ISEL
INSTITUTO SUPERIOR DE ENGENHARIA DE LISBOA

PROCESSAMENTO DE IMAGEM E BIOMETRIA

IMAGE PROCESSING AND BIOMETRICS

5. **SPATIAL FILTERING (part 2)**

Summary (part 2)

- Gradient concept

- Gradient operators
  - Roberts cross
  - Sobel
  - Prewitt

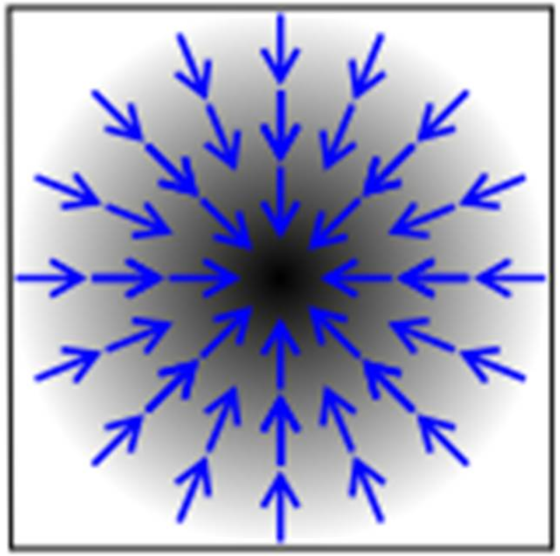
- The Canny edge detector
- Experimental Results
- MATLAB functions
- Exercises

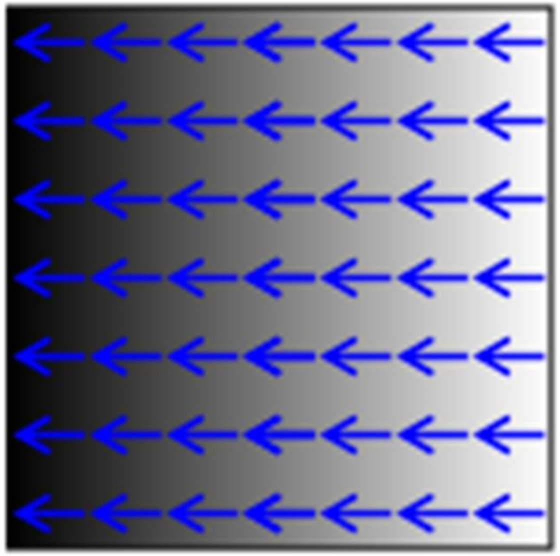
Gradient Concept (1)

- The **gradient** is a generalization of the **derivative** concept for multi-variate functions
- A **derivative** is defined as a function of a single variable
- For functions with more than one variable the **gradient** is applied
- The **gradient** is a vector-valued function
- The **derivative** is scalar-valued

Gradient Concept (2)

- The gradient points in the direction of the greatest rate of increase of the function
- Its magnitude is the slope of the graph in that direction





- The values of the function are represented in black and white
- Black represents higher values
- The corresponding gradient is represented by blue arrows

https://en.wikipedia.org/wiki/Gradient

Gradient masks – the 1st order derivative (1)

The gradient at coordinates (x,y) is defined as:

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}.$$

Magnitude (norm) of the gradient

$$\begin{aligned} \nabla f &= \mathrm{mag}(\nabla \mathbf{f}) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}. \end{aligned}$$

$$\Rightarrow \quad \nabla f \approx |G_x| + |G_y|.$$

Gradient masks – the 1st order derivative (2)

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

a
b c
d e

FIGURE 3.41 A 3 × 3 region of an image (the *z*s are intensity values). (b)–(c) Roberts cross gradient operators. (d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

Gradient masks – the 1st order derivative (3)

Common gradient operators:

- Roberts cross

https://en.wikipedia.org/wiki/Roberts_cross

- Sobel

https://en.wikipedia.org/wiki/Sobel_operator

- Prewitt

https://en.wikipedia.org/wiki/Prewitt_operator

Roberts cross (1)

- The horizontal and vertical masks are defined as

Input
Image I

−1	0
0	1

Gx

0	−1
1	0

Gy

Magnitude

$$\nabla f = \sqrt{{G_x}^2 + {G_y}^2}$$

Angle

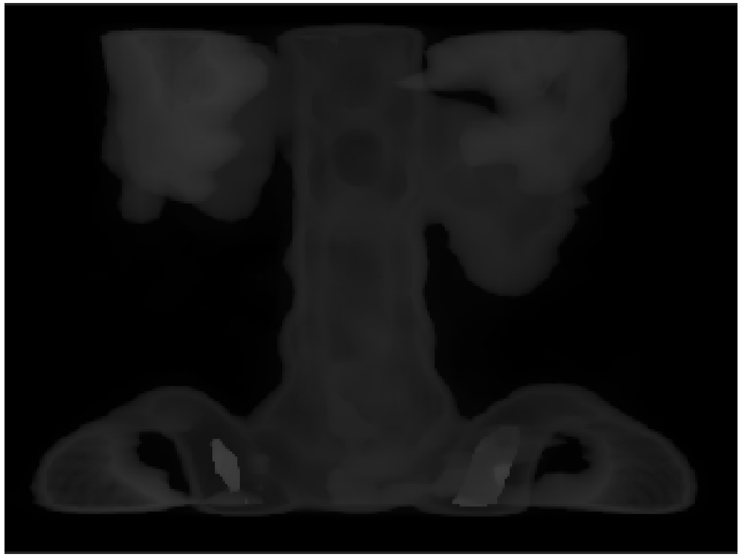
$$\theta = atan\left(\frac{G_y}{G_x}\right) - \frac{3\pi}{4}$$

Roberts cross (2)

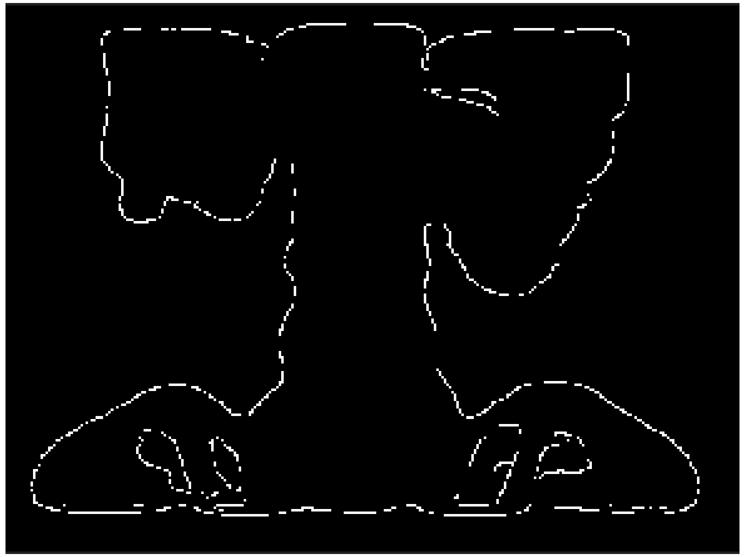
- Experimental results

```
I = imread('spine.tif');
Ir = edge(I,'roberts');
imshow(Ir)
```

Original



Roberts



Roberts cross (3)

- Experimental results: the absolute value of the gradient components

Sobel (1)

- The horizontal and vertical masks are defined as

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Gx

Gy

Magnitude

$$\nabla f = \sqrt{{G_x}^2 + {G_y}^2}$$

Angle

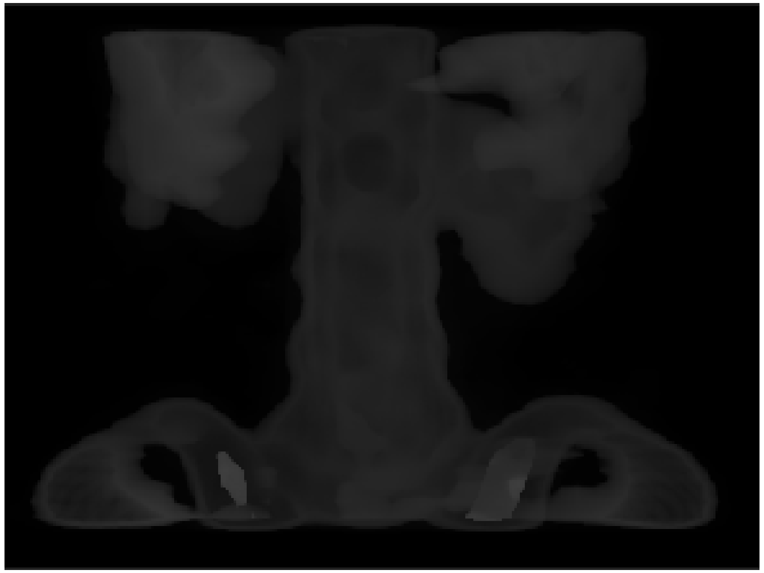
$$\theta = atan\left(\frac{G_y}{G_x}\right)$$

Sobel (2)

- Experimental results

```
I = imread('spine.tif');
Ir = edge(I,'sobel’);
imshow(Ir)
```

Original



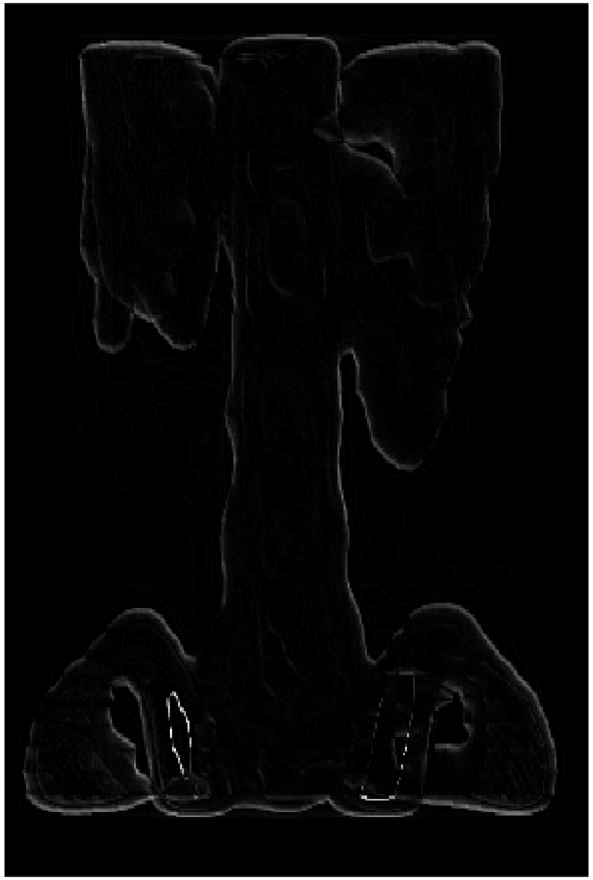
Sobel

Sobel (3)

- Experimental results: the absolute value of the gradient components

Gx

Gy



Prewitt (1)

- The horizontal and vertical masks are defined as

$$\begin{bmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{bmatrix}$$

Gx

Gy

Magnitude

$$\nabla f = \sqrt{{G_x}^2 + {G_y}^2}$$

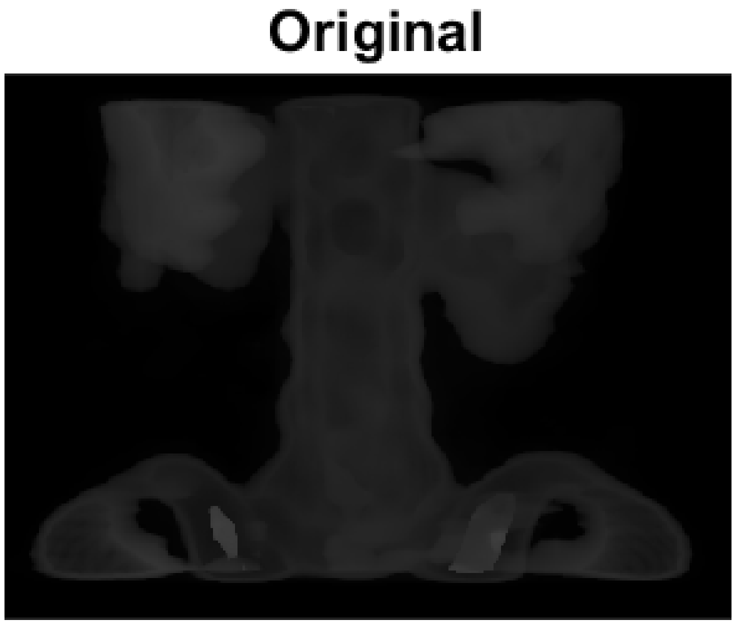
Angle

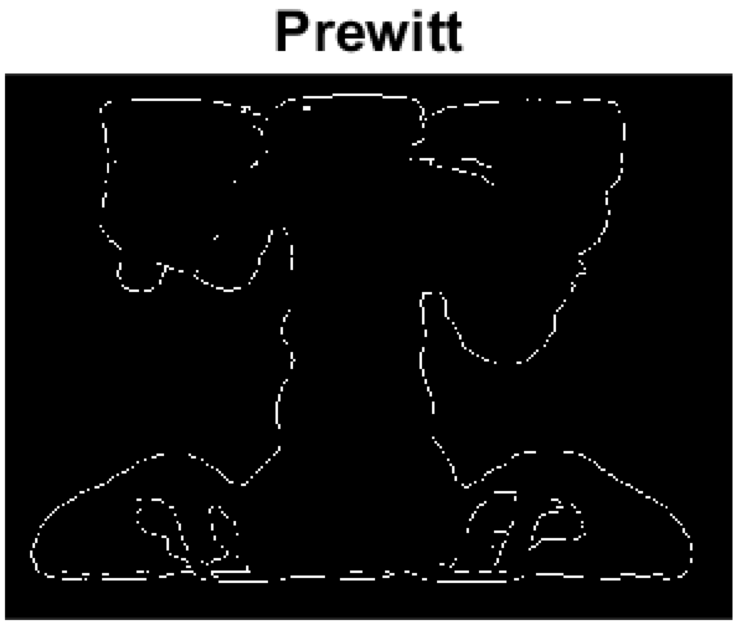
$$\theta = atan2(G_y, G_x)$$

Prewitt (2)

- Experimental results

```
I = imread('spine.tif');
Ir = edge(I,'prewitt');
imshow(Ir)
```



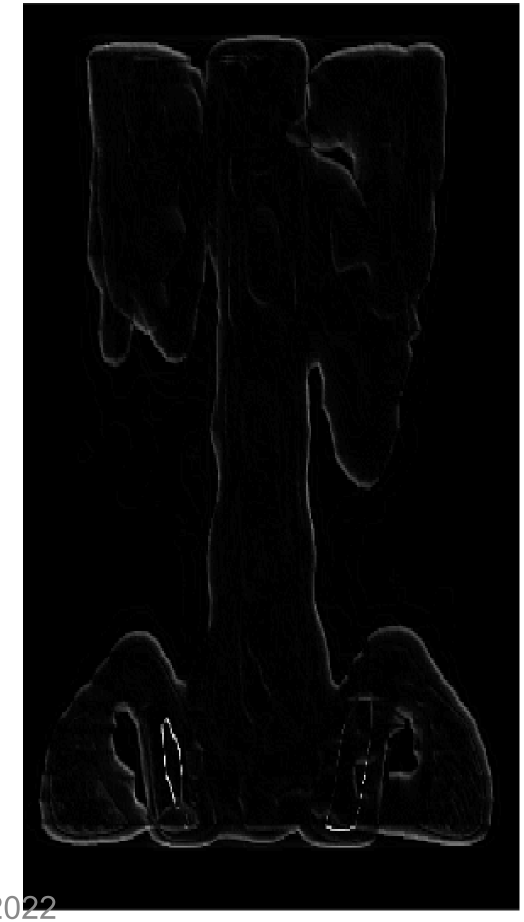


Prewitt (3)

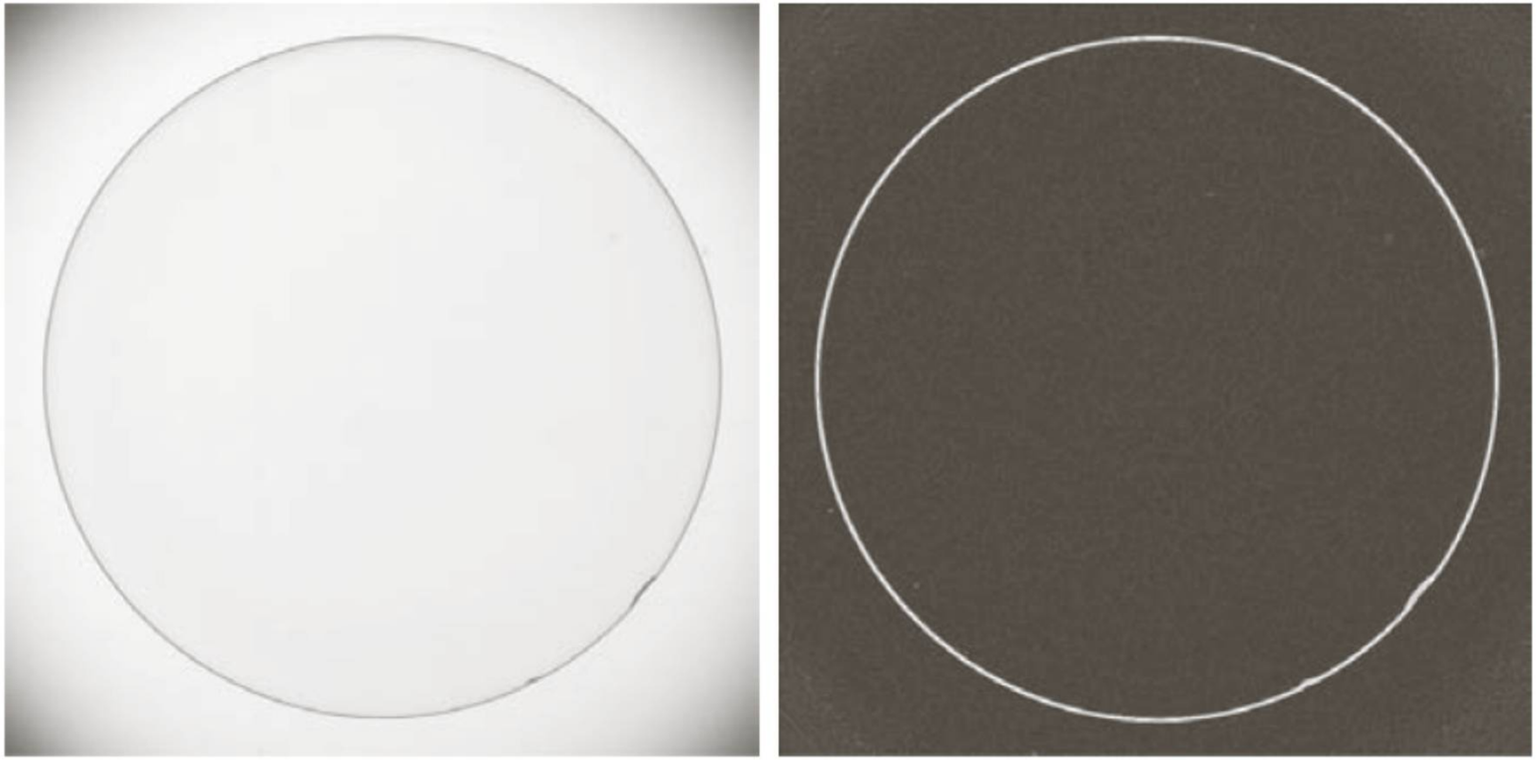
- Experimental results: the absolute value of the gradient components

Gx

Gy



Gradient – some results (1)

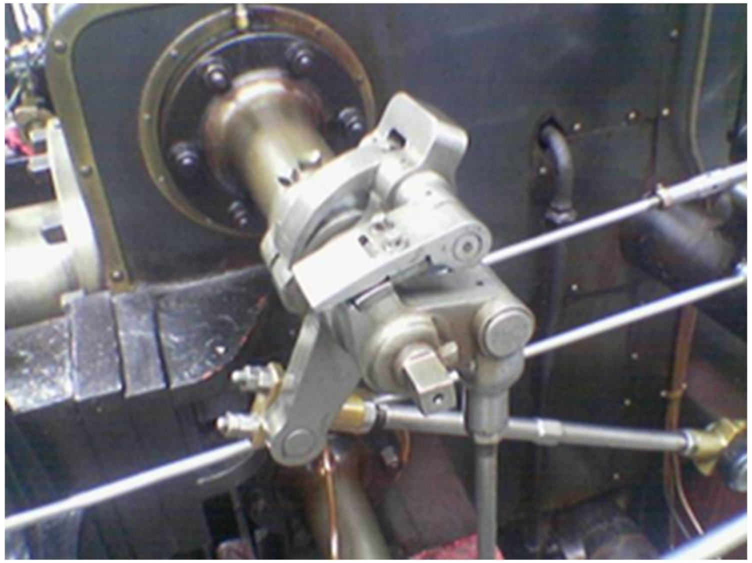


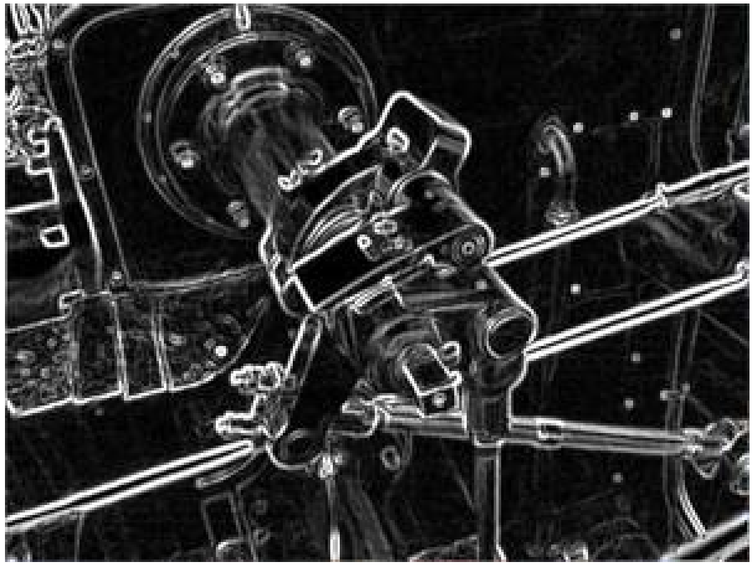
a b

FIGURE 3.42
(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Pete Sites, Perceptics Corporation.)

Gradient – some results (2)

https://en.wikipedia.org/wiki/Sobel_operator





Gradient – some results (3)

https://en.wikipedia.org/wiki/Prewitt_operator

Gradient – some results (4)

https://en.wikipedia.org/wiki/Roberts_cross

Grayscale test image of brick wall and bike rack

Gradient magnitude from Roberts cross operator

Gradient magnitude from Sobel operator

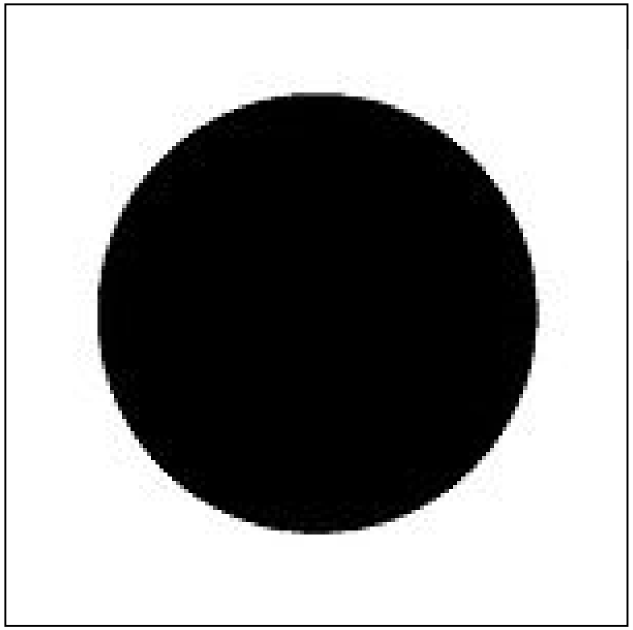
Gradient magnitude from Scharr operator

Gradient magnitude from Prewitt operator

Gradient – some results (5)

https://en.wikipedia.org/wiki/Sobel_operator

- When the sign of Gx and Gy is the same -> positive angle
- Red and yellow colors -> positive angles
- Blue and cyan colors -> negative angles



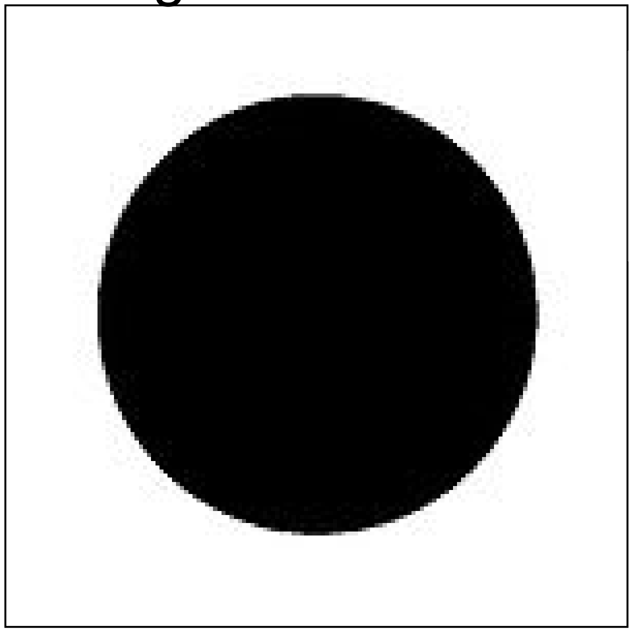
Angle

$$\theta = atan\left(\frac{G_y}{G_x}\right)$$

Gradient – some results (6)

https://en.wikipedia.org/wiki/Sobel_operator

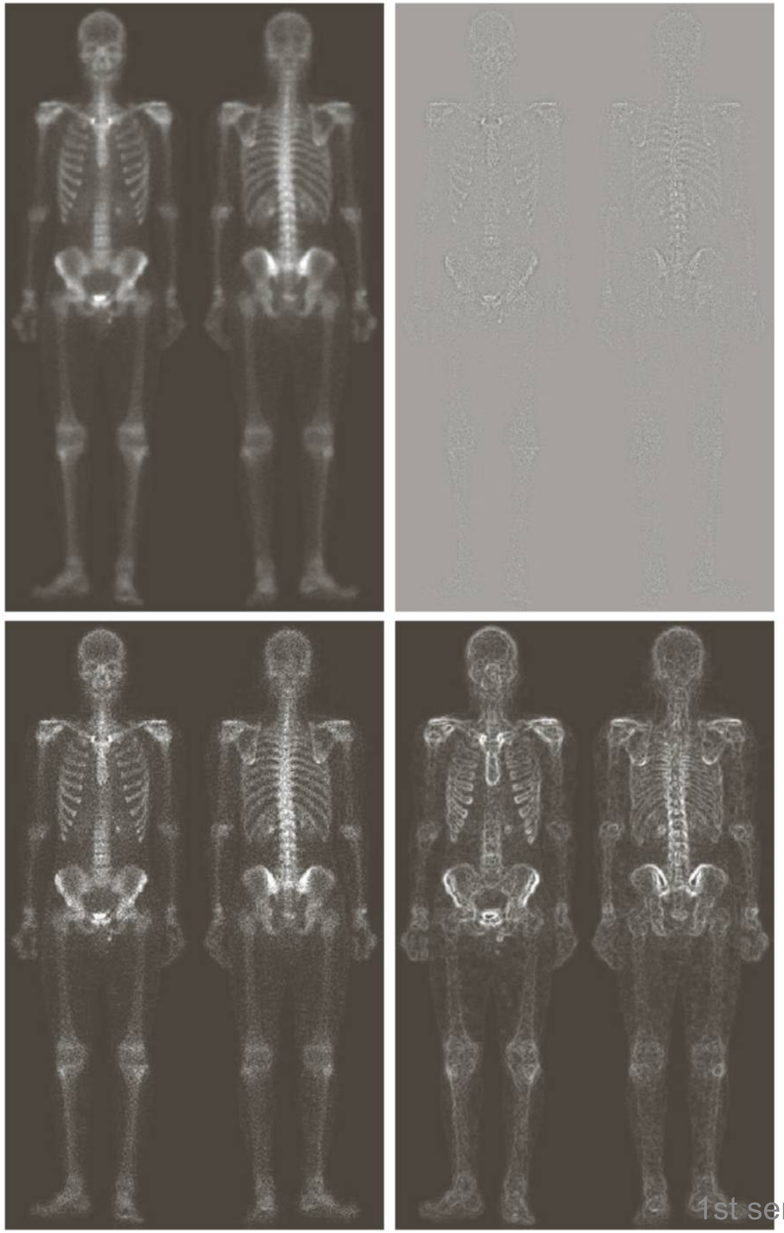
- Vertical edges -> angle = 0
- Horizontal edges -> angles of −π/2 and π/2
- Negative angle for top edge -> transition from bright to dark region
- Positive angle for the bottom edge -> transition from a dark to bright



Angle

$$\theta = atan\left(\frac{G_y}{G_x}\right)$$

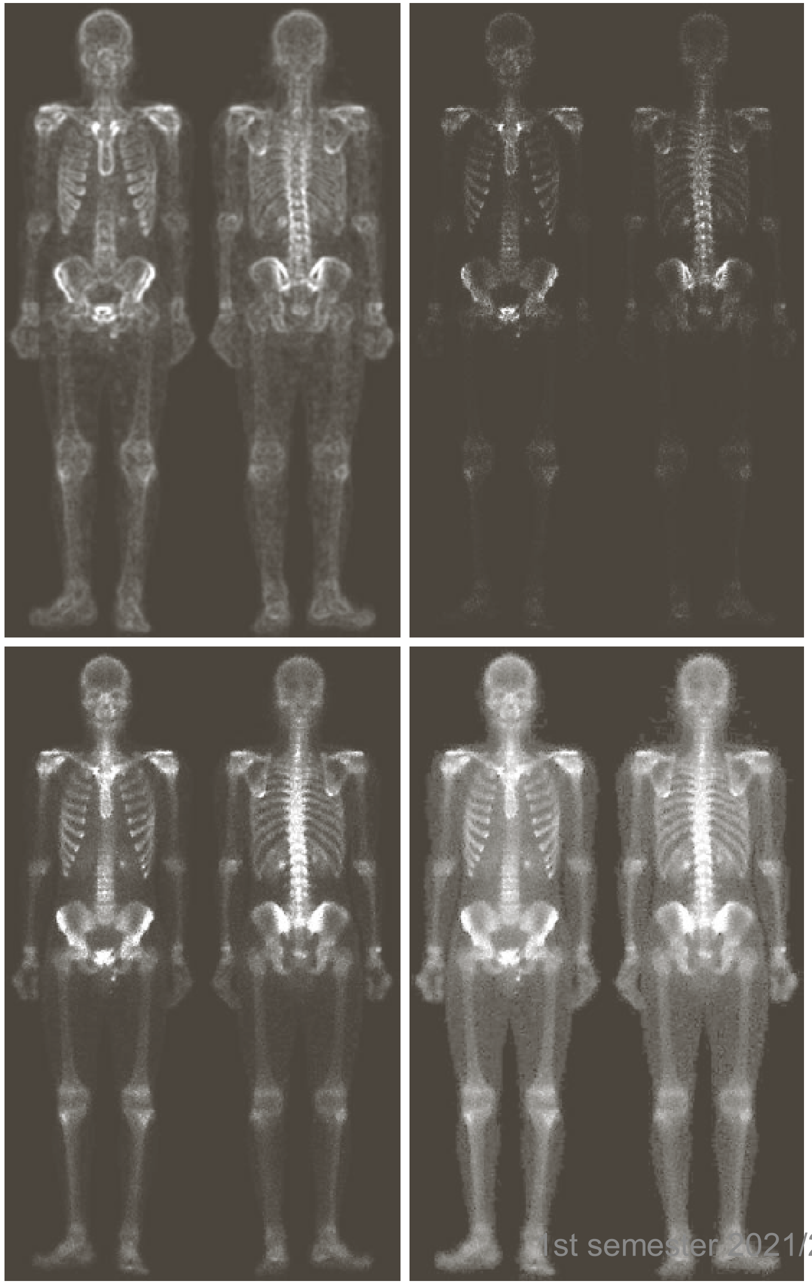
Medical Exam Image (1)



a	b
c	d

FIGURE 3.43 (a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).

Medical Exam Image (2)

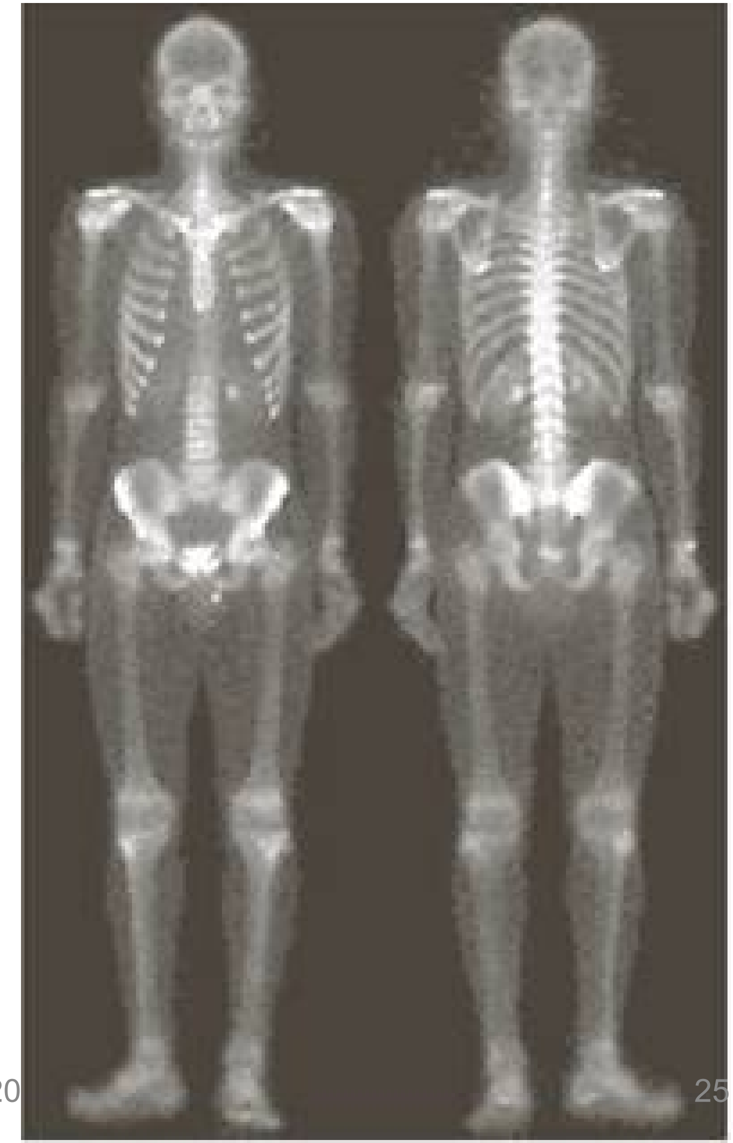


e f
g h

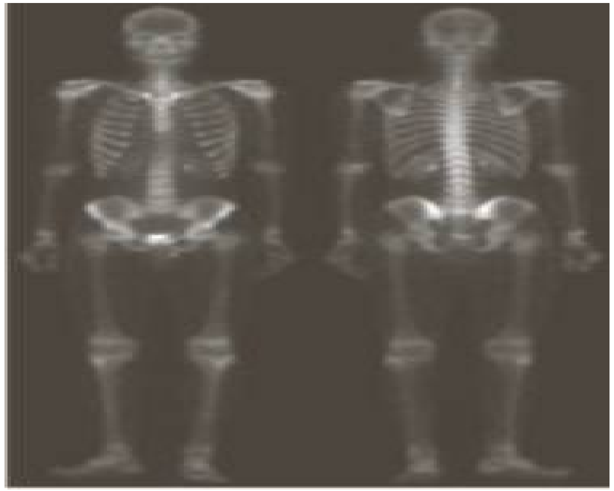
FIGURE 3.43 *(Continued)* (e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

Medical Exam Image (3)

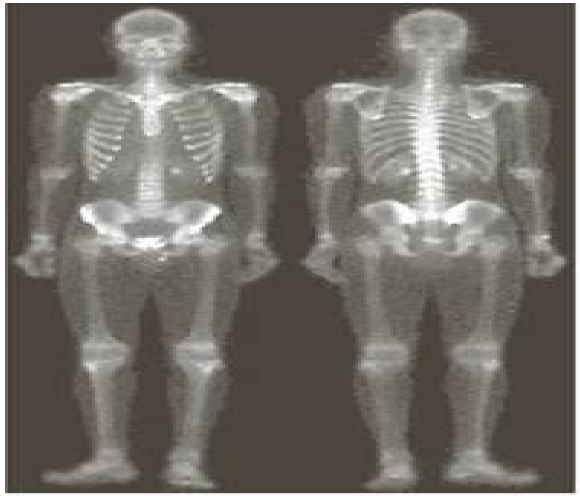
DIP



Medical Exam Image (4)



DIP Algorithm



a) Acquire image of the PET body scan

b) Laplacian of a)

c) Sharpened image by adding to the image its Laplacian c) = a) + b)

d) Sobel (gradient) of a)

e) Smoothed version (5x5 average filter) of d)

f) Mask image. f) = c) x e)

g) Add the input image to the mask: g) = a) + f)

h) Power-law intensity transformation on g)

The Canny Edge detector (1)

- https://en.wikipedia.org/wiki/Canny_edge_detector
- *A computational approach to edge detection* IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 8, 1986, pp. 679–698.

The algorithm has five steps:

1) Apply Gaussian filter to smooth the image in order to remove the noise
2) Find the intensity gradients of the image
3) Apply non-maximum suppression to get rid of spurious response to edge detection
4) Apply double threshold to determine potential edges
5) Track edge by hysteresis: Finalize the detection of edges by suppressing all the other edges that are weak and not connected to strong edges.

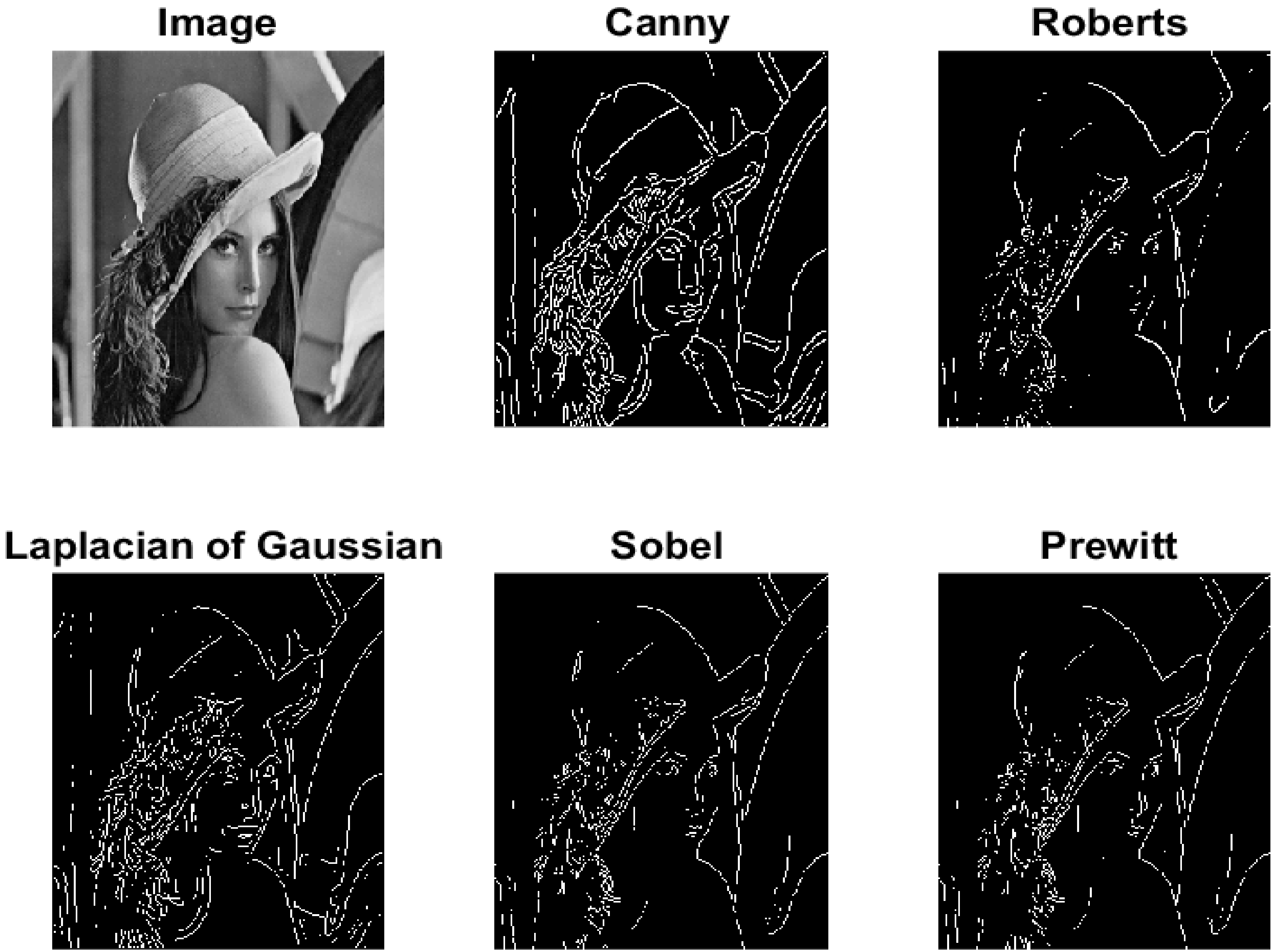
The Canny Edge detector (2)

Image

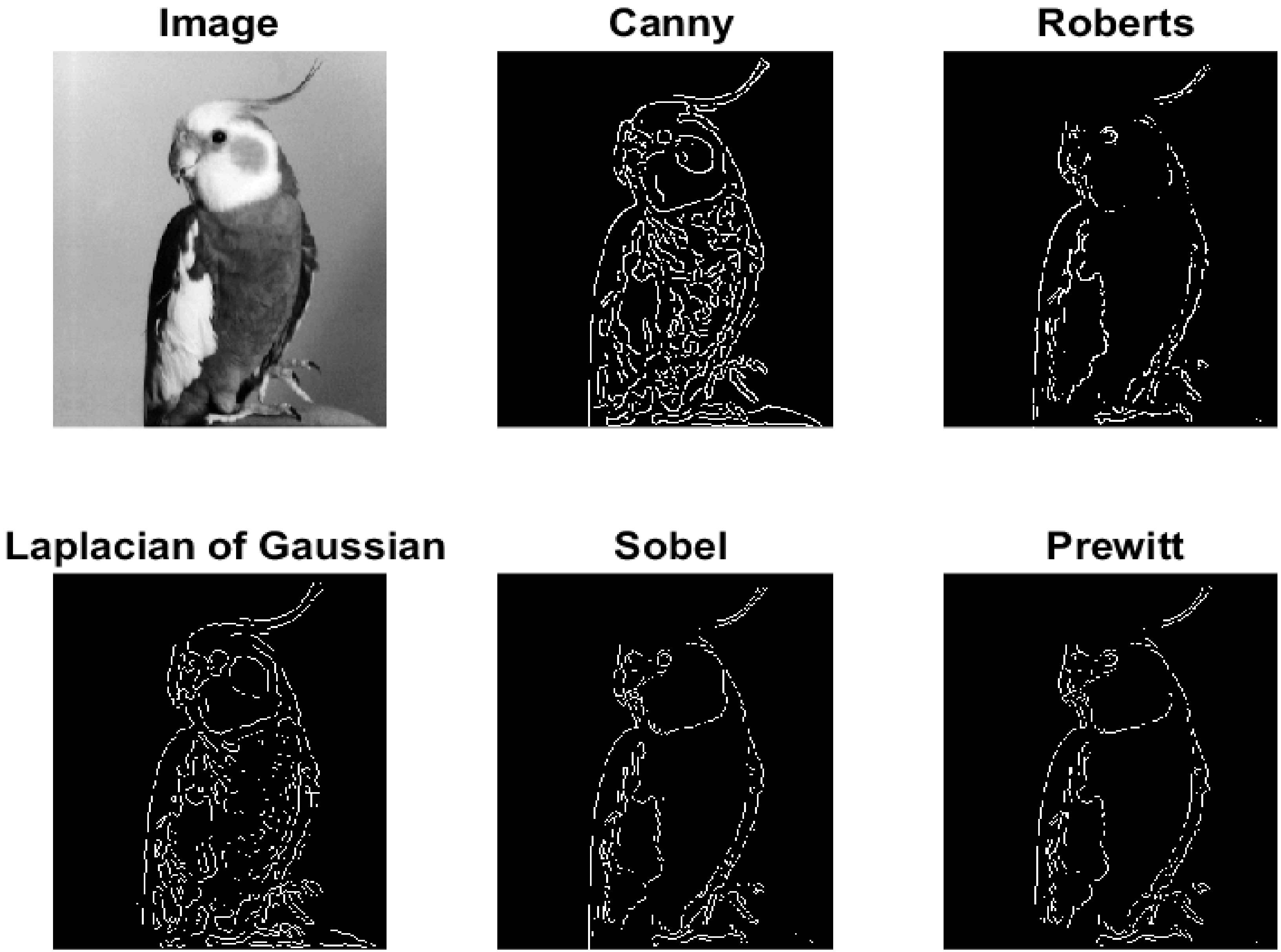
John Canny

Canny

Edge Detection (1)



Edge Detection (2)

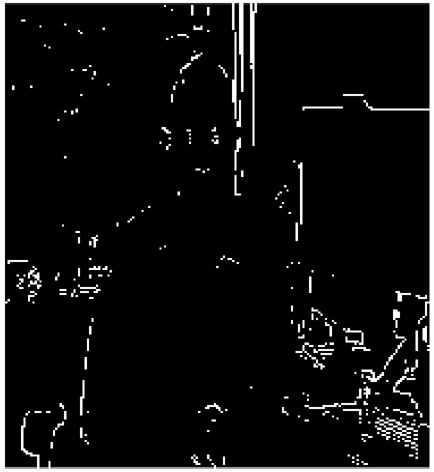


Edge Detection (3)

Image

Canny

Roberts



Laplacian of Gaussian

Sobel

Prewitt

Exercises (1)

1. {R1||TG} Considere a imagem monocromática I, quadrada de resolução espacial 8×8, com profundidade de $n = 8$ bit/pixel. A imagem possui linhas com valor constante, tal que a primeira linha tem o valor 11, a segunda tem o valor 22, a terceira tem o valor 33 e assim sucessivamente até à última linha que possui o valor 88.

4. {R1} {2,5} Considere a imagem I definida no exercício 1 e as máscaras w_1, w_2 e w_3 definidas como

$$w_1 = \begin{bmatrix} 0{,}5 \\ -0{,}5 \end{bmatrix} \quad , \quad w_2 = \begin{bmatrix} 0{,}5 & 0{,}5 \end{bmatrix} \quad e \quad w_3 = \text{máximo} \quad \{2 \times 1\}.$$

Apresente as imagens J_1, J_2 e J_3, resultantes da aplicação destas máscaras sobre a imagem I.

Exercises (2)

3. As seguintes questões abordam técnicas de filtragem espacial de imagem.

   a) Considere o operador Laplaciano.

      i) {1,0} Este operador pode ser definido por diferentes máscaras. Apresente duas dessas máscaras e indique os critérios que levam à definição das mesmas.

      ii) {1,0} Refira duas aplicações em que este operador é tipicamente aplicado com sucesso.

      iii) {1,0} Quais as diferenças entre este operador (Laplaciano) e o operador Laplacian of Gaussian (LoG)? Quais as vantagens e motivação da existência do operador LoG? Quais as desvantagens do operador LoG em relação ao Laplaciano?

   b) {2,0} Considere a imagem

$$I = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}.$$

   Apresente as imagens I_1, I_2 e I_3, resultantes da aplicação sobre I, das máscaras w_1, w_2 e w_3, respetivamente. As máscaras são definidas através de

$$w_1 = \begin{bmatrix} -1 & 1 \end{bmatrix} \quad , \quad w_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad e \quad w_3 = \text{mediana} \quad \{1 \times 3\}.$$

   Compare e comente os resultados obtidos.

Exercises (3)

5. $\{R1\}$ Considere as seguintes questões sobre técnicas de processamento digital de imagem

   i) $\{1{,}25\}$ Em que consiste a técnica *unsharp masking*? Indique em que situações deve ser aplicada.

   ii) $\{1{,}25\}$ Explique sucintamente as razões do sucesso do filtro de mediana na remoção de ruído *salt & pepper*. Este sucesso também se verifica na remoção de outros tipos de ruído (tal como o ruído Gaussiano, por exemplo)? Indique os critérios de escolha das dimensões da máscara do filtro de mediana.

   iii) $\{1{,}25\}$ Os operadores de gradiente podem ser definidos por diferentes máscaras. Apresente duas dessas máscaras e indique os critérios que levam à definição das mesmas.

3. As seguintes questões abordam técnicas de processamento digital de imagem.

   i) $\{1{,}0\}$ Determinada imagem monocromática de resolução $M \times N$ é operada, através de filtragem espacial linear, com uma máscara de dimensões $L_M \times L_N$. Indique o número total de multiplicações e de somas realizadas, no pior caso.

   ii) $\{1{,}0\}$ Pretende-se realizar a operação de *sharpening* sobre uma imagem monocromática, através da aplicação de uma única máscara de filtragem espacial. Tal é possível? Em caso afirmativo, apresente uma máscara que cumpra esse objetivo. Caso contrário, justifique a impossibilidade.

Exercises (4)

4. Considere o algoritmo que se apresenta de seguida. A função `linear_spatial_filtering` realiza filtragem espacial linear com as máscaras

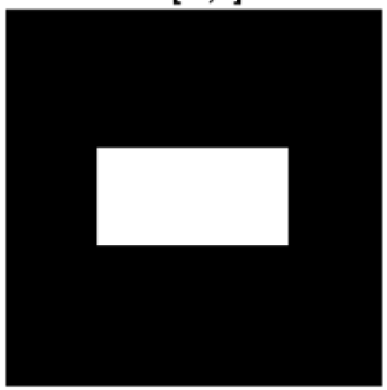
$$a = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \quad \text{e} \quad b = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}.$$

```
----------------------------------------------
Entrada:  Imagem monocromática f[m,n];
Saída:    Imagens monocromáticas g1[m,n] e g2[m,n].
----------------------------------------------
1. f1 = linear_spatial_filtering( f, a ).
2. f2 = linear_spatial_filtering( f, b ).
3. g1 = sqrt( pow(f1,2) + pow(f2,2) ).
4. g2 = atan2( f1, f2 ).
----------------------------------------------
```

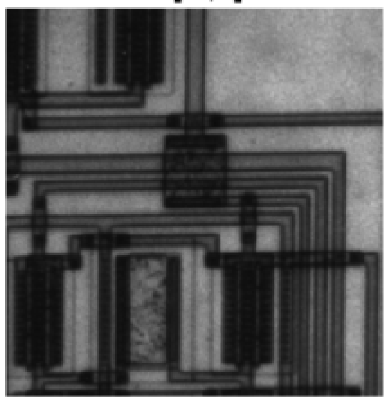
a) {1,0} Indique a funcionalidade do algoritmo. Em termos genéricos, qual a informação contida nas imagens obtidas pelo algoritmo, designadas por $g_1[m,n]$ e $g_2[m,n]$?

b) {1,0} Descreva o conteúdo da imagem de saída $g_1[m,n]$, quando na entrada se apresentam as imagens $f_1[m,n]$ e $f_2[m,n]$, apresentadas na figura.

f1[m,n]



f2[m,n]



MATLAB
Image Processing Toolbox functions

https://www.mathworks.com/products/image.html

- *filter2, imfilter,* image filtering
- *medfilt2,* median filter
- *imsharpen,* unsharp masking technique
- *imgradient,* gradient computation
- edge, edge map computation (binary image)
- *fspecial,* kernels/windows/masks for common filters

Bibliography

- The images displayed in these slides are from:

  - R. Gonzalez, R. Woods, *Digital Image Processing*, 4^{th} edition, Prentice Hall, 2018, ISBN 0133356728

  - S. Smith, *The Scientist and Engineer's Guide to Digital Signal Processing*, Newnes, 2003, ISBN 0-750674-44-X [chapter 23]

  - O. Filho, H. Neto, Processamento Digital de Imagens, Rio de Janeiro: Brasport, 1999, ISBN 8574520098.

  - Wikipedia and Mathworks web pages