Project 2023-2024: Simulation of the Double Slit Experiment and simulation of Wi-Fi signals at home

Isidoor Pinillo Esquivel

December 18, 2023

1 Exterior Complex Scaling (ECS) boundary conditions

1.1 Equivalence of complex grid and complex wave number

For homogenous the discretized Helmholtz equation ECS is equivalent to complex wave number. Let h be normal "real" grid spacing and $\tilde{h} = zh, z \in \mathbb{C}$ be the complex grid spacing. Let σ be normal real wave number and $\tilde{\sigma} = z^2\sigma$ be the complex wave number. Let u be the solution to the discretized Helmholtz equation with complex wave number on a normal grid and $\tilde{u}$ be the solution to the discretized Helmholtz equation on the complex grid.

$$\frac{\tilde{u}(\tilde{x}-\tilde{h}) - 2\tilde{u}(\tilde{x}) + \tilde{u}(\tilde{x}-\tilde{h})}{\tilde{h}^2} + \sigma\tilde{u} = 0 \Leftrightarrow \tag{1}$$

$$\frac{\tilde{u}(\tilde{x}-zh) - 2\tilde{u}(\tilde{x}) + \tilde{u}(\tilde{x}-zh)}{z^2h^2} + \sigma\tilde{u} = 0 \Leftrightarrow \tag{2}$$

$$\frac{u(x-h) - 2u(x) + u(x-h)}{h^2} + z^2\sigma u = 0 \Leftrightarrow \tag{3}$$

$$\frac{u(x-h) - 2u(x) + u(x-h)}{h^2} + \tilde{\sigma}u = 0 \tag{4}$$

It choosing $z \sim e^{i\frac{\pi}{6}}$ is equivalent to choosing $\beta = 0.5$ because of the squaring. This equivalence doesn't hold when there is a source term.

1.2 non uniform helmhotz matrix

$$(\Delta_h u)_i = \left(\frac{u_{i+1} - u_i}{h_{i+1/2}} - \frac{u_i - u_{i-1}}{h_{i-1/2}}\right)\frac{2}{h_{i+1/2} + h_{i-1/2}}$$