Project 2023-2024: Simulation of the Double Slit Experiment and simulation of Wi-Fi signals at home

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1 Implementation of Exterior Complex Scaling (ECS) Boundary Conditions

1.1 Equivalence Between Complex Grid and Complex Wave Number

The homogenous discretized Helmholtz equation ECS is equivalent to to the Helmholtz equation with a complex wave number. Let h be the normal "real" grid spacing and $\tilde{h} = zh, z \in \mathbb{C}$ be the complex grid spacing. Let σ be the normal real wave number and $\tilde{\sigma} = z^2\sigma$ be the complex wave number. Let u be the solution to the discretized Helmholtz equation with a complex wave number on a normal grid and $\tilde{u}$ be the solution to the discretized Helmholtz equation on the complex grid.

$$\frac{\tilde{u}(\tilde{x} - \tilde{h}) - 2\tilde{u}(\tilde{x}) + \tilde{u}(\tilde{x} - \tilde{h})}{\tilde{h}^2} + \sigma\tilde{u} = 0 \Leftrightarrow \tag{1}$$

$$\frac{\tilde{u}(\tilde{x} - zh) - 2\tilde{u}(\tilde{x}) + \tilde{u}(\tilde{x} - zh)}{z^2h^2} + \sigma\tilde{u} = 0 \Leftrightarrow \tag{2}$$

$$\frac{u(x - h) - 2u(x) + u(x - h)}{h^2} + z^2\sigma u = 0 \Leftrightarrow \tag{3}$$

$$\frac{u(x - h) - 2u(x) + u(x - h)}{h^2} + \tilde{\sigma}u = 0 \tag{4}$$

This equivalence does not hold when there is a source term.

1.2 Non-Uniform Helmholtz Matrix Implementation

Our implementation of the non-uniform of the Helmholtz equation uses following discretization scheme:

$$(\Delta_h u)_i = -\left(\frac{u_{i+1} - u_i}{h_{i+1/2}} - \frac{u_i - u_{i-1}}{h_{i-1/2}}\right)\frac{2}{h_{i+1/2} + h_{i-1/2}}.$$

1.3 Interpolation Matrix Construction

The interpolation matrix is based on linear interpolation on a irregular grid, we only use the real part of the complex grid to do interpolation. The restriction operation is defined through the variational property.

1.4 Validation of V-cycle Implementation with Test Problem

We test our implementation of the V-cycle on a point source problem with $\sigma = -10$.

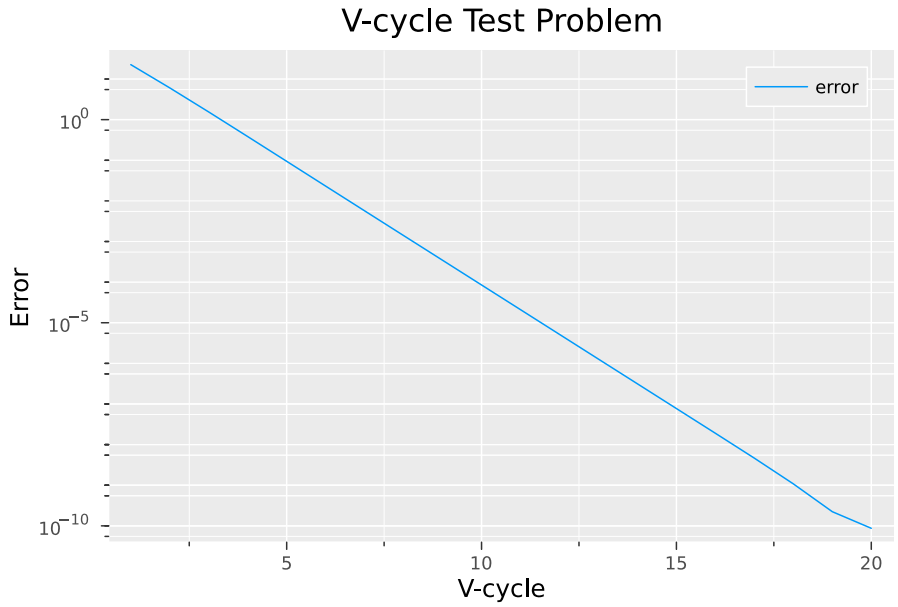

Figure 1: Convergence of the V-cycle for the test problem, demonstrating that the algorithm has been implemented correctly.

2 Simulation of Double Slit Experiment

2.1 Normal Wavenumber

In our previous project, we examined the divergence of the Helmholtz equation with a homogeneous wavenumber. It is not unexpected that introducing inhomogeneity to the wavenumber does not alter the divergence behavior of the multigrid method.

2.2 Spectral Analysis of ECS Poisson Operator

Adding ECS to the Poisson operator makes some eigenvalues complex, but this does not explain the divergent behavior of multigrid. The divergence can be explained by the spectral properties of the Helmholtz operator.

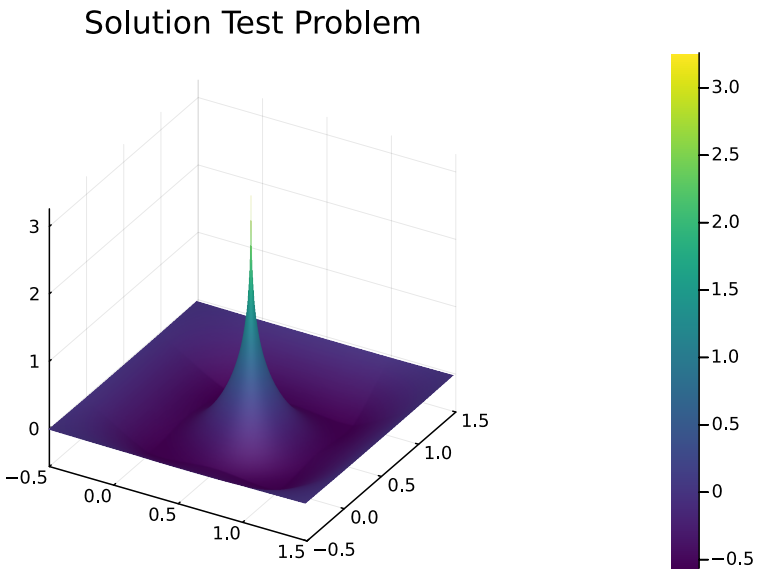

Figure 2: Computed solution for the test problem.

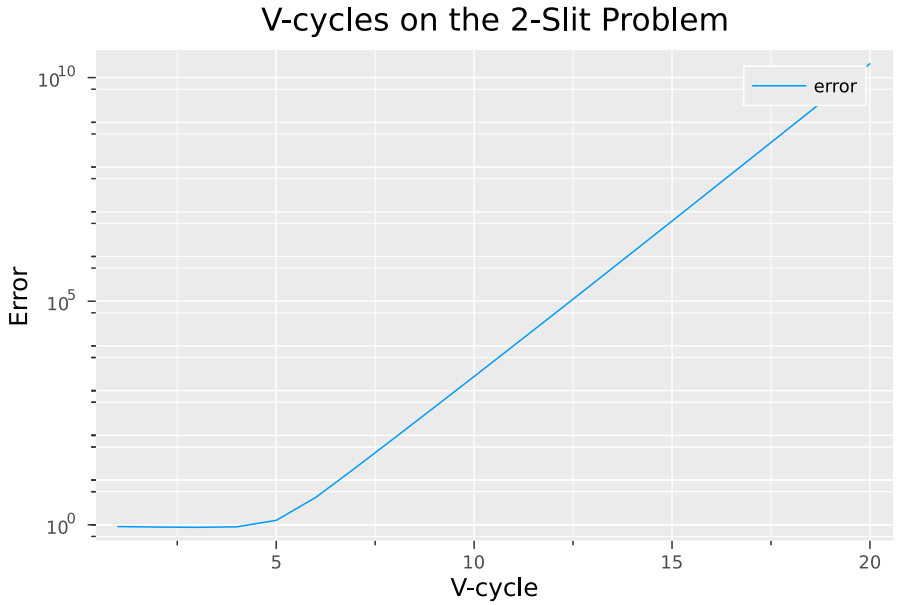

Figure 3: Divergence of V-cycles on the 2-Slit Problem.

2.3 Shifted Wavenumber

Similar to Project 1, adding a complex shift makes multigrid converge.

2.4 Preconditioned GMRES

The shifted problem can serve as a preconditioner for the non-shifted problem.

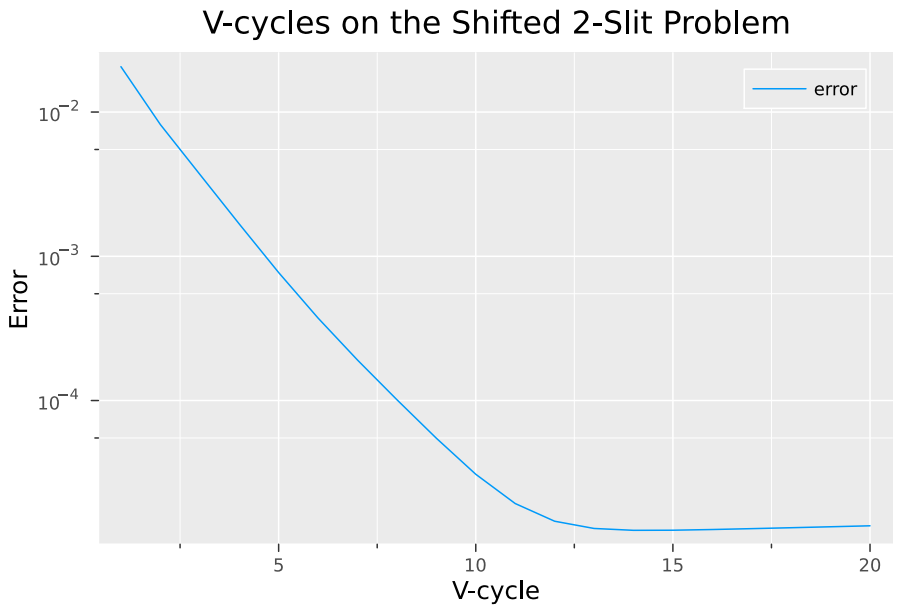

Figure 4: Convergence of V-cycles with shifted wavenumber.

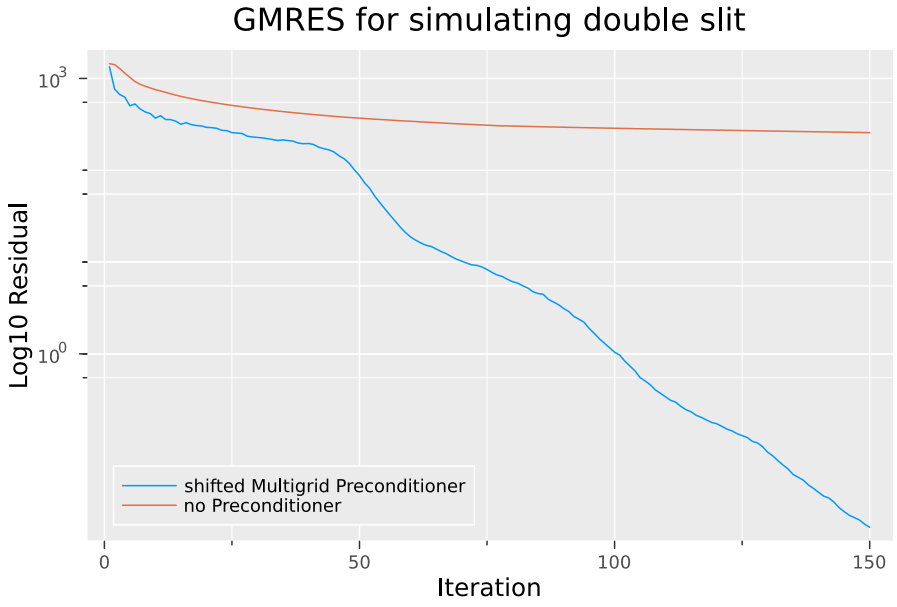

Figure 5: Convergence of Preconditioned GMRES

3 Simulation of Wi-Fi Signal Propagation at Home

3.1 Construction of Wavenumber and Source Representation

We utilized GitHub Copilot to construct the wavenumber and the source, which represents the house.

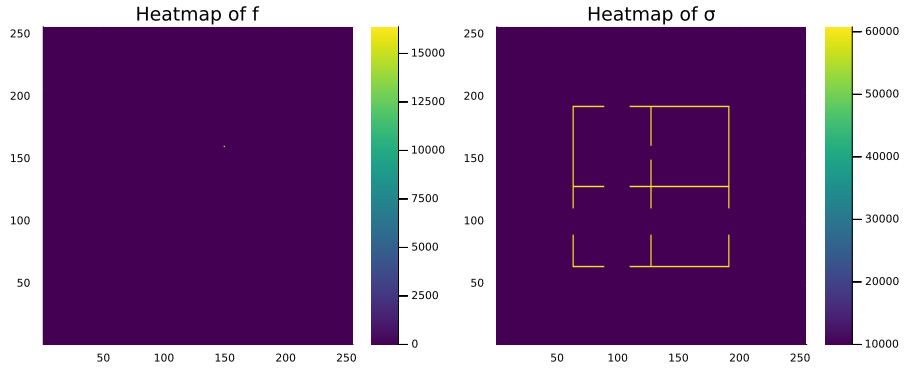

Figure 6: Setup for the Wi-Fi Signal Simulation Problem

3.2 Preconditioned GMRES for Wi-Fi Signal Simulation

This is analogous to the 2-slit problem, where we use the shifted problem as a preconditioner for the non-shifted problem. The convergence of GMRES takes longer due to the more challenging behavior of the wavenumber.

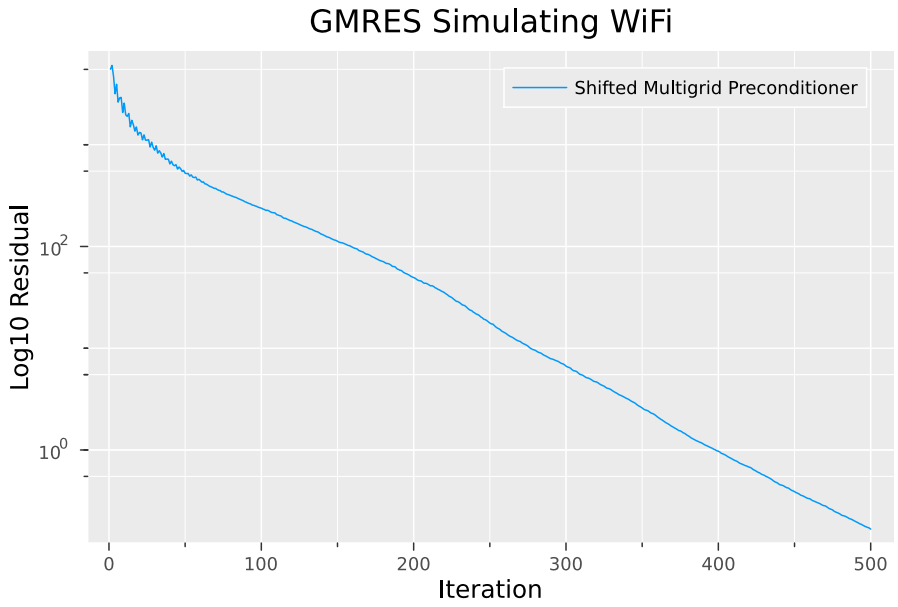

Figure 7: Convergence of Preconditioned GMRES

Below, we present the computed solution to the Wi-Fi problem.

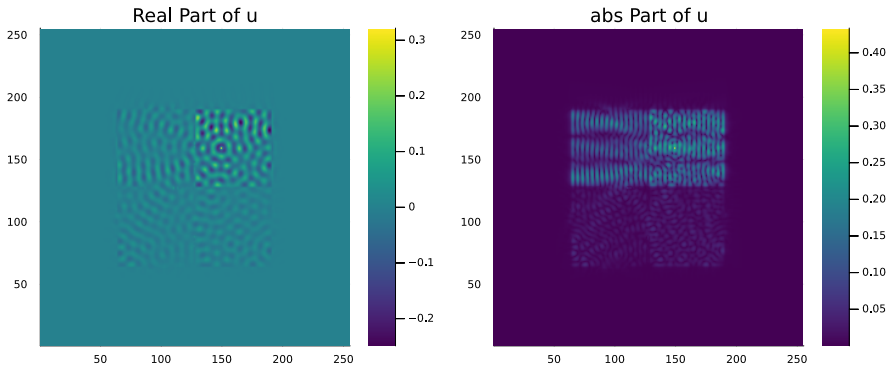

Figure 8: Computed Solution for the Wi-Fi Problem