The Analysis of return rate volatility of S&P 500 based on GARCH model

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Contents

1 Abstract

The analysis of price volatility of the stock market not only has an important academic significance but also practical significance. The price volatility gives investors a chance to make money. Therefore, the volatility has always been a focus which investors and economic researchers paying attention to.
This paper analyzes the return rate based on the data from 2016-06-09 to 2017-06-09. The result shows that the volatility of daily return rate series has the characteristic of time-varying burstiness and clustering. The series distribution has a characteristic of sharp peak and heavy tail. It presents a significant GARCH effect which shows that the past volatility has a persistent effect on the future and the effect is gradually delays. Whats more, the S&P has a large fluctuation range. The frequent trading makes the stock index futures market has a high liquidity and the high liquidity is a main factor of stock

index volatility.
Key words: return rate; ARCH model; GARCH model

2 Introduction

2.1 The research meaning

The analysis of price volatility of the stock market not only has an important academic significance but also practical significance. The price volatility gives investors a chance to make money. Investors can predict how much the stock market risk is by measuring the volatility rate. At the same time, knowing the volatility is helpful for investors to understand and control the market operating law and make a wise decision to make more profit. Therefore, the volatility has always been a focus which investors and economic researchers paying attention to.

2.2 Research objects and methods

The S&P 500, or the Standard & Poor's 500, is an American stock market index based on the market capitalizations of 500 large companies having common stock listed on the NYSE or NASDAQ. The S&P 500 index components and their weightings are determined by S&P Dow Jones Indices. It differs from other U.S. stock market indices, such as the Dow Jones Industrial Average or the Nasdaq Composite index, because of its diverse constituency and weighting methodology. It is one of the most commonly followed equity indices, and many consider it one of the best representations of the U.S. stock market, and a bellwether for the U.S. economy. The National Bureau of Economic Research has classified common stocks as a leading indicator of business cycles.
This paper is based on the data of S&P 500 from 6th, Jun, 2016 to 6th, Jun, 2017, totally 253 samples.
As for the research method of return rate volatility of S&P 500, the many results of finance time series at home and abroad use the GARCH model to explain the fiance data. Therefore, this paper uses GARCH model to check the return rate volatility of S&P 500, and we hope that we could find the characteristic of the return rate volatility of S&P 500.

3 Introduction of GARCH model

3.1 ARCH model

The ARCH model is proposed by the Engle(1982), and developed into GARCH by Bollerslev(1986) General Autoregressive Conditional Heteroskedasticity. These models are applied to various fields, especially, in financial time series.
An ARCH(m) model takes the following form

$$a_t = \sigma_t * \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots + \alpha_m a_{t-m}^2$$

$$\text{where}\epsilon_t \sim \text{i.i.d(0,1)}, \alpha_0 \geq 0, \alpha_i > 0\text{for}i = 0, 1, 2, \ldots$$

3.2 GARCH(1,1) model

An GARCH(1,1) model takes the following form

$$a_t = \sigma_t * \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

$$\text{where}\epsilon_t \sim \text{i.i.d}(0,1), \alpha_0 \geq 0, \alpha_1 > 0, \beta_1 > 0, \alpha_1 + \beta_1 < 1$$

4 Case Study Based on The S&P 500

4.1 Basic statistical analysis

This paper is based on the data of S&P 500 from 6th, Jun, 2016 to 6th, Jun, 2017, totally 253 samples. To reduce the estimate error, we do a natural log processing to the daily return rate.

$$r = log(p_t/p_{t-1})$$

The graphic1 is the scatter diagram of log return rate

return.png

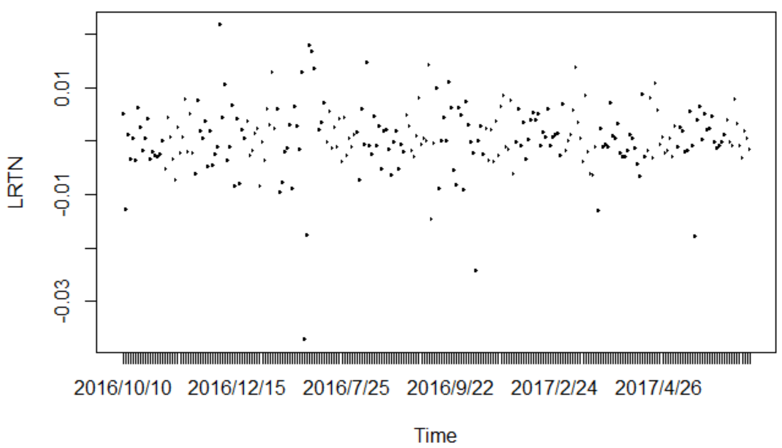

Figure 1: log return

We calculate the mean,variance,range,skewness and kurtosis.The results are shown in the table 1

Table 1: Results of the screening

Mean	Variance	Range	Skewness	Kurtosis
0.0005467607	0.006118438	0.05892841	-0.9539777	9.915377

From table 1,we can see that the skewness of log return rate is -0.954, which is left-of-center. The kurtosis is 9.915, which is far more larger than 3.

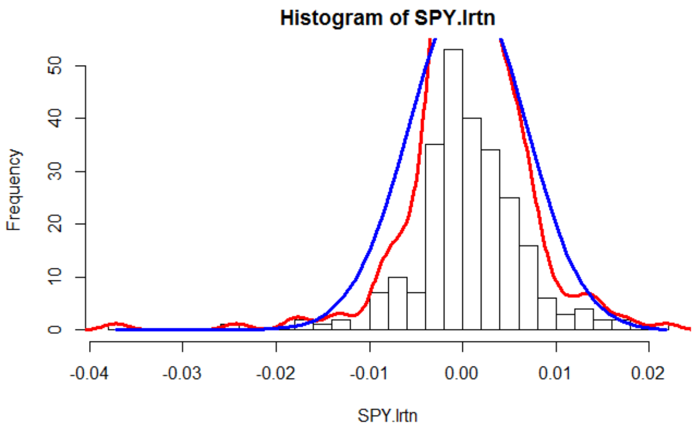

Figure 2: Histogram

We use R to get the histogram of S&P 500 return rate. Nuclear density curve is showed by the blue line, and normal distribution curve is showed by the red curve. The log return rate has a sharp crest and a heavy tail. We can conclude that the log return rate is not normal distributed and the test methods of statistical analysis to log return rate based on normal distributed are all failed.

4.2 Stationary test

We use R to plot the ACF and PACF of log return rate.

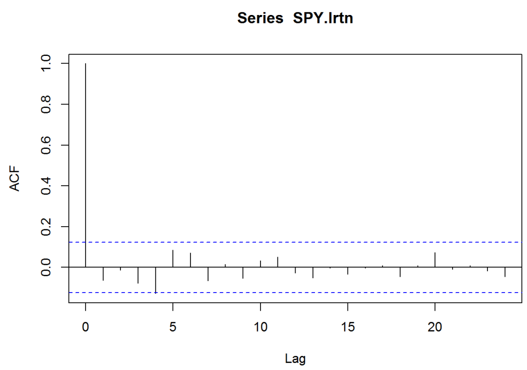

Figure 3: ACF

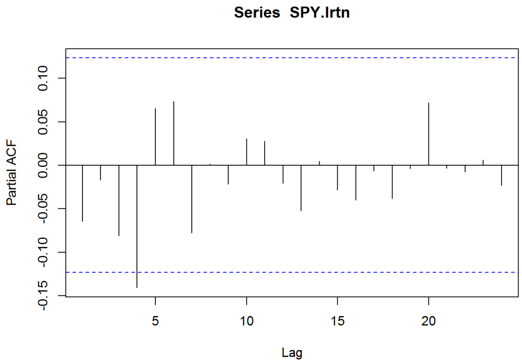

Figure 4: PACF

The 95% confidence interval is between the 2 blue lines. It seems that there is no significant autocorrelation. The most common method of stationary test is unit-root test. The result is following:

```
Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 1
  STATISTIC:
    DF: -11.6788
  P VALUE:
    t: < 2.2e-16
    n: 0.01664
```

Figure 5: D-F Test

The t-statistic is smaller than the critical value and we reject the null hypothesis, that is, there is no unit-root. It is stationary.

4.3 ARCH Test

4.3.1 Selecting Lagged Order and Mean Equation

In this paper, we use the model of time series to analyse, so we should choose the autoregressive lag order first. So the mean equation of Standard & Poors 500 yield r_t is like this:

$$r_t = c_0 + \sum_{i=1}^{n} c_i r_{t-i} + \epsilon_t$$

Do the regression of lag 1, 2, 3, 4 and 5 respectively:

Table 2: AIC of different order

Lag	1	2	3	4	5
AIC	-7.470154	-7.470421	-7.463554	-7.474055	-7.466638

According to the AIC minimum principle, we can see that the lag 4 is the best, so we choose lag 4, so the function become as:

$$r_t = c_0 + \sum_{i=1}^{4} c_i r_{t-i} + \epsilon_t$$

4.3.2 The Autocorrelation Validation of the Residual Sequences

Figures of autocorrelation of squared residuals and residuals in daily yield series by R are below:

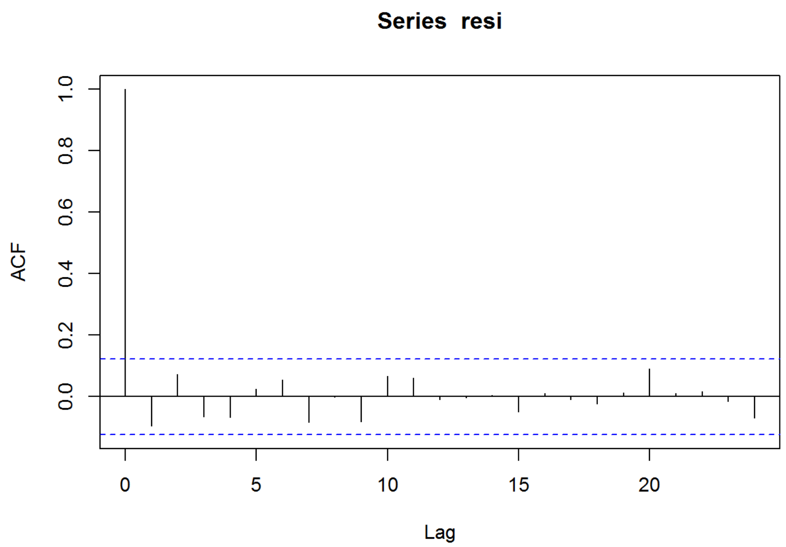

Figure 6: ACF of the series residuals

As can be seen from the sequence residuals, The correlation coefficient basically falls into the blue dashed line (95% confidence interval), so there is no significant autocorrelation between the daily yield residuals of the Standard & Poors 500. But for the residuals squares, The correlation coefficients did not fall into the blue dashed line (95% confidence interval), So The mean square of the daily return of the Standard & Poors 500 has significant autocorrelation, showing the ARCH effect.
Do the residual squared linear figure for the daily return of the Standard & Poors 500:

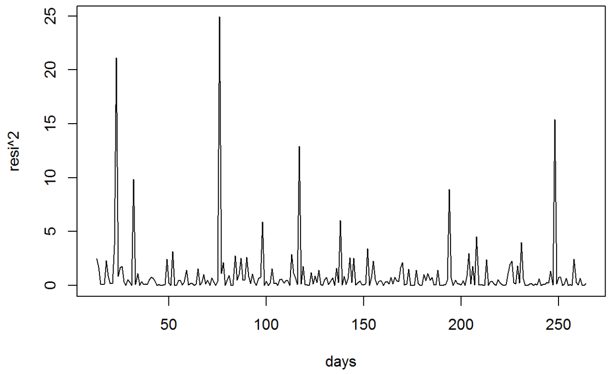

Figure 7: Residual Squared Linear Figure

It can be seen from the residual square linear graph, the residual of the regression equation fluctuates very small for a long time period, and big for other long time periods. It has obvious temporal variability and clustering. This indicates that the residual sequence may have higher order ARCH effects, suitable for modeling with the GARCH class model. Moreover, it can be observed that the volatility has weakened significantly the year the stock index futures go public.

4.3.3 ARCH Test of the Residuals

Run ARCH test for residual linear regression by R:

```
ARCH test
##
##   ARCH LM-test; Null hypothesis: no ARCH effects
##
## data:   y
## Chi-squared = 12.5, df = 20, p-value = 0.8978
```

Figure 8: ARCH Test

As can be seen, it is within the 95% confidence interval. So the residuals have the ARCH effect. So the GARCH model can be used to fit the data.

4.4 GARCH(1,1) Model Fitting

By Rwe conform that the residual sequence was fitted by GARCH (1,1) model.

From the table 3,GARCH(1,1) function:

$$r_t = 5.525 * 10^{-4} + a_t$$

$$\sigma_t^2 = 1.635 * 10^{-5} + 0.2618\alpha_{t-1}^2 + 0.2884\beta_{t-1}^2$$

Table 3: Estimation of GARCH(1,1) Model

GARCH	Estimate Coefficient	Std.Error	T-value	Pr(> $\|t\|$)
mu	5.525e-04	3.358e-04	1.646	0.09986
omega	1.635e-05	6.938e-06	2.356	0.01847
alpha1	2.618e-01	9.931e-02	2.636	0.00839
beta1	2.884e-01	2.351e-01	1.226	0.22004

Moreover, the coefficients in the equations are statistically significant, and the alpha1 and beta1 are highly significant, indicating that the yield series has significant volatility clustering. Moreover, $\alpha_1 + \beta_1 = 0.5502 < 1$ satisfies the parameter constraint condition. At the same time, because the coefficient sum is very close to 1, it shows that the impact of a conditional variance is persistent, that is, it plays an important role in all future predictions. Thus, the GARCH (1,1) process is stationary.

5 Summary

We use the Standard & Poors 500 of closing price per trading day from 2016-06-09 to 2017-06-09 as our original data. Calculate T=the daily logarithmic yield of the Standard & Poors 500. Then analyze the basic statistical characteristics and stationarity of the returns and test the residual sequence of the autoregressive model. We find that it has a strong ARCH effect so we establish GARCH (1,1) to fit it. And these are our conclusions:
1)The Standard & Poors 500 yield sequence has significant volatility clustering. Fully demonstrates the speculative atmosphere of American stock market is strong, investors' short-term investment preferences are obvious.
2)The Standard & Poors 500 returns have obvious GARCH effect. The effect of past fluctuations on the future is persistent and gradually weakened.
3)The Standard & Poors 500 fluctuation ranges very much. Frequent trading results in high liquidity of stock index futures market, which makes it fluctuates very much.

References

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