Battle of Online and Offline Consumption: Comparative Analysis of Amazon and Walmart Stocks

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1 Introduction

2 Data Processing

In our research report, we use Walmart as the representative of offline seller and Amazon as that of online seller. We focus on the daily log return of the Walmart stock and Amazon stock from January 3th, 2010 to December 30th, 2016, with 1761 observations. All the data are pulled from Wind Financial Terminal.

Let r_t be the log return of an asset at time t, and the formula we use to calculate the log return is

$$r_t = ln(\frac{p_t}{p_{t-1}}) \times 100\%$$

where p_t is the close price in day t and p_{t-1} is the close price in day t-1.

The descriptive statistics and time plot are shown in Table 1 and Firgure 1.

Table 1: Descriptive Statistics

(a) Amazon Stock

Year	Sample	Mean(%)	Sd(%)
2010	251	0.1179	2.0591
2011	252	-0.0155	2.4337
2012	250	0.1484	1.9656
2013	252	0.1839	1.6947
2014	252	-0.0995	2.0677
2015	252	0.3089	2.0582
2016	252	0.0412	1.8682
2010-2016	1761	0.0978	2.0323

(b) Walmart Stock

Year	Sample	Mean(%)	Sd(%)
2010	251	0.0068	0.8782
2011	252	0.0515	1.0469
2012	250	0.0627	1.0309
2013	252	0.0662	0.7749
2014	252	0.0445	0.8367
2015	252	-0.1229	1.3191
2016	252	0.0590	1.2030
2010-2016	1761	0.0239	1.0297

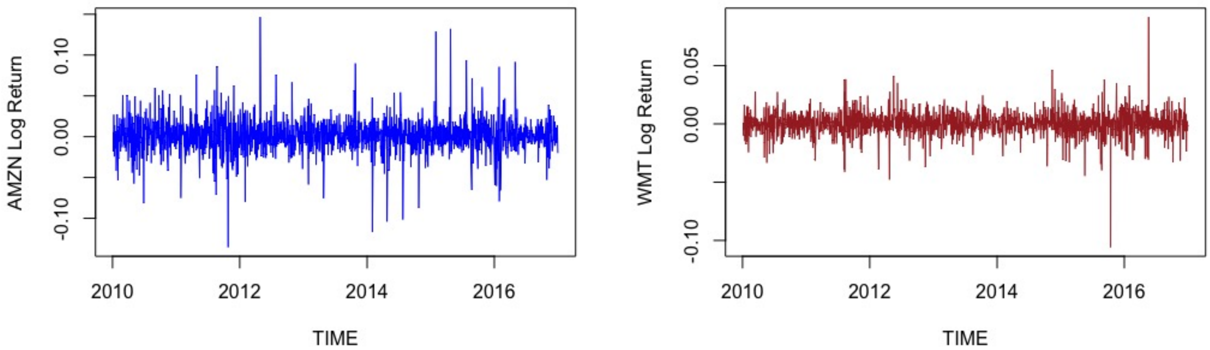

Figure 1: Timeplots of Amazon (Blue) and Walmart (Red) Stocks

From Table 1, we provide the mean log return and its standard deviation for each stock, and Figure 1 shows the time plots of the log returns for Amazon and Walmart. From the plots, the return series for both stocks appear to be stationary and random.

3 Model Establishment

3.1 Model Specification

Figure 2.(a) shows the sample ACF of the log returns for Amazon stock, which suggests no significant serial correlations. Figure 2.(b), showing the sample PACF of the log returns, also confirms our conclusion of no serial correlation. Then, we plot the sample ACF of the squared log returns for Amazon stock in Figure 2.(c). Combing the three plots, it seems that the log returns are neither serially correlated nor dependent. Similar results can also be found for Walmart stock in Figure 3 that the log returns for Walmart stock are serially uncorrelated and independent.

3.2 Mean Equation

The observations above suggest that the daily log returns of Amazon stock follow an ARMA(0,0) model, in other words, a white noise series. This is in agreement with the result suggested by the sample ACF in Figure 2(c) that all sample ACFs are close to zero. Therefore, we propose a mean equation that is simply a constant plus innovations, $r_t = \mu + a_t$, where r_t is the log return of an asset at time t, μ is the estimate we get using a volatility model. The squared series a_t^2 is then used to check for conditional heteroscedasticity (ARCH effects). We perform the usual LjungBox statistics Q(m) to the $\{a_t^2\}$ series. The null hypothesis is that the first m lags of ACF of the a_t^2 series are zero.

The LjungBox statistics of the a_t^2 series shows ARCH effects with Q(5) = 9.6519, the p value of which is close to zero.

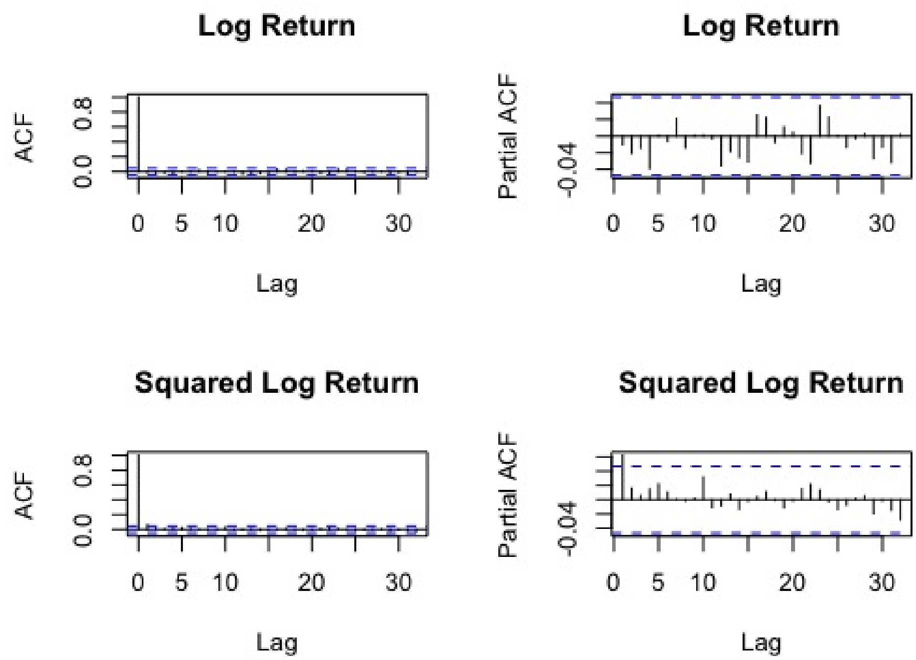

Figure 2: ACF and PACF of Amazon Stock

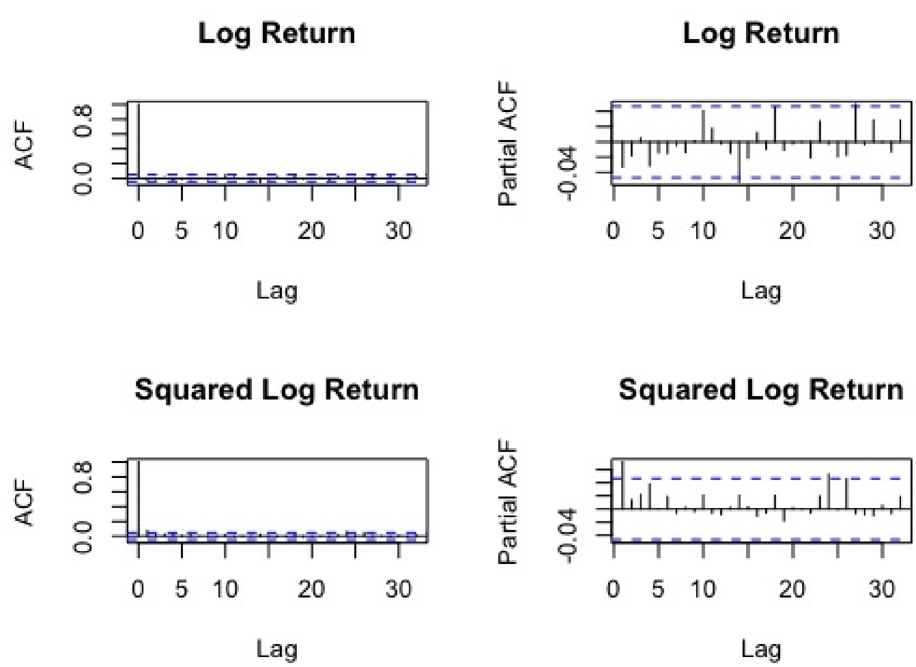

Figure 3: ACF and PACF of Walmart Stock

3.3 Volatility Equation

Here we entertain an ARCH(1) model and a GARCH(1,1) model for the volatility and we specify the model as the following:

$$r_t = \mu + a_t, a_t = \sigma_t \epsilon_t$$

$$\text{ARCH(1): } \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2$$

$$\text{GARCH(1,1): } \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

in which ϵ_t, here in our model, we assume as a Gaussian innovation that is independent and identically distributed as a Normal distribution with mean 0 and variance 1.

4 Estimation

Table 2: Ljung-Box tests for ARCH(1) and GARCH(1,1) models for AMZN Stock

		Standardized residuals			Squared standardized residuals		
		Q(10)	Q(15)	Q(20)	Q(10)	Q(15)	Q(20)
ARCH(1)	statistic	4.7873	10.2395	12.8693	3.9721	4.7376	5.1054
	p-value	0.9049	0.8044	0.8829	0.9486	0.9941	0.9997
GARCH(1,1)	statistic	4.1734	10.2365	12.8473	3.0443	4.2163	4.7107
	p.value	0.9392	0.8046	0.8838	0.9804	0.9969	0.9999

Based on the results of log likelihood, AIC, and BIC for ARCH(1) model and GARCH(1,1) model, GARCH(1,1) is slightly more appropriate.

Consequently, we obtain a GARCH(1,1) to model the volatility:

$$\sigma_t^2 = 1.0199 + 0.1286 a_{t-1}^2 + 0.6373 \sigma_{t-1}^2$$

where the standard errors of the parameters are 0.3435, 0.0315, 0.0984, respectively.

In conclusion, we propose the following mean equation and conditional heteroskedasticity model for AMZN stock

$$r_t = 0.1367 + a_t, \sigma_t^2 = 1.0199 + 0.1286 a_{t-1}^2 + 0.6373 \sigma_{t-1}^2.$$

Following the same logic as in the previous section, we select ARCH(1) model for WMT Stock. Since theres little difference between these two models based on their log likelihood, AIC, and BIC, our selection is rather subjective.

Consequently, we obtain a ARCH(1) to model the volatility:

$$\sigma_t^2 = 0.1286 + 0.1920 a_{t-1}^2$$

Table 3: Results of Estimation of Two Volatility Models for AMZN Stock

	ARCH(1)	GARCH(1,1)
mu	0.115**	0.137***
	(0.046)	(0.046)
omega	3.515***	1.020***
	(0.149)	(0.344)
alpha1	0.168***	0.129***
	(0.036)	(0.032)
beta1		0.637***
		(0.098)
Observations	1761	1761
Log Likelihood	3724.855	3720.010
Akaike Inf. Crit.	4.234	4.229
Bayesian Inf. Crit.	4.243	4.242
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 4: Ljung-Box tests for ARCH(1) and GARCH(1,1) models for WMT Stock

		Standardized residuals			Squared standardized residuals		
		Q(10)	Q(15)	Q(20)	Q(10)	Q(15)	Q(20)
ARCH(1)	statistic	9,0085	15.1256	20.0360	2.1312	2.4483	4.4991
	p-value	0.5313	0.4424	0.4557	0.9952	0.9999	0.9999
GARCH(1,1)	statistic	8.8064	14.4575	18.5378	1.6428	1.9398	4.0184
	p.value	0.5506	0.4912	0.5520	0.9984	0.9999	0.9999

Table 5: Results of Estimation of Two Volatility Models for WMT Stock

	ARCH(1)	GARCH(1,1)
mu	0.037	0.033
	(0.023)	(0.024)
omega	0.863***	0.442**
	(0.036)	(0.187)
alpha1	0.192***	0.140***
	(0.034)	(0.040)
beta1		0.445**
		(0.207)
Observations	1761	1761
Log Likelihood	2506.755	2506.020
Akaike Inf. Crit.	2.850	2.851
Bayesian Inf. Crit.	2.860	2.863
Note:	*p<0.1; **p<0.05; ***p<0.01	

where the standard errors of the parameters are 0.0358, and 0.03389, respectively.

In conclusion, we propose the following mean equation and conditional heteroskedasticity model for AMZN stock

$$r_t = 0.0372 + a_t, \sigma_t^2 = 0.1286 + 0.1920a_{t-1}^2.$$

5 Model Checking

6 Prediction

7 Conclusion

8 Appendix

References

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