ISEL

INSTITUTO SUPERIOR DE ENGENHARIA DE LISBOA

PROCESSAMENTO DE IMAGEM E BIOMETRIA

IMAGE PROCESSING AND BIOMETRICS

6. **FREQUENCY FILTERING (part 1)**

Summary (part 1)

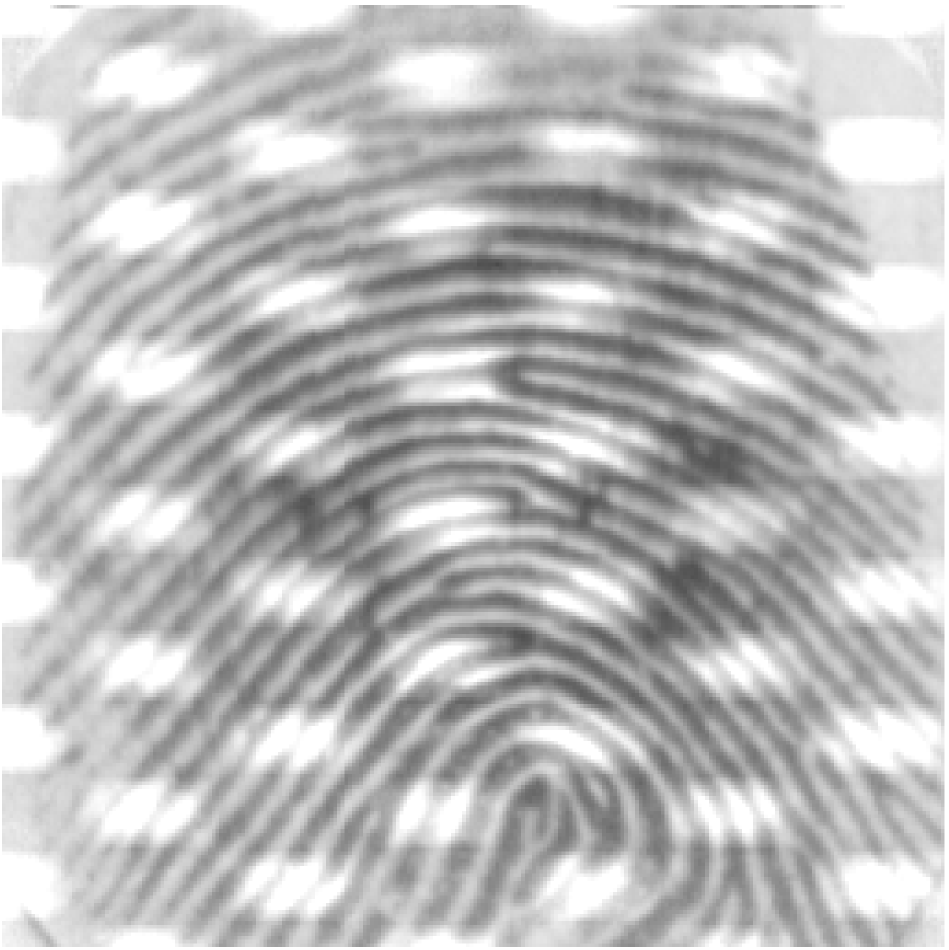
- The limitations of spatial filtering
  - Periodic noise
  - Periodic patterns
- A review on 1D signal filtering
- 2D signal filtering (image filtering)
- The DFT and IDFT
- Image spectrum, module and phase
- Some simple filtering operations

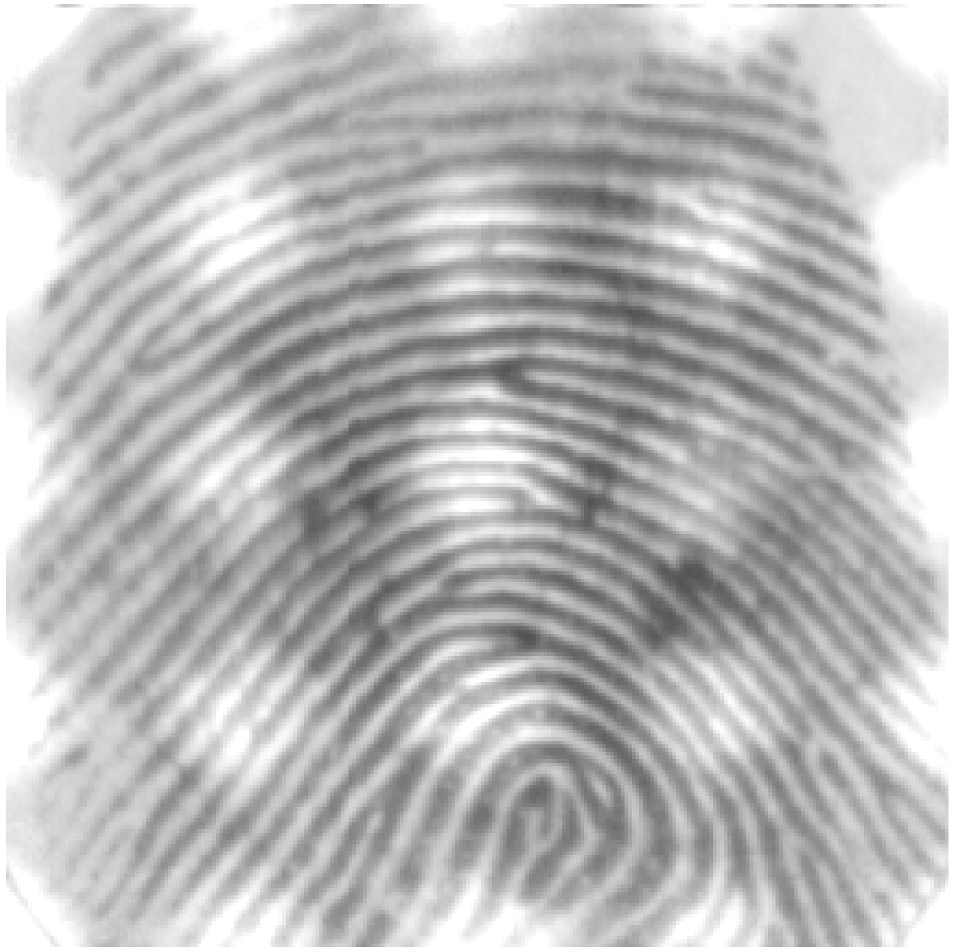
The limitations of spatial filtering (1)

FIGURE 4.21
A newspaper image of size 246 × 168 pixels sampled at 75 dpi showing a moiré pattern. The moiré pattern in this image is the interference pattern created between the ±45° orientation of the halftone dots and the north–south orientation of the sampling grid used to digitize the image.

Periodic noise/interference

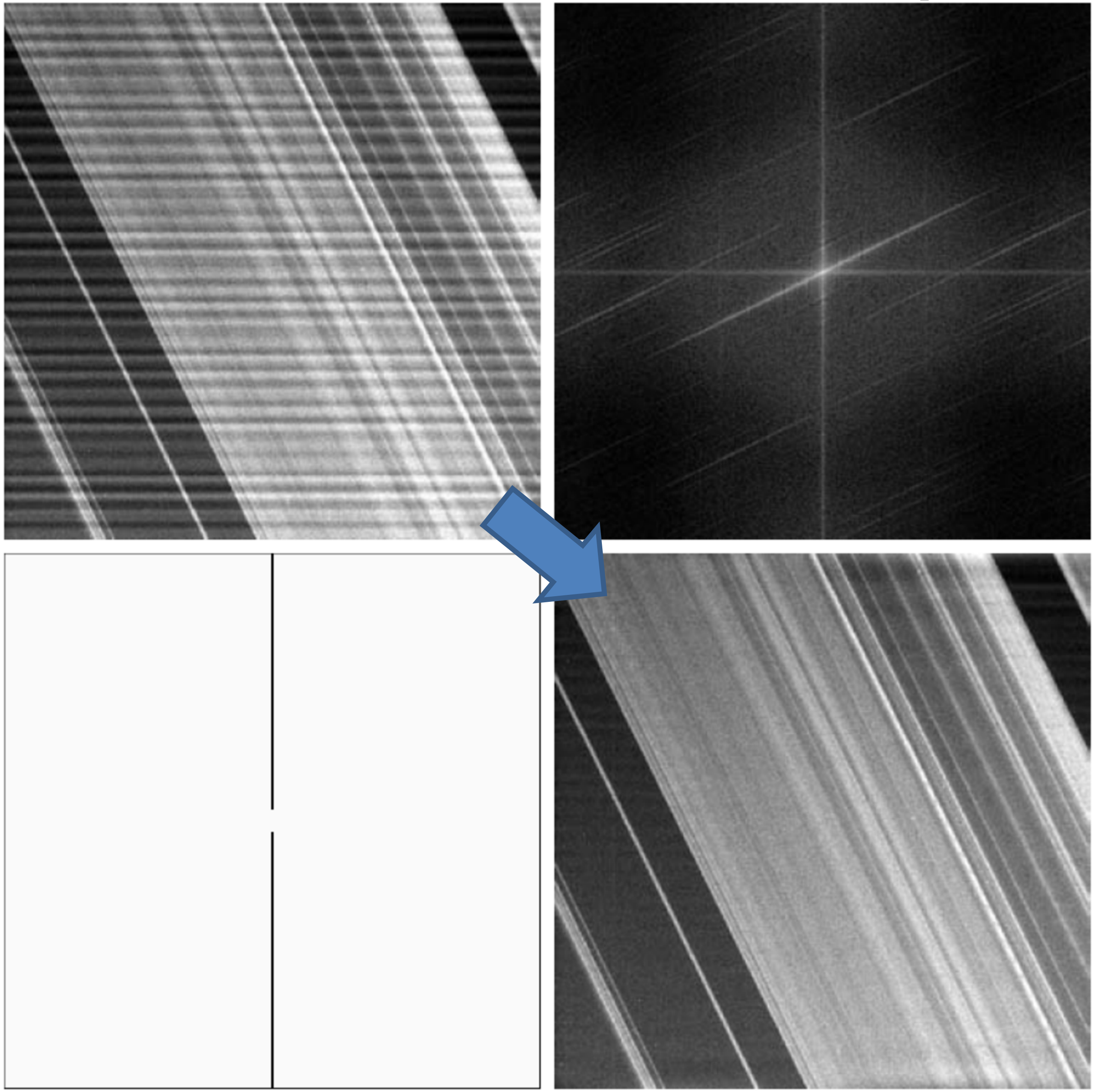
The limitations of spatial filtering (2)





How to remove the noise/interference from these images?
Use of linear or non-linear spatial filtering?!

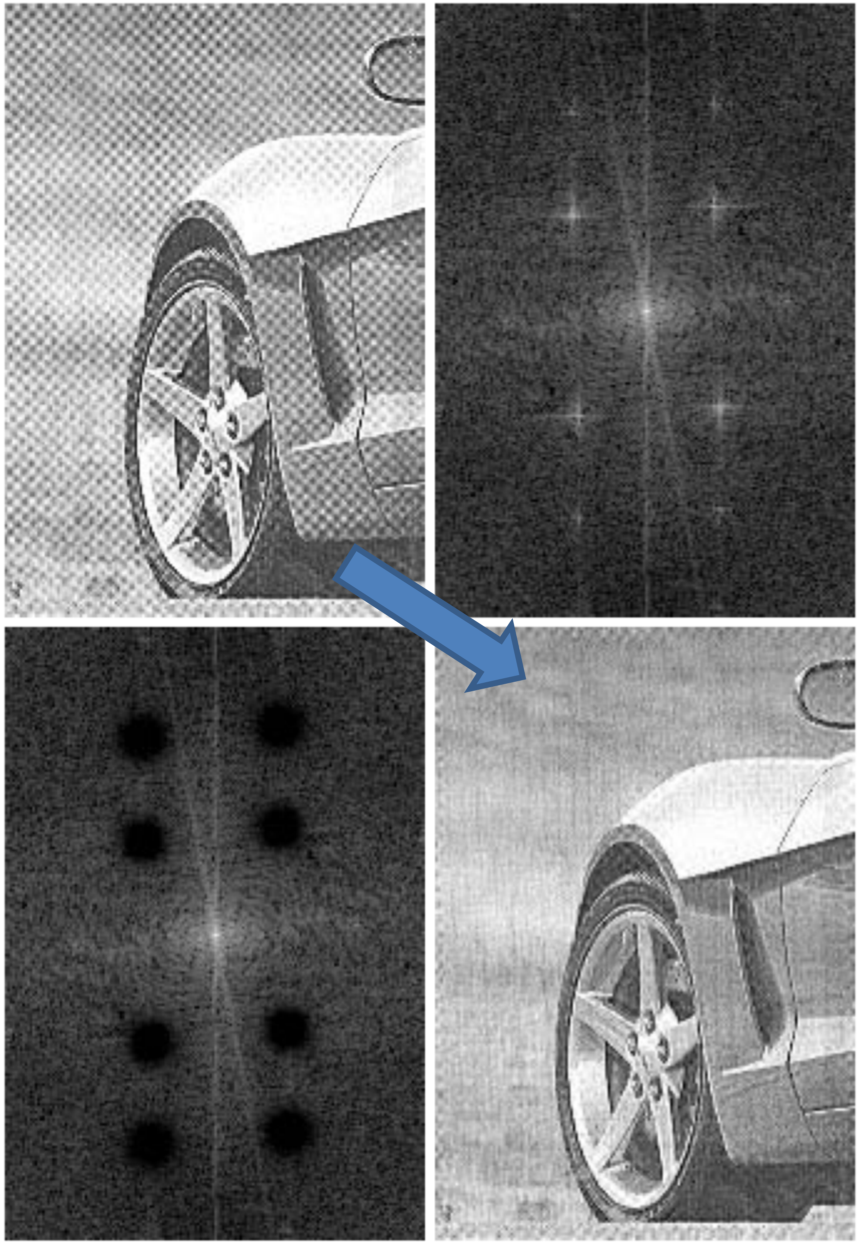
The limitations of spatial filtering (3)



a b
c d

FIGURE 4.65 (a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)

The limitations of spatial filtering (4)

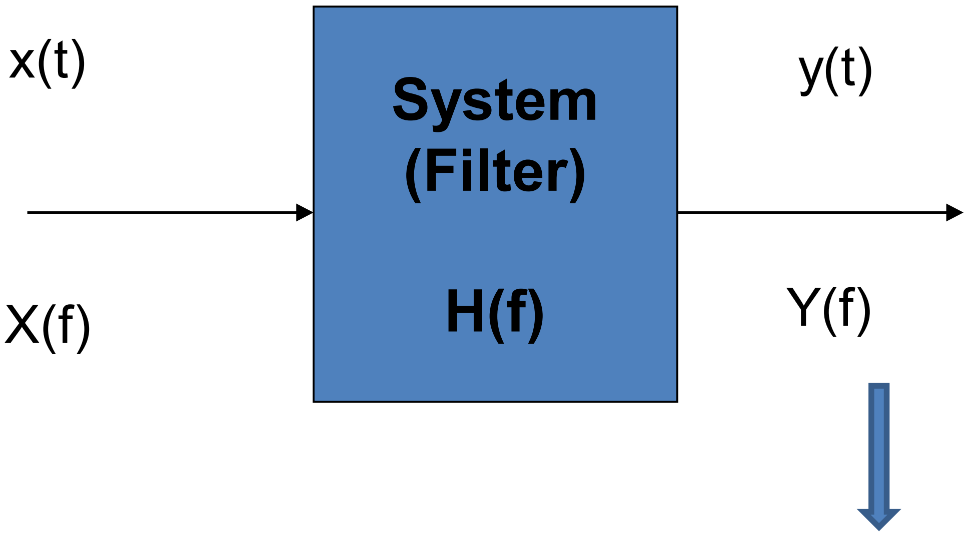


How to remove the noise/interference from these images?

Apply frequency-based filtering techniques!

A review on 1D signal filtering (1)

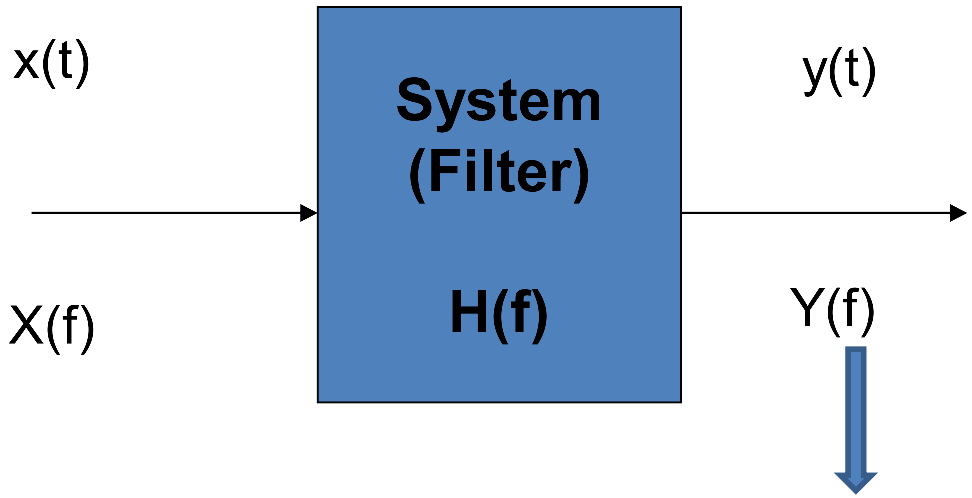
- The **frequency response H(f)** of a system, describes how it operates on the frequency domain
- It states the gain that the system applies to each frequency



$$Y(f) = X(f)H(f)$$

A review on 1D signal filtering (2)

- The frequency components of the output spectrum are the ones:
  - Found in the input signal
  - That the system has gain different from zero



$$Y(f) = X(f)H(f)$$

A review on 1D signal filtering (3)

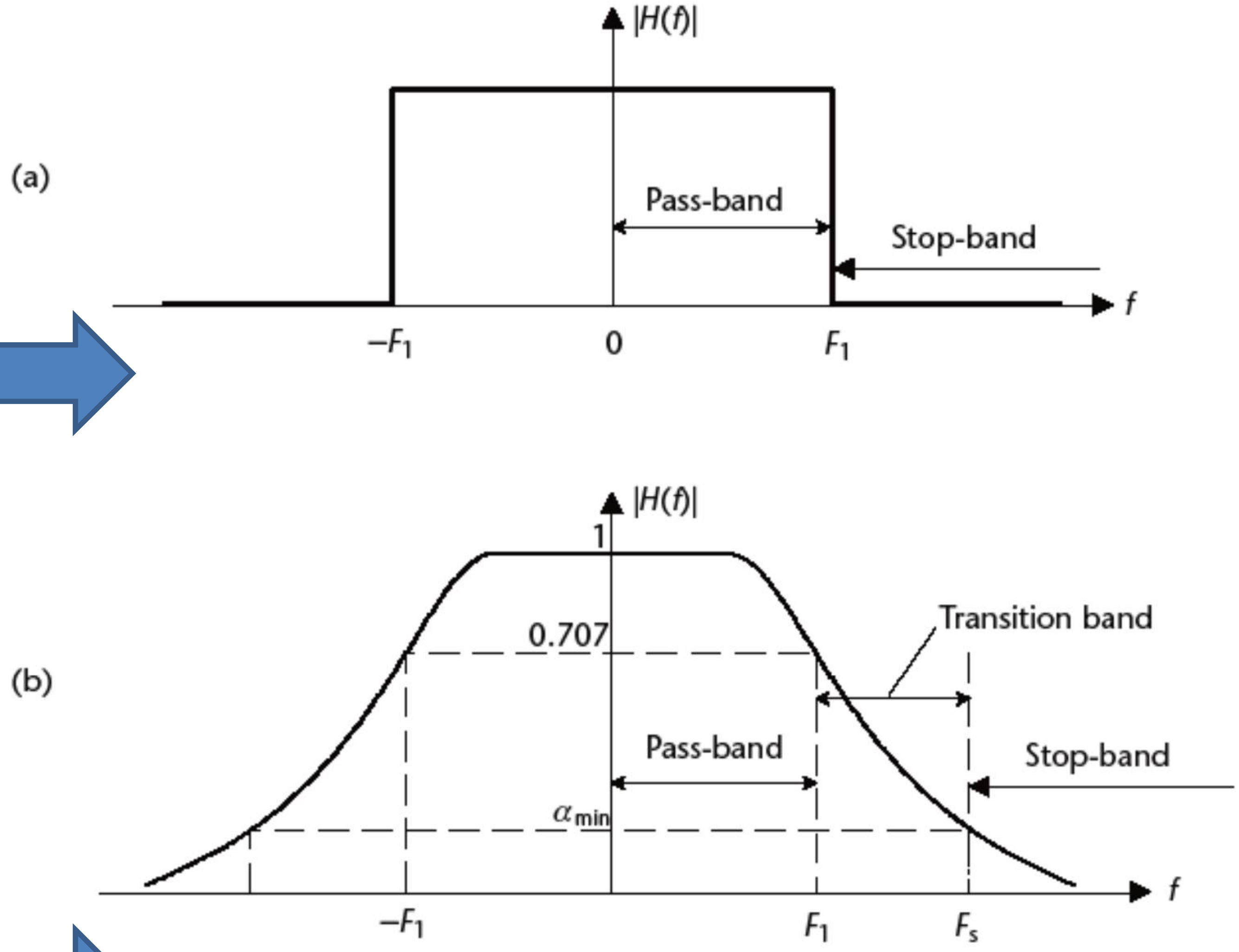
- The output spectrum is $Y(f) = X(f)H(f)$
- The frequency components of X(f) that are not present in Y(f) are **filtered** (removed) by the system
- The type of filtering is defined by the frequency response function, H(f)
- There are four common types of filtering:
  - Low-pass filter (LPF)
  - High-pass filter (HPF)
  - Band-Pass filter (BPF)
  - Band-Reject filter (BRF)

A review on 1D signal filtering (4)

Figure 2.29

Ideal
Low-Pass Filter

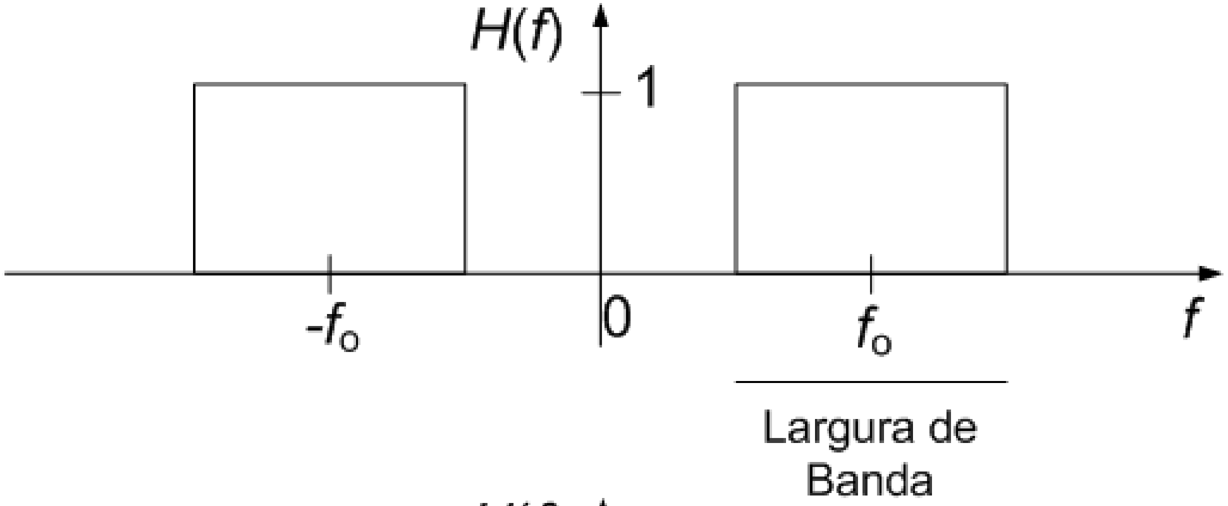
Realizable
Low-Pass Filter



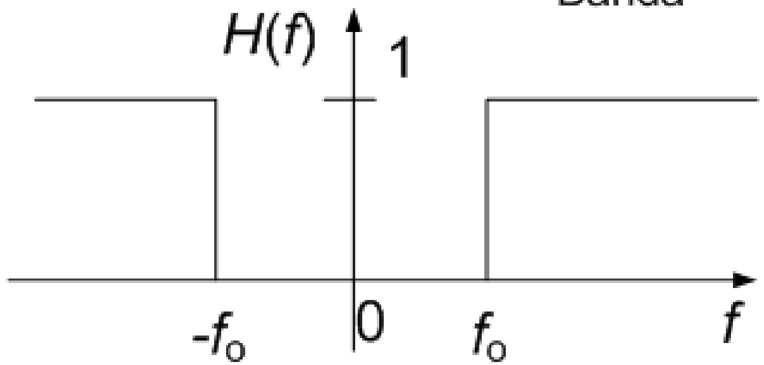
Taken from *Communication Engineering Principles*, © Ifiok Otung, published 2001 by Palgrave

A review on 1D signal filtering (5)

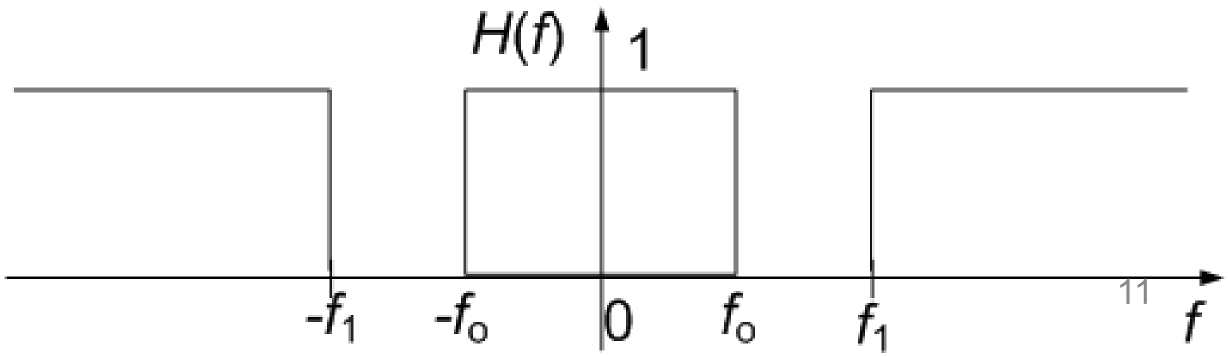
Ideal
Band-Pass Filter



Ideal
High-Pass Filter



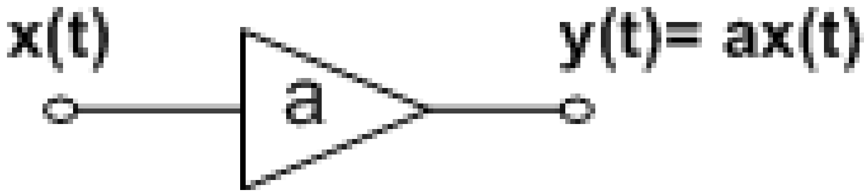
Ideal
Band-Reject Filter



A review on 1D signal filtering (6)

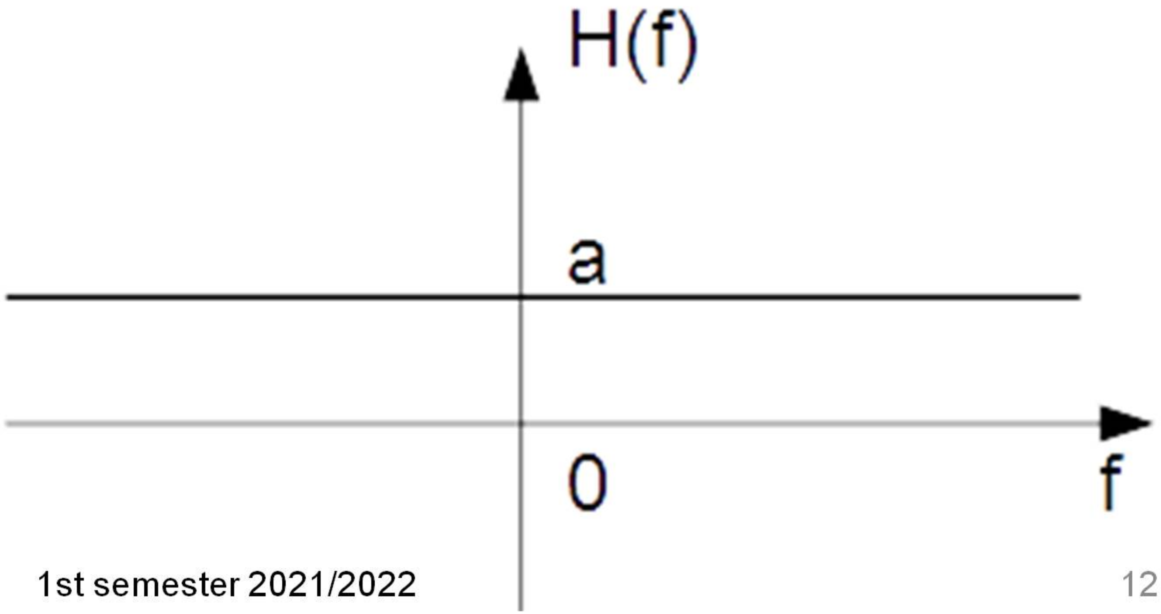
- Example: amplifier system
- All frequencies are treated the same way, since the system applies the same gain for all frequencies

$$Y(f) = H(f)X(f) = aX(f)$$



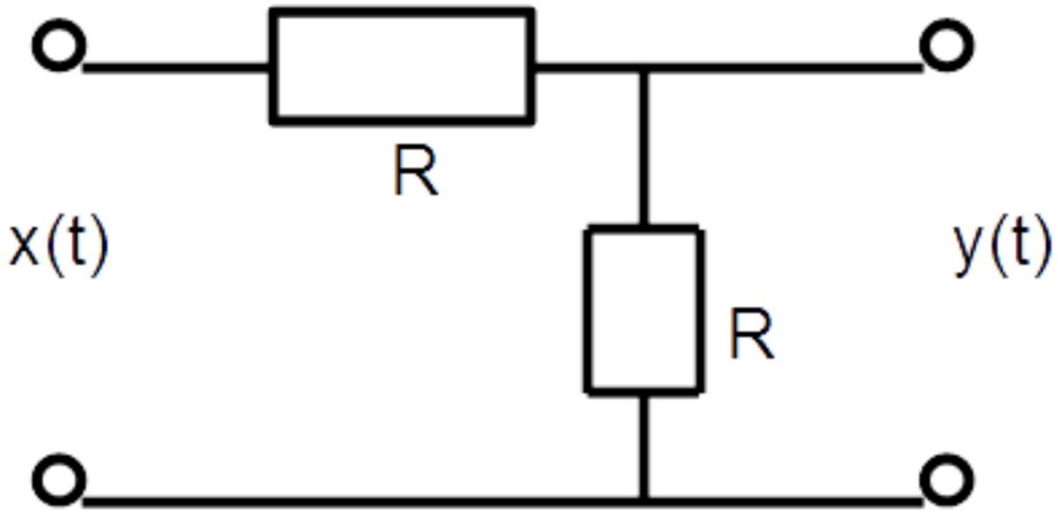
$$H(f) = \frac{Y(f)}{X(f)} = a$$

$$h(t) = a\delta(t)$$



A review on 1D signal filtering (7)

- Example: amplitude divider



$$y(t) = 0{,}5x(t)$$

$$h(t) = 0{,}5\delta(t)$$

$$Y(f) = 0{,}5X(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = 0{,}5$$

A review on 1D signal filtering (8)

Suppose you have an ideal low-pass-filter with cutoff frequency f_c = 40 kHz and unitary gain on the pass band.

Let $x(t) = 3 + 2\cos(2\pi\, 5000\, t) + 2\cos(2\pi\, 25000\, t) + 2\cos(2\pi\, 55000\, t)$ be the signal on the input of the system.

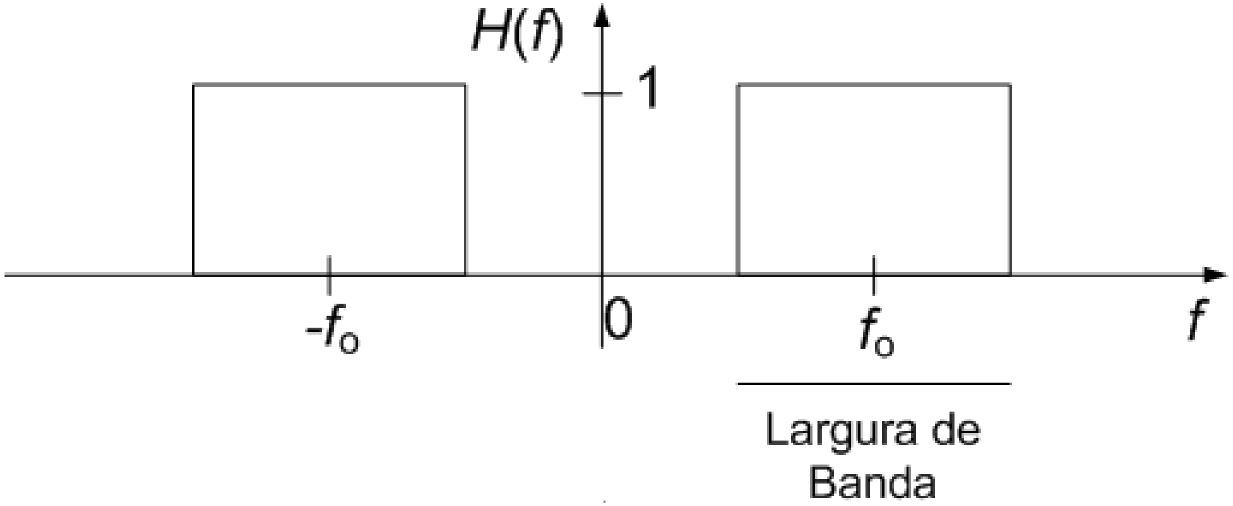
Then, the output signal is $y(t) = 3 + 2\cos(2\pi\, 5000\, t) + 2\cos(2\pi\, 25000\, t)$.

Suppose now that the filter has cutoff frequency f_c = 20 kHz and gain of 2.5, on the pass band, the output signal (with x(t) as input) is

$$y(t) = 2.5 \times 3 + 2.5 \times 2\cos(2\pi\, 5000\, t) = 7.5 + 5\cos(2\pi\, 5000\, t).$$

A review on 1D signal filtering (9)

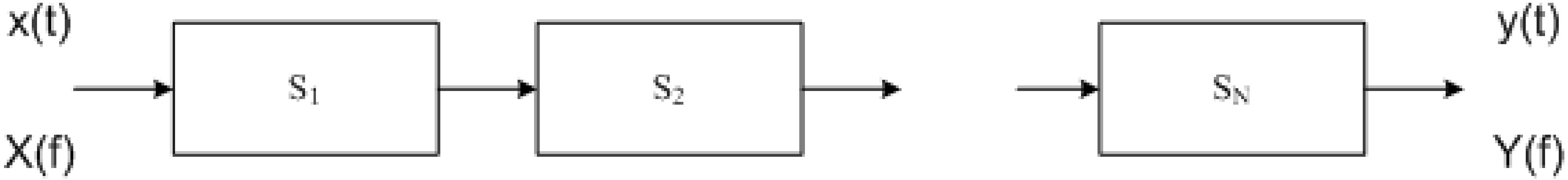
Suppose now that we have an ideal band-pass filter with central band frequency f_o = 30 kHz, and bandwith of 20 kHz, with unitary gain on the pass-band.



With x(t) = 3 + 2cos(2 π 5000 t) + 2cos(2 π 25000 t) + 2cos(2 π 45000 t) present on the input of the system, the output signal is y(t) = 2 cos(2 π 25000 t).

A review on 1D signal filtering (10)

- Cascade of systems

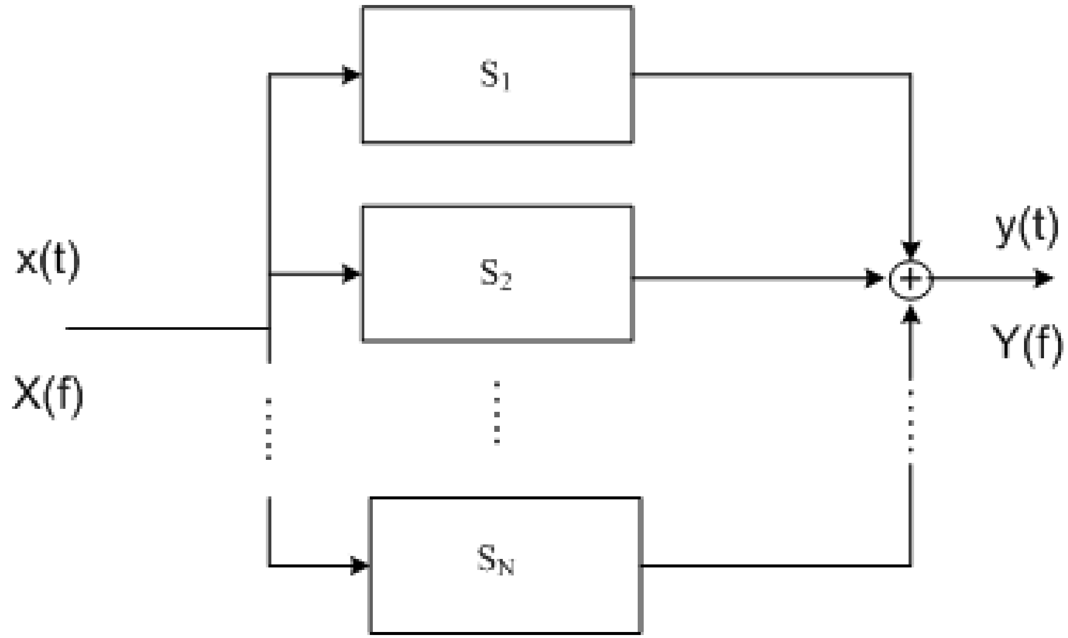


- The equivalent frequency response is the **product** of the each frequency response

$$H_{eq}(f) = H_1(f)H_2(f)..H_N(f)$$
$$= \prod_{k=1}^{N} H_k(f)$$

A review on 1D signal filtering (11)

- Parallel of systems



- The equivalent frequency response is the **sum** of the each frequency response

$$H_{eq}(f) = H_1(f) + H_2(f) + \ldots + H_N(f)$$
$$= \sum_{k=1}^{N} H_k(f)$$

2D signal filtering (image filtering) (1)

- The filtering procedure for images (2D signals) is similar
- Let f[m,n] be the image on the original spatial domain
- Let F[u,v] be the spectrum of f[m,n]
- Suppose you have a **filter H[u,v]** defined on the frequency domain
- Then, this **image F[u,v] filtered with H[u,v]**, will yield:

$$G[u,v] = F[u,v]\, H[u,v]$$

which is the spectrum of the filtered image

2D signal filtering (image filtering) (2)

- F[u,v] is the spectrum of f[m,n]
- F[u,v] is computed by the **Discrete Fourier Transform (DFT)** of the input image f[m,n]
- F[u,v] = DFT[f[m,n]]

$$F[u,v]=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f[m,n]\exp\left(-\frac{j2\pi u}{M}m\right)\exp\left(-\frac{j2\pi v}{N}n\right)$$

- F[u,v] is the **image spectrum** (represented by complex numbers)
- MATLAB: fft2.m and fftshift.m

https://www.mathworks.com/help/matlab/ref/fft2.html

https://www.mathworks.com/help/matlab/ref/fftshift.html

2D signal filtering (image filtering) (3)

- F[u,v] = DFT[f[m,n]] is the spectrum of f[m,n]

- |F[u,v]| is the **module** of the spectrum
  - Shows how the energy is spread over the (horizontal and vertical) frequency components

- arg[F[u,v]] is the **argument/phase** of the spectrum
  - Typically exhibits a rapidly changing textured pattern

$$F[u,v] = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f[m,n]\exp\left(-\frac{j2\pi u}{M}m\right)\exp\left(-\frac{j2\pi v}{N}n\right)$$

2D signal filtering (image filtering) (4)

- F[u,v] is the spectrum of f[m,n]
- f[m,n] is computed by the **Inverse Discrete Fourier Transform (IDFT)** of the spectrum F[u,v]
- f[m,n] = IDFT[F[u,v]]

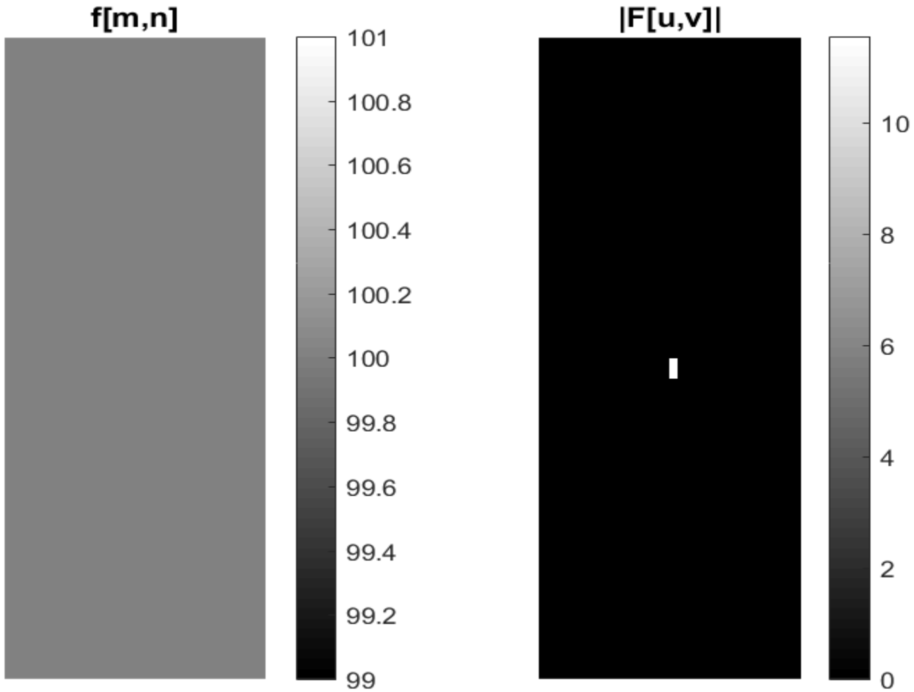
$$f[m,n] = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F[u,v] \exp\left(\frac{j2\pi m}{M} u \right) \exp\left(\frac{j2\pi n}{N} v \right)$$

- MATLAB: ifft2.m and ifftshift.m

https://www.mathworks.com/help/matlab/ref/ifft2.html

https://www.mathworks.com/help/matlab/ref/ifftshift.html

Image Spectrum – Module (1)




```
% Compute the DFT
F = fft2(f);

 % Center the spectrum
F = fftshift(F);

% Display the image
figure(1);
subplot(121); imagesc(f);
title(' f[m,n] '); axis off; colorbar;

% Display the module of the spectrum
subplot(122); imagesc( log( 1 + abs(F)) );
colormap('gray'); title(' |F[u,v]| ');
axis off; colorbar;
```

Image Spectrum – Module (2)

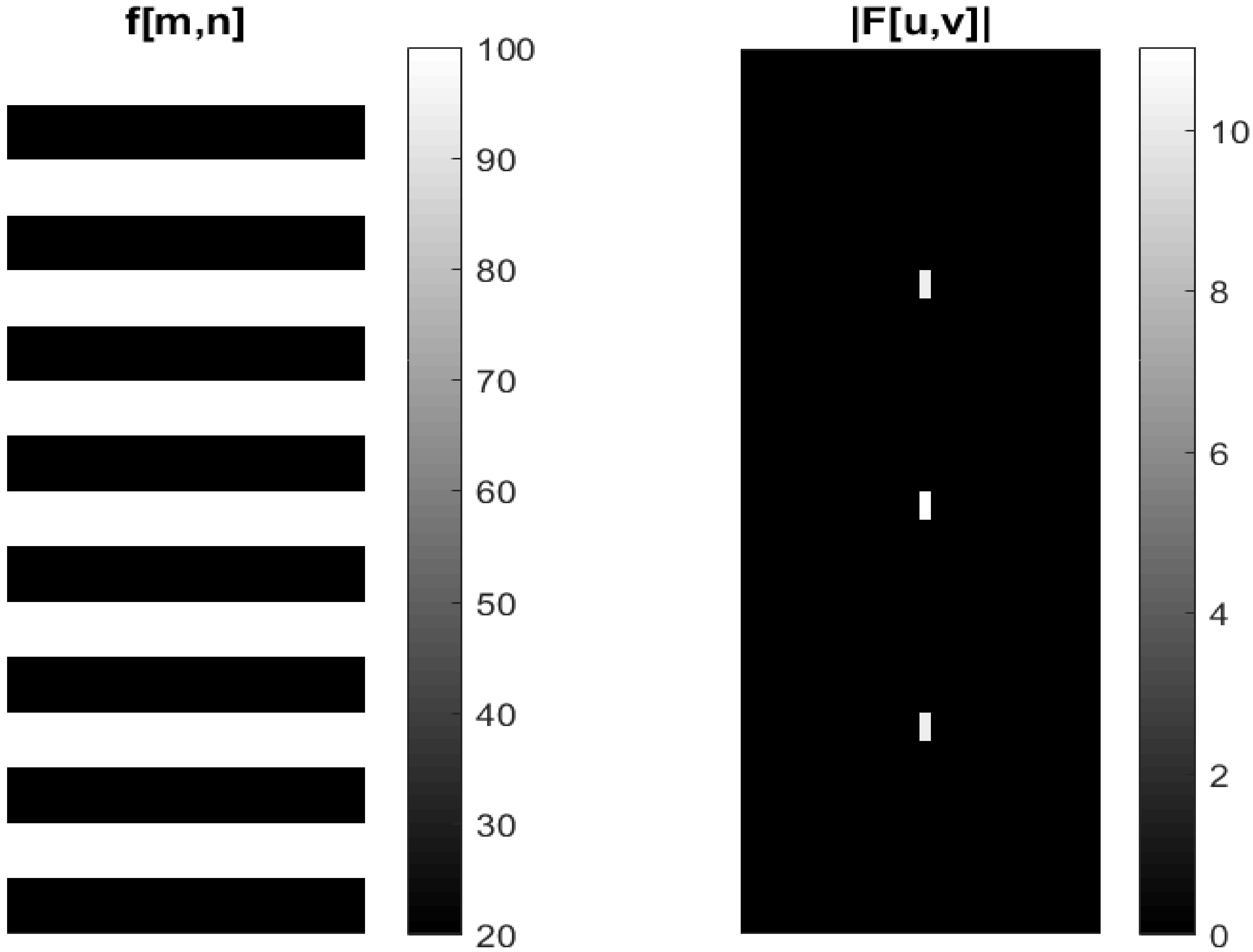


Image Spectrum – Module (3)

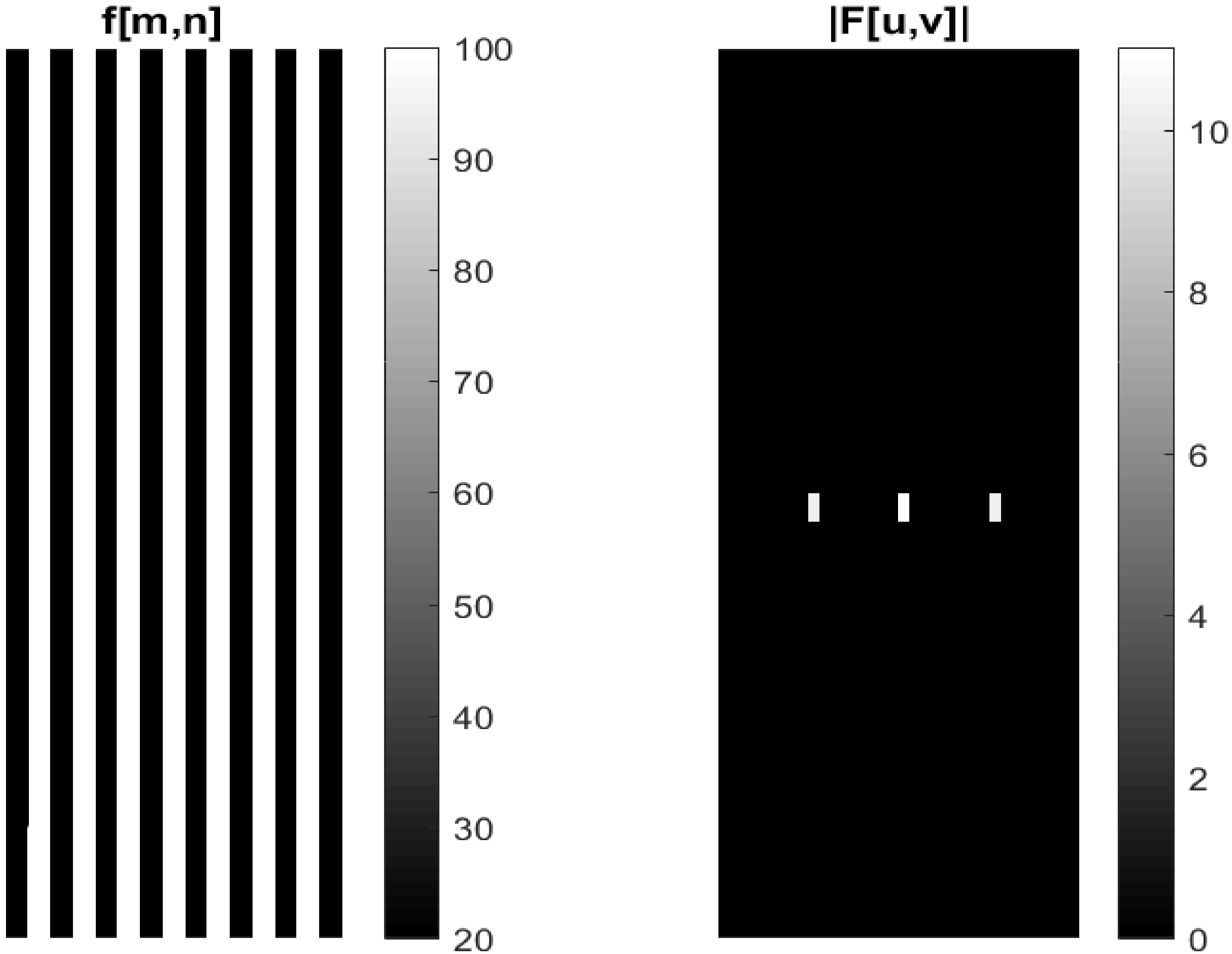


Image Spectrum – Module (4)

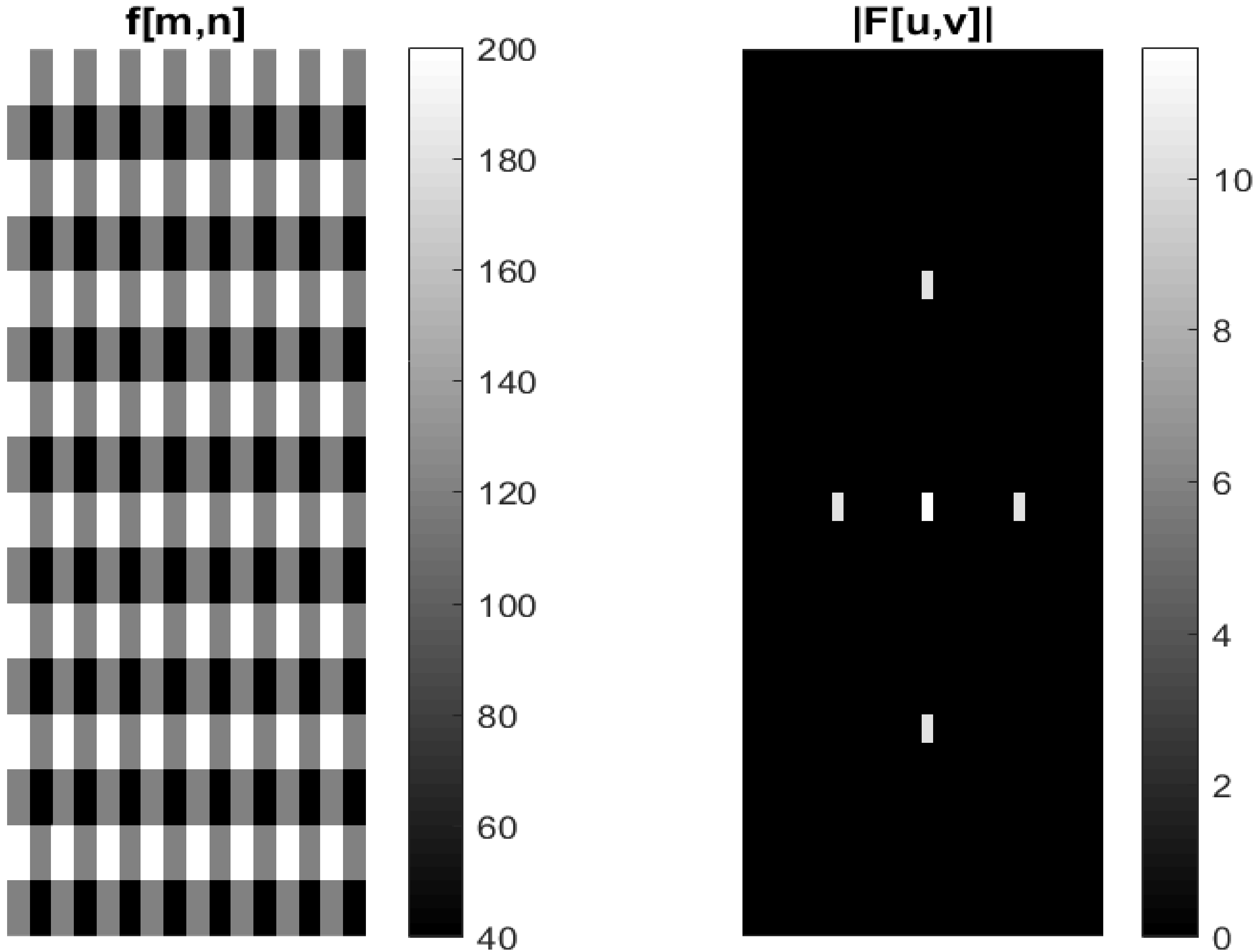


Image Spectrum – Module (5)

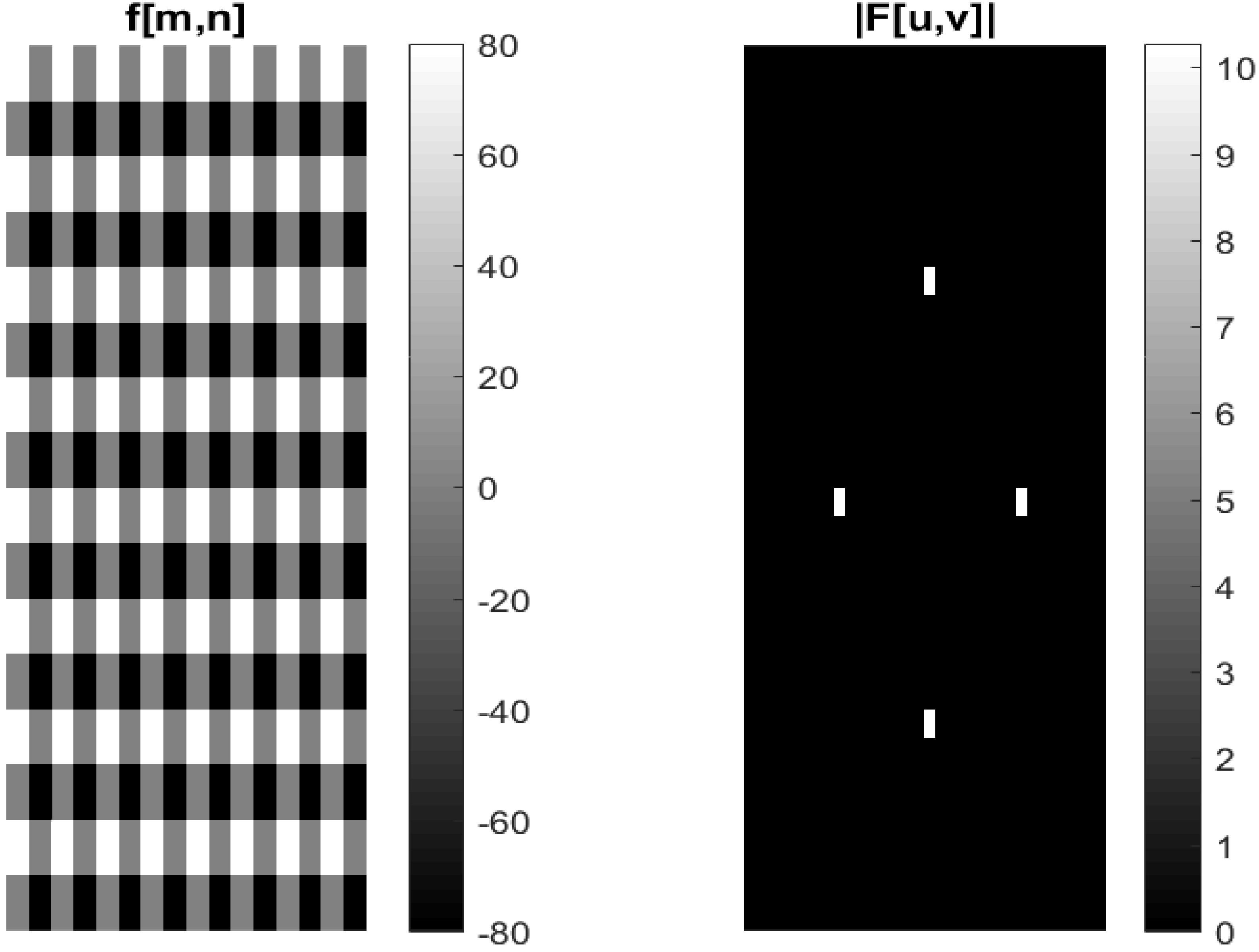


Image Spectrum – Module (6)

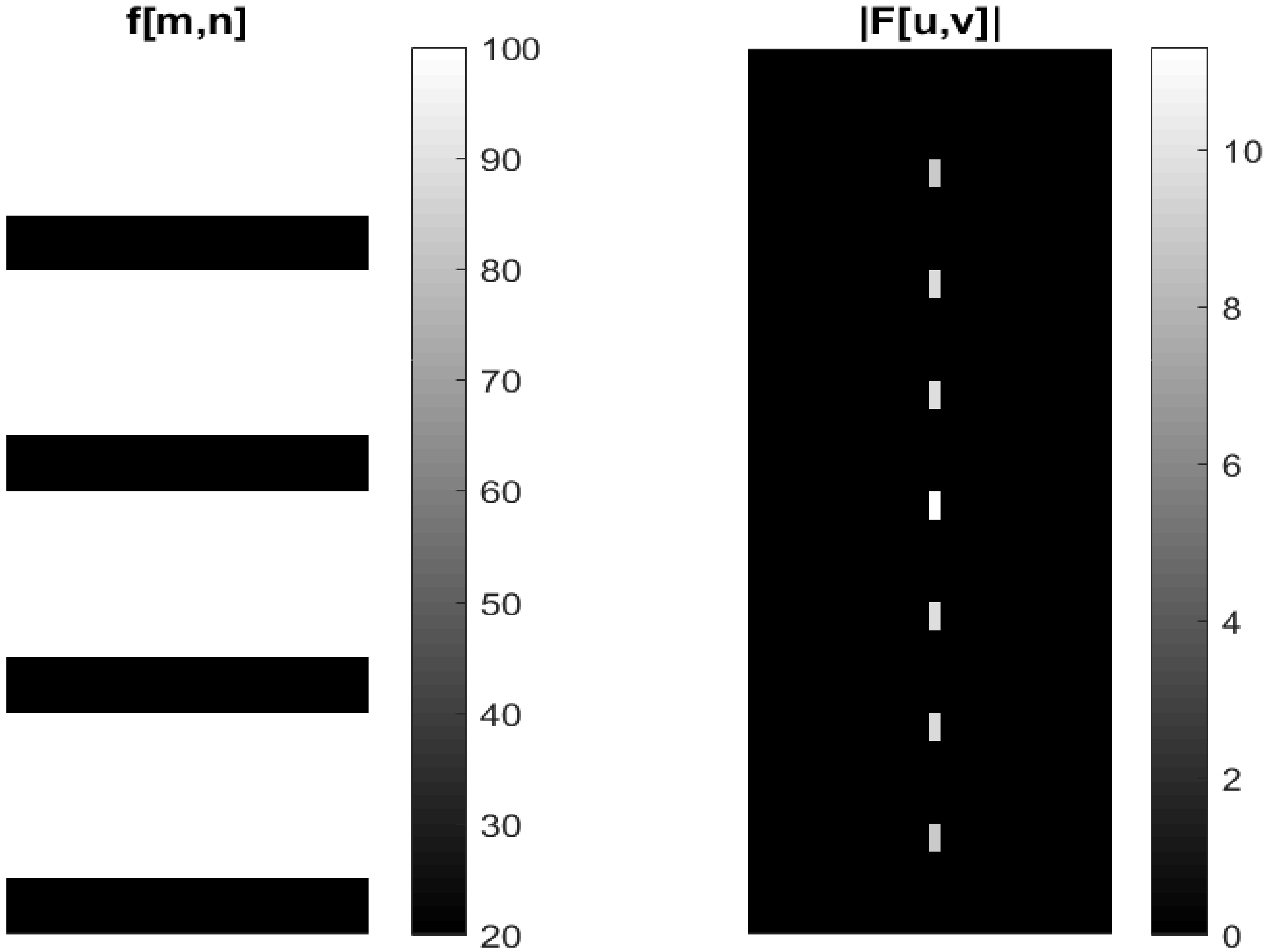


Image Spectrum – Module (7)

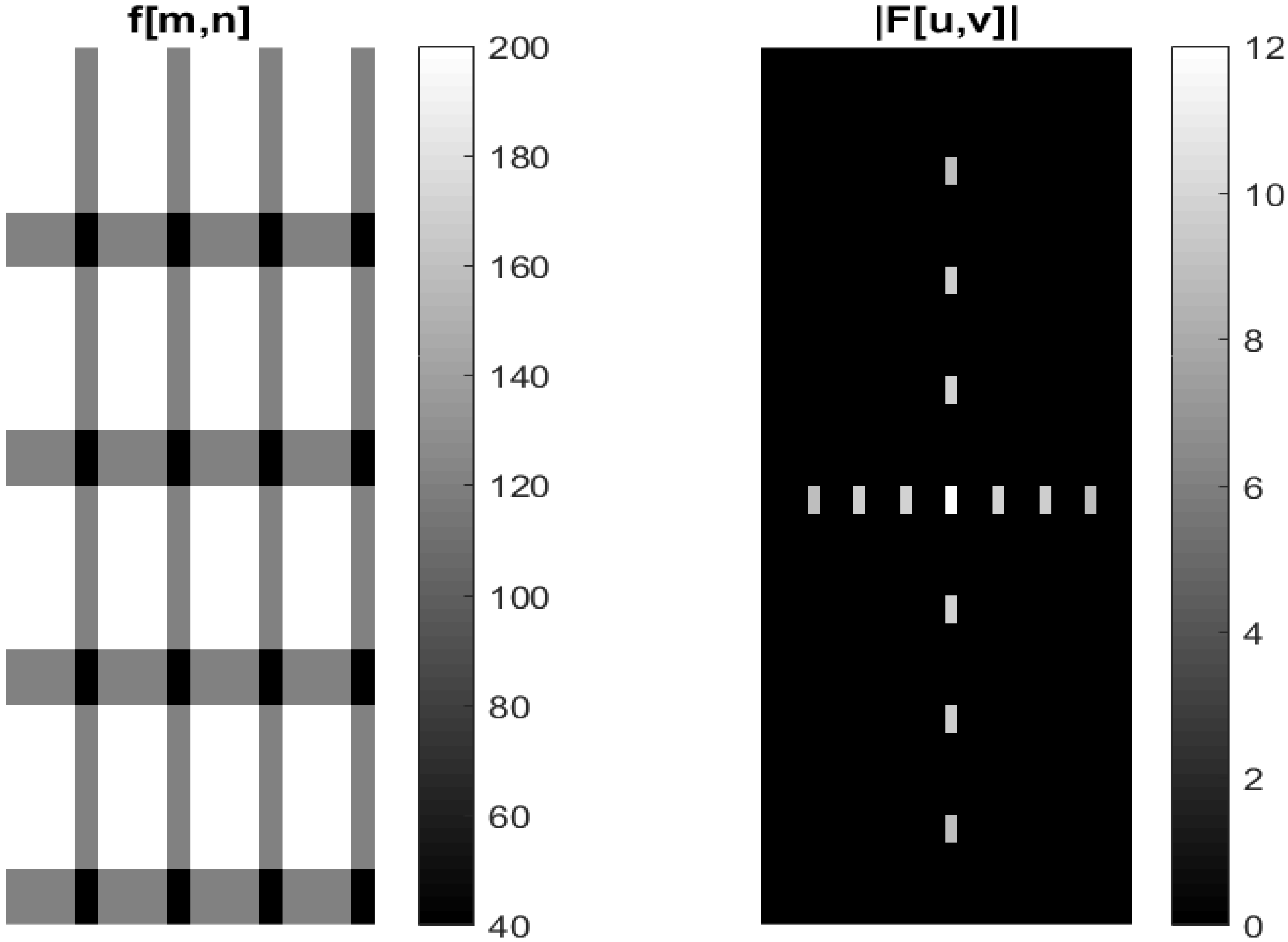


Image Spectrum – Module (8)

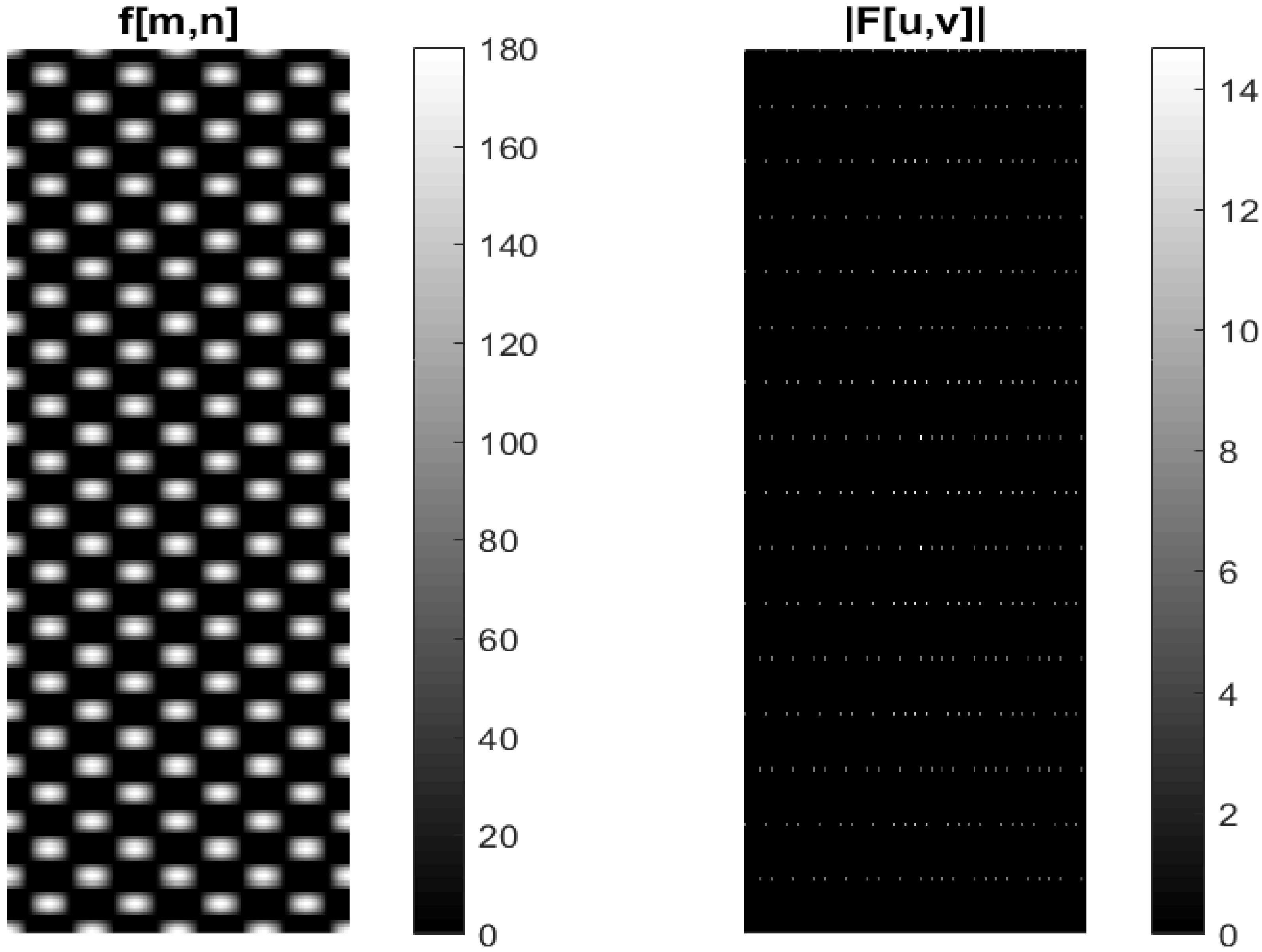
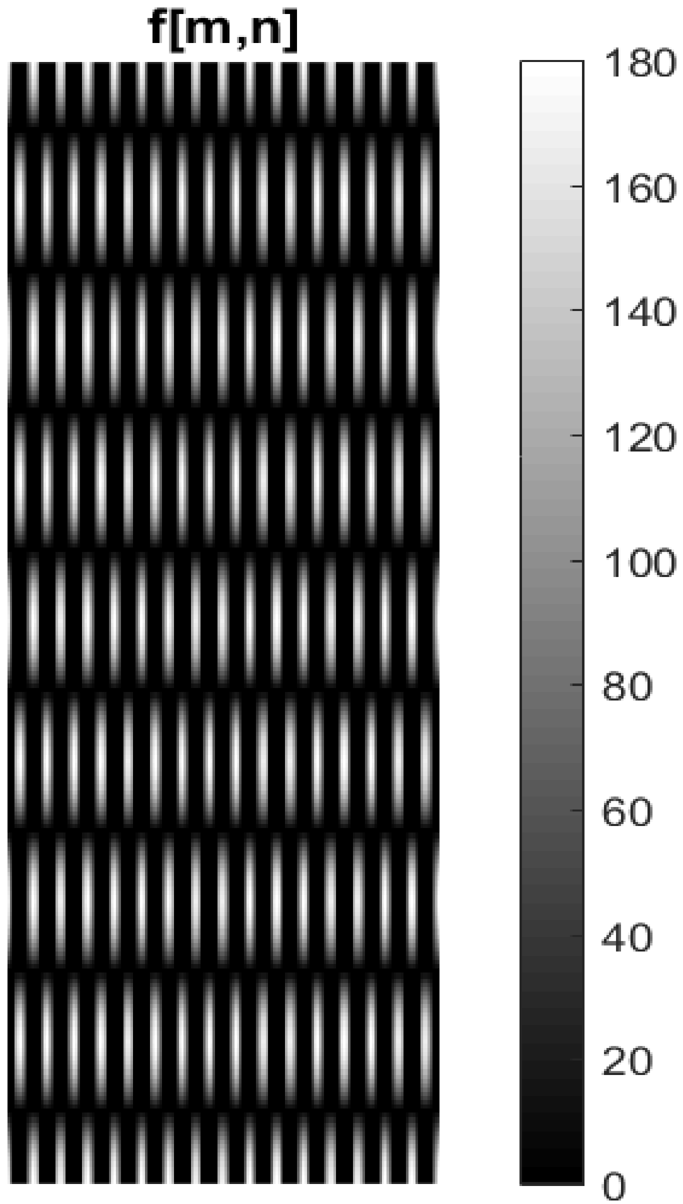


Image Spectrum – Module (9)



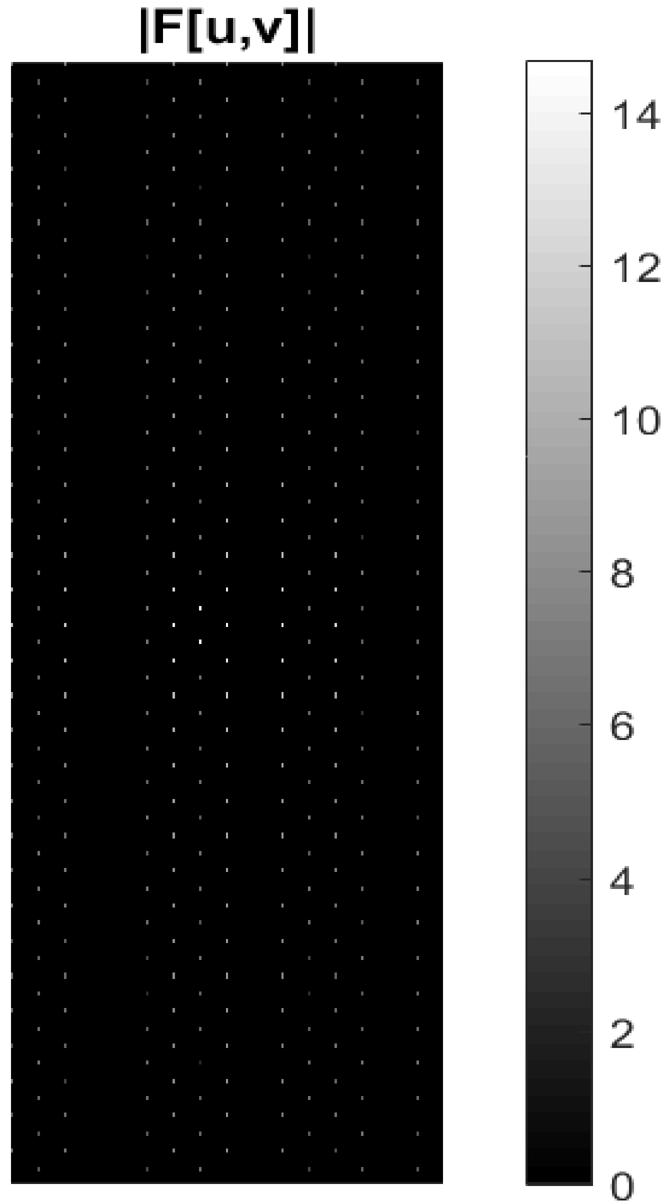
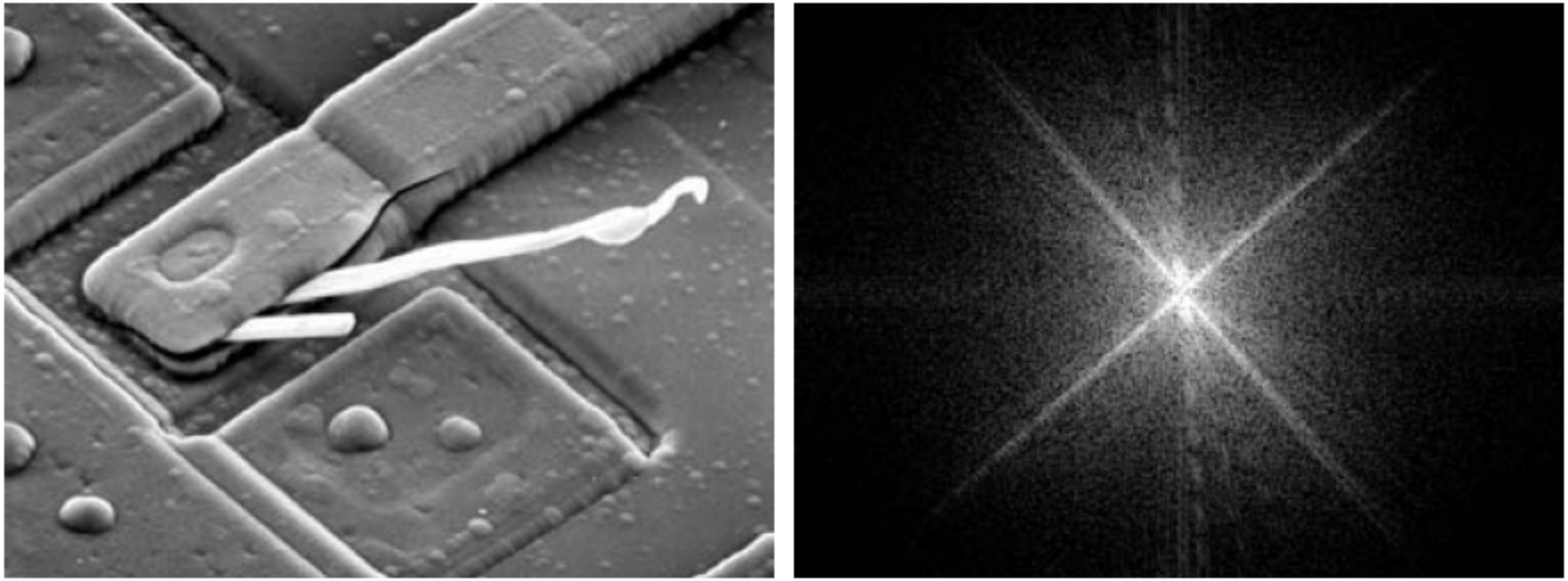


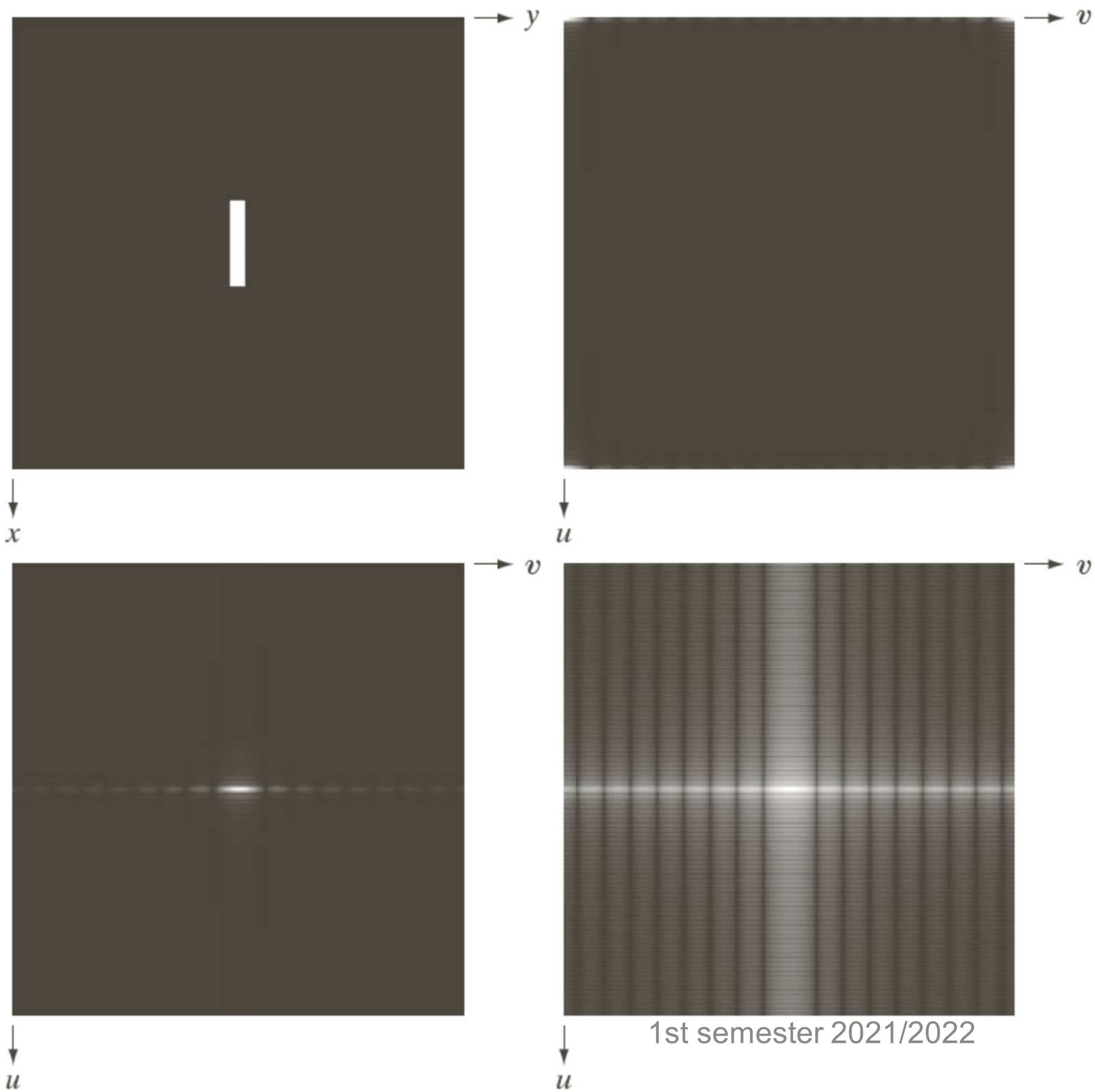
Image Spectrum – Module (10)



a b

FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

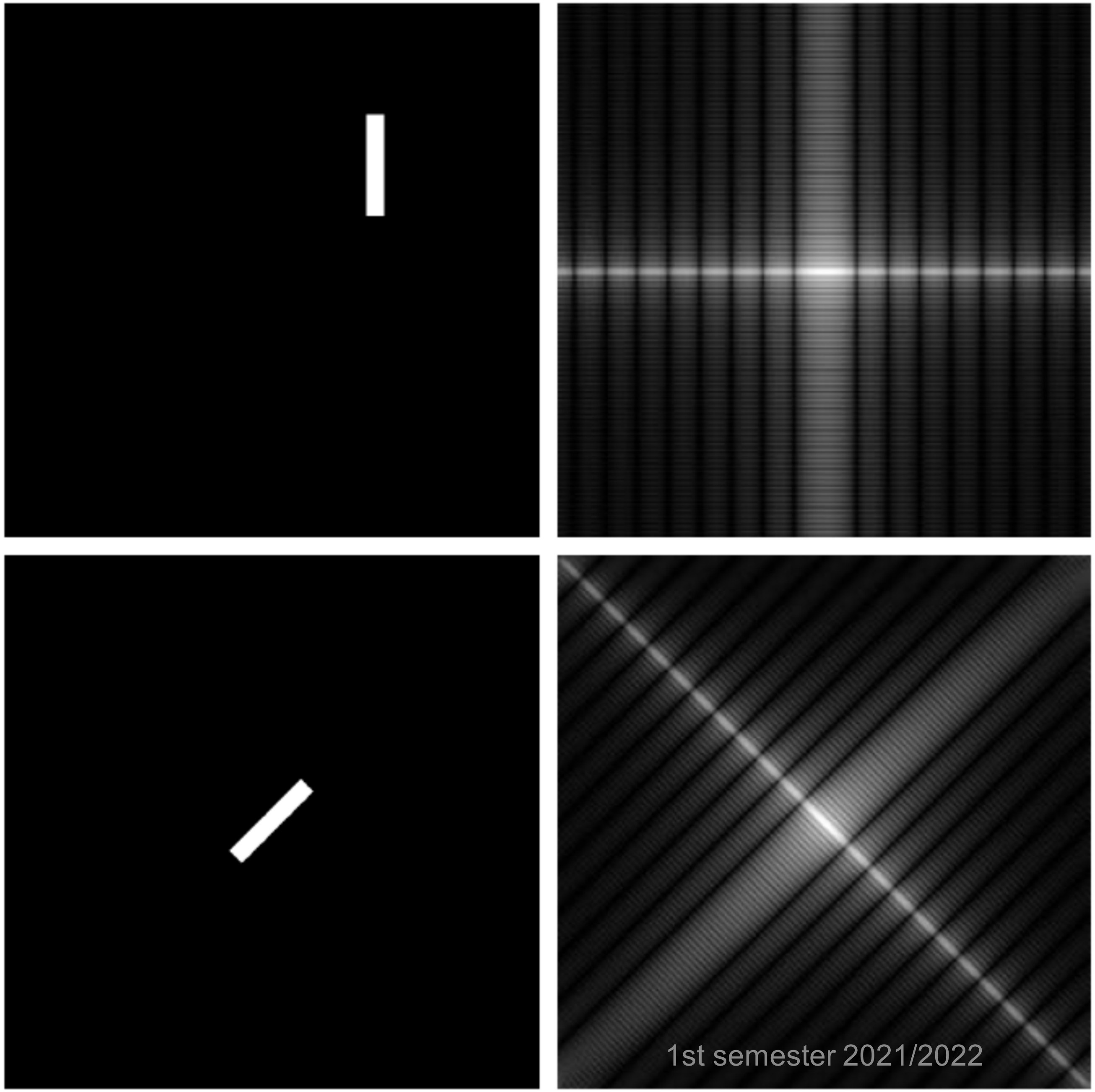
Image Spectrum Module – Non centered, centered, and centered with log transformation



a b
c d

FIGURE 4.24
(a) Image.
(b) Spectrum showing bright spots in the four corners.
(c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.

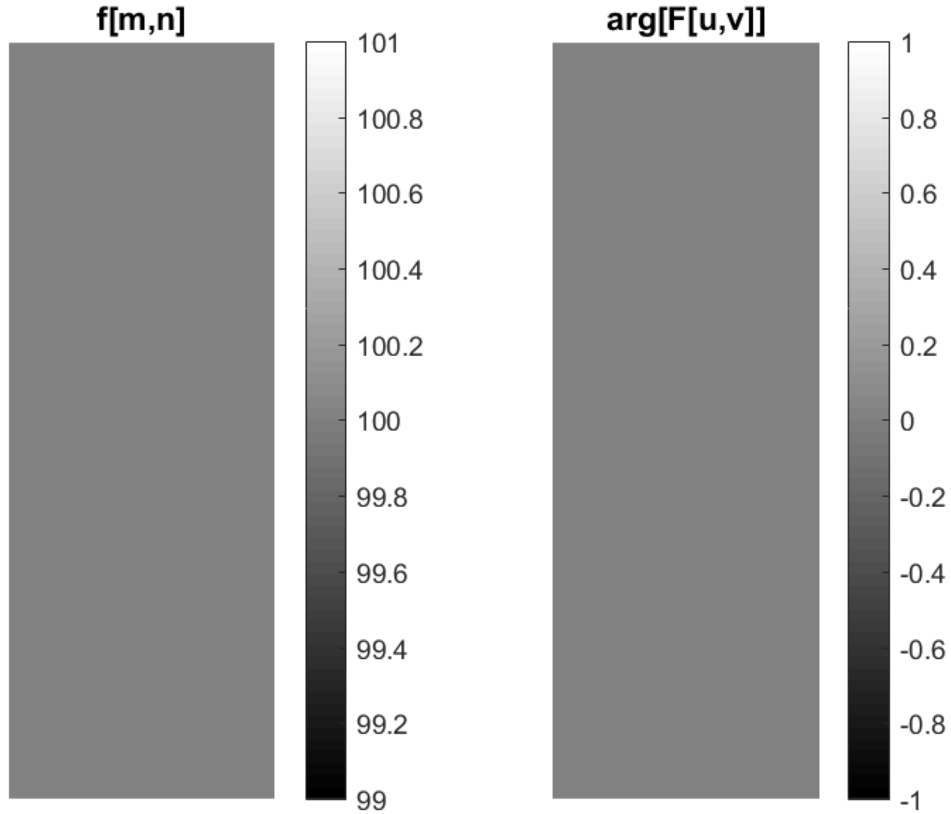
Image Spectrum – Translation and Rotation



a	b
c	d

FIGURE 4.25 (a) The rectangle in Fig. 4.24(a) translated, and (b) the corresponding spectrum. (c) Rotated rectangle, and (d) the corresponding spectrum. The spectrum corresponding to the translated rectangle is identical to the spectrum corresponding to the original image in Fig. 4.24(a).

Image Spectrum – Phase (1)




```
% Compute the DFT
F = fft2(f);

 % Center the spectrum
F = fftshift(F);

% Display the image
figure(1);
subplot(121); imagesc(f);
title(' f[m,n] '); axis off; colorbar;

% Display the argument of the spectrum
subplot(122); imagesc( angle(F) );
colormap('gray'); title( ' arg[F[u,v]] ');
colorbar; axis off;
```

Image Spectrum – Phase (2)

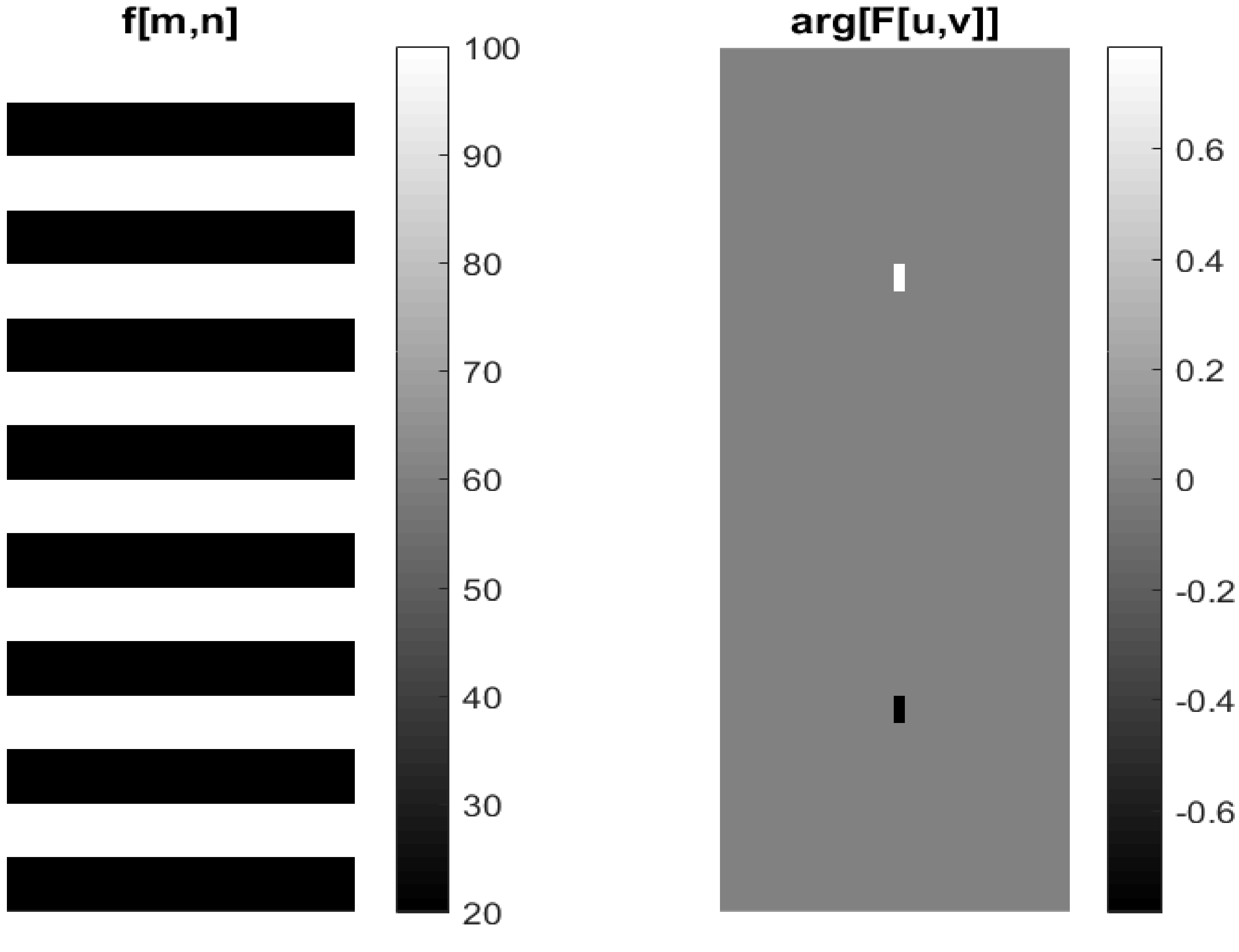


Image Spectrum – Phase (3)

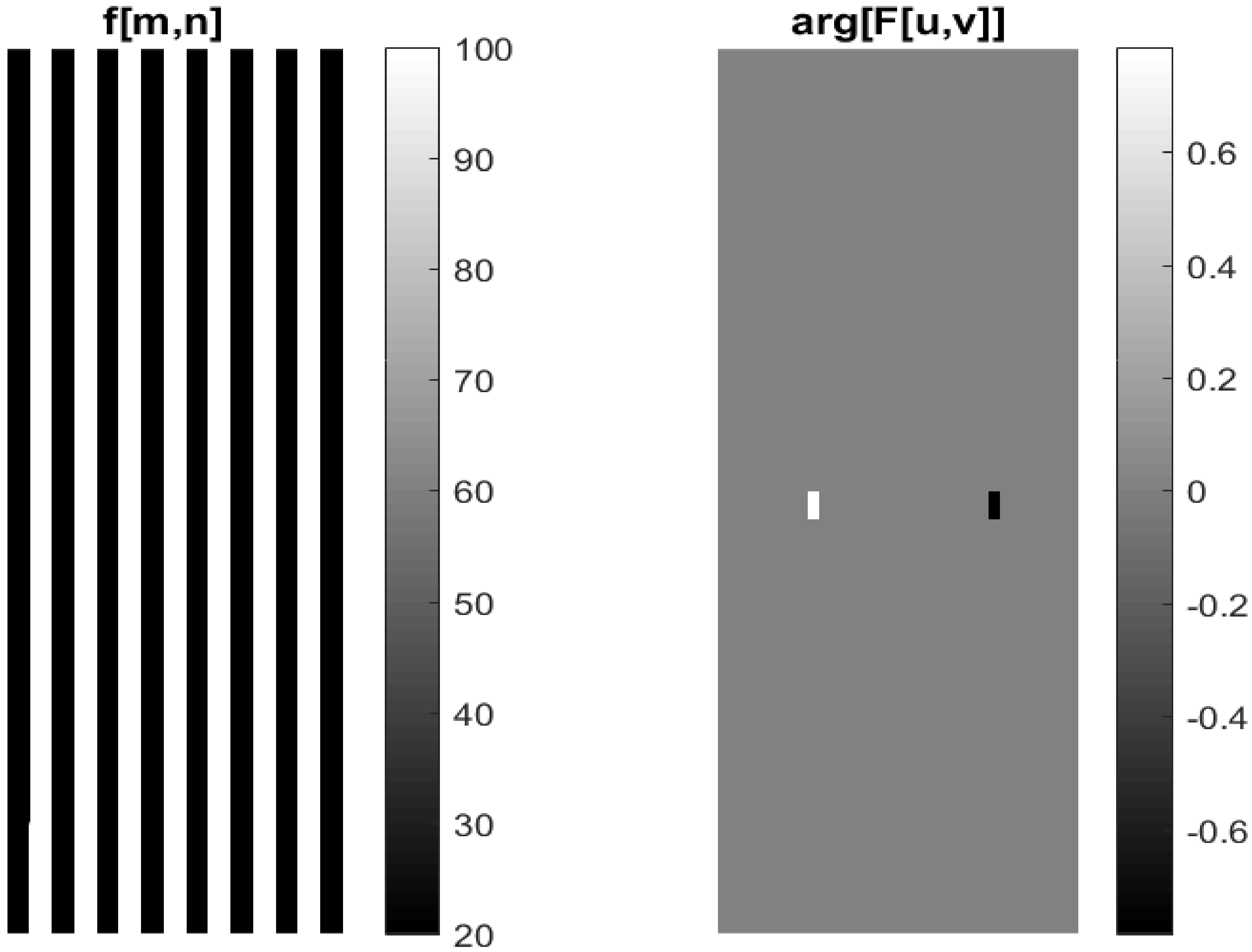


Image Spectrum – Phase (4)

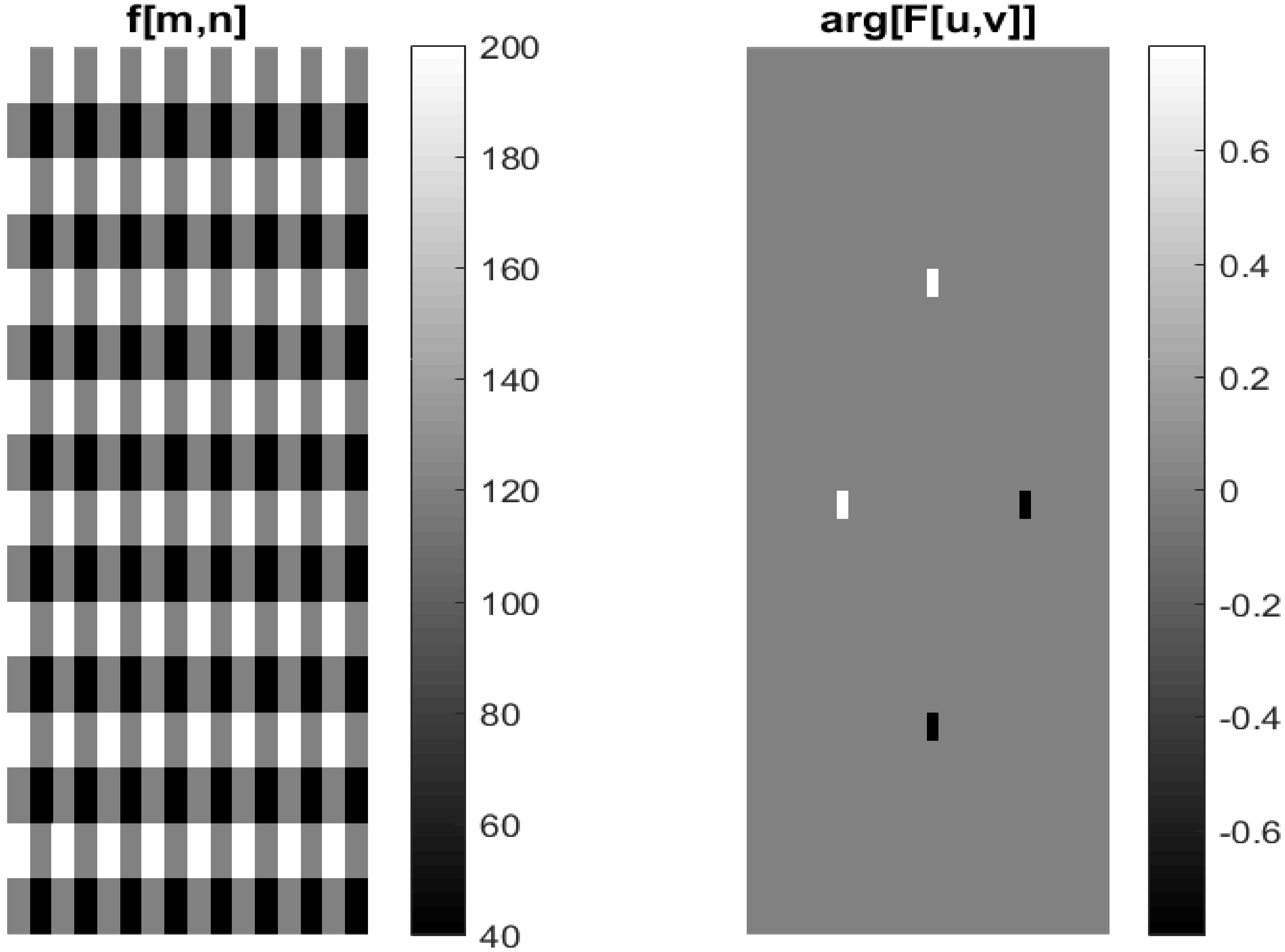


Image Spectrum – Phase (5)

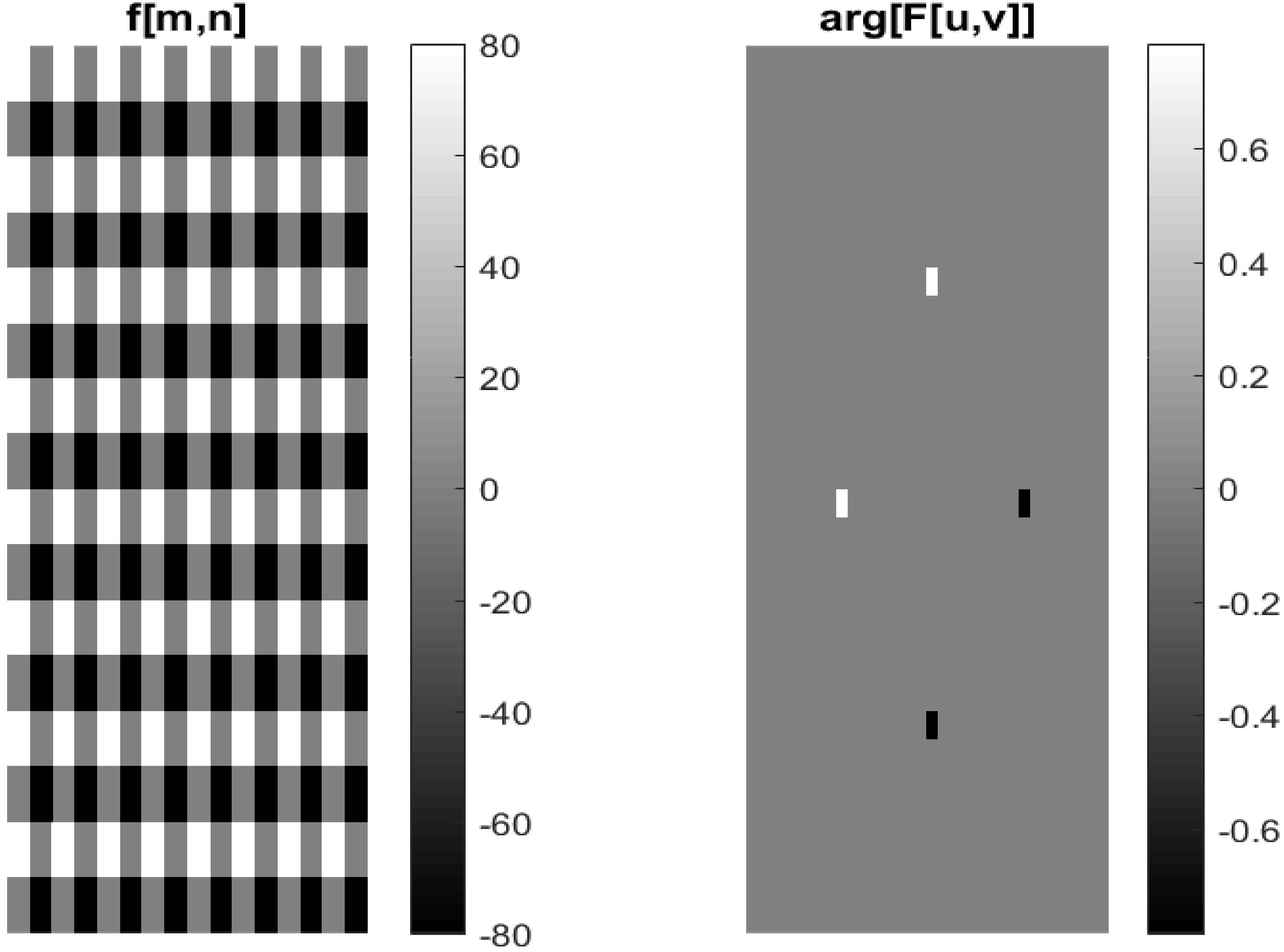


Image Spectrum – Phase (6)

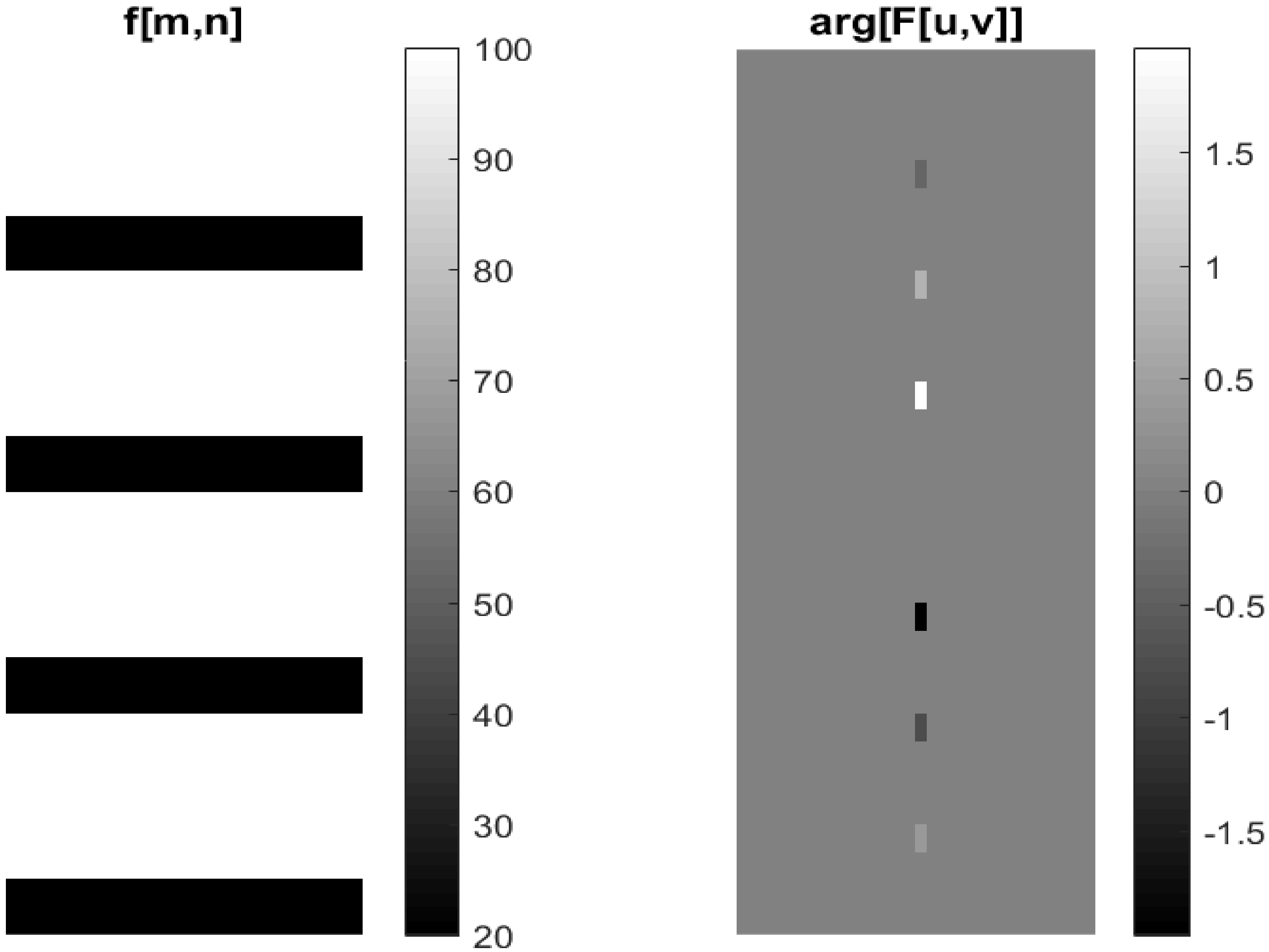


Image Spectrum – Phase (7)

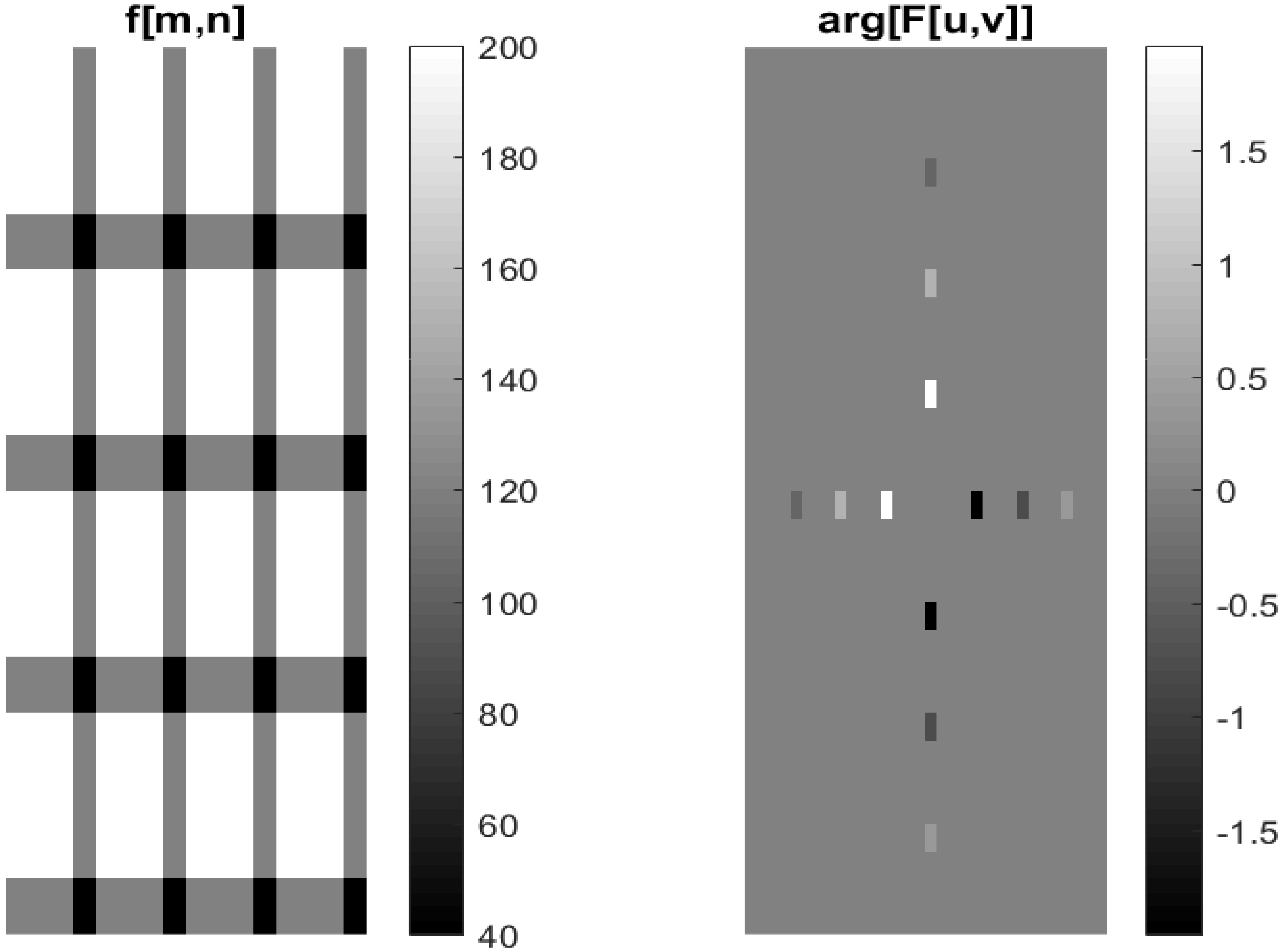
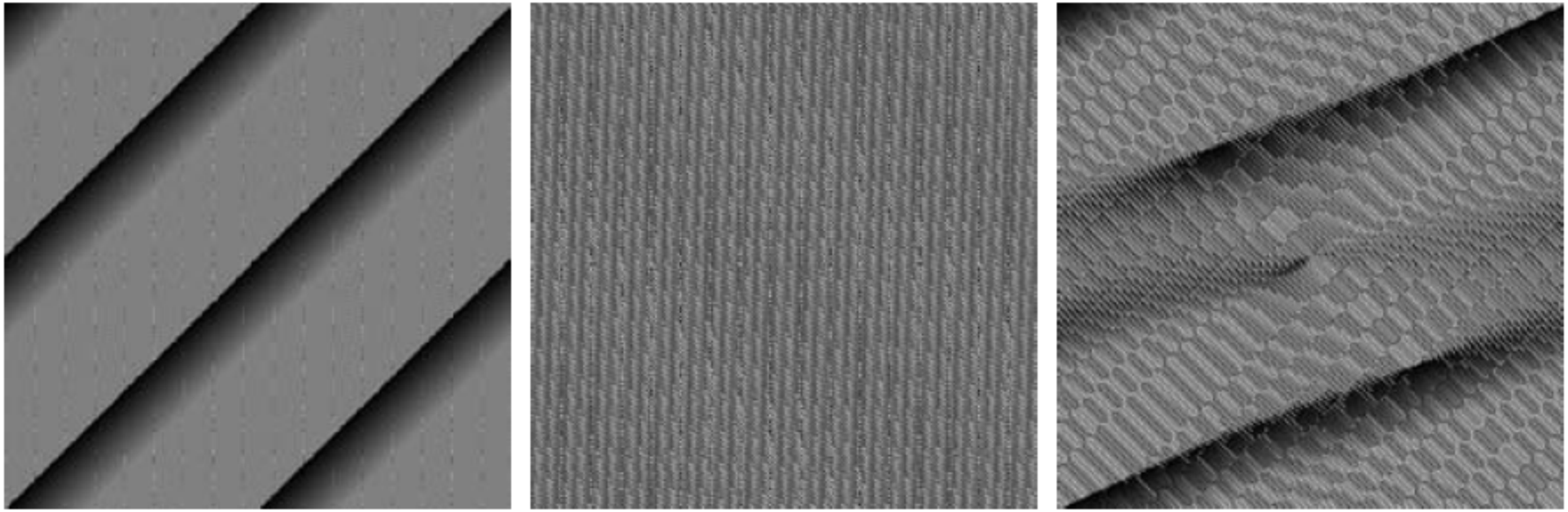


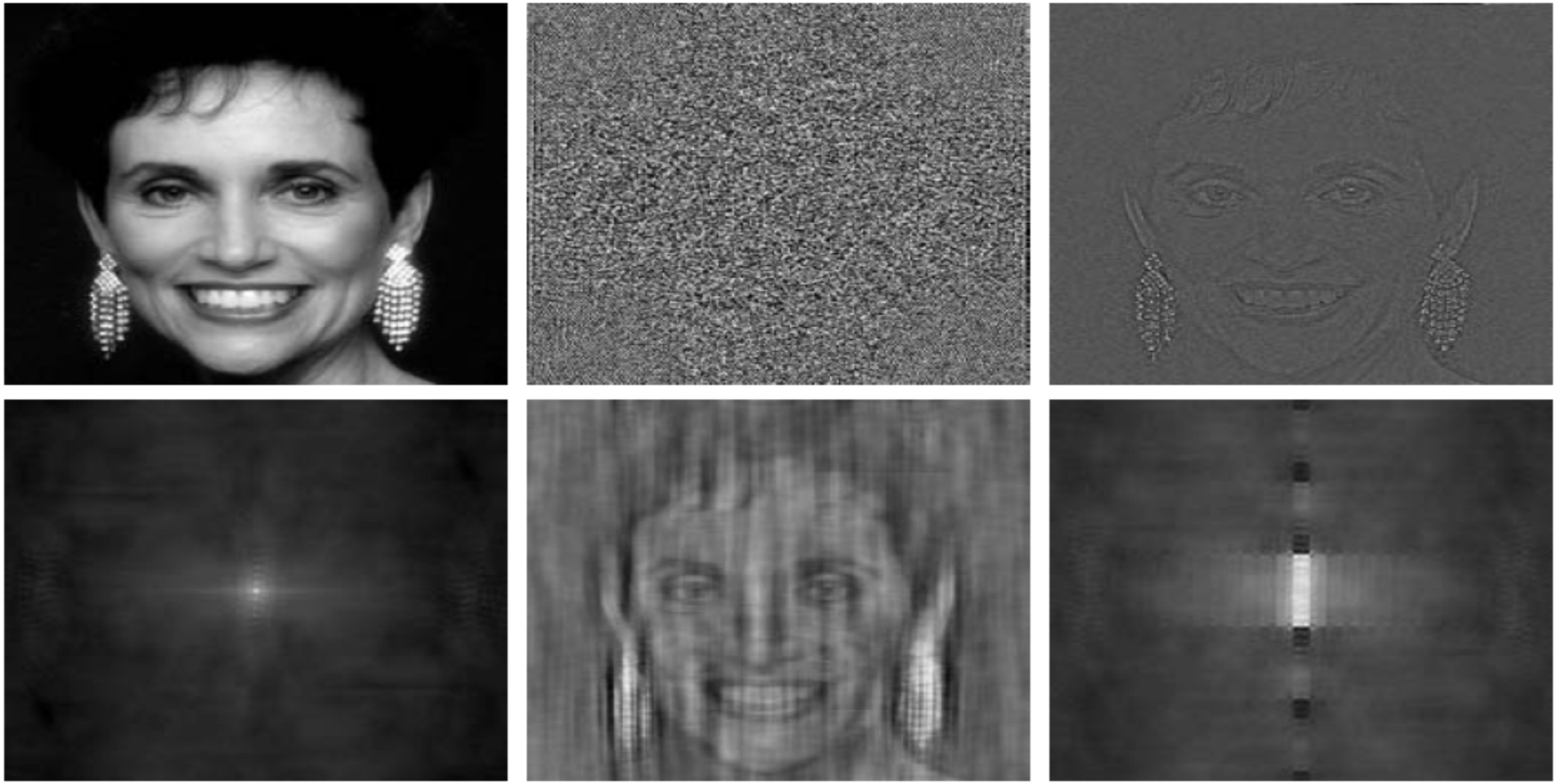
Image Spectrum – Phase (8)



a b c

FIGURE 4.26 Phase angle array corresponding (a) to the image of the centered rectangle in Fig. 4.24(a), (b) to the translated image in Fig. 4.25(a), and (c) to the rotated image in Fig. 4.25(c).

Image Spectrum – Module and Phase (1)



a	b	c
d	e	f

FIGURE 4.27 (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.

Filtering (1)

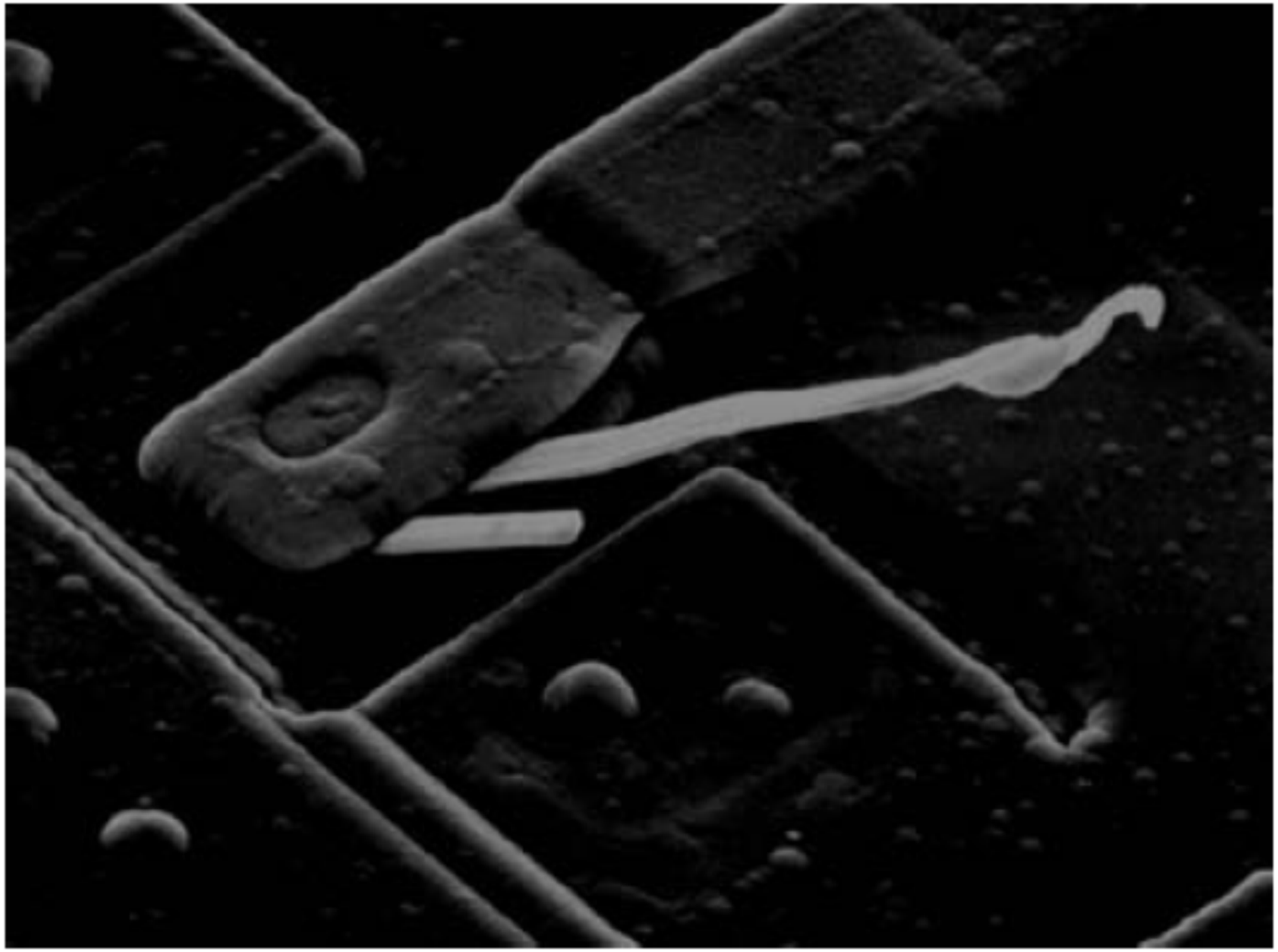


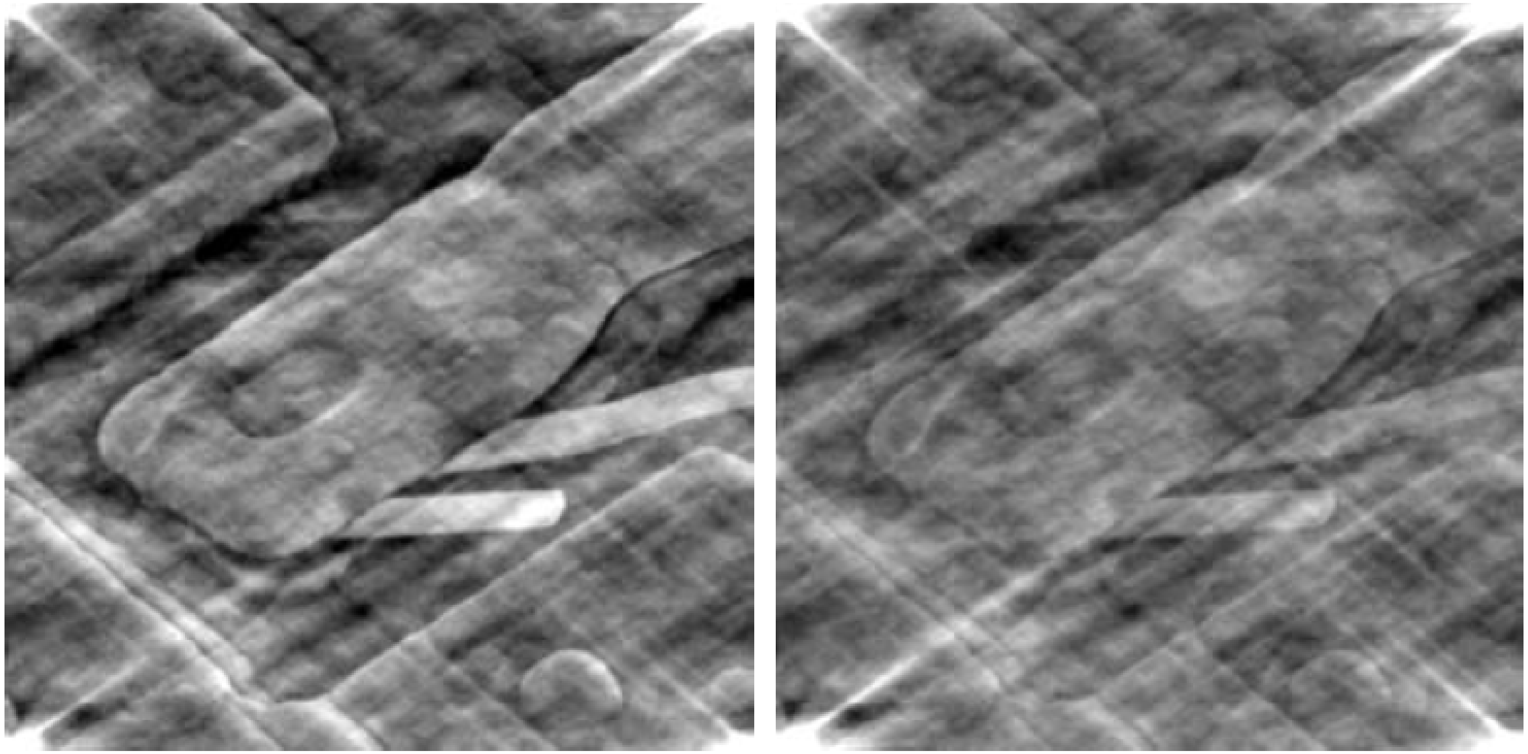
FIGURE 4.30
Result of filtering the image in Fig. 4.29(a) by setting to 0 the term $F(M/2, N/2)$ in the Fourier transform.

Filtering (2)

- The **filter H[u,v] is defined as**
  - H[u,v] = 0, u=0, v=0
  - H[u,v] = 1, other u and v

- The filtering steps are defined as follows:
  1) F[u,v] = DFT[f[m,n]]
  2) G[u,v] = F[u,v] H[u,v]
  3) g[m,n] = IDFT[G[u,v]]

Filtering (3)



a b

FIGURE 4.35
(a) Image resulting from multiplying by 0.5 the phase angle in Eq. (4.6-15) and then computing the IDFT. (b) The result of multiplying the phase by 0.25. The spectrum was not changed in either of the two cases.

Exercises (1)

6. A imagem $f[m,n]$ tem espetro $F[u,v] = \begin{bmatrix} 18 & -2-j2 & 2 & -2+j2 \\ 2 & 0 & -2 & 0 \\ -2 & 2-j2 & -2 & 2+j2 \\ 2 & 0 & -2 & 0 \end{bmatrix}$.

   (a) $\{1{,}25\}$ Apresente $F[u,v]$, $|F[u,v]|$ e arg[F[u,v]], na forma de espetro centrado.

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7. The $f[m,n]$ image has spectrum $F[u,v] = \begin{bmatrix} 13 & -1 & 5 & -1 \\ -j & j & -j & j \\ -5 & 1 & 3 & 1 \\ j & -j & j & -j \end{bmatrix}$.

   (a) $\{1.25\}$ Regarding $f[m,n]$ state: the average intensity; the power.

   (b) $\{1.25\}$ Display $|F[u,v]|$ and arg[F[u,v]], on the centered spectrum form.

Exercises (2)

6. {R1} {2.0} The image $f[m,n]$ has spectrum $F[u,v] = \begin{bmatrix} 24 & 3+j\sqrt{3} & 3-j\sqrt{3} \\ -3+j\sqrt{3} & -3-j\sqrt{3} & 0 \\ -3-j\sqrt{3} & 0 & -3+j\sqrt{3}. \end{bmatrix}$. Calculate the mean value and power of $f[m,n]$.

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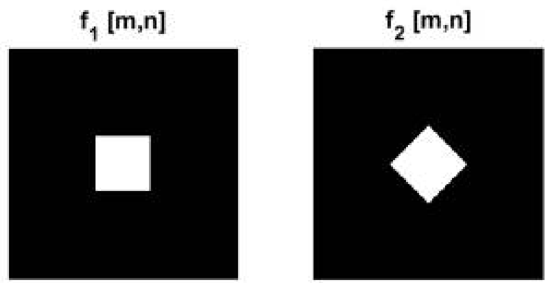
b) {1,5||1,0} As imagens $f_1[m,n]$ e $f_2[m,n]$ apresentam o mesmo objeto, antes e após rotação de 45 graus.



Relativamente ao módulo e fase dos espetros destas duas imagens, qual das seguintes afirmações é verdadeira?

i) $|F_1[u,v]| = |F_2[u,v]|$ *e* $arg[F_1[u,v]] = arg[F_2[u,v]]$.

ii) $|F_1[u,v]| = |F_2[u,v]|$ *e* $arg[F_1[u,v]] \neq arg[F_2[u,v]]$.

iii) $|F_1[u,v]| \neq |F_2[u,v]|$ *e* $arg[F_1[u,v]] = arg[F_2[u,v]]$.

iv) $|F_1[u,v]| \neq |F_2[u,v]|$ *e* $arg[F_1[u,v]] \neq arg[F_2[u,v]]$.

MATLAB
Image Processing Toolbox functions

https://www.mathworks.com/products/image.html

- *fft2.m,* the two-dimensional Fourier transform of a matrix
- *ifft2.m,* the inverse two-dimensional Fourier transform of a matrix
- *fftshift.m*, shift zero-frequency component to center of spectrum
- *ifftshift.m,* inverse zero-frequency shift
- *abs.m*, absolute value (module); norm/magnitude, for complex numbers
- *angle.m*, the angle with the origin; phase angle

Bibliography

- The images displayed in these slides are from:

  - R. Gonzalez, R. Woods, *Digital Image Processing*, 4th edition, Prentice Hall, 2018, ISBN 0133356728
  - S. Smith, *The Scientist and Engineer's Guide to Digital Signal Processing*, Newnes, 2003, ISBN 0-750674-44-X [chapter 23]
  - O. Filho, H. Neto, Processamento Digital de Imagens, Rio de Janeiro: Brasport, 1999, ISBN 8574520098
  - Wikipedia and Mathworks web pages