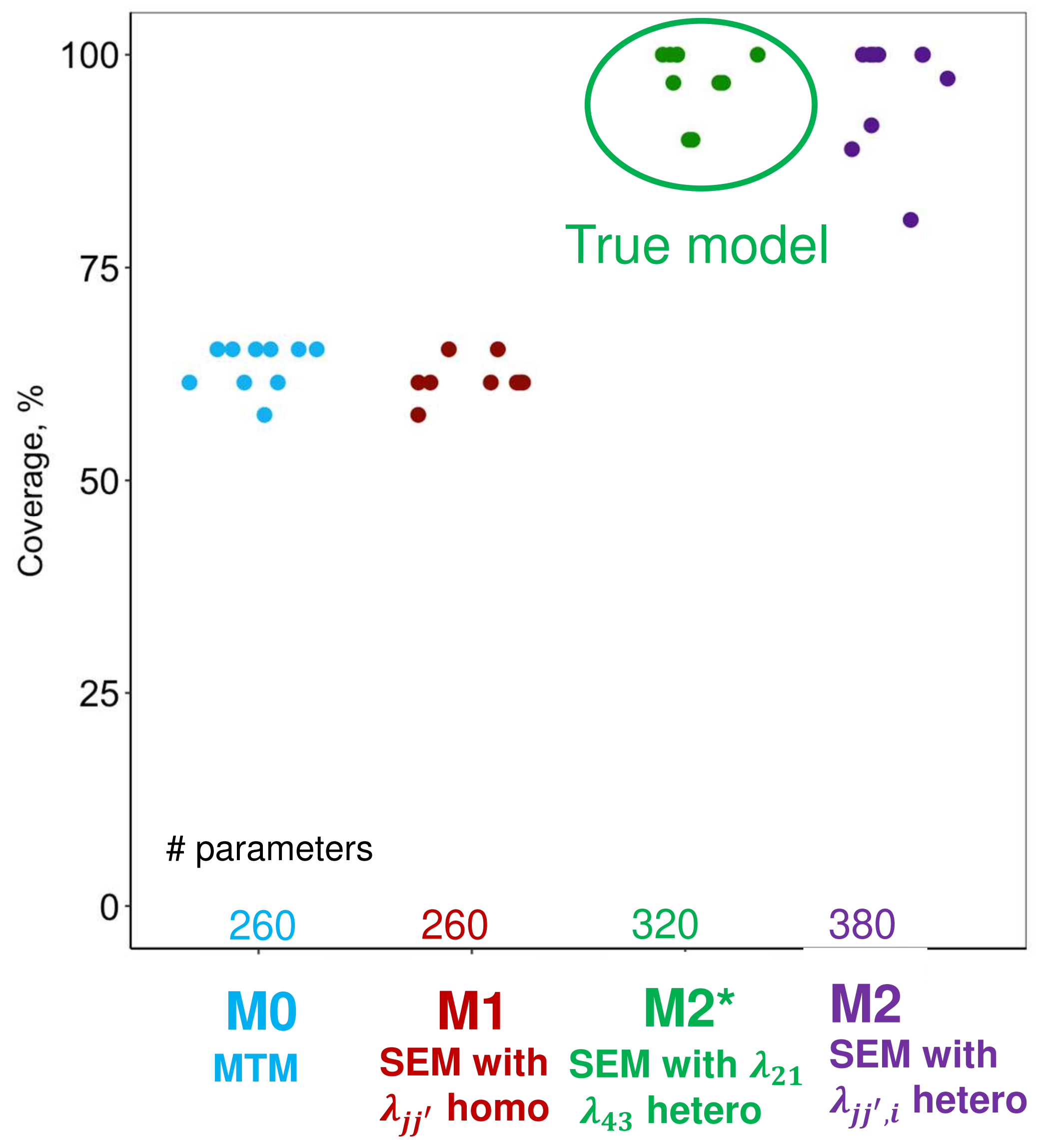


Simulation Study:
Overall Estimation Accuracy

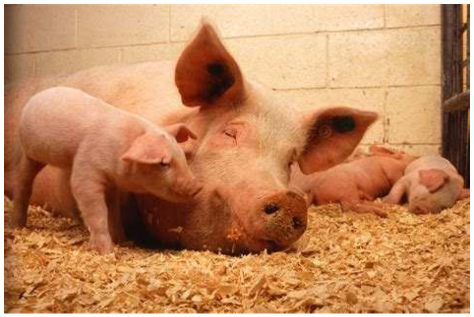
Scenario 1: Homogeneous $\lambda_{jj'}$



Scenario 2: Heterogeneous $\lambda_{21,i}$, $\lambda_{43,i}$



Revisiting…



Investigating causal biological relationships between reproductive performance traits in high-performing gilts and sows[1]

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Data

Reproductive outcomes:

1. Female body weight gain during late gestation (GAIN), kg
2. Total number born in a litter(TB)
3. Number born alive in a litter (BA)
4. Born alive average birth weight (BABW), kg
5. Wean-estrous interval (WEI), day
6. Subsequent total born (SuTB)

Data generation process:

- Complete records for 440 gilts and 200 sows
- Trt structure: 2 parities x 4 diets
- Design structure: BW blocks 222 and 97

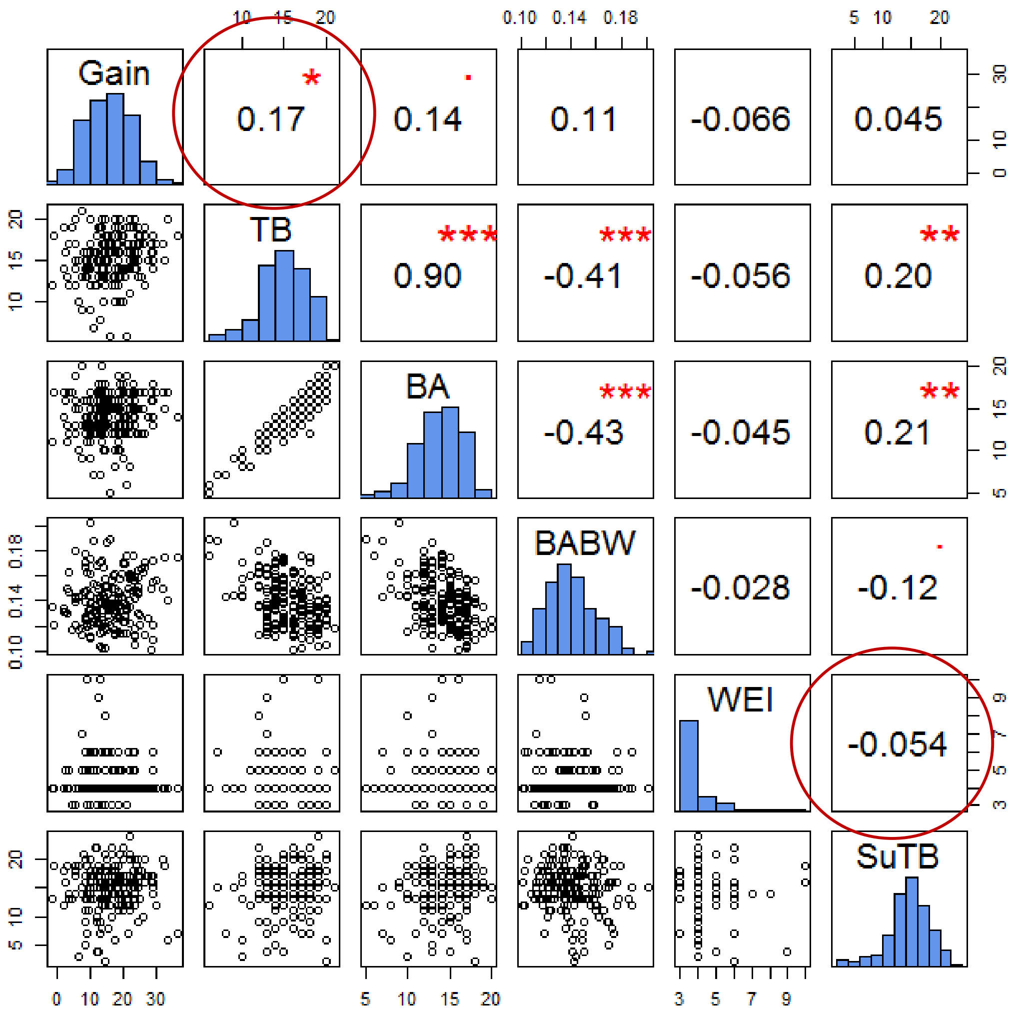
Gonçalves et. al 2016 JAS

Data Descriptives

Sows

Total complete records = 200

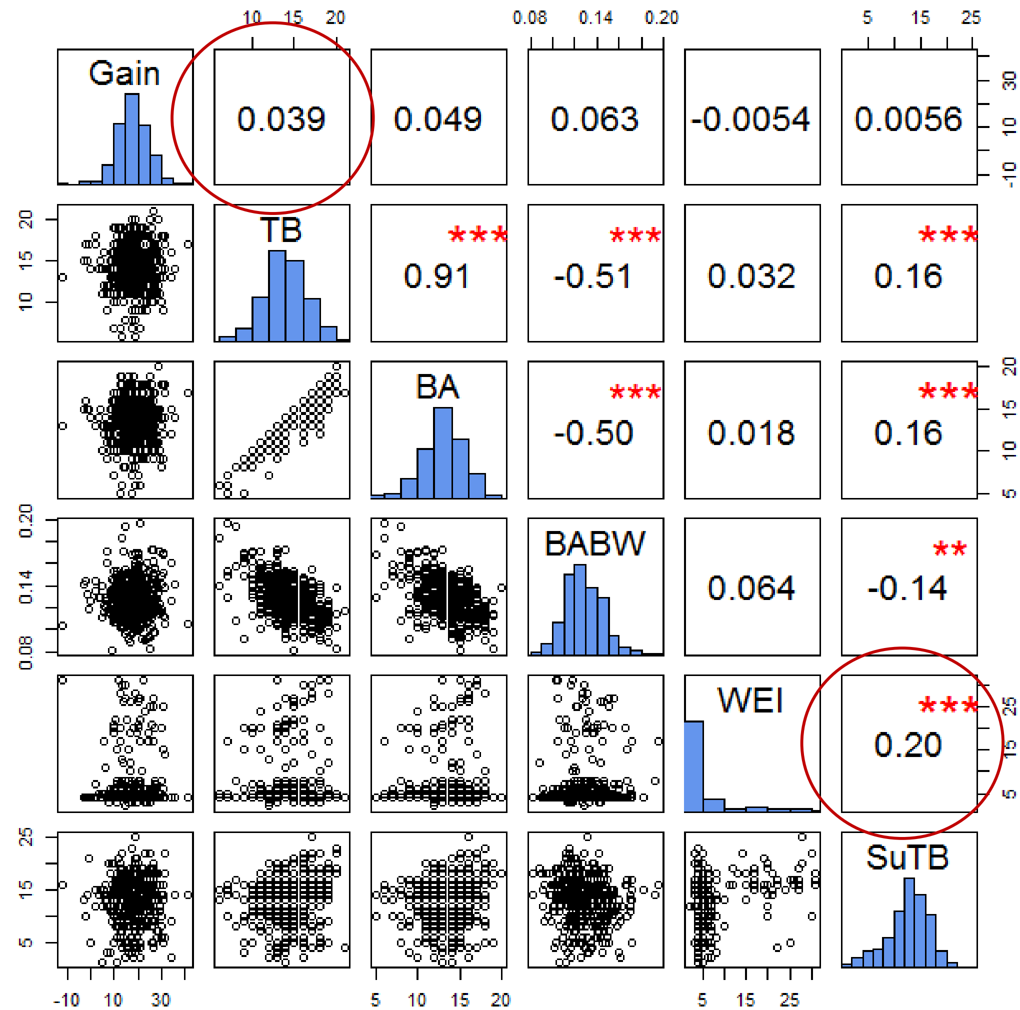
Total BW blocks = 97



Gilts

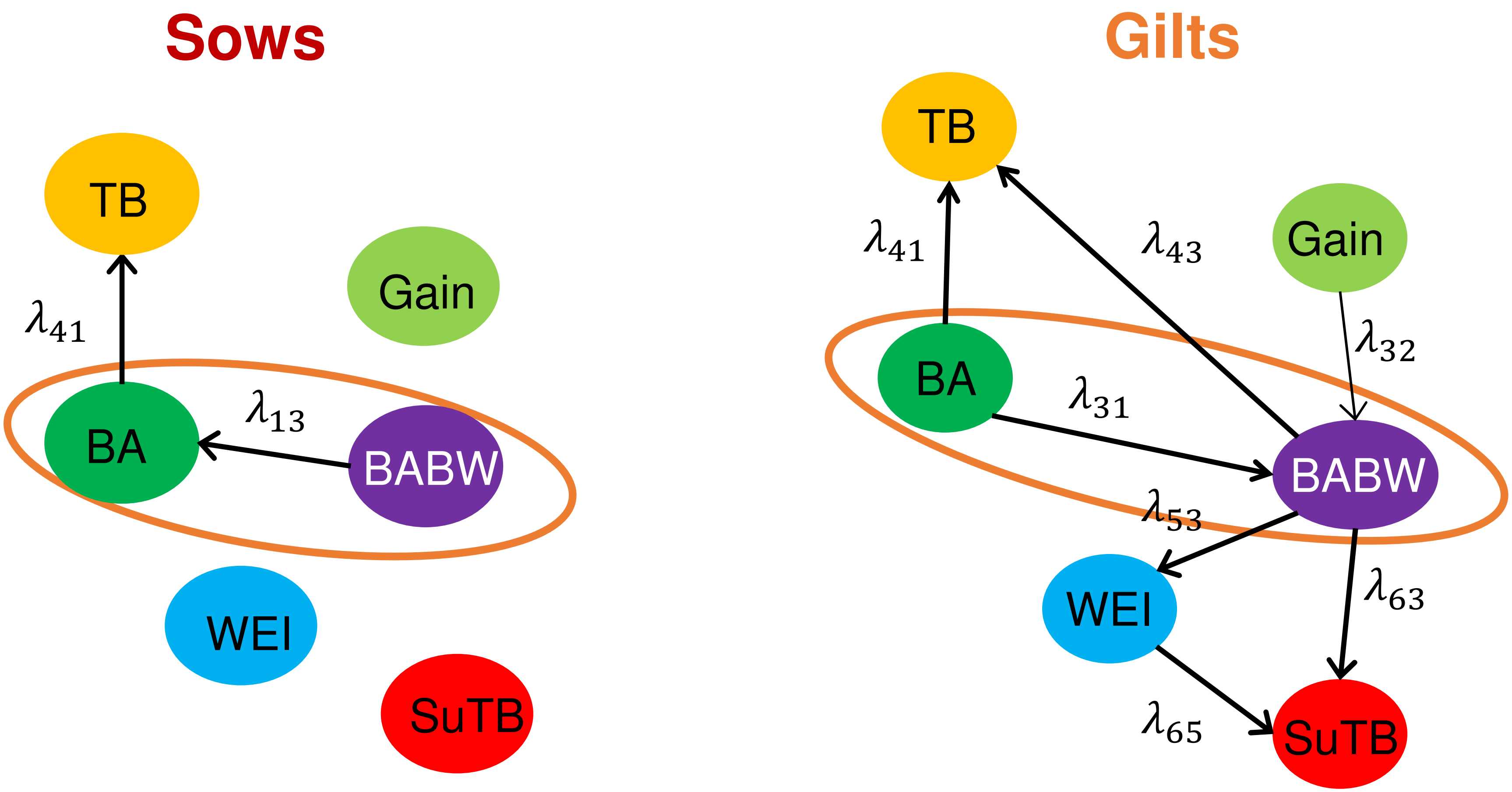
Total complete records = 440

Total BW blocks = 222



· $P < 0.05$, * $P < 0.01$, ** $P < 0.001$, *** $P < 0.0001$

Data Application: Heterogeneous Networks?



- Nested arrangement?
- Direction of link BA – BABW

Chitakasempornkul et al. 2019 JAS

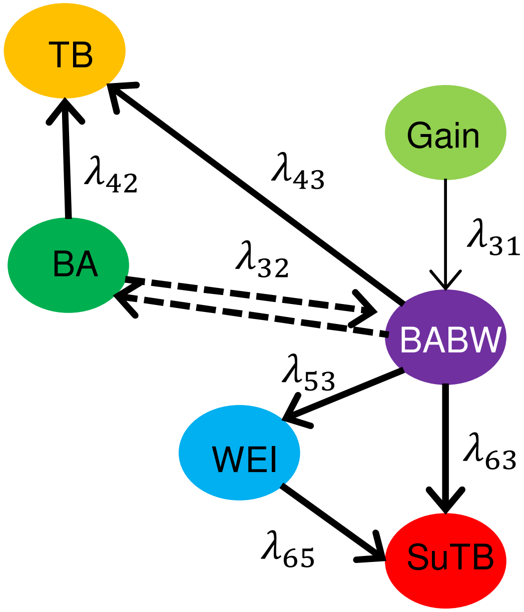
Data Application:
Alternative Models

M0: Standard Multiple-trait model

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 & 0 & 0 & 0 \\ \lambda_{31} & \lambda_{32} & 0 & 0 & 0 & 0 \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & 0 & 0 & 0 \\ \lambda_{51} & \lambda_{52} & \lambda_{53} & \lambda_{54} & 0 & 0 \\ \lambda_{61} & \lambda_{62} & \lambda_{63} & \lambda_{64} & \lambda_{65} & 0 \end{bmatrix}$$

M1: SEM with homogeneous $\lambda_{jj'}$

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{31} & \lambda_{32} & 0 & 0 & 0 & 0 \\ 0 & \lambda_{42} & \lambda_{43} & 0 & 0 & 0 \\ 0 & 0 & \lambda_{53} & 0 & 0 & 0 \\ 0 & 0 & \lambda_{63} & 0 & \lambda_{65} & 0 \end{bmatrix}$$



M2: SEM with heterogeneous $\lambda_{jj',i}$

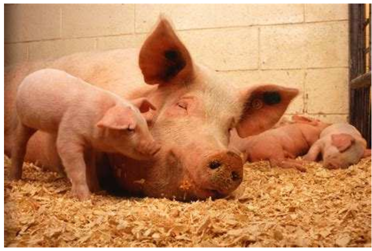
$$\boldsymbol{\lambda}_{jj',i} = \boldsymbol{x}'_{jj',i}\boldsymbol{\delta}_{jj'} + \boldsymbol{z}'_{jj',i}\boldsymbol{v}_{jj'} \qquad \boldsymbol{v}_{jj'} \sim N(\mathbf{0}, \sigma^2_{v_{jj'}}\boldsymbol{I}_q)$$

Parity BW blocks

M3: SEM with heterogeneous $\lambda_{jj',i}$ and $\sigma^2_{e_{j,i}}$

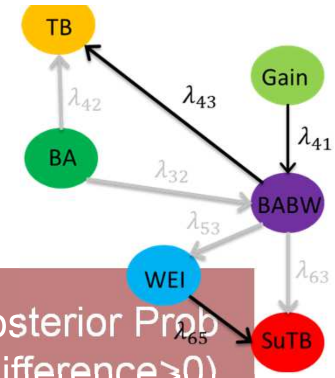
Kizilkaya and Tempelman 2005

Data Application:
Model Fit



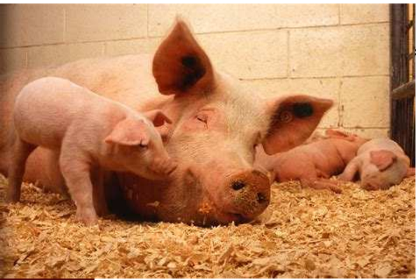
Models	Network structure	p_D	DIC
M0: Standard MTM - Fully recursive SEM		310.4	17075.1
M1: SEM with homogeneous $\lambda_{jj'}$	BA ← BABW	298.3	17044.2
	BA → BABW	297.1	17043.7
M2: SEM with heterogeneous $\lambda_{jj',i}$	BA ← BABW	389.2	17036.4
	BA → BABW	387.3	17027.4
M3: SEM with heterogeneous $\lambda_{jj',i}$ and $\sigma^2_{e_{j,i}}$	BA ← BABW	393.7	17019.1
	BA → BABW	392.5	17014.7

Data Application: Heterogeneous Networks



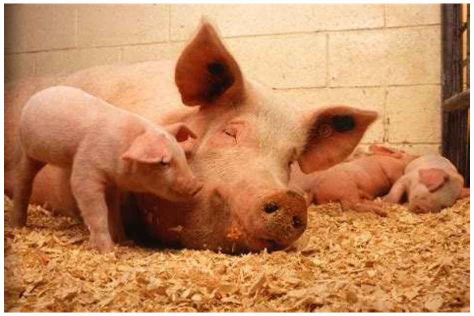
Structural coefficients	Sows	Gilts	Posterior Prob (Difference>0)
$\lambda_{BABW,Gain}$ (λ_{41}) *(g per kg)*	0.07 [0.02, 1.11]	0.03 [- 0.02, 0.57]	0.86
$\lambda_{BABW,BA}$ (λ_{32}) *(g per unit)*	-356.3 [-447.5, -267.7]	-358.1 [-421.7, -291.8]	0.02
$\lambda_{TB,BA}$ (λ_{42}) *(unit per unit)*	0.91 [0.84, 0.94]	0.86 [0.80, 0.92]	0.76
$\lambda_{TB,BABW}$ (λ_{43}) *(unit per 100g)*	-0.64 [-1.64, 0.31]	-0.14 [-0.22, -0.05]	0.78
$\lambda_{WEI,BABW}$ (λ_{53}) *(day per 100g)*	0.01 [-0.05, 0.07]	0.29 [-0.31, 0.83]	0.32
$\lambda_{SuTB,BABW}$ (λ_{63}) *(unit per 100g)*	-0.26 [-0.48, -0.06]	-0.39 [-0.64, -0.15]	0.62
$\lambda_{SuTB,WEI}$ (λ_{65}) *(unit per day)*	-0.26 [-0.48, -0.07]	0.16 [0.09, 0.23]	0.98

Data Application: Heterogeneous Networks



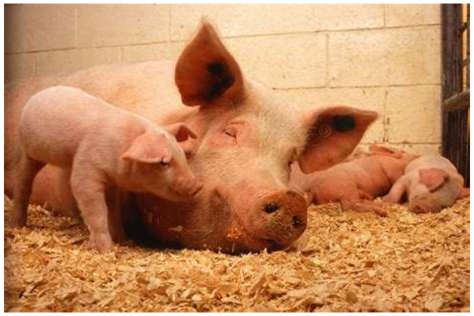
Direct effect From	To	Between-block variance component	Posterior mean [95% HPD]		Expected range of $\lambda_{jj'}$ based on empirical rule = $\left(\pm 1.96\sqrt{\sigma^2_{v_{jj'}}}\right)$
GAIN	BABW	$\sigma^2_{v\,BABW,GAIN}$	1.54×10^{-4} [2.23×10^{-10}, 5.38×10^{-4}]	(g per kg)^2	±0.02 g per kg
BA	BABW	$\sigma^2_{v\,BABW,BA}$	1.05×10^{-3} [6.22×10^{-12}, 3.91×10^{-4}]	(g per unit)^2	±0.06 g per unit
BA	TB	$\sigma^2_{v\,TB,BA}$	6.98×10^{-5} [2.18×10^{-11}, 2.68×10^{-4}]	(unit per unit)^2	±0.02 unit per unit
BABW	TB	$\sigma^2_{v\,TB,BABW}$	0.17 [7.05×10^{-8}, 0.58]	(unit per 100g)^2	±0.81 unit per 100g
BABW	WEI	$\sigma^2_{v\,WEI,BABW}$	0.27 [8.24×10^{-6}, 0.98]	(day per 100g)^2	±1.02 day per 100g
BABW	SuTB	$\sigma^2_{v\,SuTB,BABW}$	0.91 [7.21×10^{-7}, 3.44]	(unit per 100g)^2	±1.87 unit per 100g
WEI	SuTB	$\sigma^2_{v\,SuTB,WEI}$	4.64×10^{-3} [6.96×10^{-8}, 1.65×10^{-2}]	(unit per day)^2	±0.13 unit per day

Heterogeneous Networks: Concluding Remarks



- General methodological approach to extend hierarchical SEM for heterogeneous networks
- Structural coefficients specified as functions of systematic and non-systematic sources of variability
- Hierarchical Bayesian framework
  - Easily extendable to other Trt and design structures
  - Other: heterogeneous variances?
  - Designed experiments and observational studies

Heterogeneous Networks: Concluding Remarks



- Simulation study
  - Validation and frequentist properties
  - Diagnosis of network heterogeneity: model fit?
- Data application in an animal production system
  - Inference on heterogeneous reproductive networks for gilts and sows
  - Tailored management of specific subpopulations
  - Enhanced mechanistic understanding of biological processes

Thank you!


Kessinee Chitakasempornkul and yours truly

Questions? Comments?

- Guilherme Rosa, UW
- Abigail Jager, WUSTL
- Kansas State University Dept. Statistics and KSRE

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PhD Graduate Associateship (GRA) POSITION AVAILABLE
Animal Systems Modeling

The successful candidate will be involved in the development and application of quantitative models for efficient and sustainable food production systems. Areas of application are open to any agricultural livestock species and their interphase with cropping systems. The research focus will be interdisciplinary and integrative of scientific domains in alignment with the candidate's interest.

From a methodological standpoint, opportunities for novel and impactful developments relevant to systems modeling include causal inference, networks and graphical models, as well as hierarchical models, structural equations and dynamic modeling, amongst others. The successful candidate will have the opportunity to develop competency in the Bayesian framework of inference, as well as proficiency with big-data analytics and high-performance computing through the Ohio Supercomputer Center. The candidate will have the opportunity to interact regularly with a vibrant interdisciplinary collaborative environment, working on cutting edge problems in the life sciences through the Translational Data Analytics Institute.

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