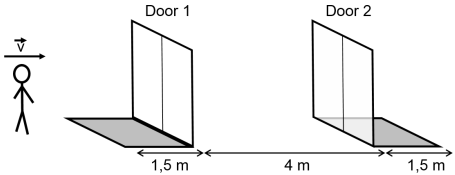
Embedded systems
Exercise session 8
Hybrid systems

Guidelines

1. Read carefully the problem statement.
2. For unspecified details, make reasonable choices.
3. Decompose the problem into some number of processes.
4. Choose your variables and **document them**.
5. For each process, determine the number of control locations needed to represent the possible behaviors.
6. Organize the communication between processes.
7. Write the invariants, activities, guards and actions.

Problem 1

A microcontroller controls two security doors at the entrance of a bank, located 4 meters apart. Both doors are equipped with a sensor that is able to detect people. The range of the sensors is depicted here



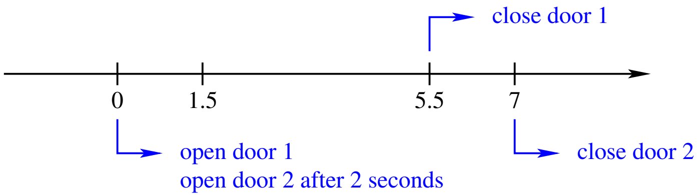
It is assumed that people always move from left to right, i.e., that they first go through door 1 and then through door 2. The microcontroller controls the doors as follows:

- Door 1 opens as soon as its sensor detects someone, at a rate of 67% per second (in other words, it takes about 1.5 s for it to become fully open). It closes, at a rate of 50% per second, when the sensor of door 2 picks up someone.
- Door 2 starts to open, at a rate of 50% per second, exactly 2 seconds after the sensor of door 1 has detected someone. It closes, at the same rate, when the sensor of door 2 stops sensing.

A person can pass through a door only if it is at least 50% open. Otherwise, he/she waits for the door to open enough. It is assumed that at most one person can use the doors at any time, in other words, a new person may approach door 1 only after the previous one has left the sensing area of door 2.

1. Describe a hybrid system modelling this situation.
2. Give the first 3 steps of the space-state exploration of this system, in the case of a person moving at a speed of 1.75 m/s.

Problem digest:



- At least 7 meters between two successive persons.
- No assumption on the speed of the person (other than $v > 0$).

Processes and variables:

- P_1: movement of the person, $x_1 =$ current position.
- P_2: door 1, $x_2 =$ percentage of openness (0–100).
- P_3: door 2, $x_3 =$ percentage of openness (0–100).

Processes and variables:

- P_1: movement of the person, $x_1 =$ current position.
- P_2: door 1, $x_2 =$ percentage of openness (0–100).
- P_3: door 2, $x_3 =$ percentage of openness (0–100).
- P_4: 2-second timer, $x_4 =$ elapsed time.

Processes and variables:

- P_1: movement of the person, $x_1 =$ current position.
- P_2: door 1, $x_2 =$ percentage of openness (0–100).
- P_3: door 2, $x_3 =$ percentage of openness (0–100).
- P_4: 2-second timer, $x_4 =$ elapsed time.

Initial conditions:

- The person is out ($x_1 \leq 0$).
- The doors are closed.

Process 1

Modes of operation:

- [1]: $x_1 \leq 0$ or $x_1 \geq 7$
- [2]: $0 \leq x_1 \leq 1.5$
- [3]: $1.5 \leq x_1 \leq 5.5$
- [4]: $5.5 \leq x_1 \leq 7$

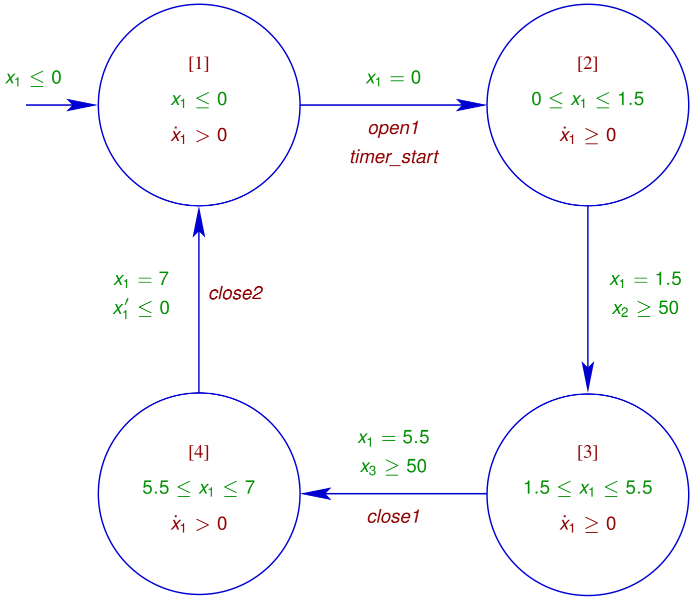
Process 1

Modes of operation:

- [1]: $x_1 \leq 0$ or $x_1 \geq 7$
- [2]: $0 \leq x_1 \leq 1.5$
- [3]: $1.5 \leq x_1 \leq 5.5$
- [4]: $5.5 \leq x_1 \leq 7$

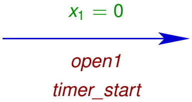
Communication:

- Values of x_2 and x_3 read to pass doors.
- Synchronization labels *open1*, *close1* with P_2, *close2* with P_3, *timer_start* with P_4.

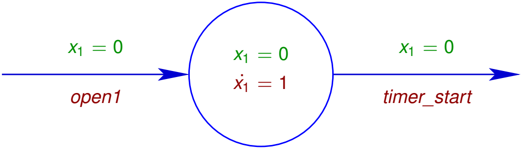


$x_1 \leq 0$
[1]
$x_1 \leq 0$
$\dot{x}_1 > 0$
$x_1 = 0$
open1
timer_start
[2]
$0 \leq x_1 \leq 1.5$
$\dot{x}_1 \geq 0$
$x_1 = 1.5$
$x_2 \geq 50$
[3]
$1.5 \leq x_1 \leq 5.5$
$\dot{x}_1 \geq 0$
$x_1 = 5.5$
$x_3 \geq 50$
close1
[4]
$5.5 \leq x_1 \leq 7$
$\dot{x}_1 > 0$
$x_1 = 7$
$x_1' \leq 0$
close2

Note: Strictly speaking,



is not correct, since it links 3 processes rather than 2. But we consider this construction to be a shorthand for



Process 2

Modes of operation:

- [1]: opening door
- [2]: closing door
- [3]: door open
- [4]: door closed

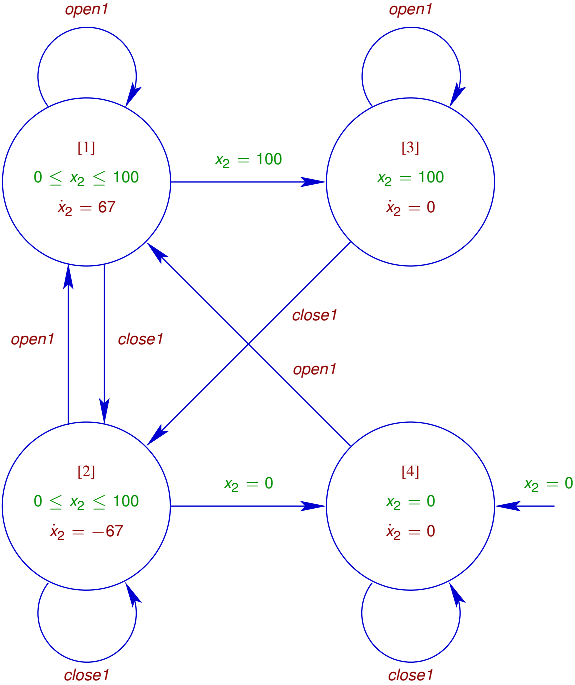
Process 2

Modes of operation:

- [1]: opening door
- [2]: closing door
- [3]: door open
- [4]: door closed

Communication:

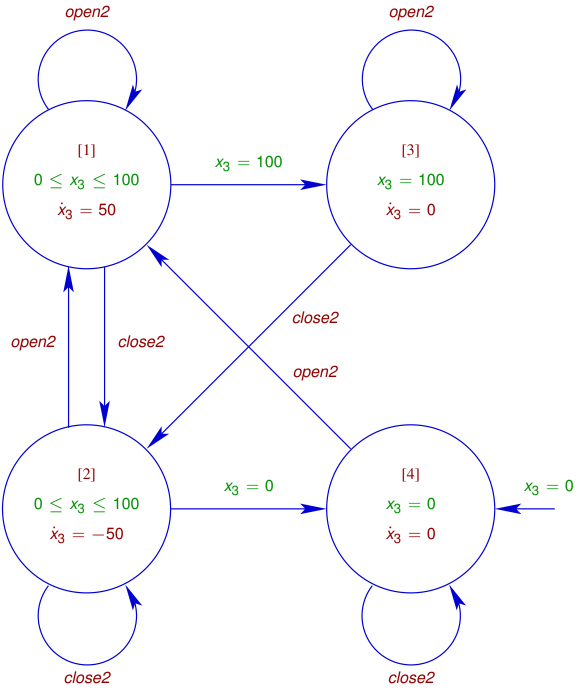
- Provides the value of x_2.
- Reacts to the labels *open1* and *close1*.



open1
open1
[1]
$0 \leq x_2 \leq 100$
$\dot{x}_2 = 67$
$x_2 = 100$
[3]
$x_2 = 100$
$\dot{x}_2 = 0$
close1
open1
close1
open1
[2]
$0 \leq x_2 \leq 100$
$\dot{x}_2 = -67$
$x_2 = 0$
[4]
$x_2 = 0$
$\dot{x}_2 = 0$
$x_2 = 0$
close1
close1

$\boxed{\text{Process 3}}$

Similar to Process 2, with x_2 replaced by x_3 and *open1*/*close1* by *open2*/*close2*.



open2
open2
[1]
$0 \leq x_3 \leq 100$
$\dot{x}_3 = 50$
$x_3 = 100$
[3]
$x_3 = 100$
$\dot{x}_3 = 0$
close2
open2
close2
open2
[2]
$0 \leq x_3 \leq 100$
$\dot{x}_3 = -50$
$x_3 = 0$
[4]
$x_3 = 0$
$\dot{x}_3 = 0$
$x_3 = 0$
close2
close2

$\boxed{\text{Process 4}}$

Modes of operation:

- [1]: idle
- [2]: timer is active

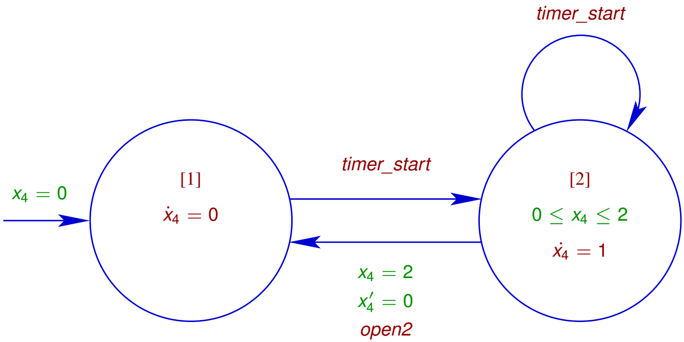
$\boxed{\text{Process 4}}$

Modes of operation:

- [1]: idle
- [2]: timer is active

Communication:

- Reacts to *timer_start*
- Triggers *open2*.


timer_start
[1]
$\dot{x}_4 = 0$
$x_4 = 0$
timer_start
[2]
$0 \leq x_4 \leq 2$
$\dot{x}_4 = 1$
$x_4 = 2$
$x_4' = 0$
open2

State-space exploration:

$([1],[4],[4],[1]) : x_1 \leq 0,\ x_2 = 0,\ x_3 = 0, x_4 = 0.$

$$\Longrightarrow \quad ([1],[4],[4],[1]) \quad : \quad \begin{array}{l} x_1 \leq 0, x_2 = 0 \\ x_3 = 0,\ x_4 = 0. \end{array}$$

$$\xrightarrow[]{\substack{open1 \\ timer\_start}} \quad ([2],[1],[4],[2]) \quad : \quad \begin{array}{l} x_1 = 0, x_2 = 0, \\ x_3 = 0,\ x_4 = 0. \end{array}$$

$$\overset{\leq \frac{1.5}{1.75}}{\Longrightarrow} \quad ([2],[1],[4],[2]) \quad : \quad \begin{array}{l} x_1 = 1.75\,t, x_2 = 67\,t \\ x_3 = 0,\ x_4 = t \\ 0 \leq t \leq \frac{1.5}{1.75} \end{array}$$

$$\longrightarrow \quad \ldots$$

Problem 2

When the sluice gates of a dam are closed, the water level in the reservoir rises at the rate of 0.4 *m/h* if it is between 10 *m* and 20 *m*, and of 0.2 *m/h* between 20 *m* and 30 *m*. Above 30 *m*, a spillway drains the extra water. When the sluice gates are open, the water level decreases at the rate of 0.5 *m/h*, independently from the water level.

A sensor constantly measures the water level in the reservoir, and sends a signal each time that this level changes by 1 *m*. The sluice gates must be opened when the level exceeds 25 *m*, and closed when it drops below 15 *m*. Each opening or closing operation of the gates needs 120 seconds to complete.

1. Construct an hybrid system modelling this problem. Initially, the water level is at 17 *m*, and the sluice gates are open.
2. Give the first 3 steps of the state-space exploration of this system.

Processes and variables:

- P_1: level of the water, $x_1 =$ current level.
- P_2: sensor, $x_2 =$ value of the level at the last signal.
- P_3: gates, $x_3 =$ percentage of closure (0–100).
- P_4: controller.

Processes and variables:

- P_1: level of the water, $x_1 =$ current level.
- P_2: sensor, $x_2 =$ value of the level at the last signal.
- P_3: gates, $x_3 =$ percentage of closure (0–100).
- P_4: controller.

Notes:

- We assume that the water level increases only when the gates are **fully** closed.
- If the water level goes outside the interval $[10, 30]$, the model may deadlock.

Process 1

Modes of operation:

- [1]: $10 \leq x_1 \leq 20$, gates fully closed, level increasing
- [2]: $20 \leq x_1 \leq 30$, gates fully closed, level increasing
- [3]: $x_1 = 30$, gates fully closed, level stable
- [4]: gates (at least partially) open, level decreasing

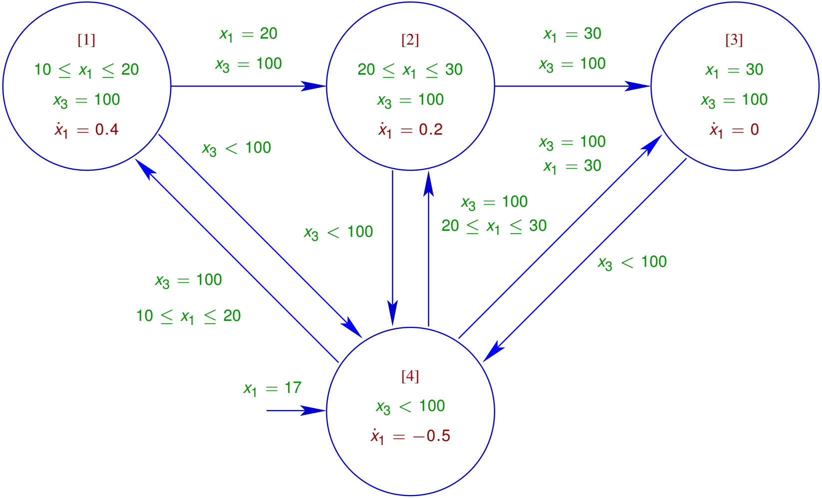
$\boxed{\text{Process 1}}$

Modes of operation:

- [1]: $10 \leq x_1 \leq 20$, gates fully closed, level increasing
- [2]: $20 \leq x_1 \leq 30$, gates fully closed, level increasing
- [3]: $x_1 = 30$, gates fully closed, level stable
- [4]: gates (at least partially) open, level decreasing

Communication:

- Value of x_3 read to know whether the gates are open or not.


[1]
$10 \leq x_1 \leq 20$
$x_3 = 100$
$\dot{x}_1 = 0.4$
[2]
$20 \leq x_1 \leq 30$
$x_3 = 100$
$\dot{x}_1 = 0.2$
[3]
$x_1 = 30$
$x_3 = 100$
$\dot{x}_1 = 0$
[4]
$x_3 < 100$
$\dot{x}_1 = -0.5$
$x_1 = 20$
$x_3 = 100$
$x_1 = 30$
$x_3 = 100$
$x_3 < 100$
$x_3 = 100$
$x_1 = 30$
$x_3 < 100$
$x_3 = 100$
$20 \leq x_1 \leq 30$
$x_3 < 100$
$x_3 = 100$
$10 \leq x_1 \leq 20$
$x_1 = 17$

$\boxed{\text{Process 2}}$

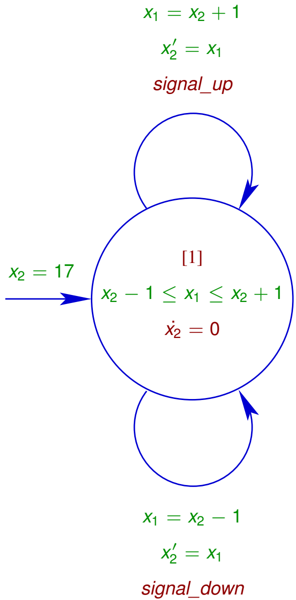
Modes of operation: A single mode from which we check the relative values of x_1 and x_2, and update x_2 whenever necessary.

Process 2

Modes of operation: A single mode from which we check the relative values of x_1 and x_2, and update x_2 whenever necessary.

Communication:

- Reads the current value of x_1.
- Synchronizes with P_4 on the labels *signal_up* and *signal_up*.


$x_1 = x_2 + 1$
$x_2' = x_1$
signal_up
$x_2 = 17$
[1]
$x_2 - 1 \leq x_1 \leq x_2 + 1$
$\dot{x_2} = 0$
$x_1 = x_2 - 1$
$x_2' = x_1$
signal_down

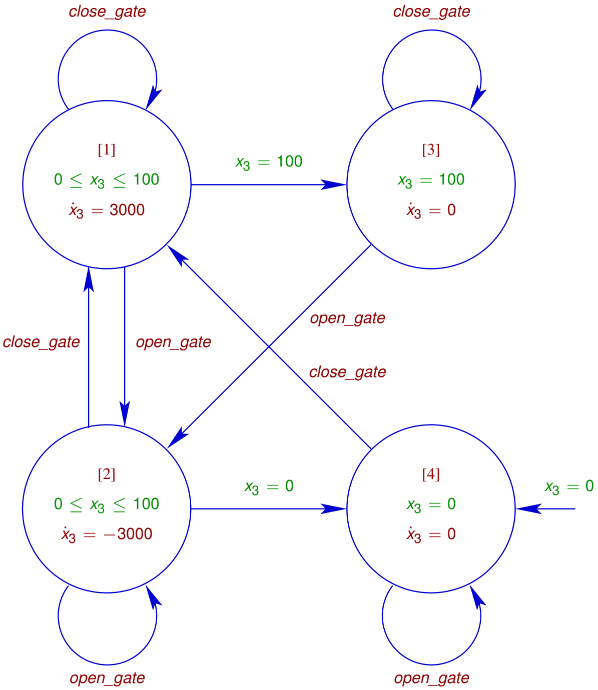
Process 3

Modes of operation:

- [1]: closing gates
- [2]: opening gates
- [3]: gates closed
- [4]: gates open

Communication:

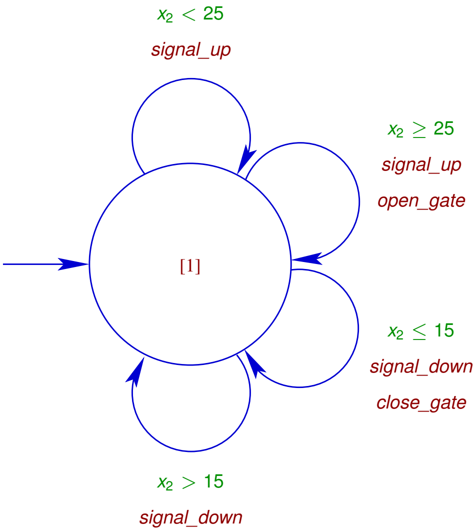
- Provides the value of x_3.
- Reacts to the labels *open_gate* and *close_gate*.



close_gate
close_gate
[1]
$0 \leq x_3 \leq 100$
$\dot{x}_3 = 3000$
$x_3 = 100$
[3]
$x_3 = 100$
$\dot{x}_3 = 0$
open_gate
close_gate
open_gate
close_gate
[2]
$0 \leq x_3 \leq 100$
$\dot{x}_3 = -3000$
$x_3 = 0$
[4]
$x_3 = 0$
$\dot{x}_3 = 0$
$x_3 = 0$
open_gate
open_gate

Process 4

Modes of operation: A single mode from which we check the value of x_2 when *signal_up* and *signal_down* are received, and and emit *open_gate* and *close_gate* orders.


$x_2 < 25$
signal_up
$x_2 \geq 25$
signal_up
open_gate
[1]
$x_2 \leq 15$
signal_down
close_gate
$x_2 > 15$
signal_down

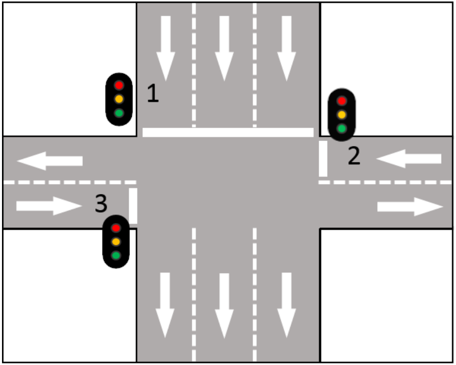
State-space exploration:

$([4],[1],[4],[1]) : x_1 = 17,\ x_2 = 17,\ x_3 = 0.$

$$
\begin{array}{lll}
\stackrel{\le 2}{\Longrightarrow} & ([4],[1],[4],[1]) & : \quad 16 \le x_1 \le 17, x_2 = 17, \\
 & & \quad\; x_3 = 0. \\
\xrightarrow{\mathit{signal\_down}} & ([4],[1],[4],[1]) & : \quad x_1 = 16, x_2 = 16, \\
 & & \quad\; x_3 = 0. \\
\stackrel{\le 2}{\Longrightarrow} & ([4],[1],[4],[1]) & : \quad 15 \le x_1 \le 16, x_2 = 16, \\
 & & \quad\; x_3 = 0. \\
\longrightarrow & \ldots &
\end{array}
$$

Problem 3

A smart traffic light system is installed at the intersection of two roads. There are three stop lights $\{1, 2, 3\}$ that can either be red, green, or orange. The state of 2 and 3 is identical at all times.



Traffic lights 2 and 3 are red and traffic light 1 is green as long as there are less than six cars waiting in front of 2 or 3. When this threshold is reached, stop light 1 becomes orange for 5 seconds before switching to red. At this time, stop lights 2 and 3 become green for 15 seconds. After that delay, they change to orange for 5 seconds and then switch to red as traffic light 1 becomes green again.

The incoming flows of cars at the three traffic lights are respectively $f_1 = 30$, $f_2 = 6$ and $f_3 = 3$ cars/minute. We also define the saturation flow of a traffic light as the rate of cars that are able to cross this light when it is green. The saturation flows of the three lights are respectively $s_1 = 1.5$, $s_2 = 0.5$ and $s_3 = 0.5$ cars/second.

1. Construct a hybrid system that models this traffic management system.
2. Give the first 3 steps of the space-state exploration of this system, when initially no cars are queueing in front of the traffic lights, and stop lights 2 and 3 are red while 1 is green.