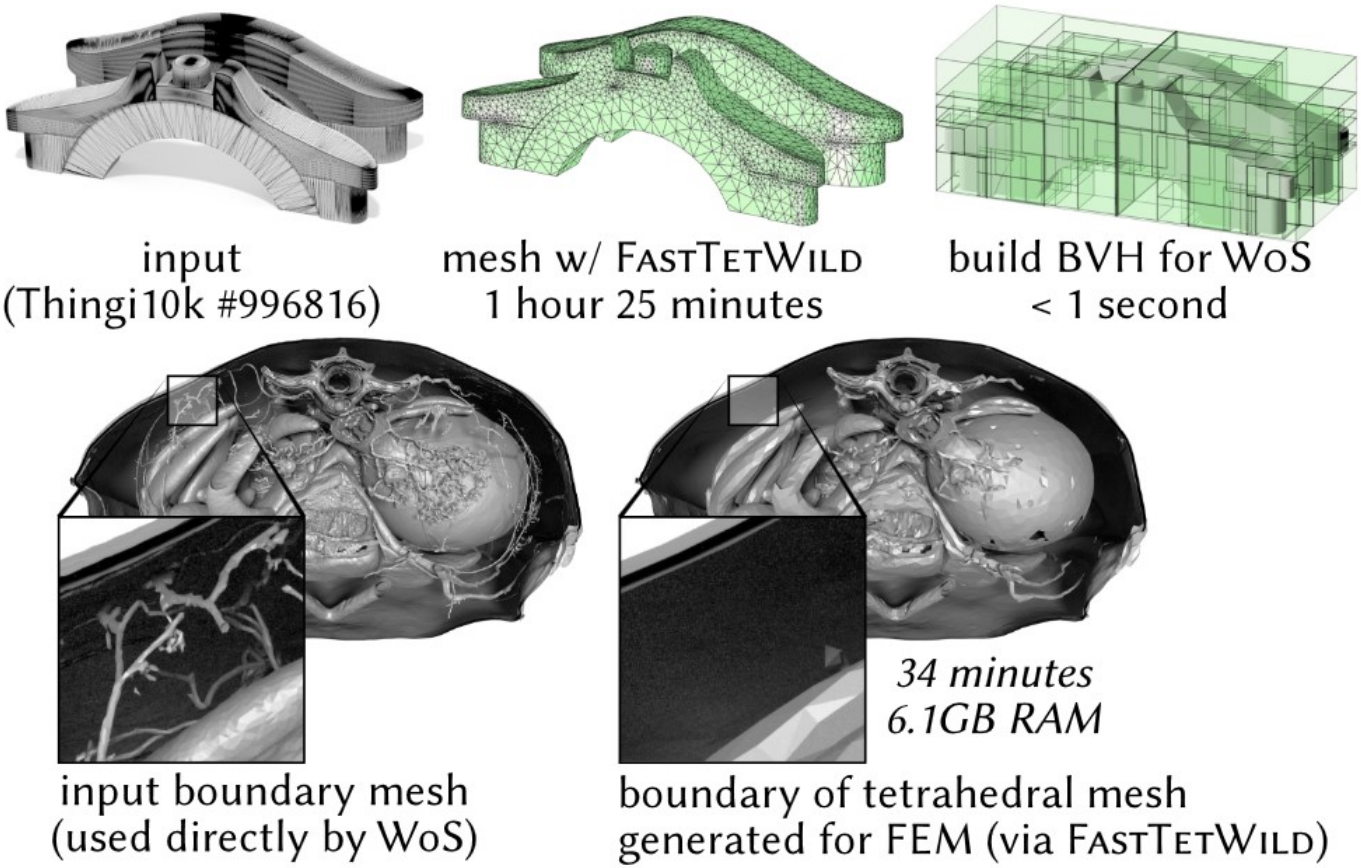
RMC voor lineaire ODEs

Isidoor Pinillo Esquivel

Grid-Free Monte Carlo

Sawhney e.a. 2022



Monte Carlo

$$\int_0^1 f(x)dx = E[f(U)] \tag{1}$$

$$\approx \frac{1}{n}\sum_{j=1}^{n} f(U_j) \tag{2}$$

met $U_j \sim \text{Uniform}(0,1)$

Waarom Monte Carlo?

- eenvoudig
- paralleliseerbaar
- dimensie onafhankelijke convergentie
- complexe geometrie
- enkel simulaties beschikbaar

Waarom ODEs?

- grid-free + tijdafhankelijkheid
- ODEs simpeler als PDEs

Stochastic Gradient Descent

$$\text{SGD} = \text{GD} + \text{unbiased gradients}$$

Stochastic Gradient Descent

$$\text{SGD} = \text{GD} + \text{unbiased gradients}$$

$$f(x) = \frac{1}{n}\sum_{j=1}^{n} f_j(x) \tag{3}$$

(4)

(5)

Stochastic Gradient Descent

$$\text{SGD} = \text{GD} + \text{unbiased gradients}$$

$$f(x) = \frac{1}{n}\sum_{j=1}^{n} f_j(x) \tag{3}$$

$$\nabla f(x) = \frac{1}{n}\sum_{j=1}^{n} \nabla f_j(x) \tag{4}$$

(5)

Stochastic Gradient Descent

$$\text{SGD} = \text{GD} + \text{unbiased gradients}$$

$$f(x) = \frac{1}{n}\sum_{j=1}^{n} f_j(x) \tag{3}$$

$$\nabla f(x) = \frac{1}{n}\sum_{j=1}^{n} \nabla f_j(x) \tag{4}$$

$$= E[\nabla f_J(x)] \tag{5}$$

Russische Roulette Voorbeeld

$$Z = U + \frac{f(U)}{1000} \tag{6}$$

$$U \sim \text{Uniform}(0, 1), \quad f \text{ duur} \tag{7}$$

(8)

(9)

Russische Roulette Voorbeeld

$$Z = U + \frac{f(U)}{1000} \tag{6}$$

$$U \sim \text{Uniform}(0, 1), \quad f \text{ duur} \tag{7}$$

$$\tilde{Z} = U + B\left(\frac{1}{100}\right)\frac{f(U)}{10} \tag{8}$$

$$B(p) \sim \text{Bernoulli}(p) \tag{9}$$

Control variate

- voorbeeld (+ tekening)

RMSE vs fout begrenzing

- verschil (+ formule)
- mss: steins paradox (yt video)
- MSE decompositie
- tabel: 0 variantie vs 0 bias

Trapezium MC

- def (+ tekening)
- kleine afleiding
- tabel: 0 variantie vs 0 bias

Unbiased non-linearity

- exponentiele voorbeeld + screenshot paper
- VRE
- Feynman-Kac formule
- Magnus series

RMC voorbeeld

$$y' = y \tag{10}$$

(11)

(12)

(13)

(14)

RMC voorbeeld

$$y' = y \tag{10}$$

$$y(t) = y(0) + \int_0^t y(s)ds \tag{11}$$

(12)

(13)

(14)

RMC voorbeeld

$$y' = y \tag{10}$$

$$y(t) = y(0) + \int_0^t y(s)ds \tag{11}$$

$$\text{wil } Y : E[Y(t)] = y(t) \tag{12}$$

$$\tag{13}$$

$$\tag{14}$$

RMC voorbeeld

$$y' = y \tag{10}$$

$$y(t) = y(0) + \int_0^t y(s)ds \tag{11}$$

$$\text{wil } Y : E[Y(t)] = y(t) \tag{12}$$

$$Y(t) = y(0) + tY(S) \tag{13}$$

$$S \sim \text{Uniform}(0, t) \tag{14}$$

RMC voorbeeld

$$Y(t) = y(0) + tY(S) \tag{15}$$
$$\tag{16}$$
$$\tag{17}$$
$$\tag{18}$$
$$\tag{19}$$

tekening

RMC voorbeeld

$$Y(t) = y(0) + tY(S) \tag{15}$$

$$\infty \text{ recursie} \tag{16}$$

(17)

(18)

(19)

tekening

RMC voorbeeld

$$Y(t) = y(0) + tY(S) \tag{15}$$

$$\infty \text{ recursie} \tag{16}$$

$$Y(t) = 1 + B(t)Y(S) \tag{17}$$

$$t < 1 \tag{18}$$

$$B(t) \sim \text{Bernoulli}(t) \tag{19}$$

tekening

RRMC voorbeeld