Logic for Computer Science

Exercises, tutorial 6

Pascal Fontaine

November 29, 2021

1. Tony, Mike and Jessica are members of an alpine club. Each member is either a skier or an alpinist or both. No alpinist likes rain but all skiers like snow. Mike likes nothing that Tony likes and likes everything that Tony doesn't like. Tony likes rain and snow.

Is there a member of the alpine club who is an alpinist but not a skier?

Solution: We can formalize the above text as follows:

1. $\forall x \,.\, C(x) \Rightarrow (S(x) \vee A(x))$
2. $\forall x \,.\, A(x) \Rightarrow \neg L(x, \text{rain})$
3. $\forall x \,.\, S(x) \Rightarrow L(x, \text{snow})$
4. $\forall y \,.\, L(t, y) \Rightarrow \neg L(m, y)$
5. $\forall y \,.\, \neg L(t, y) \Rightarrow L(m, y)$
6. $L(t, \text{rain})$
7. $L(t, \text{snow})$

Then the question is $\exists x \,.\, C(x) \wedge A(x) \wedge \neg S(x)$?

From 6 and 2, one derives $\neg A(t)$, thus Tony is not a good candidate. Form 7 and 4, we have $\neg L(m, \text{snow})$, thus, thanks to (3), $\neg S(m)$. But Mike belongs to the alphine club ($C(m)$), therefore thanks to (1), $S(m) \vee A(m)$. Finally $A(m)$ is true. Mike is our example: $C(m) \wedge A(m) \wedge \neg S(m)$ holds, and thus $\exists x \,.\, C(x) \wedge A(x) \wedge \neg S(x)$.

2. What are the relations between the following formulas?

1. $A =_{\text{def}} \forall x \exists y \, P(x, y)$

2. $B =_{\text{def}} \forall y\, P(y, a)$

3. $C =_{\text{def}} \forall x\, P(x, f(x))$

4. $D =_{\text{def}} \exists x \forall y\, P(y, x)$

Solution:

	A	B	C	D
A	$\longleftrightarrow$	-	-	-
B	$\models$	$\longleftrightarrow$	-	$\models$
C	$\models$	-	$\longleftrightarrow$	-
D	$\models$	-	-	$\longleftrightarrow$

$B \models A, \ldots$