ISEL
INSTITUTO SUPERIOR
DE ENGENHARIA DE LISBOA

PROCESSAMENTO DE IMAGEM E BIOMETRIA

IMAGE PROCESSING AND BIOMETRICS

6. **FREQUENCY FILTERING (part 2)**

Summary (part 2)

- Frequency filtering algorithm details

- Commonly used filters on the frequency domain
  - Low-pass filters (LPF)
  - High-pass filters (HPF)

- Experimental results

- Some frequency domain techniques
  - High-frequency emphasis
  - Unsharp masking
  - Homomorphic filtering

- Band-pass, band-reject, and notch filters
- Exercises

Frequency filtering algorithm (1)

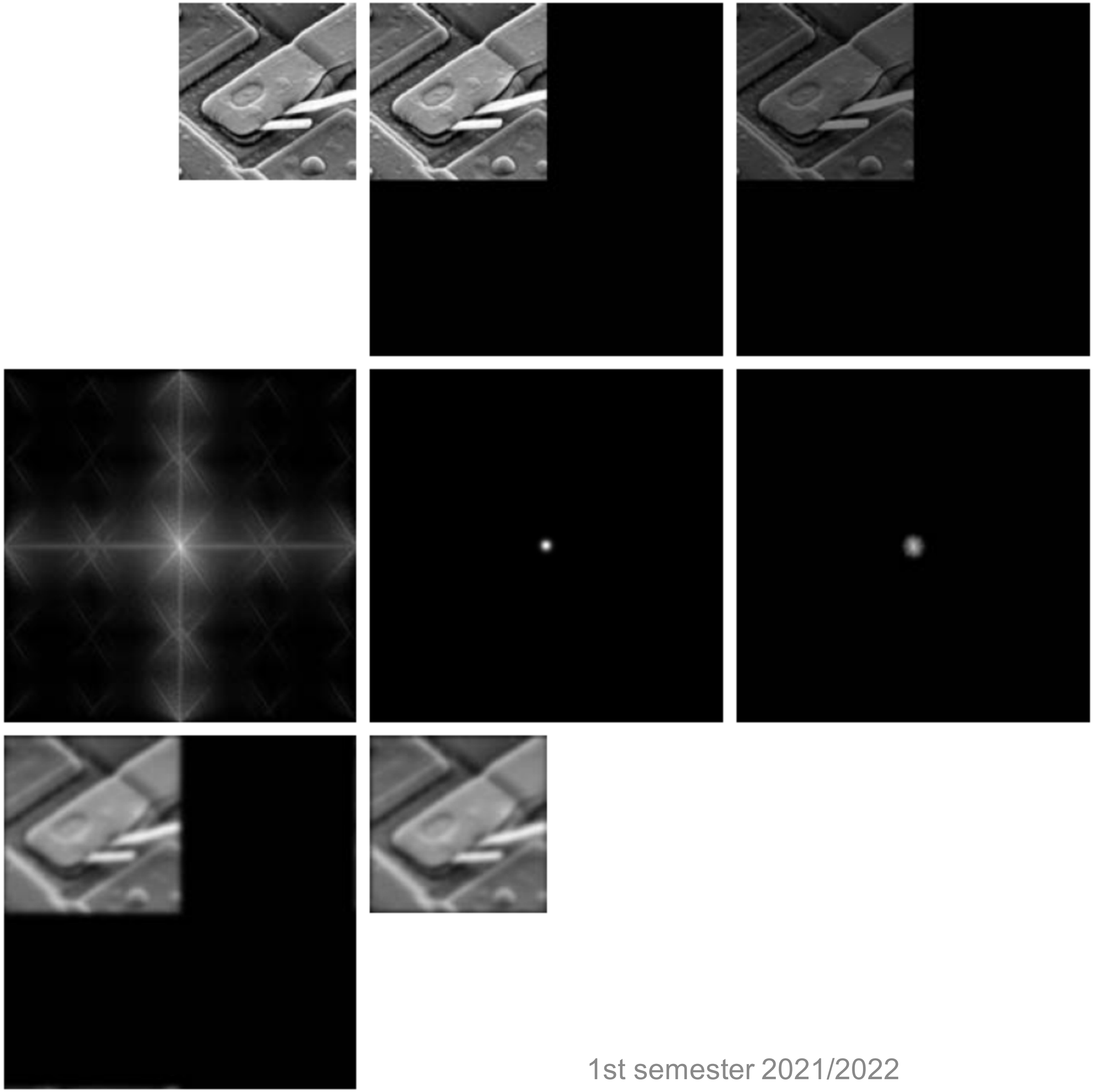


FIGURE 4.36 (a) An $M \times N$ image, f. (b) Padded image, f_p of size $P \times Q$. (c) Result of multiplying f_p by $(-1)^{x+y}$. (d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$. (f) Spectrum of the product HF_p. (g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p. (h) Final result, g, obtained by cropping the first M rows and N columns of g_p.

Frequency filtering algorithm (2)

1. Given an input image $f(x, y)$ of size $M \times N$, obtain the padding parameters P and Q from Eqs. (4.6-31) and (4.6-32). Typically, we select $P = 2M$ and $Q = 2N$.
2. Form a padded image, $f_p(x, y)$, of size $P \times Q$ by appending the necessary number of zeros to $f(x, y)$.
3. Multiply $f_p(x, y)$ by $(-1)^{x+y}$ to center its transform.
4. Compute the DFT, $F(u, v)$, of the image from step 3.
5. Generate a real, symmetric filter function, $H(u, v)$, of size $P \times Q$ with center at coordinates $(P/2, Q/2)$.† Form the product $G(u, v) = H(u, v)F(u, v)$ using array multiplication; that is, $G(i, k) = H(i, k)F(i, k)$.
6. Obtain the processed image:

$$g_p(x, y) = \left\{\text{real}\left[\Im^{-1}[G(u, v)]\right]\right\}(-1)^{x+y}$$

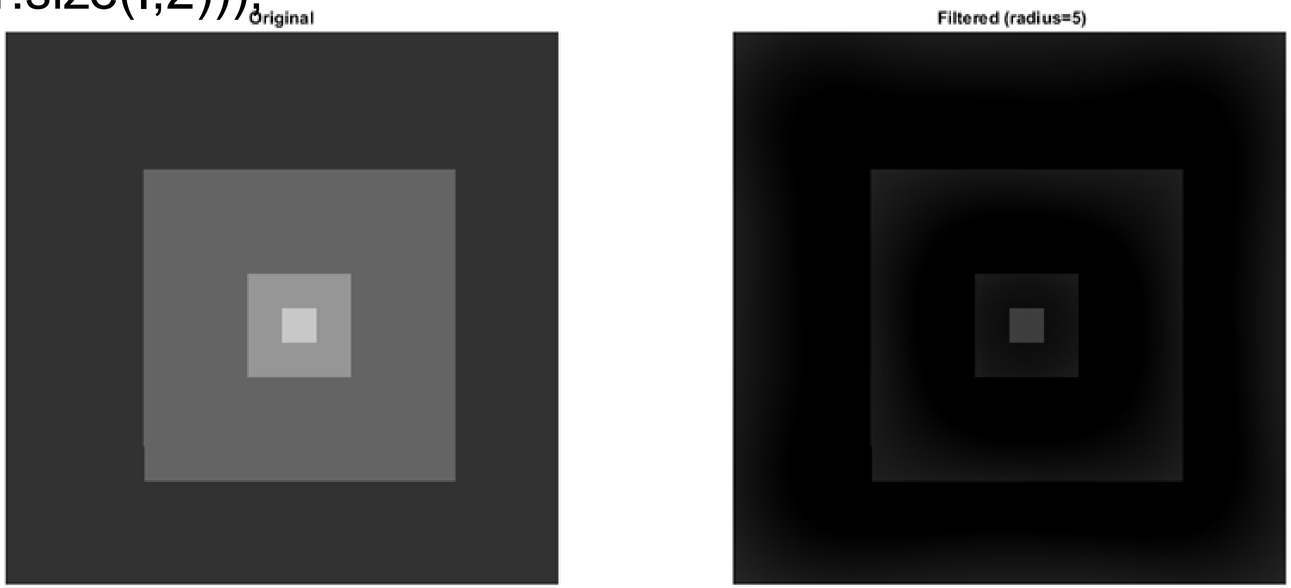
   where the real part is selected in order to ignore parasitic complex components resulting from computational inaccuracies, and the subscript p indicates that we are dealing with padded arrays.
7. Obtain the final processed result, $g(x, y)$, by extracting the $M \times N$ region from the top, left quadrant of $g_p(x, y)$.

Frequency filtering algorithm (3)

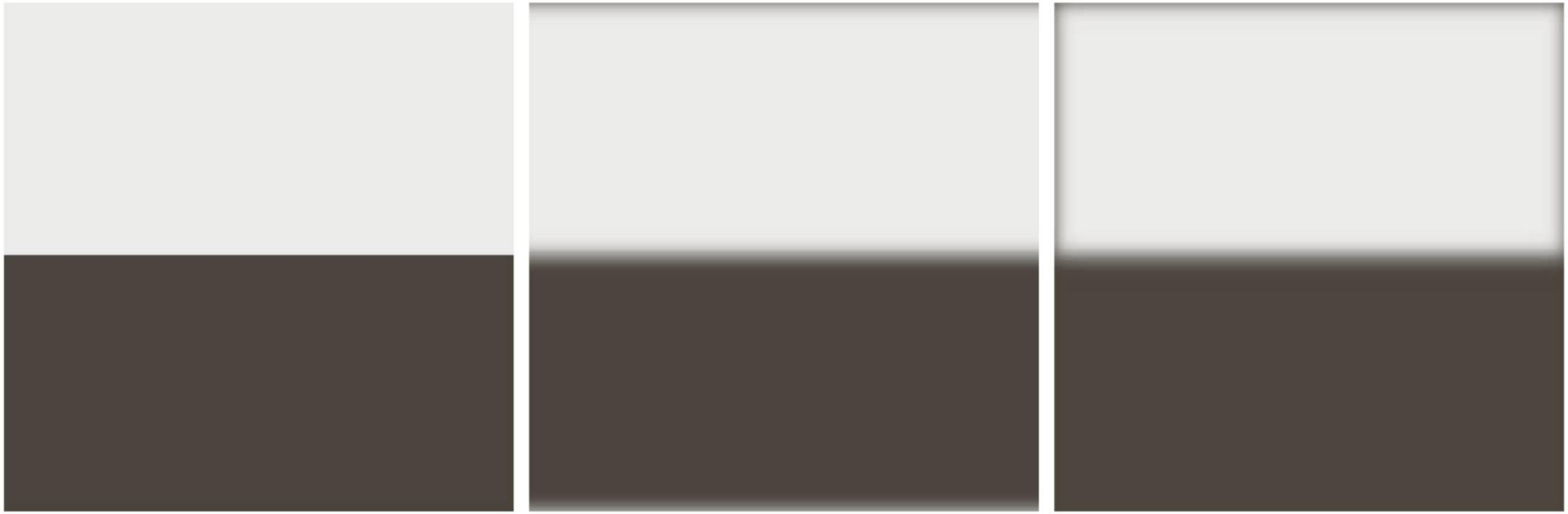
```
% Step 1 - Zero-padded image to resolution P=2M and Q=2N.
Ip = [ I, zeros(size(I,1), size(I,2));
       zeros(size(I,1), size(I,2)), zeros(size(I,1), size(I,2))];
Ip = uint8(Ip);
P = size(Ip,1);
Q = size(Ip,2);

% Step 2 - Compute the DFT of the padded image and center its spectrum.
Fp = fftshift(fft2(Ip));

% Step 3 - Set the filter mask H, on the frequency domain.
% In this case, it is an ideal high-pass filter.
H = zeros( P, Q );
for u=1 : P
   for v=1 : Q
      d = sqrt( (u - P/2)^2 + (v - Q/2)^2);
      if d > radius
         H(u,v)=1;
      end
   end
end
```

Frequency filtering algorithm (4)

```
% Step 4 - Multiply the spectrum of the padded image
% with the filter mask H.
G = Fp .* H;

% Steps 5 and 6 - Perform the inverse DFT (with centered spectrum).
% Remove residual imaginary parts and convert to unsigned integer.
If = ifft2 ( ifftshift(G) ) ;
If = uint8(real(If));

% Step 7 - Crop the resulting image
If = uint8(If( 1:size(I,1), 1:size(I,2)));
```



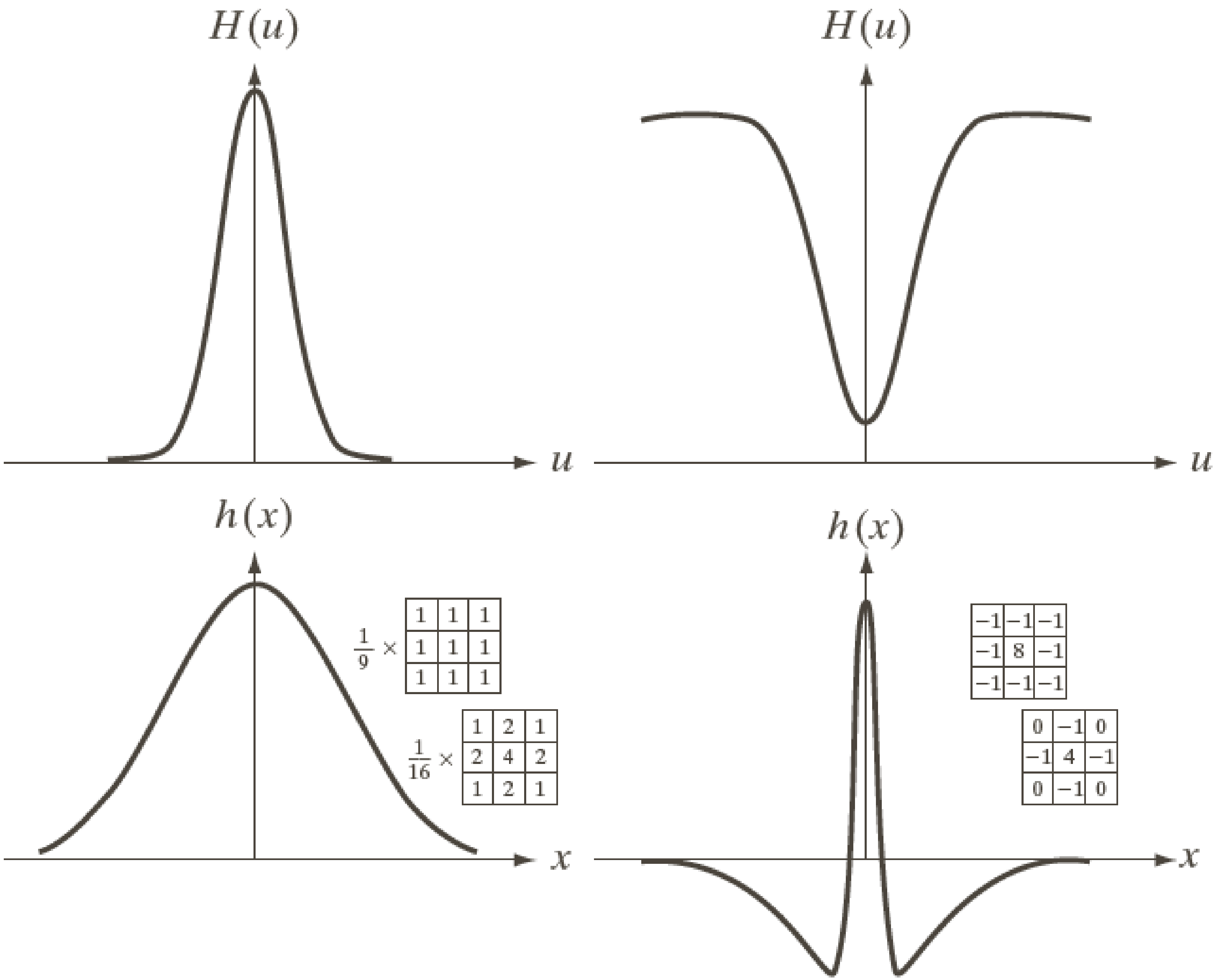
The need of zero-padding



a b c

FIGURE 4.32 (a) A simple image. (b) Result of blurring with a Gaussian lowpass filter without padding. (c) Result of lowpass filtering with padding. Compare the light area of the vertical edges in (b) and (c).

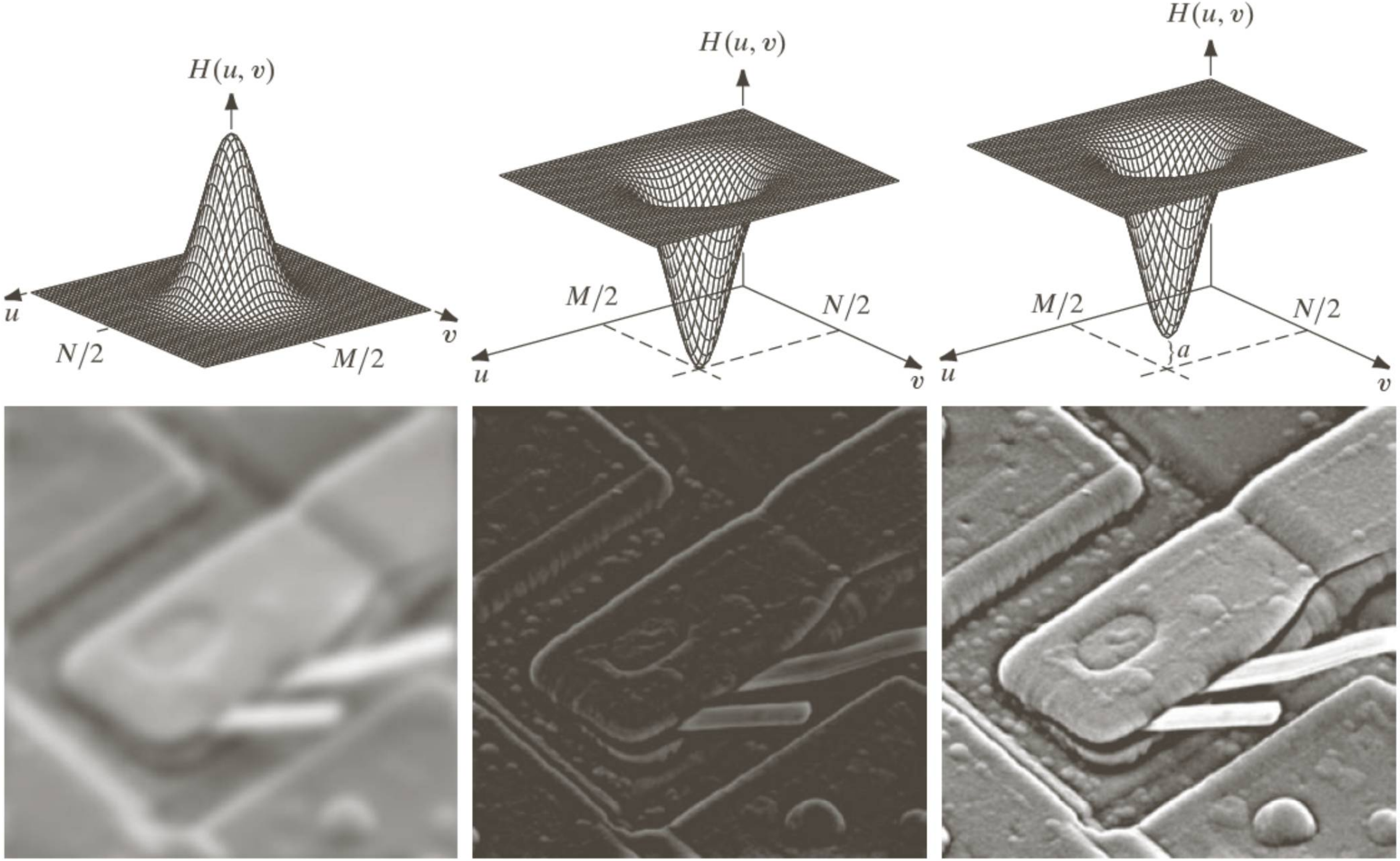
Commonly used filters (1)



a	c
b	d

FIGURE 4.37 (a) A 1-D Gaussian lowpass filter in the frequency domain. (b) Spatial lowpass filter corresponding to (a). (c) Gaussian highpass filter in the frequency domain. (d) Spatial highpass filter corresponding to (c). The small 2-D masks shown are spatial filters we used in Chapter 3.

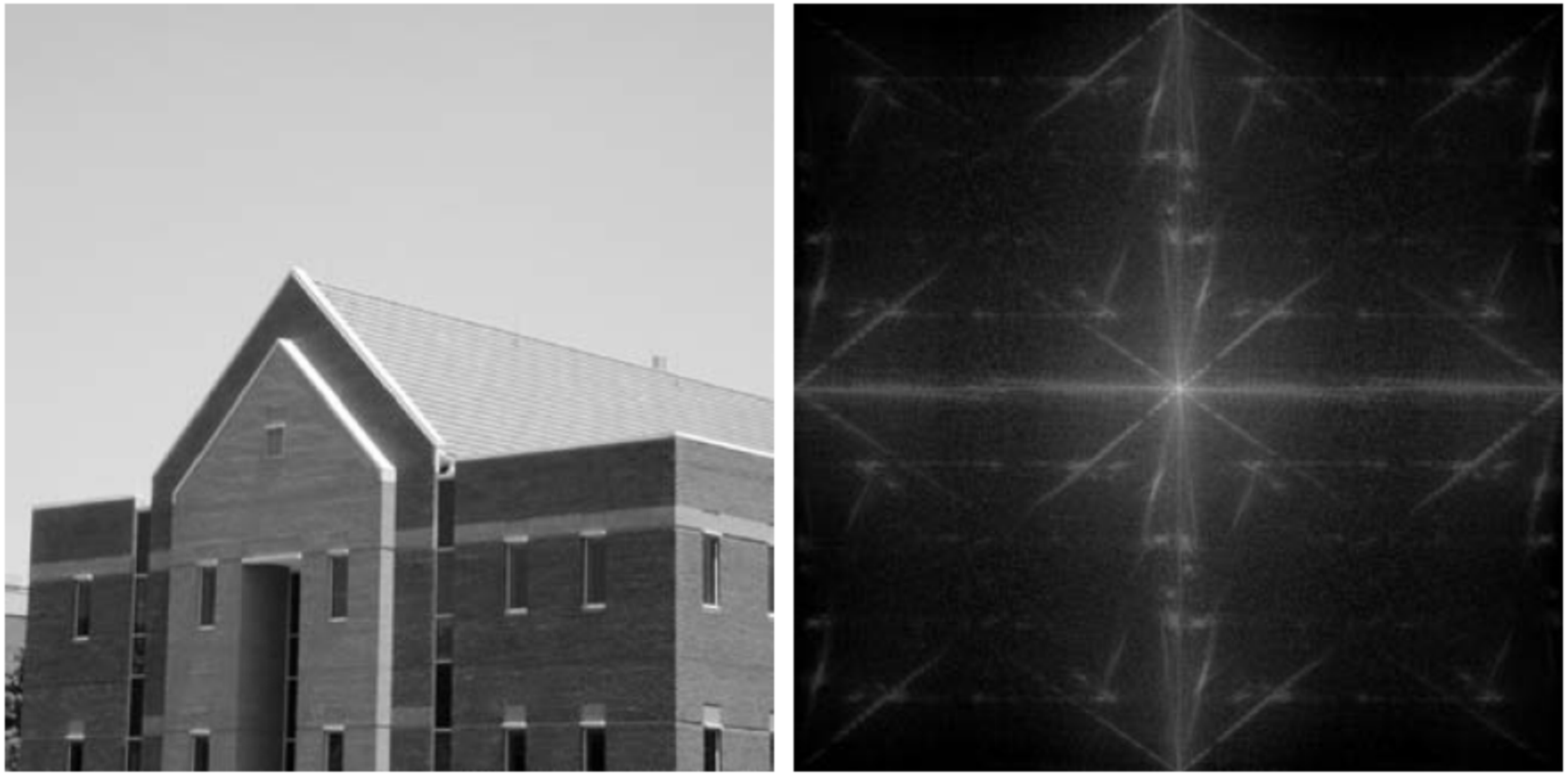
Commonly used filters (2)



a	b	c
d	e	f

FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

Commonly used filters (3)



a b

FIGURE 4.38
(a) Image of a building, and (b) its spectrum.

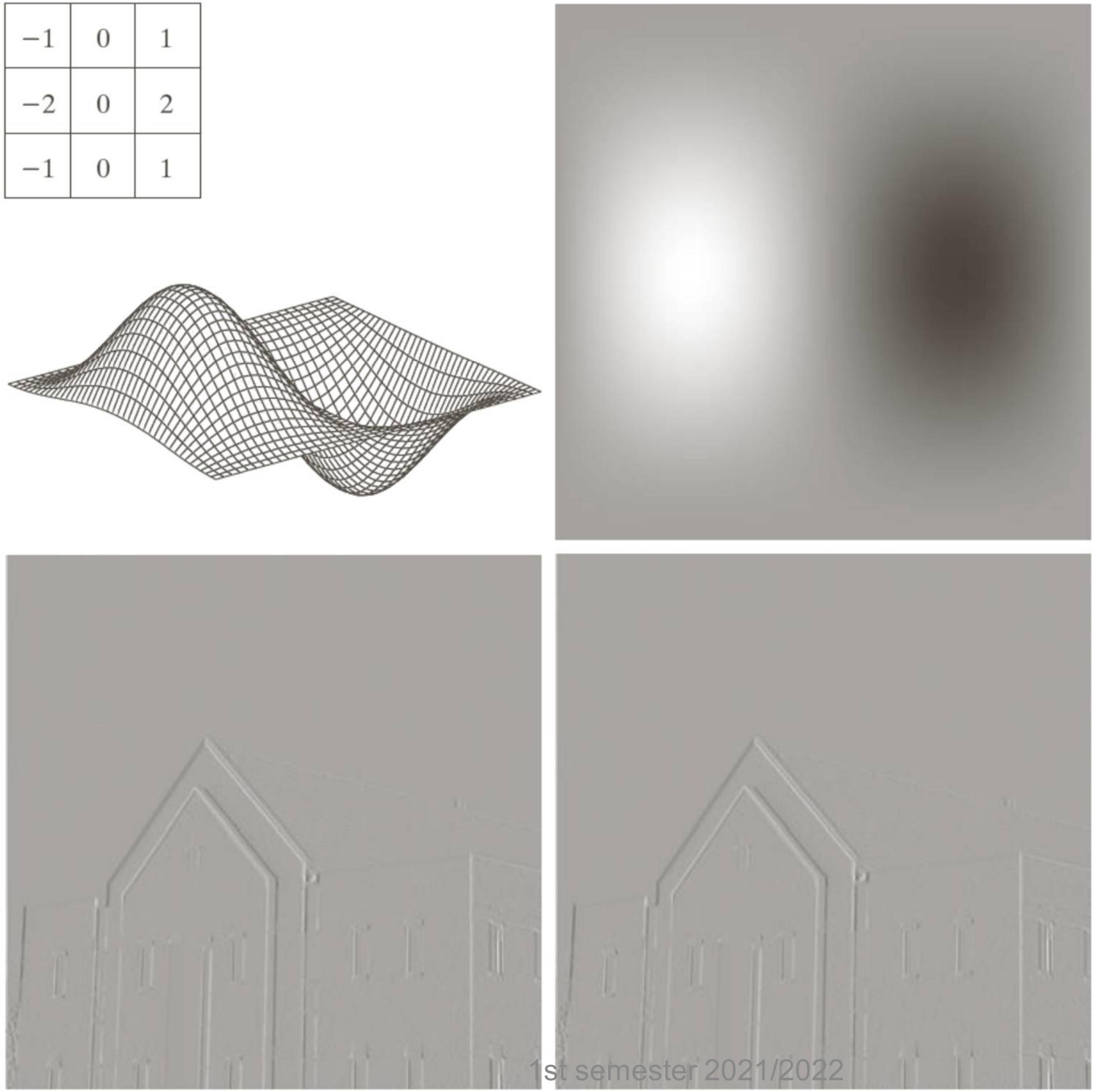
Commonly used filters (4)

Common types of Low-Pass Filters (LPF) – algorithms:

- Ideal filter
- Butterworth filter
- Gaussian filter

Commonly used filters (5)

−1	0	1
−2	0	2
−1	0	1



a b
c d

FIGURE 4.39 (a) A spatial mask and perspective plot of its corresponding frequency domain filter. (b) Filter shown as an image. (c) Result of filtering Fig. 4.38(a) in the frequency domain with the filter in (b). (d) Result of filtering the same image with the spatial filter in (a). The results are identical.

Commonly used filters – low-pass (6)

- Ideal Low-pass Filter (LPF)

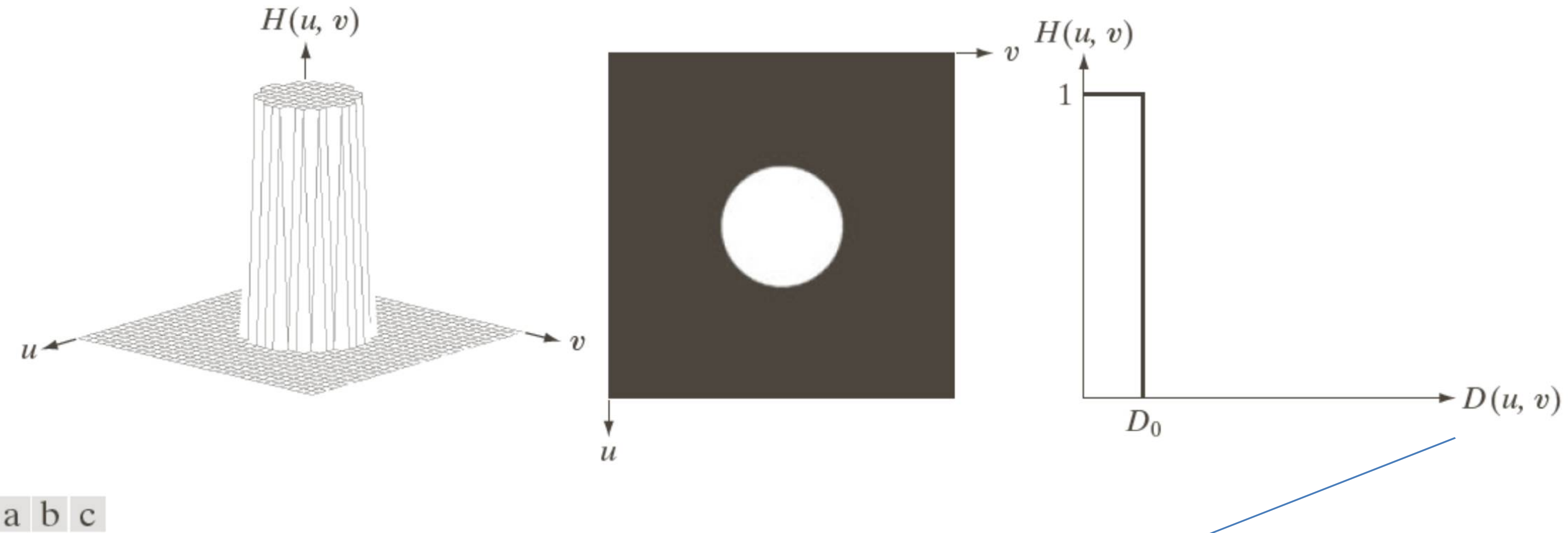
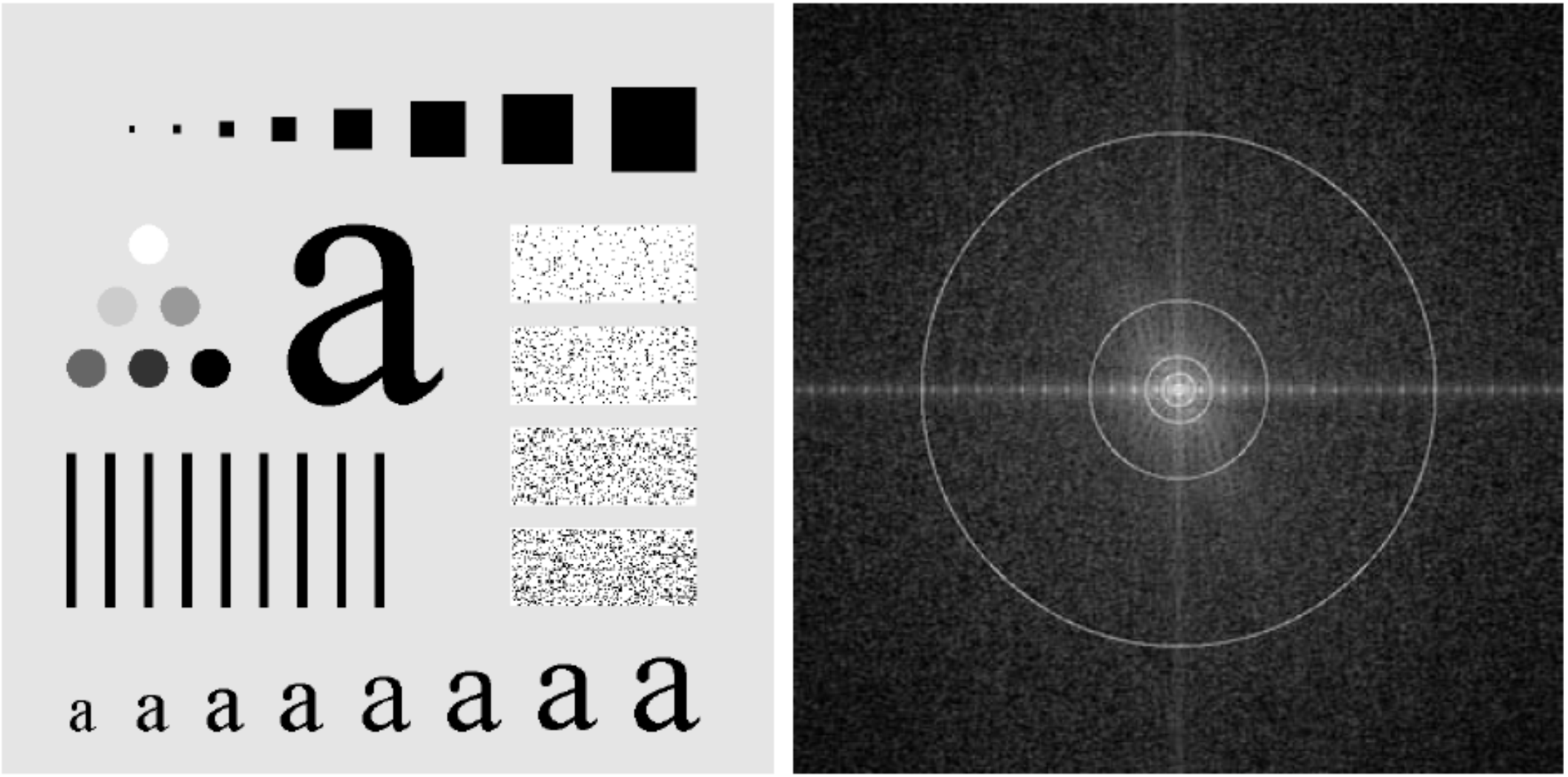


FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

$$D[u, v] = \sqrt{(u - P/2)^2 + (v - Q/2)^2},$$

Commonly used filters – low-pass (7)



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

Commonly used filters – low-pass (8)

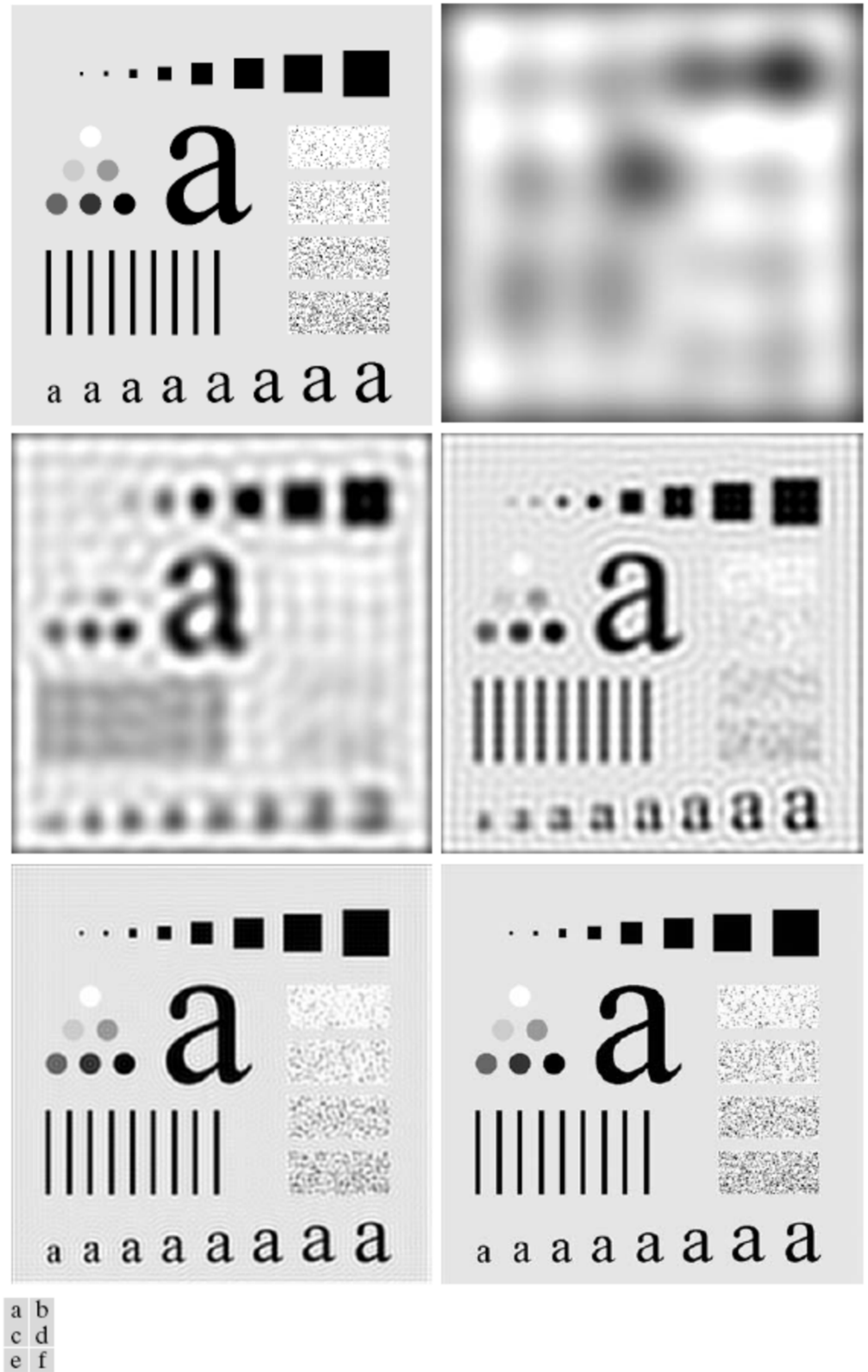


FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

Commonly used filters – low-pass (9)

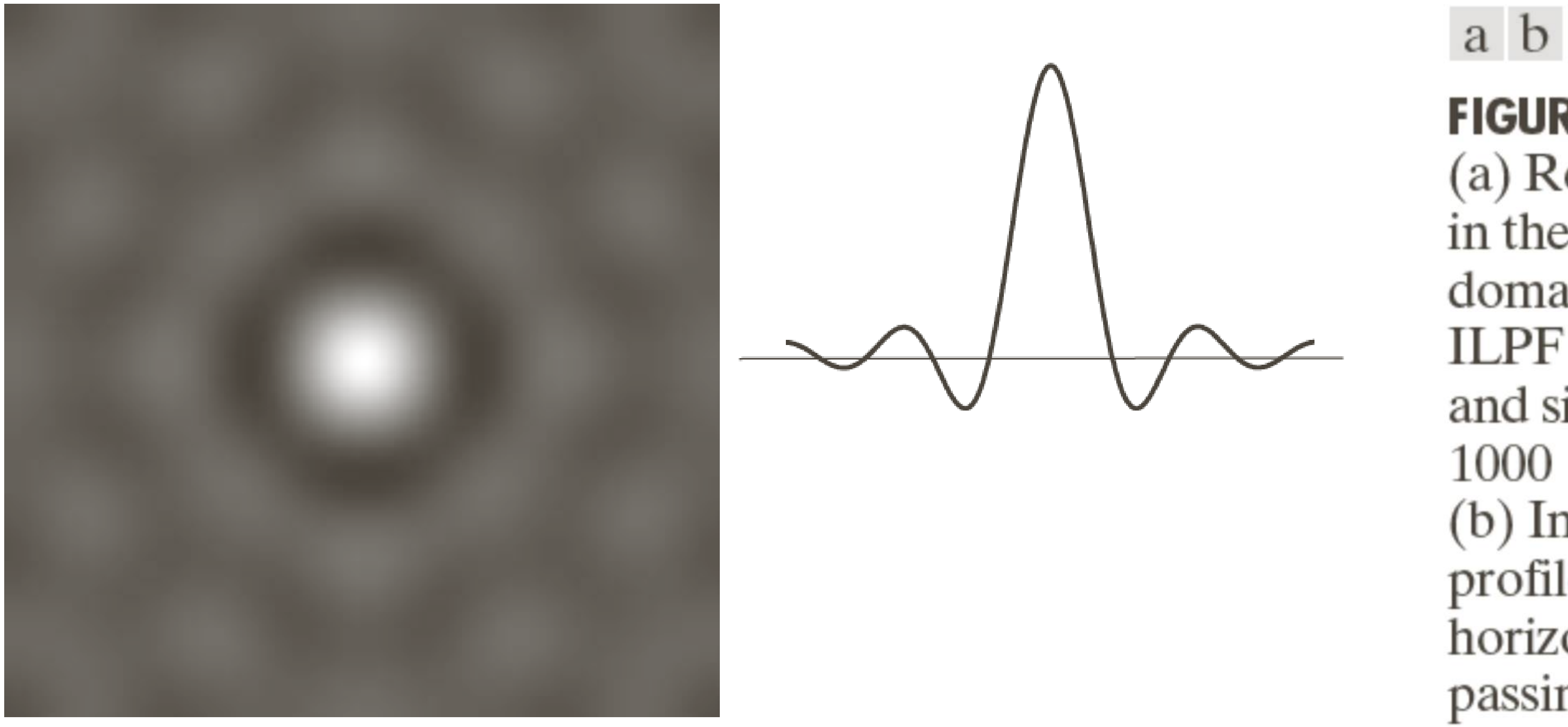


FIGURE 4.43
(a) Representation in the spatial domain of an ILPF of radius 5 and size 1000×1000.
(b) Intensity profile of a horizontal line passing through the center of the image.

Commonly used filters – low-pass (10)

- Butterworth low-pass filter (BLPF)
- ‘n’ is the filter order

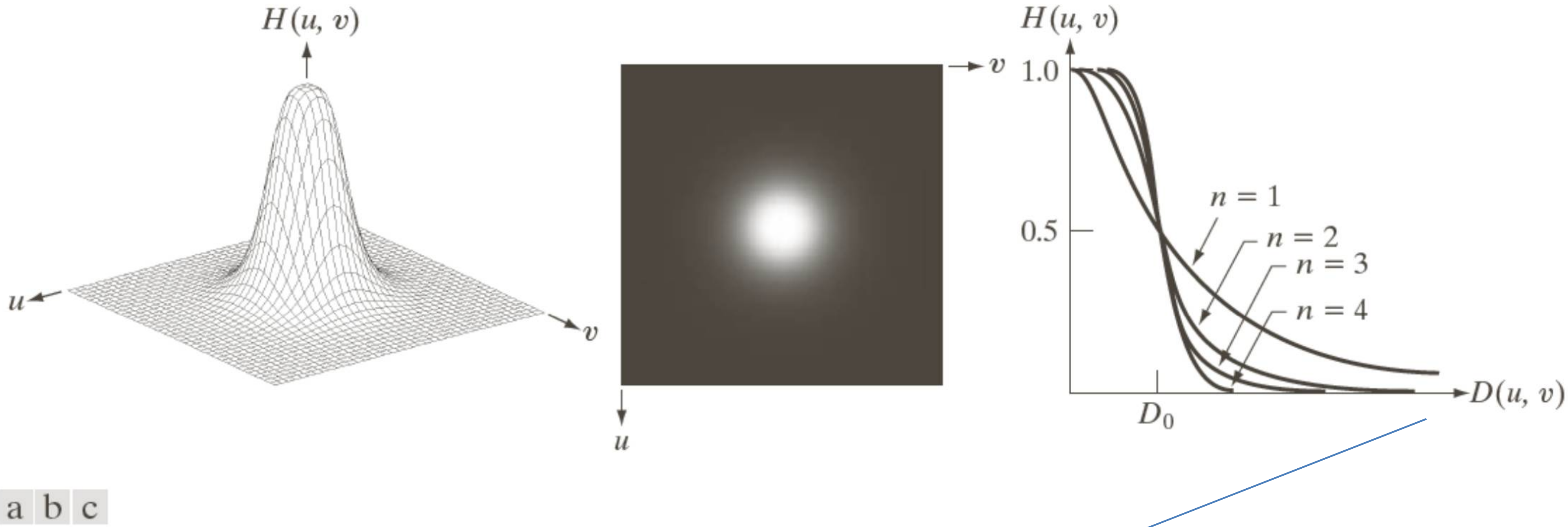


FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

$$D[u, v] = \sqrt{(u - P/2)^2 + (v - Q/2)^2},$$

Commonly used filters – low-pass (11)

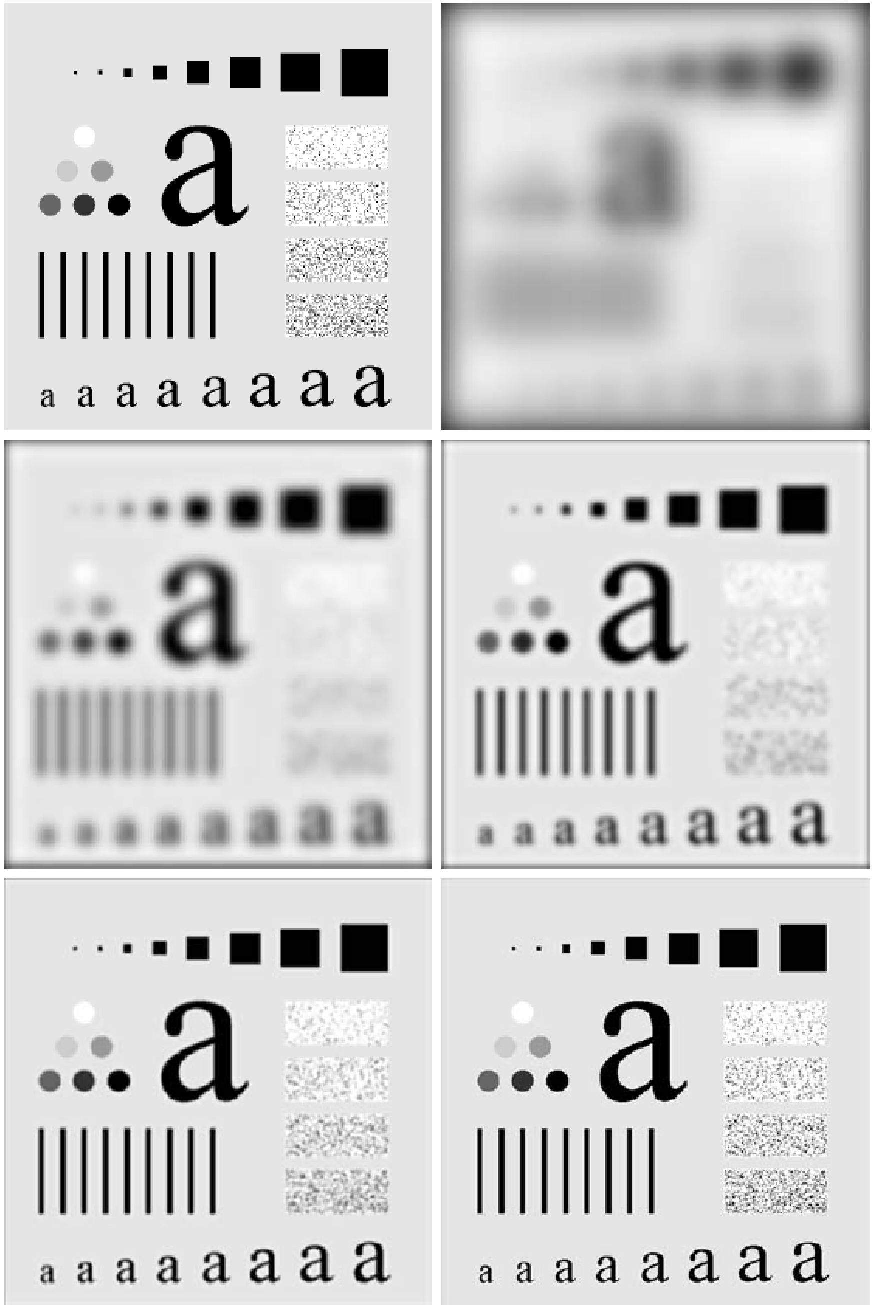
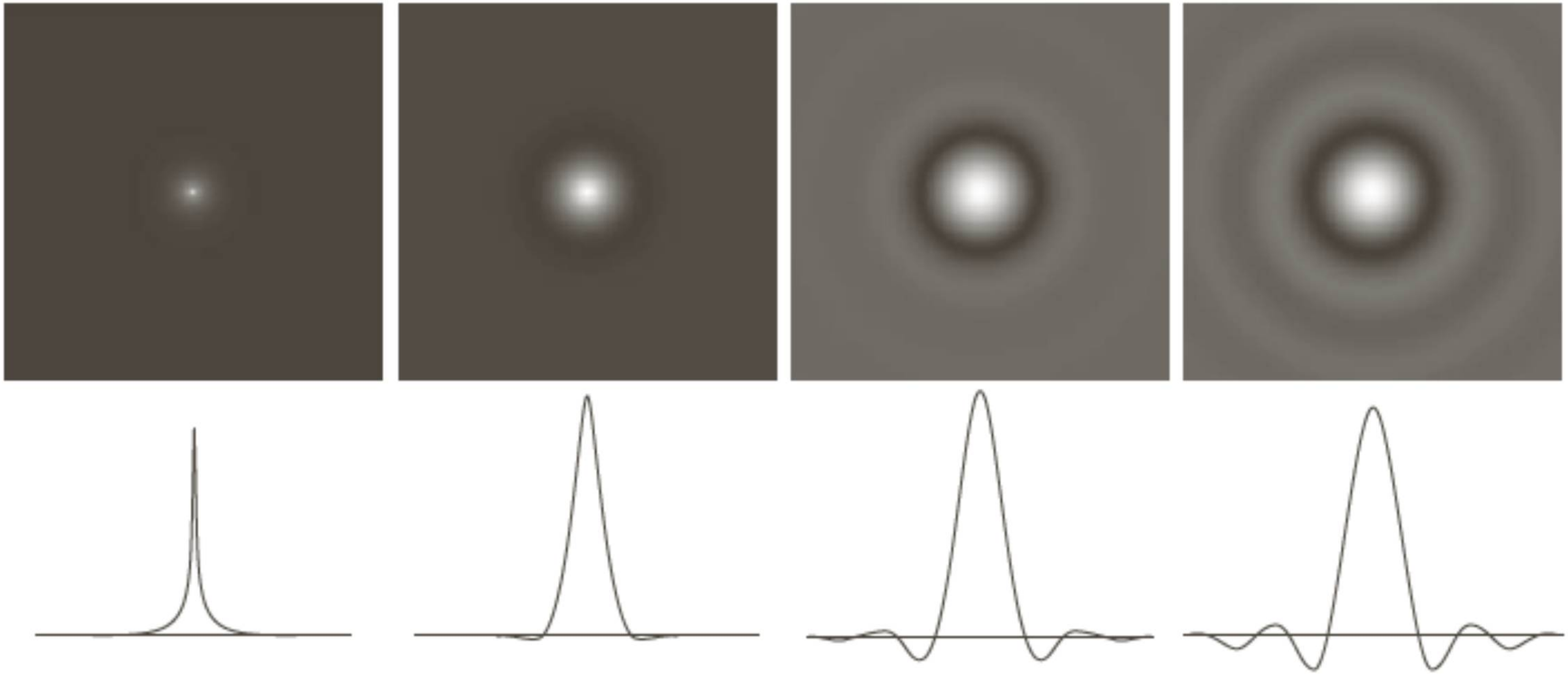


FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

Commonly used filters – low-pass (12)



a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

Commonly used filters – low-pass (13)

- Gaussian low-pass filter (GLPF)

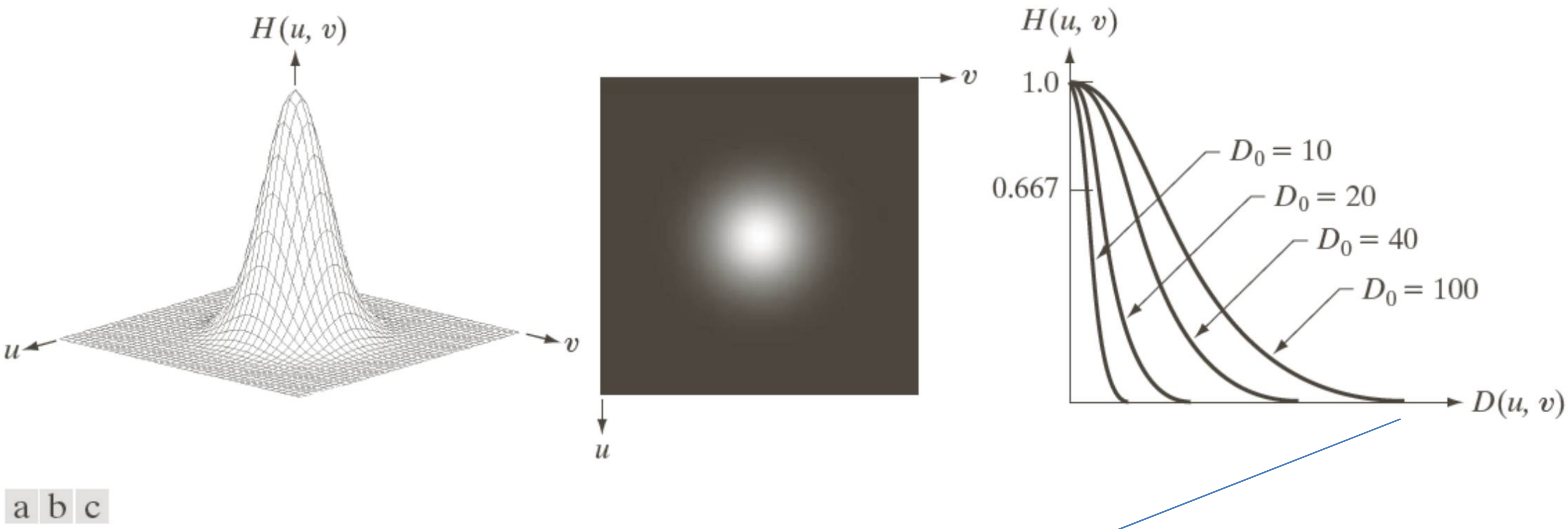
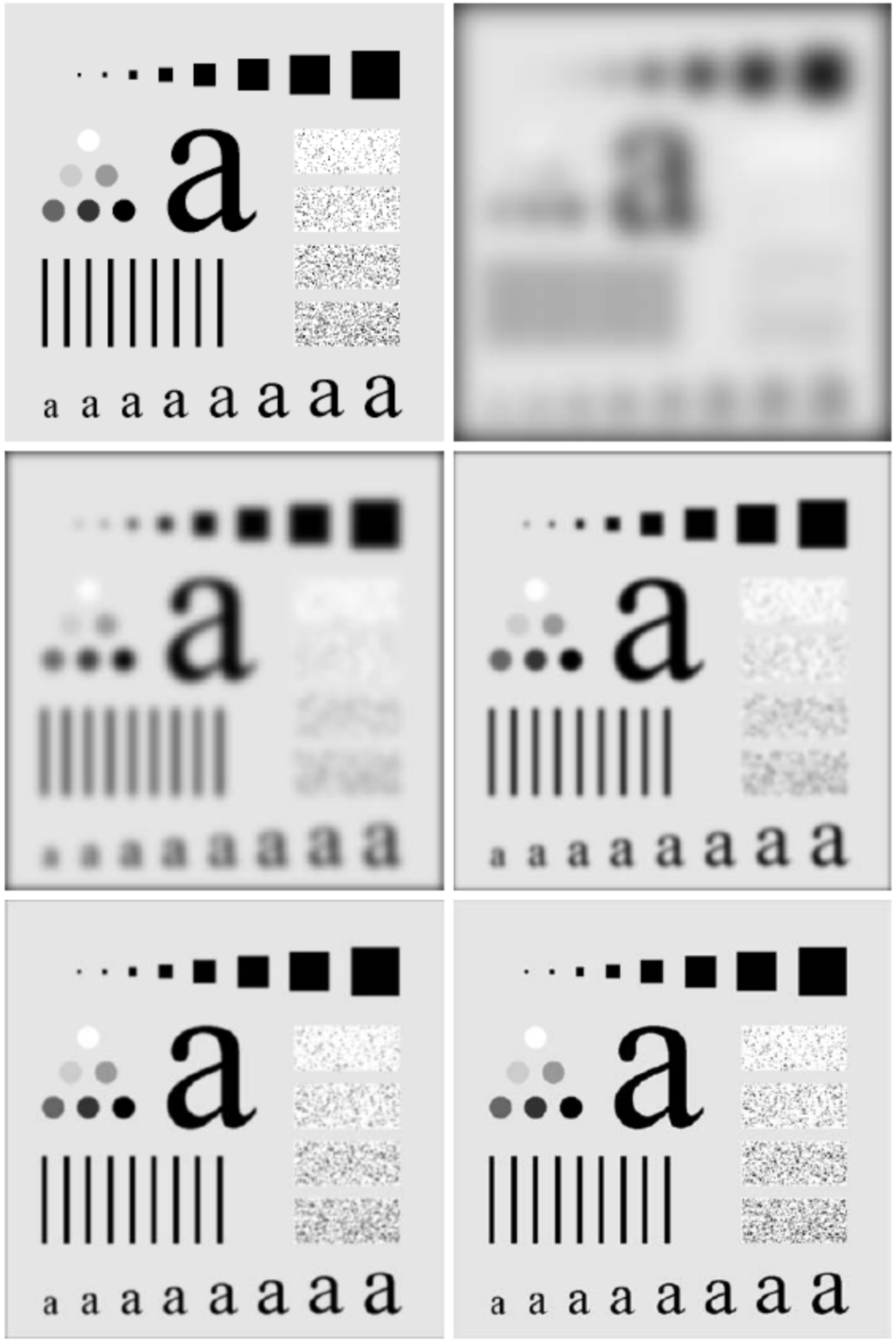


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

$$D[u, v] = \sqrt{(u - P/2)^2 + (v - Q/2)^2},$$

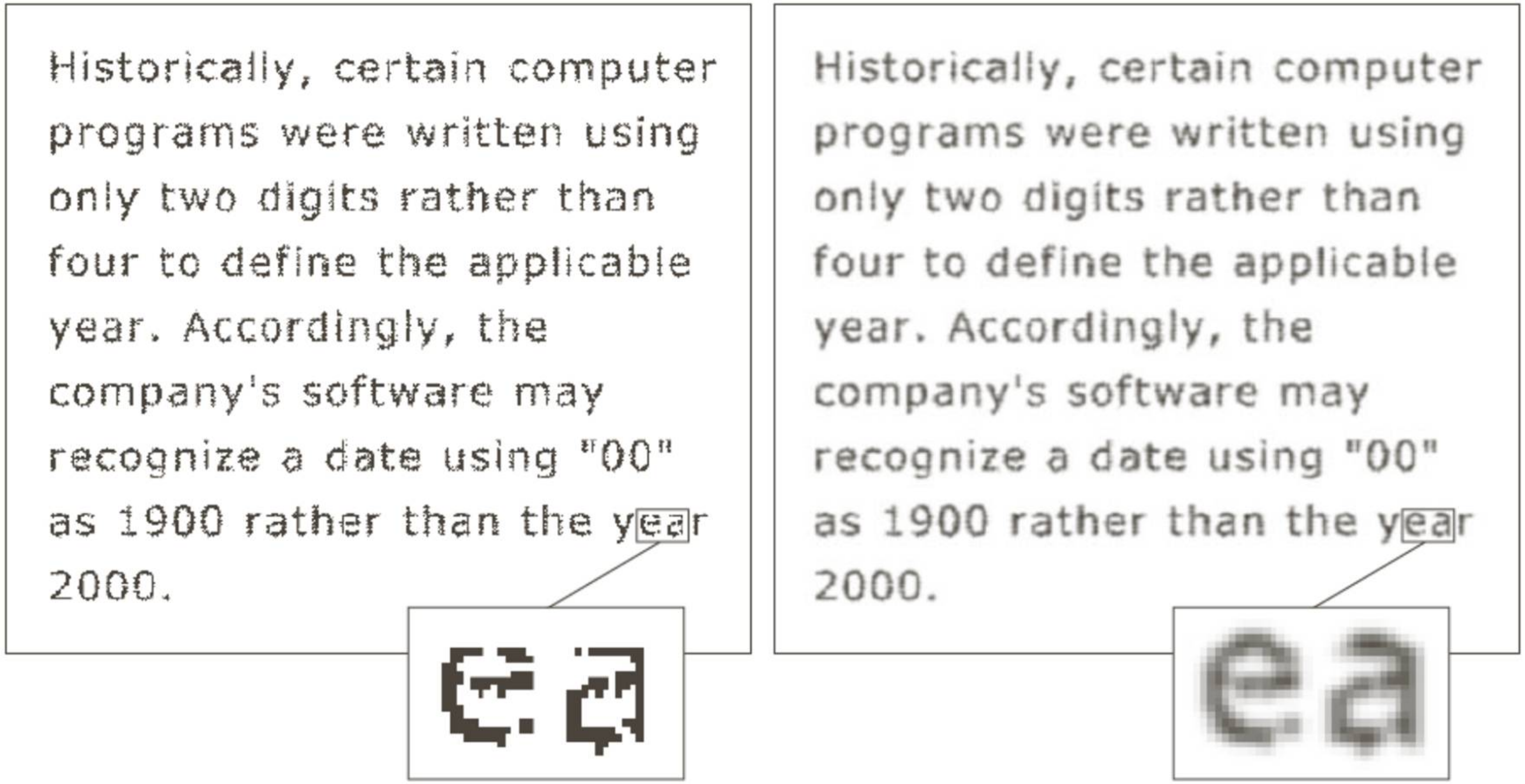
Commonly used filters – low-pass(14)



a b
c d
e f

FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Commonly used filters – low-pass (15)



a b

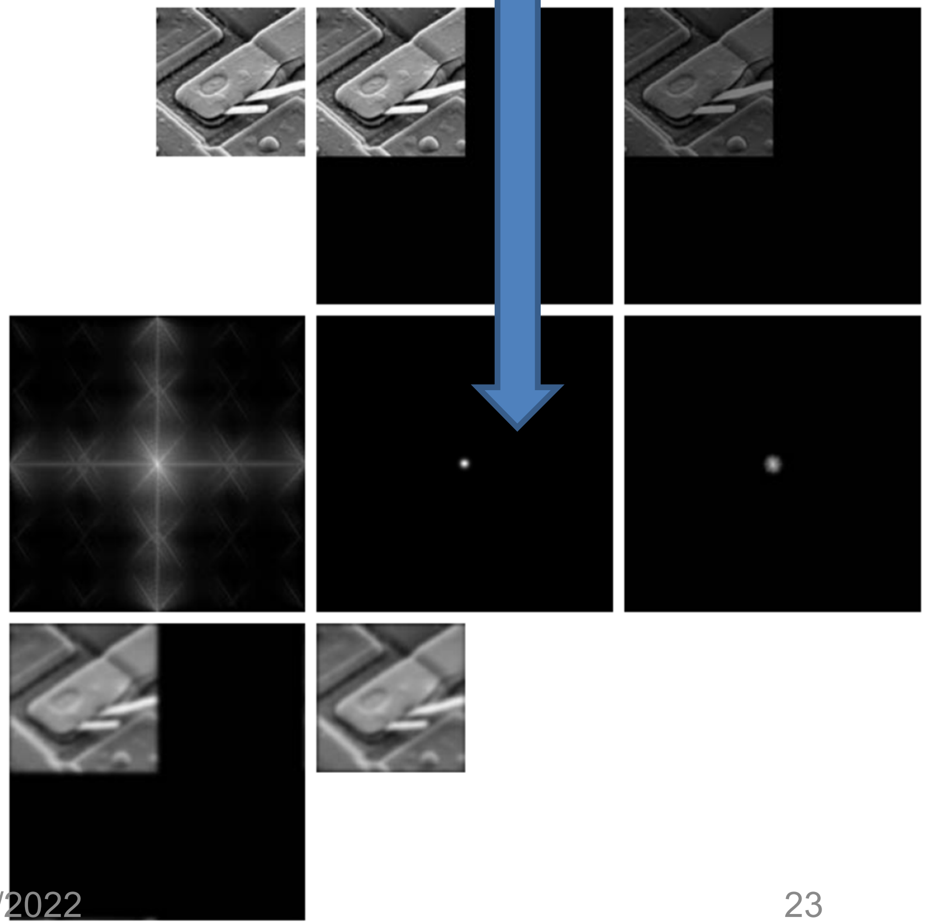
FIGURE 4.49 (a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Commonly used filters – low-pass (16)

TABLE 4.4
Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$

$$D[u,v] = \sqrt{(u - P/2)^2 + (v - Q/2)^2},$$

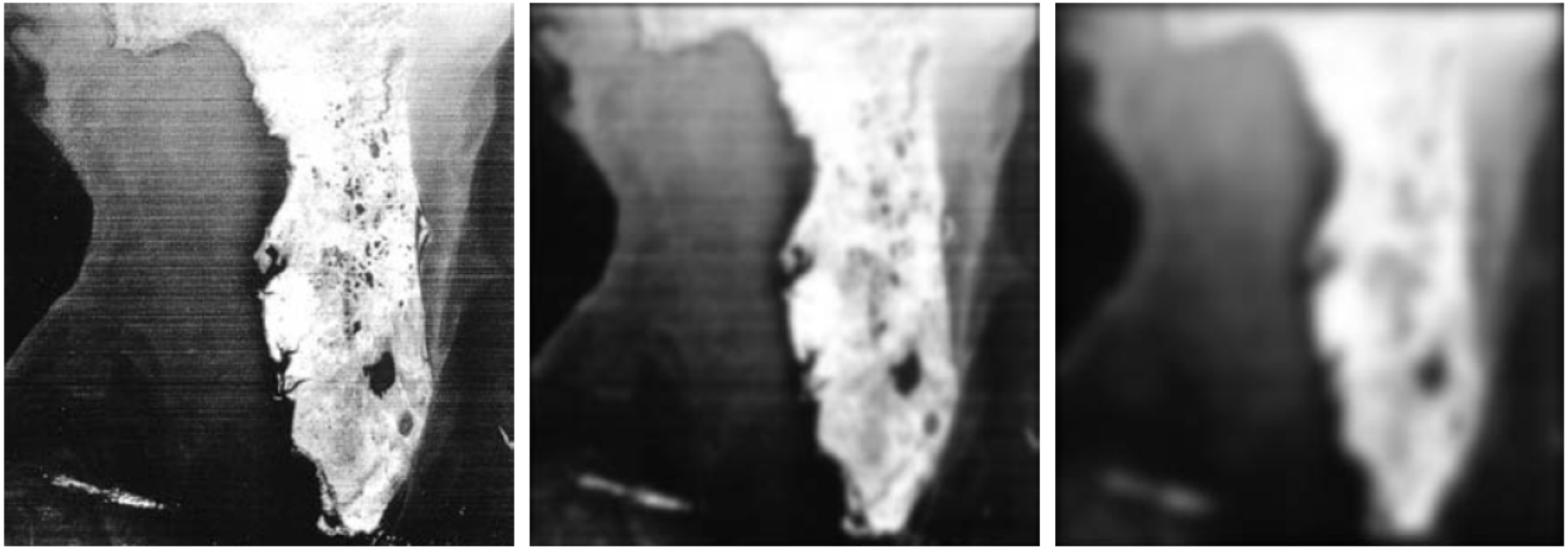


Commonly used filters – low-pass (17)

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Commonly used filters – low-pass (18)



a b c

FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

Commonly used filters – high-pass filter (HPF) (1)

Ideal

Butterworth

Gaussian

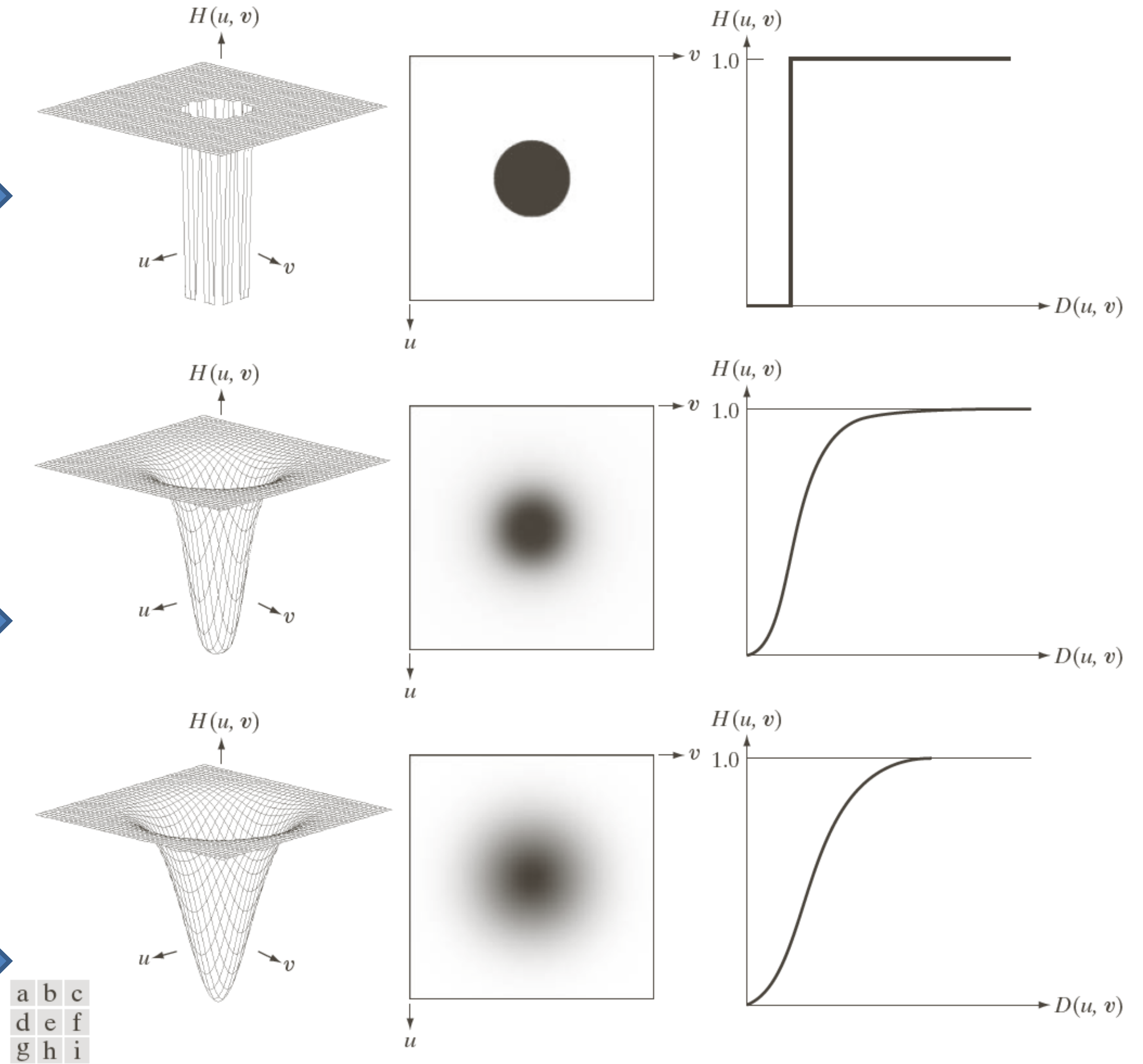


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Commonly used filters – high-pass (2)

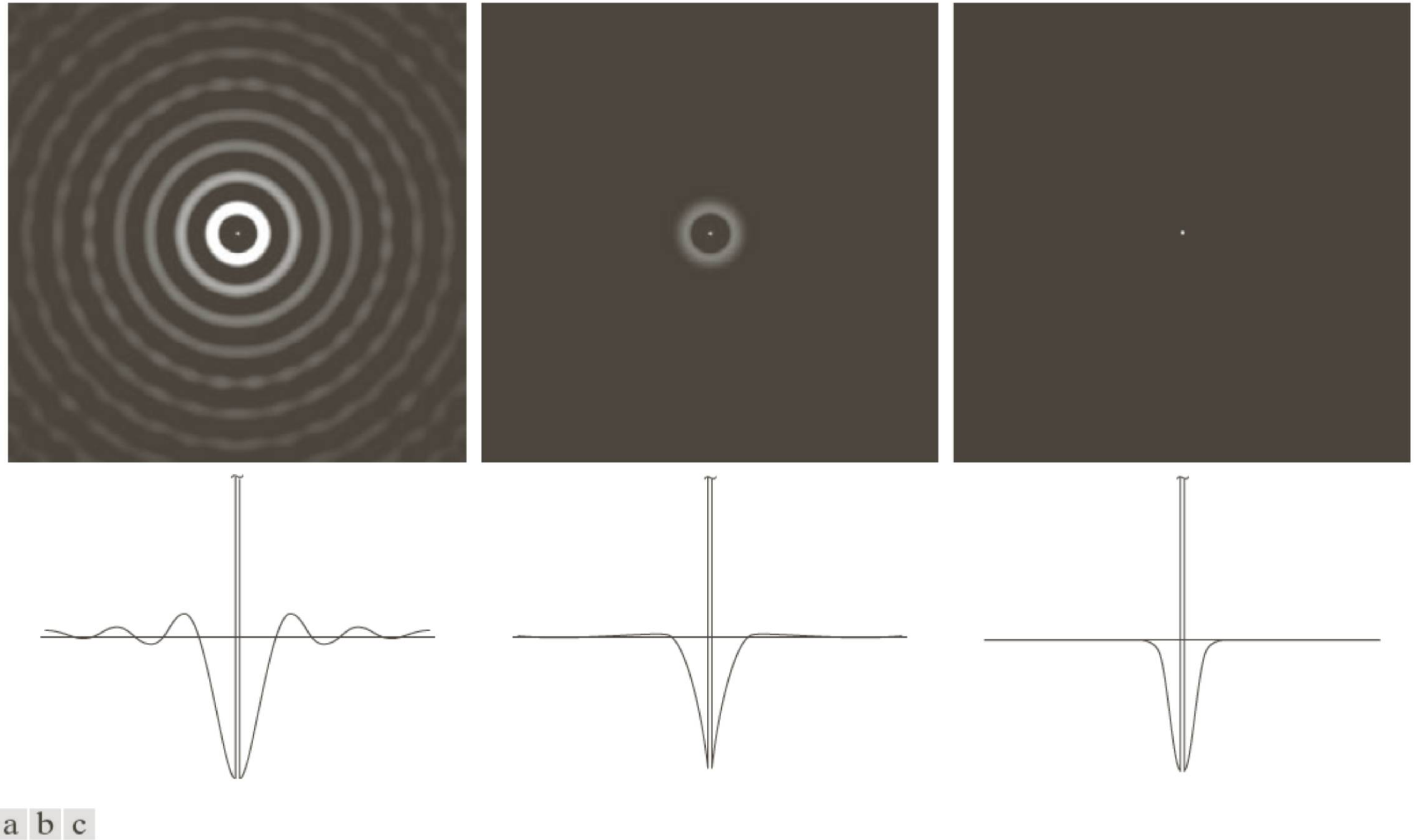


FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Commonly used filters – high-pass (3)

- Ideal High-pass Filter (IHPF)

a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60$, and 160.

Commonly used filters – high-pass (4)

- Butterworth High-pass Filter (BHPF)

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

Commonly used filters – high-pass (5)

- Gaussian High-pass Filter (GHPF)

a b c

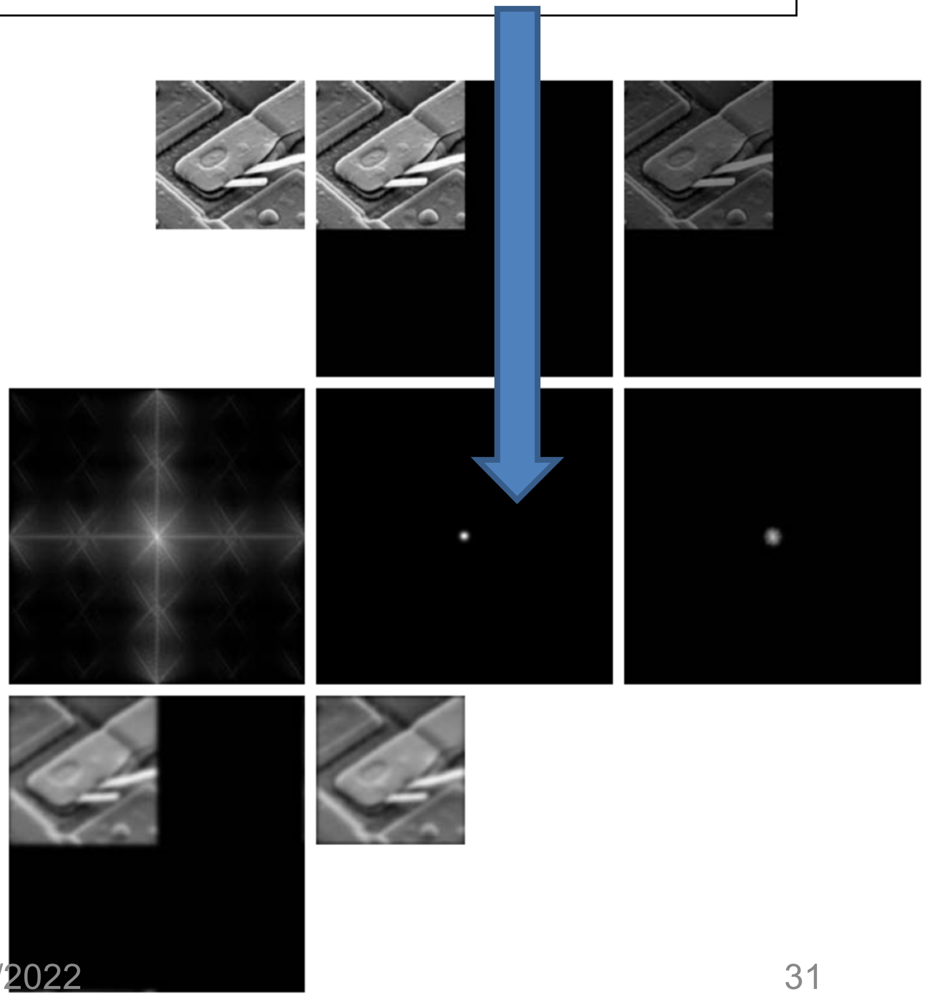
FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

Commonly used filters – high-pass (6)

TABLE 4.5
Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

$$D[u,v] = \sqrt{(u - P/2)^2 + (v - Q/2)^2},$$



Experimental results (1)

- Butterworth high-pass filter (BHPF), D_o=50, n=4
- Followed by thresholding

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Experimental results (2)

- Laplacian on the frequency domain, for sharpening
- $H[u,v] = 1 + 4\,\pi^2\,D^2[u,v]$

a b

FIGURE 4.58
(a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Experimental results (3)

1) Gaussian high-pass filter (GHPF), D_o=40
2) High-frequency emphasis
3) Histogram equalization

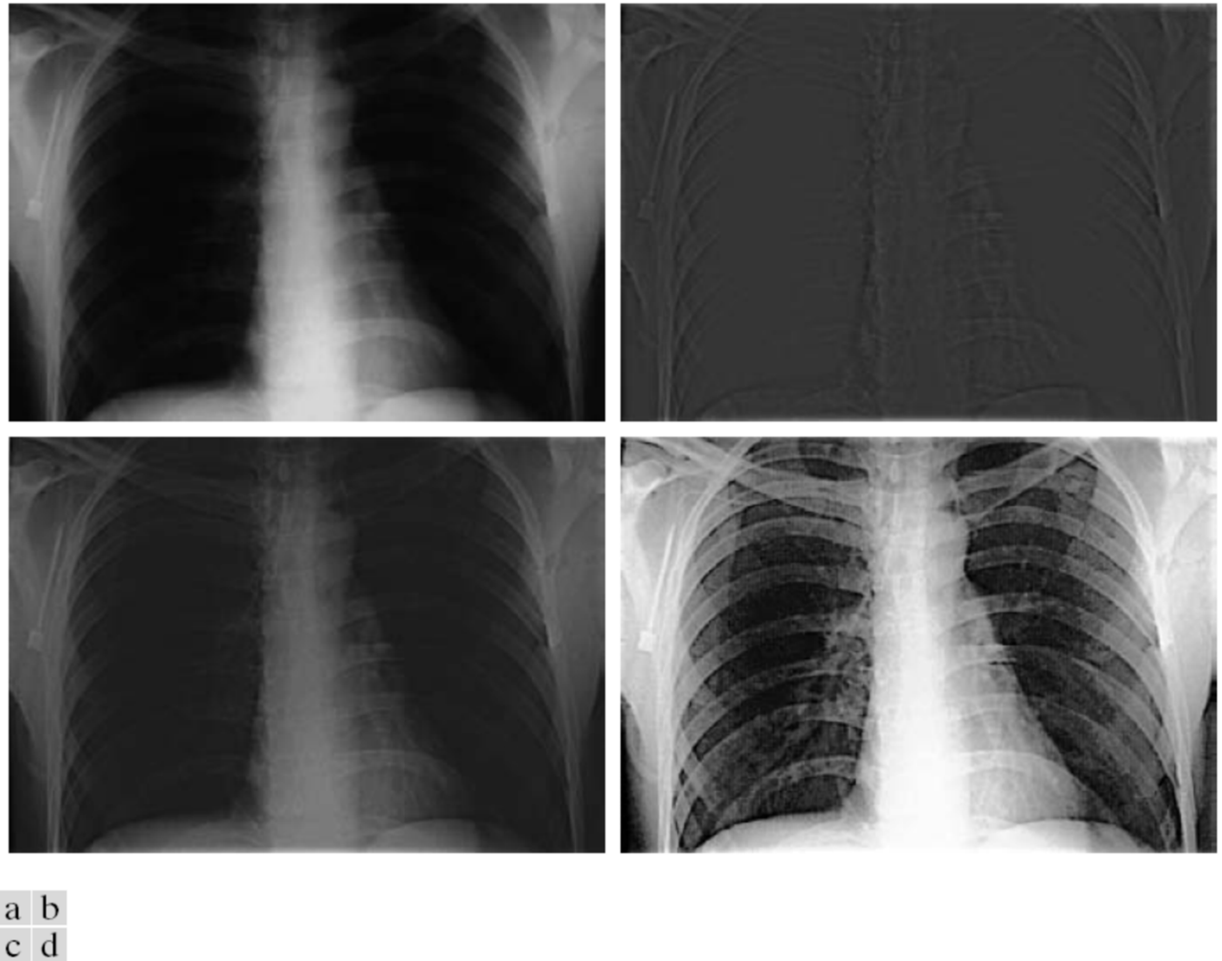


FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

High-frequency emphasis

- It sharpens an image

- Simple version
  1) $F[u,v] = DFT[\, f[m,n]\,]$
  2) $G[u,v] = (1 + k\, H_{HPF}[u,v]) \,.\, F[u,v]$
  3) $g[m,n] = IDFT[\, G[u,v]\,]$

  k is a suitable constant

- Improved version
  1) $F[u,v] = DFT[\, f[m,n]\,]$
  2) $G[u,v] = (k_1 + k_2\, H_{HPF}[u,v]) \,.\, F[u,v]$
  3) $g[m,n] = IDFT[\, G[u,v]\,]$

  k_1 and k_2 are suitable constants
  k_1, controls the offset from the origin
  k_2, defines the contribution of the mask

Unsharp masking (revisited)

- The unsharp masking is a sharpening procedure
- It enhances the edges of the image by adding a mask (of edge detail) on the original image

- The algorithm:

1. From the input image $f[m,n]$ compute its spectrum,
$F[u,v] = DFT[\ f[m,n]\ ]$

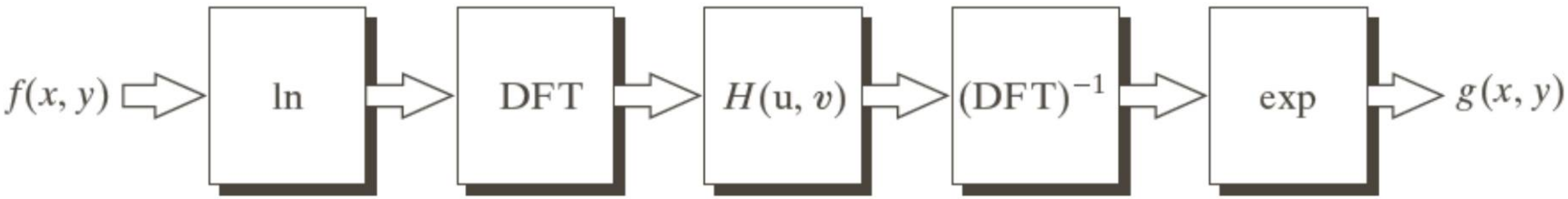
2. Compute its blurred/smoothed version,
$f_{LPF}[m,n] = IDFT[\ H_{LPF}[u,v]\ F[u,v]\ ]$

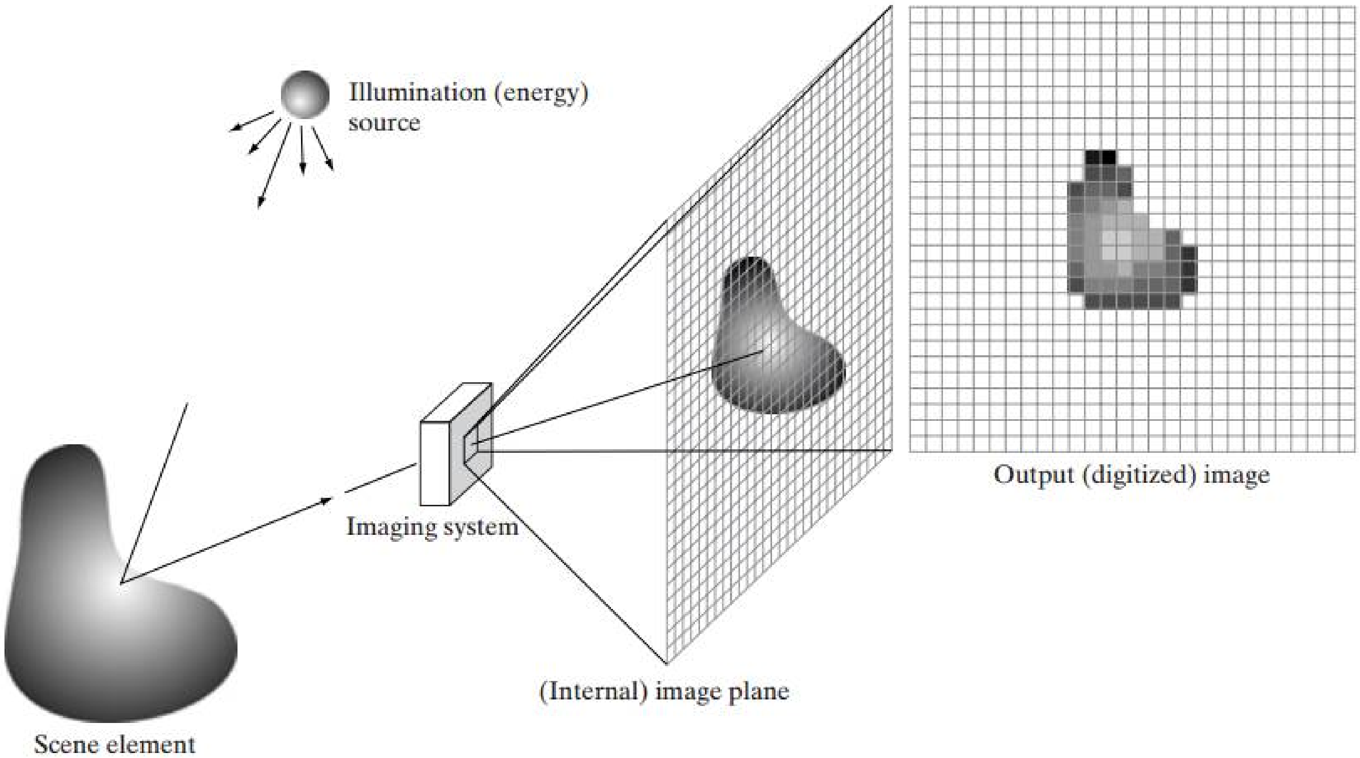
3. Compute the image mask $mask[m,n] = f[m,n] – f_{LPF}[m,n]$

4. Add a proportion 'k' of the mask to the original image yielding the final image $g[m,n] = f[m,n] + k\ mask[m,n]$

Homomorphic Filtering (1)

FIGURE 4.60 Summary of steps in homomorphic filtering.





a b c d e

FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Homomorphic Filtering (2)

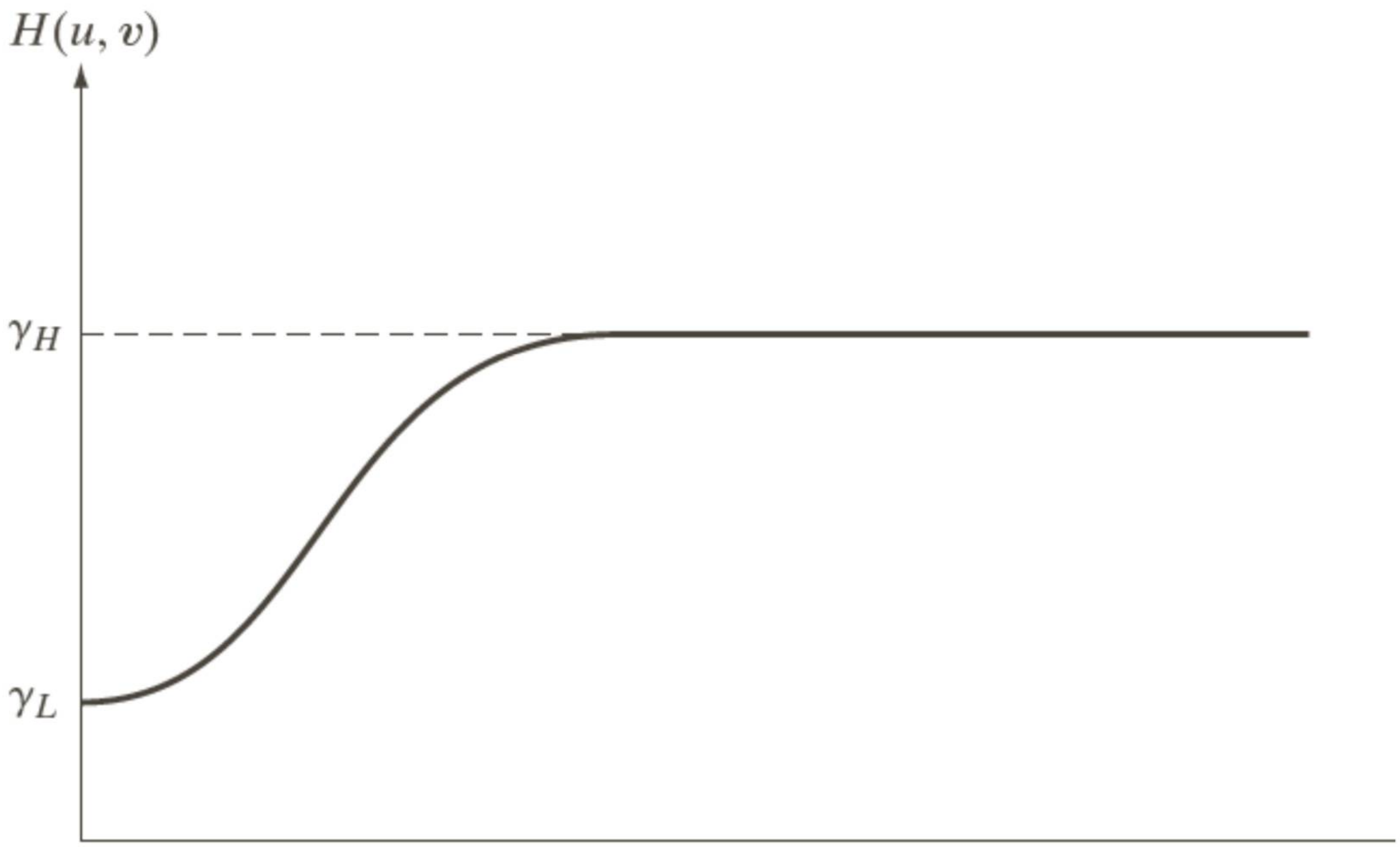
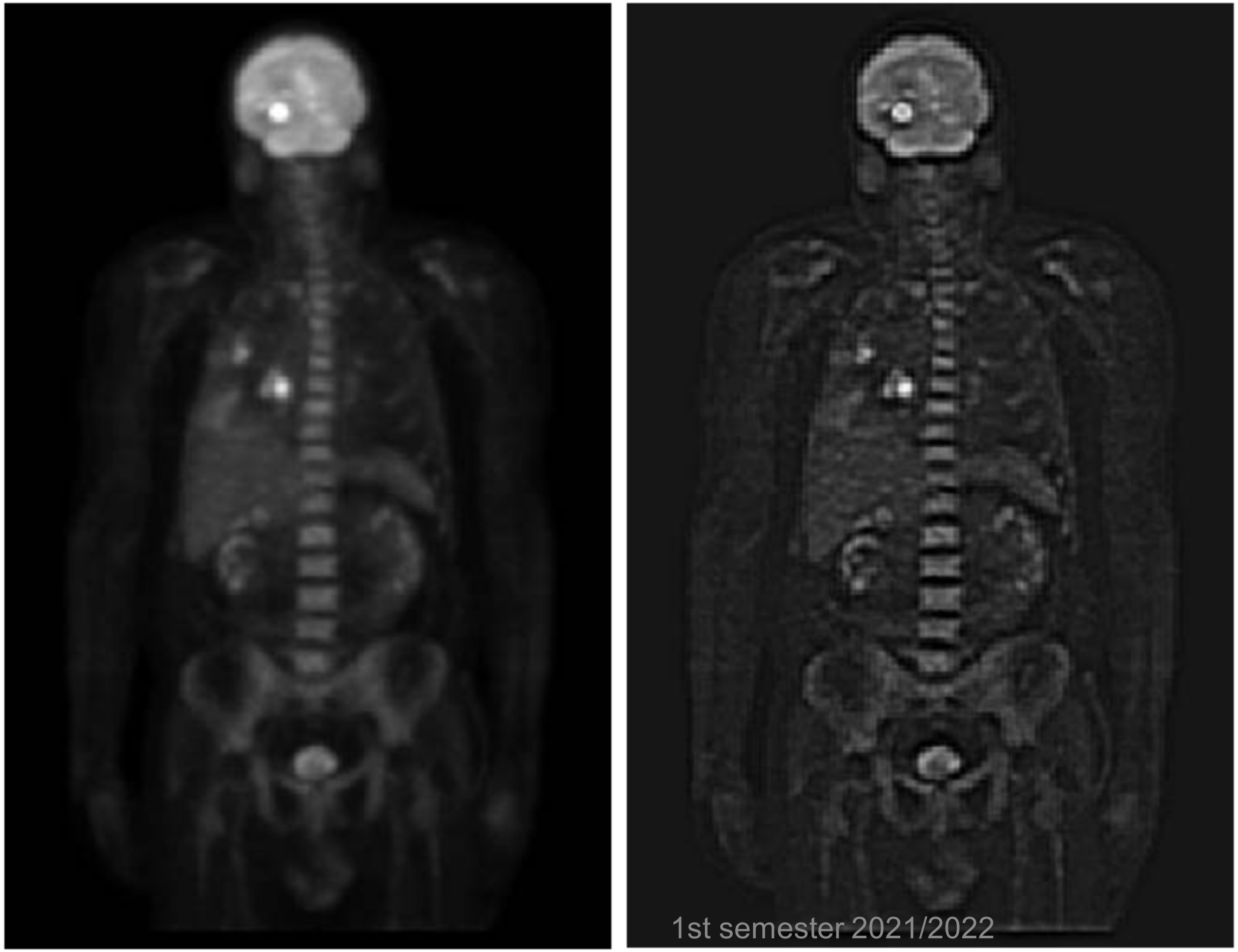


FIGURE 4.61 Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and $D(u, v)$ is the distance from the center.

Homomorphic Filtering (3)



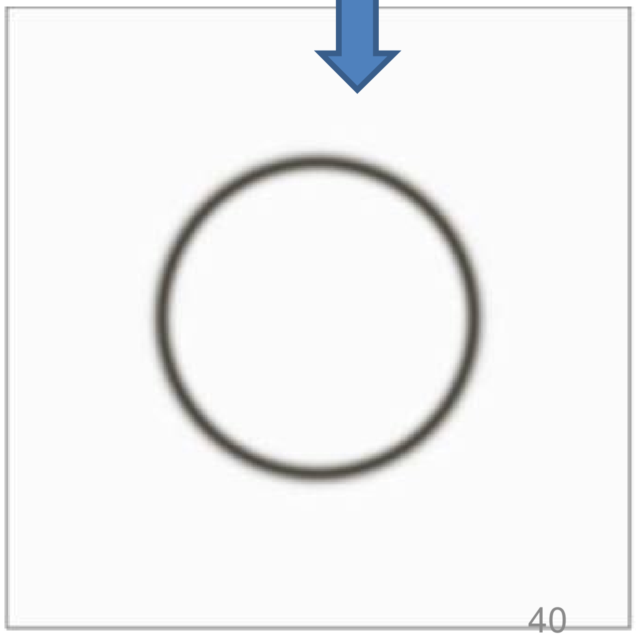
a b

FIGURE 4.62
(a) Full body PET scan. (b) Image enhanced using homomorphic filtering. (Original image courtesy of Dr. Michael E. Casey, CTI PET Systems.)

Band-Reject Filters (1)

TABLE 4.6
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$



Band-Reject Filters (2)

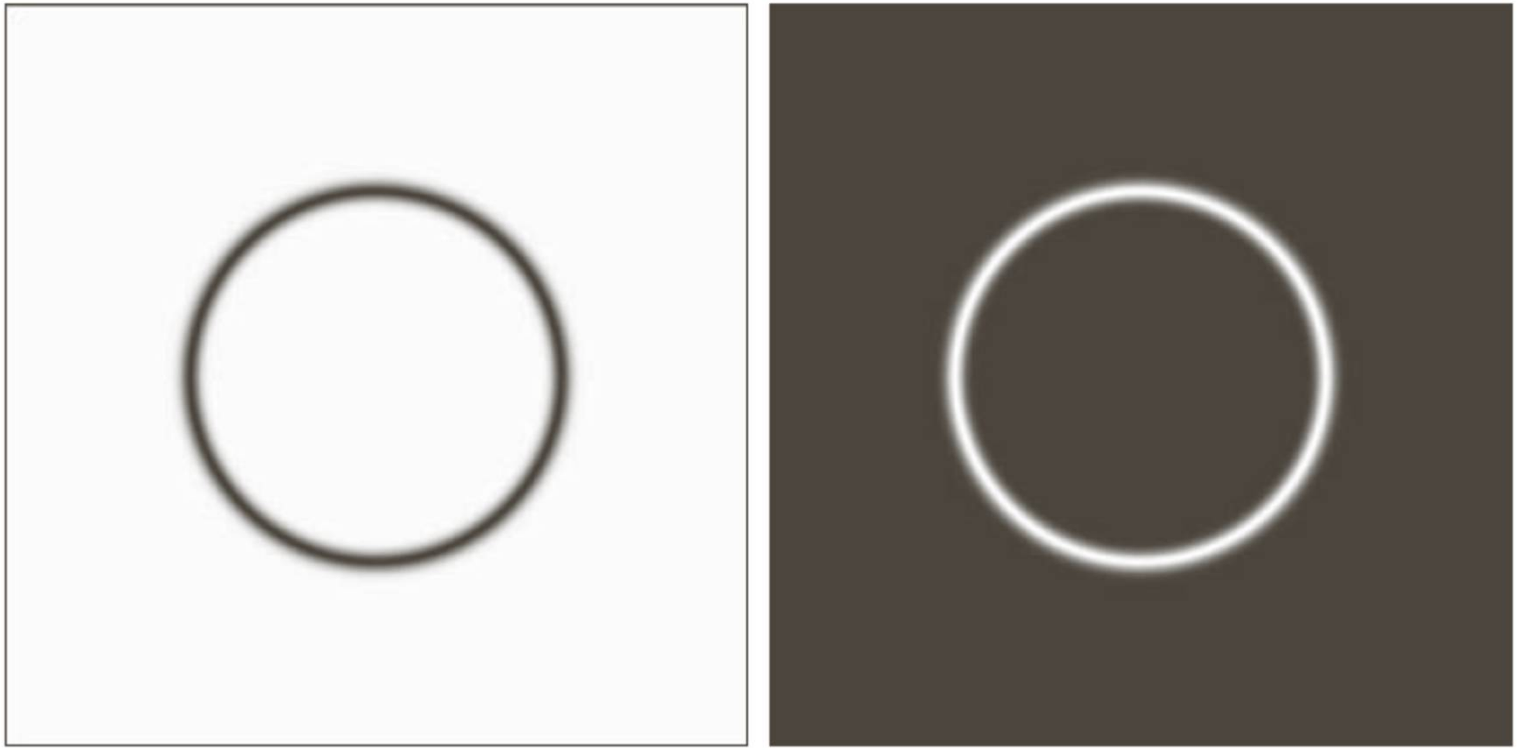


FIGURE 4.63 (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

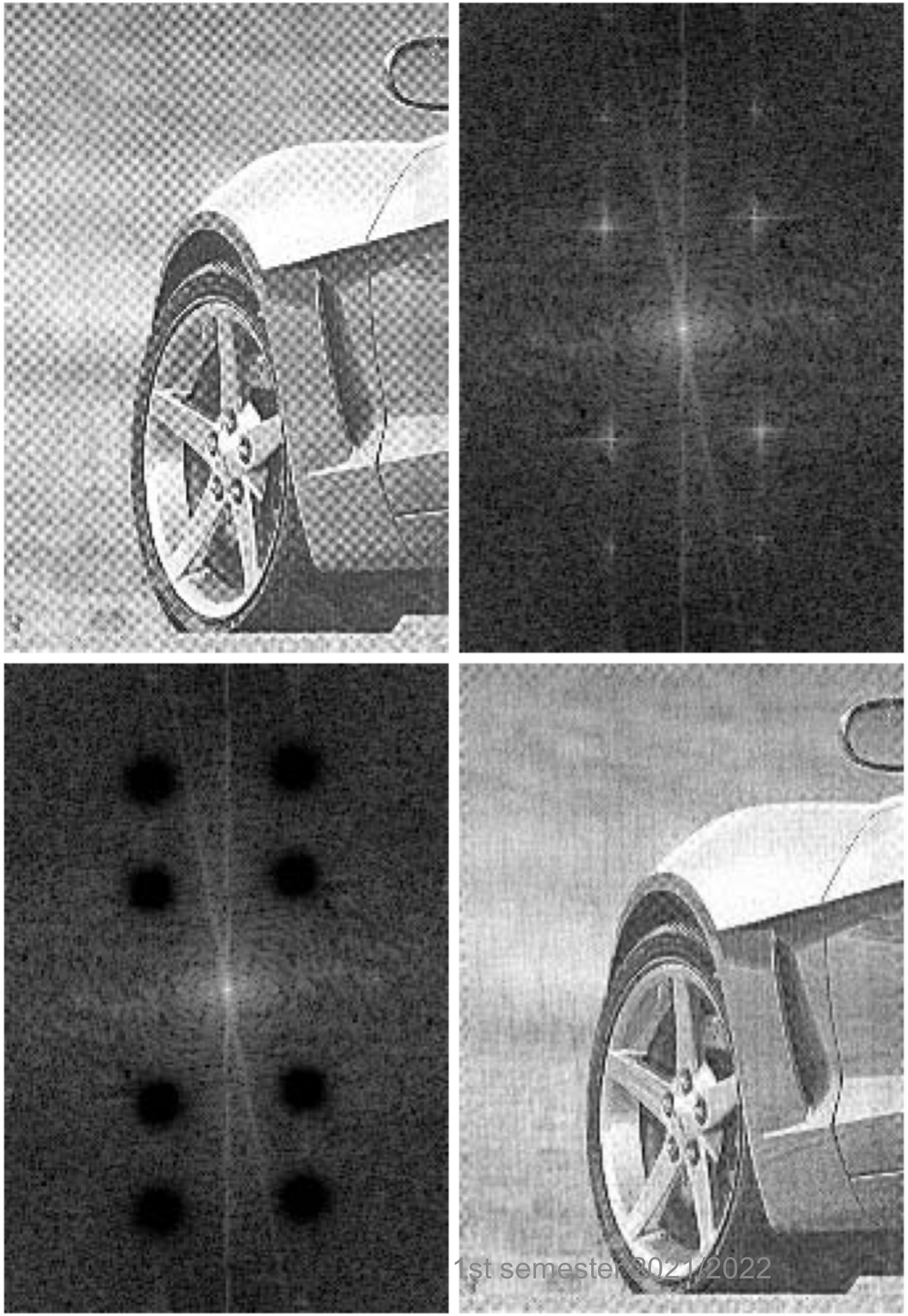
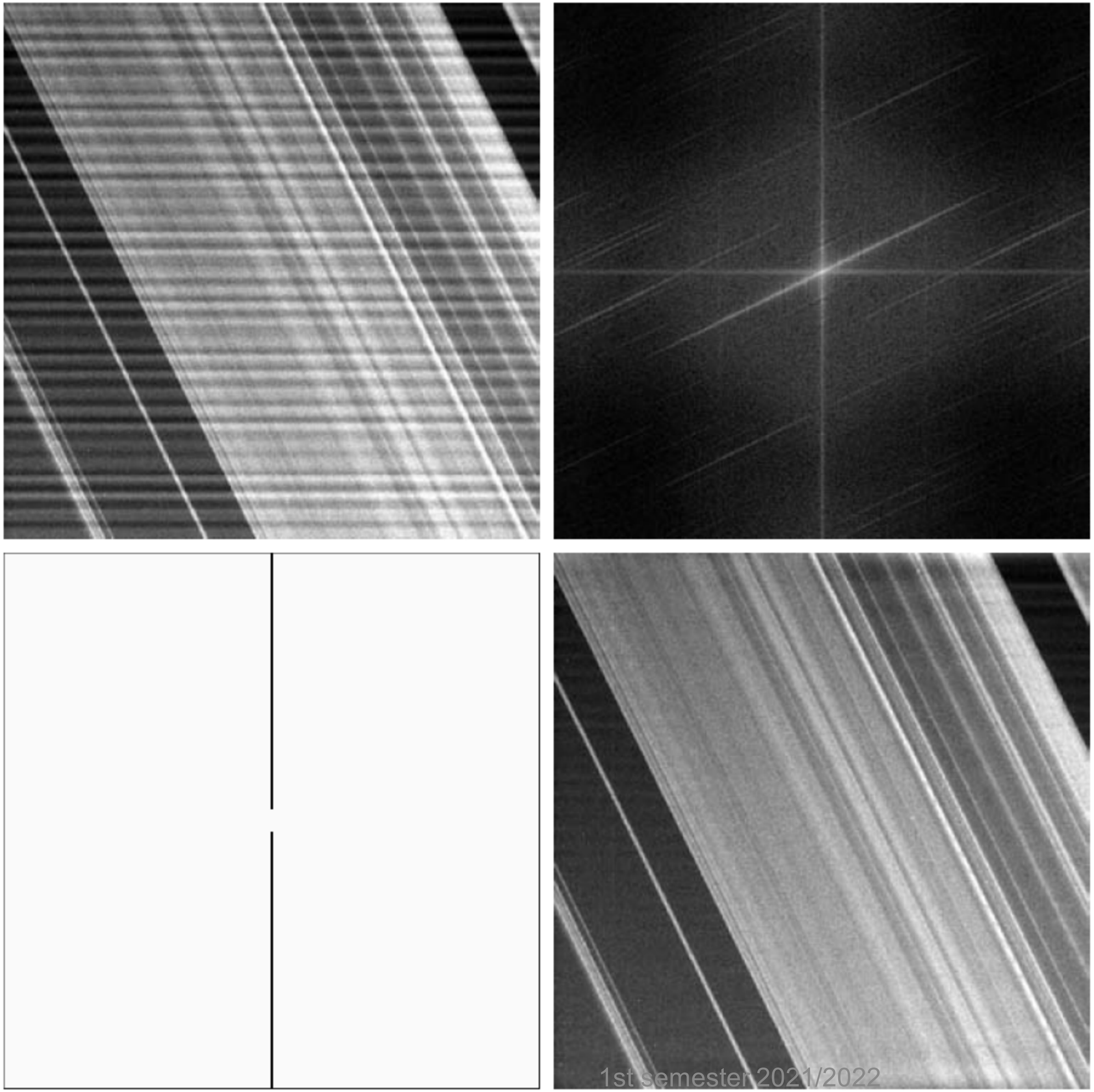
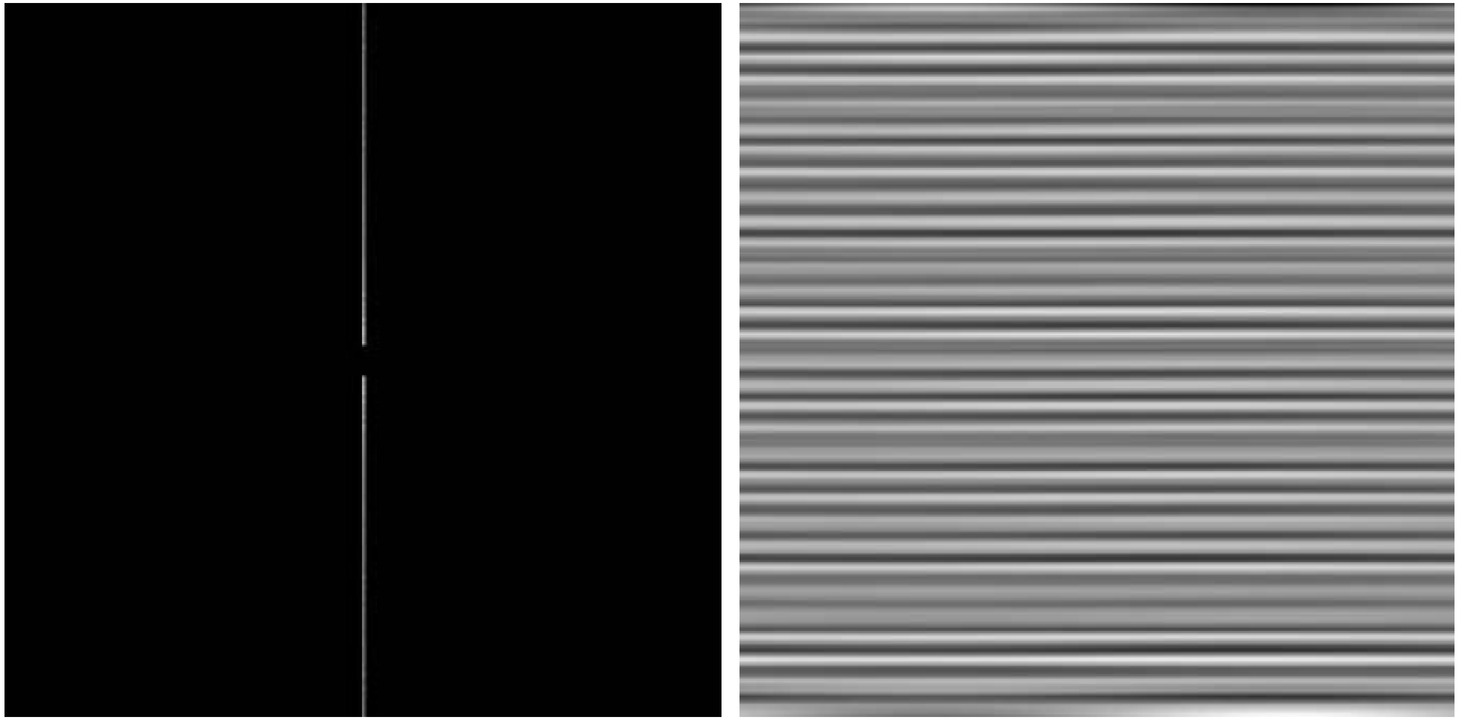


FIGURE 4.64
(a) Sampled newspaper image showing a moiré pattern.
(b) Spectrum.
(c) Butterworth notch reject filter multiplied by the Fourier transform.
(d) Filtered image.



a	b
c	d

FIGURE 4.65 (a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)



a b

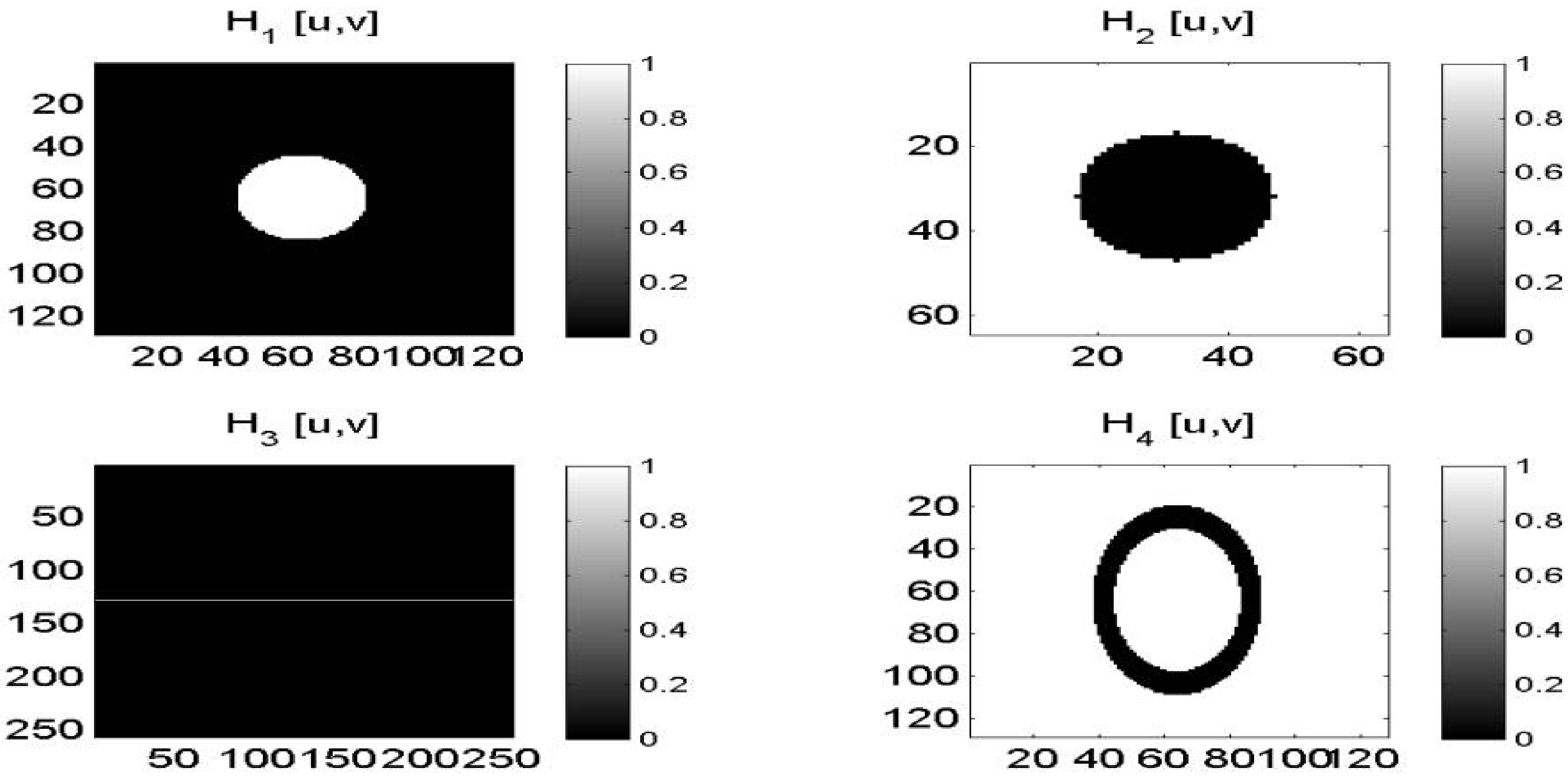
FIGURE 4.66 (a) Result (spectrum) of applying a notch pass filter to the DFT of Fig. 4.65(a). (b) Spatial pattern obtained by computing the IDFT of (a).

Exercises (1)

2. Considere o algoritmo de filtragem de imagem, no domínio da frequência.

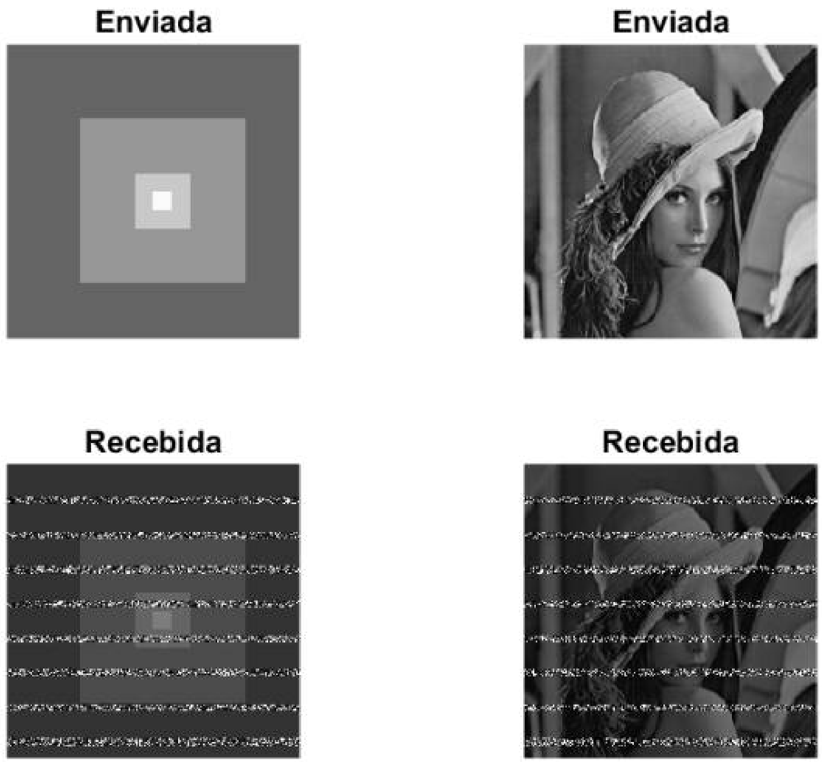
   a) {1,5} Explique, de forma sucinta, quais as ações realizadas pelo algoritmo de forma a realizar a filtragem.

   b) {1,5} Considere os quatro filtros definidos no domínio da frequência, na forma de espetro centrado, tal como se apresenta na figura. Cada filtro, $H_i[u, v]$, é apresentado como imagem cujas dimensões são uma potência inteira de 2. Para cada filtro, indique: a resolução de imagem original a que se destina; o tipo de filtragem realizado.



Exercises (2)

4. Considere um sistema de transmissão de imagem, no qual se verificou que a imagem recebida aparece sempre contaminada com o mesmo tipo de efeitos indesejados. Na figura abaixo, apresentam-se dois exemplos de transmissão de imagem, neste sistema.



a) {2,0} Caraterize os efeitos indesejados que o sistema introduz sobre as imagens. Proponha uma técnica baseada em transformações de intensidade e/ou filtragem espacial, para eliminar/minimizar estes efeitos indesejados.

b) {1,5} Suponha agora que se pretende resolver o problema da alínea anterior, recorrendo a técnicas de filtragem no domínio da frequência. Indique os procedimentos que faria para estabelecer, implementar e avaliar uma técnica adequada para este efeito.

Exercises (3)

7. {R2||TG} Considere o processamento de imagens no domínio da frequência.

   a) {1,5||1,0} Seja o seguinte algoritmo

```
Entrada:  Imagem f[m,n]; Filtro passa-alto H[u,v], especificado na frequência.
Constantes 'a' e 'b'.
Saída:    Imagem g[m,n].
----------------------------------------------
1. F[u,v] = DFT[ f[m,n] ].
2. G[u,v] = (a + bH[u,v]) . F[u,v].
3. g[m,n] = IDFT[ G[u,v] ).
4  Retornar g[m,n].
----------------------------------------------
```

   Identifique as ações efetuadas pelo algoritmo nas seguintes situações: i) a=1; b=0; ii) a=0; b=1; iii) a=0,5; b=0,5.

Exercises (4)

c) $\{1{,}5 || 1{,}0\}$ Na filtragem na frequência, após a ação de *zero padding* obtiveram-se as dimensões da imagem *padded*, designadas por P e Q. Tendo em conta que $D[u,v] = \sqrt{(u-P/2)^2 + (v-Q/2)^2}$, definem-se os filtros:

$$H_A[u,v] = \begin{cases} 1, & \text{se} \quad D[u,v] \leq 40 \\ 0, & \text{se} \quad D[u,v] > 40 \end{cases} \quad \text{e} \quad H_B[u,v] = \frac{D[u,v]}{D[u,v]+1}.$$

Para os filtros definidos por $H_A[u,v]$, $H_B[u,v]$ e $H_C[u,v] = 1 - H_A[u,v]$, indique o tipo de filtragem realizado e esboce o filtro na forma de imagem.

5. $\{1{,}0\}$ Considere o algoritmo de filtragem na frequência. Após a ação de *zero padding* obtiveram-se as dimensões da imagem *padded*, designadas por P e Q. Tendo em conta que $D[u,v] = \sqrt{(u-P/2)^2 + (v-Q/2)^2}$, definem-se os filtros:

$$H_A[u,v] = \begin{cases} 2, & \text{se} \quad D[u,v] \leq 40 \\ 1, & \text{se} \quad 40 < D[u,v] \leq 60 \\ 0, & \text{se} \quad D[u,v] > 60 \end{cases}, \quad H_B[u,v] = \frac{D[u,v]}{200} \quad \text{e} \quad H_C[u,v] = \exp\left(-\frac{D^2[u,v]}{200}\right).$$

Indique o tipo de filtragem realizado por $H_A[u,v]$, $H_B[u,v]$, $H_C[u,v]$ e $H_D[u,v] = 1 - H_A[u,v]$.

Exercises (5)

c) {1,0} Na visualização e observação, na forma de imagem, do módulo do espetro $|F[u,v]|$ de uma imagem $f[m,n]$, por vezes opta-se por visualizar a imagem dada pela transformação $\log(1+|F[u,v]|)$ em vez de visualizar diretamente $|F[u,v]|$. Quais as razões e as vantagens desta opção? Justifique.

d) Seja o seguinte algoritmo

```
Entrada: Imagem f[m,n]; Filtro passa-alto H[u,v], especificado na frequência.
Saída:   Imagem g[m,n].
1. F[u,v] = DFT[ log(f[m,n]) ].
2. G[u,v] = H[u,v]  . F[u,v].
3. g[m,n] = exp( IDFT[ G[u,v] ] ).
4  Retornar g[m,n].
```

i) {1,0} Identifique a designação e a funcionalidade do algoritmo. Para que tipo de imagem, é tipicamente aplicado com sucesso?

ii) {1,0} Comente sobre a possível existência de problemas numéricos na implementação do algoritmo.

Bibliography

- The images displayed in these slides are from:
  - R. Gonzalez, R. Woods, *Digital Image Processing*, 4th edition, Prentice Hall, 2018, ISBN 0133356728
  - S. Smith, *The Scientist and Engineer's Guide to Digital Signal Processing*, Newnes, 2003, ISBN 0-750674-44-X [chapter 23]
  - O. Filho, H. Neto, Processamento Digital de Imagens, Rio de Janeiro: Brasport, 1999, ISBN 8574520098.
  - Wikipedia and Mathworks web pages