# Project 2023-2024: Simulation of the Double Slit Experiment and simulation of Wi-Fi signals at home

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## 1 Exterior Complex Scaling (ECS) boundary conditions

### 1.1 Equivalence of complex grid and complex wave number

For homogenous the discretized Helmholtz equation ECS is equivalent to complex wave number. Let $h$ be normal "real" grid spacing and $\tilde{h} = zh, z \in \mathbb{C}$ be the complex grid spacing. Let $\sigma$ be normal real wave number and $\tilde{\sigma} = z^2\sigma$ be the complex wave number. Let $u$ be the solution to the discretized Helmholtz equation with complex wave number on a normal grid and $\tilde{u}$ be the solution to the discretized Helmholtz equation on the complex grid.

$$\frac{\tilde{u}(\tilde{x}-\tilde{h}) - 2\tilde{u}(\tilde{x}) + \tilde{u}(\tilde{x}-\tilde{h})}{\tilde{h}^2} + \sigma\tilde{u} = 0 \Leftrightarrow \tag{1}$$

$$\frac{\tilde{u}(\tilde{x}-zh) - 2\tilde{u}(\tilde{x}) + \tilde{u}(\tilde{x}-zh)}{z^2h^2} + \sigma\tilde{u} = 0 \Leftrightarrow \tag{2}$$

$$\frac{u(x-h) - 2u(x) + u(x-h)}{h^2} + z^2\sigma u = 0 \Leftrightarrow \tag{3}$$

$$\frac{u(x-h) - 2u(x) + u(x-h)}{h^2} + \tilde{\sigma}u = 0 \tag{4}$$

This equivalence doesn't hold when there is a source term.

### 1.2 non uniform helmhotz matrix

Our implementation of the non-uniform of the helmholtz equation uses at its base:

$$(\Delta_h u)_i = -\left(\frac{u_{i+1} - u_i}{h_{i+1/2}} - \frac{u_i - u_{i-1}}{h_{i-1/2}}\right)\frac{2}{h_{i+1/2} + h_{i-1/2}}.$$

### 1.3 interpolation matrix

The interpolation matrix is based on linear interpolation on a irregular grid, we only use the real part of the complex grid to do interpolation. The restriction operation is defined through the variational property.

### 1.4 Test Problem

We test our implementation of the V-cycle on a point source problem with $\sigma = -10$.

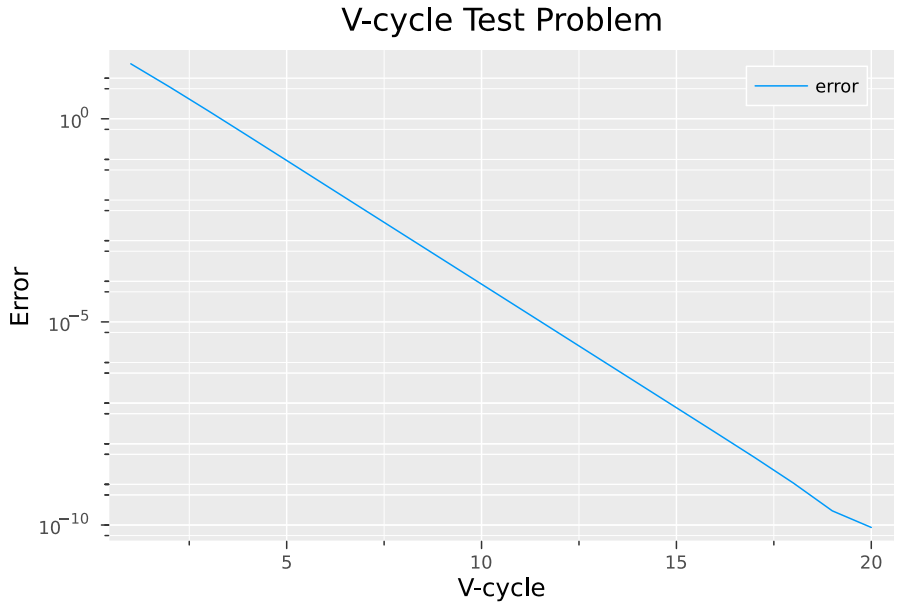


*Figure 1: Convergence of the V-cycle for the test problem, demonstrating that the algorithm has been implemented correctly.*

## 2 Double Slit Experiment

### 2.1 divergence of multigrid 2 slit

In our previous project, we examined the divergence of the Helmholtz equation with a homogeneous wavenumber. It is not unexpected that introducing inhomogeneity to the wavenumber does not alter the divergence behavior of the multigrid method.

### 2.2 analysis of ECS Poisson Operator

Adding ECS to the Poisson operator makes some eigenvalues complex, but this doesn't explain the divergent behavior of multigrid. The divergence can be explained by the spectral properties of the Helmholtz operator.

### 2.3 convergence of shifted multigrid 2 slit

Similar to Project 1, adding a complex shift makes multigrid converge.

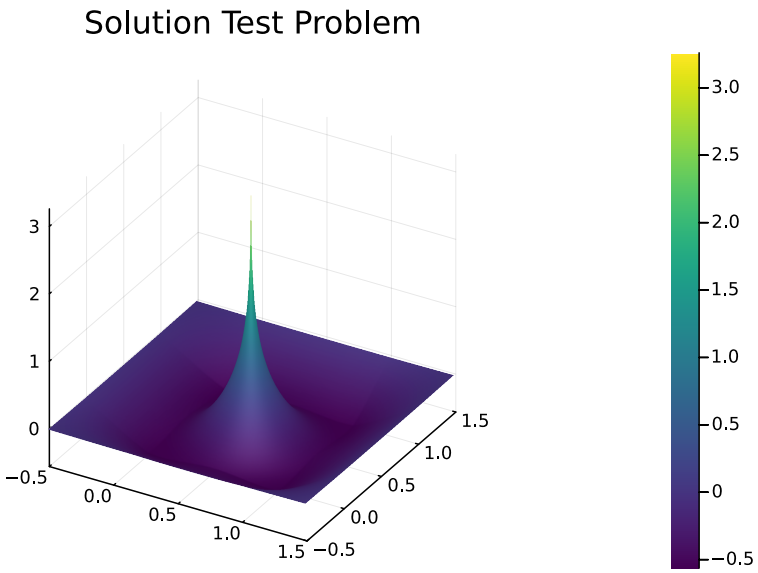


*Figure 2: Computed solution for the test problem.*

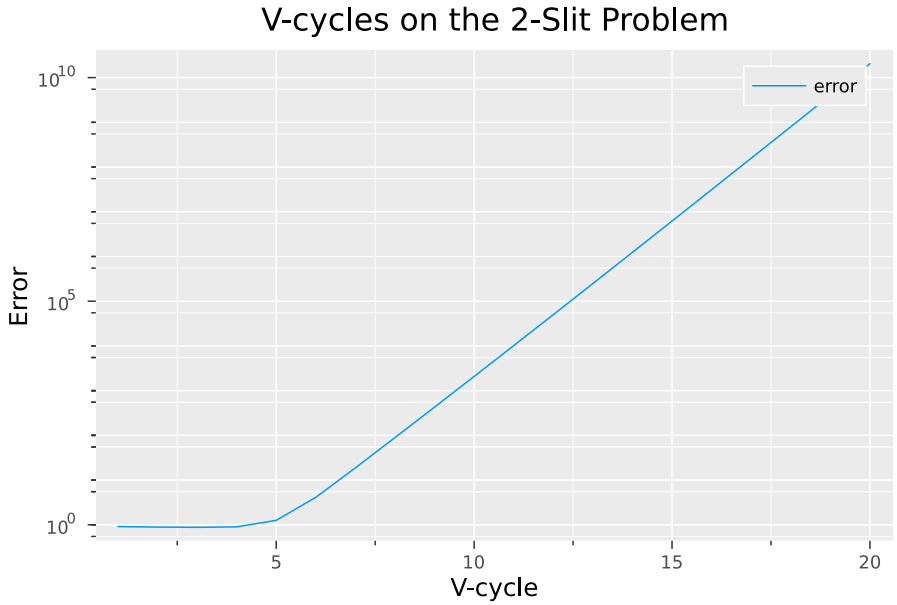


*Figure 3: Divergence of V-cycles on the 2-Slit Problem.*

### 2.4 preconditioned GMRES

The shifted problem can serve as a preconditioner for the non-shifted problem.

## 3 Wi-Fi signals at home

### 3.1 wavenumber and source

We utilized GitHub Copilot to construct the wavenumber and the source, which represents the house.

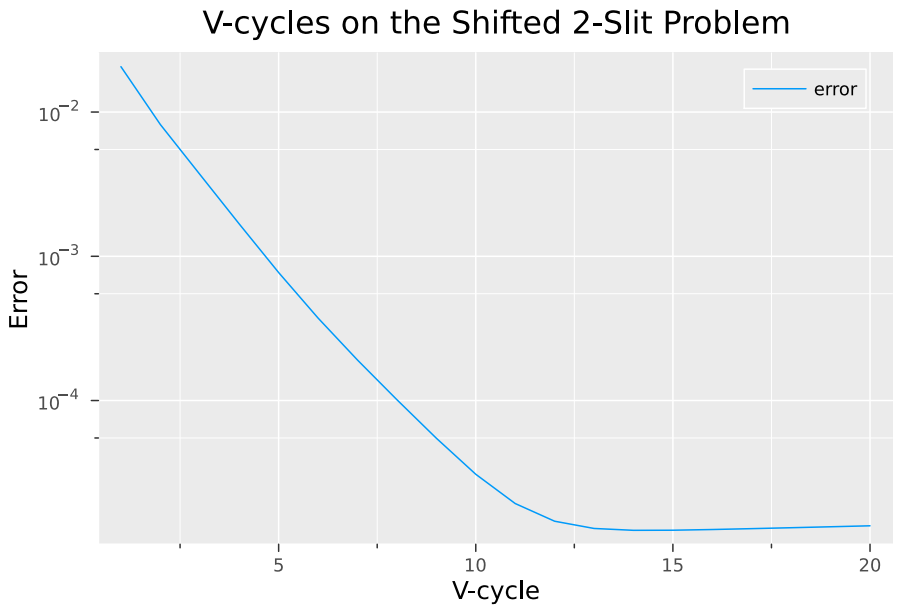


*Figure 4: Convergence of V-cycles with shifted wavenumber.*

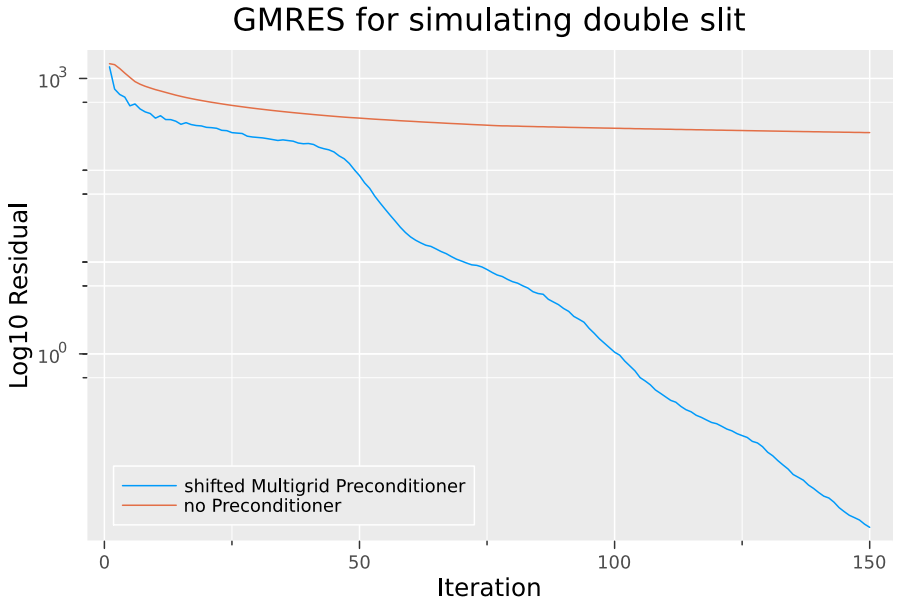


*Figure 5: Convergence of Preconditioned GMRES*

## 3.2 preconditioned gmres

This is analogous to the 2-slit problem, where we use the shifted problem as a preconditioner for the non-shifted problem. The convergence of GMRES takes longer due to the more challenging behavior of the wavenumber.
Below, we present the computed solution to the Wi-Fi problem.

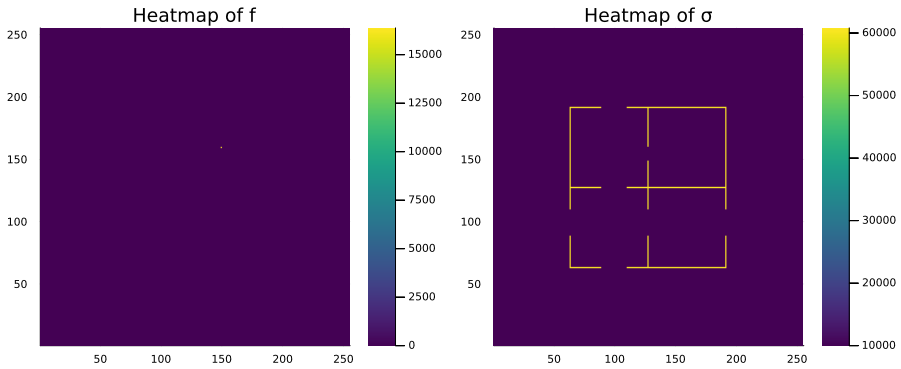


*Figure 6: Setup for the Wi-Fi Signal Simulation Problem*

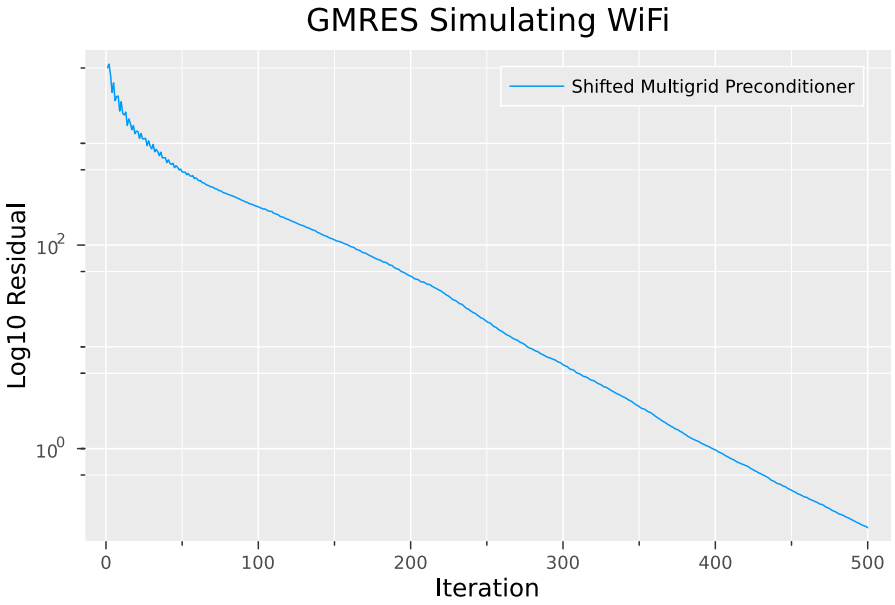


*Figure 7: Convergence of Preconditioned GMRES*

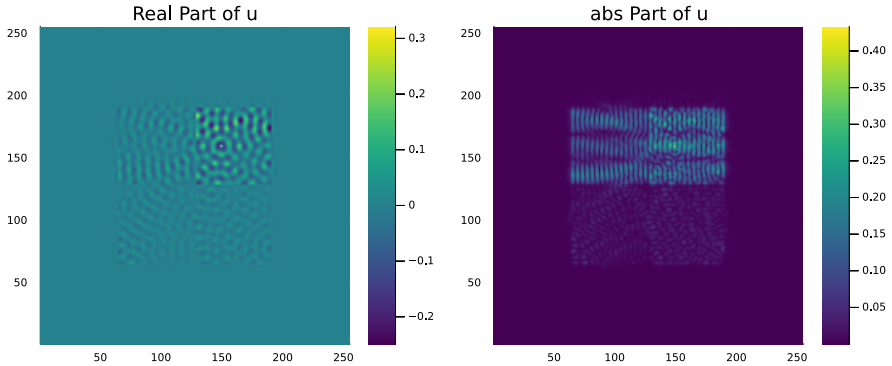


*Figure 8: Computed Solution for the Wi-Fi Problem*