Lecture 2: The ARCH Model

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1 The Volatility of Asset Returns

This lecture focuses on the volatility of asset returns. Here volatility refers to the **conditional variance** of a time series. That is, for a return series $\{r_t\}$, we are now interested in

$$\sigma_t^2 = \mathrm{Var}(r_t \mid F_{t-1})$$

where F_{t-1} is the information set at time $t-1$.

1.1 Characteristics of volatility

In practice, we have observed some stylized facts about volatility, and some volatility models are proposed to characterize them. These properties of volatility include:

1. There exist volatility clusters. That is, volatility may be high for certain time periods and low for other periods. Figure 1 shows the daily changes in the log of the NYSE U.S. 100 stock price index. As seen, a cluster of tranquil periods from 2003 to 2007 is followed by a cluster of drastic volatile periods from 2008 to 2010.

2. Volatility evolves over time in a continuous manner. That is, volatility jumps are rare. In Figure 2, the change of the volatility of the annualized real GDP growth rate of the U.S. is relatively smooth.

3. Volatility does not diverge to infinity. That is, volatility varies within some fixed range. Statistically speaking, this means that volatility is often stationary.

4. Volatility seems to react differently to a big price increase or a big price drop, referred to as the leverage effect.

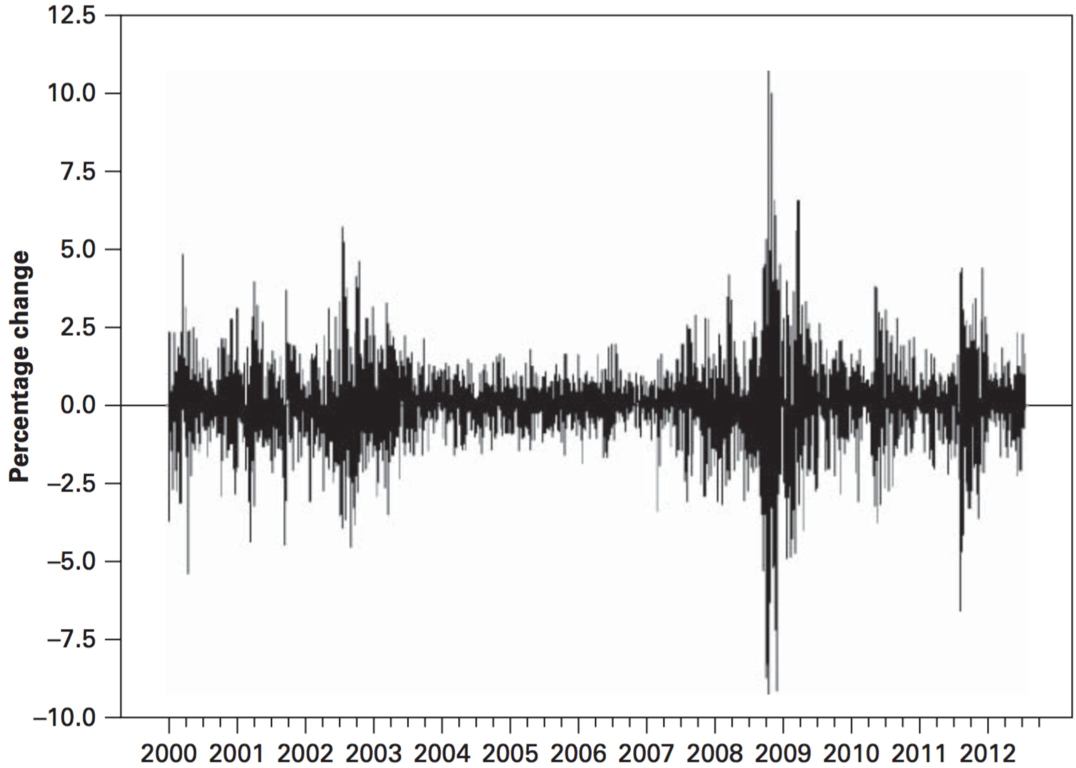

Figure 1: Percentage Change in the NYSE U.S. 100

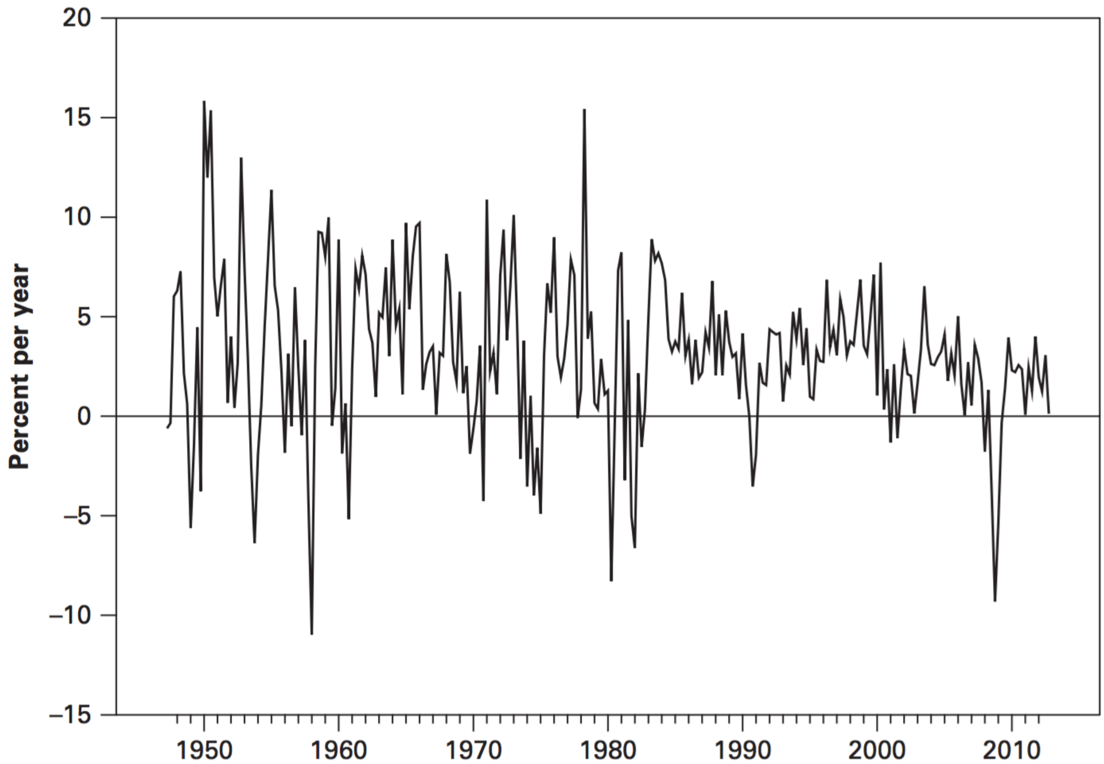

Figure 2: Annualized Growth Rate of Real GDP

2 The Structure of a Volatility Model

2.1 The basic idea of building a volatility model

Consider the log return series $\{r_t\}$. The basic idea of a volatility model is that $\{r_t\}$ may appear to be either serially uncorrelated or serially correlated with a minor order, but $\{r_t\}$ is a dependent series and the dependence arises from its conditional variance.

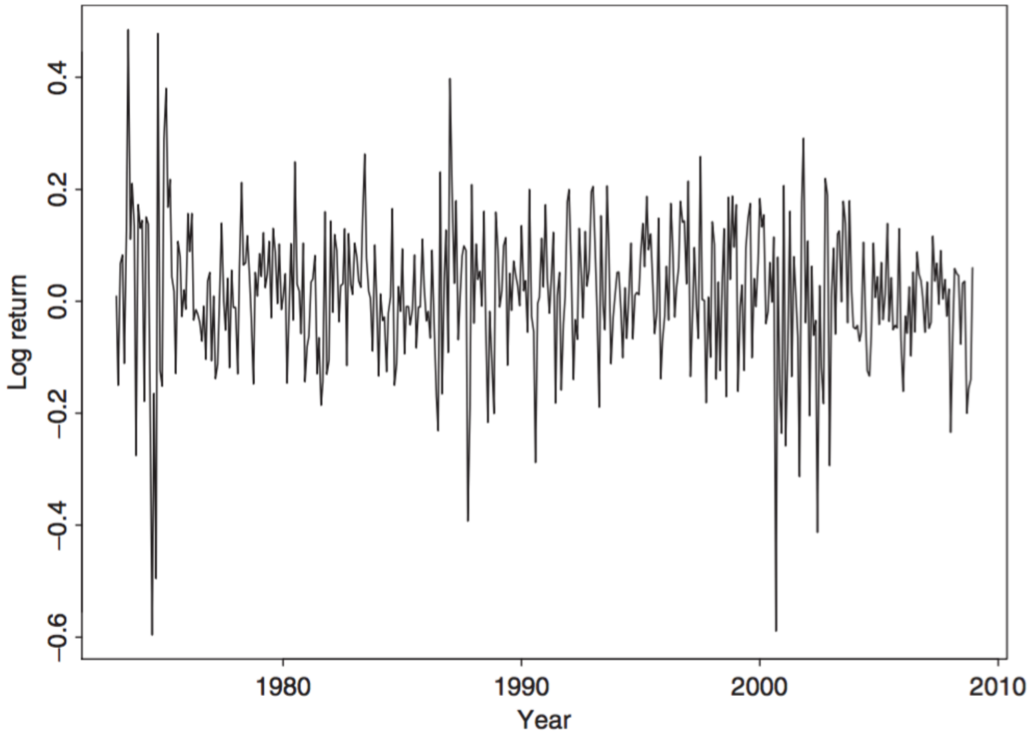

Figure 3: Time plot of monthly log returns of Intel stock from January 1973 to December 2008

For illustration, consider the monthly log stock returns of Intel Corporation from January 1973 to December 2008 shown in Figure 3.

Figure 4 displays the sample ACF and PACF of the log return.

- Figure 4(a) shows the sample ACF of the log return series $\{r_t\}$, which suggests no significant serial correlations except for a minor one at lag 7.
- Figure 4(b) shows the sample ACF of the squared log returns $\{r_t^2\}$.
- Figure 4(c) shows the sample ACF of the absolute log returns,

These two plots clearly suggest that the monthly log returns are not serially independent. Combining the three plots, it seems that the log returns are indeed serially uncorrelated but dependent. Volatility models attempt to capture such dependence in the return series.

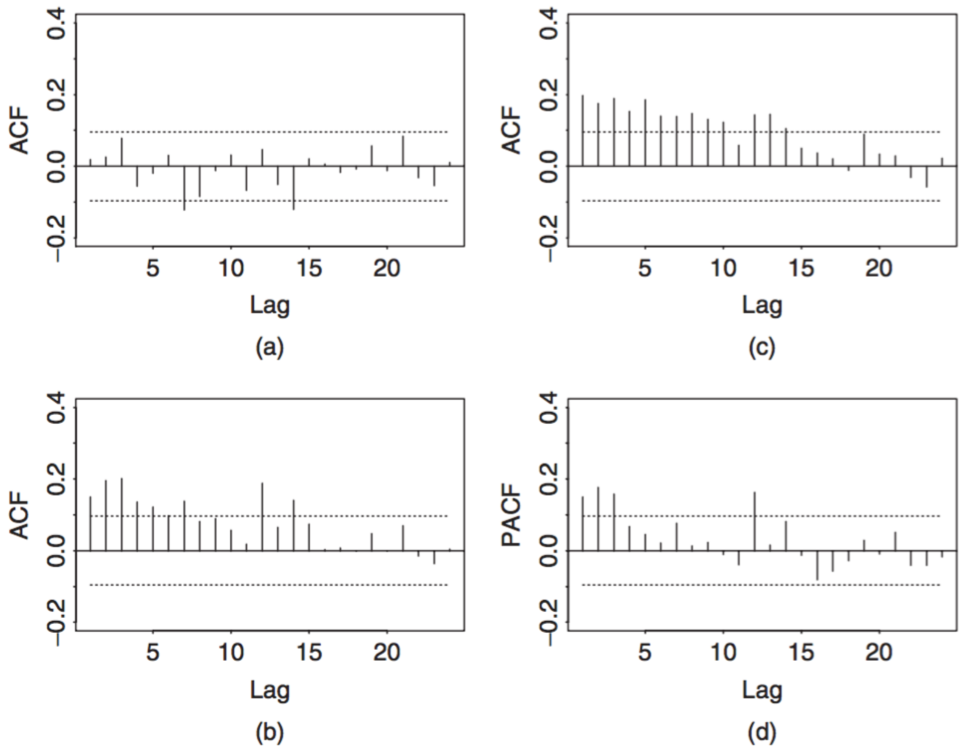

Figure 4: Sample ACF and PACF of various functions of monthly log stock returns of Intel Corporation from January 1973 to December 2008: (a) ACF of the log returns, (b) ACF of the squared log returns, (c) ACF of the absolute log returns, and (d) PACF of the squared log returns.

2.2 The mean equation and the volatility equation

2.3 The procedure of building a volatility model

2.4 Testing for the presence of ARCH effect

3 The ARCH Model

3.1 The ARCH(m) Model

3.2 The properties of ARCH models

3.3 Order determination

3.4 Estimation

3.5 Model checking

4 Applications with R