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University of Oxford

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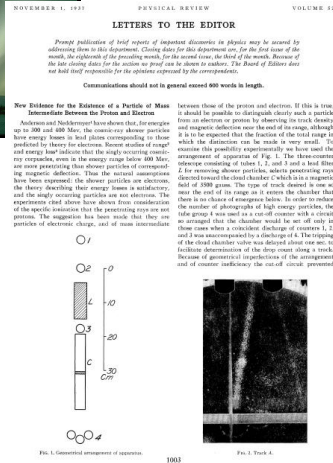
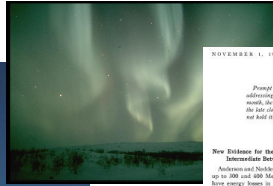
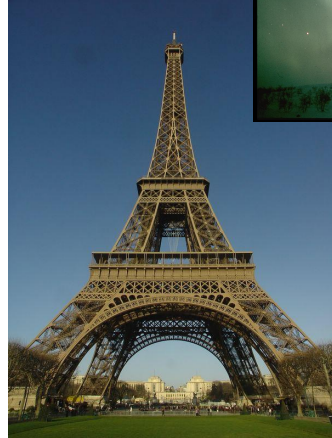
- Properties of the muon
- Muon production and facilities
- Muon-matter interactions
- Applications in solid-state science
 - Magnetism
 - Superconductivity
 - Charge transport
 - Semiconductor defects
- Complementarity with neutrons

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Muons: origins and properties

- Generated in upper atmosphere as H^+ in cosmic rays hits molecules
 - Muons survive to sea level ($1 \text{ cm}^{-2} \text{ min}^{-1}$) – identified in 1936

321m



Birth of the cloud chamber: C.T.R. Wilson



Muons: origin and properties

- Fundamental (indivisible) particle

- charge +1 (μ^+) or -1 (μ^-)

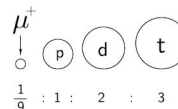
- mass \approx 200 electron, 1/9 proton

- Spin $\frac{1}{2}$

- Spontaneous decay: $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ $\tau_{\mu^+} \approx 2.2 \mu\text{s}$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \quad \tau_{\mu^-} \text{ shorter due to nuclear capture}$$

- In practical terms it is the positive form that is important (lifetime) – but why?



	charge	spin	mass	moment	$\gamma / 2\pi$ (MHz T ⁻¹)	lifetime (μs)
e	$\pm e$	1/2	m_e = 0.51 MeV	657 μp	28×10^3	∞
μ	$\pm e$	1/2	207 m_e = 105.7 MeV	3.18 μp	135.5	2.19
p	$\pm e$	1/2	1836 m_e = 938 MeV	μp	42.6	∞

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Muon production and polarisation

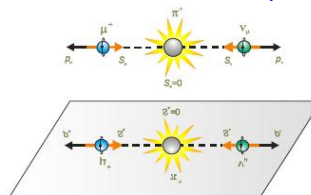
- Cosmic sources too feeble to be practical – use artificial sources
- Create from pions (π^+), in turn produced by firing high-energy protons ($> 500 \text{ MeV}$) at target containing nuclei of intermediate mass (C, Be)

$$p + p \rightarrow \pi^+ + p + n$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

- Most of π^+ that lead to useable μ^+ are at rest in the *surface* of target
- Conservation of spin (s) and momentum (p) for decay of π^+ at rest (s=0, p=0) leads to 100% polarisation of μ^+ spin opposite to momentum (parity violation means that mirror-image process doesn't occur and neutrino has spin antiparallel to its momentum)

- Kinetic energy $\mu^+ \approx 4.1 \text{ MeV}$
- Half-life $\pi^+ 26 \text{ ns}$



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- So why is this useful?

Muon implantation (*not* scattering!)

- μ^+ behaves like a light proton in terms of implantation in solids
 - 4.1 MeV kinetic energy rapidly lost by ionisation and e^- scattering \rightarrow keV (ns)
 - Final stages of energy loss involve e^- loss and capture \rightarrow 100s eV (ps)
 - Can end up in state with e^- captured (muonium - $\mu^+ e^-$)
 - If ends up positive, comes to rest at site favoured by charge – e.g. near O (*c.f.* O-H)
 - Thermalisation does not degrade spin polarisation appreciably

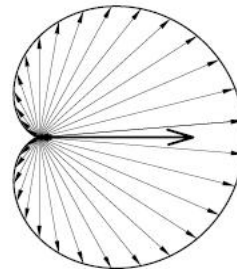


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Muon decay

- The muon never emerges from solid – decays, $\tau_{\mu^+} \approx 2.2 \mu\text{s}$
 - Of the decay products, it is the positrons (e^+) that can be detected directly
 - Angular distribution of positrons reflects muon spin polarisation at point of decay (parity violation again, plus momentum distribution of decay products)

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

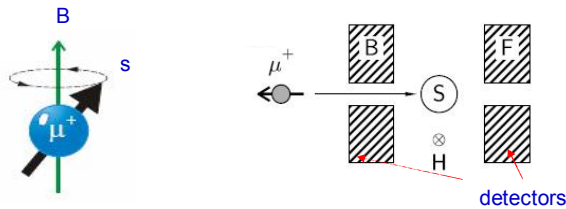


– So...?

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Muon as local probe of internal fields

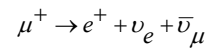
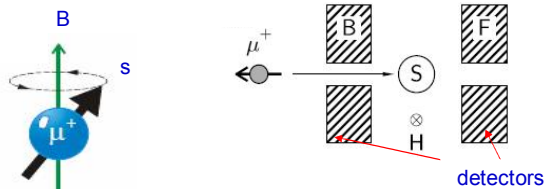
- Implanted μ^+ (Larmor) precesses about any component of field (B) transverse to the direction of implantation
 - Angular frequency $\omega_\mu = \gamma_\mu B$ ($\gamma_\mu = ge/2 m_\mu$) – between e^- (esr) and H^+ (nmr)
 - Typically falls in μ s region (MHz)
 - μ^+ polarisation rotates between forwards and backwards direction (relative to direction of implantation) and distribution of detected positrons will reflect this



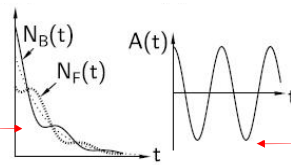
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Muon as local probe of internal fields

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Positron signal in forward and backward detector – decaying with $\tau_{\mu^+} \approx 2.2 \mu$ s



Asymmetry function $A(t)$:

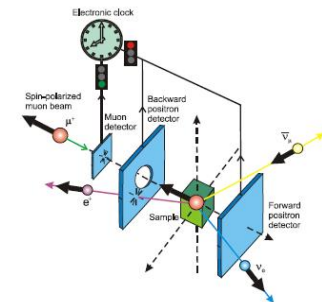
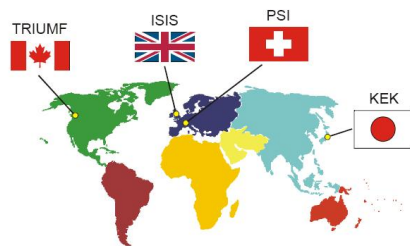
$$A(t) = (N_B(t) - N_F(t)) / (N_B(t) + N_F(t))$$

Commonly normalize to $A_{\max} \sim 0.25$:

$$G(t) = A(t) / A_{\max}$$

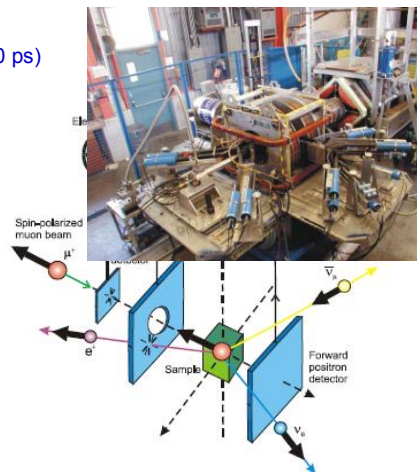
The muon experiment: facilities

- Principal distinction: continuous or pulsed source
 - Continuous sources at the Paul Scherrer Institute (PSI) (Villigen, Switzerland) and the TRIUMF Meson Facility (Vancouver, Canada)
 - Pulsed sources at the KEK Meson Science Laboratory (Japan), and the ISIS Facility of the Rutherford Appleton Laboratory (UK)
- Most common instrument configuration: longitudinal
 - Detectors forward and backward with respect to initial muon polarisation
 - Any magnetic field applied along the same direction



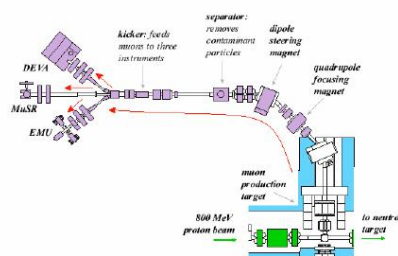
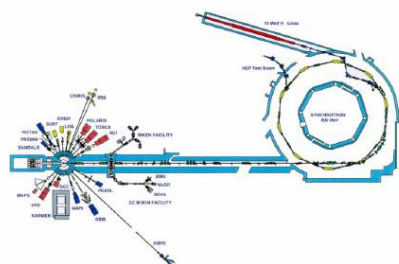
The muon experiment: facilities

- Continuous sources
 - Each incoming muon detected – and clock started
 - Stop clock when corresponding positron detected (meanwhile reject other μ^+)
 - Repeat to accumulate statistics: $A(t)$, $G(t)$
 - Advantages:
 - Can detect events at very short time (~ 100 ps)
 - Disadvantages
 - Relatively low intensities/weak signal
 - Often not extended to long times



The muon experiment: facilities

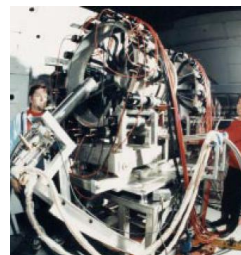
- Pulsed sources
 - Pulsed proton beam from synchrotron or linac directed on target
 - Pulses of muons produced with width set by proton pulse (~ 10 ns) and π^+ lifetime
 - Repetition period must be much longer than muon lifetime (typically 20 ms)
 - Accumulate statistics over many pulses: $A(t)$, $G(t)$



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The muon experiment: facilities

- Pulsed sources
 - Pulsed proton beam from synchrotron or linac directed on target
 - Pulses of muons produced with width set by proton pulse (~ 10 ns) and π^+ lifetime
 - Repetition period must be much longer than muon lifetime (typically 20 ms)
 - Accumulate statistics over many pulses: $A(t)$, $G(t)$
 - Advantages:
 - Use all of muons in pulse – relatively intense
 - Background very low
 - Disadvantages:
 - Cannot observe at shorter times than the pulse width



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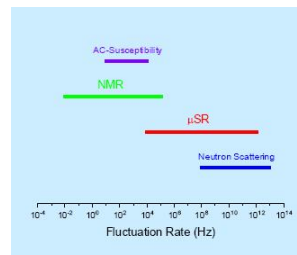
Science with implanted muons

- Muons implanted as μ^+ or muonium ($\mu^+ e^-$) after electron capture
- Muons highly sensitive to static and dynamic magnetic fields
 - Particular applications to systems with very small or dilute moments
 - Local probe – particularly good at sensing short-range effects
 - Works well in zero field – less perturbation of the system
 - Time-window $10^4 - 10^{12}$ Hz complements other techniques
 - Not element specific (nmr nuclei, neutron absorbers)
 - No spatial information – applicable to crystals, powders, films



Average picture provided by susceptibility – can be misleading

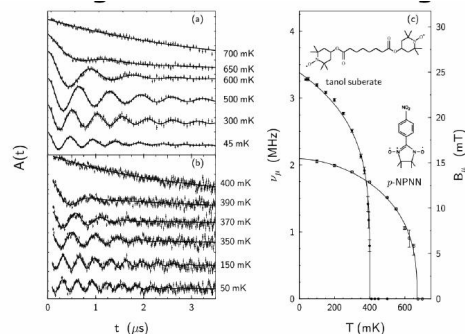
Muons more likely to reveal nature of *local* magnetic environment and tell these two apart



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Probing magnetism with muons

- Start with case of 'simple' ordered magnet
- Implanted muons 'feel' static internal field component and precess
 - This is μ SR – muon spin *rotation*
 - $\omega_\mu = \gamma_\mu B$; $\gamma_\mu/2\pi = 135 \text{ MHz T}^{-1}$
 - With longitudinal geometry and no applied field, $G(t)$ oscillates
 - Frequency gets smaller as magnet warmed to ordering temperature
 - Typically able to measure to 10^{-5} T ; moment unknown unless muon site known

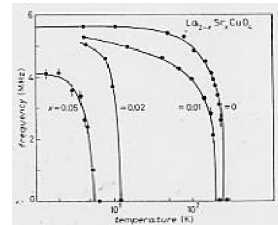
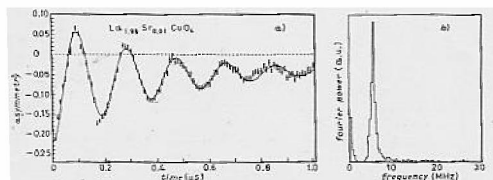


Blundell *et al*, *Europhys. Lett.* **31** (1995) 573

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Probing magnetism with muons

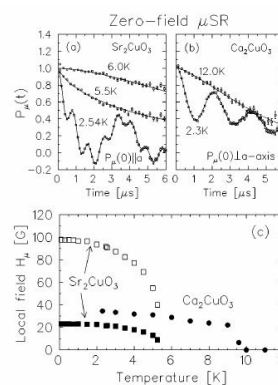
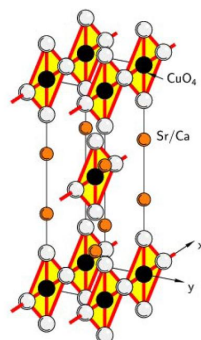
- Muons particularly good at detecting *weak* moments (c.f. neutrons)
 - Detection of spin-freezing in high- T_c materials $\text{La}_{1-x}\text{Sr}_x\text{CuO}_4$ ($s = 1/2$)
 - Budnick *et al*, *Europhys. Lett.* **5** (1988) 651



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Probing magnetism with muons

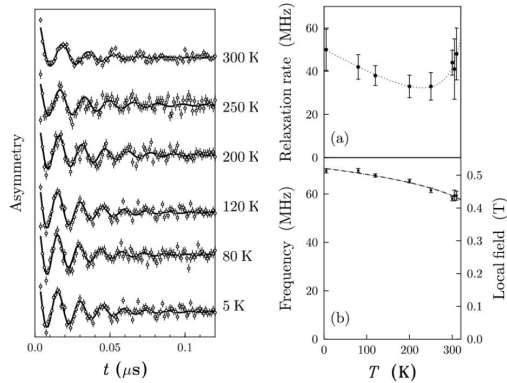
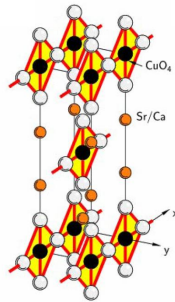
- Exploration of effects of lower-dimensionality in cuprates
 - huge activity on cuprate chain and ladder materials e.g. Sr_2CuO_3 and Ca_2CuO_3
 - -Cu-O-Cu-O-Cu-O- chains well separated - J'/J small
 - Moment below T_N scales with J'/J – does it disappear as $J'/J \rightarrow 0$?
 - Kojima *et al*, *PRL* **78** (1997) 1787
 - See what happens when chains pushed further apart



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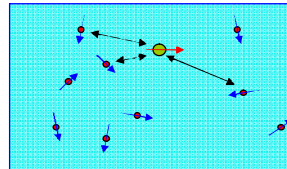
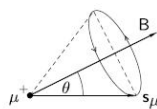
Probing magnetism with muons

- Attempt to make LaSrCoO_3 by reduction of LaSrCoO_4 with CaH_2
 - Obtain target material with Sr_2CuO_3 structure – and chains are further apart.
 - However, internal field and ordering T *very* high (>300 K rather than 5 -10 K)
 - Closer analysis reveals $\text{LaSrCoO}_3\text{H}_{0.7}$ – H between chains – J' very strong
 - Hayward *et al*, *Science* **295** (200) 1882



μSR with less than perfect order

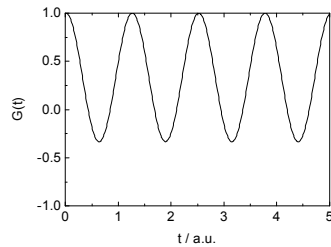
- What happens when the field is not entirely uniform?
 - e.g. array of frozen, randomly oriented *nuclear* moments (in materials that have nuclear moments)
 - e.g. array of frozen, randomly oriented *electronic* moments – in spin glass
 - Uemura *et al*, *PRB* **31** (1985) 546



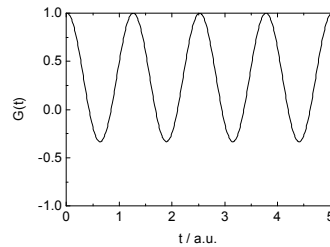
- For one component of the field: $G(t) = \cos^2 \theta + \sin^2 \theta \cos(\gamma_\mu B t)$
- Average over completely random orientations: $G(t) = \frac{1}{3} + \frac{2}{3} \cos(\gamma_\mu B t)$

Adding up all the components

- Add up the contributions from different field strengths: 1...



individual component

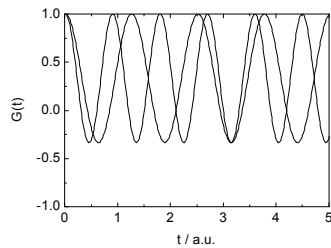


sum

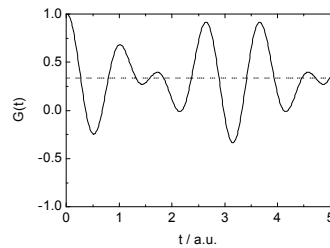
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Adding up all the components

- Add up the contributions from different field strengths : 1,2...



individual component

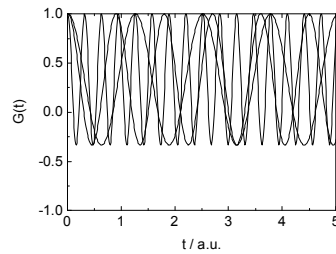


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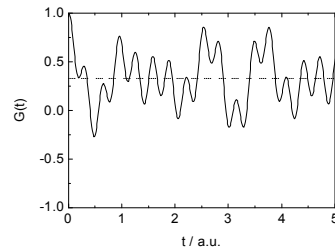
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Adding up all the components

- Add up the contributions from different field strengths: 1,2,3...



individual component

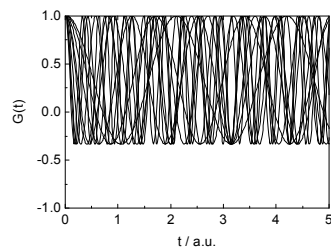


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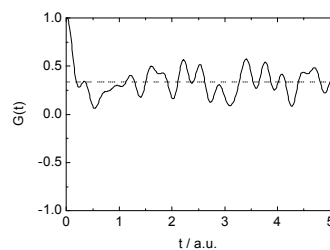
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Adding up all the components

- Add up the contributions from different field strengths : 1,2,3...many



individual component



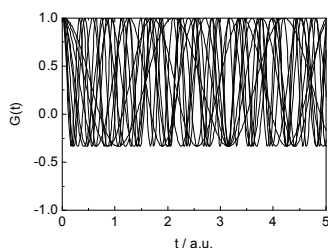
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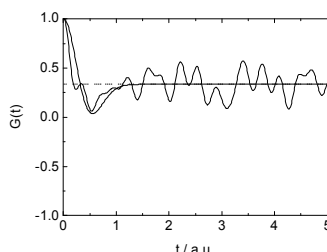
Adding up all the components

- Add up the contributions from different field strengths : 1,2,3...many
 - introduce a continuous Gaussian* distribution of width Δ/γ_μ :

$$G(t) = \frac{1}{3} + \frac{2}{3} e^{\frac{-\Delta^2 t^2}{2}} (1 - \Delta^2 t^2)$$



individual component



sum

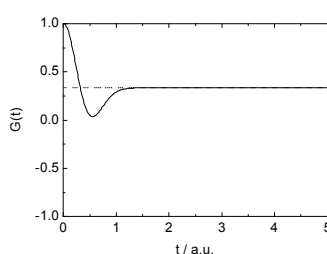
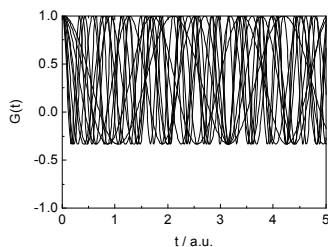
* appropriate for *concentrated* collection of dipolar fields – for *dilute* systems *e.g.* some spin-glasses, this is Lorentzian; Walstedt and Walker, *PRB* **9** (1974) 4857; Crook and Cywinski, *JPCM* **9** (1997) 1149

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Adding up all the components

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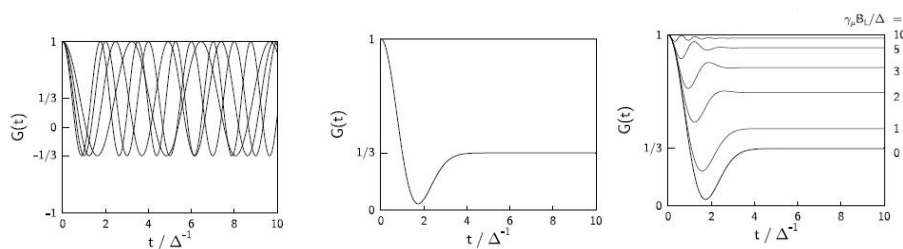
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'R' is also for 'Relaxation'

- Kubo-Toyabe relaxation function for frozen static moments in zero field and longitudinal geometry
i.e. a form of muon spin *relaxation*

$$G(t) = \frac{1}{3} + \frac{2}{3} e^{-\frac{\Delta^2 t^2}{2}} (1 - \Delta^2 t^2)$$

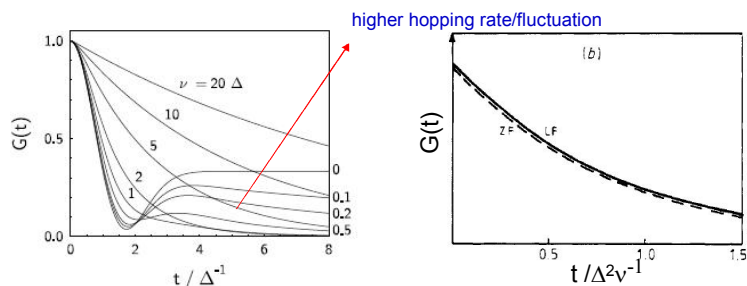


- At long times, $G(t)$ recovers to 1/3 initial value – reflects 1/3 net component of random moments along longitudinal direction – no contribution to relaxation
- An applied field increases the value of this field and hence the '1/3 tail'

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Fluctuating moments

- The muon spin often experiences fluctuations in the field – either because the field moves (e.g. in a paramagnet) or the muon hops
 - Hayano *et al*, *PRB* **20** (1979) 850
- Assume rate of change of direction $p(t) = \exp(-\nu t)$
- Field orientation moves randomly at this rate within distribution $P(B_i)$
 - For *fast* relaxation rates: $G(t) = \exp(-\lambda_z t)$; $\lambda_z = 2\Delta^2/\nu$ ('nmr' - motional narrowing – ν)
 - For *slow* relaxation rates: $G(t) = \frac{1}{2} \exp(-\frac{2}{3} \nu t)$ – recover 1/3 tail
 - Full behaviour can either be simulated or approximated by analytic function (dynamical KT function) - Keren *PRB* **50** (1994) 10039
- Applied field doesn't make much difference to signal from paramagnet



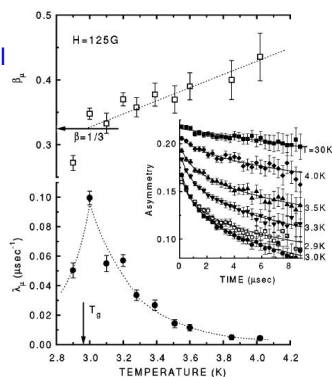
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Probing the energy landscape

- Spin-glasses provide insights into glassy dynamics in general
 - Range of environments leads to distribution of relaxation times
 - No longer simple exponential decay for $G(t)$ (implies one relaxation time)
 - Observe stretched exponential for many SGs (universal?): $G(t) \approx \exp(-\lambda_\mu t)^\beta$
 - e.g. 0.5 at% Mn in Ag (Ag has weak nuclear moment – passive matrix)
 - Campbell *et al*, *PRL* **72** (1994) 1291; Keren *et al* *PRL* **77** (1996) 1386

Stretched exponential
exponent vs T

Relaxation rate
 λ vs T

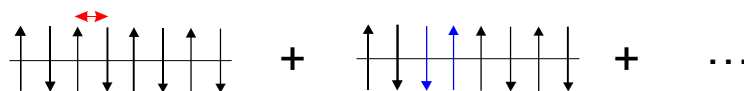


(Unnormalised)
asymmetry $A(t)$ vs T

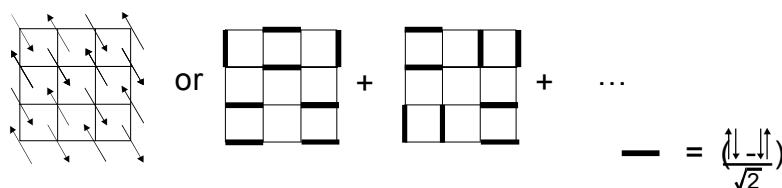
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Opening a very old can of worms

- Conventional (Néel) ground state may not be correct



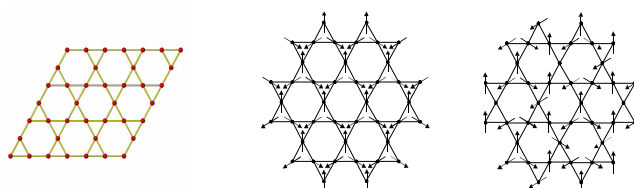
- = RVB (resonating valence bond) or spin fluid ground state
- Similarly for layered magnets



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Pushing the boundaries of magnetism

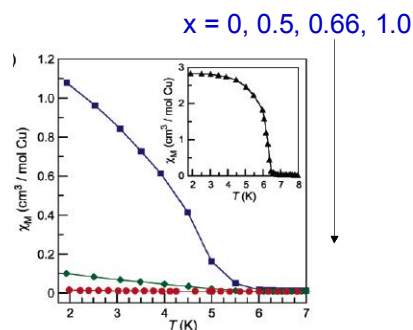
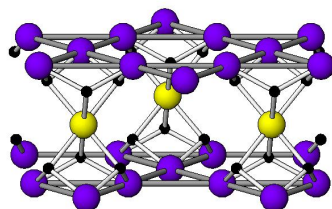
- Classical model works well most of the time – Néel order on cooling
- Quantum fluctuations more significant for
 - Small spin ($S = \frac{1}{2}$)
 - Fewer neighbours - chains and planes
 - Frustrated interactions
- $S = \frac{1}{2}$ kagome antiferromagnet brings all these together
 - Any good examples out there?



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Herbertsmithite - a perfect $S = \frac{1}{2}$ kagome afm?

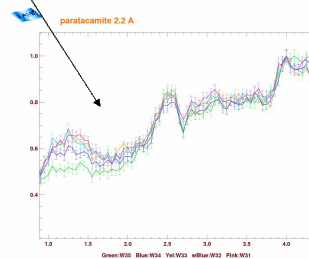
- Parent compound: γ - $\text{Cu}_4(\text{OH})_6\text{Cl}_2$ - dope with Zn: $\text{Zn}_x\text{Cu}_{4-x}(\text{OH})_6\text{Cl}_2$
 - Parent compound has pyrochlore structure
 - Zn selects sites between kagome layers (no JT distortion)
 - For Cu_3Zn compound, yields undistorted kagome layers separated by Zn
 - Zn severs weak FM component of Cu-Cu exchange ($|\theta| \uparrow$ as %Zn \uparrow)
 - Intra-plane Cu-O-Cu 119° ; inter-plane Cu-O-Cu 97°
 - Shores et al, JACS 127 (2005) 13462



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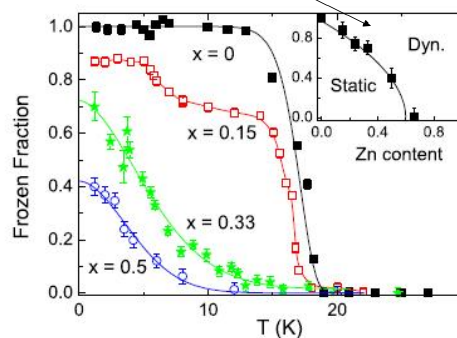
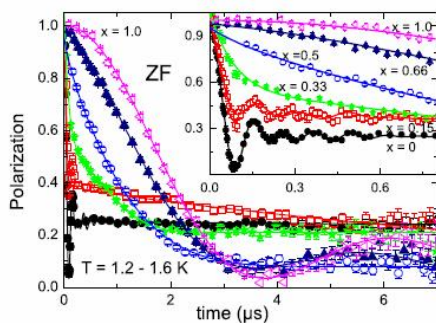
Neutron probe of correlations...

- No long-range order to 20 mK in $x = 1$ (pure kagome)
- Traces of excitation around 7 meV (but only for some of the spins)
 - Short-range correlation visible at 2 K but not 60 K
 - Scan along Q for data in energy range 7 – 8 meV; $T = 2$ to 60 K
 - Is this the energy required to break up spin singlets?



Spin freezing in paratacamite series - $\text{Cu}_{4-x}\text{Zn}_x$

- Track magnetic behaviour with x using μSR (Mendels *et al*, PRL **98** (2007) 077204)
 - For $x \leq 0.15$ distinct oscillations plus paramagnetic term
 - For $x = 0.33$, $x = 0.5$ freezing transition broader and at lower T
 - Higher values of x only dynamic down to lowest T (50 mK – cf 300 K for θ)



Superconductors

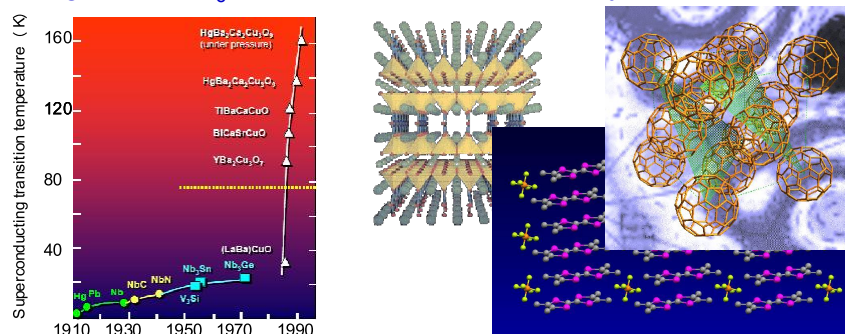
- Superconductors – why the fuss?
 - 'superconductivity is perhaps the most remarkable physical property in the universe' *David Pines*
 - It's also one of the most useful – really and potentially



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Superconductors

- Striking leaps in T_c in the past decades – but why?

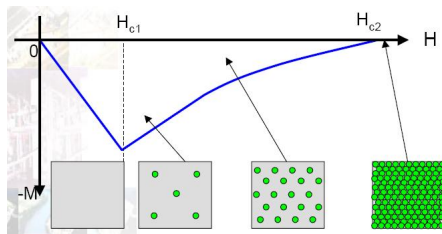


- Tremendous range of materials that are now known to superconduct
 - metals and alloys
 - oxides, especially cuprates
 - fullerenes
 - molecular solids
 - where next?

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Superconductors

- What to look for? What provides clues about the underlying physics?
- Two characteristic length scales:
 - Coherence length (ξ) – scale for variation of sc wavefunction
 - Penetration depth (λ) – controls ability of sc to screen magnetic fields

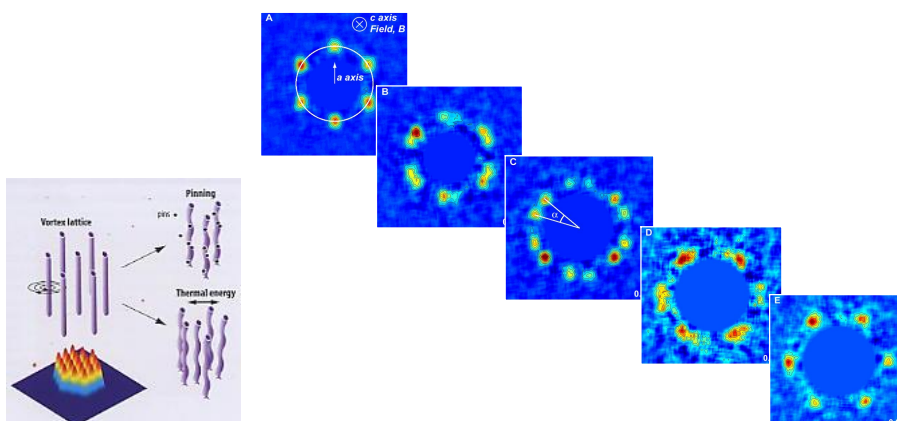


- If λ is much greater than ξ ($\lambda > \xi/\sqrt{2}$) flux can penetrate entire sample
 - Does so as quantized flux lines ($h/2e$) called vortices which may form lattice
 - Behaviour of flux lattice a good test for theories of superconductivity
 - How to study?

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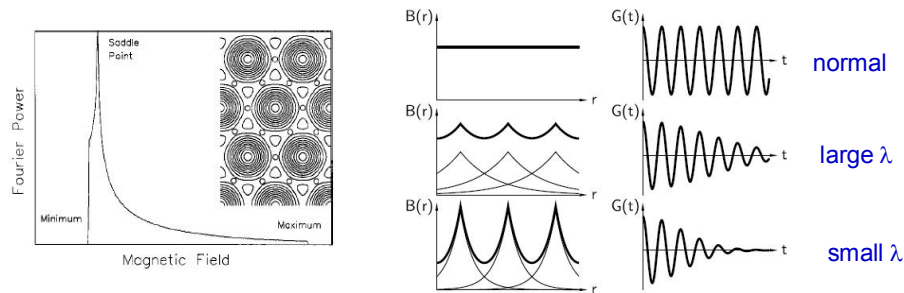
SANS

- Flux lines can scatter neutrons just as moments do
- Typical spacing between vortices puts scattering in SANS territory
 - E.g. MgB_2 (note – 98 mg Xtal – $0.75 \text{ mm}^2 \times 30 \text{ mm}$, 95% ^{11}B enriched)
 - Cubitt and Dewhurst, Phys. Rev. Lett. 91 (2003) 47002



Muons and superconductors

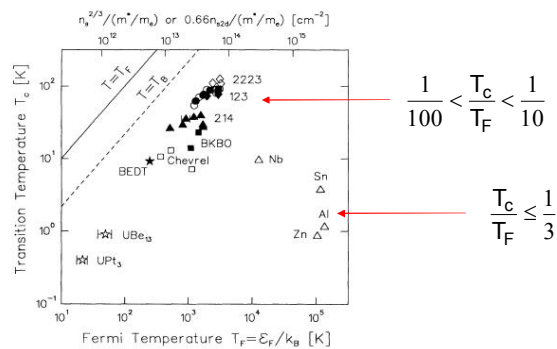
- Muons can probe internal field distribution
 - apply transverse B to sample
 - In normal (non-sc) state, uniform field and simple muon precession signal
 - If vortices form, field felt by muon depends on implantation site relative to vortex
 - For larger λ , internal field variation less so relaxation rate (σ) less: $\sigma \approx 1/\lambda^2$
 - Generally difficult to get bulk measure of λ (don't need good x'tals as with SANS)



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Muons and superconductors

- λ for clean sample at $T = 0$ can provide estimate for Fermi temperature (T_F)
- Plot T_c against T_F to provide clear distinction between ordinary and 'exotic' sc
- Common physics for exotic superconductors? But what?
 - Uemura *et al*, *PRL* **66** (1991) 2665



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Probe of diffusion in solids

- Transport in solids – ions, atoms, electrons – in key technologies
 - Batteries, fuel cells, sensors, catalysts, conducting polymers
- Muons can probe such motion in several ways
 - relaxation of mobile μ^+ to study motion of light particles e.g. mimic of H^+
 - relaxation of static μ^+ to study motion of other species



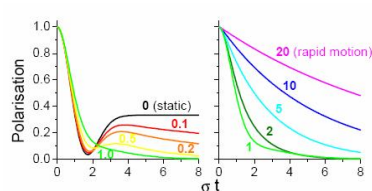
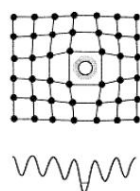
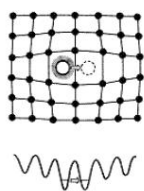
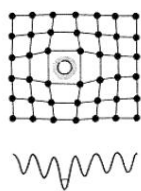
Illustrative figs
of technologies
etc



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Relaxation of diffusing muons

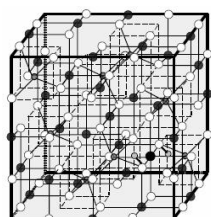
- μ^+ implanted in inorganic solid causes local lattice distortion
 - Muon spin senses local nuclear moments – KT relaxation
 - Hopping leads to relaxation of KT function (relaxation of '1/3 tail')
 - Thermal assistance of motion includes phonons: overall $v = v_0 \exp(-E_a/kT)$
 - Quantum diffusion (tunnelling) at very low temperature
 - Storchak and Prokof'ev, *Rev. Mod. Phys.* **70** (1998) 929



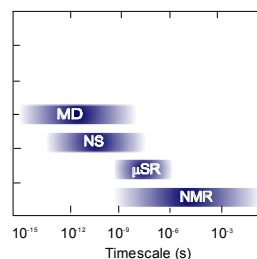
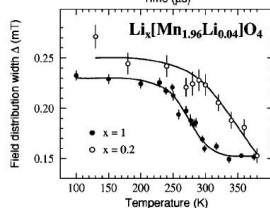
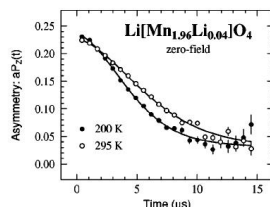
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Relaxation by diffusing ions

- $\text{Li}_x[\text{Mn}_{2-y}\text{Li}_y]\text{O}_4$ could be a key component (cathode) for Li batteries
 - Function depends on Li^+ flow in the spinel structure – optimise wrt x, y
 - μ^+ implants near O; observe dynamic KT form
 - Width of field (Δ) decreases above 230 K with $x=1, y=0.04$ – Li^+ motion
 - Li^+ motion only becomes significant above 300 K for $x=0.2, y=0.04$
 - Kaiser *et al*, *PRB* **62** (2000) R9236



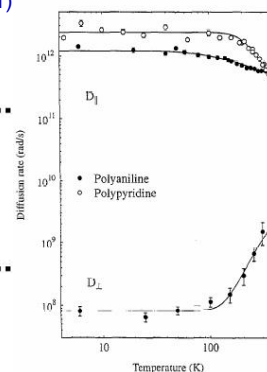
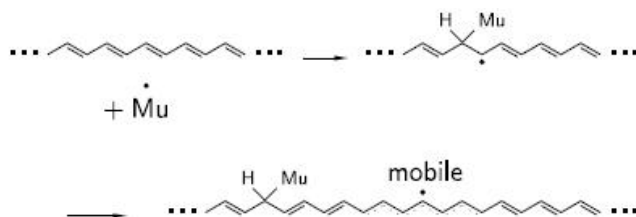
○ oxygen (32e)
● manganese (15d)
● lithium (8a)
● Mn vacancy
○ proton (96g)



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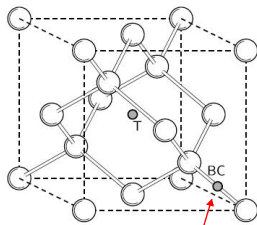
Electron motion in polymers

- μ^+ implanted in conducting polymer generally forms muonium (μ^+e^-) which reacts with the polymer to produce mobile spin (soliton)
 - Soliton moves up and down chain but cannot pass defect
 - Muon polarisation relaxes with each visit (μ^+e^- hyperfine coupling)
 - Hence probe mechanism of charge transport
 - Nagamine *et al*, *PRL* **53** (1984) 1763; Pratt *et al* *PRL* **79** (1997) 2855; Pratt *et al*, *Syn. Met.* **101** (1999) 323; Blundell *et al*, *Syn Met.* **119** (2001)

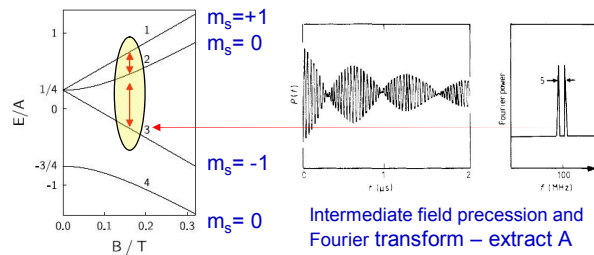


Semiconductor defects

- Defects dominate much of the useful physics of semiconductors
 - H is a particularly important defect; mimic with muonium ($\text{H}, \text{H}^+, \text{H}^-\mu^+, \mu^+e^-, \mu^+2e^-$)
 - Muonium studies most insightful for *individual* H defects – *very* sensitive
 - In low magnetic field see precession – transitions within the triplet
 - In higher field, measurement of precession signal yields A – sensitive to site
 - Patterson, *Rev. Mod. Phys.* **60** (1998) 69



Si: most tightly bound site for μ^+e^-



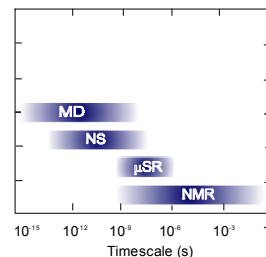
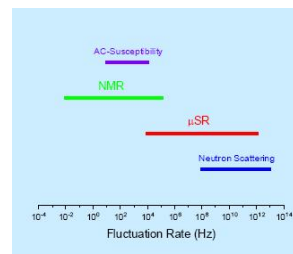
Breit-Rabi diagram for μ^+e^-
(isotropic A; $A_{\text{vac}}=4.463\text{ GHz}$)

Intermediate field precession and
Fourier transform – extract A

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Summary

- What muons can do
 - Very sensitive probe of internal magnetic fields
 - Ordering temperature
 - Type of order – ‘regular’ or glassy
 - Fluctuations in paramagnets and glassy systems
 - Complements other techniques in dynamics
 - time range $10^4 - 10^{12}$ Hz
 - Superconductors
 - Characterisation of flux lattices – measure of penetration depth
 - Diffusion in solids
 - Mimic of light particles and diffusion mechanisms
 - Probe of diffusion of ions in solids, electrons in conducting polymers
 - Defects in semiconductors
 - Probe of nature of defect sites (H) in semiconductors



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- **Books and reviews**

- J. Chappert in 'Muons and Pions in materials research', eds J.Chappert and R.I. Grynspan...
- A. Schenck, 'Muon spin rotation spectroscopy' (1985) (Bristol, Hilger)
- S.F.J. Cox, 'Implanted muon studies in condensed matter science', *J.Phys.C:Solid State Phys.* 20 (1987) 3187
- S.J. Blundell, 'Spin polarised muons in condensed matter physics', *Contemp. Phys* (arXiv:cond-mat/0207699v)
- P. Dalmas de Roetier and A. Yaouanc, 'Muon spin rotation and relaxation in magnetic materials', *J.Phys.: Cond. Mat.* 9 (1997) 9113
- Patterson, Semiconductor defects, *Rev. Mod. Phys.* 60 (1998) 69
- Storchak and Prokofev, Quantum diffusion, *Rev. Mod. Phys.* 70 (1998) 929

- **Web resources**

- ISIS web site – esp. <http://www.isis.rl.ac.uk/muons/trainingcourse/index.htm>
- TRIUMF web site: <http://www.triumf.ca>
- PSI web site: <http://www.psi.ch>