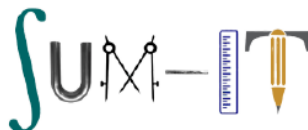




INTEGRATION 2020

(The annual techno-cultural-sports fest organized by the students of Indian Statistical Institute, Kolkata)

Presents



SUM - IT 2020

Sample Problems for Class XI

1. State True or False:

- (a) 2020 can be written as sum of two perfect squares.
- (b) 2020 can be written as sum of two perfect cubes.

2. A sequence $\{a_n\}$ is defined by $a_1 = 2$ and $a_{n+1} = 2^{a_n}$ for $n \geq 1$. What is the remainder when $(a_1 + a_2 + \dots + a_{20})$ is divided by 20?

- A. 10 B. 12 C. 14 D. 16

3. If complex numbers a and b satisfy:

$$\begin{cases} a^4 + a^2b^2 + b^4 = 2020 \\ a^2 + ab + b^2 = 101 \end{cases}$$

then the real part of $2ab$ is:

- A. 81 B. 0 C. -81 D. 20

4. $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies:

$$f(f(x)(y+z)) = x(f(y)+z) \quad \forall x, y, z \in \mathbb{R}.$$

Then which of the following is true?

- (a) f is one-one.
- (b) f is onto.

- A. (a) B. (b) C. Both (a) and (b) D. None of (a) and (b)

5. Let t_n be the hundreds digit of $23n$ for $n \in \mathbb{N}$. Find the number of n less than 1000 so that t_n and t_{n+1} are different.

6. Suppose $a \in \mathbb{R}$ is such that

$$a + \frac{1}{a} = \frac{\sqrt{5} + 1}{2}.$$

Find the value of:

$$a^{2020} + \frac{1}{a^{2020}}.$$

7. For a fixed $n \in \mathbb{N}$ find a polynomial $p(x)$ with one coefficient $\frac{1}{n}$ and other coefficients rational so that $p(x) \in \mathbb{Z}$ whenever $x \in \mathbb{Z}$.
8. Suppose for $a, b \in \mathbb{R}$, $f : [a, b] \rightarrow \mathbb{R}$ is a continuous function such that $f(a) = f(b)$. Show that there exist constants $c_1, c_2 \in (a, b)$ such that $c_2 - c_1 = \frac{b-a}{2}$ and $f(c_1) = f(c_2)$.
9. Little Dennis has seen his mom to hide a box of delicious sweets from him inside a strange refrigerator. But he could not see on which shelf it is kept. The shape of the refrigerator is like a pyramid: 1 chamber on the topmost shelf, 2 chambers on the shelf below that and so on. In the shelf where the box is kept, each chamber is connected with exactly three other chambers except the chamber containing the box, which is connected to only one chamber. Dennis also knows that his mom will try to keep the box as high as possible. But he can search only one shelf before getting caught. Can you help Dennis to find on which shelf the box is kept?
10. In the country of Sumitland, there are n cities each connected with one another by bus routes. Each bus route is coloured using two colours: red and blue. Find the smallest value of n for which we can find 2020 distinct cities $C_1, C_2, \dots, C_{2020}$ in Sumitland so that we can travel through the cities $C_1, C_2, \dots, C_{2020}$ in that order using 2019 bus routes of same colour.