ISEL
INSTITUTO SUPERIOR
DE ENGENHARIA DE LISBOA

PROCESSAMENTO DE IMAGEM E BIOMETRIA

IMAGE PROCESSING AND BIOMETRICS

8. **BINARY IMAGE PROCESSING**

Summary

- Binary image processing
  - Morphology
  - Binary Images
  - Structuring Elements
  - Common morphologic operations
  - Morphologic reconstruction
  - Grayscale morphology

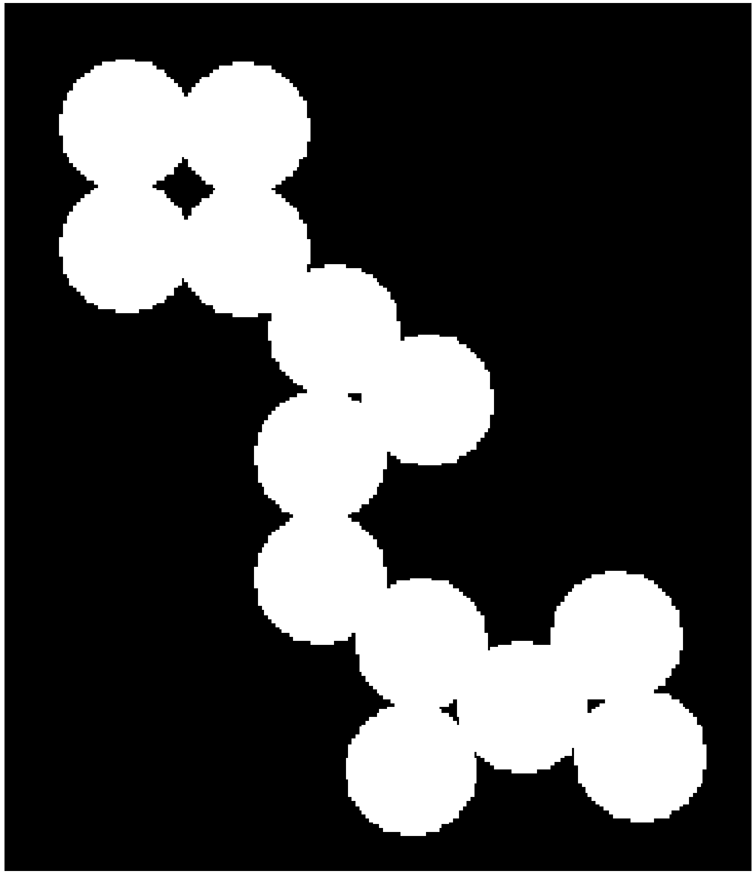
Morphology (1)

Morphology
https://en.wikipedia.org/wiki/Morphology_(biology)

Morphology *is a branch of biology dealing with the study of the form and structure of organisms and their specific structural features*

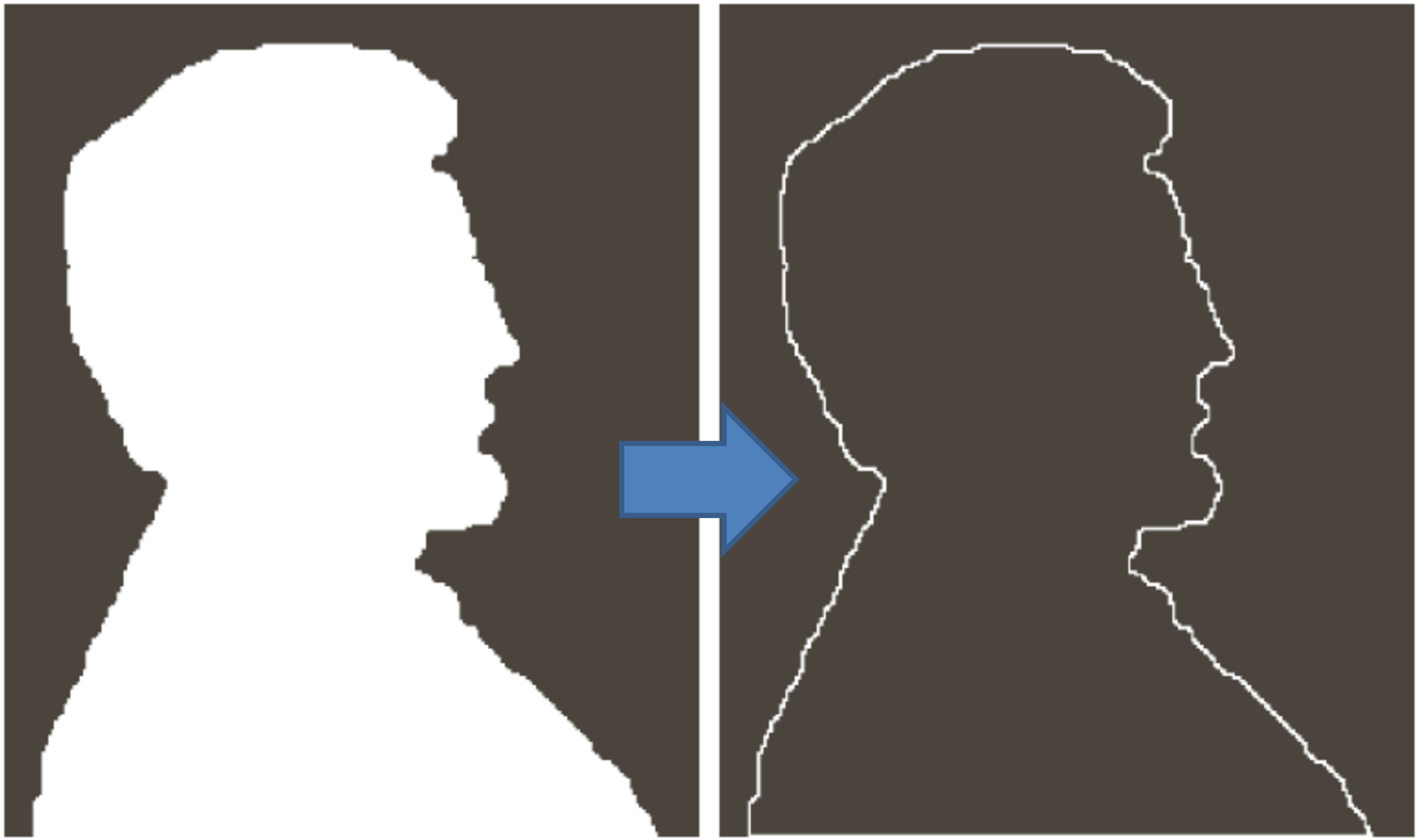
Morphology (2)

- Morphology is the study of shape
- Binary images are analyzed by the *shape* of their contents




Morphology (3)

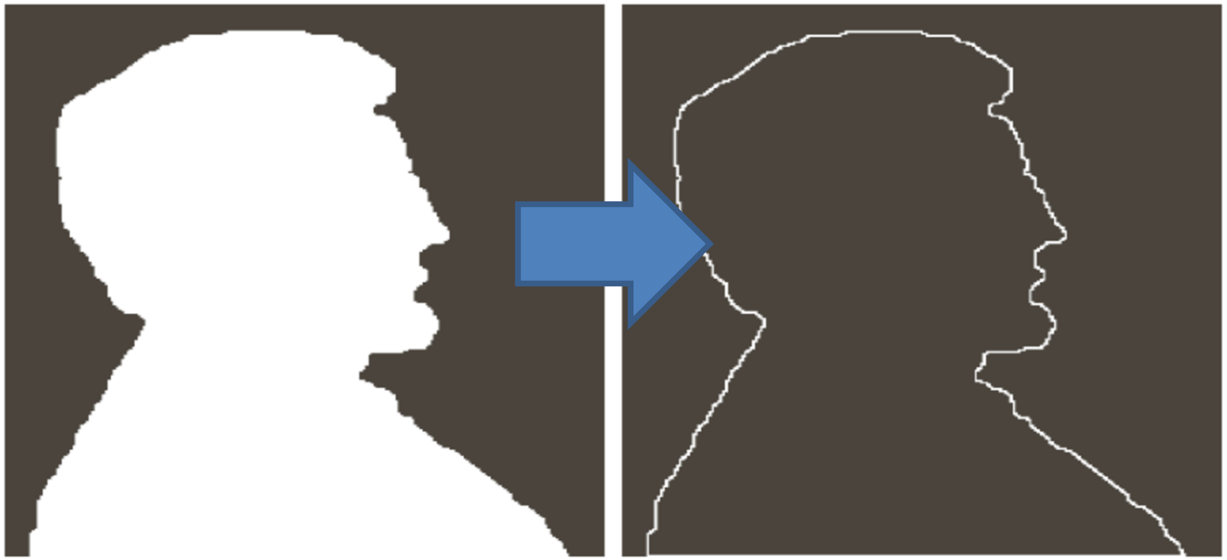
- The shape is analyzed by using a ***structuring element*** and an ***operation*** over the binary image



Binary Image Processing (1)

- The input of a typical binary image processing algorithm:
  - Binary image
  - Structuring Element (SE) - defines the shape
  - Operation (what to do with the shape)

- The output of a typical binary image processing algorithm:
  - Binary image



Binary Image Processing (2)

- The Structuring Element (SE) defines the (arbitrary) shape to process

- There are four basic morphologic operations
    1) **Erosion** – removes the boundaries of objects, as defined by the SE (imerode.m)

    2) **Dilation** – enlarges the boundaries of objects, as defined by the SE (imdilate.m)

    3) **Opening** – Erosion followed by dilation using the same SE (imopen.m)

    4) **Closing** - Dilation followed by erosion using the same SE (imclose.m)

Structuring Elements (SE)

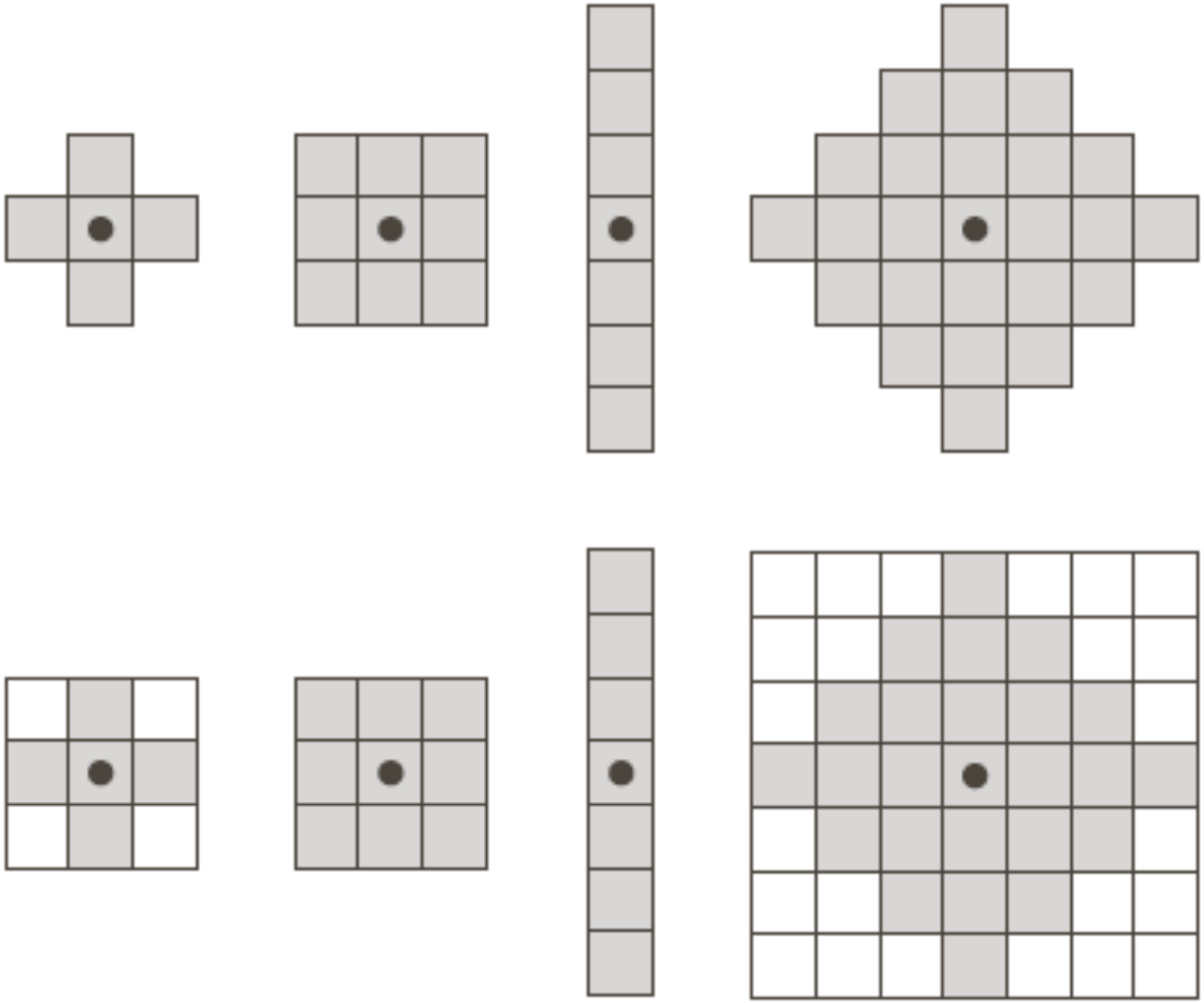


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Erosion Operation (1)

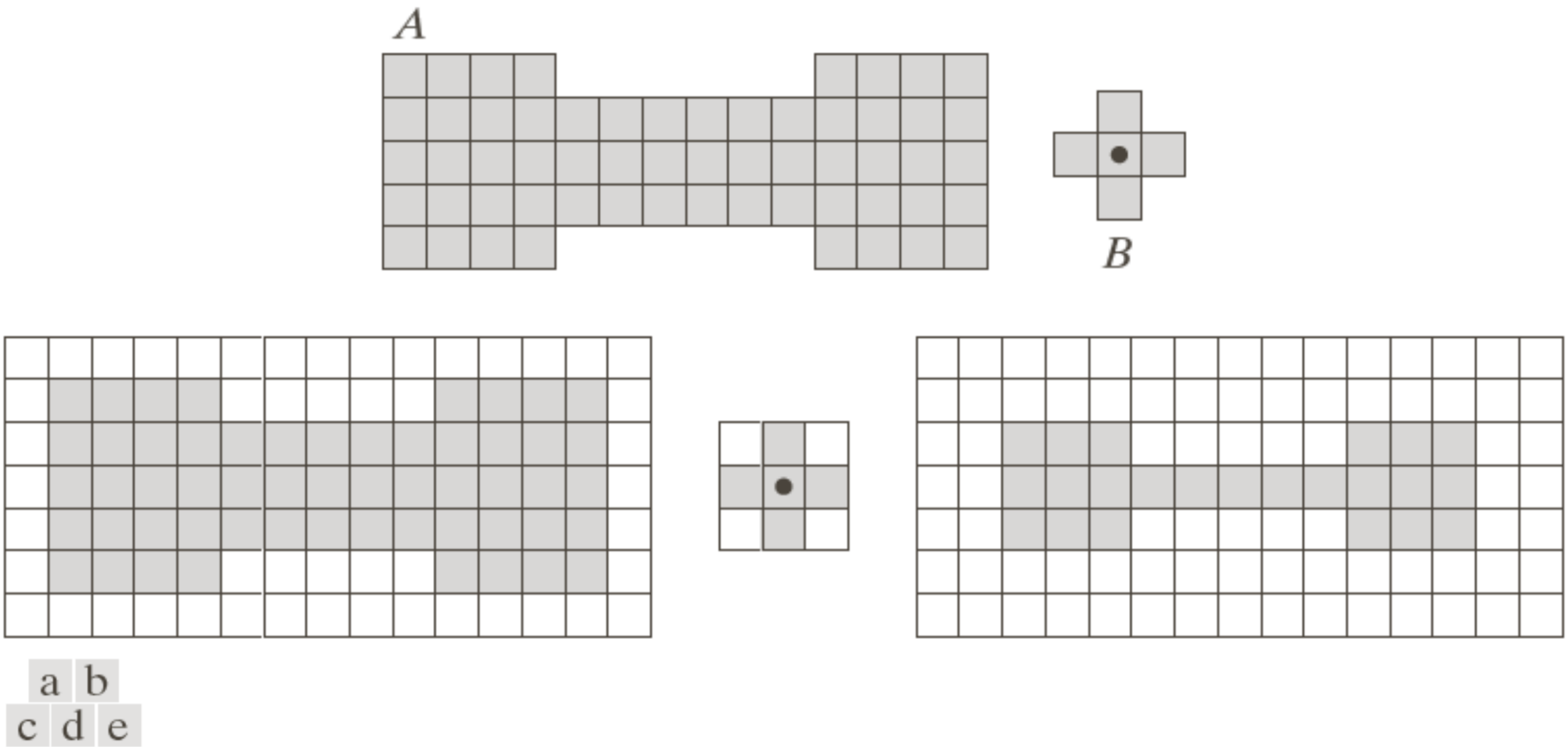


FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

Erosion Operation (2)

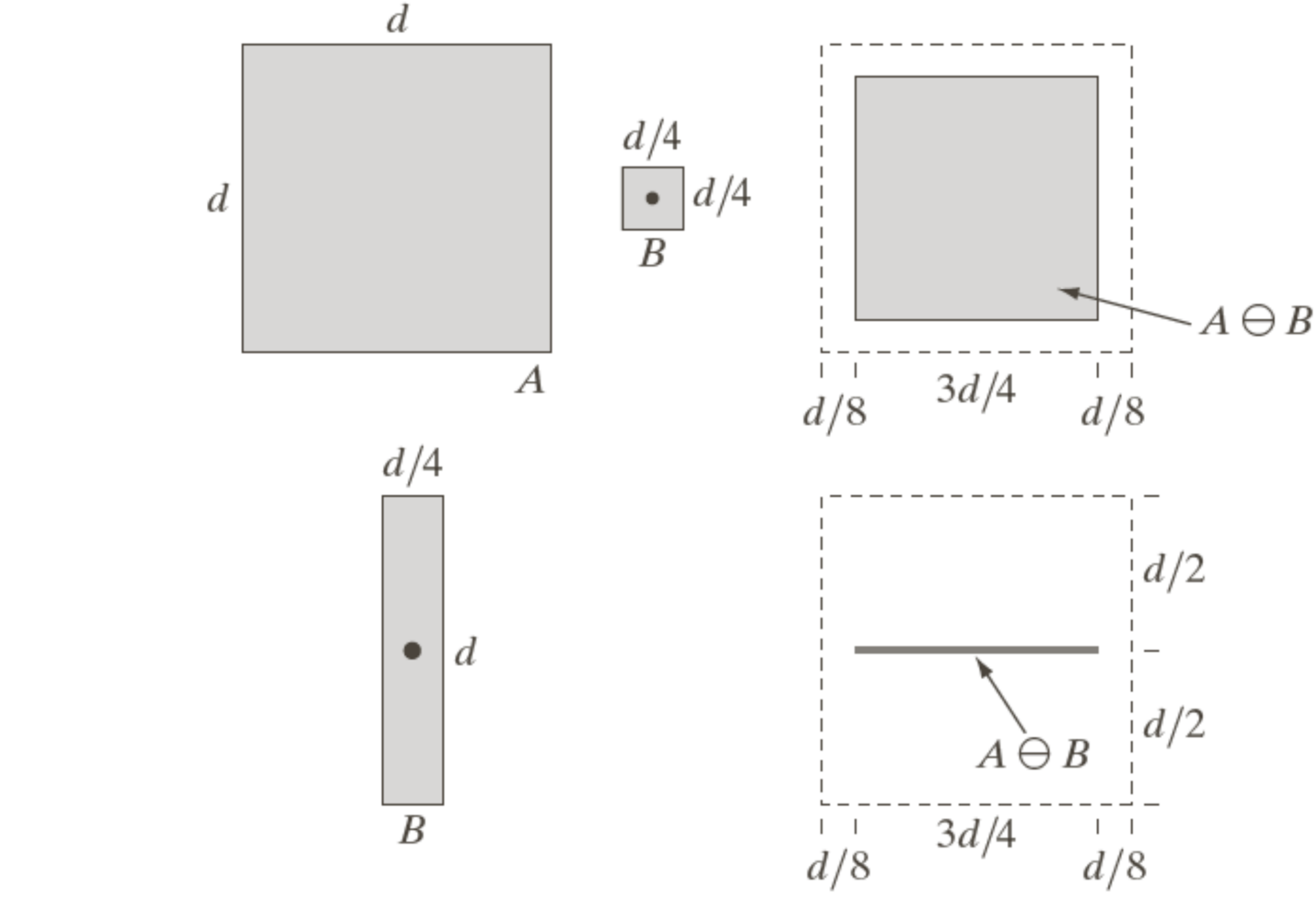


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Erosion Operation (3)

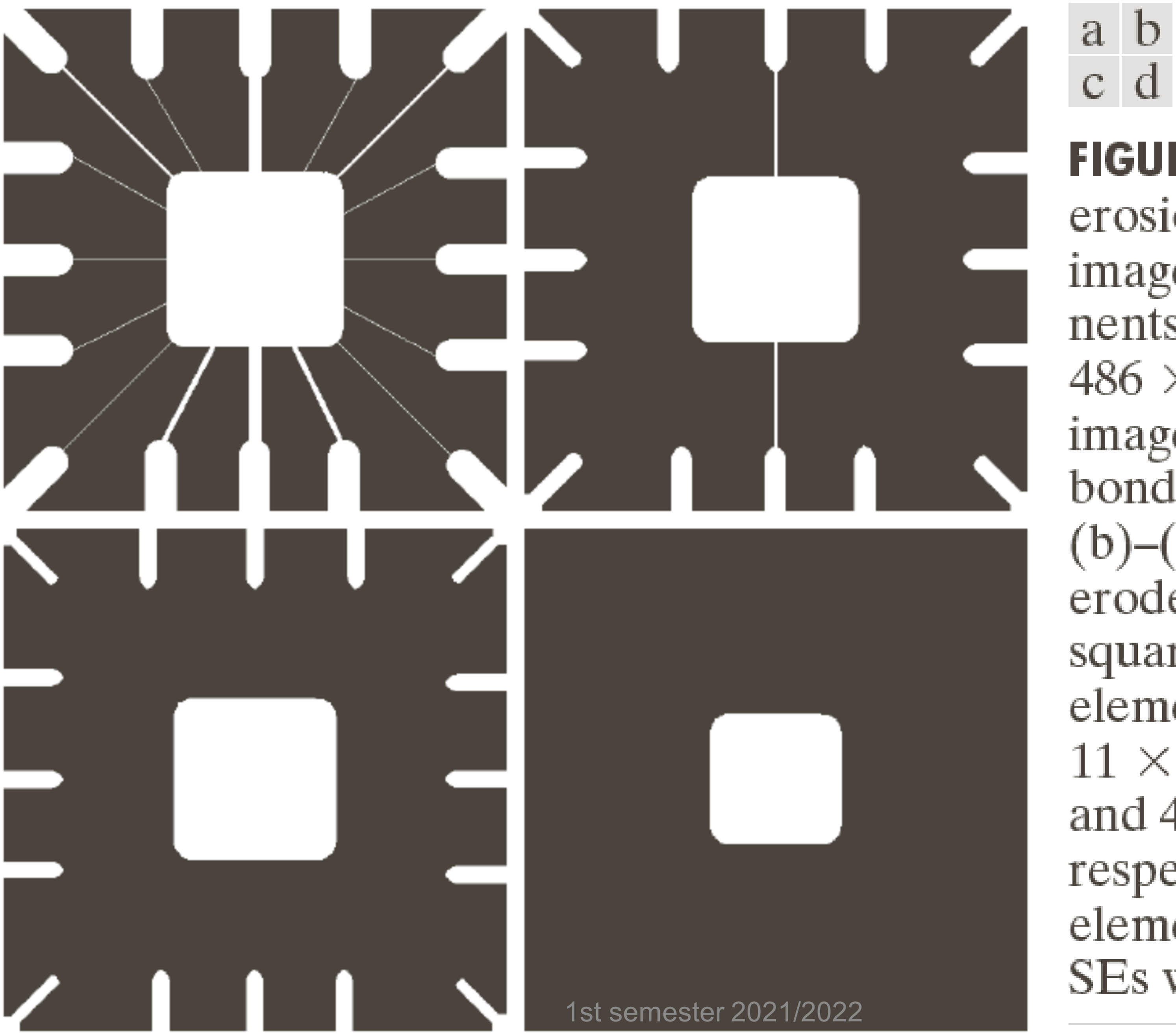
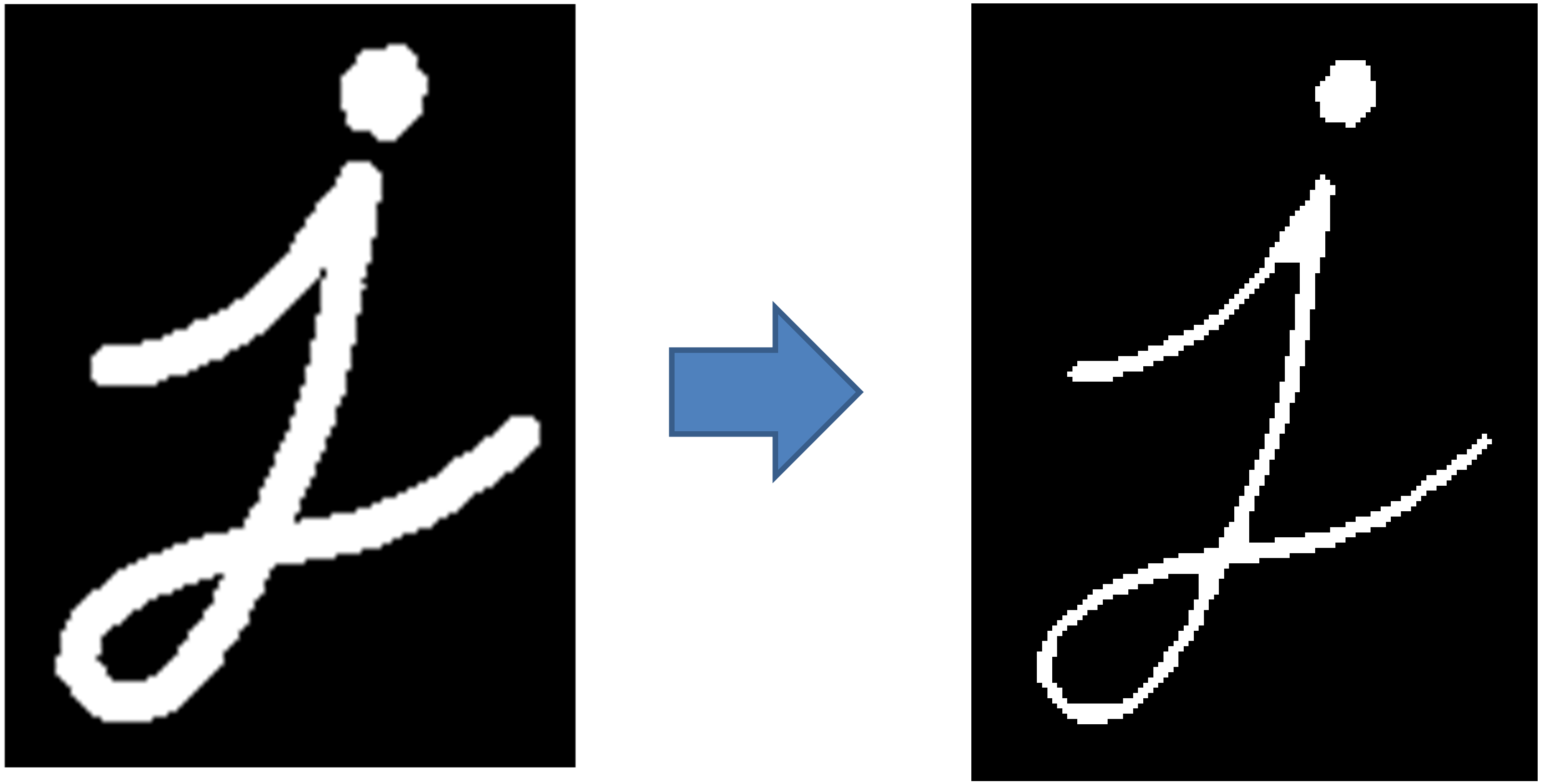


FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.

Erosion Operation (4)



https://docs.opencv.org/3.4/d9/d61/tutorial_py_morphological_ops.html

Erosion Operation (5)

- Erosion
  - Provides a thinner image
  - The boundaries of the objects are eroded (like in Nature) as a function of the SE
  - It is a ‘filtering’ operation in which all the outer elements with less details than the SE are removed

Dilation Operation (1)

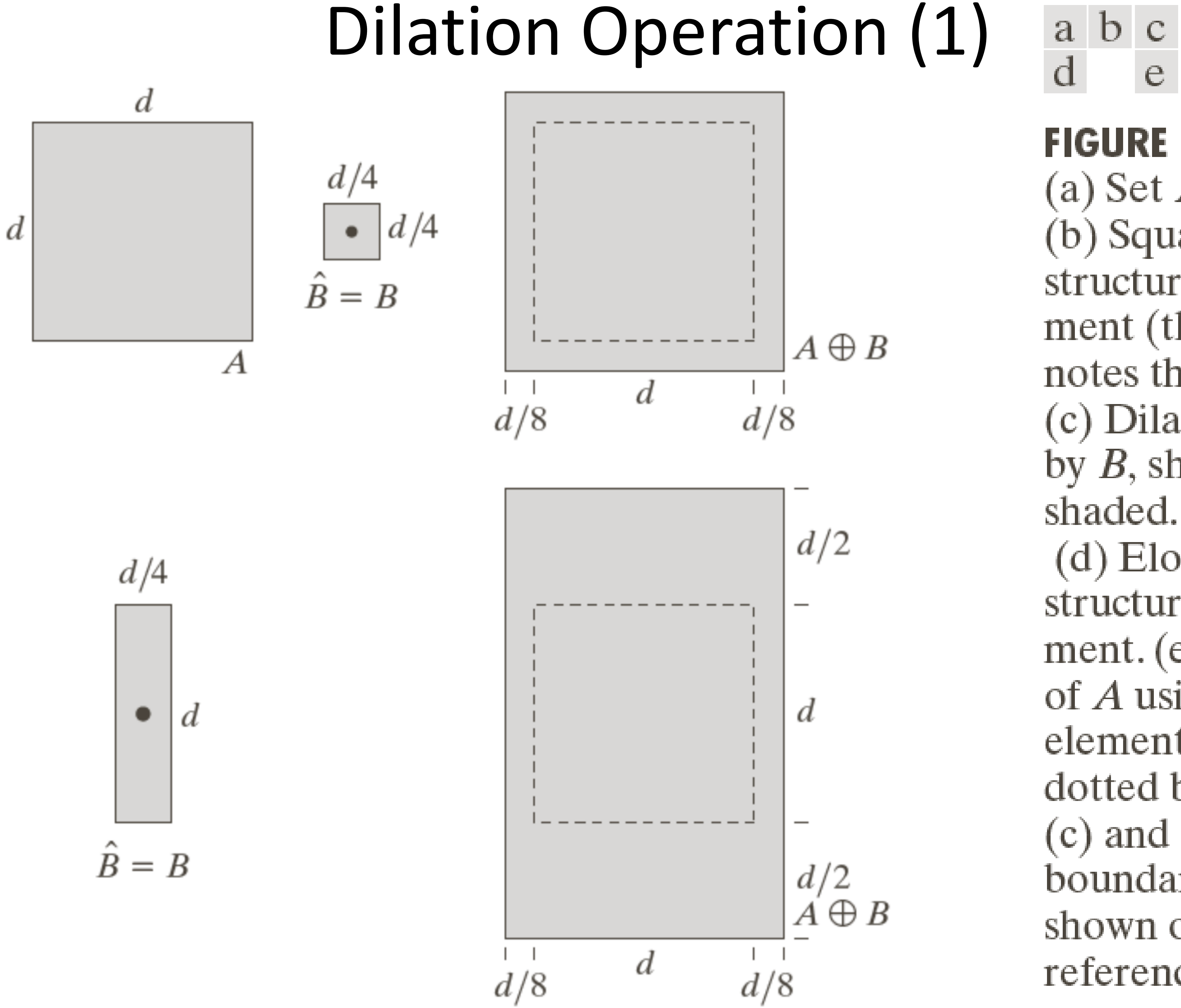
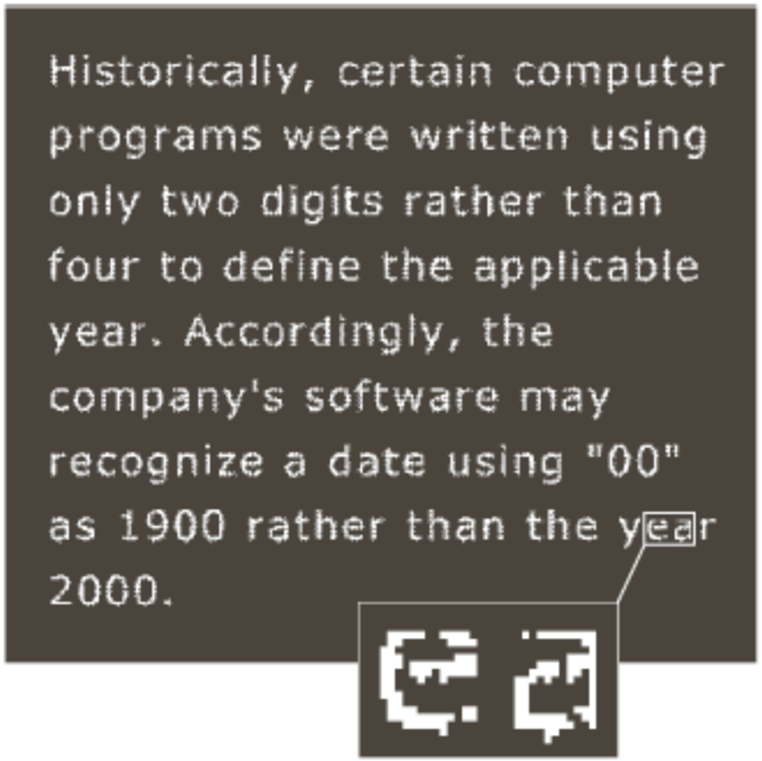
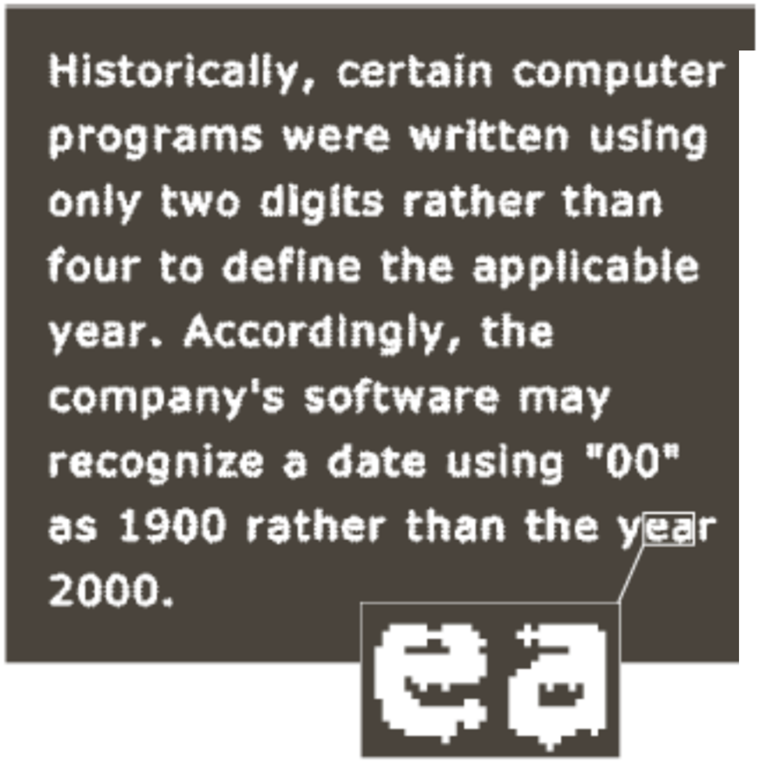


FIGURE 9.6 (a) Set A. (b) Square structuring element (the dot denotes the origin). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Dilation Operation (2)



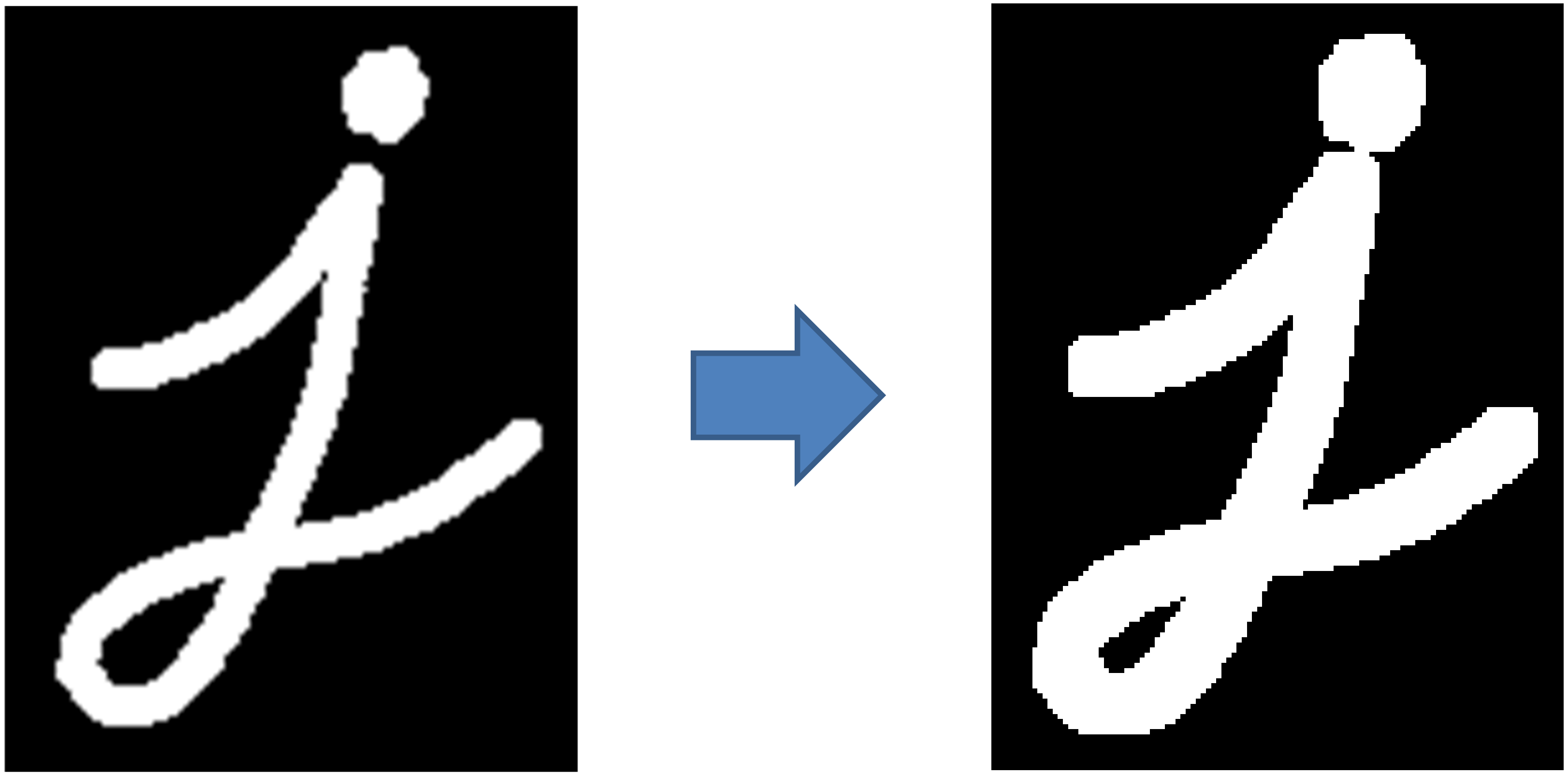
0	1	0
1	1	1
0	1	0



a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Dilation Operation (3)



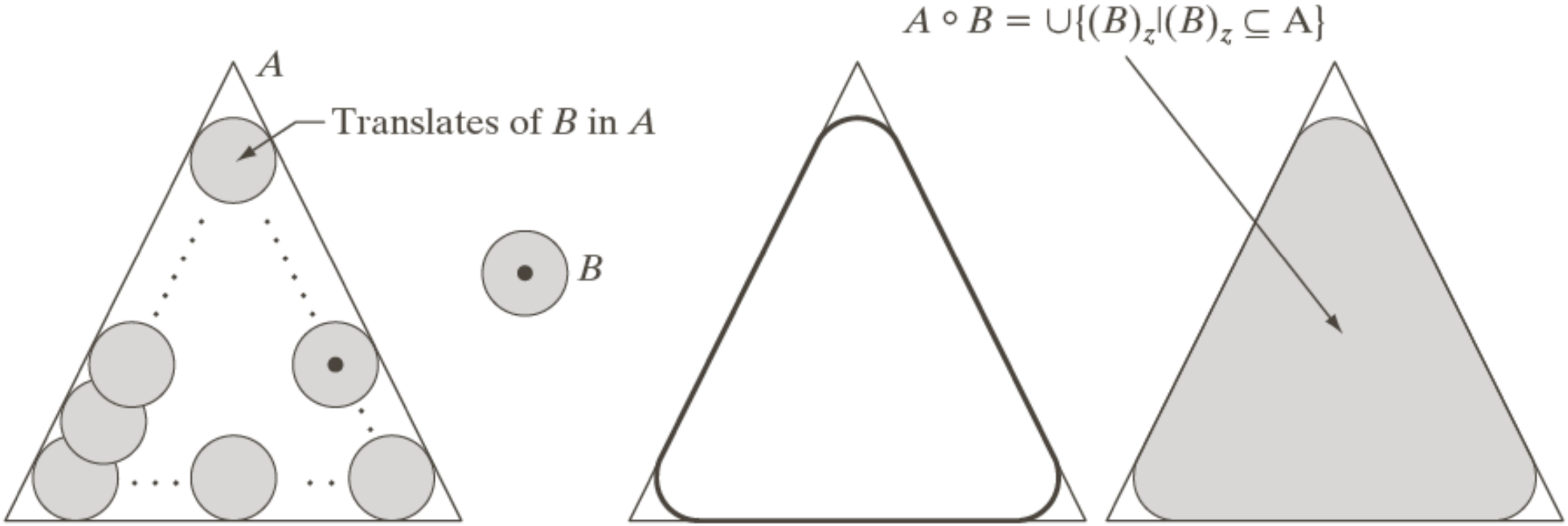
https://docs.opencv.org/3.4/d9/d61/tutorial_py_morphological_ops.html

Dilation Operation (4)

- Dilation
    - Provides a thicker image
    - The boundaries of the objects are dillated as a function of the SE
    - It is a ‘fatening’ operation in which the boundaries of an object grow ticker as functions of the SE

Opening Operation

- First erode, then dilate
- The dilation of the erosion

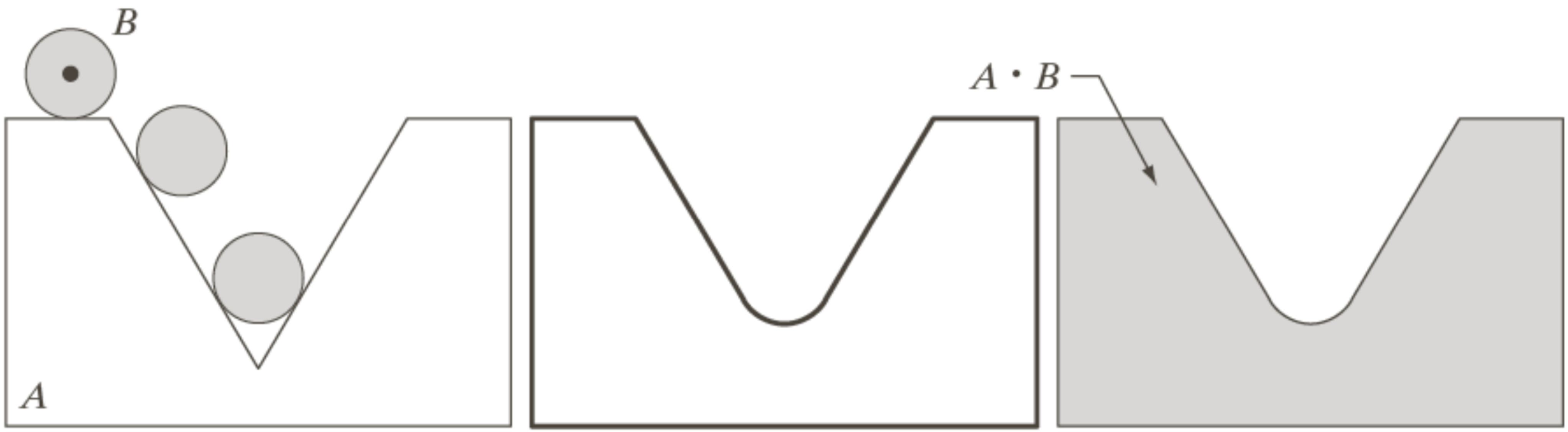


a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Closing Operation

- First dilate, then erode
- The erosion of the dilation



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

Opening and Closing Operations (1)

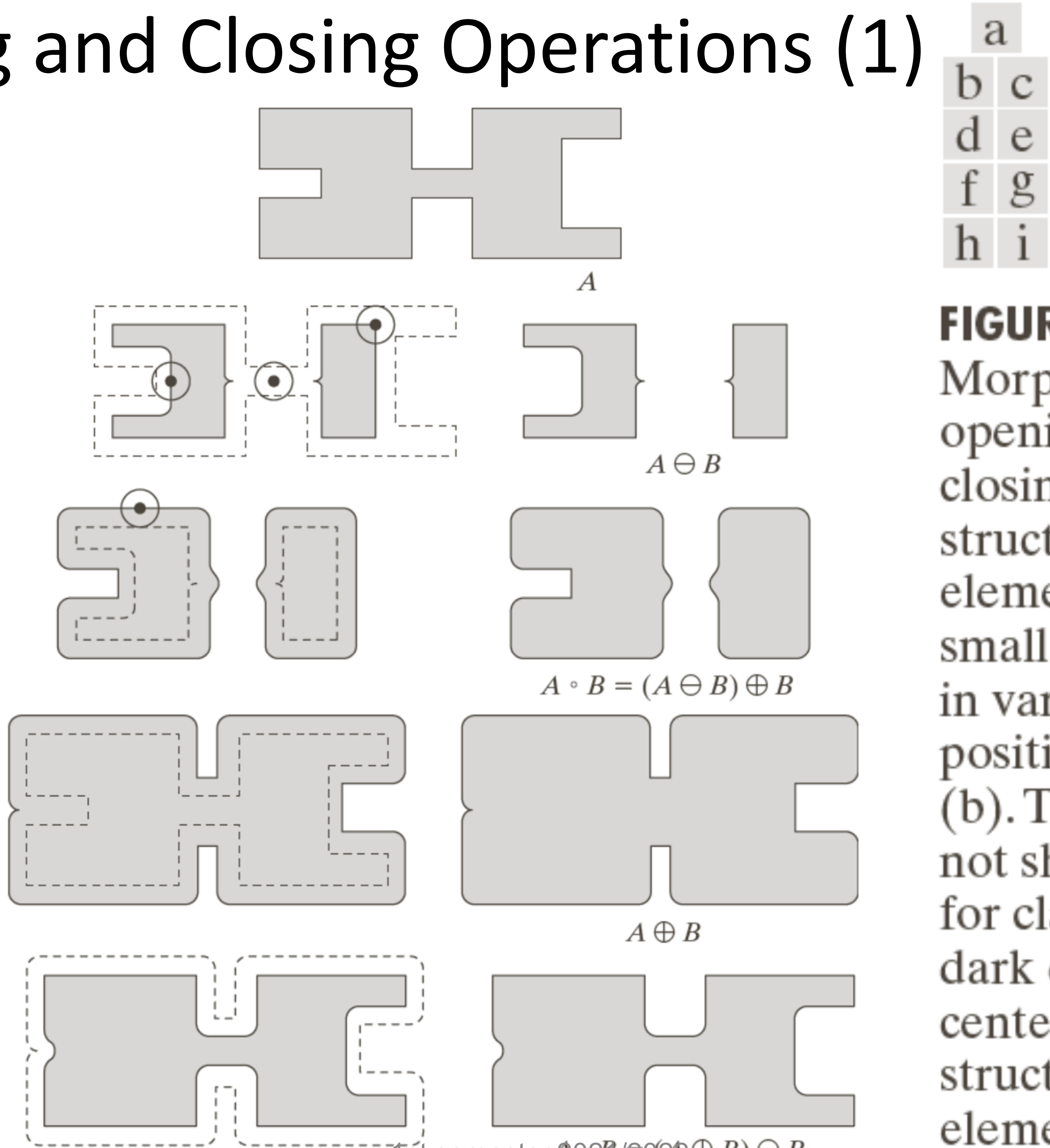
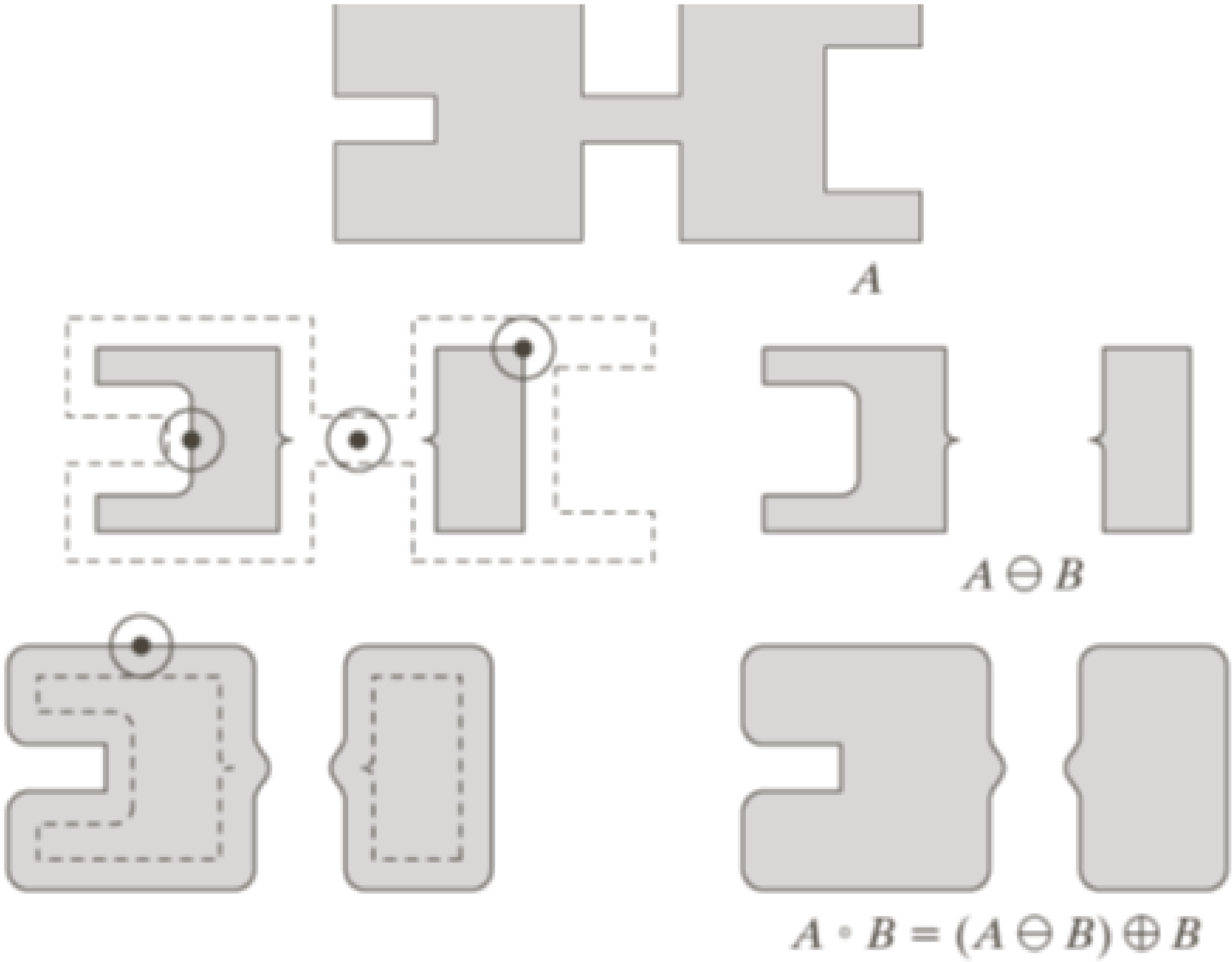
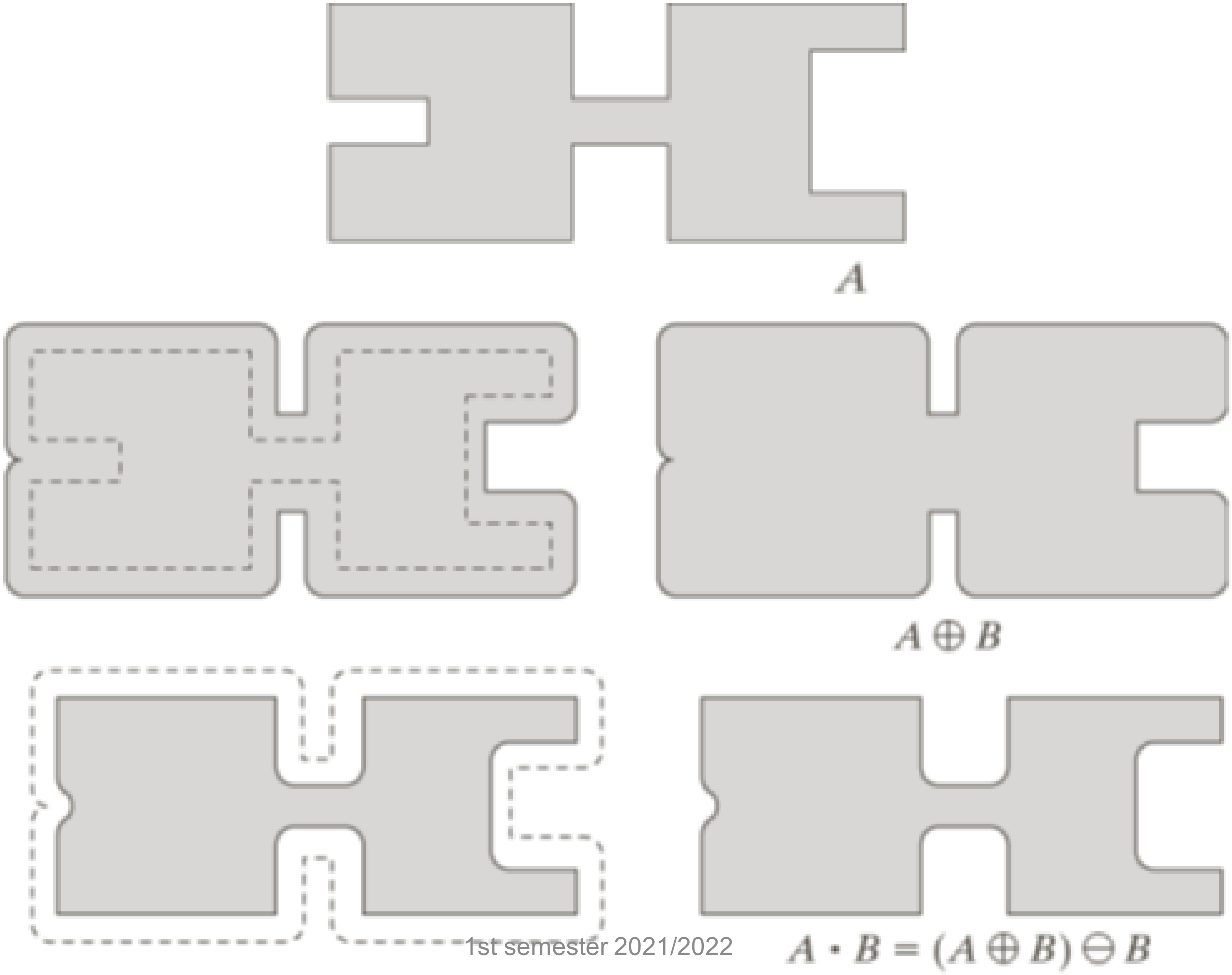


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Opening and Closing Operations (2)



Opening and Closing Operations (3)

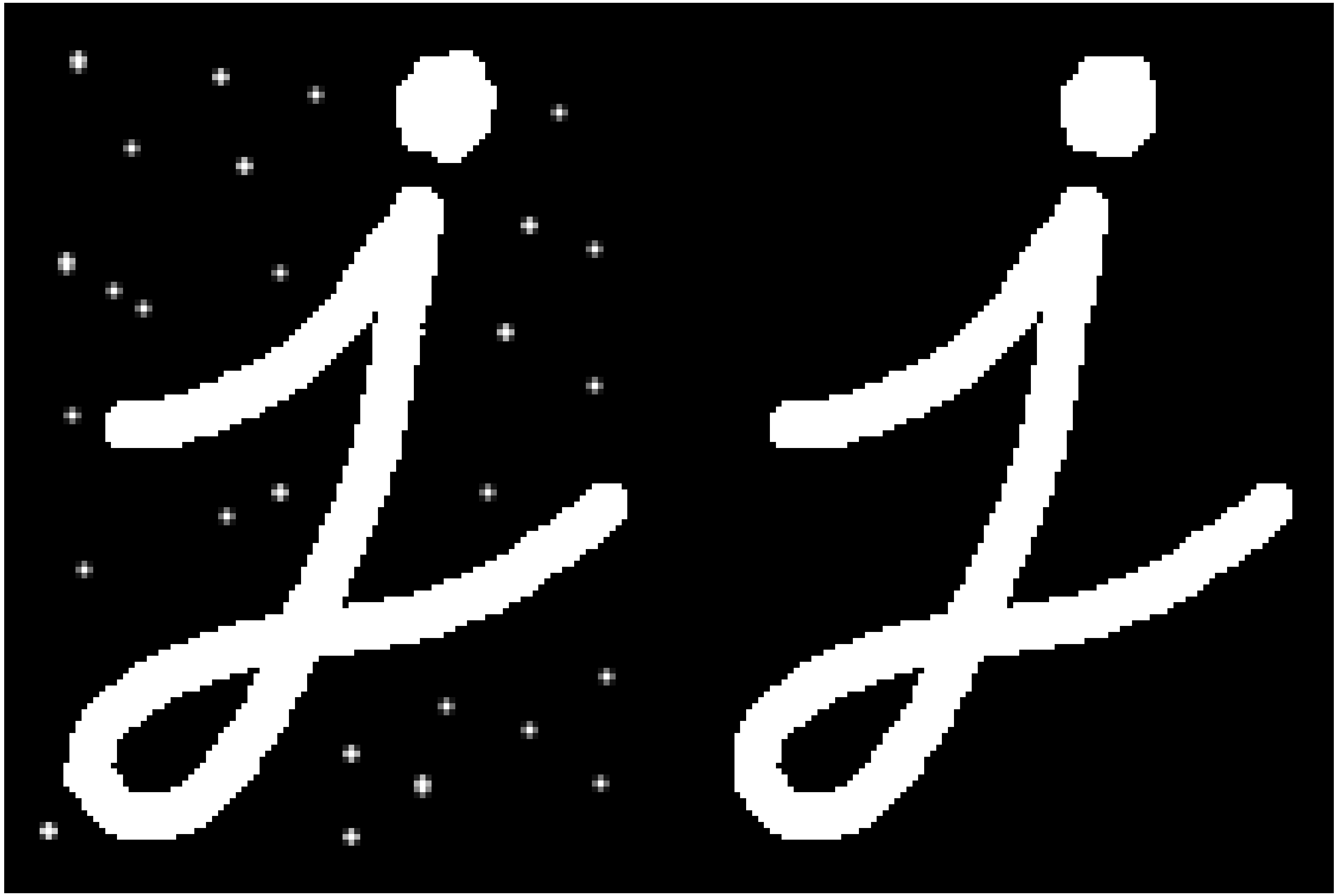


a b
c d
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Opening Operation

- Opening operation removes noise



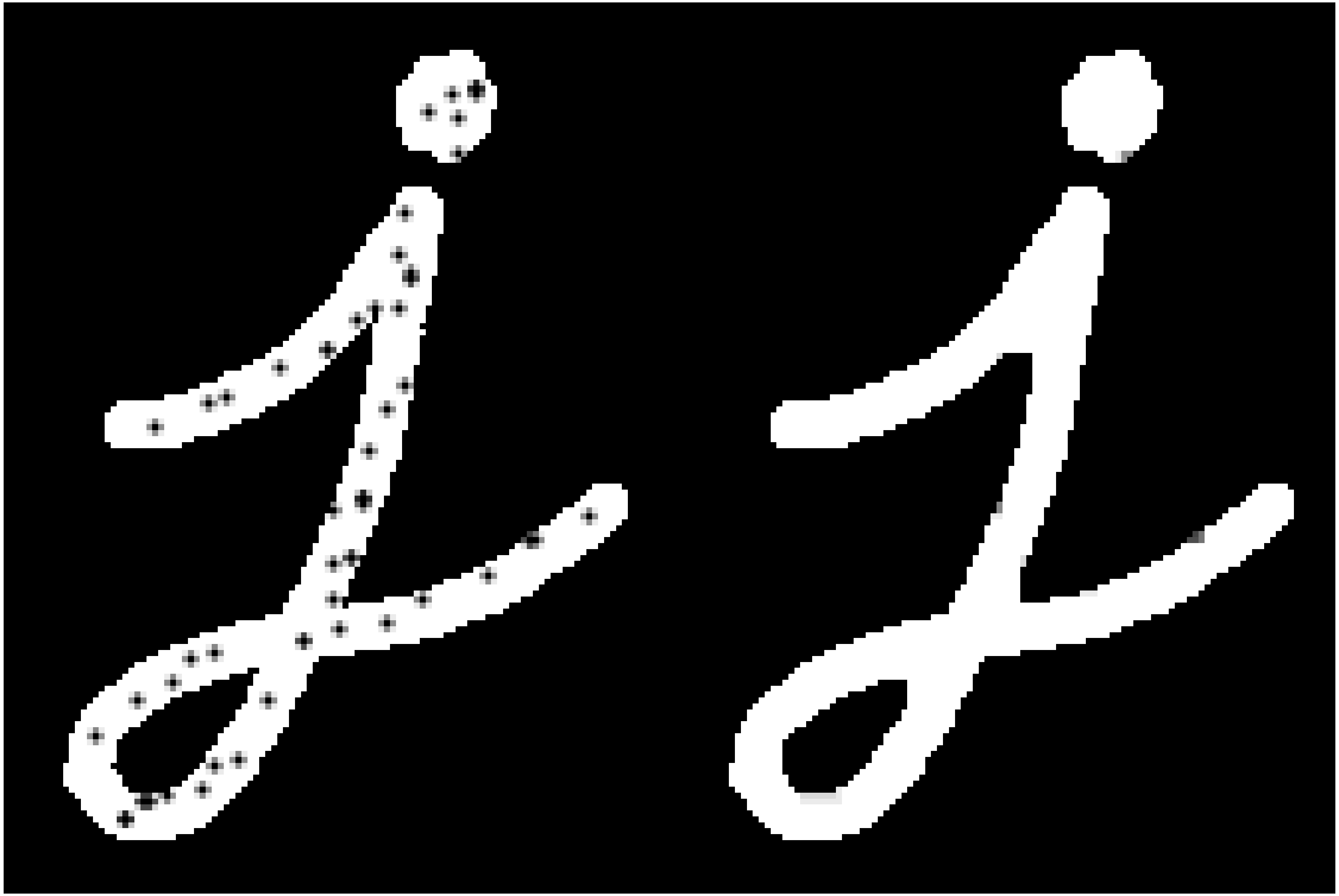
https://docs.opencv.org/3.4/d9/d61/tutorial_py_morphological_ops.html

Opening Operation

- Opening
    - First erode, then dilate
    - “Opens” the image
    - Smooths the objects contours
    - Breaks components into small parts
    - Removes small perturbations

Closing Operation

- Closing operation removes small holes



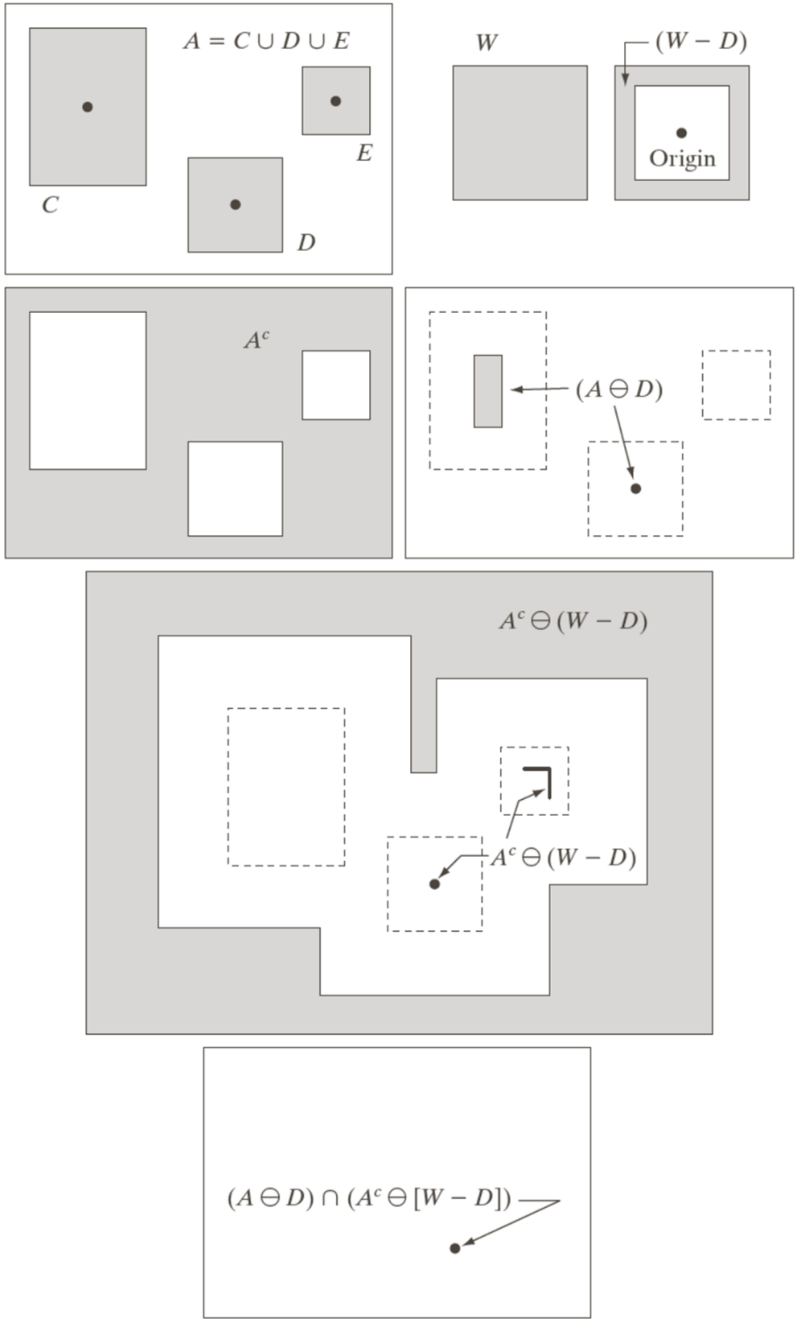
Closing Operation

- Closing
  - First dilate, then erode
  - “Closes” the image
  - Smooths the objects contours
  - Joins small parts
  - Joins thin lines
  - Fills gaps

Common Morphologic Operations

- Hit-or-Miss Transformation
- Boundary Extraction
- Hole Filling
- Extraction of Connected Components
- Convex Hull
- Thining
- Tickening
- Morphologic Gradient
- Top Hat
- Black Hat / Bottom Hat

Hit-or-Miss Transformation (1)



a	b
c	d
e	
f	

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

MATLAB
bwhitmiss.m

https://www.mathworks.com/help/images/ref/bwhitmiss.html

Hit-or-Miss Transformation (2)

https://www.mathworks.com/help/images/ref/bwhitmiss.html

MATLAB bwhitmiss.m

BW2 = bwhitmiss(BW,SE1,SE2)

performs the hit-miss operation defined by the structuring elements SE1 and SE2

The hit-miss operation

preserves pixels in binary image BW whose neighborhoods match the shape of SE1

and

don't match the shape of SE2

Hit-or-Miss Transformation (3)

```
bw = [0 0 0 0 0 0
      0 0 1 1 0 0
      0 1 1 1 1 0
      0 1 1 1 1 0
      0 0 1 1 0 0
      0 0 1 0 0 0]
```

```
interval = [0 -1 -1
            1  1 -1
            0  1  0];
```

Perform hit-miss operation.

```
bw2 = bwhitmiss(bw,interval)
```

```
bw2 = 6x6 logical array

   0   0   0   0   0   0
   0   0   0   1   0   0
   0   0   0   0   1   0
   0   0   0   0   0   0
   0   0   0   0   0   0
   0   0   0   0   0   0
```

Boundary Extraction (1)

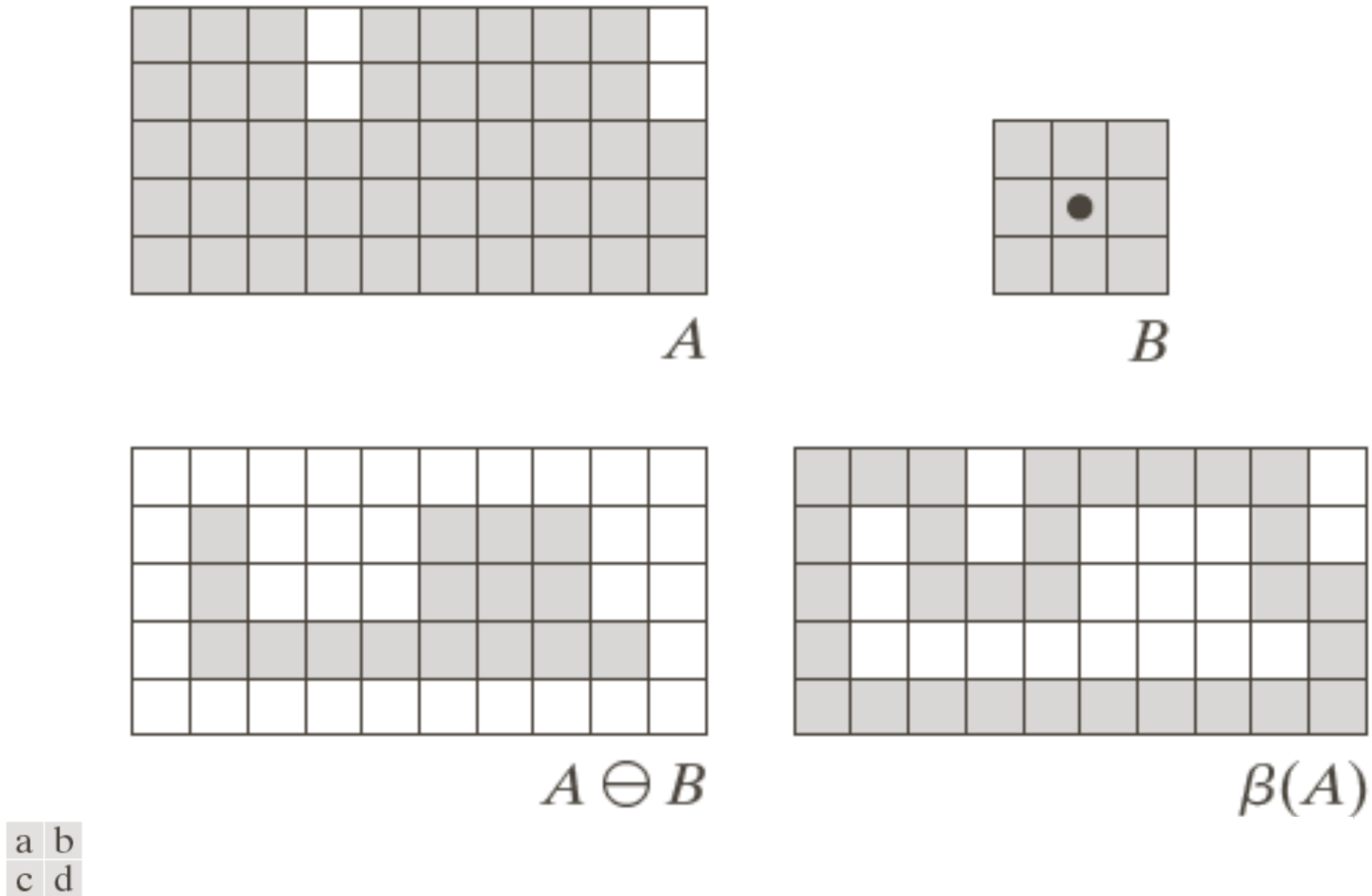


FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

Boundary Extraction (2)

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Hole Filling (1)

- Conditional Dilation

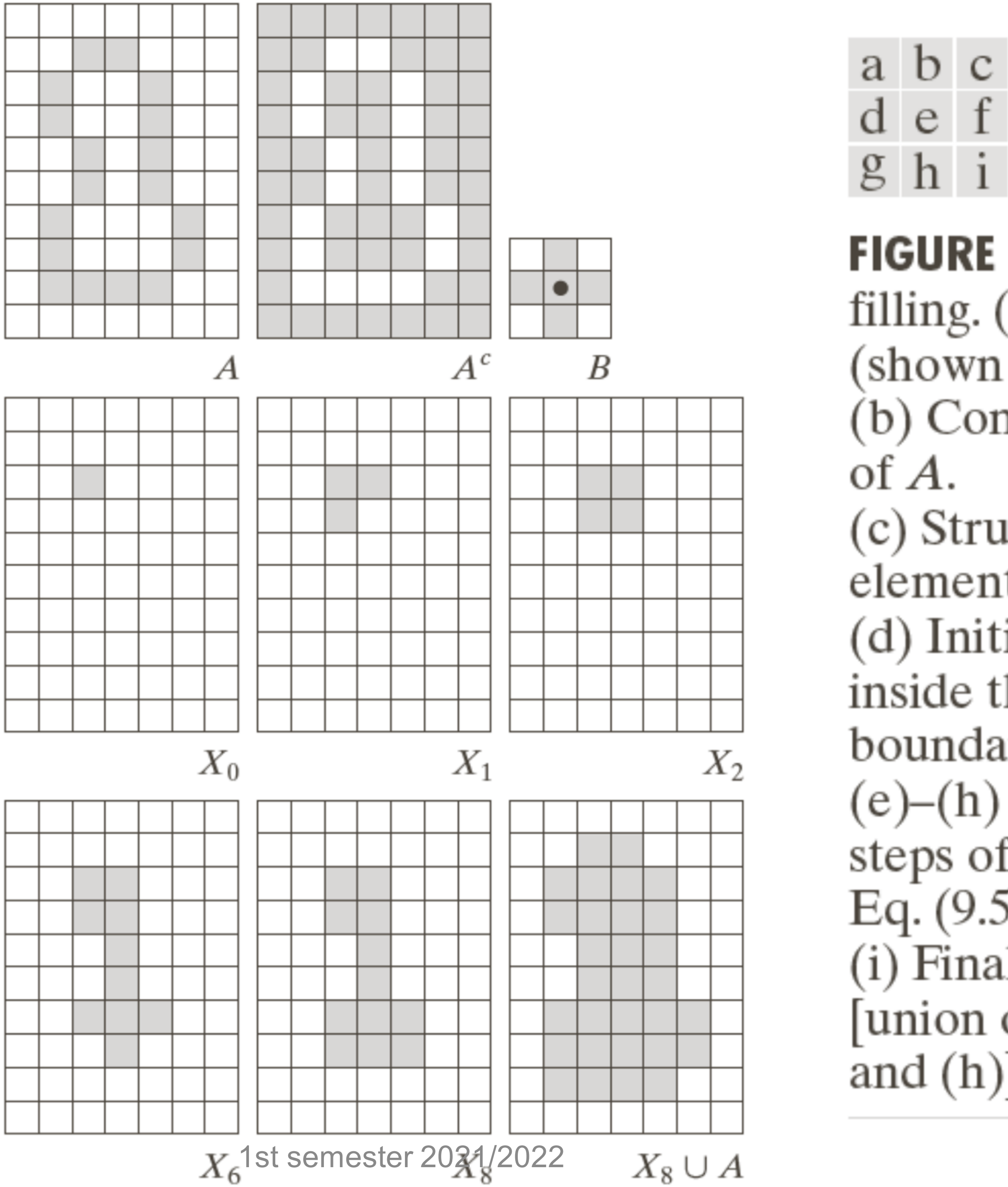


FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Hole Filling (2)

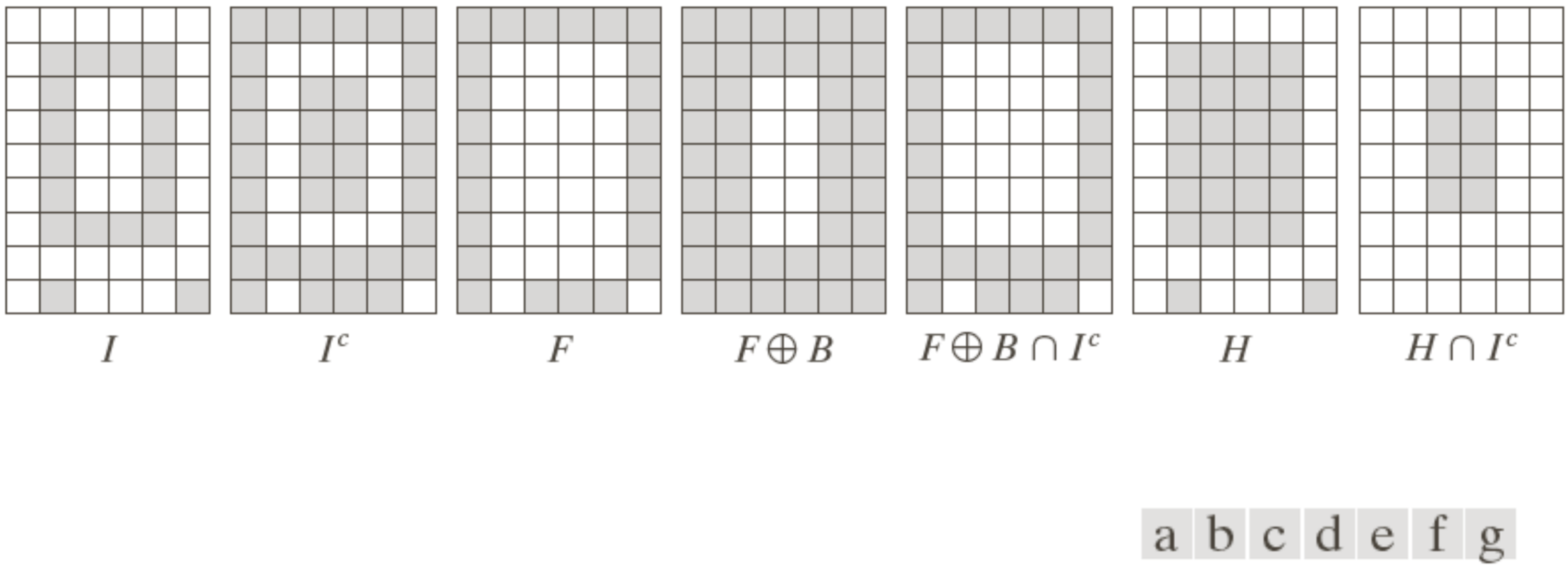


FIGURE 9.30 Illustration of hole filling on a simple image.

Hole Filling (3)

a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

Extraction of Connected Components (1)

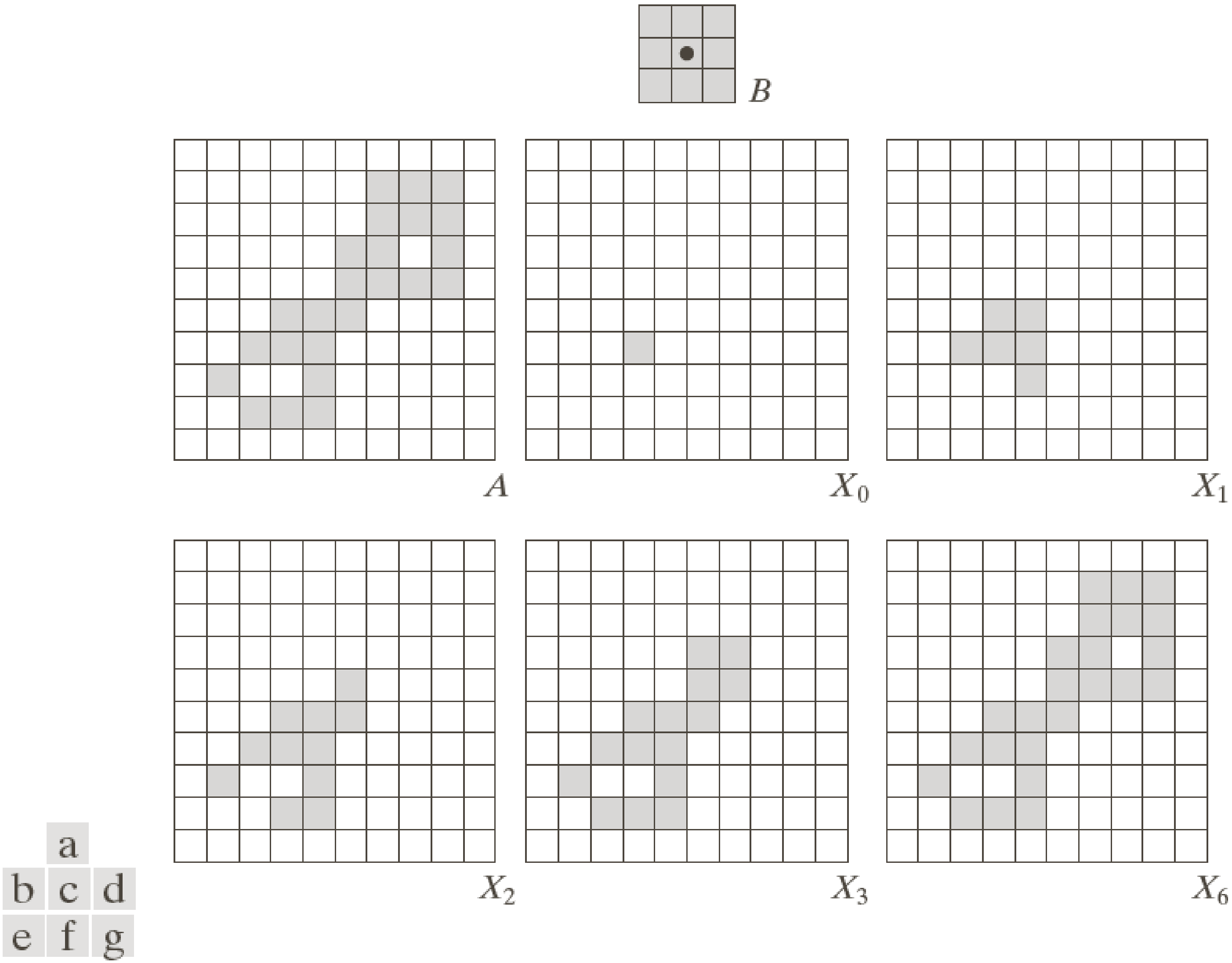
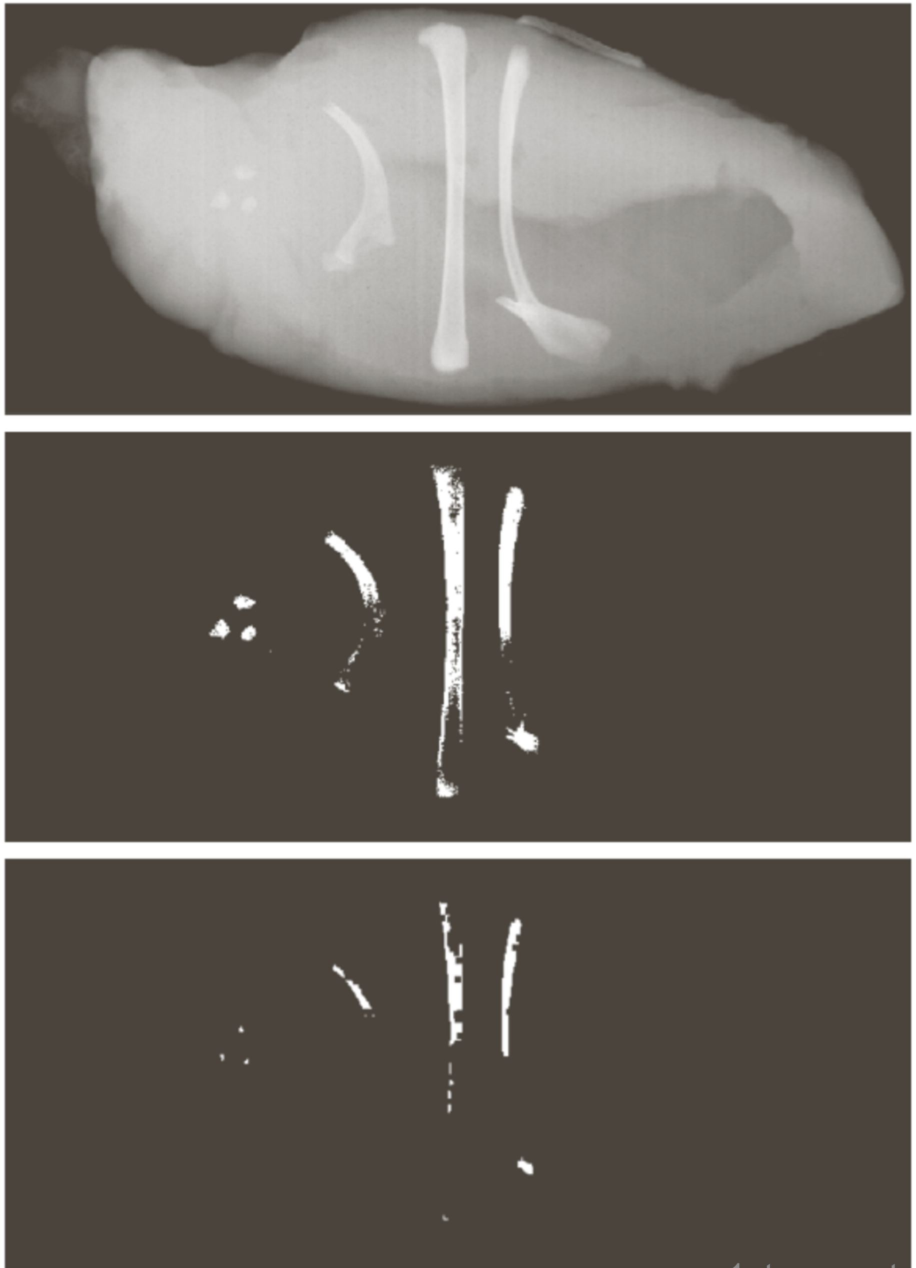


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).

Extraction of Connected Components (2)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Convex Hull (1)

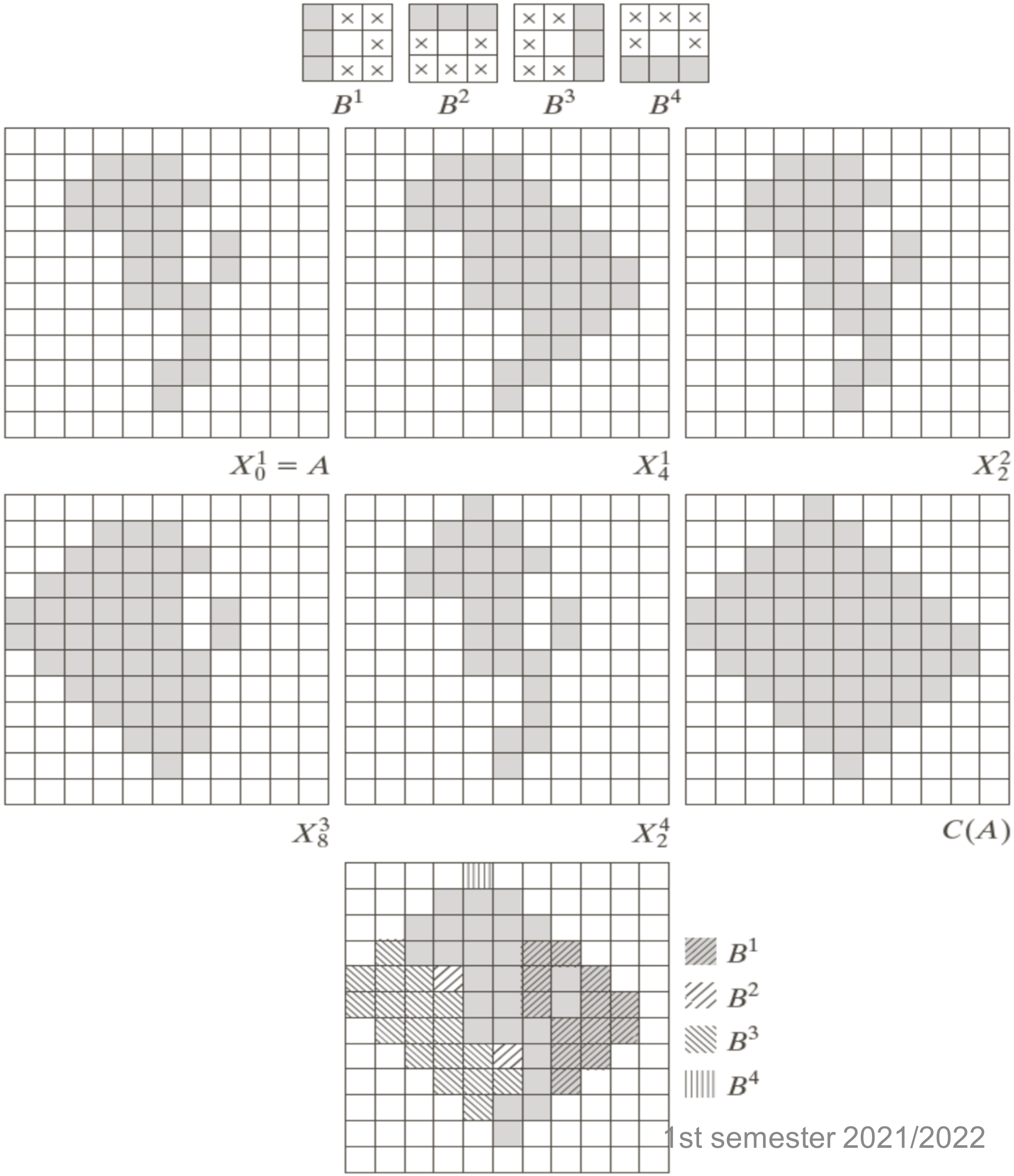


FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

Convex Hull (2)

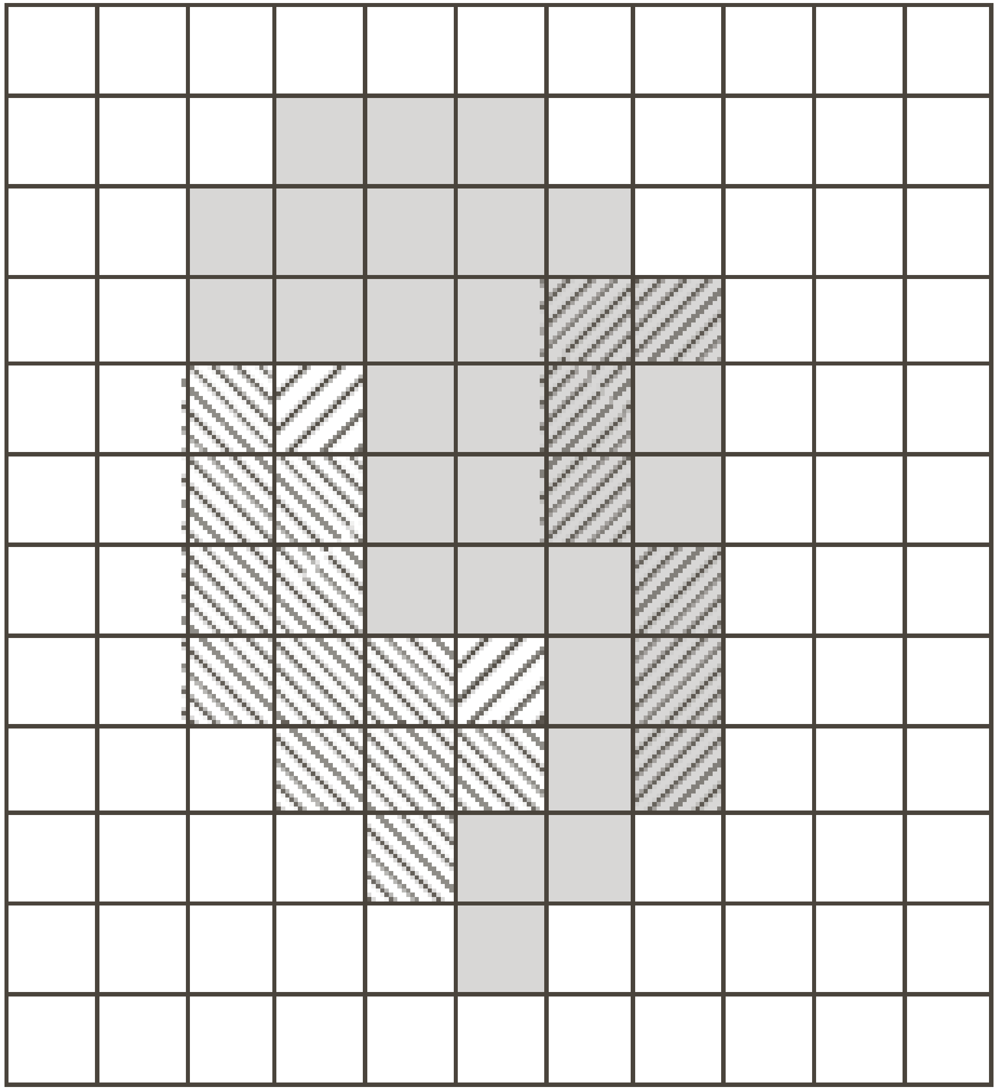
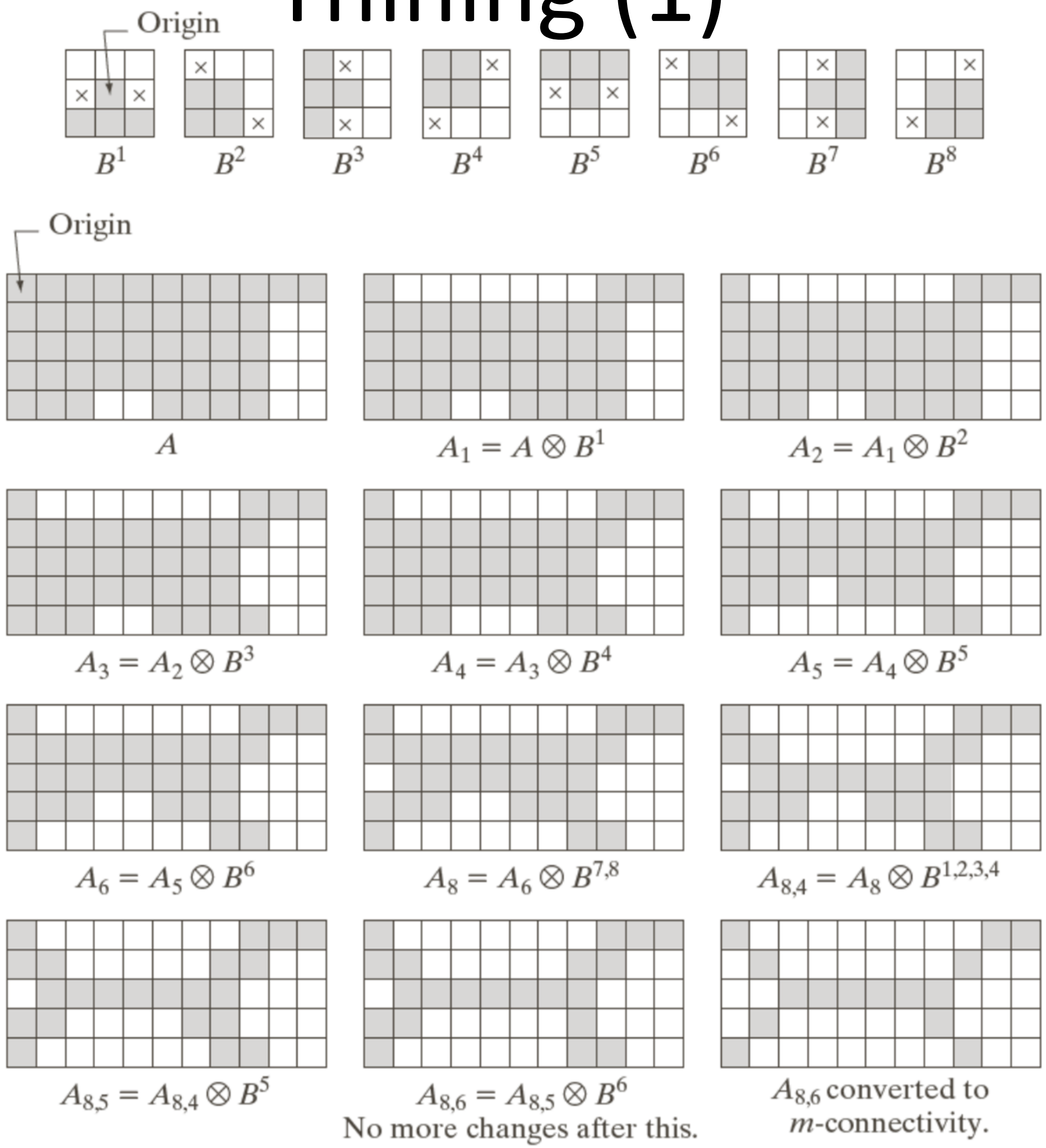


FIGURE 9.20 Result of limiting growth of the convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

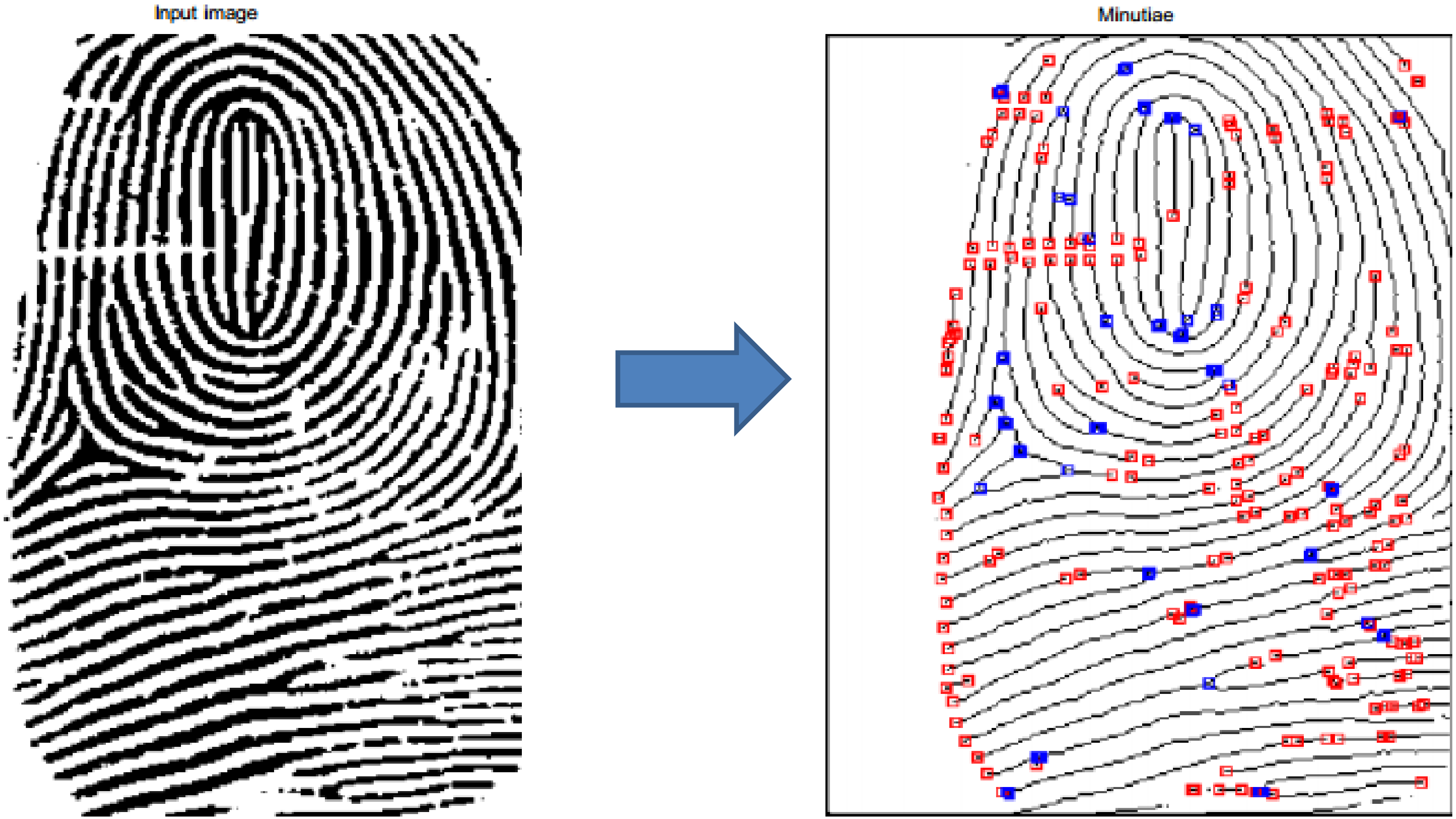
Thining (1)



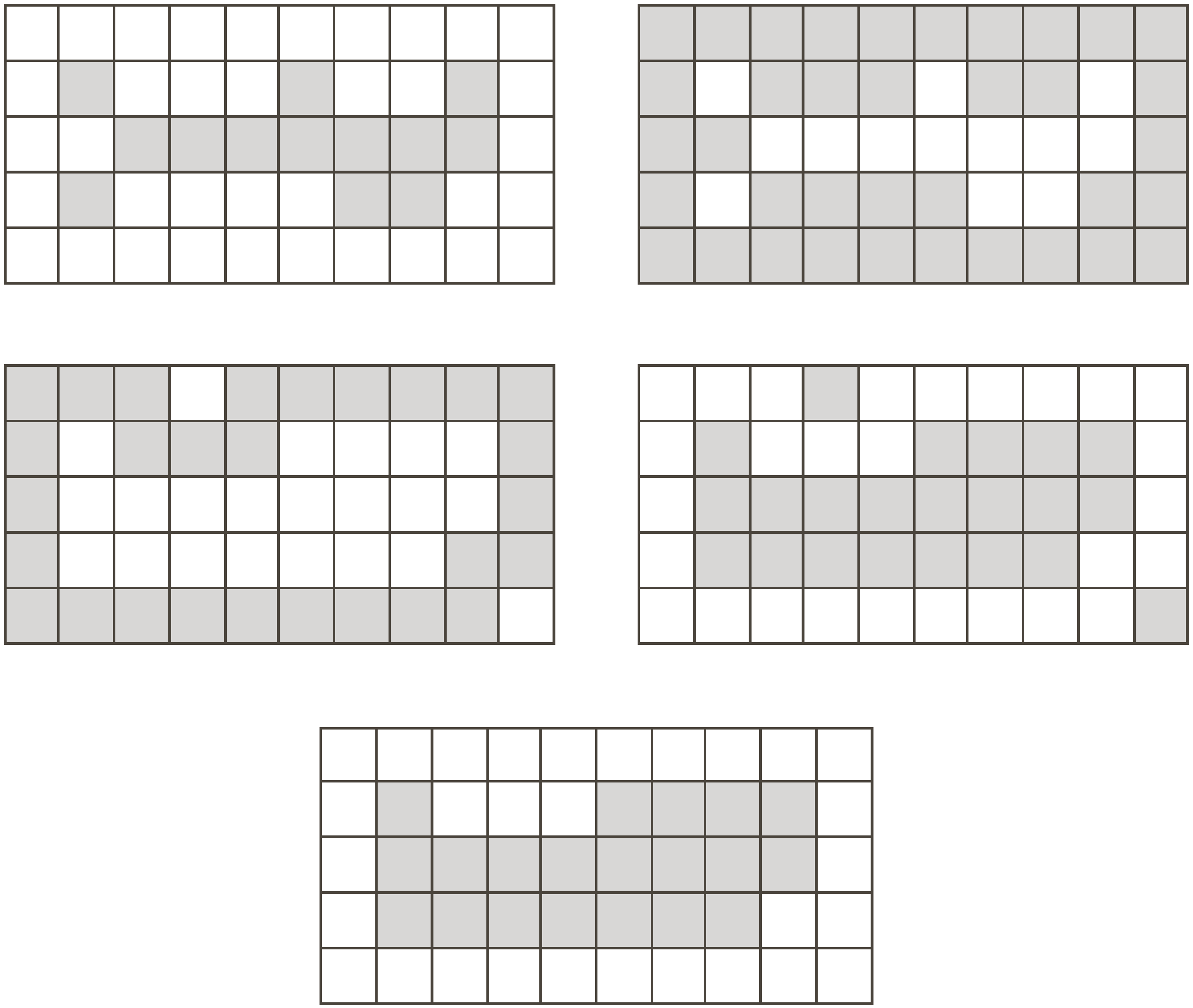
a
b c d
e f g
h i j
k l m

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

Thining (2)



Thining and Tickening

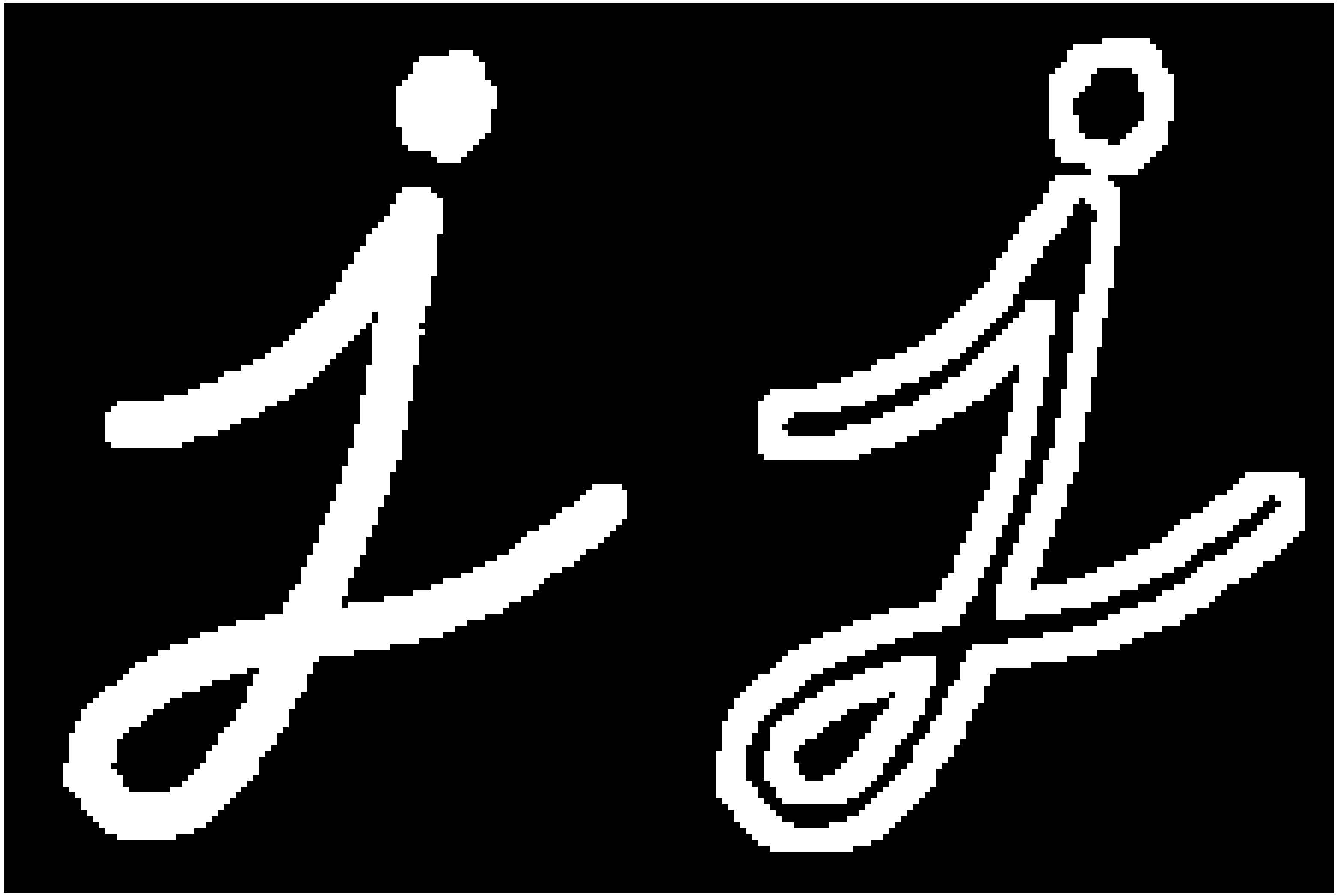


a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Morphologic Gradient

- The **morphologic gradient** is the difference between dilation and erosion of an image

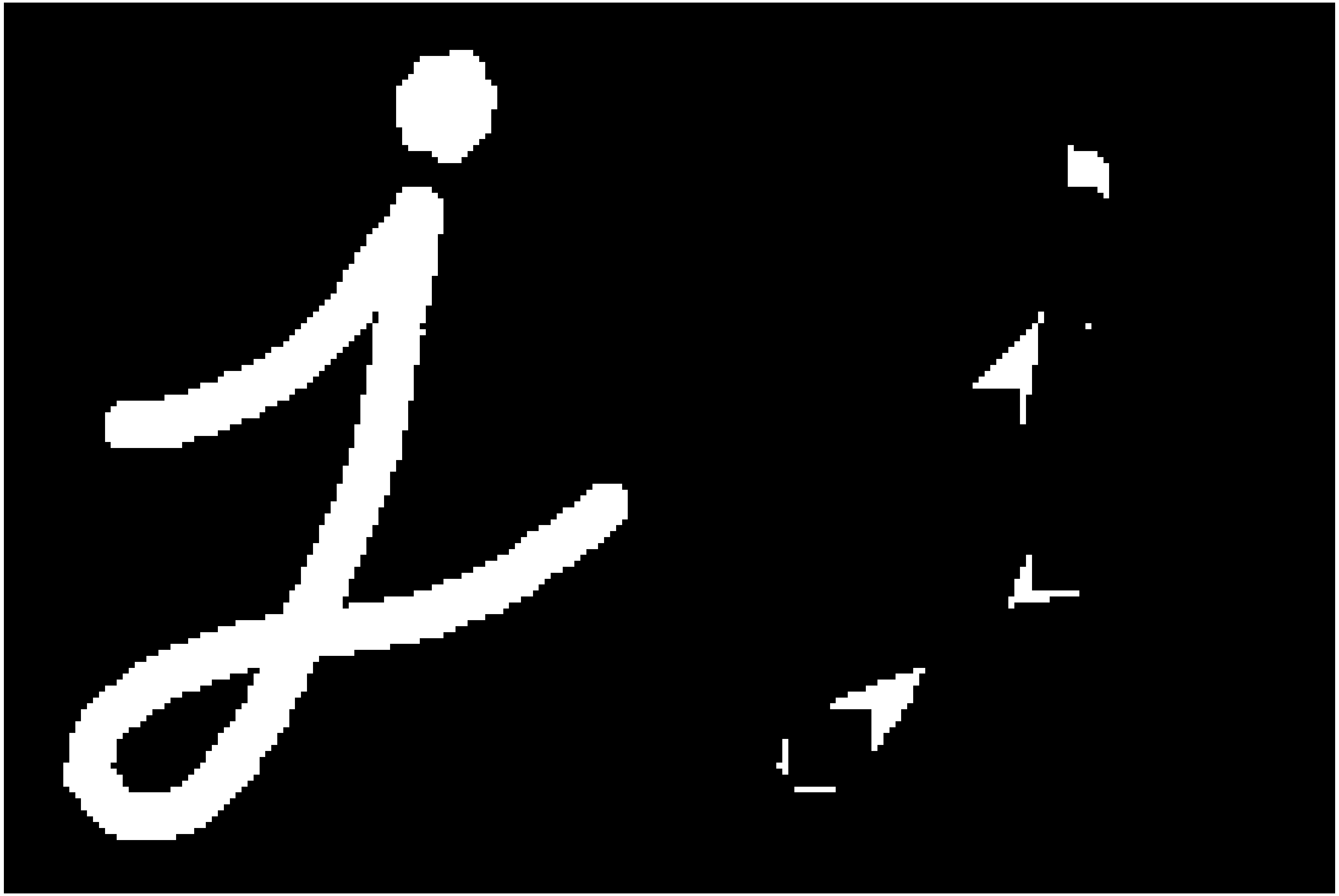


Top Hat

- The **Top Hat** is the difference between the input image and its opening

Black Hat or Bottom Hat

- The **Black Hat** is the difference between the input image and its closing



Morphologic Reconstruction (1)

- Two images: the *marker* (starter points) and the *mask* (constrains the transformation)
- One SE
- Geodesic dilation

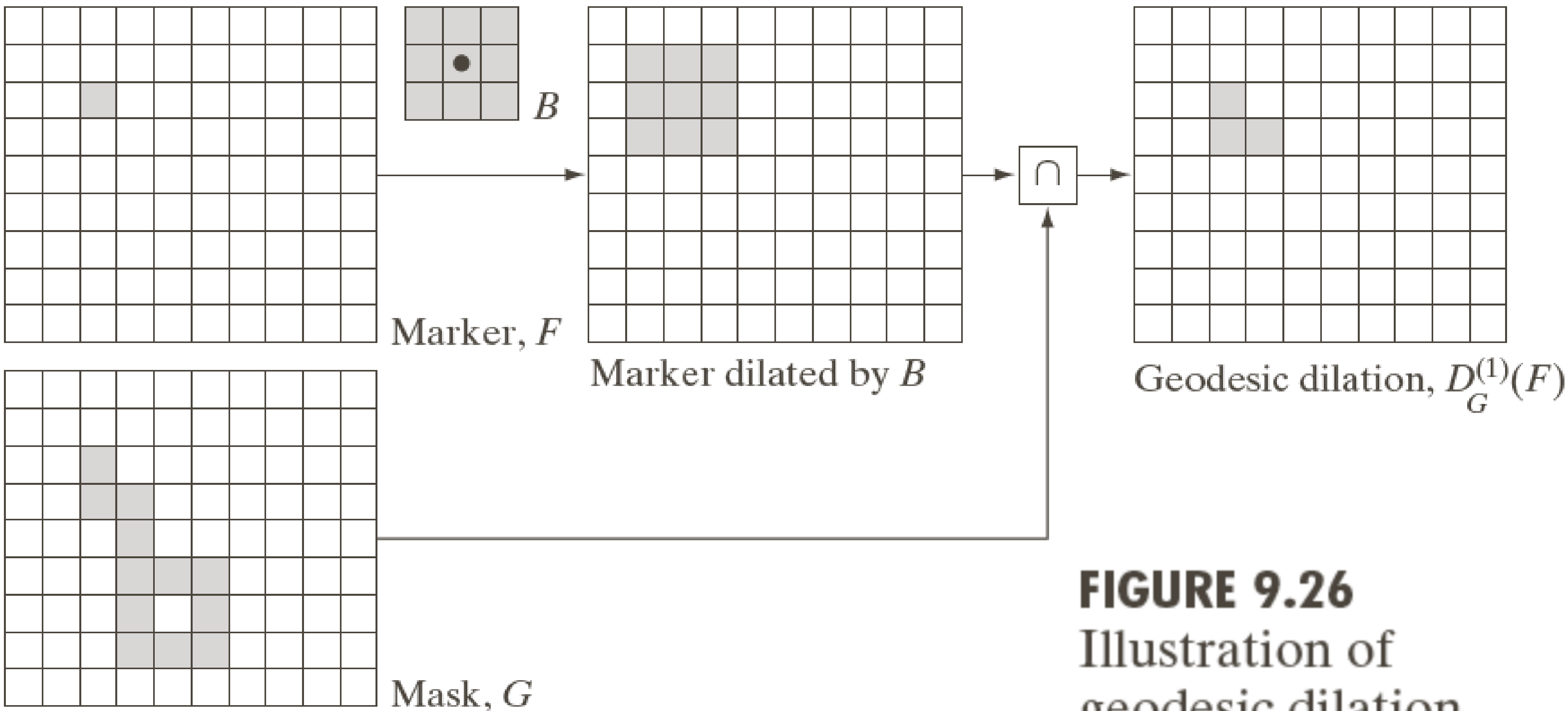


FIGURE 9.26
Illustration of geodesic dilation.

Morphologic Reconstruction (2)

- Two images: the *marker* (starter points) and the *mask* (constrains the transformation)
- One SE
- Geodesic erosion

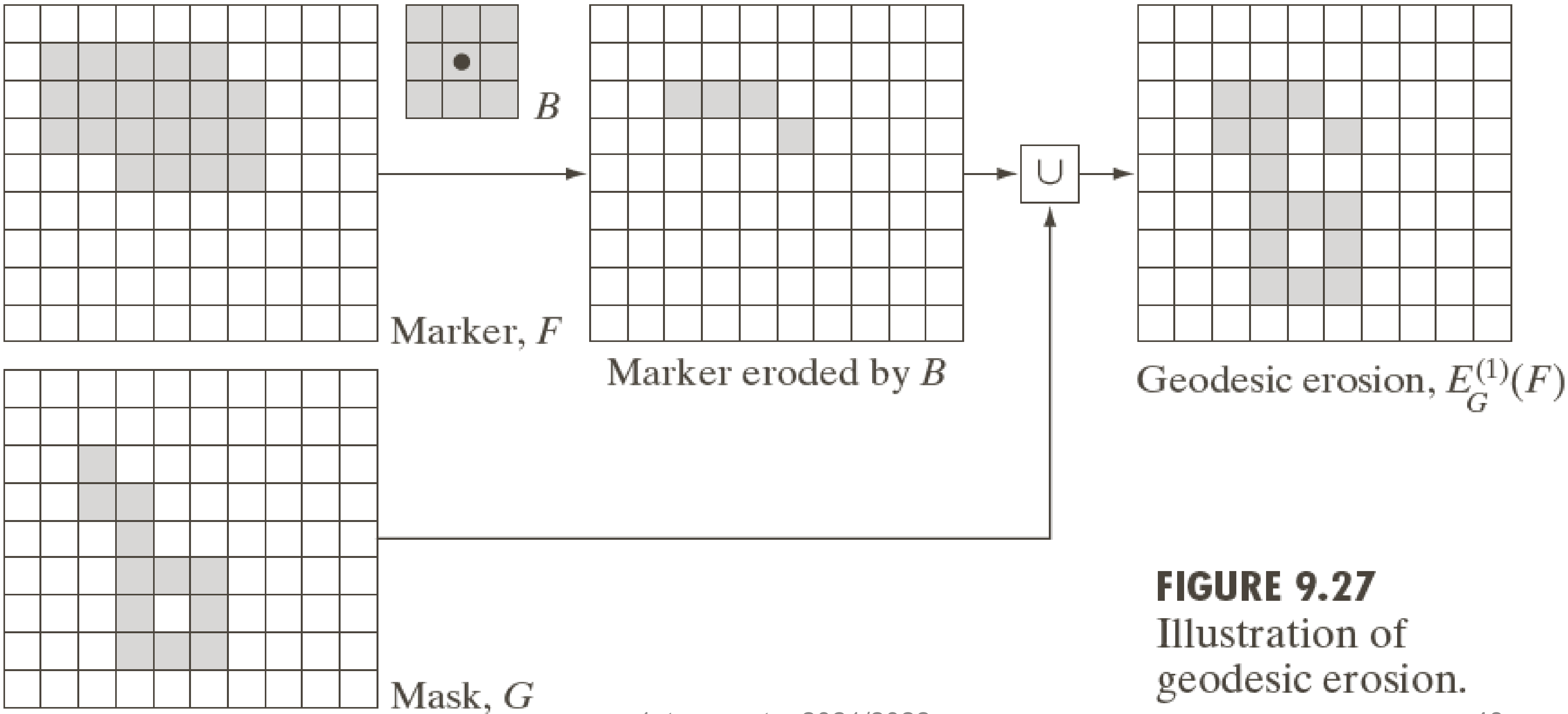
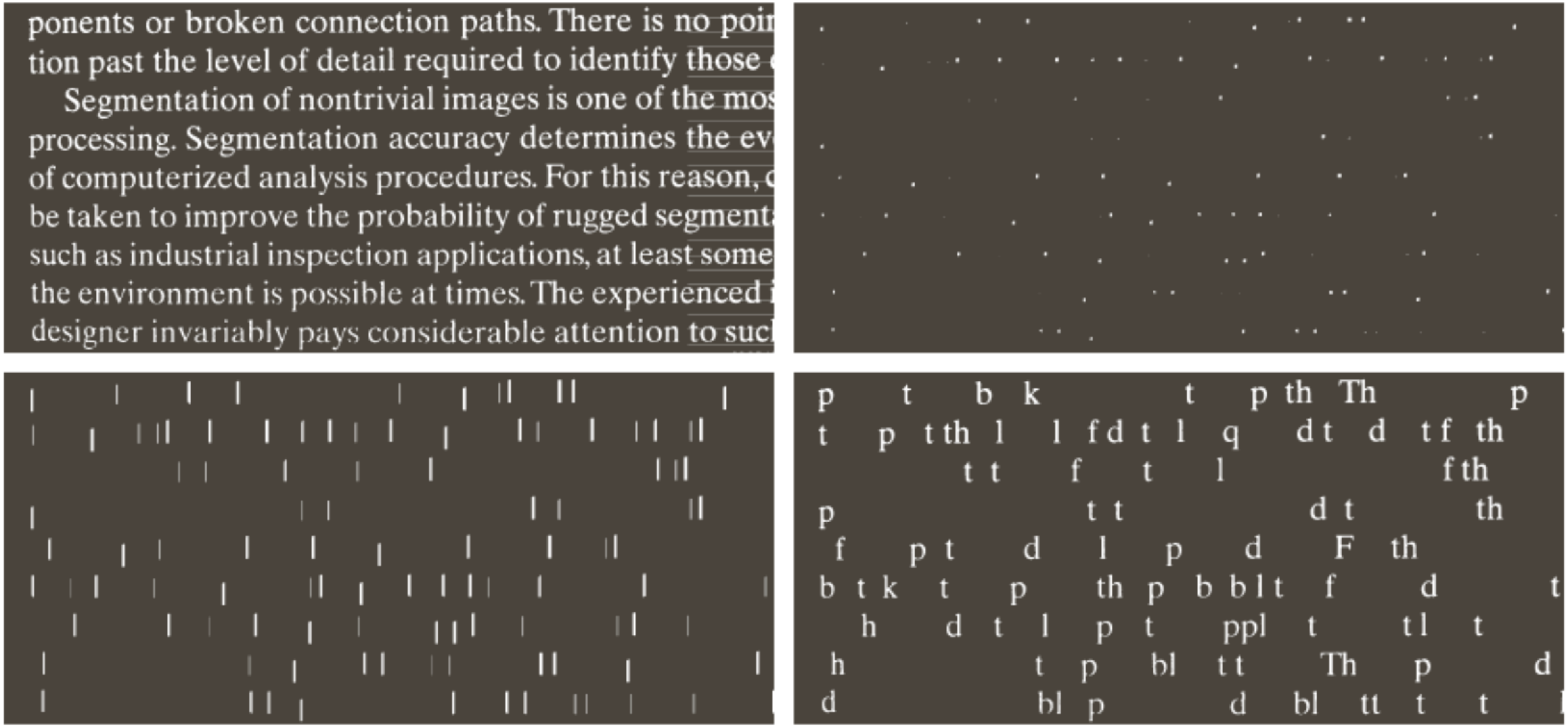


FIGURE 9.27
Illustration of geodesic erosion.

Opening by Reconstruction



a b
c d

FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

Morphologic Operations – summary (1)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

TABLE 9.1 Summary of morphological operations and their properties.

(*Continued*)

Morphologic Operations – summary (2)

TABLE 9.1 *(Continued)*

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \dots$	Fills holes in A; $X_0 =$ array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \dots$	Finds connected components in A; $X_0 =$ array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \dots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\dots((A \otimes B^1) \otimes B^2)\dots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\dots(A \odot B^1) \odot B^2 \dots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

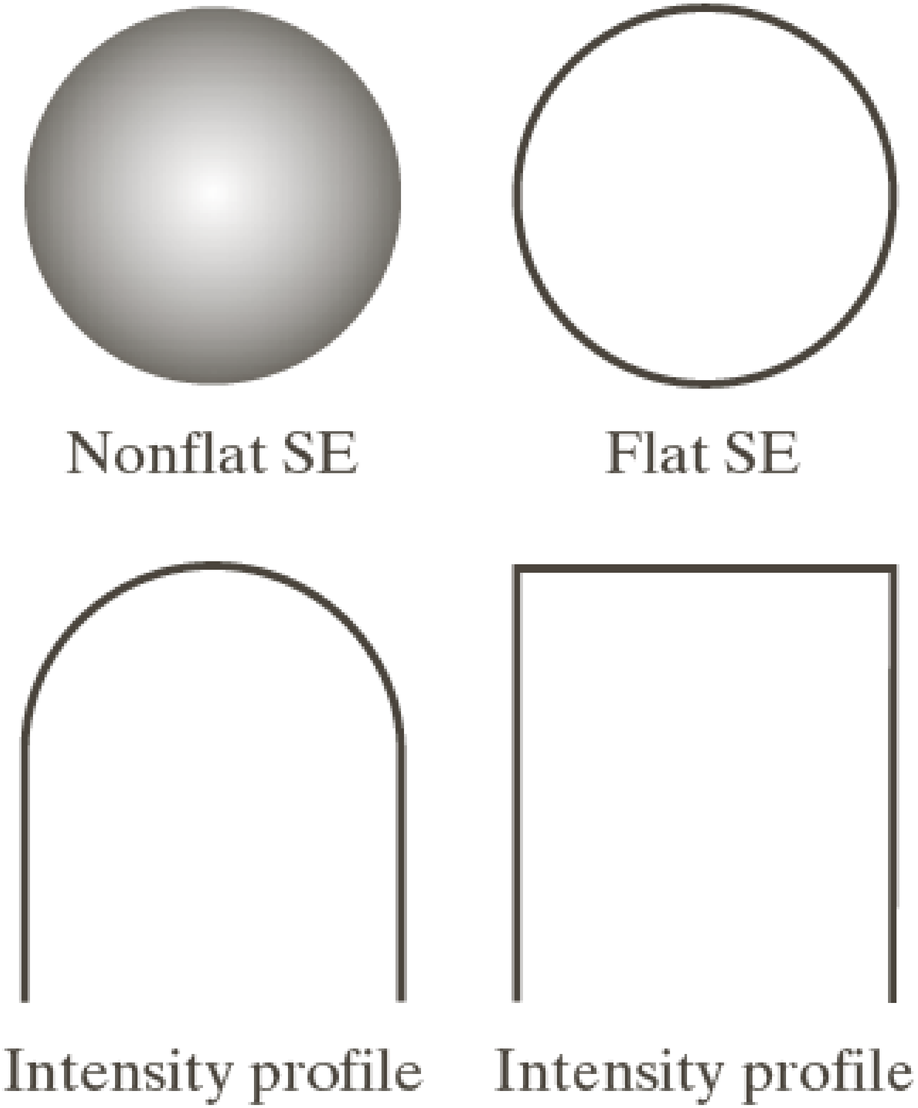
(Continued)

Morphologic Operations – summary (3)

TABLE 9.1 *(Continued)*

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8}(X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and *mask* images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right];$ $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right];$ $E_G^{(0)}(F) = F$	
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B.
Hole filling	$H = \left[R_{I^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F.

Grayscale Morphology (1)



a b
c d

FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

Grayscale Morphology (2)

- The Structuring Element (SE) is now a grayscale small image

- There are four basic morphologic operations
  1) **Erosion** – find minimum values

  2) **Dilation** – find maximum values

  3) **Opening** – remove small bright details

  4) **Closing** – the opposite of opening

Grayscale Morphology (3)

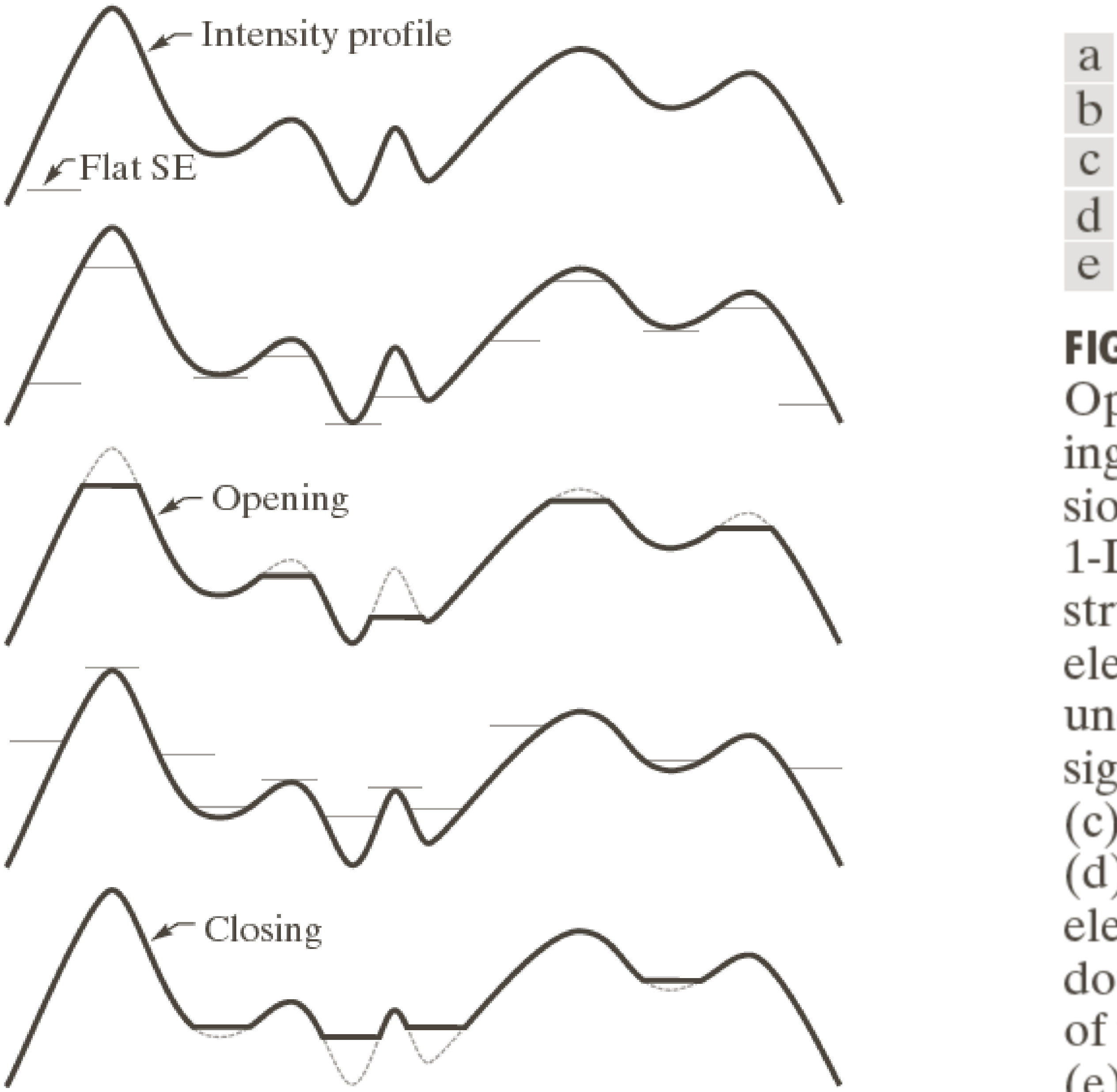
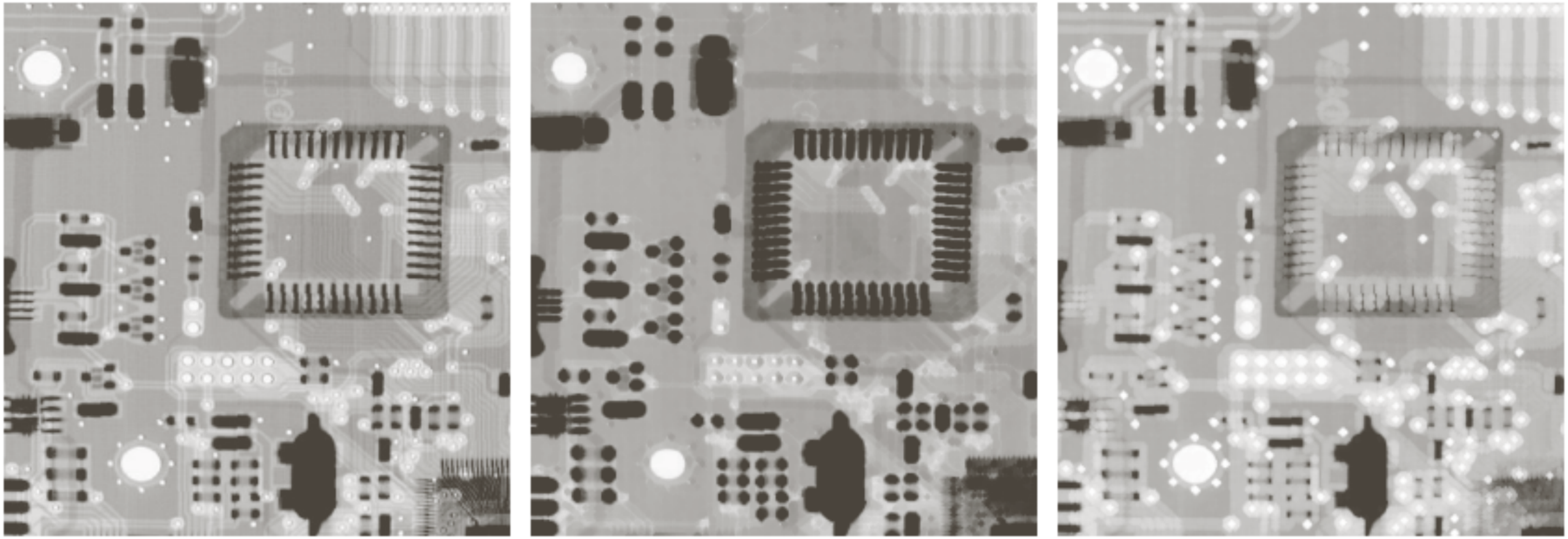


FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

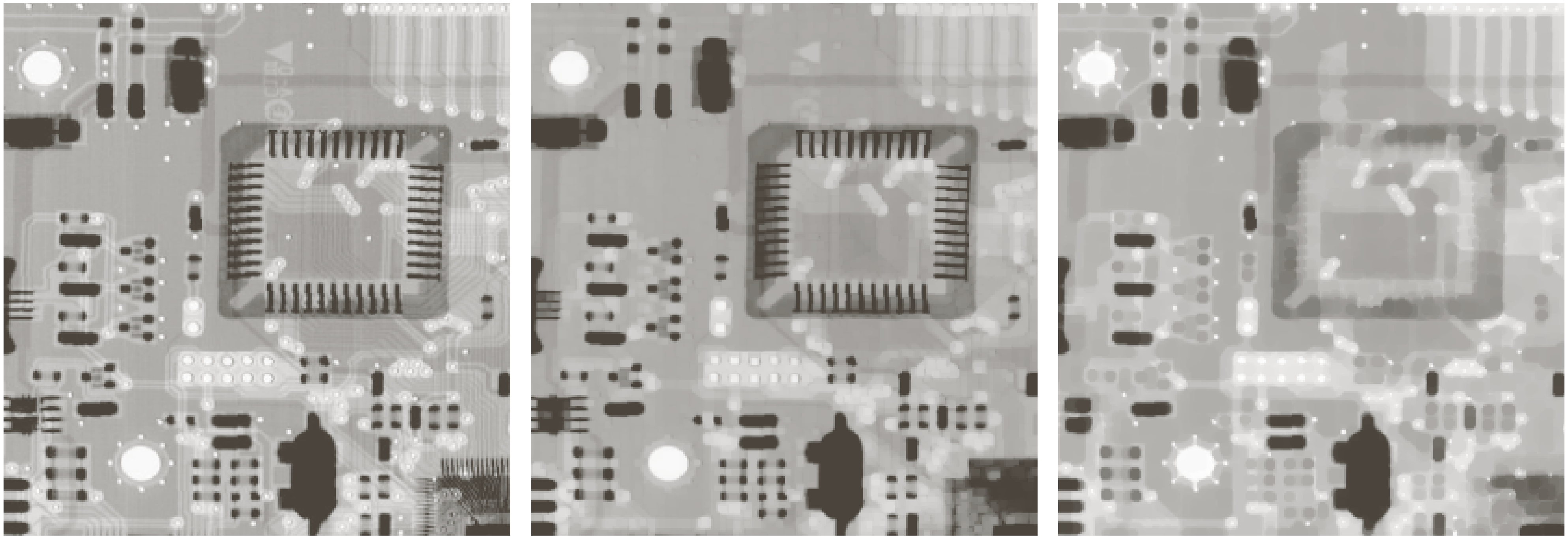
Grayscale Morphology (4)



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

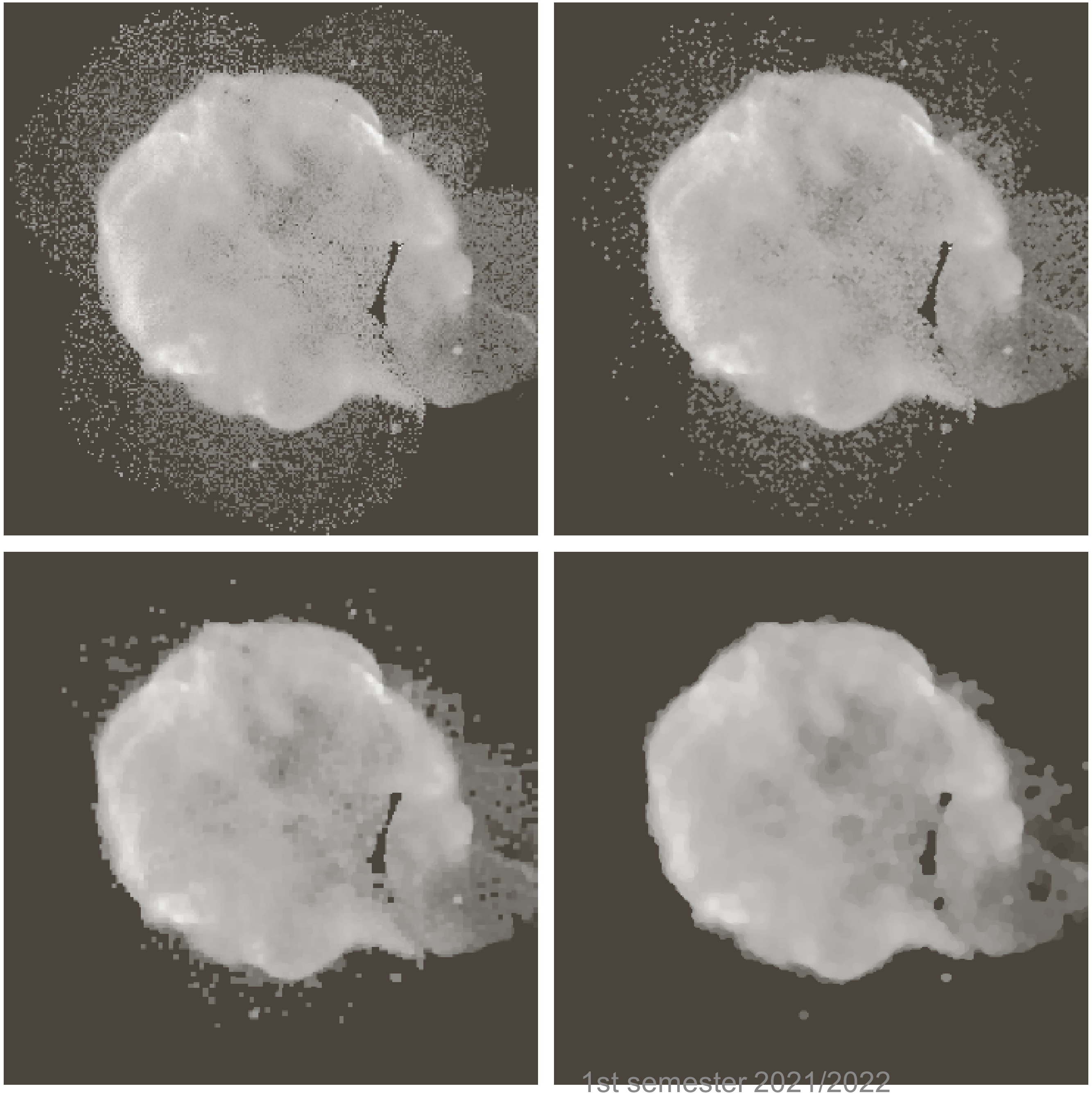
Grayscale Morphology (5)



a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

Grayscale Morphology (6)



a	b
c	d

FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

Grayscale Morphology (7)

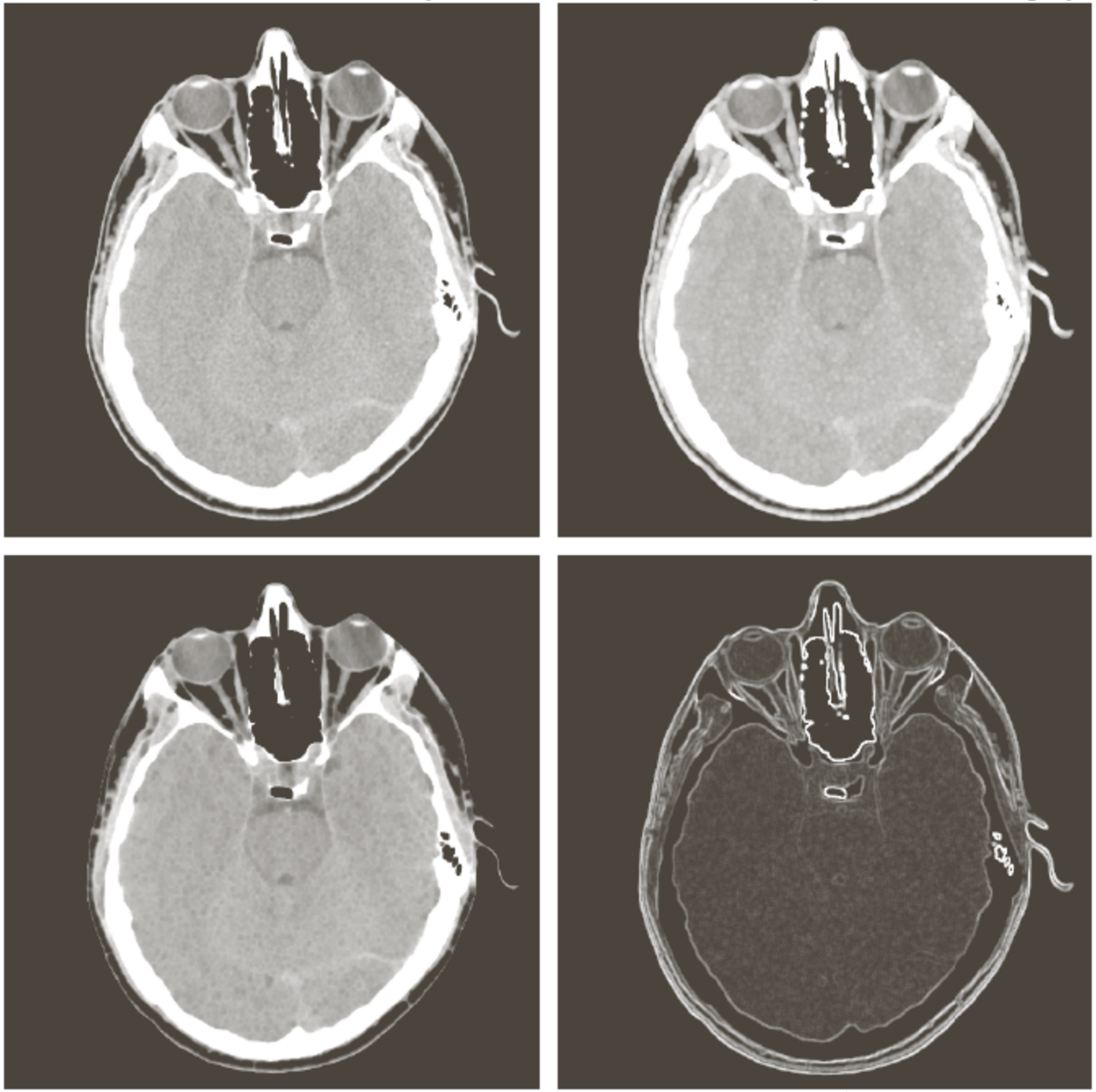
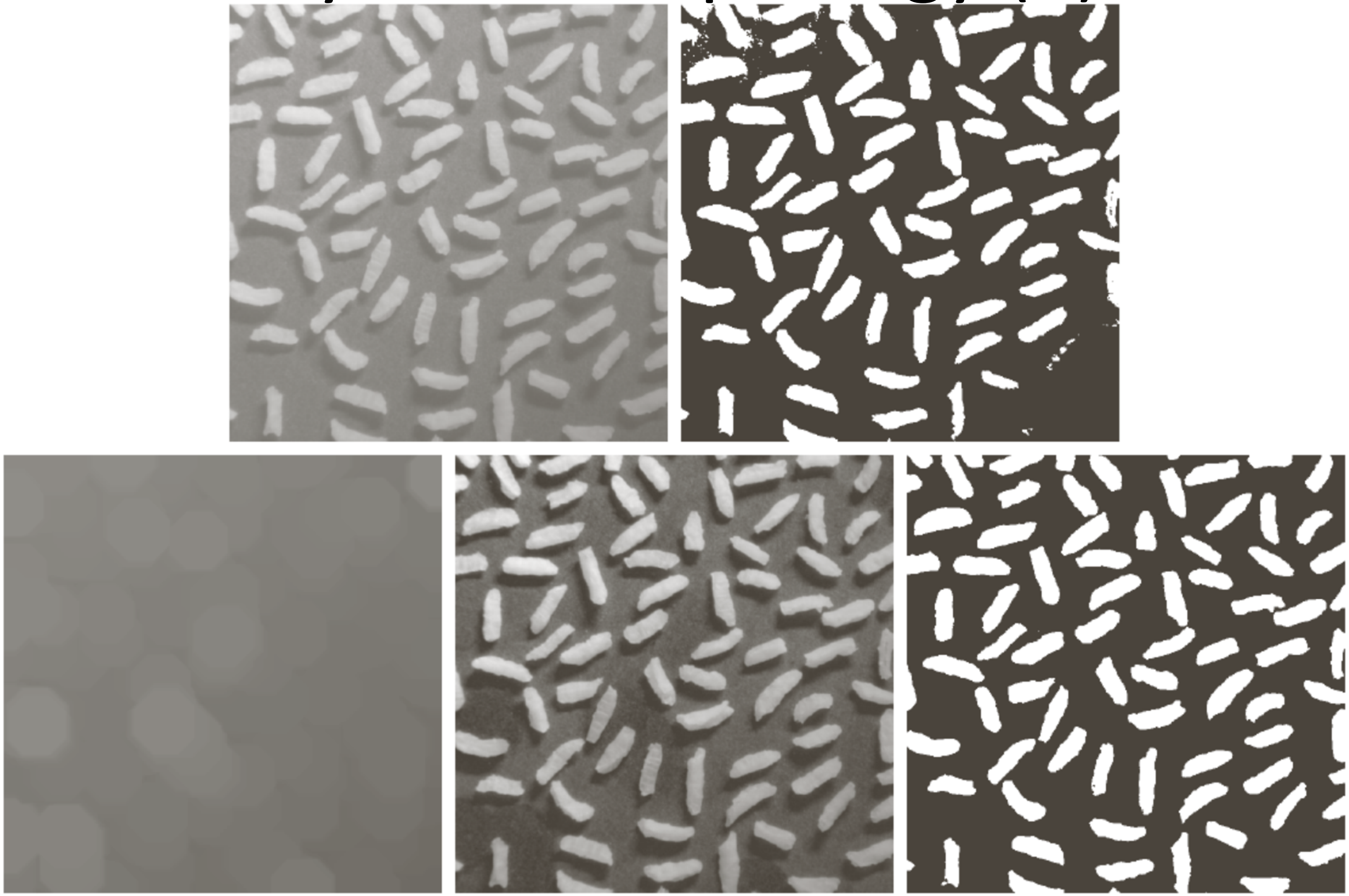


FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

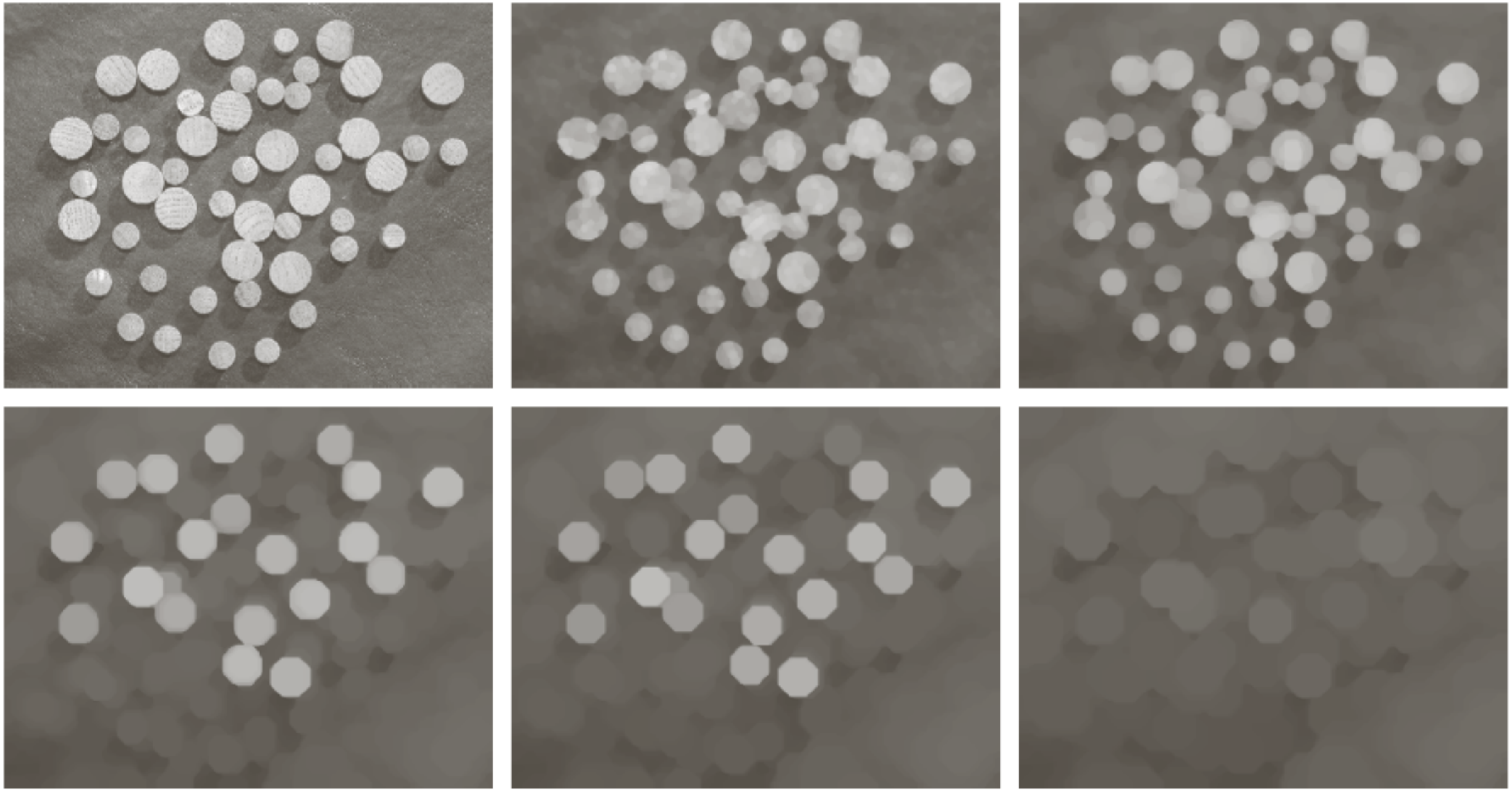
Grayscale Morphology (8)



a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

Grayscale Morphology (9)



a b c
d e f

FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

a b c
d e f
g h i

FIGURE 9.44 (a) Original image of size 1134 × 1360 pixels. (b) Opening by reconstruction of (a) using a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same line. (d) Top-hat by reconstruction. (e) Top-hat. (f) Opening by reconstruction of (d) using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

MATLAB Functions

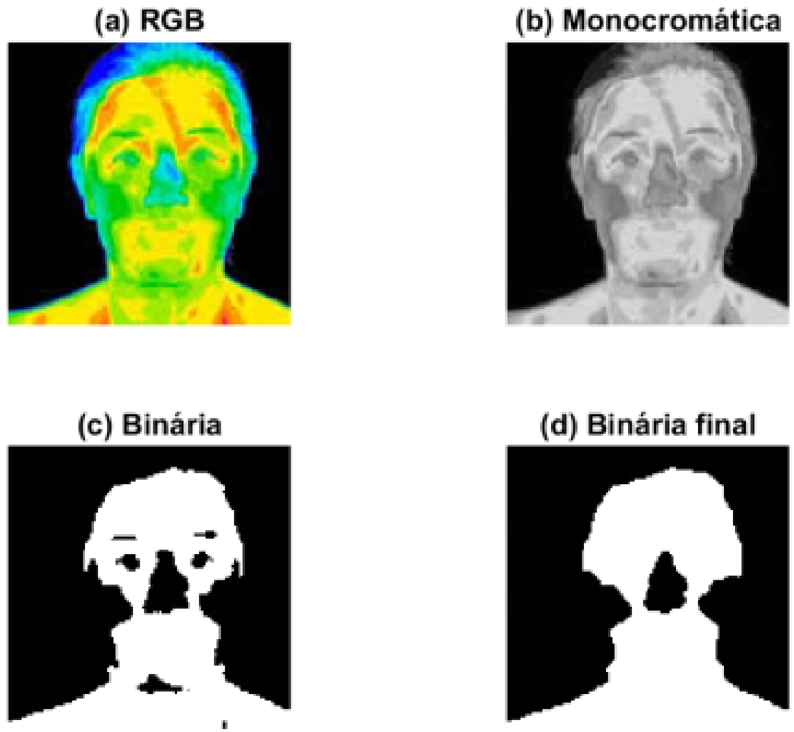
- strel.m, defines a structuring element
- imerode.m, erosion
- imdilate.m, dilation
- imopen.m, opening
- imclose.m, closing
- bwhitmiss.m, hit-and-miss transformation
- bwmorph.m, (many) morphological operations on binary images

https://www.mathworks.com/help/images/morphological-filtering.html

Exercises (1)

In Portuguese. Exercício 4 do 2.º teste parcial, verão 2016/2017, 5 de junho de 2017

4. A figura apresenta quatro imagens:



(a) imagem RGB que representa um termograma facial;

(b) versão monocromática de (a);

(c) versão binária de (b);

(d) imagem binária com uma mancha que representa a área facial de temperaturas mais elevadas (excluindo as zonas frias da imagem).

a) {1,5} Caraterize o dispositivo e o processo físico que realiza a aquisição e a geração da imagem (a). Indique uma forma possível de obter a imagem (b), a partir da imagem (a).

b) {2,0} Indique como se obtém a imagem (c), a partir da imagem (b). Indique um procedimento detalhado para transformar a imagem (c) na imagem (d).

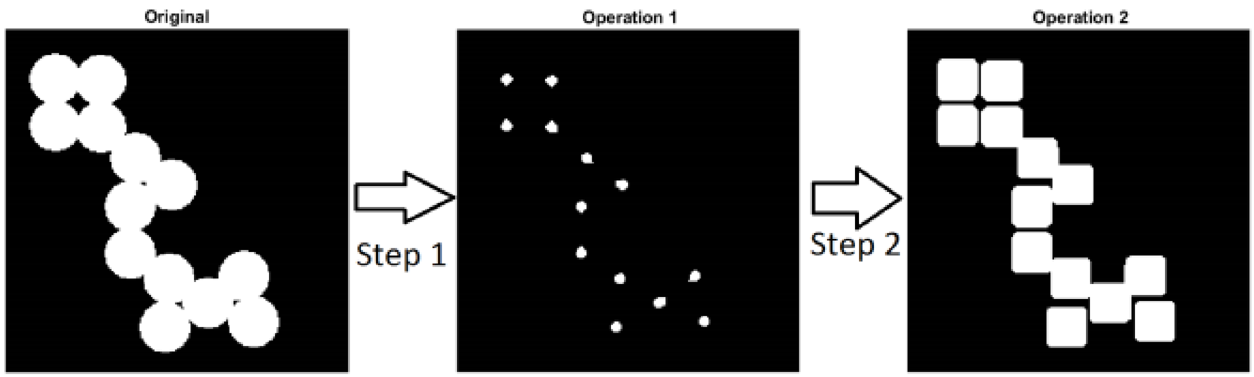
Exercises (2)

In Portuguese. Exercício 8 do teste de época normal, verão 2015/2016, 4 de julho de 2016

8. {R2} Tenha em conta as técnicas de processamento morfológico de imagem.

   i) {1,5} Quais as vantagens e desvantagens do processamento morfológico, em relação ao processamento espacial?

   ii) {2,0} A figura apresenta uma sequência de duas operações morfológicas. Descreva as operações efetuadas no passo 1 e no passo 2.



Exercises (3)

In Portuguese. Exercício 9 do teste de época normal, verão 2016/2017, 22 de junho de 2017

9. {R2||TG} A figura apresenta quatro imagens:

(a) imagem monocromática de impressão digital;

(b) versão binária de (a);

(c) imagem binária resultante do processamento de (b);

(d) imagem binária resultante do processamento de (c);

a) {1,25||1,0} Caraterize o dispositivo e o processo físico que realiza a aquisição e a geração da imagem (a). Indique uma forma possível de obter a imagem (b), a partir da imagem (a).

b) {2,0||1,0} Indique como se obtém: a imagem (c), a partir da imagem (b); a imagem (d), a partir da imagem (c).

Exercises (4)

In Portuguese. Exercício 7 do teste de época de recurso, verão 2016/2017, 10 de julho de 2017

7. A figura apresenta cinco imagens obtidas ao longo do processamento realizado por determinado algoritmo.

a) {1,0} Caraterize o tipo de ruído presente nas imagens (a) e (b). Indique duas formas distintas de obter uma versão monocromática de determinada imagem RGB.

b) {1,0} Indique como se obtém: a imagem (c), a partir da imagem (b); a imagem (d) a partir da imagem (c); a imagem (e) a partir da imagem (d).

Bibliography

- The images displayed in these slides are from:

  - R. Gonzalez, R. Woods, *Digital Image Processing*, 4th edition, Prentice Hall, 2018, ISBN 0133356728

  - S. Smith, *The Scientist and Engineer's Guide to Digital Signal Processing*, Newnes, 2003, ISBN 0-750674-44-X [chapter 23]

  - O. Filho, H. Neto, Processamento Digital de Imagens, Rio de Janeiro: Brasport, 1999, ISBN 8574520098.

  - Wikipedia and Mathworks web pages