

Project 2023-2024: Simulation of the Double Slit Experiment and simulation of Wi-Fi signals at home

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1 Exterior Complex Scaling (ECS) boundary conditions

1.1 Equivalence of complex grid and complex wave number

For homogenous the discretized Helmholtz equation ECS is equivalent to complex wave number. Let h be normal "real" grid spacing and $\tilde{h} = zh, z \in \mathbb{C}$ be the complex grid spacing. Let σ be normal real wave number and $\tilde{\sigma} = z^2\sigma$ be the complex wave number. Let u be the solution to the discretized Helmholtz equation with complex wave number on a normal grid and \tilde{u} be the solution to the discretized Helmholtz equation on the complex grid.

$$\frac{\tilde{u}(\tilde{x} - \tilde{h}) - 2\tilde{u}(\tilde{x}) + \tilde{u}(\tilde{x} + \tilde{h})}{\tilde{h}^2} + \sigma\tilde{u} = 0 \Leftrightarrow \quad (1)$$

$$\frac{\tilde{u}(\tilde{x} - zh) - 2\tilde{u}(\tilde{x}) + \tilde{u}(\tilde{x} + zh)}{z^2h^2} + \sigma\tilde{u} = 0 \Leftrightarrow \quad (2)$$

$$\frac{u(x - h) - 2u(x) + u(x + h)}{h^2} + z^2\sigma u = 0 \Leftrightarrow \quad (3)$$

$$\frac{u(x - h) - 2u(x) + u(x + h)}{h^2} + \tilde{\sigma}u = 0 \quad (4)$$

This equivalence doesn't hold when there is a source term.

1.2 non uniform helmhotz matrix

Our implementation of the non-uniform of the helmholtz equation uses at its base:

$$(\Delta_h u)_i = - \left(\frac{u_{i+1} - u_i}{h_{i+1/2}} - \frac{u_i - u_{i-1}}{h_{i-1/2}} \right) \frac{2}{h_{i+1/2} + h_{i-1/2}}.$$

1.3 interpolation matrix

The interpolation matrix is based on linear interpolation on a irregular grid, we only use the real part of the complex grid to do interpolation. The restriction operation is defined through the variational property.

1.4 Test Problem

We test our implementation of the vcycle on a point source problem with $\sigma = -10$.

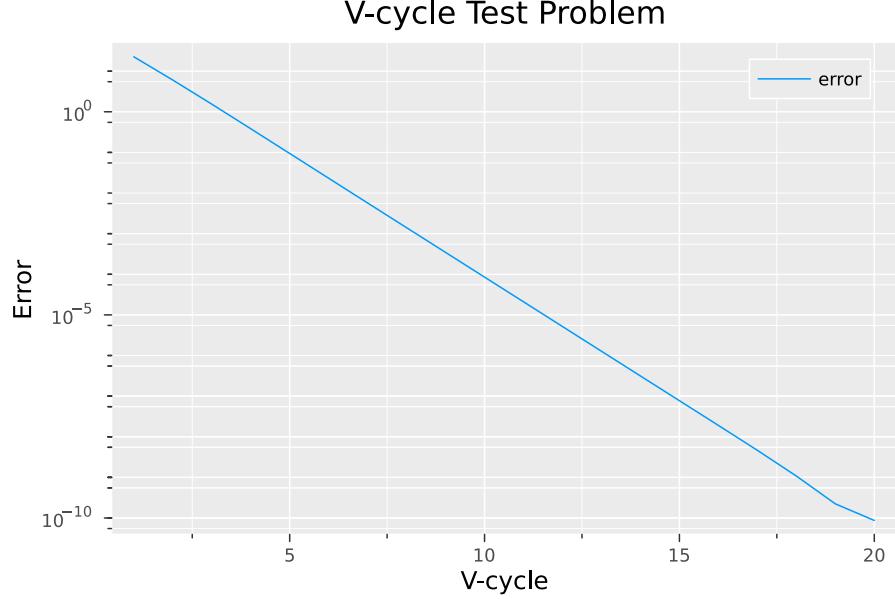


Figure 1: Convergence of the Vcycle for the test problem.

Solution Test Problem

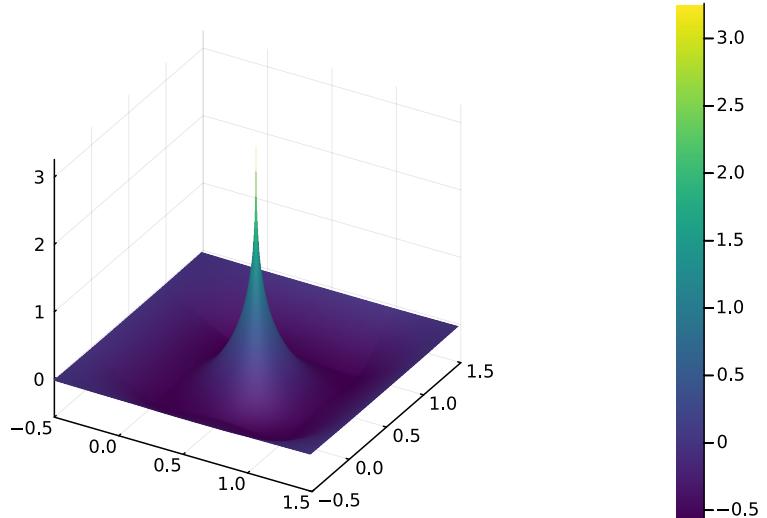


Figure 2: Computed solution for the test problem.

2 Double Slit Experiment

2.1 divergence of multigrid 2 slit

In previous project we studied divergence of the helmholtz equation with homogeneous wavenumber. Making the wavenumber inhomogeneous doesn't change the divergence of the multigrid method.

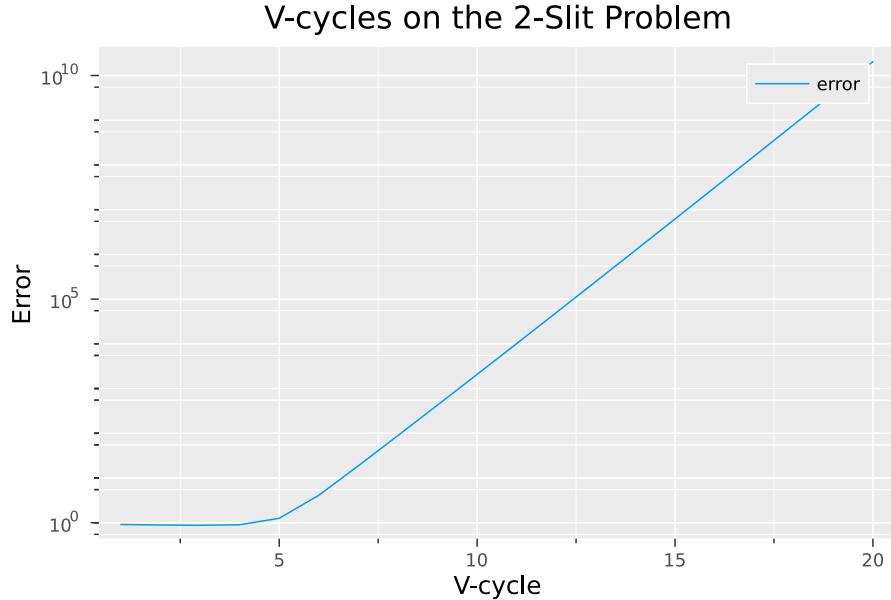


Figure 3: convergence vcycles

2.2 analysis of ECS Poisson Operator

Adding ECS to the Poisson operator make some eigenvalues complex but this doesn't explain divergent behavior of multigrid. What would explain the divergence are the spectral properties of the Helmholtz operator.

2.3 convergence of shifted multigrid 2 slit

As in project 1 adding a complex shift helps multigrid converge.

2.4 preconditioned GMRES

The shifted problem can be used as a preconditioner to the non-shifted problem.

3 Wi-Fi signals at home

3.1 wavenumber and source

We used github copilot to construct the wavenumber and the source representing the house.

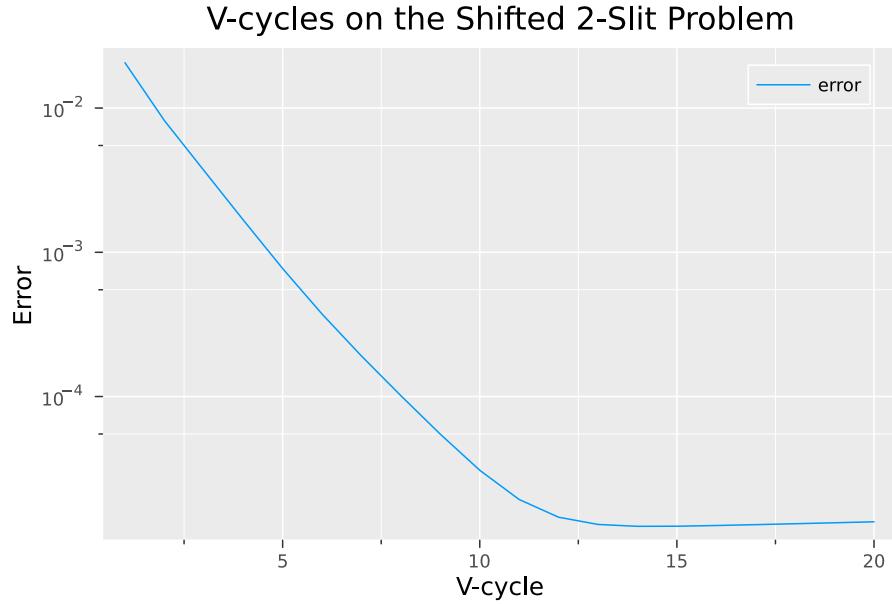


Figure 4: convergence vcycles shifted

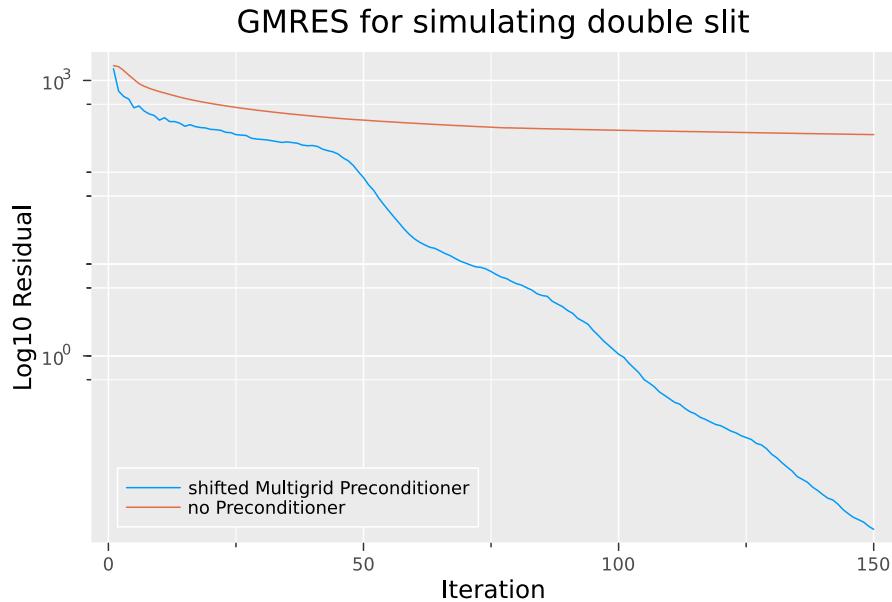


Figure 5: convergence preconditioned GMRES

3.2 preconditioned gmres

This is analogous to the 2 slit problem, we use the shifted problem as a preconditioner to the non-shifted problem. GMRES takes longer to converge because of worse behaving wavenumber.

Here we plot the computed solution to the wifi problem.

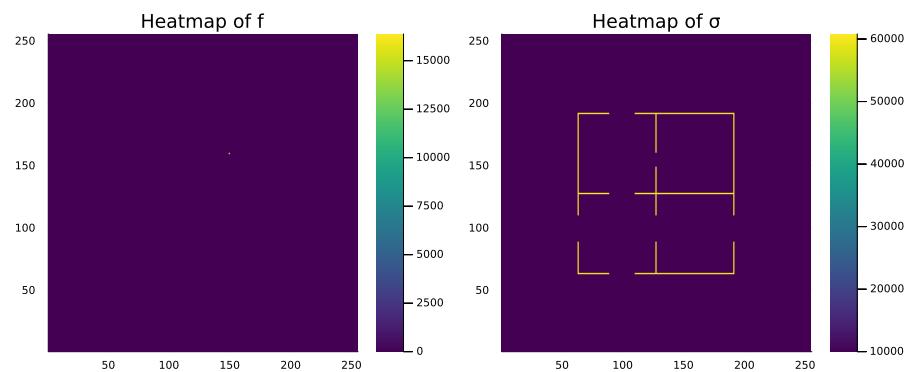


Figure 6: setup for wifi problem

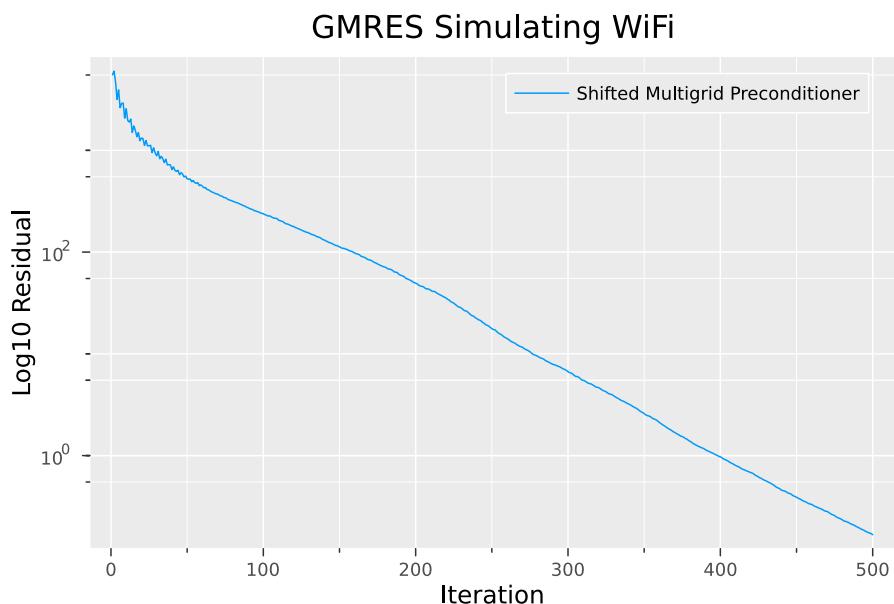


Figure 7: convergence plot

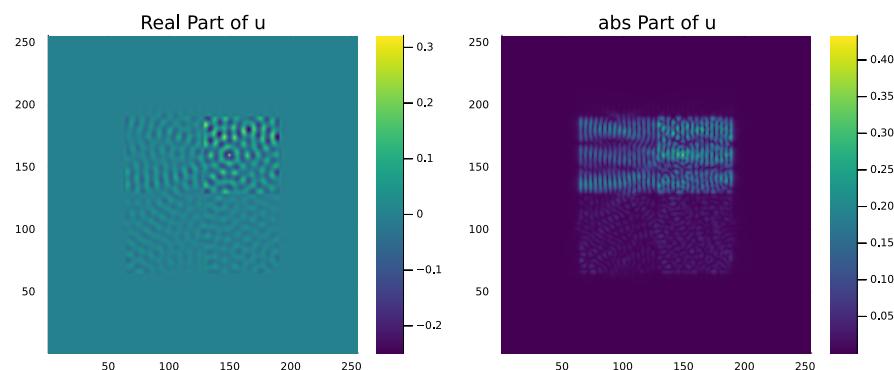


Figure 8: solution wifi