A Study on the Volatility of Stock Returns of Intel Corporation

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May 16, 2017

1 Introduction

This document is to show an application of modeling volatility of stock returns using ARCH and GARCH models. The detailed explanation is in Chapter 4 in Tsay (2013).

2 Data Description

Basic description

The data source that we consider is the monthly log stock returns of Intel Corporation from January 1973 to December 2009 for 444 observations. Table 1 shows the descriptive statistics, and Figure 1 displays the time plot of the returns.

Table 1: The descriptive statistics of the monthly log stock returns of Intel Corp.

	nobs	Minimum	Maximum	1. Quartile	3. Quartile	Stdev	Skewness	Kurtosis
1	444.00	-0.60	0.49	-0.05	0.10	0.13	-0.55	3.12

Checking stationarity of the Intel stock returns series

Figure 2 shows that sample ACF of the log returns and the absolute log returns. There is no significant serial correlations in the log returns except for some minor ones at lags 7 and 14. The Ljung-Box statistic for $\log(r_t)$ shows that $Q(12) = 18.68$, and the p-value is 0.0967, confirming that there is no serial correlation at the first 12 lags. On the other hand, the Ljung-Box statistic for $|\log(r_t)|$ shows that $Q(12) = 124.91$ with the p-value being 0, rejecting the null hypothesis of no serial correlation. Consequently, the monthly log returns of Intel stock are serially correlated but dependent.

Detecting ARCH effect

We can use the Ljung-Box test for a_t^2 to test whether there is an ARCH effect in the innovation series calculated from the log return series. The Ljung-Box statistic, $Q(12)$, is 92.94 with the p-value being 0, rejecting the null hypothesis and confirming the ARCH effects.

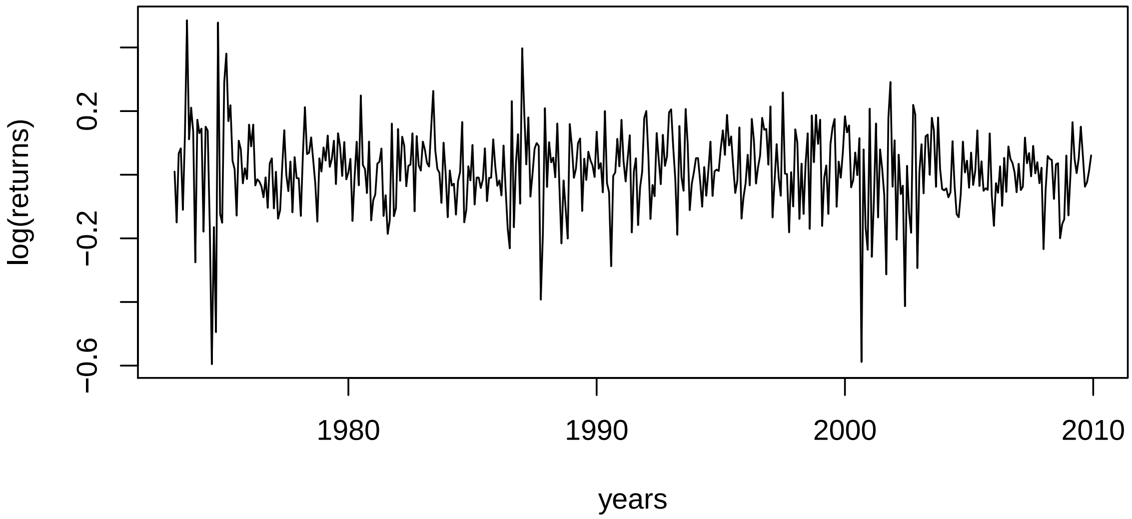

Figure 1: Time plot of the monthly log returns of Intel stock

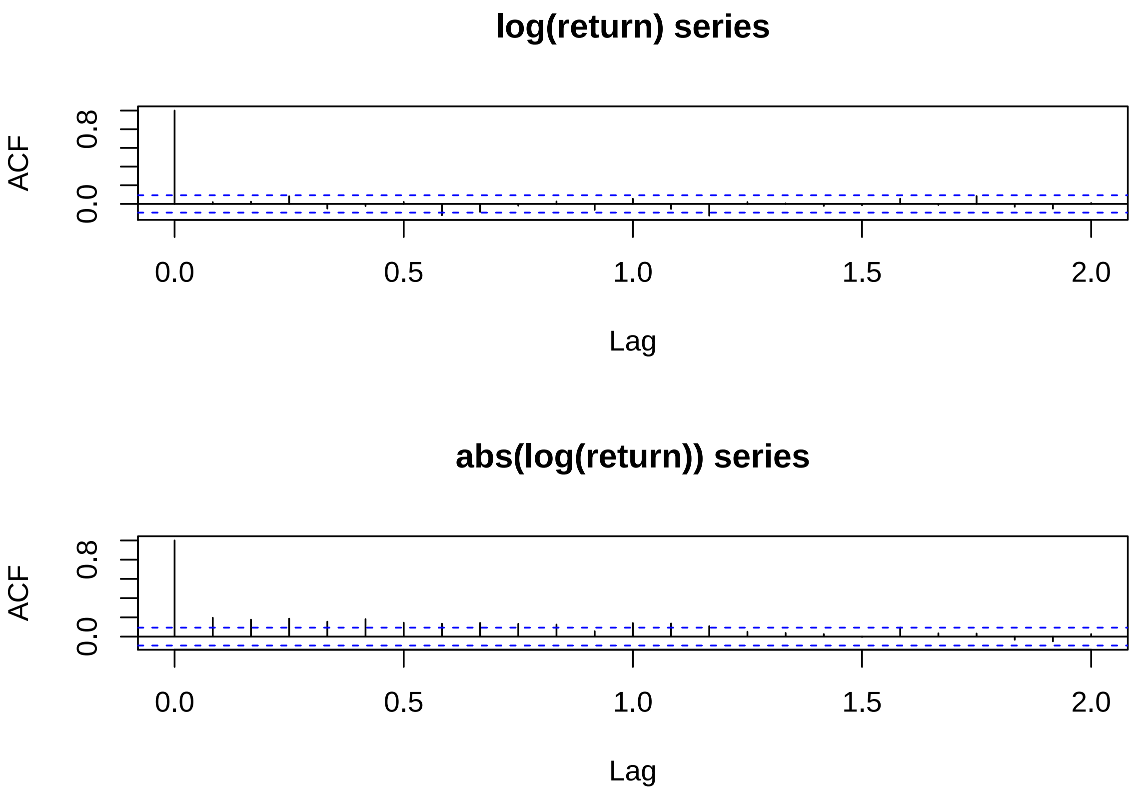

Figure 2: Sample ACF of the monthly log returns of Intel stock

3 The Volatility Model

The ARCH model

We first build an ARCH(3) model with the Gaussian ϵ_t and then estimate an ARCH(1) model because only α_1 is significant in the ARCH(3) model. Then, we replace the normal distribution with a Student-t distribution. All the results are reported in Table 2.

$$r_t = \mu + a_t,\ a_t = \sigma_t \epsilon_t,\ \epsilon_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \alpha_2 a_{t-2}^2 + \alpha_3 a_{t-3}^2 \tag{1}$$

Table 2: The Estimation Results of ARCH Models

	Dependent variable:		
	(1)	(2)	(3)
mu	0.013**	0.013**	0.017***
	(0.006)	(0.005)	(0.005)
omega	0.010***	0.011***	0.012***
	(0.001)	(0.001)	(0.002)
alpha1	0.233**	0.375***	0.277***
	(0.112)	(0.113)	(0.107)
alpha2	0.075		
	(0.047)		
alpha3	0.052		
	(0.045)		
shape			5.970***
			(1.530)
Observations	444	444	444
Log Likelihood	−303.961	−299.925	−315.090
Akaike Inf. Crit.	−1.347	−1.337	−1.401
Bayesian Inf. Crit.	−1.301	−1.310	−1.364
Note:	*p<0.1; **p<0.05; ***p<0.01		

The log-likelihood function and AIC point to the ARCH(1) model, while BIC points to the ARCH(3) model. We check the model adequacy by testing autocorrelation in the stardardized residuals and squared residuals.

The GARCH model

While the ARCH models fit the data well, there is still some correlation in the squared standardized residuals at the lags after 10. Instead of adding many lagged terms in an ARCH model, we entertain a GARCH(1, 1,) model for the log return series for the sake of parsimony.

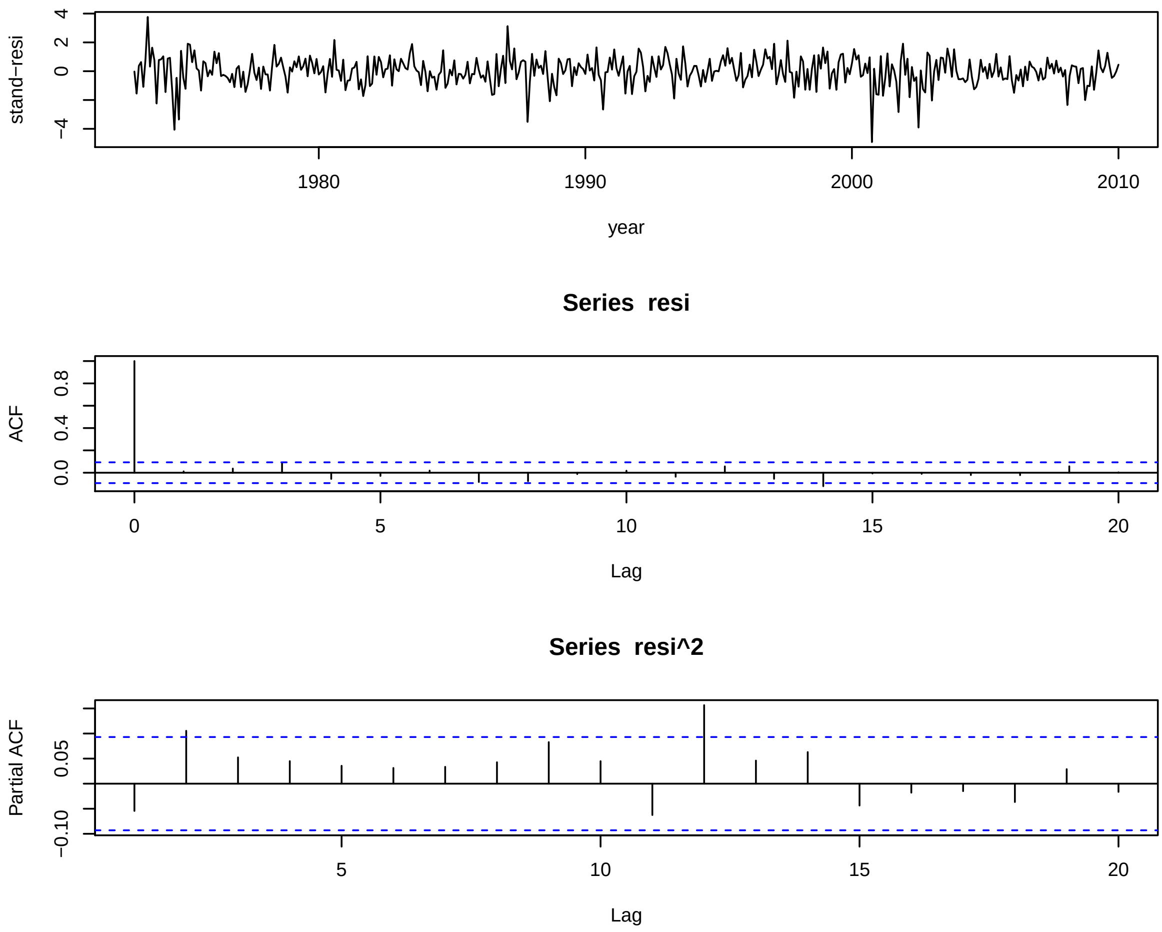

Figure 3: Model checking of the Gaussian ARCH(1) model

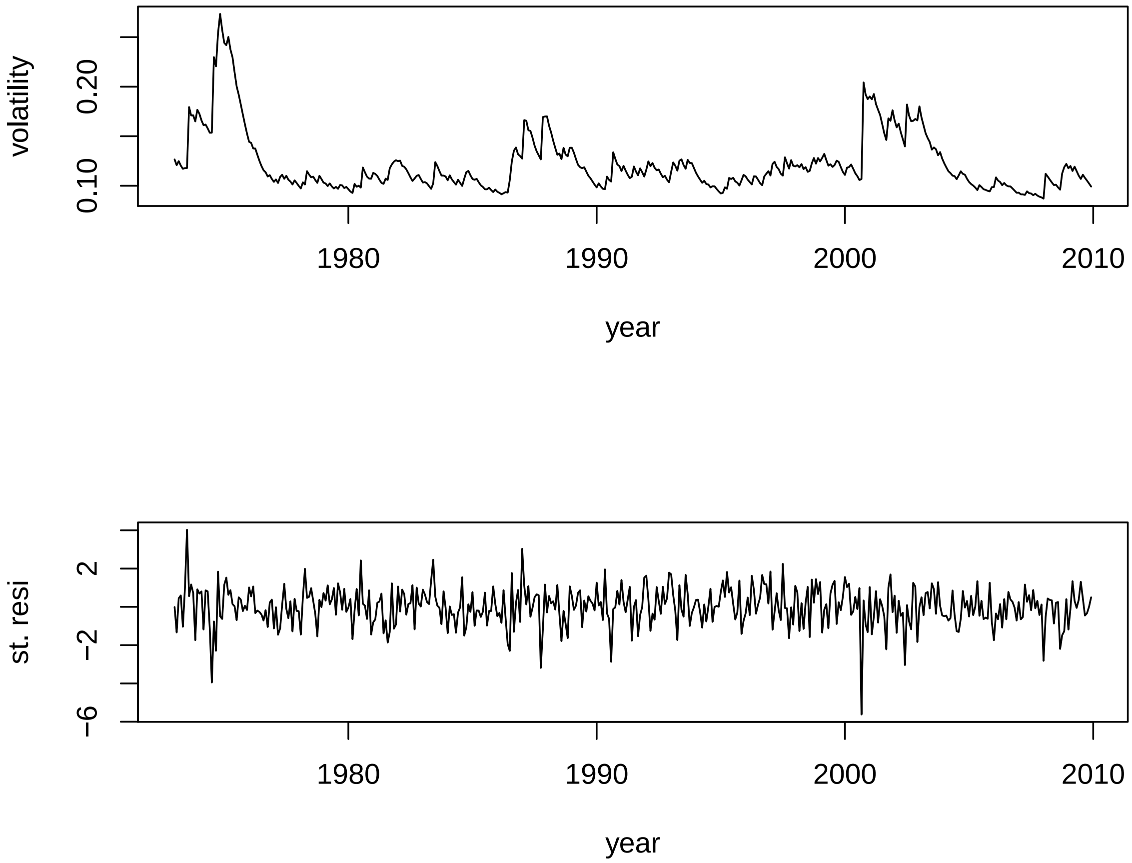

Figure 4: Time plots for a fitted Gaussian GARCH(1, 1) model

A GARCH(1, 1) model is as follows

$$r_t = \mu + a_t,\ a_t = \sigma_t \epsilon_t,\ \epsilon_t \sim N(0, 1)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

We plot the fitted volatility and the standardized residuals from the GARCH(1, 1) model in Figure 4. Next, we check model adequacy the ACF of $\tilde{a}_t$ and $\tilde{a}_t^2$ in Figure 5. And add the predicative intervals to the log return series in Figure 6.

4 Conclusion

References

Ruey S. Tsay. *An Introduction to Analysis of Financial Data with R.* John Wiley & Sons, Inc., Hoboken, New Jersey, 2013.

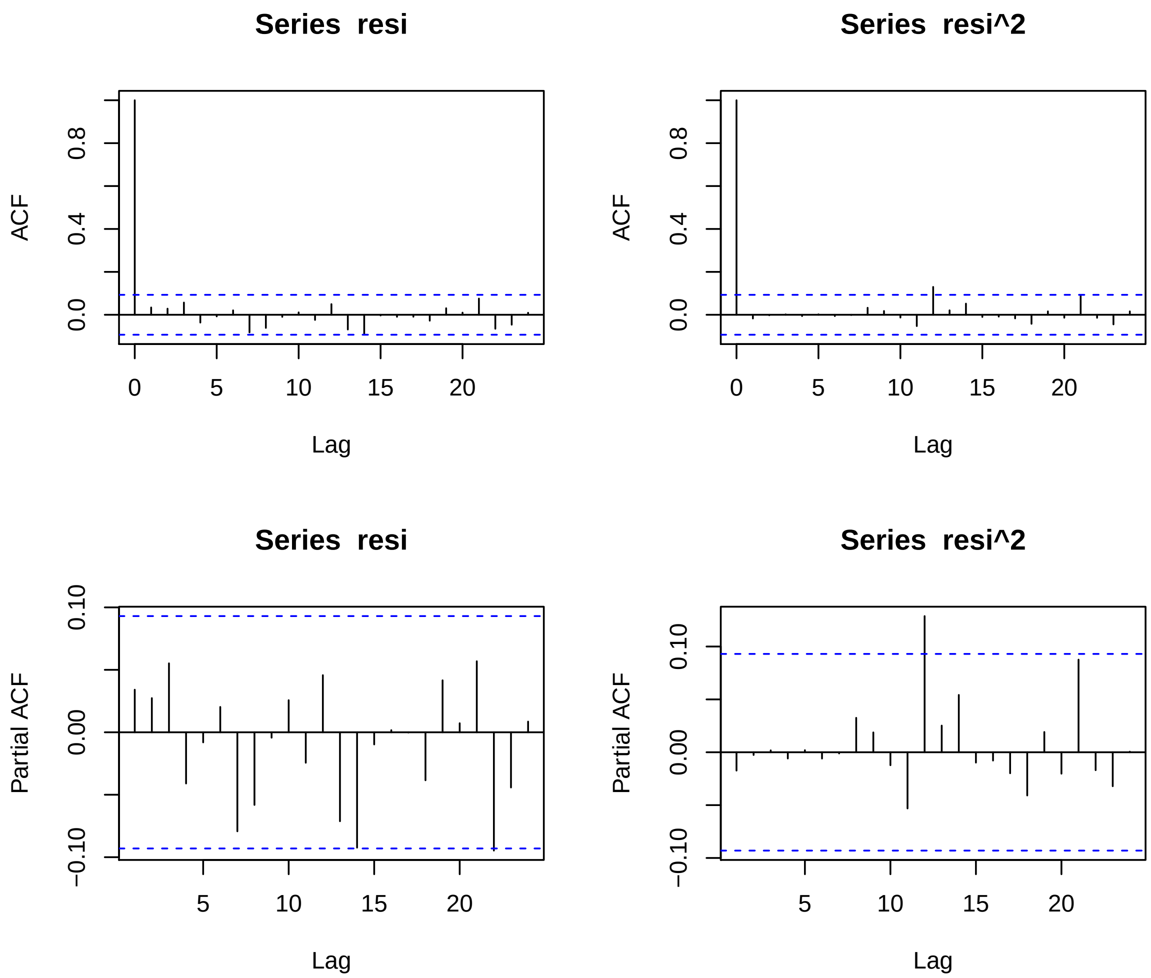

Figure 5: Sample ACF and PACF of the standardized residuals and their squared series of a Gaussian GARCH(1,1) model

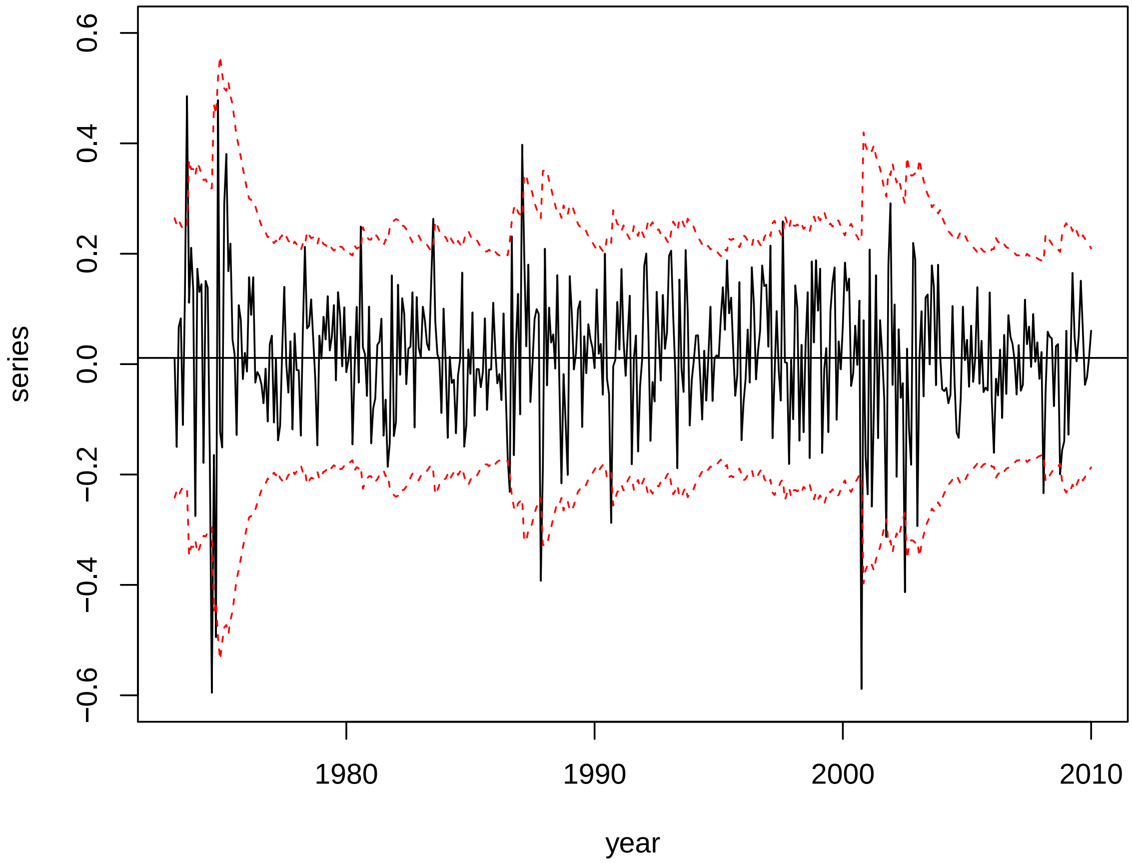

Figure 6: Time plot of the monthly log returns of Intel stock with the point-wise predicative intervals