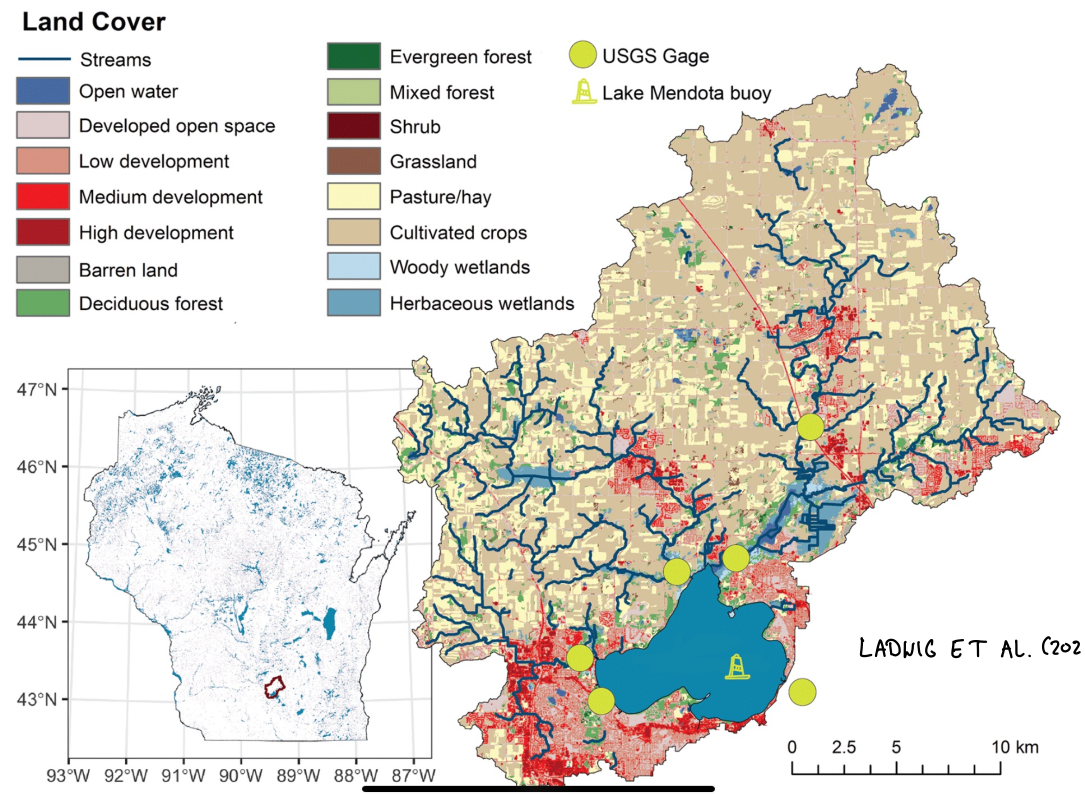
Local Lake Analysis

- model = GOTM
- GCM = GFDL-ESM
- scenario = SSP5-8.5 (2015-2100)

- Lake Mendota, Wisconsin, USA
- eutrophic
- dimictic
- about 25 m deep



Schmidt Number

Mass of the water column

$$M = \int \varrho A \, dz$$

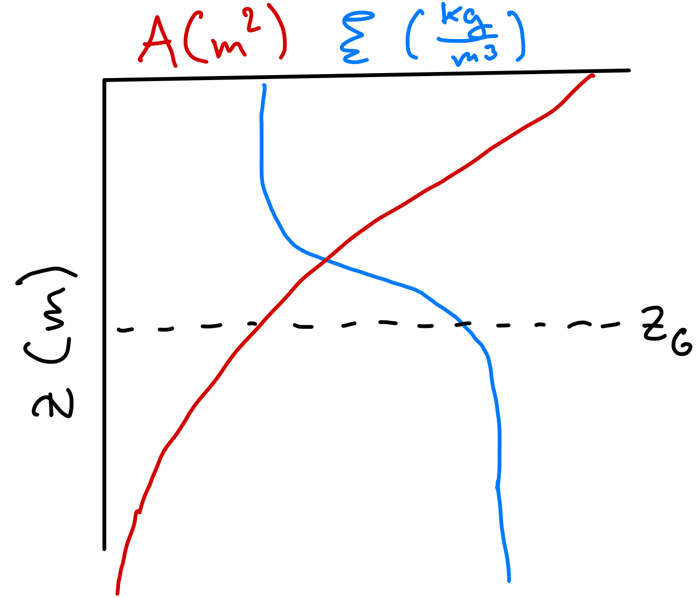
Center of mass/gravity

$$z_G = \frac{1}{\int \varrho A \, dz} \int z \varrho A \, dz$$

$$= \frac{1}{M} \int z \varrho A \, dz$$

Mean density

$$\hat{\varrho} = \frac{1}{V} \int \varrho A \, dz$$



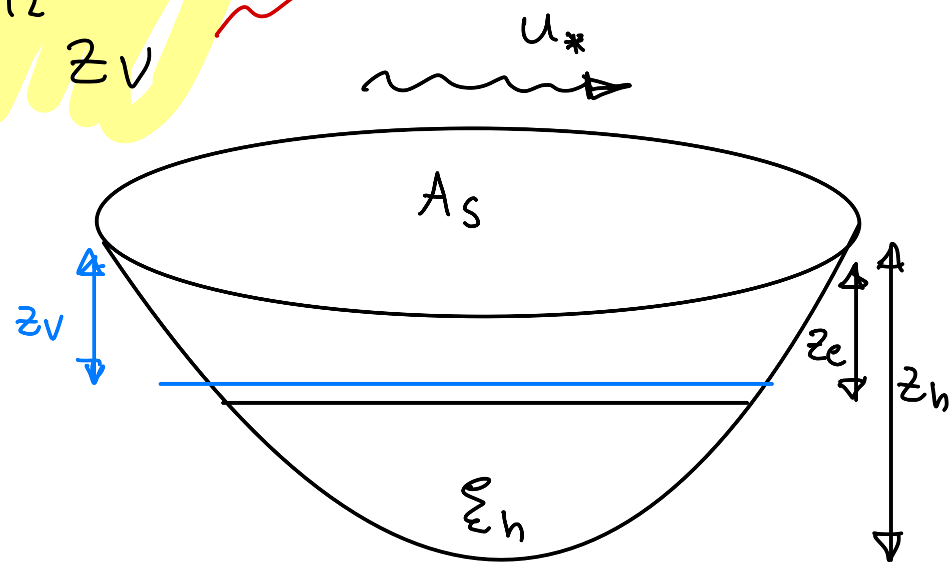
$$S_t = \frac{g}{A_0} \int (z - z_G)(\varrho - \hat{\varrho}) A \, dz$$

Lake Number

$$L_N = \frac{S_t\,(z_e + z_h)}{2\,\xi_h\,u_*^2\,A_s^{1/2}\,z_v}$$

Stabilizing forces

Mixing forces



Center of volume

$$z_v = \frac{1}{V}\int z\,A\,dz$$

Coupled Model

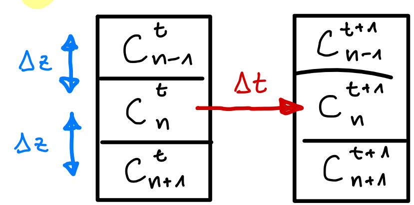
(1) ISIMIP temperature output as input for vertical mixing (Hondzo & Stefan, 1993)

$$K_z = a_k \left(N_z^2\right)^{-0.43} \quad \text{WITH} \quad a_k = 0.00706\, A_S^{0.56}$$

$$N_z^2 = \frac{g}{\varepsilon_z} \frac{d\varepsilon_z}{dz}$$

(2) Build our custom model

$$\frac{\partial C}{\partial t} = K \frac{\partial^2 C}{\partial z^2} \xrightarrow[\text{(FTCS)}]{\text{numerics}} C_n^{t+1} = C_n^t + K\frac{\Delta t}{\Delta z^2}\left(C_{n+1}^t - 2C_n^t + C_{n-1}^t\right)$$



(explicit)

(3) Be creative

$$\boxed{\frac{\partial T}{\partial t} = K \frac{\partial T^2}{\partial z^2}}$$

Taylor expansion:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + O(a^4)$$

central differencing:

$n \sim$ SPACE

$t \sim$ TIME

$$T_{n+1} = T_n + \Delta z \frac{\partial T}{\partial z} + \frac{\Delta z^2}{2!}\frac{\partial^2 T}{\partial z^2} + \frac{\Delta z^3}{3!}\frac{\partial^3 T}{\partial z^3} + O(\Delta z^4)$$

$$T_{n-1} = T_n - \Delta z \frac{\partial T}{\partial z} + \frac{\Delta z^2}{2!}\frac{\partial^2 T}{\partial z^2} - \frac{\Delta z^3}{3!}\frac{\partial^3 T}{\partial z^3} + O(\Delta z^4)$$

$$T_{n+1} + T_{n-1} = 2T_n + \Delta z^2 \frac{\partial^2 T}{\partial z^2} + O(\Delta z^4)$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{T_{n+1} - 2T_n + T_{n-1}}{\Delta z^2} + O(\Delta z^4)$$

forward in time:

$$\frac{\partial T}{\partial t} = \frac{T^{t+1} - T^t}{\Delta t}$$

FTCS scheme:

$$\boxed{T_n^{t+1} = T_n^t + \Delta t\, K \left(\frac{T_{n+1}^t - 2T_n^t + T_{n-1}^t}{\Delta z^2}\right)}$$