Battle of Online and Offline Consumption: Comparative Analysis of Amazon and Walmart Stocks

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1 Introduction

Online consumption is a relative new and prevailing concept raised in recent decade. Amazon is arguably one of the most successful online firms. As of this writing, its market cap is over \$460 billion, almost twice than the large and well-known offline retailer, Wal-Mart's, with market cap about \$240 billion.

Amazon, as a business model, has many potential advantages relative to a physical operation. It held out the potential of lower inventory and distribution costs and reduced overhead. Consumers could find the books products they were looking for more easily and a broader variety could be offered for sale in the first place. It could accept and fulfill orders from almost any domestic location with equal ease. And most purchases made on its site would be exempt from sales tax.

On the other hand, it is also acknowledged that there are some limitations of online operations. Customers would have to wait for their orders to be received, processed, and shipped. Because they couldn't physically inspect a product before ordering, Amazon would have to make its returns and redress processes transparent and reliable, and offer other ways for consumers to learn as much about the product as possible before buying. The task to judge the performance of online consumption against offline consumption is compelling. And the methods of judgments comparisons can be diverged. In this term paper, our group conducts time series data models with conditional heteroscedasticity, which are widely applied in financial data analysis, to depict the stock volatility of Amazon and Wal-Mart's.

The historical stock price data presents a diverged trend that seems to reveal a competitive relation between Amazon and Wal-Mart. Admittedly the share price cannot represent complete information about the firms, such as market shares. Nevertheless, this is a direct and prevalent way to evaluate firms' performance through numerical results. And the maximize likelihood method (MLE) estimations will also provide helpful suggestions about their prospective performance.

We organize our discussion as follows. The next section lays out some basic facts about the historical stock price data: statistics and time plots. Section 3 discusses how to determine the specifications of models.

Section 4 conducts MLE method to estimate the parameters of models. Section 5 explores future performance and compares our prediction with recent data. A short concluding section follows.

2 Data Processing

In our research report, we use Walmart as the representative of offline retail industry and Amazon as that of online retail industry. We focus on the daily log return of Walmart stock and Amazon stock from January 3th, 2010 to December 30th, 2016, with 1761 observations. All the data are pulled from Wind Terminal.

We compute the daily log return based on the daily close price. Let r_t be the log return of an asset at time t, and the formula we use to calculate the log return is

$$r_t = ln(\frac{p_t}{p_{t-1}}) \times 100\%$$

where p_t is the close price in day t and p_{t-1} is the close price in day t-1.

The descriptive statistics and time plots are shown in Table 1 and Firgure 1.

Table 1: Descriptive Statistics

(a) Amazon Stock

Year	Sample	Mean(%)	Sd(%)
2010	251	0.1179	2.0591
2011	252	-0.0155	2.4337
2012	250	0.1484	1.9656
2013	252	0.1839	1.6947
2014	252	-0.0995	2.0677
2015	252	0.3089	2.0582
2016	252	0.0412	1.8682
2010-2016	1761	0.0978	2.0323

(b) Walmart Stock

Year	Sample	Mean(%)	Sd(%)
2010	251	0.0068	0.8782
2011	252	0.0515	1.0469
2012	250	0.0627	1.0309
2013	252	0.0662	0.7749
2014	252	0.0445	0.8367
2015	252	-0.1229	1.3191
2016	252	0.0590	1.2030
2010-2016	1761	0.0239	1.0297

From Table 1, we can easily see that both returns of these two stocks display an increasing trend from 2010 to 2016. However, the average log return of Amazon stock is four times greater than Walmart stock, while the log return of Walmart stock is more stable than Amazon's. From the timeplots in Figure 1, the log return series for both stocks appear to be relatively stationary over times, fluctuating around the mean value.

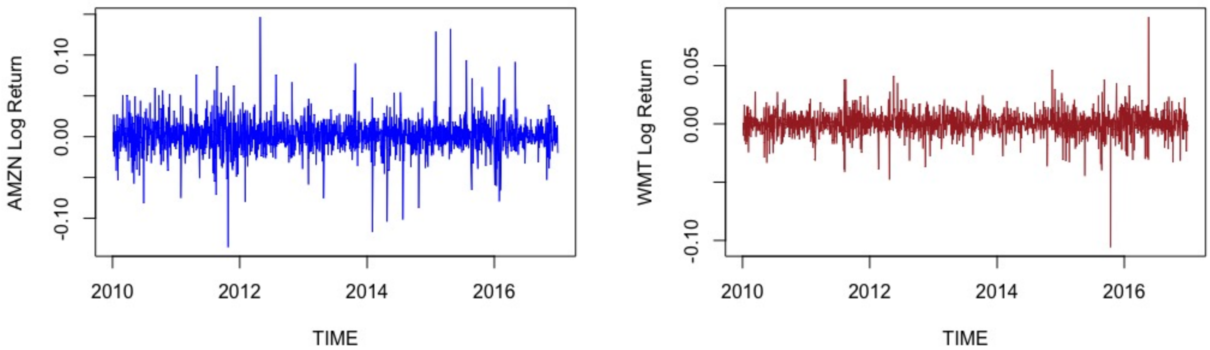

Figure 1: Timeplots of Amazon (Blue) and Walmart (Red) Stocks

3 Model Establishment

3.1 Model Specification

First of all, we determine the model to capture the log return fluctuation of Amazon stock. Figure 2.(a) shows the sample ACF of the log returns, which suggests that there is no significant serial correlations and the series is stationary. Observing the squared log returns for Amazon stock in Figure 2.(c) we find that the log returns are not serially correlated but dependent, which indicates the ARCH effect in the return series. Similar results can also be found for Walmart stock in Figure 3 that there also exists an ARCH effect in Walmart's return series.

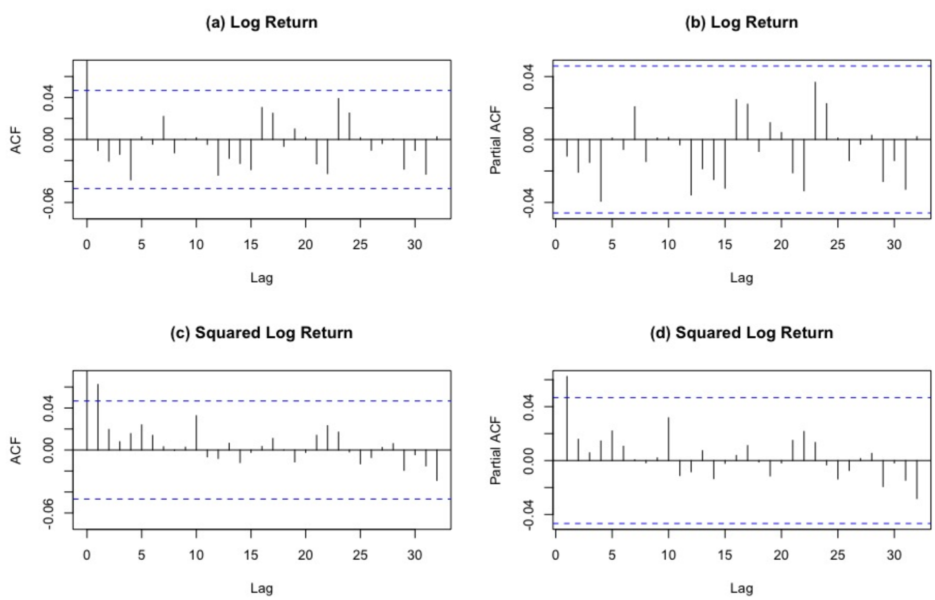

Figure 2: ACF and PACF of Amazon Stock

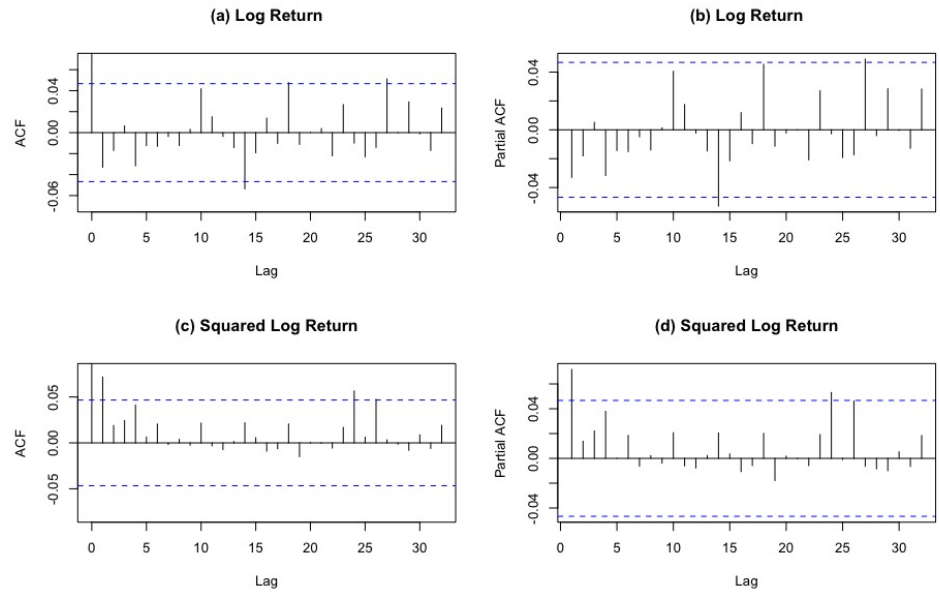

Figure 3: ACF and PACF of Walmart Stock

3.2 Mean Equation

According to the Figure 2.(b), the daily log returns of Amazon stock follow an ARMA(0,0) model because there is no cutoff in the PACF of log return. Therefore, we propose a mean equation that is simply a constant plus innovations, $r_t = \mu + a_t$, where r_t is the log return of an asset at time t, μ is the estimate of mean log return.

The a_t^2 series is then used to check for conditional heteroscedasticity (ARCH effects). To consolidate the observation results in Figure 2.(c), the ARCH effect exists in log return series, we perform the LjungBox statistics Q(m) to the $\{a_t^2\}$ series. The null hypothesis is that the first m lags of ACF of the a_t^2 series are zero. The results of LjungBox test shows an ARCH effect exists in this log return series with Q(5) = 9.6519, the p-value of which is close to zero.

Using the same method above to analyze the ACF and PACF of WMT log returns shown in Figure 3, we conclude that the WMT log returns' mean equation is also an ARMA(0,0) process. And there exists an ARCH effect in WMT's log return series. Although someone may argues that Figure 3 shows significant correlation at lag-14 and lag-27, the EACF results in Table 2 provides the evidence that ARMA(0,0) is proper.

Table 2: EACF of Log Returns of Walmart Stock

AR/MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	o	o	o	o	o	o	o	o	o	o	o	o	o	x
1	x	o	o	o	o	o	o	o	o	o	o	o	o	o
2	x	x	o	o	o	o	o	o	o	o	o	o	o	o
3	x	x	x	o	o	o	o	o	o	o	o	o	o	o
4	x	x	x	x	o	o	o	o	o	o	o	o	o	o
5	x	x	x	x	x	o	o	o	o	o	o	o	o	o
6	x	x	x	o	x	x	o	o	o	o	o	o	o	o
7	x	x	o	x	x	x	x	o	o	o	o	o	o	o

3.3 Volatility Equation

Based on the Figure 2.(d), there is a sudden cutoff at lag-1 squared a_t^2. Hence, we entertain an ARCH(1) model and a GARCH(1,1) model for the volatility and we specify the model as the following:

$$r_t = \mu + a_t, a_t = \sigma_t \epsilon_t$$

$$\text{ARCH(1):}\ \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2$$

$$\text{GARCH(1,1):}\ \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

in which ϵ_t, we assume as a Gaussian innovation that is independent and identically distributed follows a Normal distribution with mean zero and variance one.

Similar results will be derived for the log return series of Walmart stock. We entertain an ARCH(4) model and a GARCH(1,1) model as the alternative volatility equations:

$$r_t = \mu + a_t, a_t = \sigma_t \epsilon_t$$

$$\text{ARCH(4):}\ \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \alpha_2 a_{t-2}^2 + \alpha_3 a_{t-3}^2 + \alpha_4 a_{t-4}^2$$

$$\text{GARCH(1,1):}\ \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

where $\epsilon_t \sim N(0, 1)$.

4 Model Comparison and Estimation

To determine the more adequate model for Amazon stock, we perform the Ljung-Box test for the standardized residuals and the squared standardized residuals. However, there is no evident difference between ARCH(1) model and GARCH(1,1) model according to the test results in Table 3. Then we use log likelihhod and information criteria to compare models and derive the estimation results in Table 4.

Table 3: Ljung-Box tests for ARCH(1) and GARCH(1,1) models for AMZN Stock

		Standardized residuals			Squared standardized residuals		
		Q(10)	Q(15)	Q(20)	Q(10)	Q(15)	Q(20)
ARCH(1)	statistic	4.7873	10.2395	12.8693	3.9721	4.7376	5.1054
	p-value	0.9049	0.8044	0.8829	0.9486	0.9941	0.9997
GARCH(1,1)	statistic	4.1734	10.2365	12.8473	3.0443	4.2163	4.7107
	p.value	0.9392	0.8046	0.8838	0.9804	0.9969	0.9999

Table 4: Results of Estimation of Two Volatility Models for AMZN Stock

	ARCH(1)	GARCH(1,1)
μ	0.115^{**}	0.137^{***}
	(0.046)	(0.046)
α_0	3.515^{***}	1.020^{***}
	(0.149)	(0.344)
α_1	0.168^{***}	0.129^{***}
	(0.036)	(0.032)
β_1		0.637^{***}
		(0.098)
Observations	1761	1761
Log Likelihood	-3724.855	-3720.010
Akaike Inf. Crit.	4.234	4.229
Bayesian Inf. Crit.	4.243	4.242
Note:	*p<0.1; **p<0.05; ***p<0.01	

Based on the results in Table 4, GARCH(1,1) model is more appropriate because it has the largest value of Log Likelihood and the smallest value of AIC and BIC. And all estimated parameters are highly significant under 1% significance level.

Consequently, we propose an ARMA(0,0)+GARCH(1,1) model to depict the log return series of Amazon stock:

$$r_t = 0.137 + a_t$$

$$a_t = \sigma_t \epsilon_t, \epsilon_t \sim N(0,1)$$

$$\sigma_t^2 = 1.02 + 0.129 a_{t-1}^2 + 0.637 \sigma_{t-1}^2$$

Table 5: Ljung-Box tests for ARCH(4) and GARCH(1,1) models for WMT Stock

		Standardized residuals			Squared standardized residuals		
		Q(10)	Q(15)	Q(20)	Q(10)	Q(15)	Q(20)
ARCH(4)	statistic	9.6623	15.0804	19.5692	1.4971	1.9193	3.6344
	p-value	0.4706	0.4456	0.4852	0.9989	0.9999	0.9999
GARCH(1,1)	statistic	8.8064	14.4575	18.5378	1.6428	1.9398	4.0184
	p.value	0.5506	0.4912	0.5520	0.9984	0.9999	0.9999

Following the same logic above and combining the results in Table 5 and Table 6, the ARCH(4) model is more adequate for volatility in log returns of Walmart stock. Thus, we propose an ARMA(0,0)+ARCH(4) model to depict the log return series of Walmart stock:

$$r_t = 0.024 + a_t$$

$$a_t = \sigma_t \epsilon_t, \epsilon_t \sim N(0,1)$$

$$\sigma_t^2 = 0.773 + 0.13 a_{t-1}^2 + 0.034 a_{t-3}^2 + 0.112 a_{t-4}^2$$

Table 6: Results of Estimation of Two Volatility Models for WMT Stock

	ARCH(4)	GARCH(1,1)
μ	0.024	0.033
	(0.023)	(0.024)
α_0	0.773***	0.442**
	(0.042)	(0.187)
α_1	0.130***	0.140***
	(0.035)	(0.040)
α_2	0.000	
	(0.018)	
α_3	0.034	
	(0.024)	
α_4	0.112***	
	(0.040)	
β_1		0.445**
		(0.207)
Observations	1761	1761
Log Likelihood	-2498.390	-2506.020
Akaike Inf. Crit.	2.844	2.851
Bayesian Inf. Crit.	2.863	2.863
Note:	*p<0.1; **p<0.05; ***p<0.01	

5 Prediction

6 Conclusion

7 Appendix

References

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