ISEL
INSTITUTO SUPERIOR
DE ENGENHARIA DE LISBOA

PROCESSAMENTO DE IMAGEM E BIOMETRIA

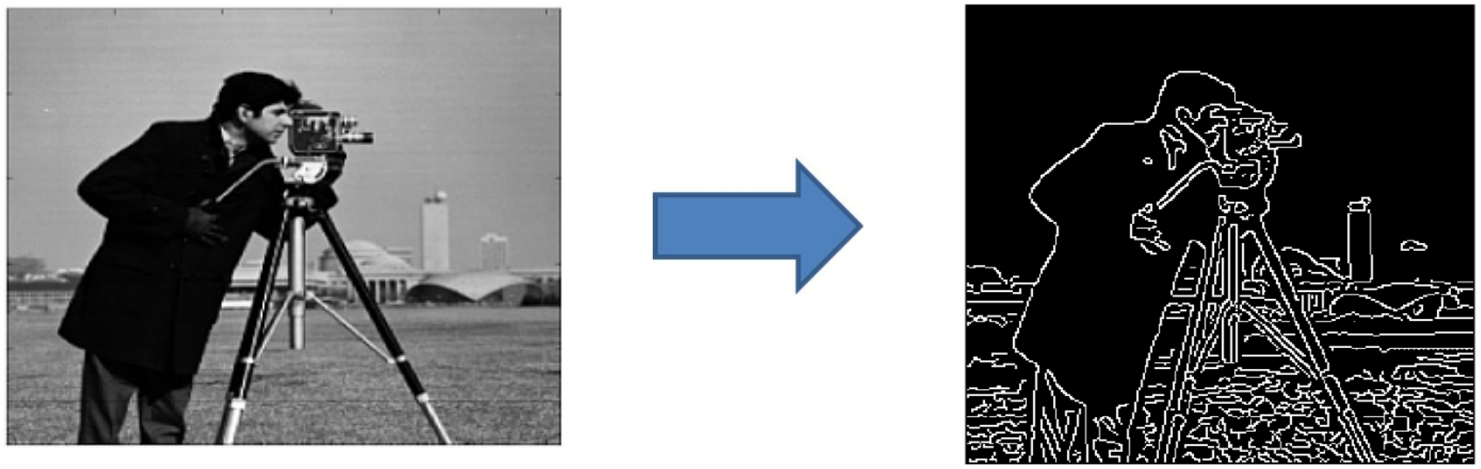
IMAGE PROCESSING AND BIOMETRICS

4. **BASIC OPERATIONS AND INTENSITY TRANSFORMATION OPERATIONS**

Summary

- Digital Image Processing (DIP) with
  - basic (logical and arithmetic) operations
  - intensity transformation operations
- Some examples of these operations
- Analysis of some experimental results
- Implementation using a look-up table
- Exercises

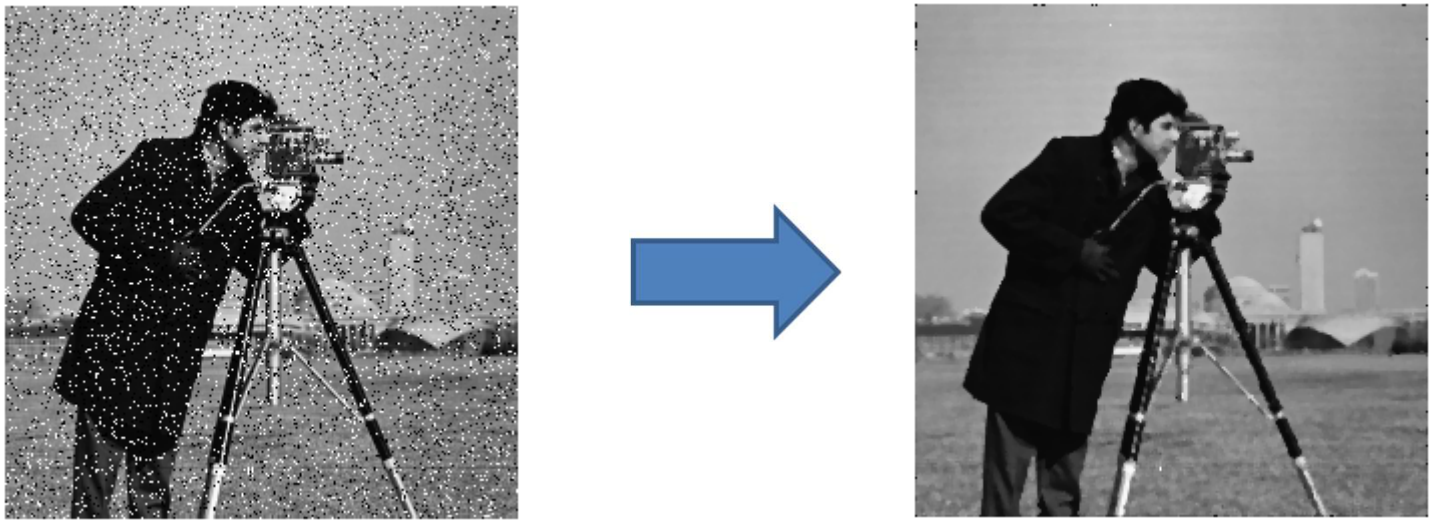
Digital Image Processing (DIP)



- DIP refers to the set of algorithms that process a digital image on a digital computer

- A DIP algorithm may produce as output:
  - an image or a set of images
  - a set of measures/descriptors of the input image
  - a combination of both previous outputs mentioned above

Digital Image Processing (DIP)



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Digital Image Processing (DIP)

Entropy
=
7.0097 bit/symbol

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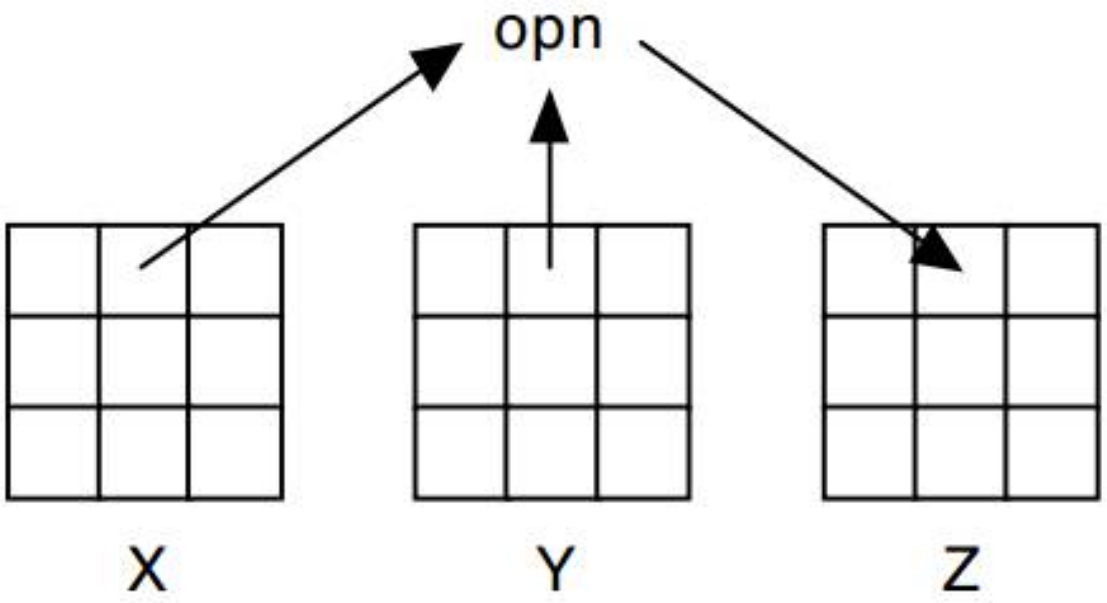
Basic operations (1)

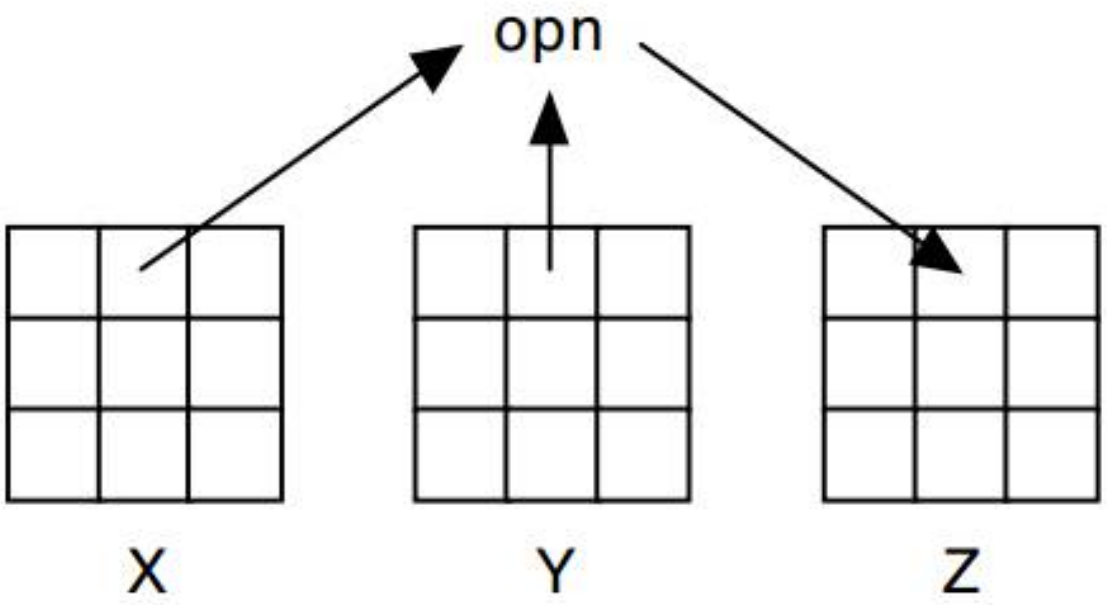
- An image is represented by a matrix with non-negative integer values (pixels) with ‘n’ bits per pixel:
  - n=8, monochrome image
  - n=1, binary image



Z = X opn Y

Basic operations (2)

- *opn* is
  - a **logical** (AND, OR, XOR,…) operation
  - an **arithmetic** (+, -, *, /) operation

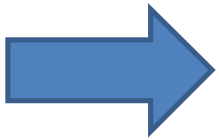


Z = X opn Y

Basic operations (3)

$$X = \begin{bmatrix} 200 & 100 & 100 \\ 0 & 10 & 50 \\ 50 & 250 & 120 \end{bmatrix} \qquad Y = \begin{bmatrix} 100 & 220 & 230 \\ 45 & 95 & 120 \\ 205 & 100 & 0 \end{bmatrix}$$

$$Z = X + Y = \begin{bmatrix} 300 & 320 & 330 \\ 45 & 105 & 170 \\ 255 & 350 & 120 \end{bmatrix}$$



Overflow Problem ?!

Some values are above 255!

Basic operations (4)

Overflow Problem – Scale back to the 0…255 range

$$Z = \begin{bmatrix} 300 & 320 & 330 \\ 45 & 105 & 170 \\ 255 & 350 & 120 \end{bmatrix} \Rightarrow \begin{bmatrix} 213 & 230 & 238 \\ 0 & 50 & 105 \\ 175 & 255 & 63 \end{bmatrix}$$

$$g = \frac{255}{f_{max} - f_{min}}(f - f_{min}),$$

$f_{max} = 350$
$f_{min} = 45$

Basic operations (5)

(a)

(b)

(c)

Z = X and Y

Basic operations (6)

(a) (b) (c)

Z = X or Y

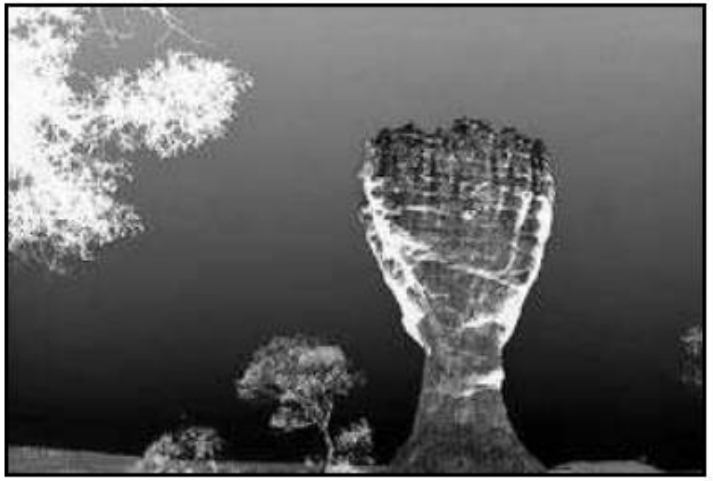
Basic operations (7)

(a) (b) (c)

Z = X xor Y

Basic operations (8)

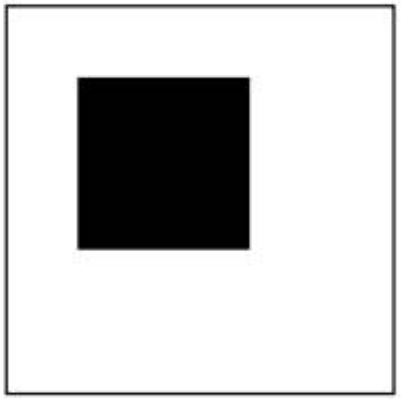
(a)



(b)

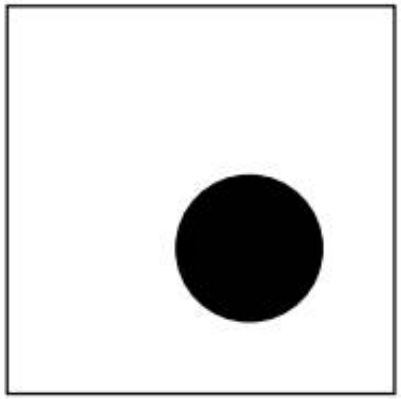
(a) X
(b) not X

Basic operations (9)



X

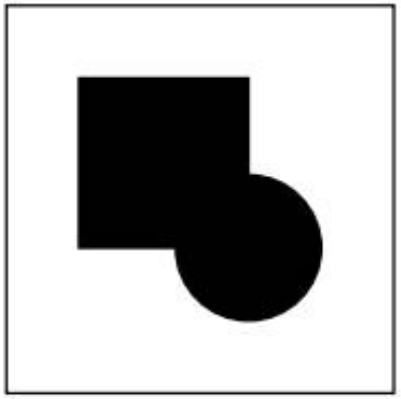
(a)



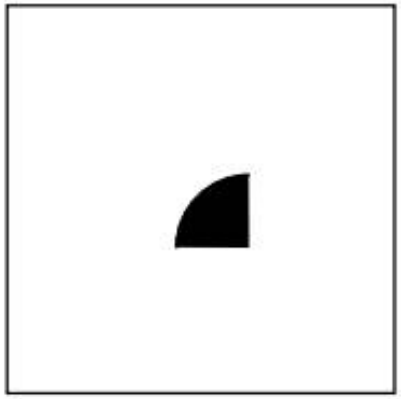
Y

(b)

Binary Images

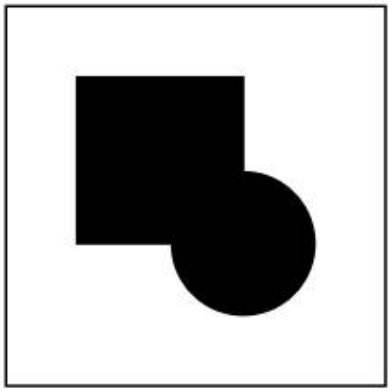


X and Y



X or Y

Basic operations (10)



X xor Y

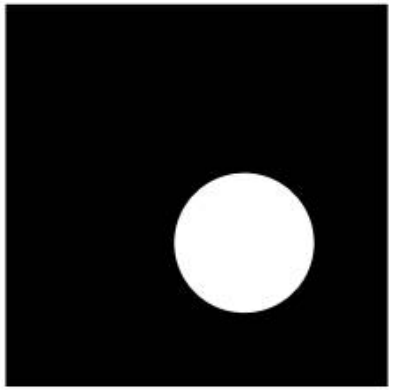
(e)

(not X) and Y

(f)

not X

(g)



not Y

(h)

Basic operations (11)

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Intensity Transformations (1)

- The intensity transformation is a DPI technique
- It applies a function T to each pixel of the input image, yielding the output pixel value

$$g(x,y) = T [f(x,y)]$$

f(x,y), is the input image pixel at coordinates (x,y)
g(x,y), is the output image pixel at coordinates (x,y)

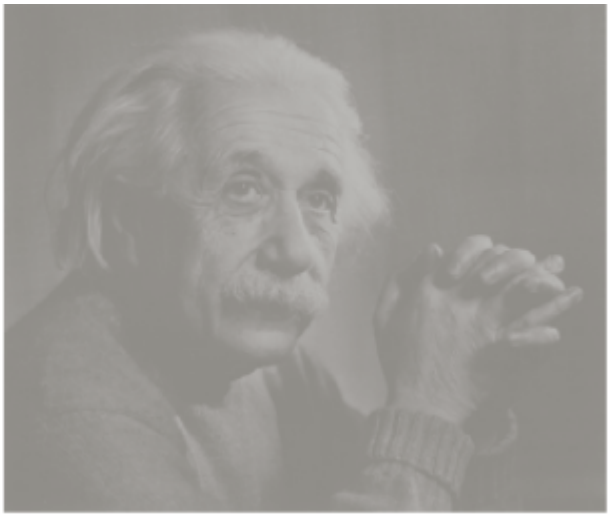
T defines completly the intensity transformation

Intensity Transformations (2)

- The intensity transformations operations are applied to many different operations such as:
  - contrast adjustment
  - brightness adjustment
  - negative version
  - binary version (conversion from a monochrome image to a two-level image)
  - …

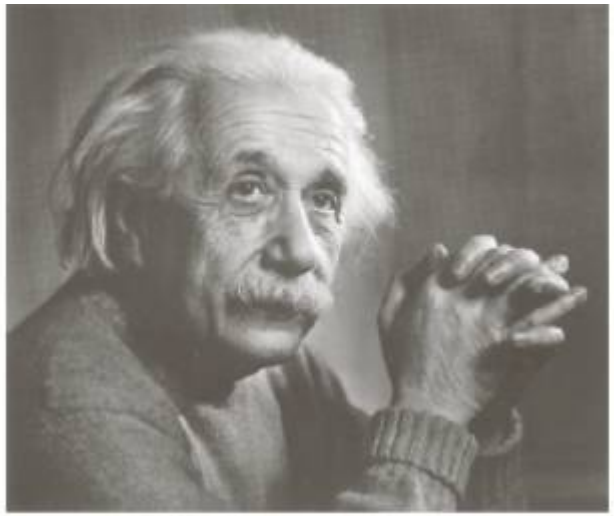
Intensity Transformations (3)

- Increase on the contrast



f(x,y)

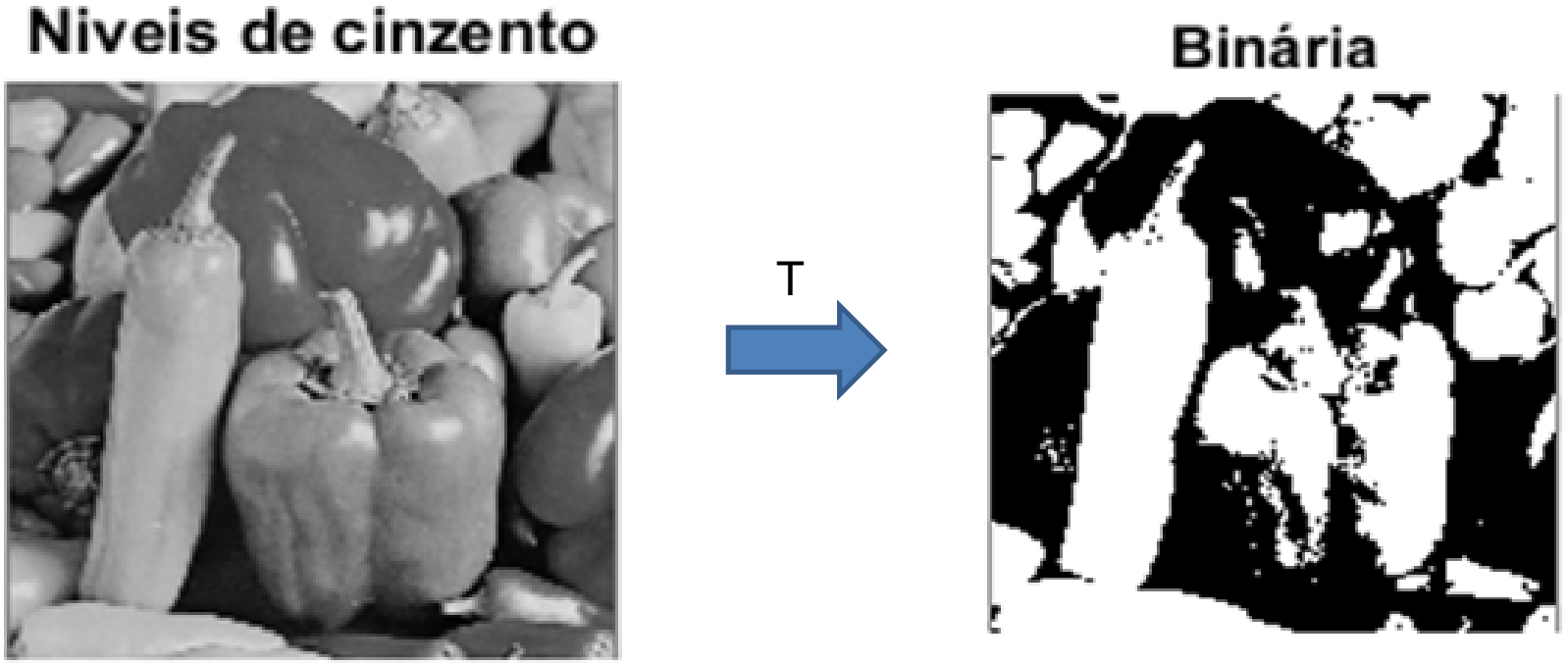
T



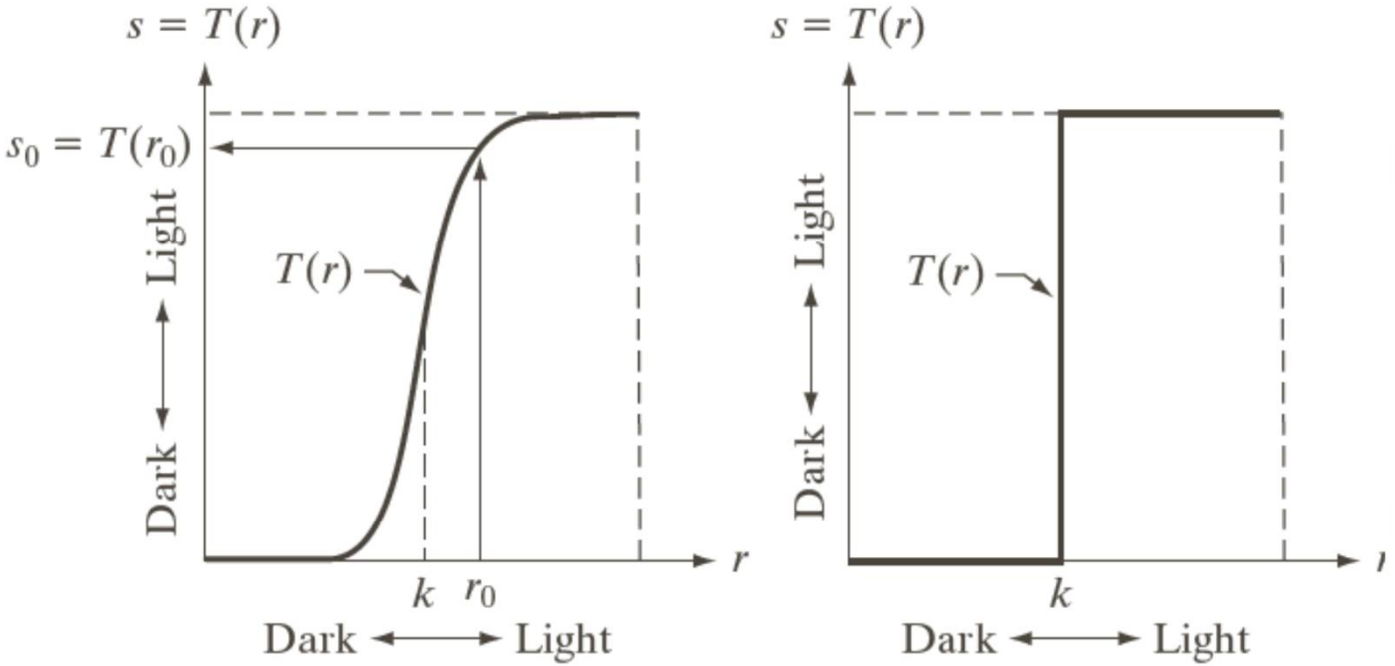
g(x,y)

Intensity Transformations (4)

- Binary version (conversion from a monochrome image to a two-level image)



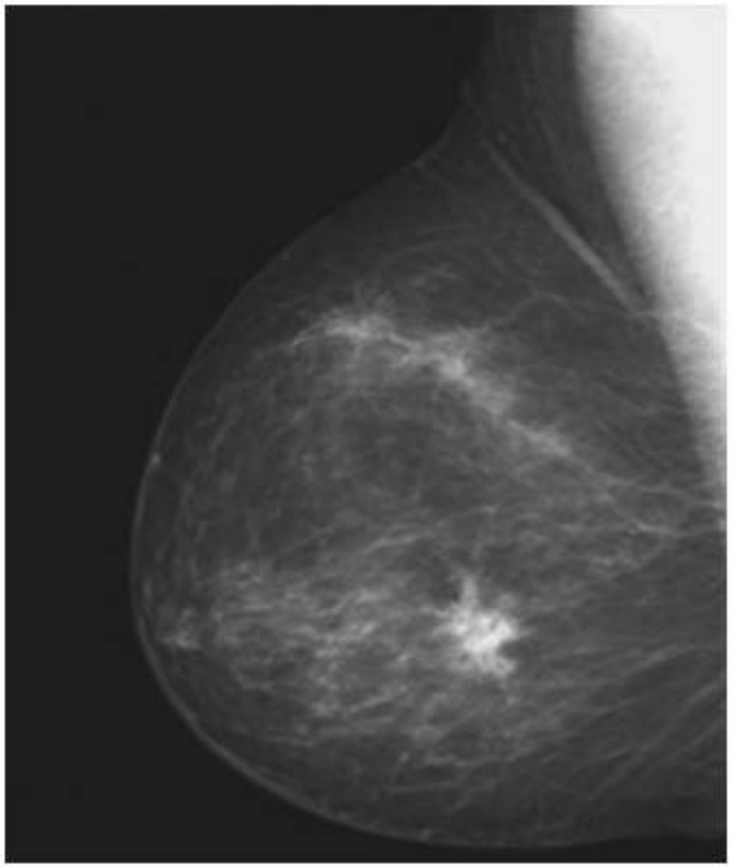
Intensity Transformations (5)

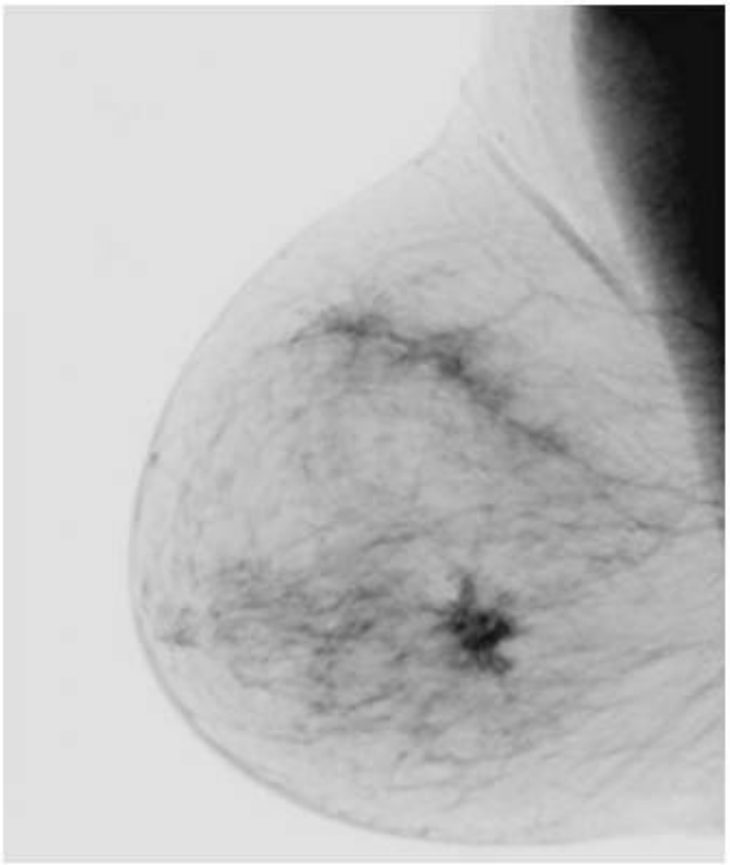


a b

FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

Intensity Transformations (6)





a b

FIGURE 3.4
(a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Intensity Transformations (7)

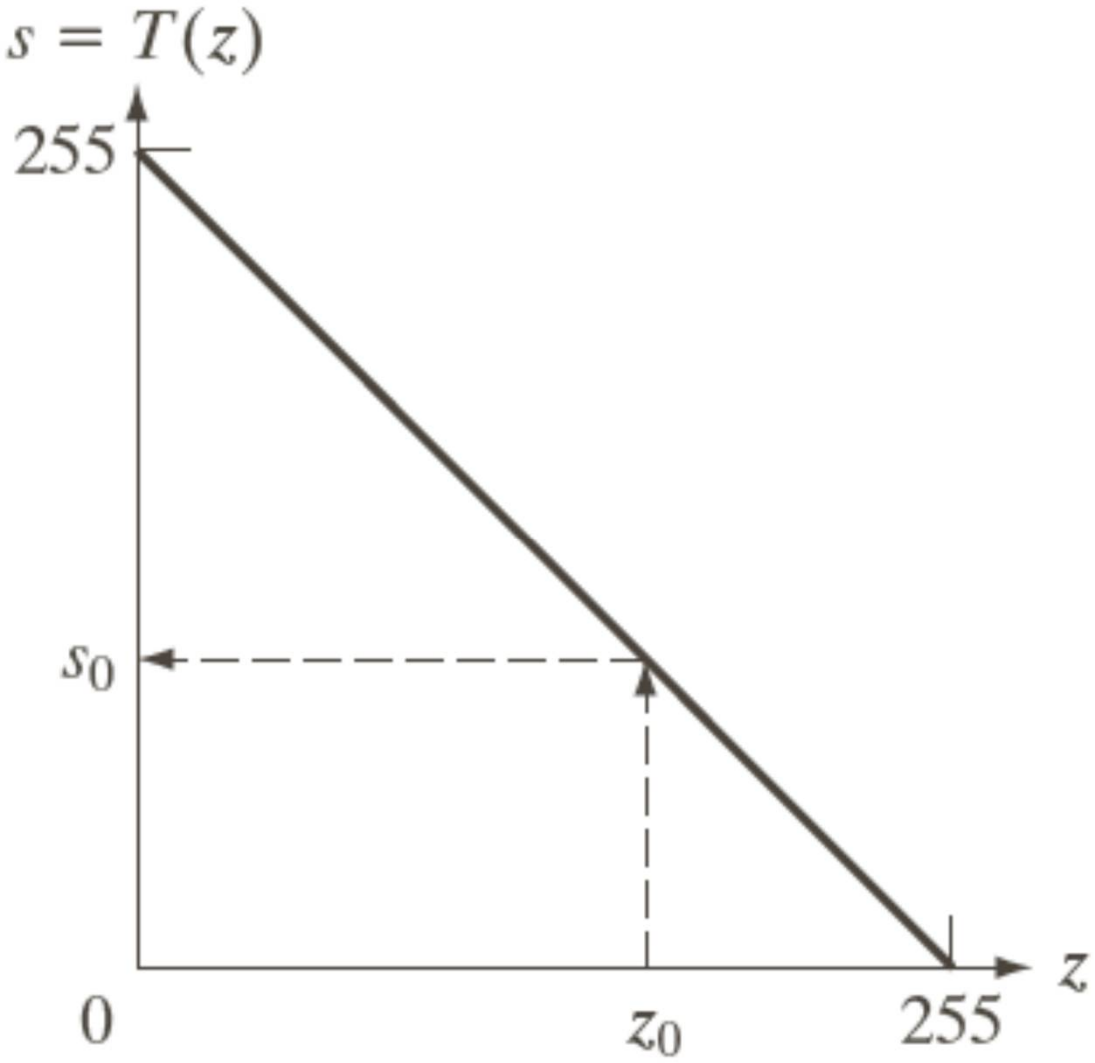
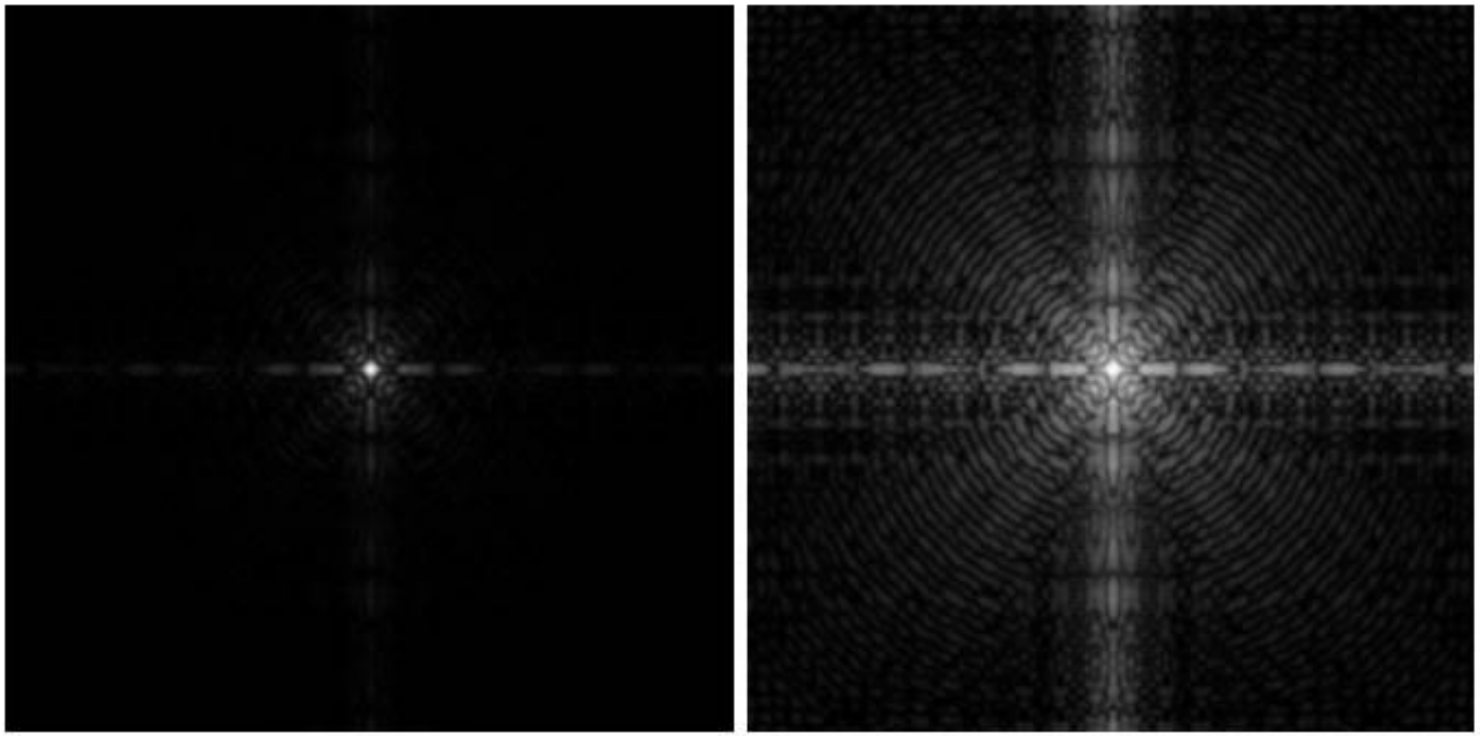


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

Intensity Transformations (8)

a b

FIGURE 3.5
(a) Fourier spectrum.
(b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.



Intensity Transformations (9)

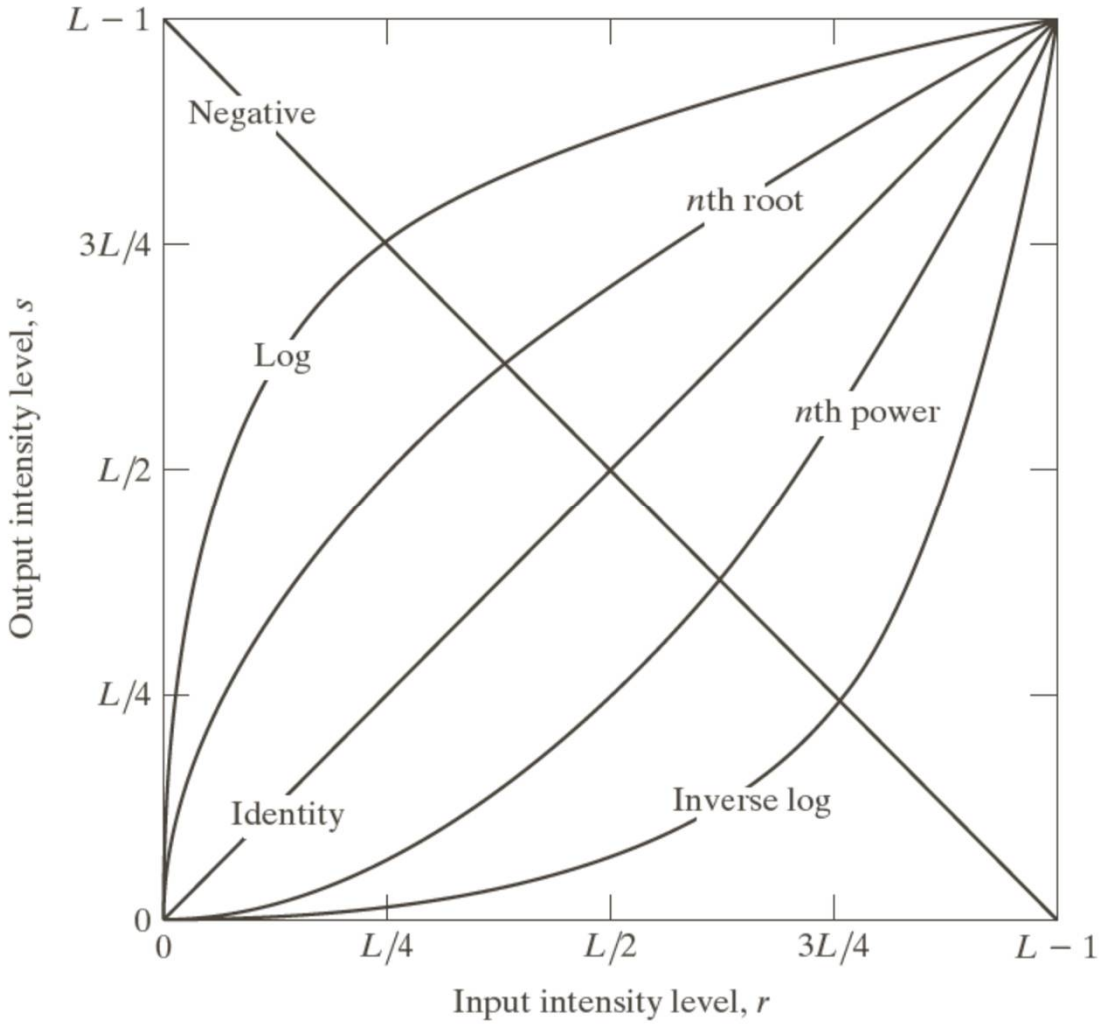


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Intensity Transformations (10)

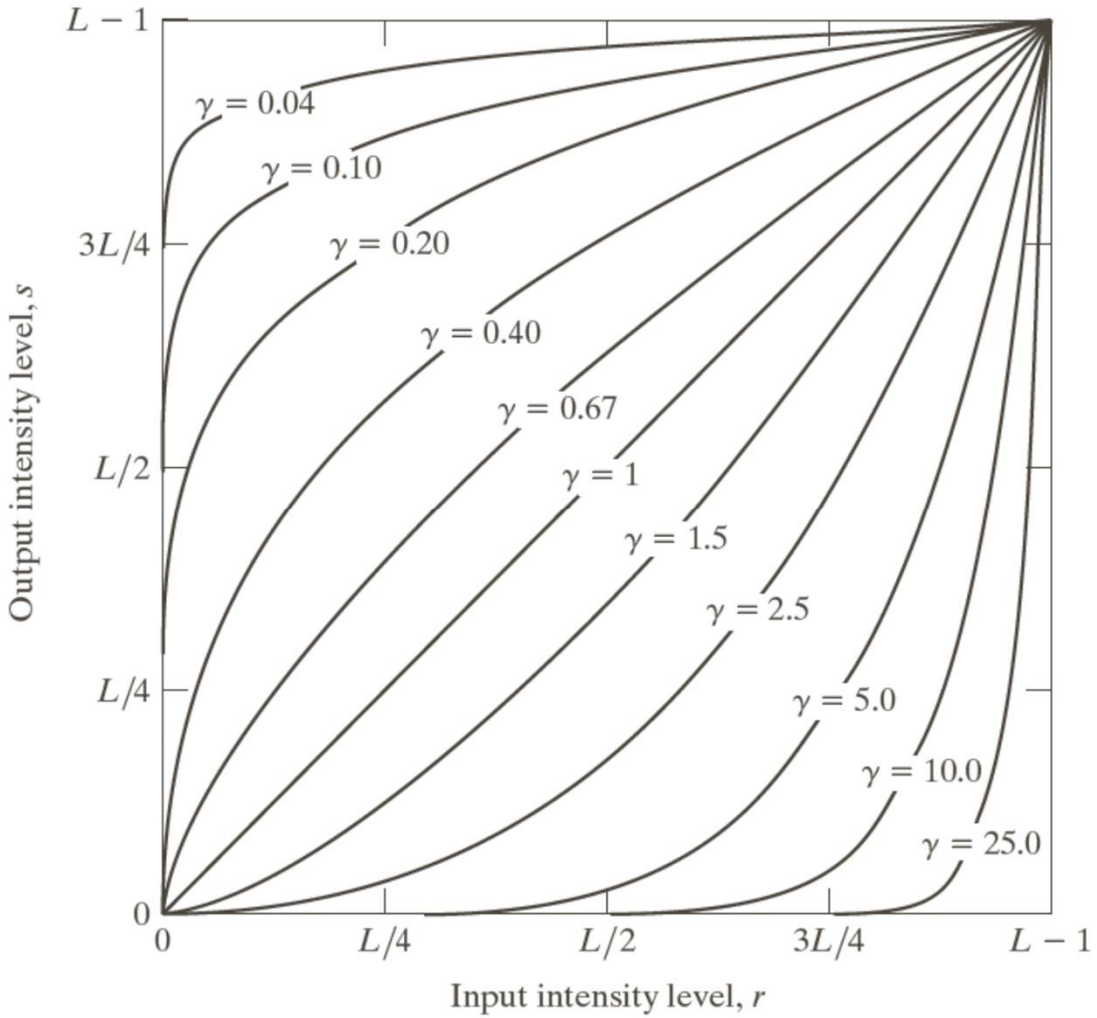
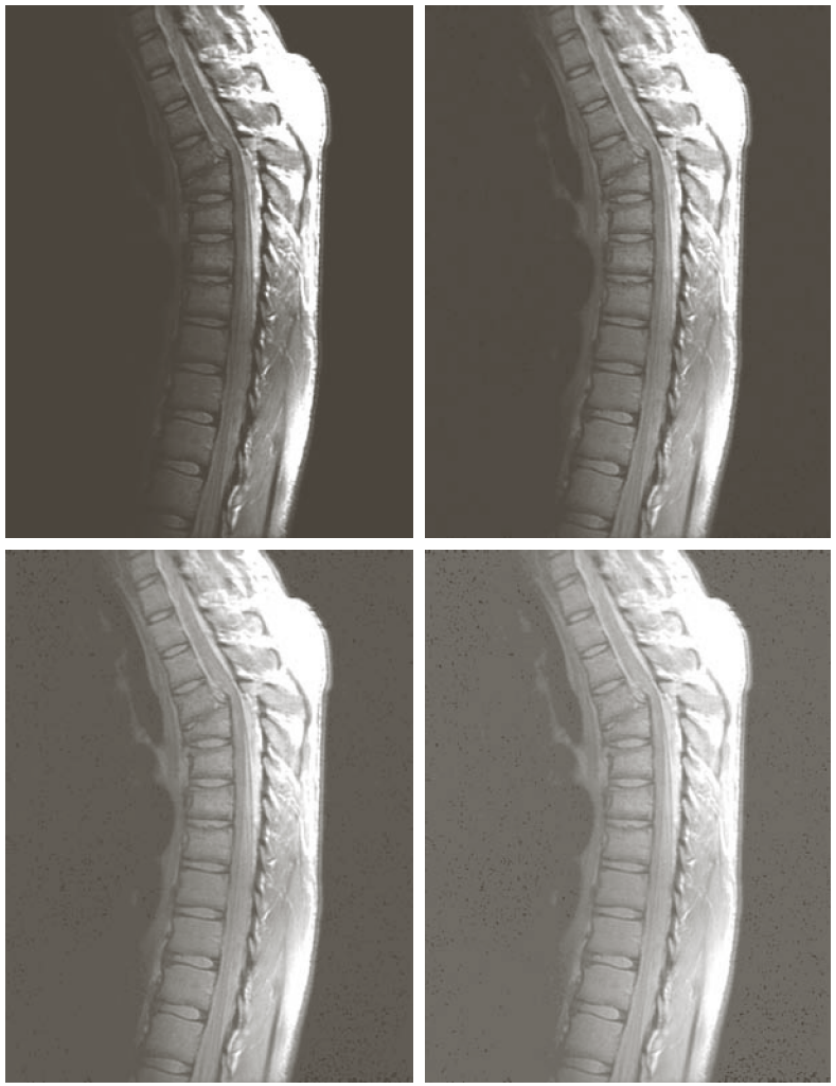


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

Intensity Transformations (11)



a b
c d

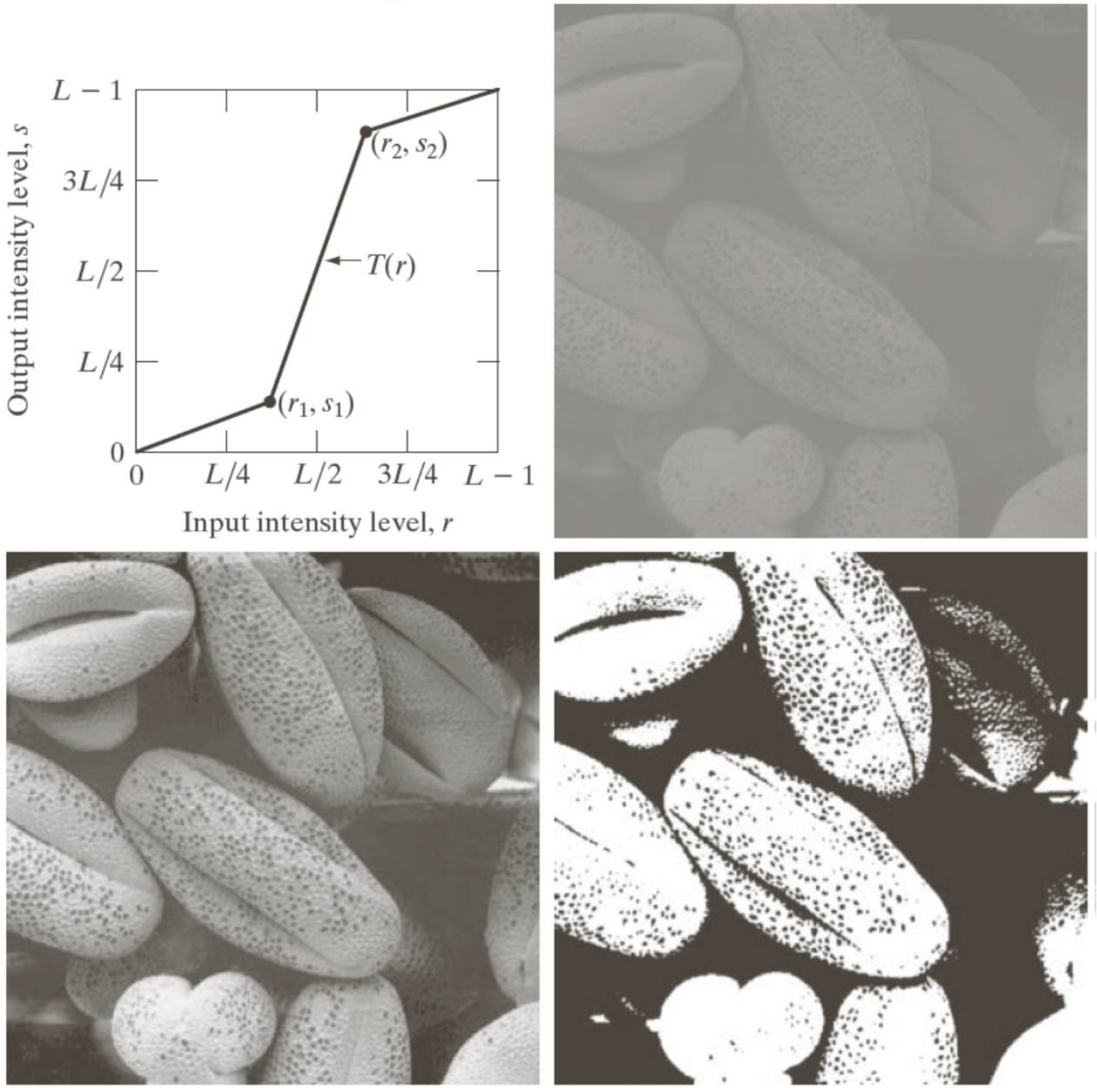
FIGURE 3.8
(a) Magnetic resonance image (MRI) of a fractured human spine.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6$, 0.4, and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Intensity Transformations (12)

a b
c d

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0$, 4.0, and 5.0, respectively. (Original image for this example courtesy of NASA.)

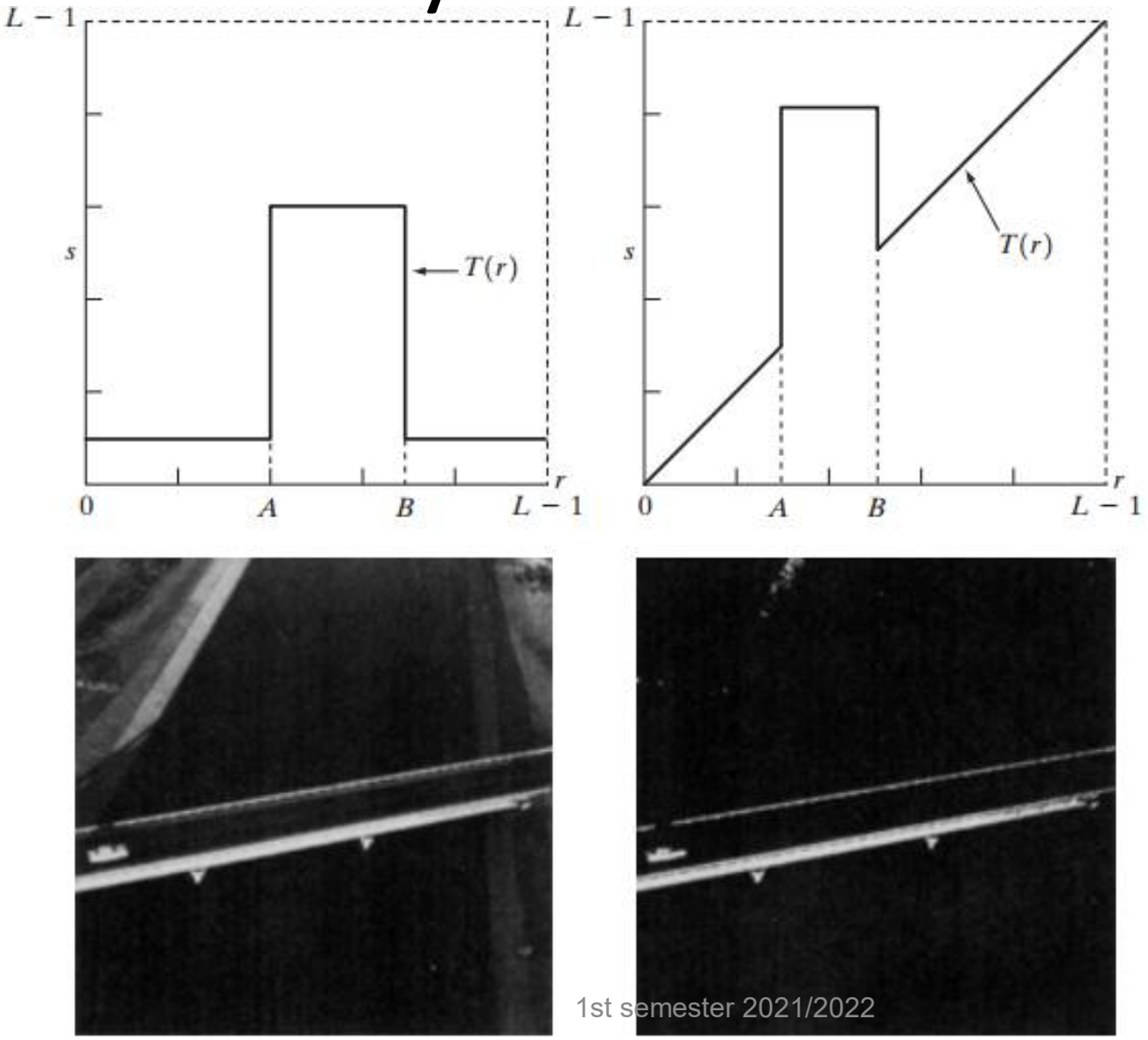
Intensity Transformations (13)



a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

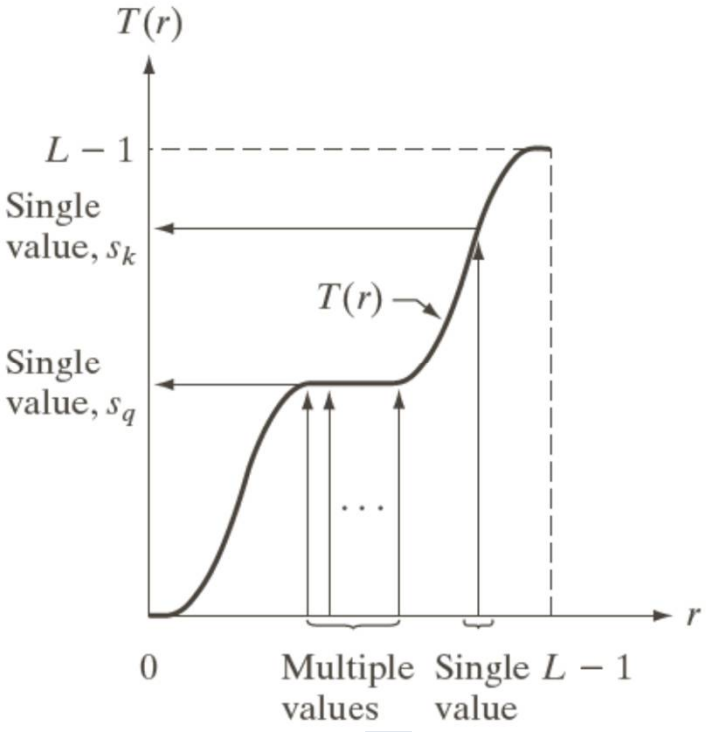
Intensity Transformations (14)



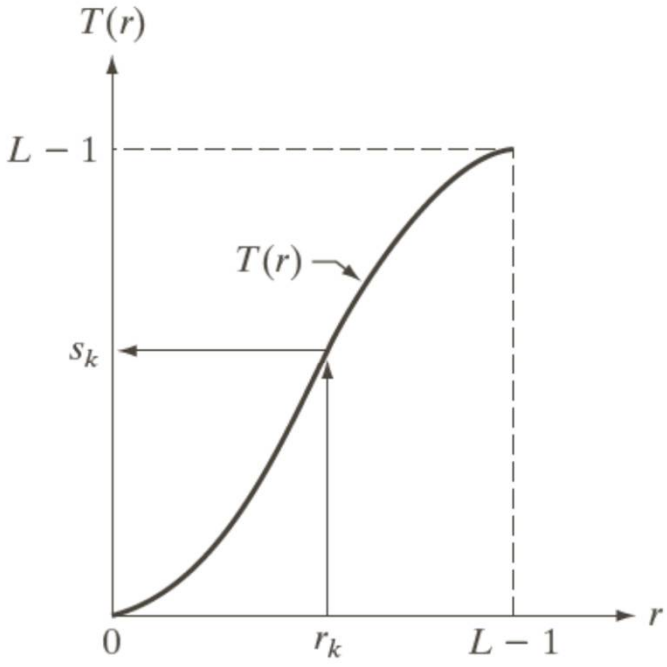
a b
c d

FIGURE 3.11
(a) This transformation highlights range $[A, B]$ of gray levels and reduces all others to a constant level. (b) This transformation highlights range $[A, B]$ but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

Intensity Transformations (15)



Non-invertible function



Invertible function

a b

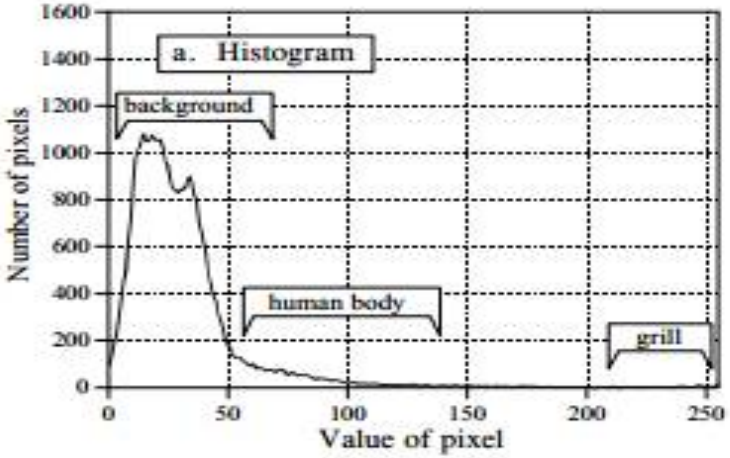
FIGURE 3.17
(a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Intensity Transformations (16)

a. Original IR image

b. With grayscale transform

Intensity Transformations (17)



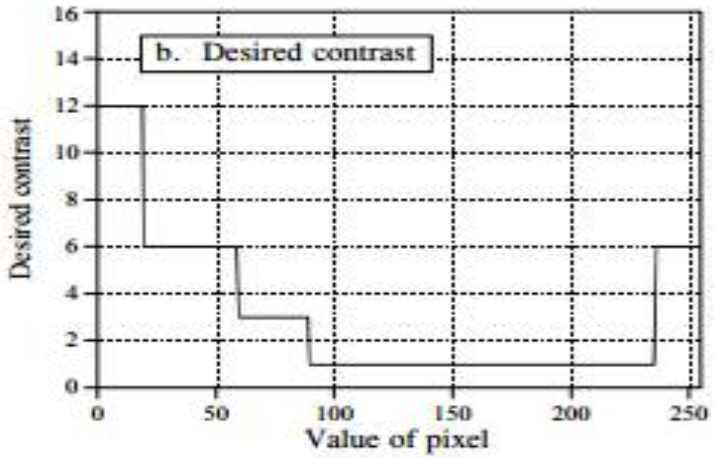
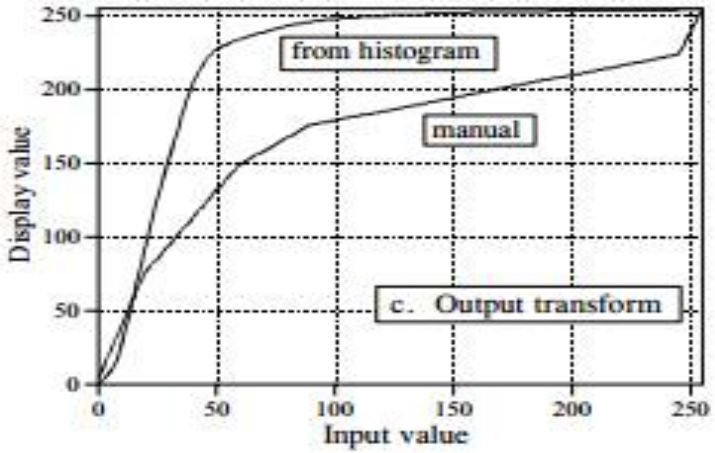
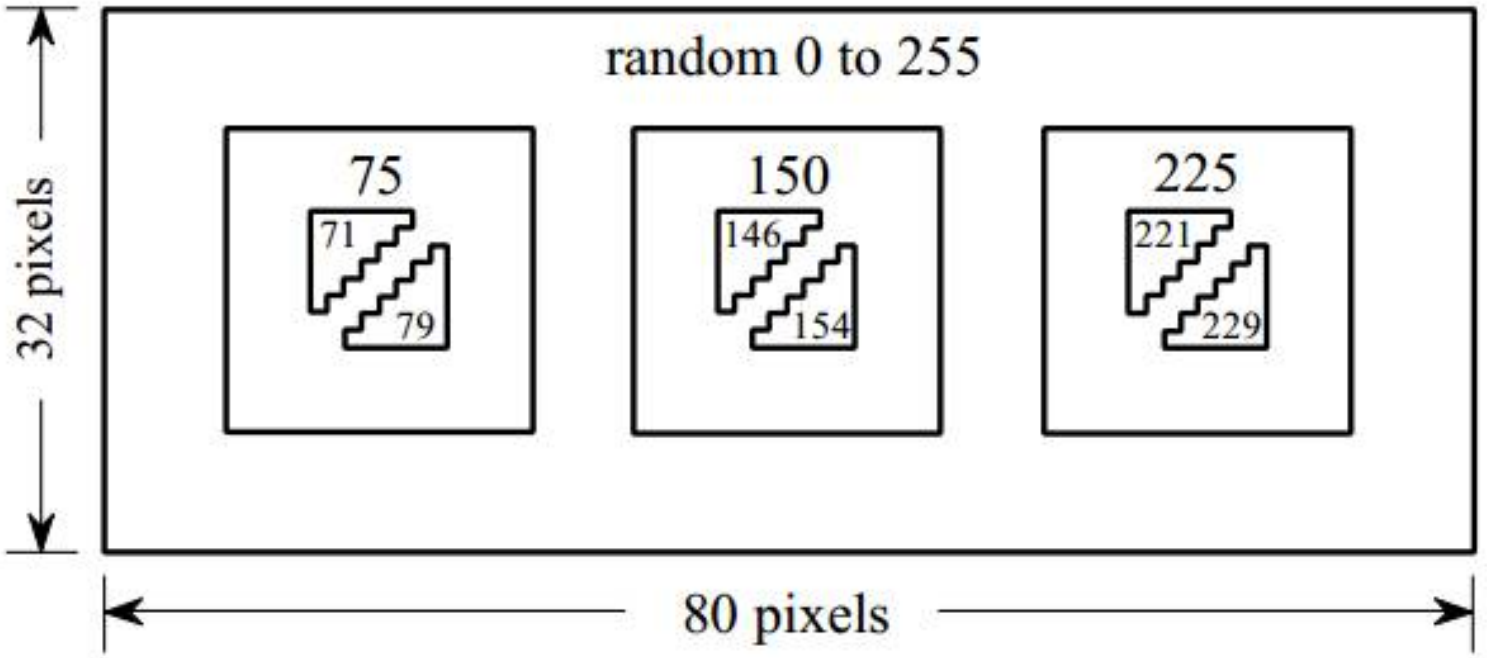


FIGURE 23-14
Developing a grayscale transform. Figure (a) is the histogram of the raw image in Fig. 23-13a. In (b), a curve is manually generated indicating the desired contrast at each pixel value. The LUT for the output transform is then found by integration and normalization of (b), resulting in the curve labeled *manual* in (c). In histogram equalization, the histogram of the raw image, shown in (a), is integrated and normalized to find the LUT, shown in (c).



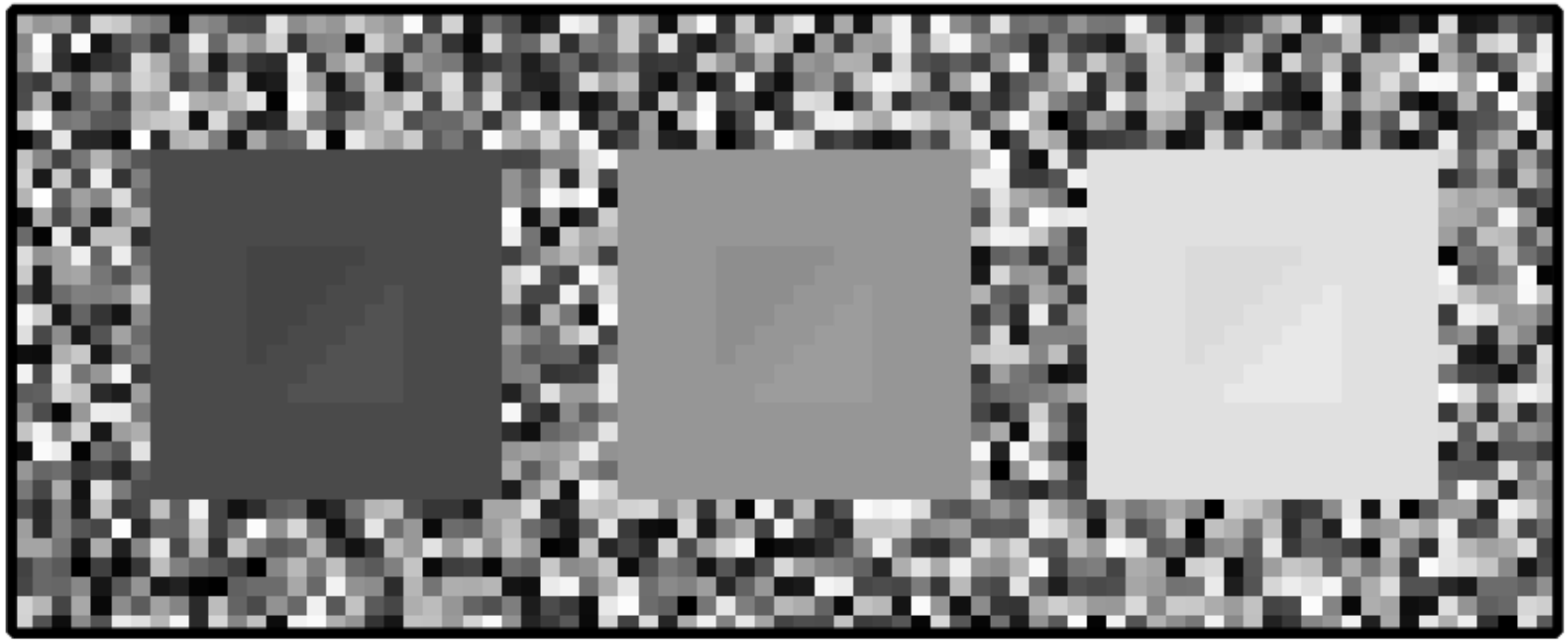
Some Numerical Results (1)



Test Image

FIGURE 23-11
Brightness and contrast test image. This is the structure of the digital image used in Fig. 23-12. The three squares form dark, medium, and bright objects, each containing two low contrast triangles. This figure indicates the digital numbers (DN) of the pixels in each region.

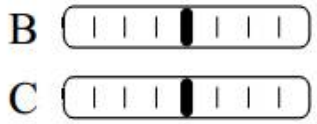
Some Numerical Results (2)

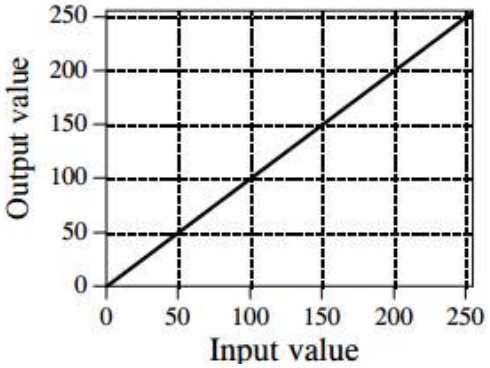


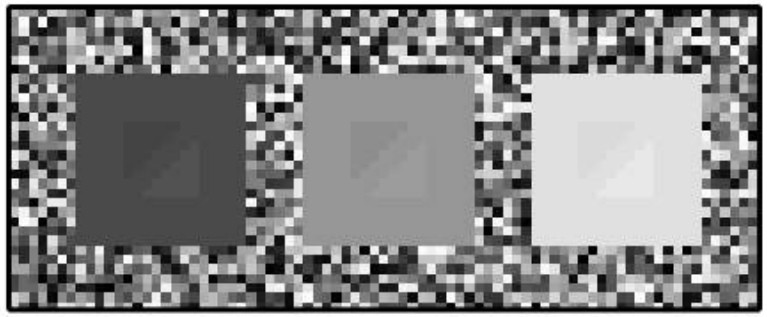
Test image

Some Numerical Results (3)

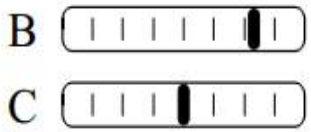
a. Normal

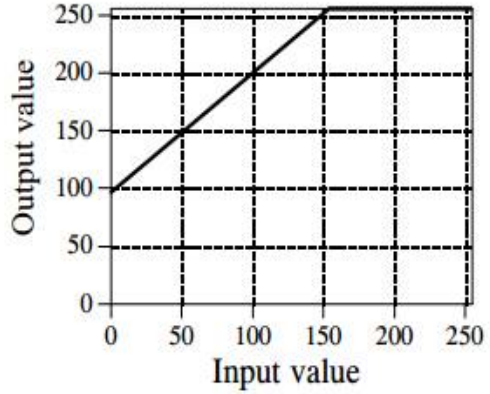


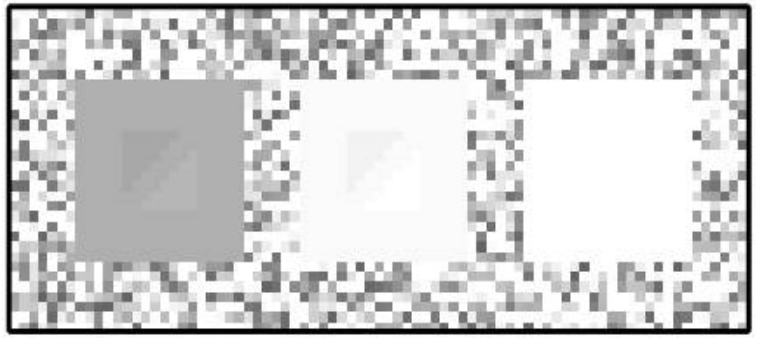




b. Increased brightness

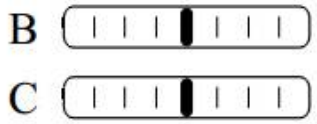


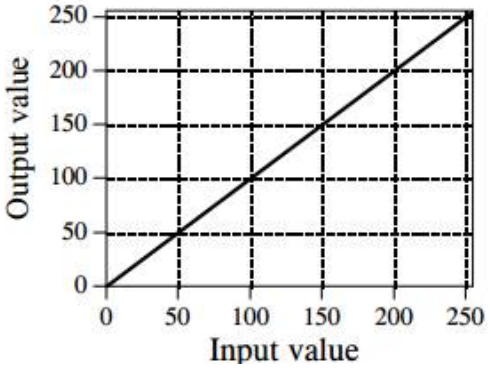


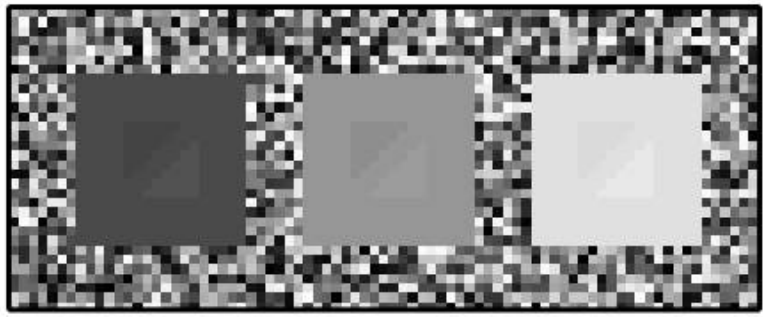


Some Numerical Results (4)

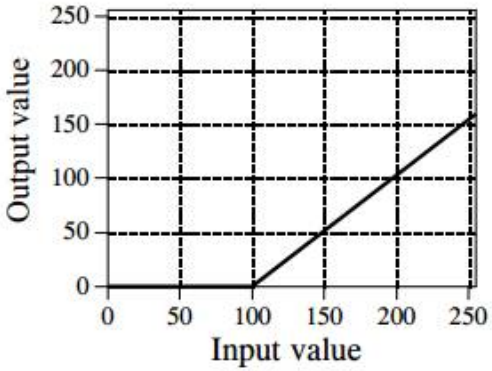
a. Normal

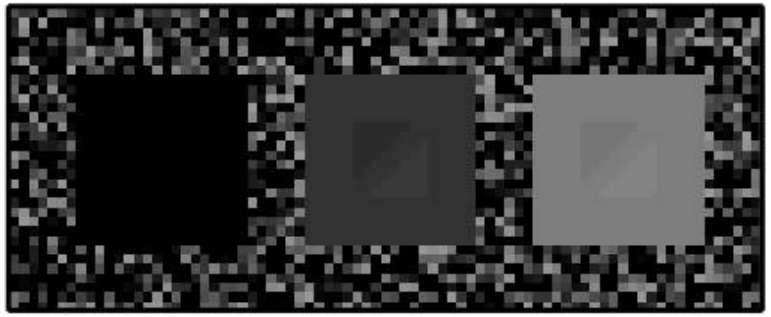






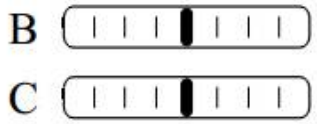
c. Decreased brightness

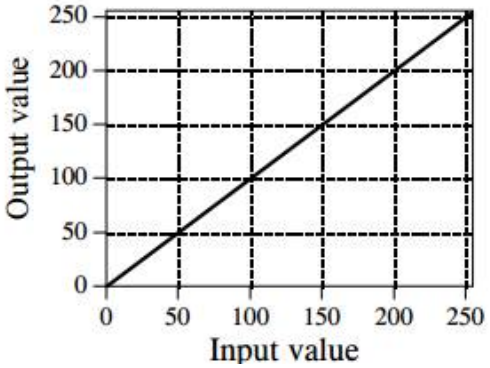


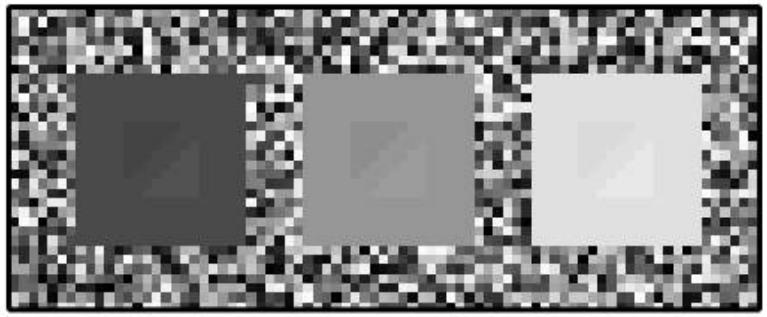


Some Numerical Results (5)

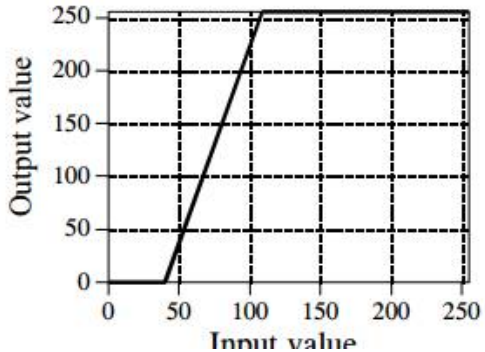
a. Normal

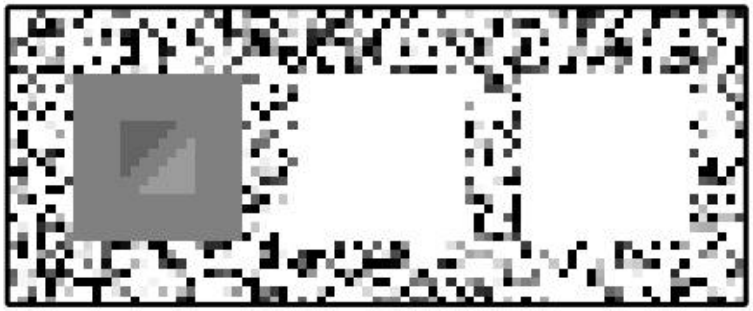






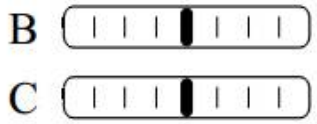
d. Slightly increased contrast at DN 75

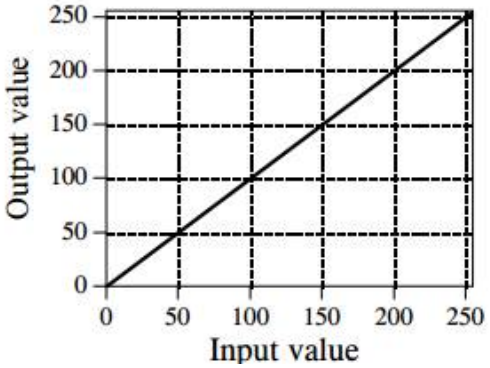


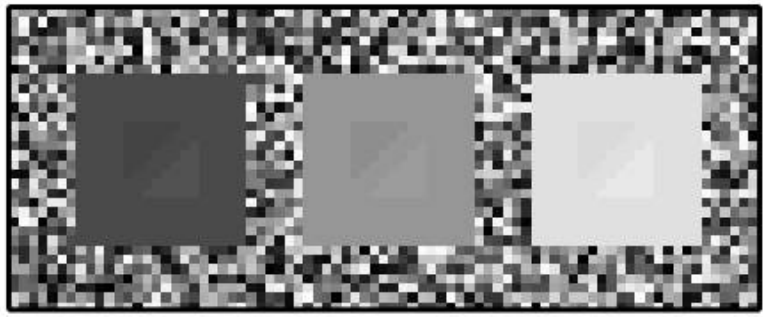


Some Numerical Results (6)

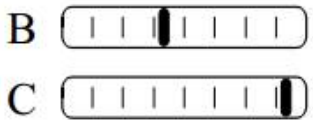
a. Normal

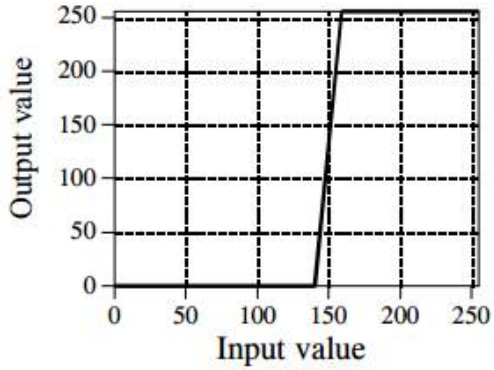


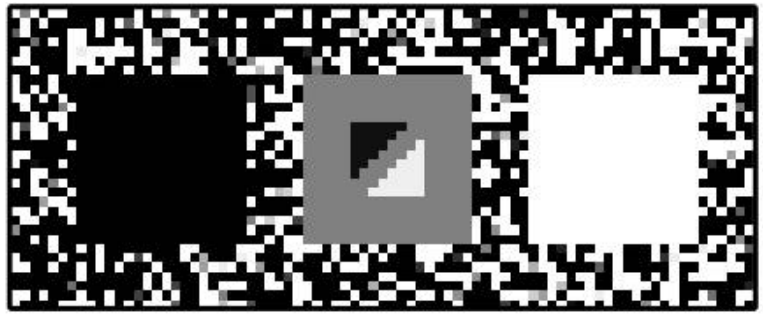




e. Greatly increased contrast at DN 150

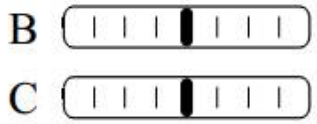


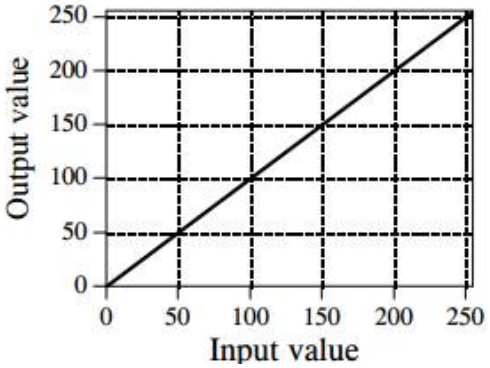


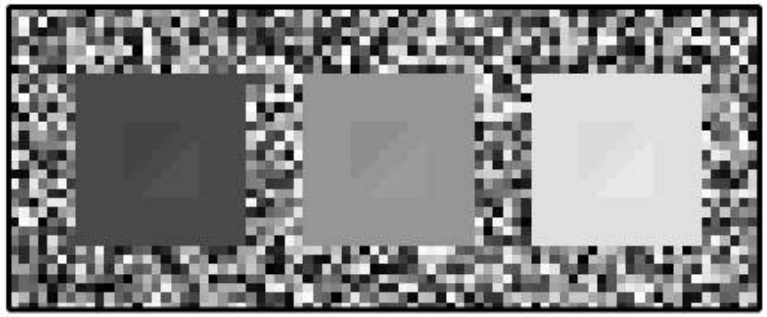


Some Numerical Results (7)

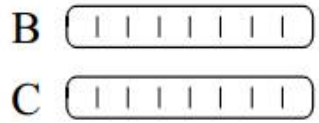
a. Normal

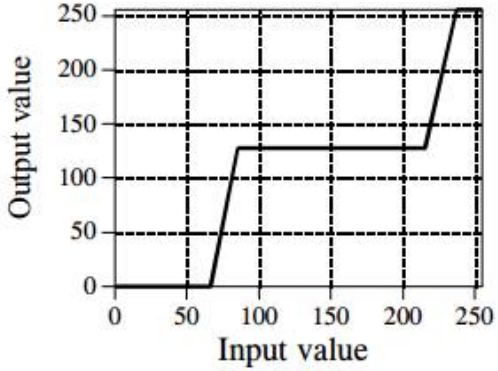


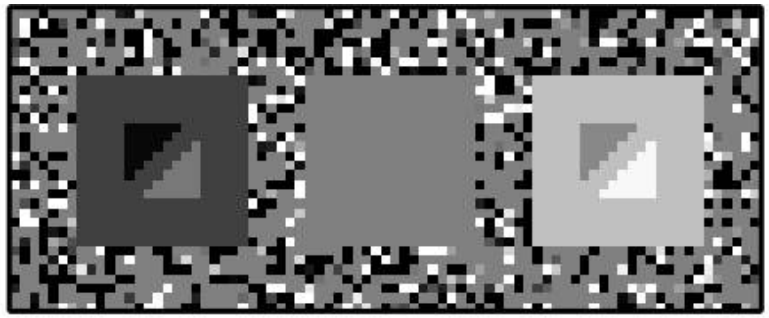




f. Increased contrast at both DN 75 and 225







A note on the implementation of intensity transformations

- The intensity transformations are usually carried out by a *lookup table;*
  - each index refers to the input pixel value
  - the value at each index is the output pixel value

- For instance, to compute the negative version of a monochrome image with n=8 bit/pixel we have

$$T[x] = 255 - x$$

$$g(x,y) = 255 - f(x,y)$$

- Thus, the *lookup* table is a 256-vector with the contents 255, 254, 253, …., 1, 0

Intensity Transformations Dependent on the Input Image

The intensity transformations functions T can be:

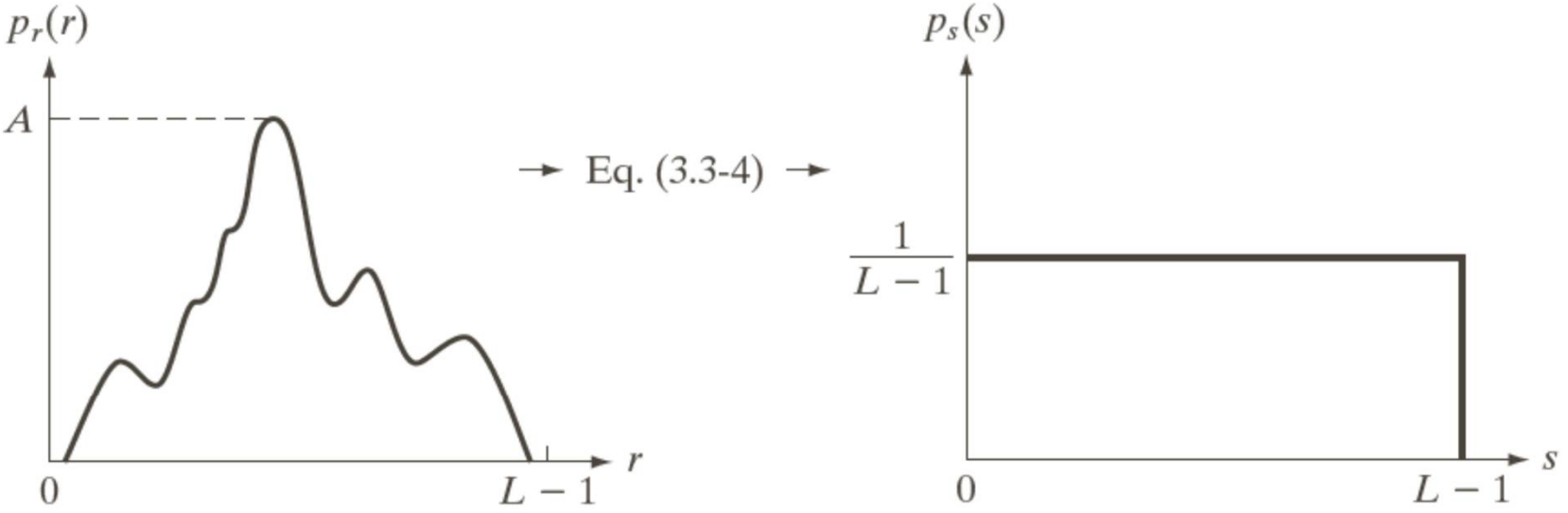
- *independent* of the image contents
- *dependent* on the image contents

Some functions that depend on the image contents, can perform two common operations:

- Histogram Equalization (HE) – change the image in such a way that it has an uniform histogram, increasing the contrast

- Histogram Specification (HS) – change the image in such a way that its histogram will be as close as possible to the histogram of the reference image

Histogram Equalization (1)



a b

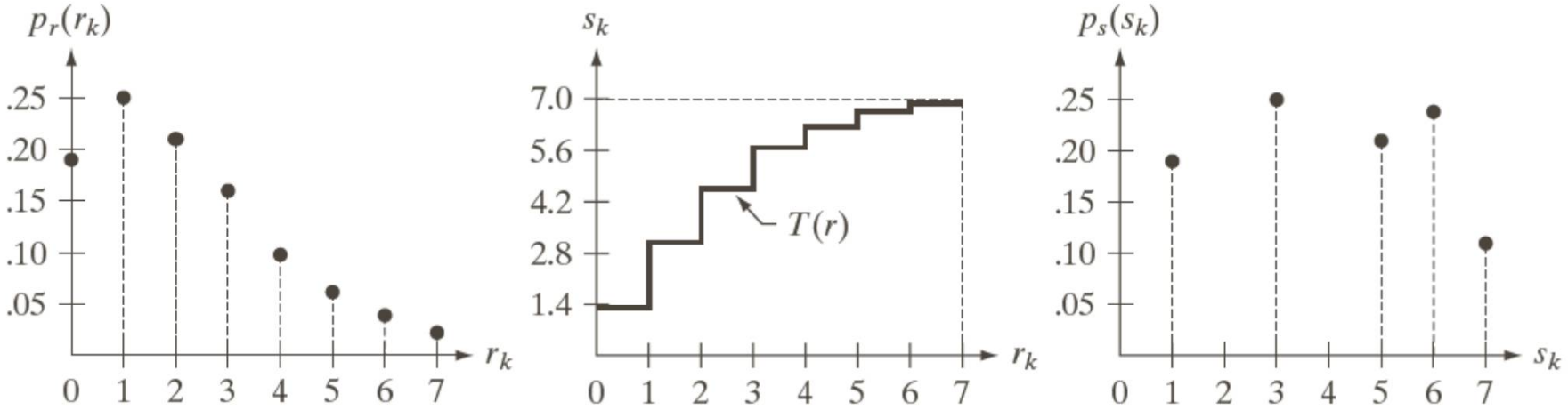
FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Equalization (2)

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1
Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

Histogram Equalization (3)



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram Equalization (4)

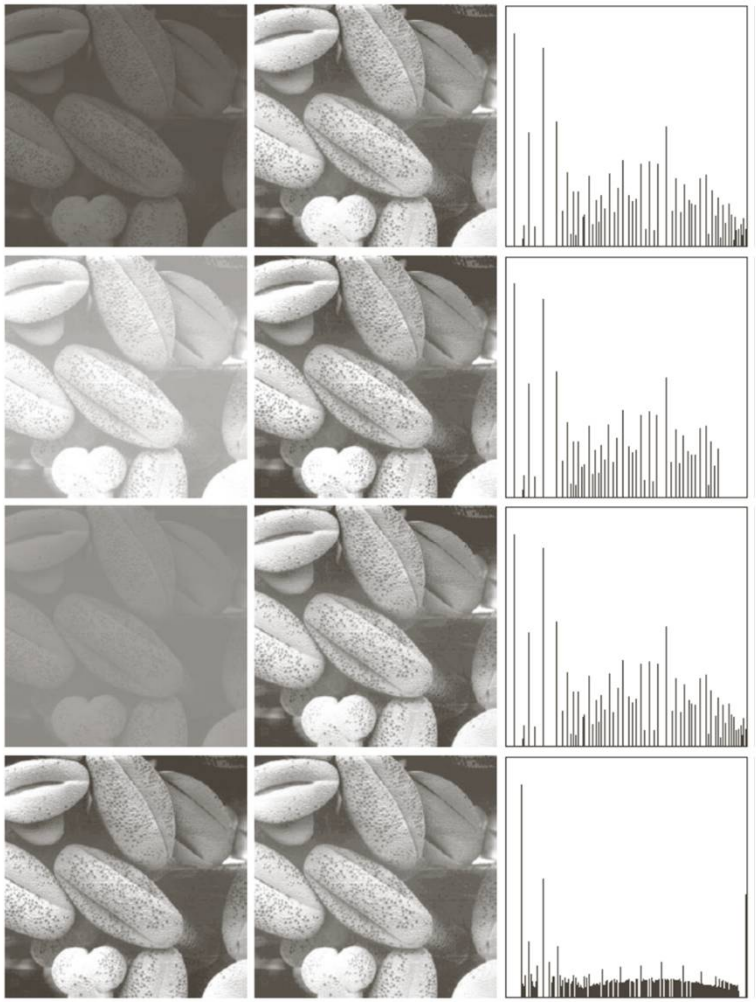


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

For the first 3 cases, we have a clear improvement on the contrast of the image

Histogram Equalization (5)

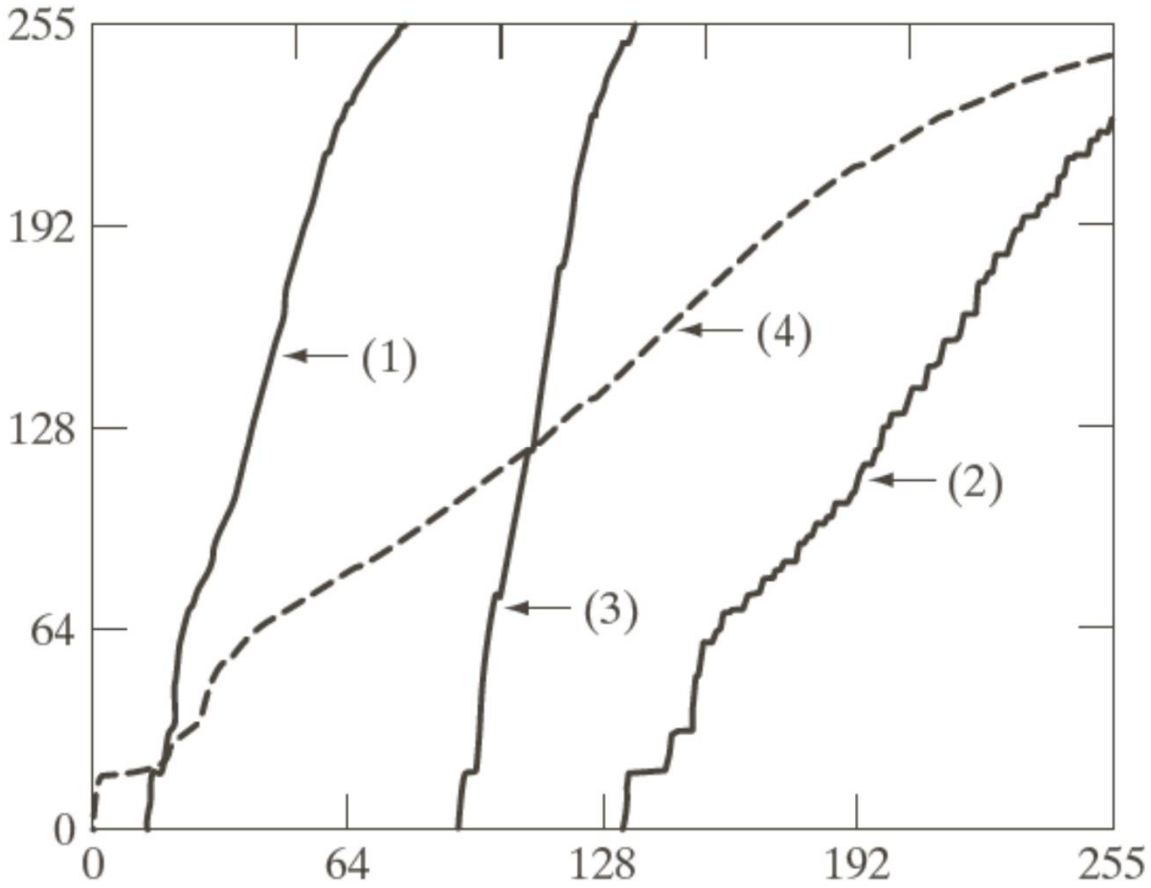
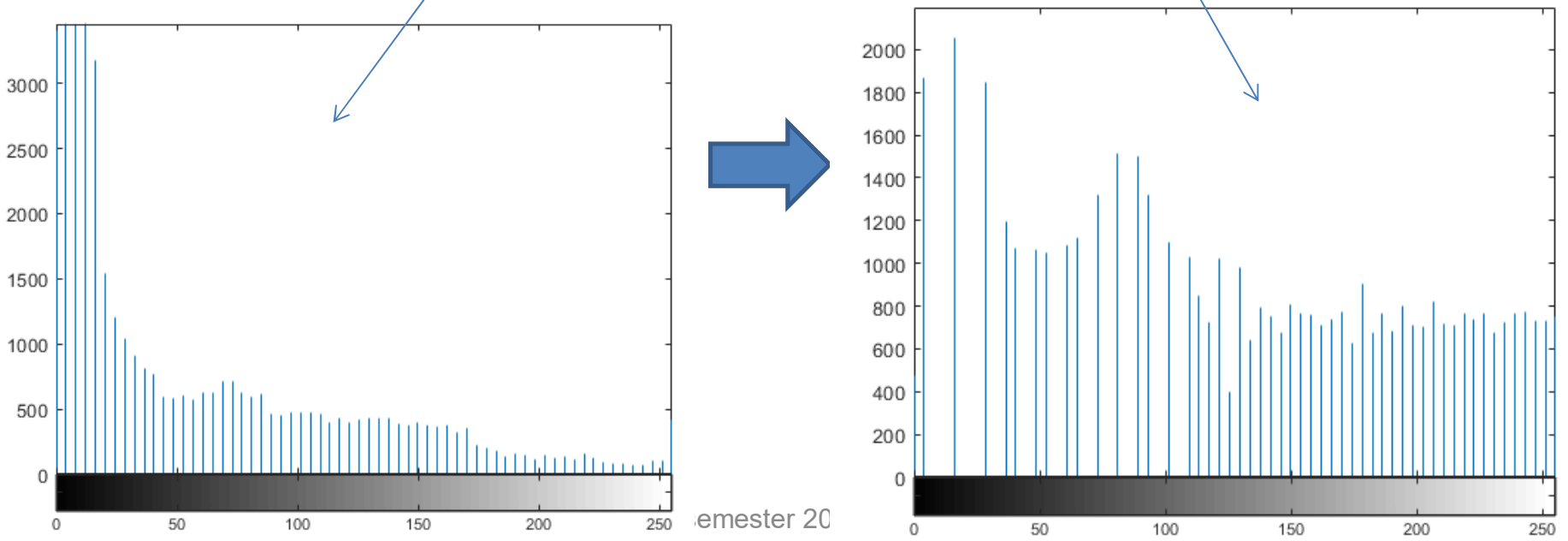


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

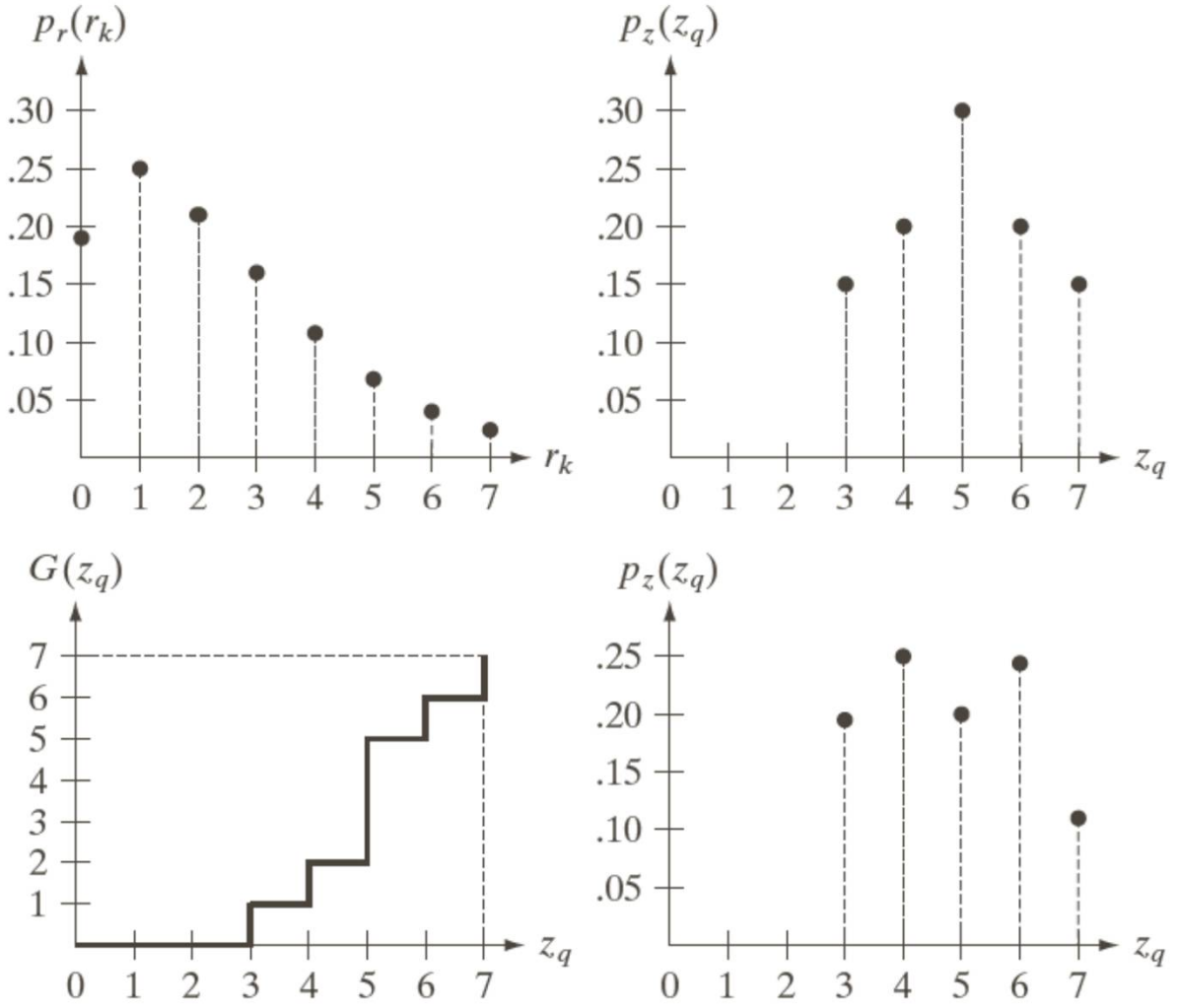
Histogram Equalization (6)

```
I = imread('tire.tif');
J = histeq(I);
imshowpair(I,J,'montage');
axis off
```

Histogram Equalization (7)



Histogram Specification (1)



a b
c d

FIGURE 3.22
(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

Histogram Specification (2)

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

TABLE 3.2
Specified and actual histograms (the values in the third column are from the computations performed in the body of Example 3.8).

Histogram Specification (3)

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

TABLE 3.3
All possible values of the transformation function G scaled, rounded, and ordered with respect to z.

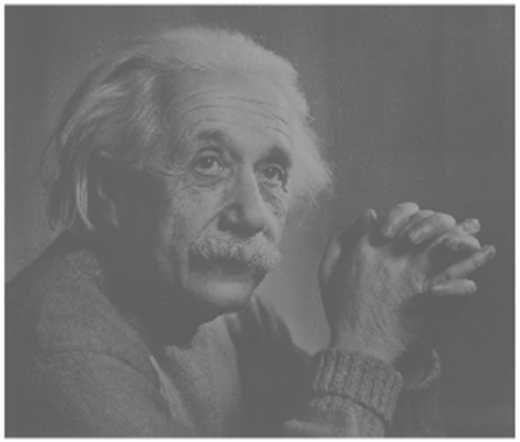
Histogram Specification (4)

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

TABLE 3.4
Mappings of all the values of s_k into corresponding values of z_q.

Histogram Specification (5)

Original Image



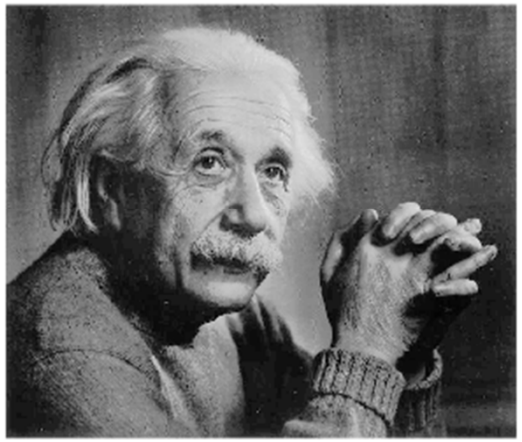
Input image
with low contrast

Reference Image

Reference image

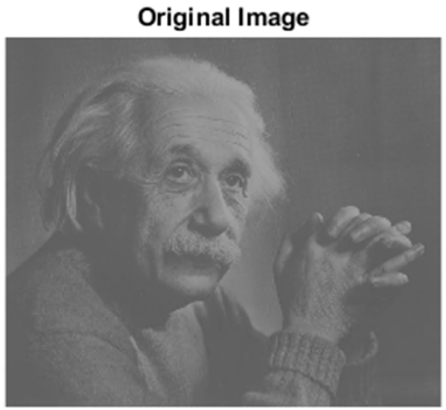
With different spatial resolution, but the same depth resolution as the input image

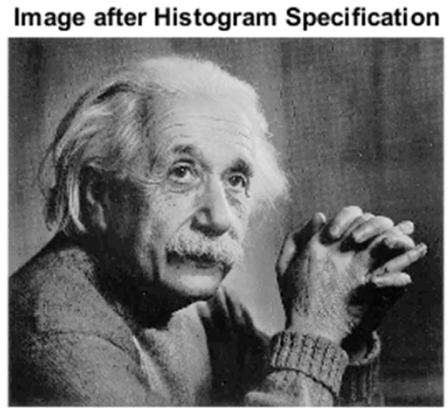
Image after Histogram Specification

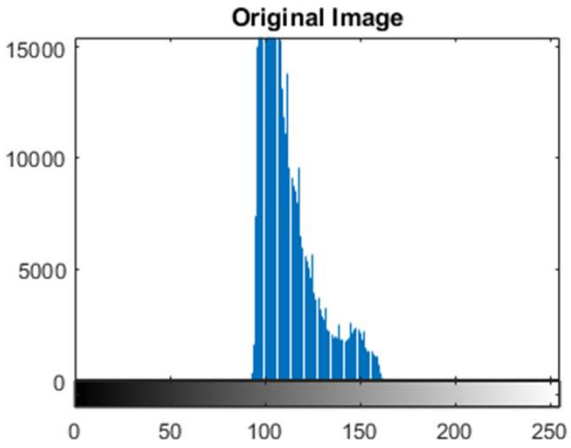


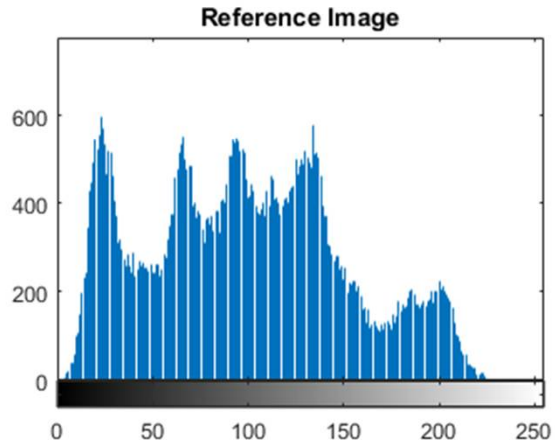
Output image
with better contrast

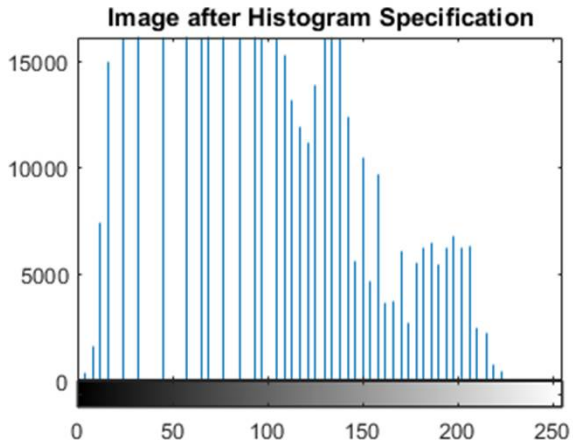
Histogram Specification (6)



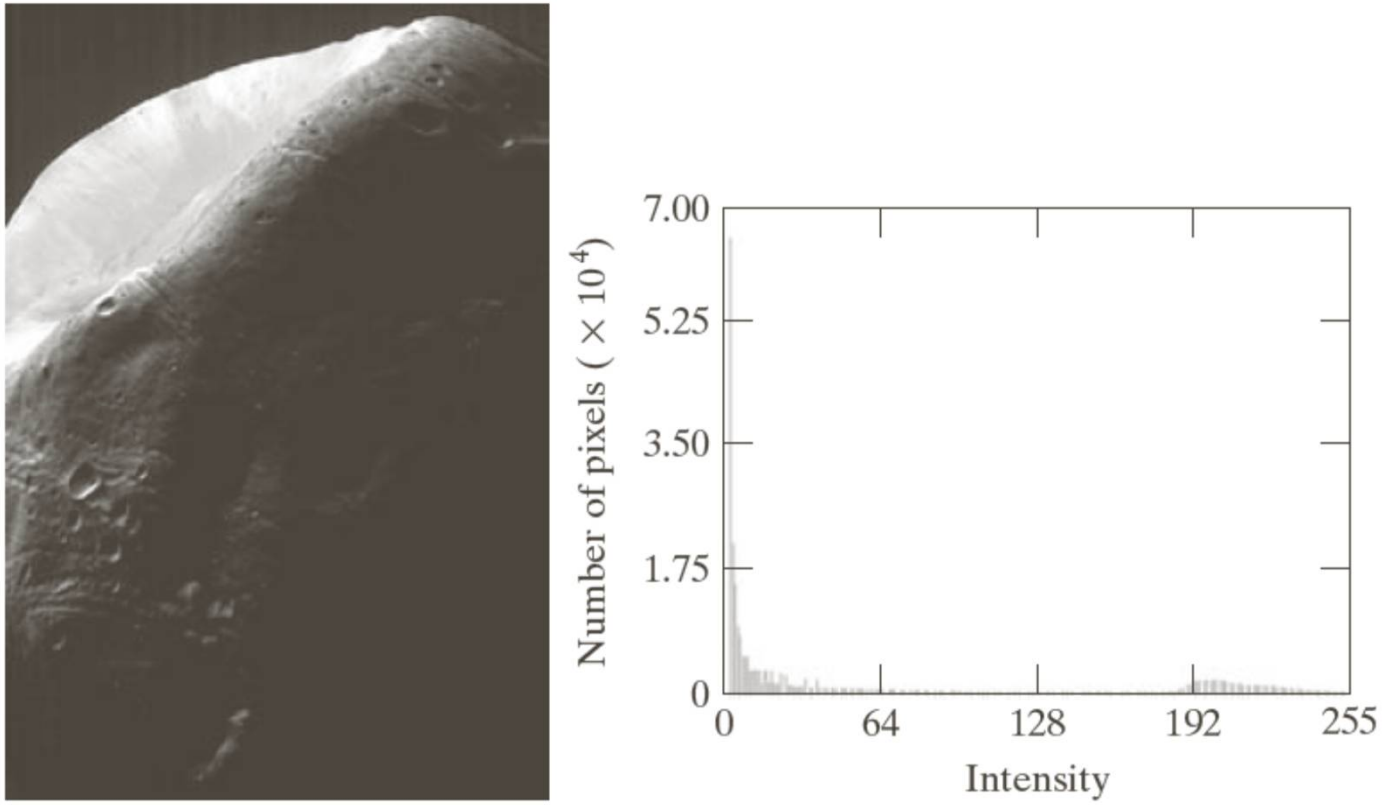








Some experimental results (1)

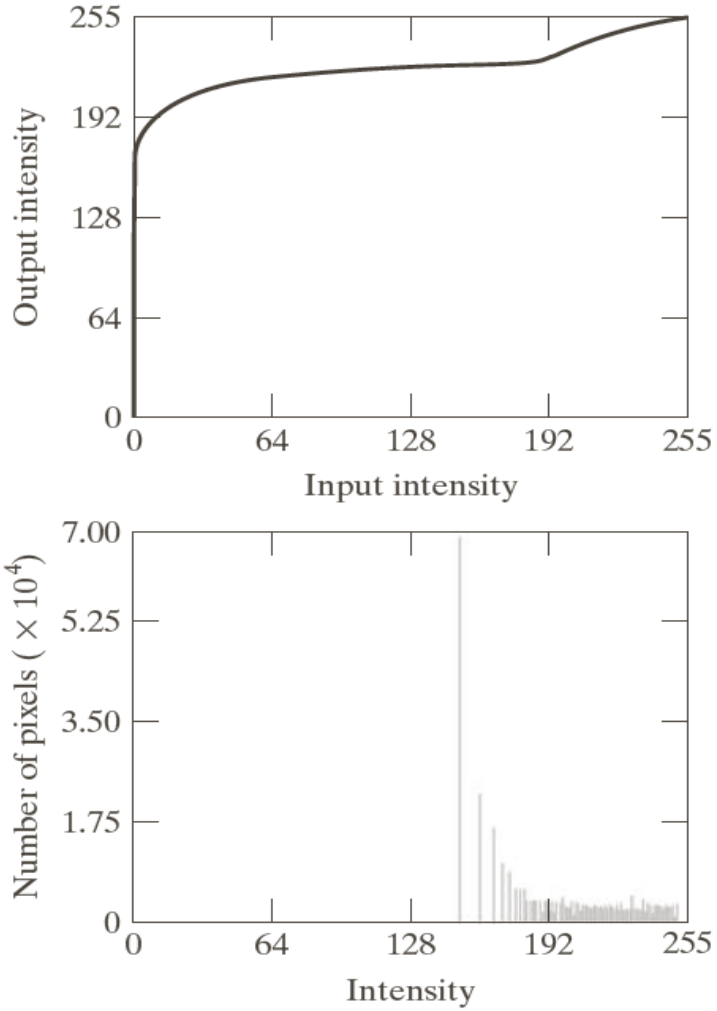


a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

Test Image and its histogram

Some experimental results (2)

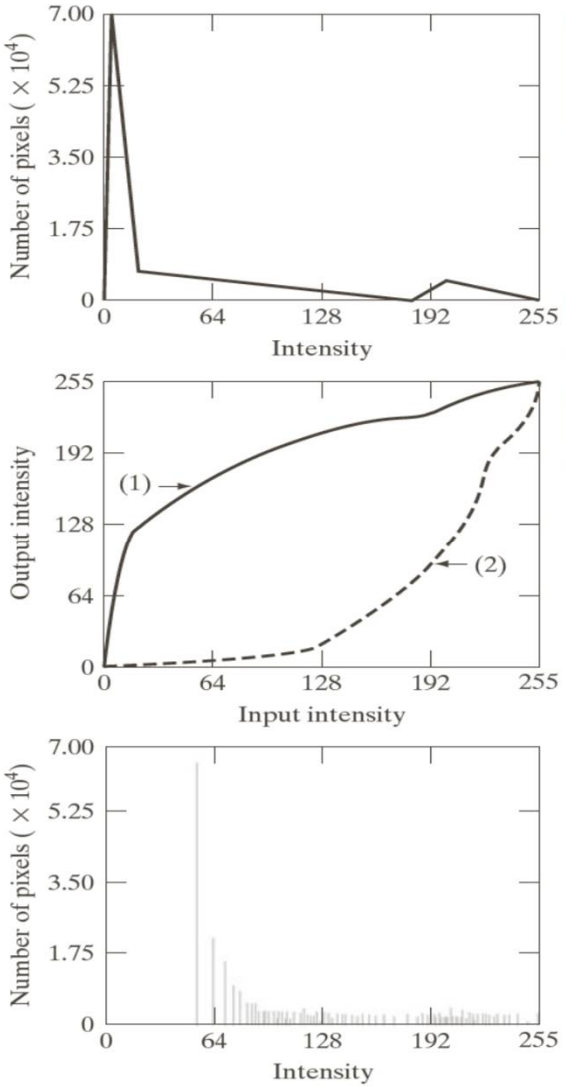


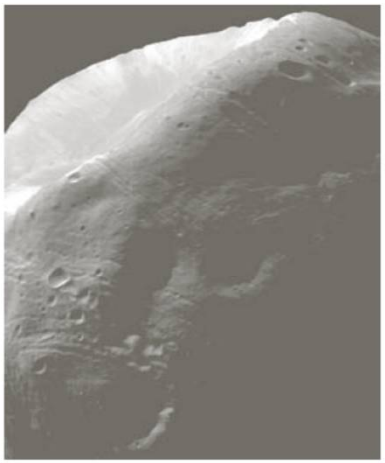
a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

HE does not work properly for this image

Some experimental results (3)



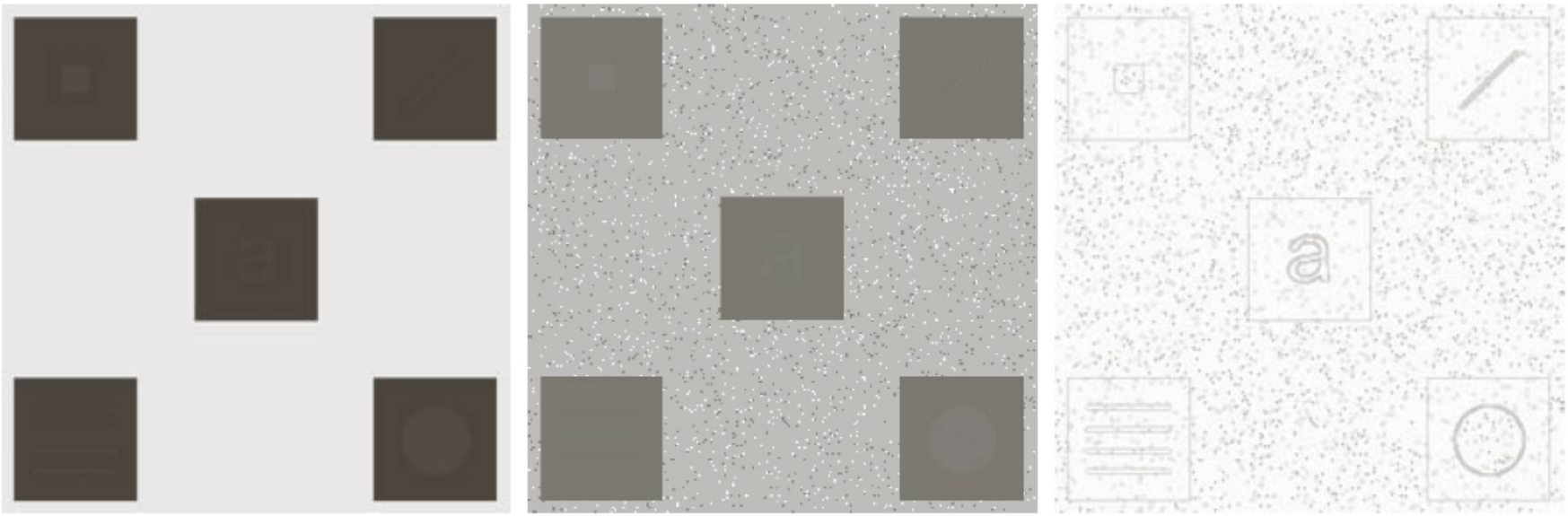


a c
b
d

FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

HS provides a better solution, in this case

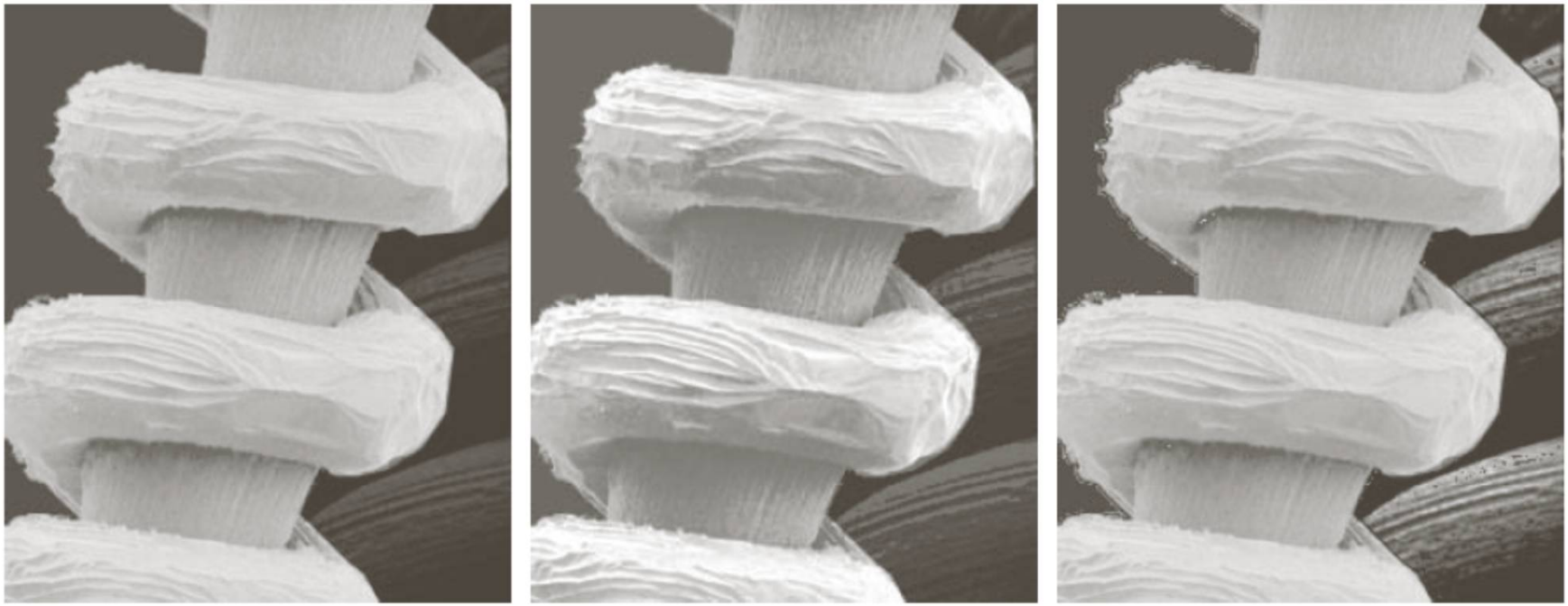
Some experimental results (4)



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.

Some experimental results (5)



a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Exercises (1)

- In Portuguese
- Exercício 2 do 1.º teste parcial, verão 2016/2017, 24 de abril de 2017

2. A tabela apresenta o histograma da imagem monocromática I, de resolução $M \times M$, com 8 níveis de cinzento.

Pixel	0	1	2	3	4	5	6	7
Ocorrências	0	128	0	30	20	10	10	58

a) {1,5} Relativamente à imagem I, indique: o valor de M; o número de bit por *pixel*; o valor mínimo de intensidade; o valor médio de intensidade; o valor máximo de intensidade; a energia; a potência.

b) Considere as transformações de intensidade definidas por

$$T_1[x] = \begin{cases} 7 - x, & 0 \leq x \leq 4 \\ x, & 5 \leq x \leq 7 \end{cases} \qquad \text{e} \qquad T_2[x] = \begin{cases} 1, & x = 0 \\ 2x - 1, & 1 \leq x \leq 4 \\ 0{,}5x, & 5 \leq x \leq 7. \end{cases}$$

i) {1,0} Apresente as tabelas de *lookup* que realizam as transformações T_1 e T_2.

ii) {1,0} Considere a função transformação de intensidade T_{12}, resultante da aplicação em sequência/série das transformações T_1 e T_2. É possível realizar T_{12} através de uma única tabela de *lookup*? Em caso afirmativo, apresente a tabela. Caso contrário, justifique a impossibilidade.

iii) {1,0} Suponha que se aplica T_1 sobre a imagem I, resultando na imagem I_1. Relacione os valores do brilho e do contraste de I e I_1 (indique se aumenta ou se diminui).

c) {1,5} Apresente um esboço da função T que realiza a equalização de histograma sobre I. Apresente a respetiva tabela de *lookup*.

Exercises (2)

- In Portuguese
- Exercício 2 do teste de época normal, verão 2016/2017, 22 de junho de 2017

2. {R1||TG} Considere a transformação de intensidade $T_1[x] = \begin{cases} 2x, & 0 \leq x \leq 45 \\ x, & 46 \leq x \leq 90 \\ x/2, & 91 \leq x \leq 255 \end{cases}$ definida para imagens com profundidade de $n = 8$ bit/pixel.

   a) {1,25||1,0} Descreva a tabela de *lookup* que realiza T_1.

   b) {1,25||1,0} Apresente a imagem I_1, resultante da aplicação de T_1 sobre a imagem I definida no exercício 1.

   c) A tabela apresenta o histograma da imagem monocromática quadrada K, com profundidade de $n = 8$ bit/pixel.

Pixel	0	10	23	90	133	255
Ocorrências	0	128	128	128	640	0

   i) {1,25||1,0} Indique a resolução da imagem K. Apresente o histograma da imagem K_1, resultante da aplicação de T_1 sobre a imagem K.

   ii) {1,25||1,0} Relacione os valores de brilho e de contraste das imagens K e K_1.

Exercises (3)

- In Portuguese
- Exercício 1 do teste de época de recurso, verão 2016/2017, 10 de julho de 2017

1. A imagem I com resolução espacial 128×128, tem 256 níveis de cinzento e histograma apresentado na tabela. A soma dos valores de todos os *pixel* da imagem é $\sum_{m=0}^{127}\sum_{n=0}^{127} I[m,n] = 1\,256\,452$.

Pixel	0	A	23	90	133	255
Ocorrências	4300	1280	1280	1280	B	0

a) {1,0} Determine os valores de A e B.

b) {1,0} Apresente o histograma da versão negativa de I, designada por I_n.
Sugestão: caso não tenha respondido à alínea anterior, considere a partir de agora valores genéricos para A e B.

c) {1,0} Compare as imagens I e I_n, relativamente ao brilho e ao contraste (idêntico, inferior ou superior).

Exercises (4)

- In Portuguese
- Exercício 2 do teste de época de recurso, verão 2016/2017, 10 de julho de 2017

2. A figura apresenta o histograma da imagem monocromática I de resolução $M \times M$, em que M pode ser escrito na forma $M = 2^k$, sendo k um número inteiro positivo.

   a) {1,0} Determine os valores de M e k. Qual a resolução em profundidade de I?

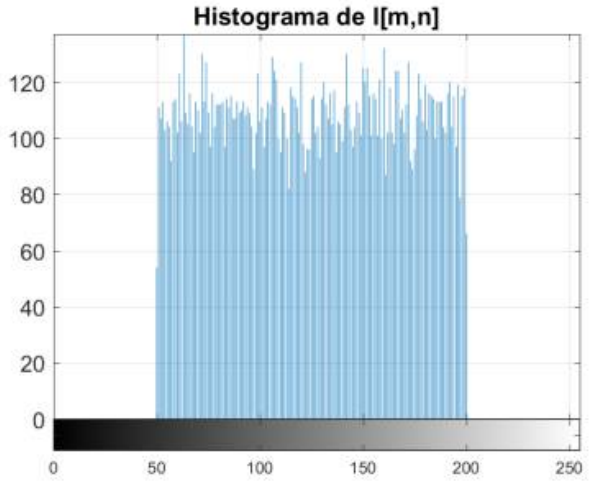
   b) {1,0} Apresente uma estimativa do valor médio de intensidade de I.

   c) Considere as transformações de intensidade definidas por

$$T_1[x] = \begin{cases} x/4, & 0 \leq x \leq 50 \\ x, & 51 \leq x \leq 150 \\ x/2, & 151 \leq x \leq 255 \end{cases} \quad \text{e} \quad T_2[x] = \begin{cases} x, & 0 \leq x \leq 50 \\ 180, & 51 \leq x \leq 150 \\ x, & 151 \leq x \leq 255. \end{cases}$$

   i) {1,0} Esboce as funções T_1 e T_2. Descreva as tabelas de *lookup* que realizam T_1 e T_2.

   ii) {1,0} Esboce os histogramas das imagens I_1 e I_2, resultantes da aplicação de T_1 e T_2, respetivamente, sobre a imagem I.



MATLAB
Image Processing Toolbox functions

https://www.mathworks.com/products/image.html

- *bitand, bitor, bitxor,* …. perform logical operations
- *intlut*, apply a lookup table to a matrix of integers (image)
- *imadjust,* contrast adjustment
- *graythresh,* compute the optimal threshold for binarization
- *im2bw,* convert a monochrome image to its binary version
- *histeq*, histogram equalization
- *adapthisteq*, adaptive histogram equalization, with limited contrast
- *imhistmatch*, histogram specification

Bibliography

- The images displayed in these slides are from:
  - R. Gonzalez, R. Woods, *Digital Image Processing*, 4th edition, Prentice Hall, 2018, ISBN 0133356728
  - S. Smith, *The Scientist and Engineer's Guide to Digital Signal Processing*, Newnes, 2003, ISBN 0-750674-44-X [chapter 23]
  - O. Filho, H. Neto, Processamento Digital de Imagens, Rio de Janeiro: Brasport, 1999, ISBN 8574520098.