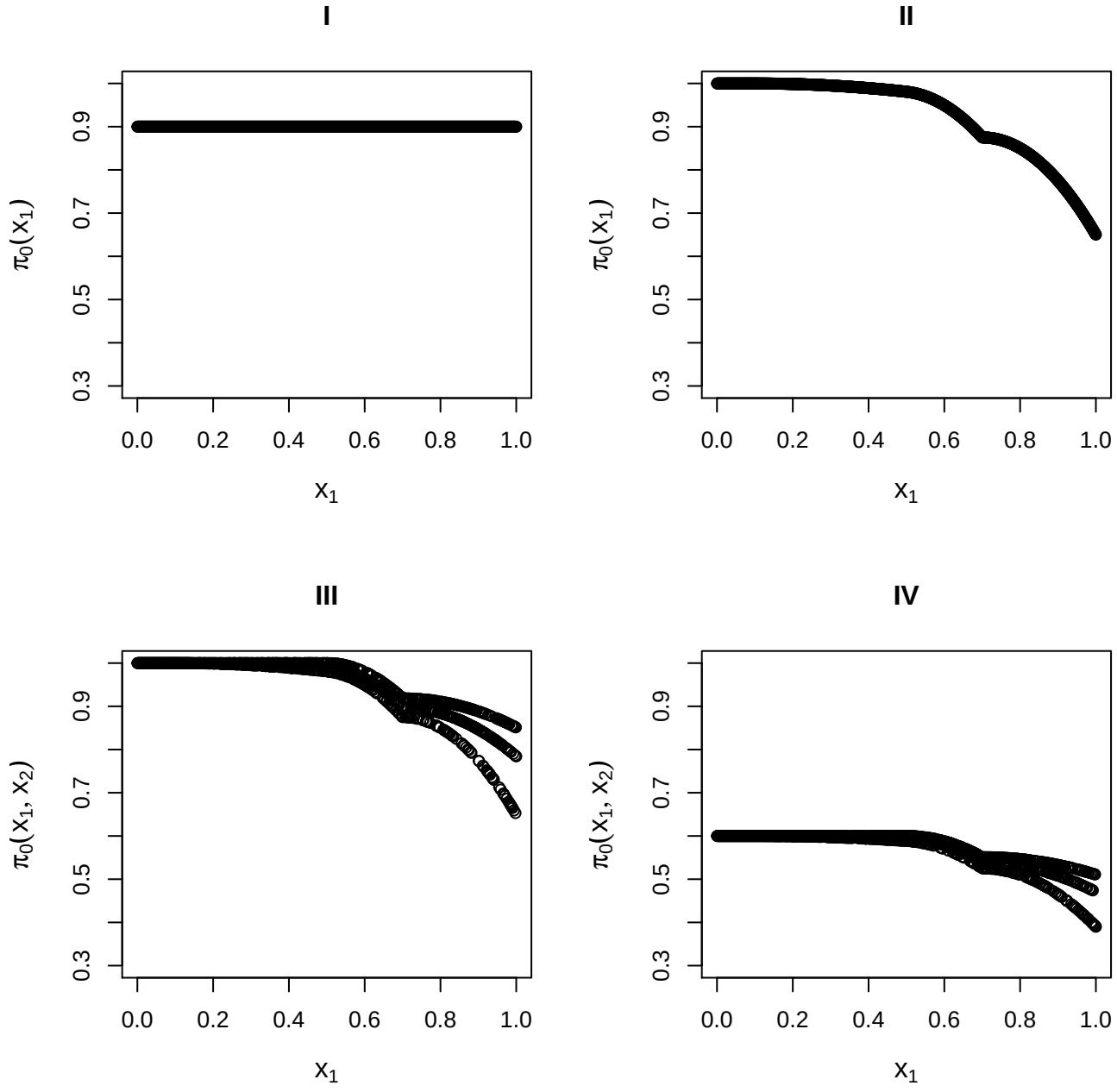
Simulations: Overview

The following 4 functions are considered for $\pi_0(x)$:



We performed 200 simulations in each scenario.
We estimated false discovery rates (FDR) and true positive rates (TPR) percentages for a nominal FDR of 5%. We considered both the theoretical and empirical nulls for the Scott method. For III and IV, a dummy variable was used for x_2, along with linear or spline terms (with 3 df) for x_1.

Independent test statistics

We first generated independent test statistics.
For the beta distribution, we generated the p-values directly from Beta(1,20). For the other distributions, we generated the test statistics and calculated the p-values from them. For the t-test, we considered 2 groups of 6 (so 2x6 = 10 df) and used the t-statistics instead of the z-statistics for the Scott method. For the chisquared test, 1 df corresponds to a 2x2 table, 4 df to a 3x3 table. We used the z-statistics obtained from back-transforming the p-values for the Scott method for the beta and the chisquared cases.
BL = Boca-Leek, Scott T = Scott theoretical null, Scott E = Scott empirical null

1,000 tests

			FDR					TPR				
$\pi_0(x)$	Dist. under H_1	Reg. model	BL	Scott T	Scott E	Storey	BH	BL	Scott T	Scott E	Storey	BH
I	Beta(1,20)	Linear	5.0	90.0	84.0	5.2	3.9	0.2	100.0	95.9	0.2	0.1
II	Beta(1,20)	Linear	4.8	92.6	85.9	4.8	4.1	0.2	100.0	98.0	0.1	0.1
II	Beta(1,20)	Spline	6.5	92.6	86.6	4.8	4.1	0.2	100.0	98.3	0.1	0.1
III	Beta(1,20)	Linear	5.2	94.9	88.9	5.4	5.4	0.2	100.0	97.5	0.2	0.2
III	Beta(1,20)	Spline	6.2	94.9	89.4	5.4	5.4	0.3	100.0	97.6	0.2	0.2
IV	Beta(1,20)	Linear	6.4	56.7		5.1	3.4	12.2	100.0		5.4	0.3
IV	Beta(1,20)	Spline	7.9	56.7		5.1	3.4	15.4	100.0		5.4	0.3
I	Norm	Linear	5.0	5.2	6.6	4.9	4.4	51.0	50.9	49.7	50.8	49.7
II	Norm	Linear	5.4	5.7	8.1	5.3	4.9	48.5	63.5	61.3	47.6	47.0
II	Norm	Spline	5.6	5.9	8.3	5.3	4.9	49.3	63.5	61.5	47.6	47.0
III	Norm	Linear	5.8	5.9	9.9	5.4	5.1	45.1	60.3	57.9	44.0	43.4
III	Norm	Spline	5.9	6.0	10.1	5.4	5.1	45.6	60.9	58.2	44.0	43.4
IV	Norm	Linear	5.0	4.9	2.4	4.7	2.8	71.6	71.8	60.6	71.2	65.4
IV	Norm	Spline	5.2	5.0	2.4	4.7	2.8	72.0	71.9	60.7	71.2	65.4
I	T	Linear	5.7	21.3	23.4	5.5	4.8	15.7	55.4	56.9	15.2	13.6
II	T	Linear	4.8	20.7	23.8	5.0	4.4	13.0	64.5	65.5	11.6	10.6
II	T	Spline	4.7	21.1	24.5	5.0	4.4	13.8	64.8	65.6	11.6	10.6
III	T	Linear	6.2	26.8	31.0	5.9	5.4	9.4	54.6	54.7	8.2	7.6
III	T	Spline	6.8	27.3	31.3	5.9	5.4	10.0	55.2	55.3	8.2	7.6
IV	T	Linear	5.0	9.3	2.8	4.7	2.9	52.5	72.9	44.4	52.0	40.3
IV	T	Spline	5.4	9.3	2.8	4.7	2.9	53.0	73.0	44.6	52.0	40.3
I	Chisq 1 df	Linear	5.0	90.0	85.5	4.8	4.4	51.2	100.0	98.7	50.9	49.7
II	Chisq 1 df	Linear	4.8	92.6	89.4	4.8	4.4	48.3	100.0	99.6	47.1	46.3
II	Chisq 1 df	Spline	5.0	92.6	90.0	4.8	4.4	48.9	100.0	99.6	47.1	46.3
III	Chisq 1 df	Linear	5.0	94.9	93.8	4.9	4.8	44.3	100.0	99.7	43.1	42.5
III	Chisq 1 df	Spline	5.3	94.9	93.9	4.9	4.8	44.8	100.0	99.7	43.1	42.5
IV	Chisq 1 df	Linear	5.1	56.7		4.7	2.8	71.6	100.0		71.1	65.1
IV	Chisq 1 df	Spline	5.3	56.7		4.7	2.8	71.9	100.0		71.1	65.1
I	Chisq 4 df	Linear	5.3	90.0	83.5	5.4	4.8	30.8	100.0	95.3	30.6	29.6
II	Chisq 4 df	Linear	5.3	92.6	89.6	5.3	5.0	28.4	100.0	98.5	27.5	26.7
II	Chisq 4 df	Spline	5.4	92.6	89.9	5.3	5.0	29.2	100.0	98.6	27.5	26.7
III	Chisq 4 df	Linear	5.9	94.9	92.4	5.4	5.3	24.8	100.0	98.3	24.0	23.4
III	Chisq 4 df	Spline	5.9	94.9	93.0	5.4	5.3	25.2	100.0	98.7	24.0	23.4
IV	Chisq 4 df	Linear	5.1	56.7	55.9	4.7	2.8	52.3	100.0	98.8	51.7	44.5
IV	Chisq 4 df	Spline	5.5	56.7	55.9	4.7	2.8	52.7	100.0	98.8	51.7	44.5

10,000 tests

			FDR					TPR				
$\pi_0(x)$	Dist. under H_1	Reg. model	BL	Scott T	Scott E	Storey	BH	BL	Scott T	Scott E	Storey	BH
I	Beta(1,20)	Linear	3.7	90.0	90.0	3.7	3.6	0.0	100.0	100.0	0.0	0.0
II	Beta(1,20)	Linear	3.1	92.6	92.6	3.1	3.0	0.0	100.0	100.0	0.0	0.0
II	Beta(1,20)	Spline	3.1	92.6	92.6	3.1	3.0	0.0	100.0	100.0	0.0	0.0
III	Beta(1,20)	Linear	4.0	94.9	94.9	3.5	3.5	0.0	100.0	100.0	0.0	0.0
III	Beta(1,20)	Spline	4.5	94.9	94.9	3.5	3.5	0.0	100.0	100.0	0.0	0.0
IV	Beta(1,20)	Linear	4.4	56.9		4.8	2.5	1.2	100.0		0.5	0.0
IV	Beta(1,20)	Spline	5.0	56.9		4.8	2.5	2.0	100.0		0.5	0.0
I	Norm	Linear	5.0	5.0	5.9	5.0	4.5	50.6	50.6	52.1	50.7	49.6
II	Norm	Linear	4.9	5.2	5.3	4.9	4.6	48.4	63.9	62.9	47.3	46.6
II	Norm	Spline	4.9	5.2	5.3	4.9	4.6	48.8	64.0	63.0	47.3	46.6
III	Norm	Linear	4.9	5.2	5.5	4.9	4.7	44.2	60.2	59.3	43.5	43.0
III	Norm	Spline	4.9	5.2	5.4	4.9	4.7	44.4	60.6	59.7	43.5	43.0
IV	Norm	Linear	4.8	5.0	2.3	4.8	2.8	71.3	71.8	62.2	71.2	65.3
IV	Norm	Spline	4.8	5.0	2.3	4.8	2.8	71.3	71.8	62.2	71.2	65.3
I	T	Linear	5.2	21.7	20.8	5.1	4.7	14.1	55.3	53.2	14.1	12.6
II	T	Linear	4.6	20.0	19.9	4.9	4.5	11.5	65.7	65.4	10.2	9.2
II	T	Spline	4.5	20.2	20.1	4.9	4.5	12.0	65.7	65.4	10.2	9.2
III	T	Linear	4.9	24.7	26.8	5.2	5.2	6.8	62.5	63.7	6.0	5.5
III	T	Spline	4.8	24.8	26.9	5.2	5.2	7.0	62.6	63.9	6.0	5.5
IV	T	Linear	4.8	9.3	1.2	4.8	2.9	51.8	72.8	28.5	51.6	40.2
IV	T	Spline	4.8	9.3	1.2	4.8	2.9	51.9	72.9	28.6	51.6	40.2
I	Chisq 1 df	Linear	5.0	90.0	90.0	5.0	4.5	50.7	100.0	100.0	50.6	49.6
II	Chisq 1 df	Linear	4.9	92.6	92.6	5.0	4.6	48.2	100.0	100.0	47.2	46.4
II	Chisq 1 df	Spline	4.8	92.6	92.6	5.0	4.6	48.6	100.0	100.0	47.2	46.4
III	Chisq 1 df	Linear	5.0	94.9	94.9	5.0	4.8	44.0	100.0	100.0	43.2	42.7
III	Chisq 1 df	Spline	5.0	94.9	94.9	5.0	4.8	44.2	100.0	100.0	43.2	42.7
IV	Chisq 1 df	Linear	4.8	56.9		4.8	2.8	71.1	100.0		71.0	65.2
IV	Chisq 1 df	Spline	4.8	56.9		4.8	2.8	71.2	100.0		71.0	65.2
I	Chisq 4 df	Linear	5.0	90.0	90.0	5.0	4.5	29.7	100.0	100.0	29.7	28.7
II	Chisq 4 df	Linear	4.9	92.6	92.6	5.0	4.7	28.0	100.0	100.0	27.1	26.5
II	Chisq 4 df	Spline	4.9	92.6	92.6	5.0	4.7	28.4	100.0	100.0	27.1	26.5
III	Chisq 4 df	Linear	5.2	94.9	94.9	5.2	5.0	24.3	100.0	100.0	23.6	23.2
III	Chisq 4 df	Spline	5.2	94.9	94.9	5.2	5.0	24.4	100.0	100.0	23.6	23.2
IV	Chisq 4 df	Linear	4.7	56.9	57.1	4.7	2.8	51.8	100.0	100.0	51.7	44.8
IV	Chisq 4 df	Spline	4.7	56.9	57.1	4.7	2.8	51.9	100.0	100.0	51.7	44.8

Dependent test statistics - 1,000 tests

We next generated independent test statistics. We used multivariate normal and t distributions (10 df for the t-distribution). We considered block-diagnonal matrices with the number of blocks equal to 20 or 10 and the within-block correlation, ρ, of 0.2, 0.5, or 0.9. Thus, 20 blocks meant a block size of 50 tests (lesser dependence) and 10 blocks a block size of 100 tests (more dependence).
BL = Boca-Leek, Scott T = Scott theoretical null, Scott E = Scott empirical null

			FDR					TPR				
$\pi_0(x)$	Dist. under H_1	Reg. model	BL	Scott T	Scott E	Storey	BH	BL	Scott T	Scott E	Storey	BH
I	N, 20 blocks, ρ=0.2	Linear	5.3	6.2	6.8	5.0	4.4	51.5	51.4	48.4	51.3	50.1
II	N, 20 blocks, ρ=0.2	Linear	5.2	6.9	8.0	5.1	4.6	48.6	63.4	59.3	47.6	46.5
II	N, 20 blocks, ρ=0.2	Spline	5.7	8.3	9.2	5.1	4.6	49.2	63.3	59.6	47.6	46.5
III	N, 20 blocks, ρ=0.2	Linear	5.5	7.6	9.3	5.2	4.8	45.1	60.0	56.0	44.0	43.2
III	N, 20 blocks, ρ=0.2	Spline	5.7	9.6	10.6	5.2	4.8	45.9	60.2	56.3	44.0	43.2
IV	N, 20 blocks, ρ=0.2	Linear	5.3	5.3	2.5	4.9	2.9	71.8	71.9	61.0	71.4	65.6
IV	N, 20 blocks, ρ=0.2	Spline	5.6	5.5	2.5	4.9	2.9	72.0	71.9	61.1	71.4	65.6
I	N, 20 blocks, ρ=0.5	Linear	6.4	10.0	10.7	6.0	5.2	52.0	51.7	47.6	51.6	50.3
II	N, 20 blocks, ρ=0.5	Linear	6.1	12.4	13.5	5.7	5.1	48.4	62.8	57.6	47.3	46.2
II	N, 20 blocks, ρ=0.5	Spline	7.1	18.7	20.4	5.7	5.1	49.5	62.6	58.0	47.3	46.2
III	N, 20 blocks, ρ=0.5	Linear	5.6	11.5	15.9	5.2	4.6	45.4	59.6	56.6	44.0	43.2
III	N, 20 blocks, ρ=0.5	Spline	6.6	19.9	23.6	5.2	4.6	46.2	59.0	56.9	44.0	43.2
IV	N, 20 blocks, ρ=0.5	Linear	5.8	6.1	2.8	5.3	3.1	72.1	72.3	59.4	71.6	65.7
IV	N, 20 blocks, ρ=0.5	Spline	6.5	6.4	3.0	5.3	3.1	72.4	72.2	59.6	71.6	65.7
I	N, 20 blocks, ρ=0.9	Linear	9.0	17.6	36.2	6.9	5.3	53.8	53.3	57.9	52.6	50.4
II	N, 20 blocks, ρ=0.9	Linear	7.8	20.0	47.5	6.4	4.9	49.6	63.8	68.0	48.0	46.2
II	N, 20 blocks, ρ=0.9	Spline	18.2	34.5	53.6	6.4	4.9	52.2	64.4	69.8	48.0	46.2
III	N, 20 blocks, ρ=0.9	Linear	6.4	23.1	48.8	5.1	4.0	47.3	60.5	67.9	46.1	44.0
III	N, 20 blocks, ρ=0.9	Spline	21.5	38.4	60.5	5.1	4.0	51.0	60.9	69.7	46.1	44.0
IV	N, 20 blocks, ρ=0.9	Linear	7.7	8.4	6.9	6.1	3.1	73.1	73.2	57.4	72.2	65.9
IV	N, 20 blocks, ρ=0.9	Spline	11.8	10.0	8.0	6.1	3.1	74.4	72.8	57.8	72.2	65.9
I	N, 10 blocks, ρ=0.2	Linear	5.4	7.8	6.1	5.1	4.4	51.6	51.6	47.3	51.2	49.9
II	N, 10 blocks, ρ=0.2	Linear	5.0	9.3	8.8	4.8	4.3	48.2	63.0	59.8	47.2	46.1
II	N, 10 blocks, ρ=0.2	Spline	5.5	13.3	11.1	4.8	4.3	49.1	62.8	59.8	47.2	46.1
III	N, 10 blocks, ρ=0.2	Linear	5.2	8.6	9.8	5.0	4.5	44.6	59.5	56.4	43.4	42.7
III	N, 10 blocks, ρ=0.2	Spline	5.8	14.3	13.2	5.0	4.5	45.2	59.2	56.6	43.4	42.7
IV	N, 10 blocks, ρ=0.2	Linear	5.3	5.7	2.4	5.0	2.9	71.8	71.8	60.4	71.4	65.5
IV	N, 10 blocks, ρ=0.2	Spline	5.7	5.9	2.5	5.0	2.9	72.1	71.8	60.5	71.4	65.5
I	N, 10 blocks, ρ=0.5	Linear	7.3	17.1	15.9	6.5	5.4	51.9	51.8	48.8	51.7	50.0
II	N, 10 blocks, ρ=0.5	Linear	5.9	20.3	19.9	5.3	4.5	48.3	62.6	61.0	46.8	45.6
II	N, 10 blocks, ρ=0.5	Spline	8.6	32.5	27.7	5.3	4.5	49.2	63.3	61.4	46.8	45.6
III	N, 10 blocks, ρ=0.5	Linear	5.8	17.4	17.7	4.9	4.2	44.2	58.1	54.3	43.0	42.0
III	N, 10 blocks, ρ=0.5	Spline	8.6	32.7	30.2	4.9	4.2	45.0	58.1	55.6	43.0	42.0
IV	N, 10 blocks, ρ=0.5	Linear	6.3	7.5	3.3	5.5	3.2	72.4	72.4	59.0	71.9	65.8
IV	N, 10 blocks, ρ=0.5	Spline	7.6	8.3	3.8	5.5	3.2	72.7	72.1	59.3	71.9	65.8
I	N, 10 blocks, ρ=0.9	Linear	14.1	30.6	45.6	6.6	4.1	55.5	54.7	65.6	53.3	50.2
II	N, 10 blocks, ρ=0.9	Linear	13.3	35.5	55.9	5.9	3.3	51.1	66.5	75.8	49.0	46.1
II	N, 10 blocks, ρ=0.9	Spline	35.1	49.9	67.5	5.9	3.3	56.1	67.4	77.6	49.0	46.1
III	N, 10 blocks, ρ=0.9	Linear	13.3	33.7	66.4	5.4	3.3	45.6	58.1	75.7	43.4	40.7
III	N, 10 blocks, ρ=0.9	Spline	40.7	51.5	73.0	5.4	3.3	52.0	61.6	77.4	43.4	40.7
IV	N, 10 blocks, ρ=0.9	Linear	11.2	12.4	12.0	7.0	3.1	74.0	73.5	63.9	72.5	65.8
IV	N, 10 blocks, ρ=0.9	Spline	19.2	15.6	13.8	7.0	3.1	76.2	73.3	64.3	72.5	65.8

			FDR					TPR				
$\pi_0(x)$	Dist. under H_1	Reg. model	BL	Scott T	Scott E	Storey	BH	BL	Scott T	Scott E	Storey	BH
I	T, 20 blocks, ρ=0.2	Linear	1.7	9.1	7.4	1.5	0.9	8.0	51.6	57.8	7.6	5.7
II	T, 20 blocks, ρ=0.2	Linear	3.2	13.9	7.3	3.2	1.8	8.0	63.8	61.0	6.8	4.5
II	T, 20 blocks, ρ=0.2	Spline	3.7	14.7	8.5	3.2	1.8	9.2	63.9	61.3	6.8	4.5
III	T, 20 blocks, ρ=0.2	Linear	2.6	13.8	9.6	2.1	1.3	4.3	59.4	60.1	3.4	2.3
III	T, 20 blocks, ρ=0.2	Spline	3.6	15.1	11.0	2.1	1.3	5.2	59.7	60.3	3.4	2.3
IV	T, 20 blocks, ρ=0.2	Linear	2.7	5.4	2.9	2.4	1.0	55.4	71.8	65.1	54.4	44.3
IV	T, 20 blocks, ρ=0.2	Spline	3.0	5.4	2.8	2.4	1.0	56.0	71.9	65.1	54.4	44.3
I	T, 20 blocks, ρ=0.5	Linear	1.7	10.3	11.0	1.5	1.0	8.6	51.6	57.4	8.2	5.9
II	T, 20 blocks, ρ=0.5	Linear	3.5	16.3	11.9	3.3	2.1	7.7	64.2	61.7	6.6	4.5
II	T, 20 blocks, ρ=0.5	Spline	4.7	19.5	16.6	3.3	2.1	9.1	63.9	62.1	6.6	4.5
III	T, 20 blocks, ρ=0.5	Linear	3.2	17.6	13.0	2.3	1.5	5.0	59.3	59.0	3.6	2.6
III	T, 20 blocks, ρ=0.5	Spline	4.4	23.4	20.5	2.3	1.5	5.6	59.6	59.5	3.6	2.6
IV	T, 20 blocks, ρ=0.5	Linear	2.7	5.5	3.0	2.3	1.0	55.3	71.9	64.7	54.3	44.4
IV	T, 20 blocks, ρ=0.5	Spline	3.2	5.8	3.1	2.3	1.0	55.8	71.9	64.8	54.3	44.4
I	T, 20 blocks, ρ=0.9	Linear	3.0	14.5	29.0	1.5	0.9	11.5	51.7	64.1	9.9	6.2
II	T, 20 blocks, ρ=0.9	Linear	3.8	20.9	45.7	2.3	1.9	10.2	64.9	70.6	7.7	5.0
II	T, 20 blocks, ρ=0.9	Spline	15.8	32.1	54.6	2.3	1.9	14.2	64.7	70.5	7.7	5.0
III	T, 20 blocks, ρ=0.9	Linear	5.2	23.9	49.7	3.2	1.4	7.3	60.7	63.5	5.6	3.1
III	T, 20 blocks, ρ=0.9	Spline	19.0	35.1	60.6	3.2	1.4	10.6	61.7	65.5	5.6	3.1
IV	T, 20 blocks, ρ=0.9	Linear	3.6	6.6	7.5	2.4	1.0	56.1	72.2	67.5	54.6	44.3
IV	T, 20 blocks, ρ=0.9	Spline	8.6	7.5	8.0	2.4	1.0	58.4	72.0	67.2	54.6	44.3
I	T, 10 blocks, ρ=0.2	Linear	1.8	9.9	7.8	1.6	0.8	8.3	51.3	57.2	8.0	5.9
II	T, 10 blocks, ρ=0.2	Linear	3.4	15.0	8.1	3.4	1.5	7.3	63.1	61.3	6.4	4.3
II	T, 10 blocks, ρ=0.2	Spline	4.0	16.7	9.9	3.4	1.5	8.6	63.2	61.5	6.4	4.3
III	T, 10 blocks, ρ=0.2	Linear	2.2	15.2	9.5	1.6	1.2	3.7	58.7	59.4	3.0	1.9
III	T, 10 blocks, ρ=0.2	Spline	2.7	18.0	12.7	1.6	1.2	4.2	58.5	59.7	3.0	1.9
IV	T, 10 blocks, ρ=0.2	Linear	2.6	5.5	2.8	2.4	1.0	54.8	71.5	64.6	53.9	43.9
IV	T, 10 blocks, ρ=0.2	Spline	3.0	5.6	2.8	2.4	1.0	55.4	71.5	64.7	53.9	43.9
I	T, 10 blocks, ρ=0.5	Linear	2.2	13.5	14.2	1.6	0.9	9.3	50.8	57.4	8.5	6.1
II	T, 10 blocks, ρ=0.5	Linear	3.3	19.2	13.6	3.4	1.7	7.9	63.1	61.2	7.0	4.4
II	T, 10 blocks, ρ=0.5	Spline	6.2	27.6	21.3	3.4	1.7	9.9	63.5	61.3	7.0	4.4
III	T, 10 blocks, ρ=0.5	Linear	2.3	23.4	21.5	1.3	0.7	4.4	58.0	59.5	3.0	2.1
III	T, 10 blocks, ρ=0.5	Spline	3.8	35.9	31.4	1.3	0.7	5.6	58.1	60.1	3.0	2.1
IV	T, 10 blocks, ρ=0.5	Linear	3.1	6.1	3.4	2.5	1.0	54.4	71.4	63.5	53.4	43.2
IV	T, 10 blocks, ρ=0.5	Spline	4.3	6.6	3.8	2.5	1.0	55.3	71.2	64.0	53.4	43.2
I	T, 10 blocks, ρ=0.9	Linear	7.7	23.0	38.0	1.6	1.0	14.9	51.5	70.9	11.4	6.7
II	T, 10 blocks, ρ=0.9	Linear	10.1	31.5	50.0	4.1	1.7	12.4	65.4	76.2	11.1	6.0
II	T, 10 blocks, ρ=0.9	Spline	41.7	43.6	60.7	4.1	1.7	22.4	68.2	78.9	11.1	6.0
III	T, 10 blocks, ρ=0.9	Linear	12.7	36.2	62.9	2.2	1.3	11.0	60.5	77.2	5.8	2.6
III	T, 10 blocks, ρ=0.9	Spline	43.0	48.4	71.0	2.2	1.3	19.3	62.9	78.7	5.8	2.6
IV	T, 10 blocks, ρ=0.9	Linear	6.2	9.2	11.1	3.2	1.0	56.3	72.1	68.3	54.2	42.4
IV	T, 10 blocks, ρ=0.9	Spline	15.1	10.8	11.8	3.2	1.0	59.3	71.8	68.3	54.2	42.4