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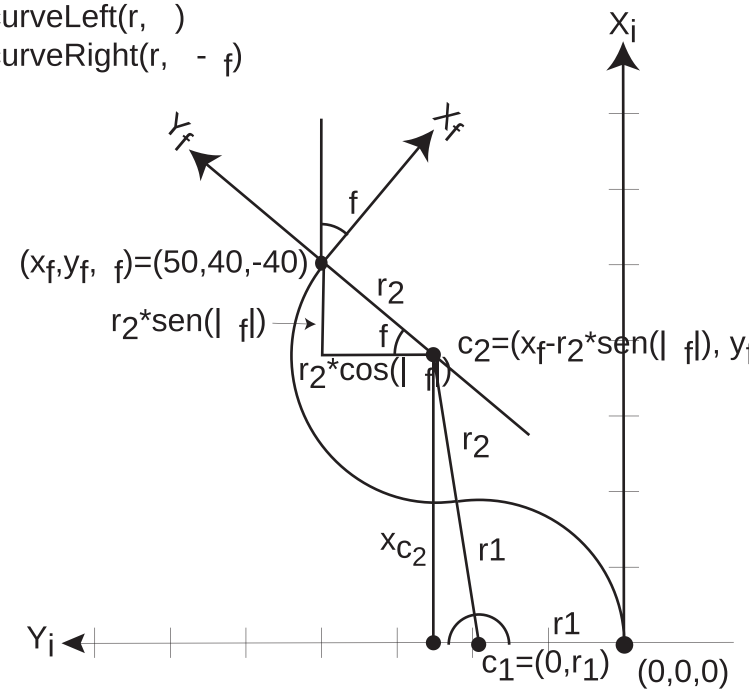
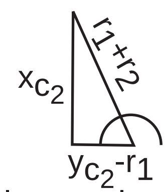
Theoretical trajectory 3

Consider a final point$(x_f, y_f, {}_f)$.
When $x_f, > y_f$ and $45 > {}_f > -90$ and $y_{c_2} > y_{c_1}$,
a possible trajectory is:

curveLeft(r,)
curveRight(r, - $_f$)





1. Calculate r_1 and r_2 using the distance d_{12} between points c_1 and c_2, which define the center points of two circunference arcs of each curve:

   $c_1 = (x_{c_1}, y_{c_1})$,
   $x_{c_1} = 0$ e $y_{c_1} = r_1$, then
   $c_1 = (0, r_1)$.

and,

   $c_2 = (x_{c_2}, y_{c_2})$,
   $x_{c_2} = x_f + r_2 * sen(| {}_f|)$ and $y_{c_2} = y_f - r_2 * cos(| {}_f|)$, imply
   $c_2 = (x_f + r_2 * sen(| {}_f|), y_f - r_2 * cos(| {}_f|))$.

then,

$$d_{12} = ((x_f + r_2 * sen(| {}_f|))^2 + (r_1 - y_f + r_2 * cos(| {}_f|))^2)^{0,5} = r_1 + r_2.$$

As the equation of d_{12} has two unknown variables r_1 e r_2, there are three solutions to get only one unknown variable:

i. $r_1 = r_2 = r \Rightarrow (2*r)^2 = (x_f + r*sen(| {}_f|))^2 + (r*(cos(| {}_f|)+1) - y_f))^2$.

ii. $r_1 = C \Rightarrow (r_2 + C)^2 = (x_f + r_2 * sen(| {}_f|))^2 + (C + r_2 * cos(| {}_f|) - y_f))^2$.

iii. $r_2 = C \Rightarrow (r_1 + C)^2 = (x_f + C * sen(| {}_f|))^2 + (r_1 + C * cos(| {}_f|) - y_f))^2$.

2. Calculate angle ,
   $= arcsen((x_f - r_2 * sen(| {}_f|))/(r_1 + r_2))$
   $= 180 -$

3. For the example (50,40,-40) and considering equation 1.i, we get
$(2*r)^2 = (y_f - r*(1+cos(| {}_f|)))^2 + (x_f - r*sen(| {}_f|))^2$,

calculating the coefficients of resolvent formula ,
$a = 2 - 2*cos(| {}_f|)$;
$b = 2*y_f*(1+cos(| {}_f|)) + 2*x_f*sen(| {}_f|)$;
$c = -(x_f^2 + y_f^2)$

from the resolvent formula, we get,
   r=19,1

from equation on point 2,
   =180-arcsen((50-19,1*sen(40))/(2*19,1)
   =99,4

4. The theoretical trajectory is,

   curveLeft(19.1, 99.4)
   curveRight(19.1, 139.4)

Note: In practice, robot is not able to do a curve with a radius of 19.1cm due the manobrability constraints of robot.