Uncertainty Calibration and Exemplar Identification for Heterogeneous Treatment Effects with Individualized Bayesian Causal Forests (iBCF)

Jennifer Starling, Dan Thal, Lauren Vollmer, Irina Degtiar, Erin Lipman, Peter Mariani, and Mariel Finucane

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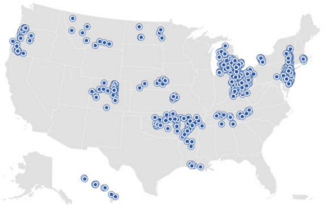
Motivation: Improving primary care

The Center for Medicare and Medicaid Innovation (CMMI) is a group in the Center for Medicare and Medicaid Services (CMS), with a bipartisan charge to decrease primary care spending and improve care for patients.

- CMMI designs **alternative payment models** to reward healthcare providers for delivering high-quality, cost-efficient care
- **Practices** voluntarily participate in these plans
- **Primary Care First** (PCF) program is currently underway



Source: Centers for Medicare & Medicaid Services

Evaluating alternative payment models

Goals:

- Determine if the program worked overall (✓)
- Assess if the program worked for specified subgroups of interest (✓)
- Evaluate how well the program worked for each practice
    - With appropriate uncertainty
    - Well-calibrated data-driven subgroup analysis
    - Reward practices that perform better than expected

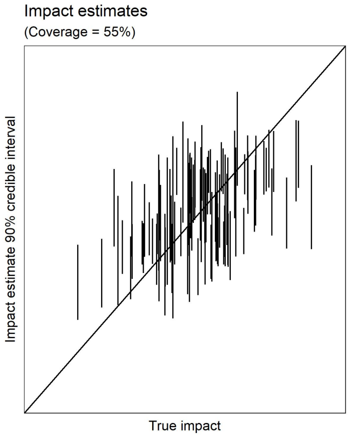
Practice impacts as potential outcomes

We can frame practice impacts in the potential outcomes framework. Let

- $y_i(1)$ be a practice's outcome under the program
- $y_i(0)$ be a practice's outcome absent participation

The practice-level impact is $y_i(1) - y_i(0)$.

Challenge 1: Uncertainty calibration for practice impact estimates



From Dorie (2019) on the 2016 ACIC data competition:

- ”Methods that flexibly model the response surface perform better overall than methods that fail to do so”
- ”Good coverage was difficult for most methods to achieve even when bias was low...we don’t feel like we have strong advice about how to optimize this aspect of performance”

Challenge 2: Identifying exemplar practices

A "high-performing" practice is one who

- Reduces expenditures
- Improves various outcome measures

Many unmeasured factors contribute to a practice's outcomes and response to payment programs.

- Energy and enthusiasm of practitioners
- Community support
- Participation in other programs
- Staff turnover

Data generating process

We estimate key data characteristics from real Medicare data, such as σ_y, ICC, and practice sizes.

We use these to simulate data that shares important features of the real data, such as

- Error variance
- Practice size

We generate non-linear control and impact functions using Gaussian covariates, scaled to mimic our real data. We sample bivariate practice-level random effects.

Introducing notation

Our data consists of n observations, indexed by i, where each observation represents a primary care practice.

Let

- y_i be the response (ex. expenditures, outcomes)
- z_i be a binary treatment indicator
- x_i are a vector of covariates for observation i
- $\pi(x_i)$ are propensity score estimates
- w_i are inverse practice sizes, which act as weights

BART model

BART is a Bayesian 'sum-of-trees' model introduced by Chipman, George, and McCulloch (2010). The BART model statement is:

$$y_i = f(\mathbf{x}_i) + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2)$$

$$f(\mathbf{x}) = \sum_{j=1}^{m} g(\mathbf{x}, T_j, M_j = \{\mu_{j1} \ldots \mu_j l\})$$

We can also think of g as a basis function parameterized by the binary tree defined by (T_j, M_j).

BART prior is composed of priors on σ^2, terminal node values μ_{jl}, and tree structures T_j.

Bayesian Causal Forests (BCF)

Models response surface as the sum of two BART fits.

$$y_i = \underbrace{\mu(x_i, \pi(x_i)) + \tau(x_i) z_i}_{f(x_i)} + \epsilon_i, \quad \epsilon_i \sim N\left(0, \frac{\sigma^2}{w_i}\right)$$

Using the potential outcomes framework, the treatment effect is

$$\tau(x_i) = f(x_i, 1) - f(x_i, 0)$$

Causal assumptions for Bayesian Causal Forests

- No interference between observations
- No unmeasured confounders
- Enough overlap to estimate impacts everywhere in covariate space

Under these conditions, $E\left[y_i(z) \mid t_i, x_i\right] = E\left[y_i \mid x_i, z_i = z\right]$, so we can express the causal estimand as:

$$\tau(t_i, x_i) = E\left[y_i \mid t_i, x_i, z_i = 1\right] - E\left[y_i \mid t_i, x_i, z_i = 0\right]$$

Benefits and shortcomings of Bayesian Causal Forests

Benefits:

- Deconfounding, through flexible modeling and less shrinkage of control covariates
- De-noising, through Bayesian shrinkage of impact estimates
- Flexible tree model tailored for learning impact heterogeneity
- Inclusion of propensity score estimates in control fit to mitigate bias

Shortcomings:

- Under-coverage
- Impact estimates for each practice are solely determined by x's

Introducing the iBCF model

$$y_i = \mu(x_i, \pi(x_i)) + \tau(x_i)z_i + \mathbf{u_i(1 - z_i) + (u_i + v_i)z_i} + \epsilon_i$$

$$\epsilon_i \sim N\left(0, \frac{\sigma^2}{w_i}\right)$$

Let

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_u^2 & \rho\sigma_u\sigma_v \\ \rho\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix}\right)$$

where

- u_i are random effects unrelated to treatment
- v_i are random impacts

The impact for practice i is $\tau(x_i) + v_i$.

iBCF is designed for weighted data

- For simplicity, write $\mathrm{Var}(u_i + v_i) = \sigma_u + \sigma_v + 2\rho\sigma_u\sigma_v = \sigma_b^2$
- The likelihood factorizes

$$y \sim \prod_{i|z_i=0}^{n_C} N\left(f(x_i), \frac{\sigma_y^2}{w_i} + \sigma_u^2\right) \times \prod_{i|z_i=1}^{n_T} N\left(f(x_i), \frac{\sigma_y^2}{w_i} + \sigma_b^2\right)$$

Weights allow us to separately identify σ_y versus the variance from the random effects.

- σ_y is the portion of the error variance that goes to zero as practice size $\rightarrow \infty$
- Even absent uncertainty, there is still remaining heterogeneity among the practices

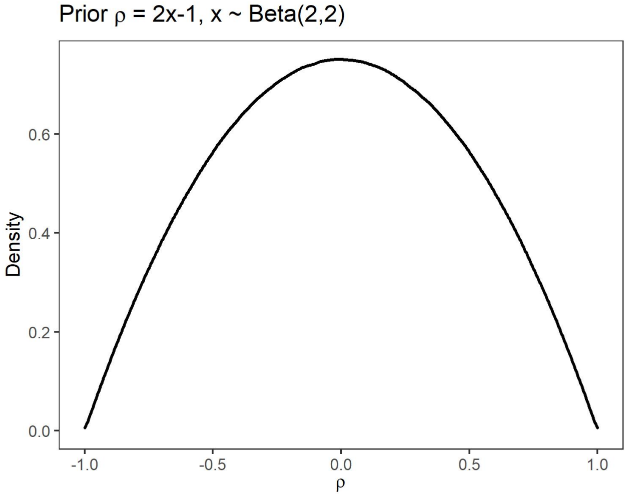
iBCF priors

We choose priors as follows:

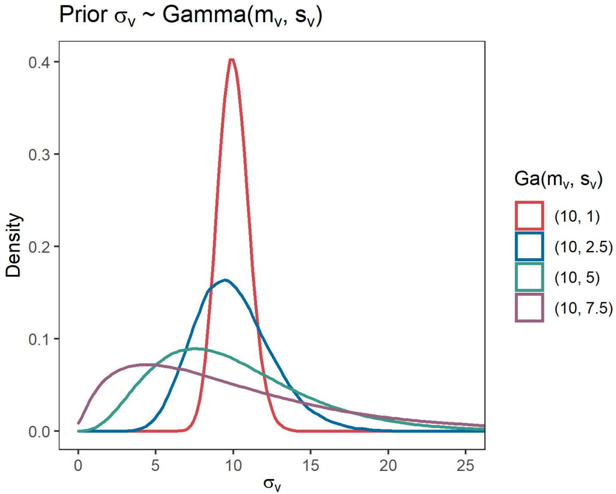
$$\begin{aligned}
\mathsf{Var}(u_i) = \sigma_u &\quad \sim C^+(1) \\
\mathsf{Var}(u_i + v_i) = \underbrace{\sigma_u + \sigma_v + 2\rho\sigma_u\sigma_v}_{\sigma_b} &\quad \sim C^+(1) \\
x = \frac{\rho - 1}{2} &\quad \sim Beta(2, 2) \\
\sigma_v &\quad \sim \mathsf{Gamma}(m_v, s_v)
\end{aligned}$$

We elicit the Gamma prior mean and sd for σ_v, which is not identified by the data.

Prior for $\rho = \text{Corr}(u_i, v_i)$



Prior for $\sigma_v = \text{Var}(v_i)$



Fitting the iBCF model

We fit iBCF using a modified version of BCF's backfitting algorithm.

Updating variance components:

1. MH step to draw σ_y; no longer conjugate
2. Posterior draws of $\sigma_u^2 = \mathsf{Var}(u_i) \mid y_C$ and $\sigma_b^2 = \mathsf{Var}(u_i + v_i) \mid y_T$
3. Posterior draws of ρ and σ_v (posterior=prior)

Drawing practice random effects:

1. Posterior draw of pair $(u_i, u_i + v_i)$
2. Calculate posterior draw for v_i as $v_i = (u_i + v_i) - u_i$

Data-driven hyperparameter tuning for $\sigma_v \sim Ga(m_v, s_v)$

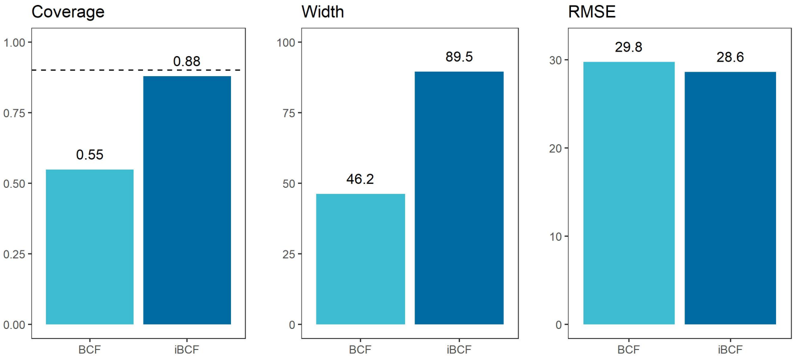
Let $\text{Var}(u_i + v_i) = \sigma_b^2$ and recall that the likelihood factorizes:

$$y \sim \prod_{i|z_i=0}^{n_C} N\left(f(x_i), \frac{\sigma_y^2}{w_i} + \sigma_u^2\right) \times \prod_{i|z_i=1}^{n_T} N\left(f(x_i), \frac{\sigma_y^2}{w_i} + \sigma_b^2\right)$$

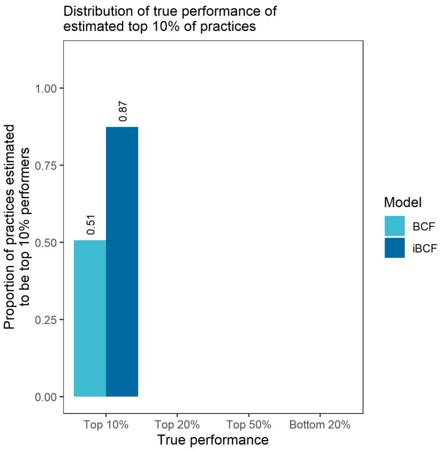
- Estimate residuals by fitting BART models to treated and control data
- Estimate σ_u and σ_b as intercepts from regressing $r_i^2 \sim (1/w_i)$ for treated and control observations (with positivity constraints)
- Solve the quadratic equation $\sigma_v^2 = \sigma_b^2 - \sigma_u^2 - 2\rho\sigma_u\sigma_v$ using $\hat{\sigma}_u$ and $\hat{\sigma}_b$ and a range of specified ρ values

We let $m_v = \hat{\sigma}_v$ and $s_v = 0.25 m_v$.

Practice-specific impact estimates: coverage, interval width, and RMSE



Identifying practices with exemplar outcomes

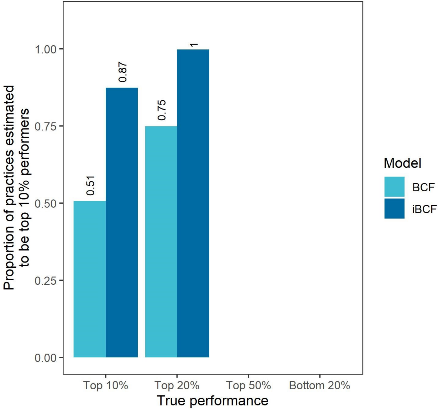


Of the practices we estimate to have outcomes in the top 10%,

- Only 51% have true top-10% outcomes under BCF
- 87% have true top-10% outcomes under iBCF

Identifying practices with exemplar outcomes

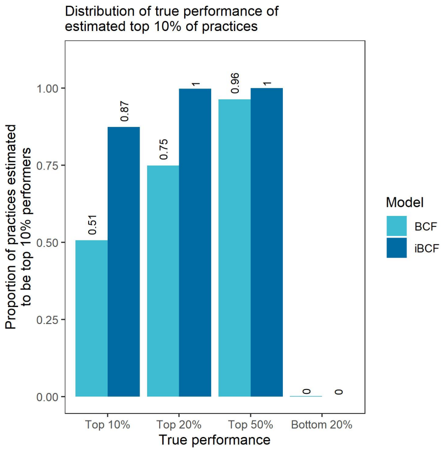
Distribution of true performance of estimated top 10% of practices



Of the practices we estimate to have outcomes in the top 10%,

- Only 75% are truly in the top 20% under BCF
- 100% are truly in the top 20% under iBCF

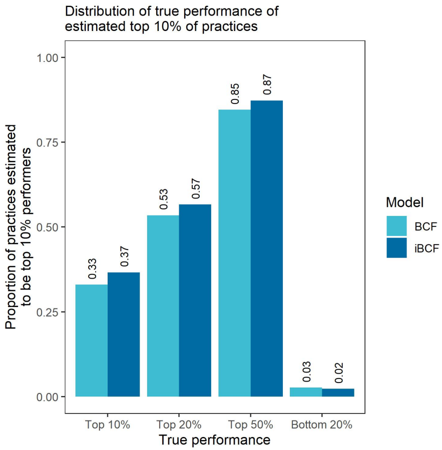
Identifying practices with exemplar outcomes



Of the practices we estimate to have outcomes in the top 10%,

- No practices are in the bottom 20% for either method.
- BCF does have 4% of practices in the bottom half

Identifying practices with exemplar impacts



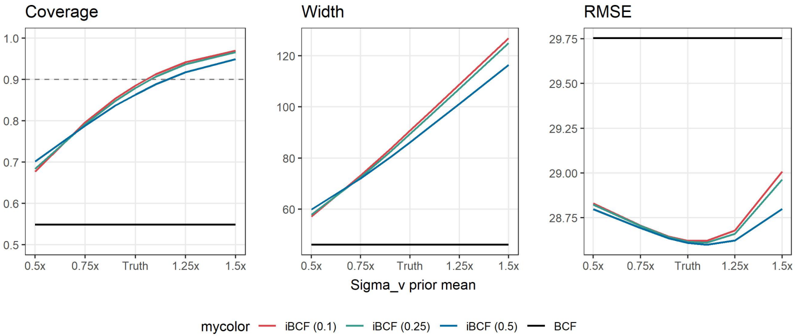
Of the practices we estimate to have impacts in the top 10%,

- 37% are truly in the top 10% under iBCF
- 33% are truly in the top 10% under BCF

Modest gains due to fairly small proportion of total impact variability accounted for by v_i in Medicare setting.

Sensitivity analysis for $\sigma_v \sim Ga(m_v, s_v)$ hyperparameters

- As $\sigma_v \to 0$, iBCF reverts towards BCF specification.
- BCF implies the prior $\sigma_v = 0$.



Summary

Novel approach for estimating observation-level impacts

- Improved uncertainty calibration
- Successfully identify top-performing practices
- iBCF performed well in the recent ACIC data challenge

Results are sensitive to the choice of hyperpriors.

- Refining our data-driven method assists with tuning
- Provides a more sensible prior than assuming $\sigma_v = 0$

References

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- Chipman, H.A. and George, E.I. and McCulloch, R.E. (2010). BART: Bayesian Additive Regression Trees. Annals of Applied Statistics, 4(1) 266-298.
- Hill, J. L. (2011). Bayesian Nonparametric Modeling for Causal Inference. Journal of Computational and Graphical Statistics, 20 (1), 217–240.

Appendix

Data generating process - details

We estimate the following quantities from real Medicare data.

- Quantiles for practice sizes
- σ_y^2, residual variance
- ICC (within-practice variance / total variance)
- σ_u estimated from ICC $= \frac{\sigma_u^2}{\sigma_u^2+\sigma_y^2}$
- σ_v estimated using our pre-regression technique
- $\text{Var}(\tau(x_i))$ fitting BCF to real data

Data generating process - details

We then generate data using our estimated values as follows:

- Weights drawn from real practice size quantiles
- Covariates drawn from standard normal distribution
- $\mu(x_i, \pi_i)$ non-linear function of covariates, with some (measured) confounders
- $\tau(x_i)$ Non-linear function of covariates, scaled to mimic Var($\tau(x_i)$)
- Draw random effects u_i, v_i from MVN with specified ρ

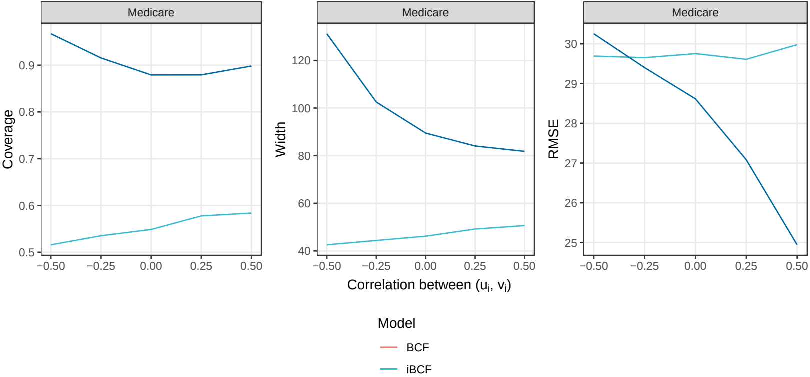
Causal assumptions for Bayesian Causal Forests - Details

- SUTVA (Stable Unit Treatment Value Assumption)
  - No interference between units, i.e.
  - The response of an observation depends only on its treatment, not on the treatment of other observations around it
- Strong ignorability, consisting of two conditions:
  - No unmeasured confounders: $(y_i(0), y_i(1)) \perp (z_i \mid t_i, X_i)$
  - Enough overlap to estimate treatment effects everywhere in covariate space: $0 < Pr(z_i = 1 \mid t_i, x_i) < 1$

Under these conditions, $E\left[y_i(z) \mid t_i, x_i\right] = E\left[y_i \mid x_i, z_i = z\right]$, so we can express the causal estimand as:

$$\tau(t_i, x_i) = E\left[y_i \mid t_i, x_i, z_i = 1\right] - E\left[y_i \mid t_i, x_i, z_i = 0\right]$$

Sensitivity to ρ in the simulated data



Data-driven hyperparameter tuning for $\sigma_v \sim Ga(m_v, s_v)$

Let $\text{Var}(u_i + v_i) = \sigma_b^2$ and recall that the likelihood factorizes:

$$y \sim \prod_{i|z_i=0}^{n_C} N\left(f(x_i), \frac{\sigma_y^2}{w_i} + \sigma_u^2\right) \times \prod_{i|z_i=1}^{n_T} N\left(f(x_i), \frac{\sigma_y^2}{w_i} + \sigma_b^2\right)$$

1. Estimate σ_u using control observations ($\sigma_i^2 = \frac{\sigma_y^2}{w_i} + \sigma_u^2$).
   - Fit a BART model to the control observations; let r_i^2 be the estimated squared residuals.
   - Fit linear model (I(r_i^2) $\sim$ 1/size)). The intercept gives $\hat{\sigma}_u^2$. (Use glmnet with λ=0 and lower.limit=0 for positivity constraints)
2. Repeat for treated observations to estimate σ_b ($\sigma_i^2 = \frac{\sigma_y^2}{w_i} + \sigma_b^2$).
3. Solve quadratic equation to estimate σ_v^2 for a range of ρ values.
   - $\sigma_v^2 = \sigma_b^2 - \sigma_u^2 - 2\rho\sigma_u\sigma_v$
   - Let $m_v = \hat{\sigma}_v^2$ and $s_v = 0.25m_v$.

Data-driven hyperparameter tuning for $\sigma_v \sim Ga(m_v, s_v)$

The quadratic equation is derived as follows. Let $b_i = u_i + v_i$.

$$\begin{aligned}
\sigma_v^2 &= \mathsf{Var}(v_i) \\
&= \mathsf{Var}(b_i - u_i) \\
&= \sigma_u^2 + \sigma_b^2 - 2\mathsf{Cov}(u_i, b_i) \\
&= \sigma_u^2 + \sigma_b^2 - 2\mathsf{Cov}(u_i, u_i + v_i) \\
&= \sigma_u^2 + \sigma_b^2 - 2(\sigma_u^2 + \mathsf{Cov}(u_i, v_i)) \\
&= \sigma_u^2 + \sigma_b^2 - 2(\sigma_u^2 + \rho\sigma_u\sigma_v) \\
&= \sigma_u^2 + \sigma_b^2 - 2(\sigma_u^2 + \rho\sigma_u\sigma_v) \\
&= \sigma_b^2 - \sigma_u^2 - 2\rho\sigma_u\sigma_v
\end{aligned}$$

Fitting the BART model using Bayesian Backfitting

The BART model is fit using an iterative MCMC called 'Bayesian Backfitting.'

- Trees T_j for $j \in \{1, \ldots, m\}$ are updated one by one, using residuals from fits of other trees
  - Each tree is drawn using a MH step to select a node and propose a birth or death move
- Tree leaves μ are drawn from conditional posterior
- Error variance σ is drawn from conditional posterior

The μ_{jl} and σ updates are easy; priors are conjugate.