

Unbiased Monte Carlo for Recursive Integrals

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Abstract

We will write this at the end.

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1 Introduction

Follows $y' = y$ example + modifications This introduces recursive Monte Carlo and important Monte Carlo techniques used extensively later.

2 Essentials

Example 2.1 (non-linearity)

$$y' = y^2$$

Example 2.2 (analytic functions)

$$e^{E[X]}$$

Example 2.3 (half variance phenomenon)

randomized trapezium rule for introduction of the half variance phenomenon and an argument by integrating a polynomial for intuition

Technique 2.4 (tail recursion)

discuss problems with implementing recursion and solutions

Example 2.5 (coupled recursion)

example with $y' = y$

Example 2.6 (recursion in recursion)

maybe induction in induction proof example

3 1D Recursive Integrals

Example 3.1 (general linear recursive integral)

We have algo in mind for this case based on coupled recursion on disjunct sets.

Theorem 3.2 (green functions)

green function stuff that we will be needing, we aren't sure in how much detail we're going to go.

Example 3.3 (IVP)

An IVP example probably using DRRMC maybe compare it to parareal. Maybe also non-linear algo

Example 3.4 (BVP)

A BVP example using yet another algo that hopefully has the half variance phenomenon.

4 Higher D Recursive Integrals

Example 4.1 (problems with geometry)

2D integral that is difficult because of its geometry

Example 4.2 (numerical green functions)

There will be probably some green functions that we need that don't have an analytic expression yet.

Example 4.3 (recursive Brownian motion)

WoS like way to simulate Brownian motion which is related to the green function of the heat equation

Example 4.4 (heat equation)

a geometric robust way to solve the heat equation and a higher order method to solve the heat equation

5 Appendix

Derivation of the green functions and some expressions.