INTEGRATION 2020

(The annual techno-cultural-sports fest organized by the **students of Indian Statistical Institute, Kolkata**)

Presents

SUM - IT 2020

Sample Problems for Class IX

1. State True or False:
   (a) 2020 can be written as sum of two perfect squares.
   (b) 2020 can be written as sum of two perfect cubes.

2. $\triangle ABC$ is a right angled triangle with right angle at A. The circles with diameters AB and AC intersect again at $D\,(\neq A)$. If $AB = 10$ and $AC = 24$, the length of AD can be expressed as $\frac{p}{q}$ for relatively prime positive integers p and q. Find $p + q$.

3. What is the value of
$$\left\lfloor \frac{2^{31} + 3^{31}}{2^{29} + 3^{29}} \right\rfloor$$
where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x?
   A. 4 B. 5 C. 8 D. 9

4. In $\triangle ABC$, $AB = 5$, $BC = 6$, $CA = 7$. A circle with center on BC touches AB and AC at D and E respectively.
   (a) Length of AD is:
   A. 3.5 B. 4 C. 4.5 D. None of these
   (b) Radius of the circle is:
   A. $\sqrt{6}$ B. $2\sqrt{6}$ C. $3\sqrt{6}$ D. None of these

5. What is the value of:
$$\sqrt[3]{72 + 32\sqrt{5}} + \sqrt[3]{72 - 32\sqrt{5}}?$$
   A. $2\sqrt{5}$ B. $4\sqrt{5}$ C. 4 D. 6

6. For $n \in \mathbb{N}$, let $d(n)$ denote the number of positive divisors of n. Define $d^{(2)}(n) = d(d(n))$, $d^{(3)}(n) = d(d(d(n)))$ and so on. Find the smallest positive integer k so that $d^{(k)}(2020^{2020}) = 2$.

7. Find the number of ordered pair of integers (a, b) taken from the set $S = \{300, 301, \ldots, 399\}$ so that a and b can be added without carrying over.

8. You are given an angle with measure $\frac{3\pi}{7}$ radians (or $\frac{3\pi}{13}$ radians). Using only a compass and an unmarked ruler describe a method to trisect the given angle.

9. Harry has a rectangular box with mirrors fitted on the inner walls. The box is 57 cm long and 82 cm wide and has four pin holes at the four base corners. Denote the base rectangle by $ABCD$ so that $AB = CD = 82$ cm and $BC = DA = 57$ cm. A light ray enters the box through corner A making 45° angle with the walls. Through which corner it will come out?

10. Five persons A, B, C, D, E are standing on the edge of a volcanic cliff and in front of them, there is a hanging bridge. All of them want to cross the bridge before the volcano erupts. But unfortunately, only two people can cross the bridge at the same time. It is already very dark and they have only one lantern with them and one must carry it while crossing the bridge. It is extremely risky to cross the bridge without carrying the lantern. A, B, C, D and E can cross the bridge in 1, 3, 5, 7 and 9 minutes respectively and the volcano is going to erupt after just 25 minutes. Can you tell them how to cross the bridge in order to save everyone?