Financial Data Analysis

Homework 2

Due date: April 3, 2017

1. **AR and MA Models**

   (a) Consider an AR(2) model,

   $$x_t = \phi_0 + \phi_2 x_{t-2} + a_t,$$

   i. Calculate the mean and variance of x_t.
   ii. Calculate the autocorrelation function.
   iii. Derive the condition that this AR(2) model is stationary.

   (b) Consider a MA(2) model

   $$x_t = c_0 + a_t - \theta_2 a_{t-2},$$

   i. Calculate the mean and variance of x_t.
   ii. Calculate the autocorrelation function.
   iii. Is this MA(2) model stationary? Why?

   (c) Describe two differences between an AR(2) model and an MA(2) model of a time series.

   Use 5% level in all tests.

2. Consider the growth rates of the real quarterly gross domestic product (GDP) of Canada from the second quarter of 1980 to the second quarter of 2011 for 125 data points. The following Figure 1 shows the PACF of the GDP growth rates.

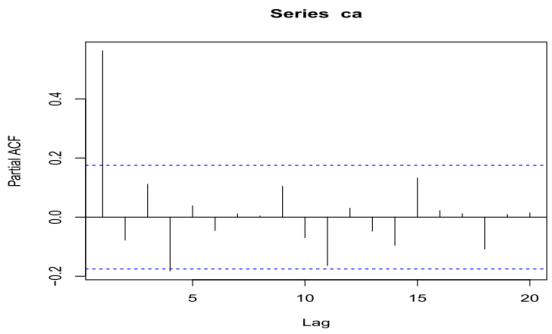

Figure 1: The sample partial autocorrelation function of the quarterly growth rates of Canadian gross domestic product from 1980.II to 2011.II.

   (a) Specify two possible AR models for the growth rate series and briefly justify your choices.

(b) The order selection via AIC is given below. The criterion selects an AR(4) model. An AR(4) model is estimated. Write down the fitted model, including the residual variance.

```
######### Canadian GDP ###
> dim(qgdp)
[1] 126   5
> y=log(qgdp[,3:5])
> head(y)
        uk       ca       us
1 12.05778 13.34518 15.59190
...
6 12.02211 13.38773 15.60021
> ca=diff(y$ca)
> pacf(ca)   ### See Figure 1
> m0=ar(ca,method="mle")
> m0$order
[1] 4
> m1=arima(ca,order=c(4,0,0))
> m1
Call: arima(x = ca, order = c(4, 0, 0))
Coefficients:
         ar1      ar2     ar3      ar4  intercept
      0.6485  -0.1757  0.2334  -0.2068     0.0060
s.e.  0.0880   0.1037  0.1032   0.0899     0.0011

sigma^2 estimated as 3.898e-05:  log likelihood = 456.85,  aic = -901.
```

(c) Describe how to verify the adequacy of this AR(4) model?

(d) Consider the fitted AR(4) model. Does it imply the existence of business cycles in the Canadian economy? Why?

(e) If business cycles exist, compute the periods of all possible cycles.

3. Consider the monthly US unemployment rate from January 1948 to November 2011 in the file **m-unrate-4811.txt**. The data are seasonally adjusted and obtained from the Federal Reserve Bank at St Louis.

(a) Does the monthly unemployment rate have a unit root? Why?

(b) Build a time series model for the monthly unemployment rates.

(c) Check the fitted model for adequacy.

(d) Revise your model if it is not adequate. Maybe you can consider seasonal factors.

(e) Forecast the unemployment rate for the December 2011 and the first three months of 2012. (Note that there are more than one model that fits the data well. You only need an adequate model.)

(f) Does the fitted model imply the existence of business cycles? Why?