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Magnetic Neutron Diffraction

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What's a magnetic structure?

Why study magnetic order?

Generic description (k-vector formalism)

- Related to representation/symmetry analysis (not covered)
- Alternative to Shubnikov symmetry, which are *TABULATED* crystallographic magnetic space groups : this is however restricted to *commensurate* structures

The magnetic structure factor

Practical aspects (Experimental choices / software, and methods for magnetic structure determination)

Note on available software:

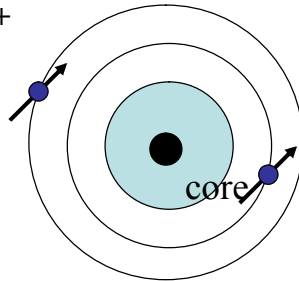
Fullprof : General, k-vector formalism

GSAS: Contains Shubnikov symmetry, but restricted to *commensurate* structures

What's a magnetic structure?

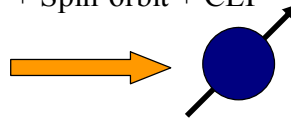
(atomic) magnetic moments (\mathbf{m}) arise from quantum effects in atoms/ions with unpaired electrons

Ni^{2+}



« Quantum description »

Intra-atomic electron correlation
Hund's rule (maximum total S)
+ Spin-orbit + CEF



« Classical description »

$$\mathbf{m} = g_J \mathbf{J} \quad (\mathbf{J} = \mathbf{L} + \mathbf{S} \text{ 4f-rare earths})$$

$$\mathbf{m} = g_S \mathbf{S} \quad (\text{3d-transition metals})$$



What's a magnetic structure?

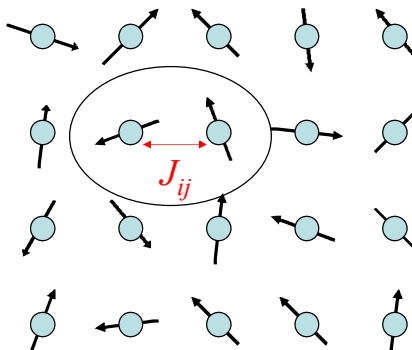
Magnet: crystal containing magnetic atoms

$$kT \gg J_{ij}$$

$$E_{ij} = -J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$\langle \mathbf{S}_i \rangle = 0$$

**paramagnetic
(disordered) state**



Temperature (entropy) overcomes magnetic energy:

Entropy essentially dominated by local
magnetic moment
fluctuations



What's a magnetic structure?

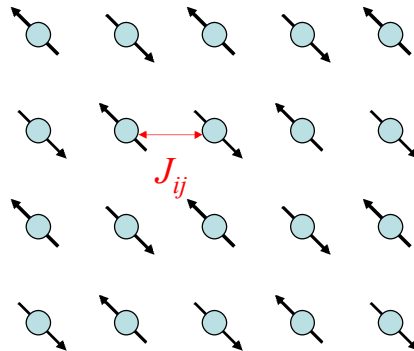
Magnet: crystal containing magnetic atoms

$$kT < J_{ij},$$

$$E_{ij} = -J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$\langle \mathbf{S}_i \rangle \neq 0$$

Exemple here: $J_{ij} > 0$
Antiferromagnetic coupling
(AF)



Magnetic energy overcomes the entropy :

⇒ Quasi-static configuration of magnetic moment

with small fluctuations that are made cooperative by the magnetic exchange (spin waves excitations)



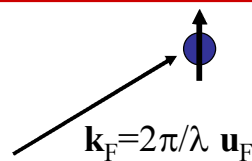
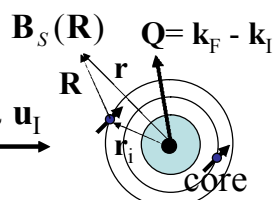
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Magnetic scattering

$$\mu_N = -\gamma \mu_N \sigma$$



$$\mathbf{k}_I = 2\pi/\lambda \mathbf{u}_I$$



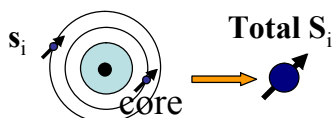
$$\mathbf{k}_F = 2\pi/\lambda \mathbf{u}_F$$

Dipolar interaction term with one electron

$$\langle \mathbf{k}_F | V_M | \mathbf{k}_I \rangle = \int e^{i\mathbf{k}_I \cdot \mathbf{r}} \left[-\gamma \mu_N \sigma \cdot \frac{\mu_0}{4\pi} \nabla \left(\frac{2\mu_B S \times \mathbf{R}}{R^3} \right) \right] e^{-i\mathbf{k}_F \cdot \mathbf{r}} d\mathbf{r}$$

$$\langle \mathbf{k}_F | V_M | \mathbf{k}_I \rangle = 4\pi \cdot \mathbf{Q} \times \left(\sum_i \exp(i\mathbf{Q} \cdot \mathbf{r}_i) \cdot \mathbf{s}_i \right) \times \mathbf{Q}$$

$$f(\mathbf{Q}) \cdot \mathbf{S}_i$$

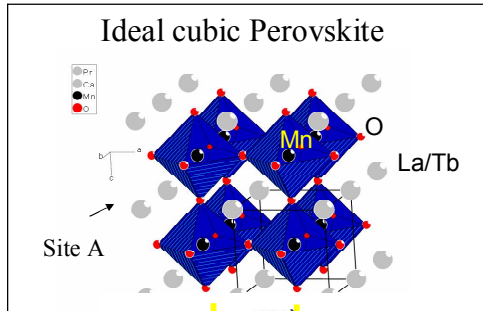


Total \mathbf{S}_i

⇒ vector scattering amplitude for one atom

$$\langle \mathbf{k}_F | V_M | \mathbf{k}_I \rangle = 4\pi \cdot f(\mathbf{Q}) \cdot \mathbf{Q} \times \mathbf{S}_i \times \mathbf{Q}$$

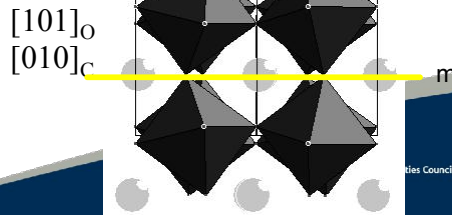
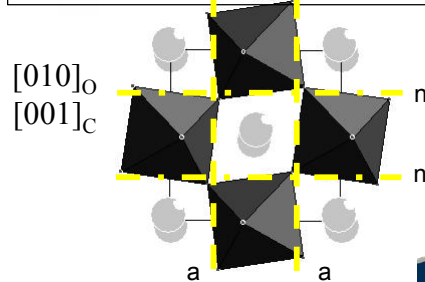
Example: $R_{1-x}D_x\text{MnO}_3$ manganites (R^{3+} : La, Pr... D^{2+} : Ca, Sr...)



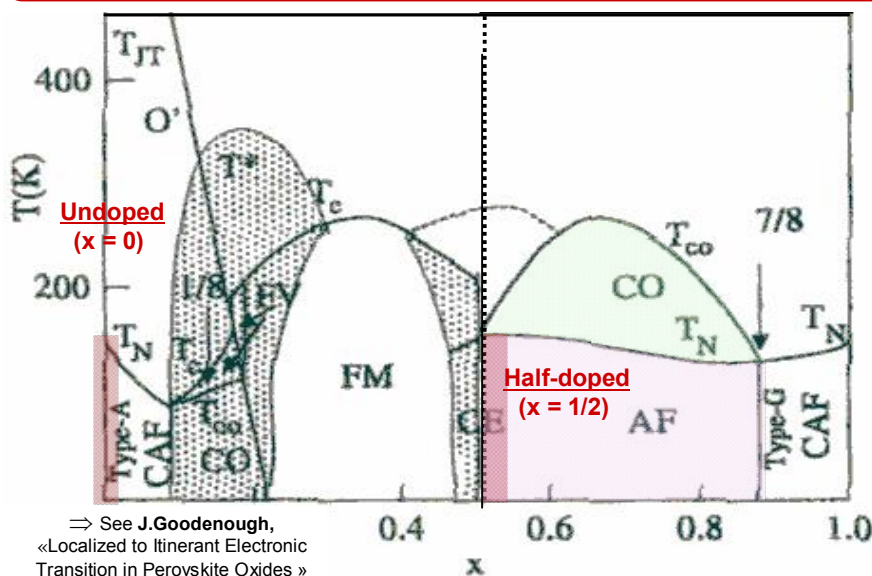
LaMnO_3 $T_N=150\text{K}$

TbMnO_3 $T_N=41\text{K}$

$Pnma$

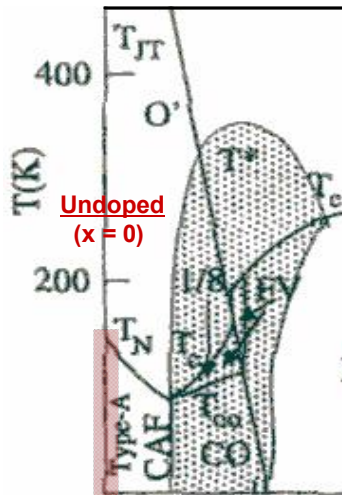


Example: $R_{1-x}D_x\text{MnO}_3$ manganites (R^{3+} : La, Pr... D^{2+} : Ca, Sr...)



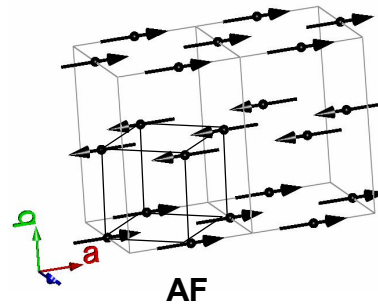
⇒ See J. Goodenough,
«Localized to Itinerant Electronic
Transition in Perovskite Oxides »
Ed. J. B. Goodenough, Springer (2001)

Example: $R_{1-x}D_x\text{MnO}_3$ manganites (R^{3+} : La, Pr... D^{2+} : Ca, Sr...)



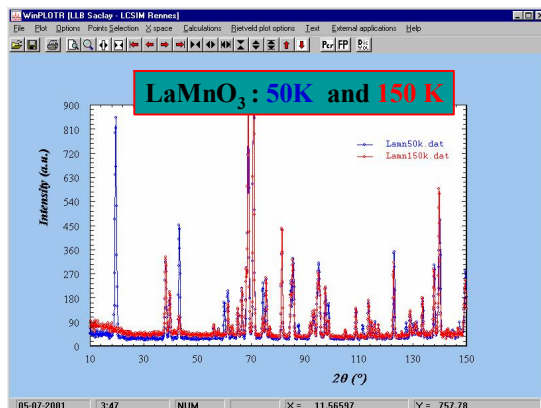
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LaMnO_3 $T_N=150\text{K}$

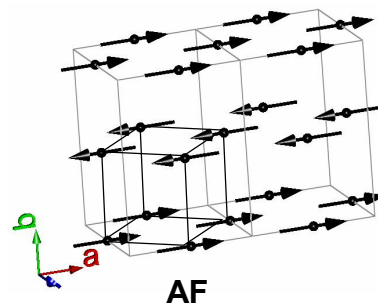


AF
Commensurate
structure

Example: $R_{1-x}D_x\text{MnO}_3$ manganites (R^{3+} : La, Pr... D^{2+} : Ca, Sr...)



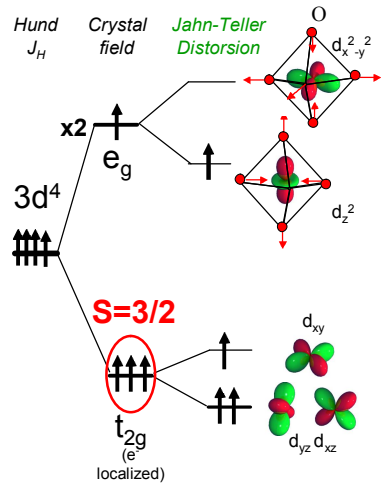
LaMnO_3 $T_N=150\text{K}$



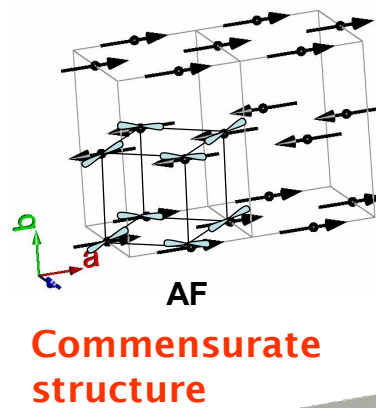
AF
Commensurate
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Example: RMnO_3 manganites

Electronic structure of Mn^{3+} ($x=0$)

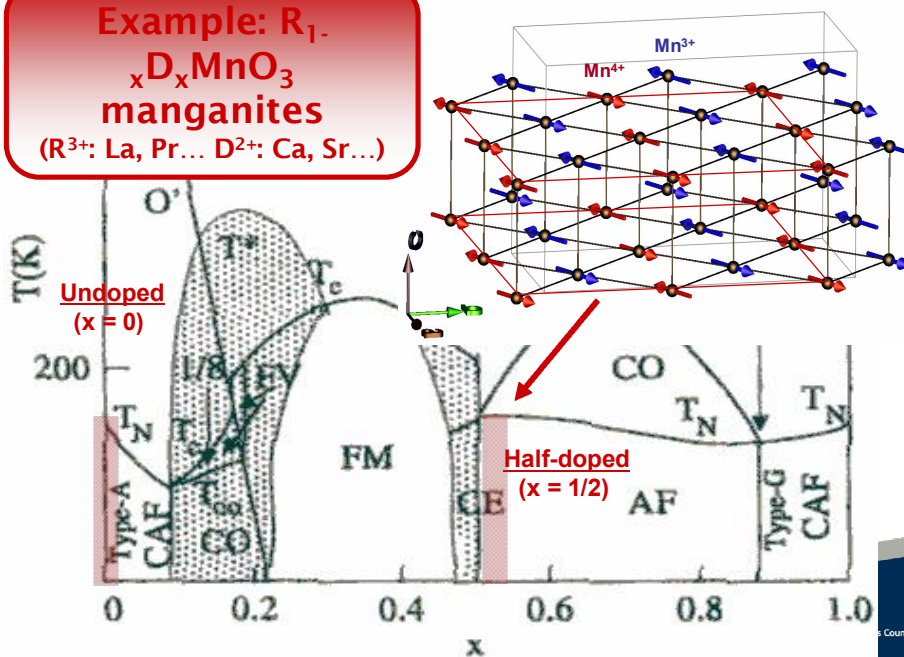


LaMnO_3 $T_N=150\text{K}$



Example: $\text{R}_{1-x}\text{D}_x\text{MnO}_3$ manganites

(R^{3+} : La, Pr... D^{2+} : Ca, Sr...)



Example: $R_{1-x}D_x\text{MnO}_3$ manganites
(R^{3+} : La, Pr... D^{2+} : Ca, Sr...)

First NPD study of $\text{La}_{1/2}\text{Ca}_{1/2}\text{MnO}_3$:
Wollan and Khoeler
Phys. Rev. **100**, 545 (1955)

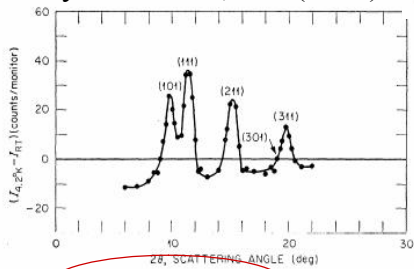
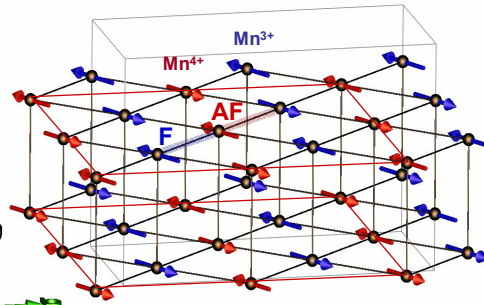
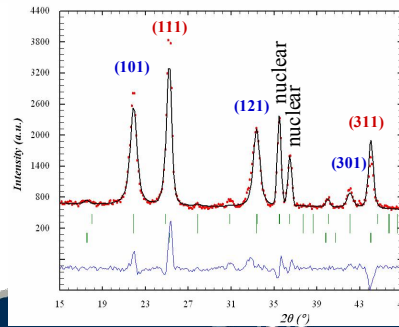


FIG. 7 Temperature difference pattern taken with Soller slits for (0.5 La, 0.5 Ca)MnO₃ No. 66. *CE*-type antiferromagnetic structure.

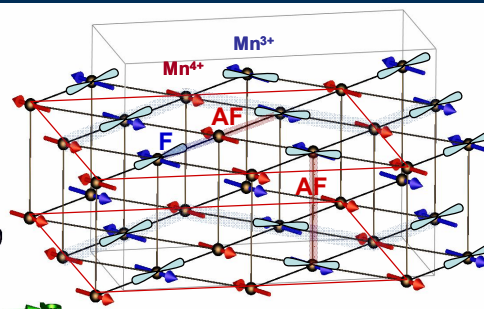
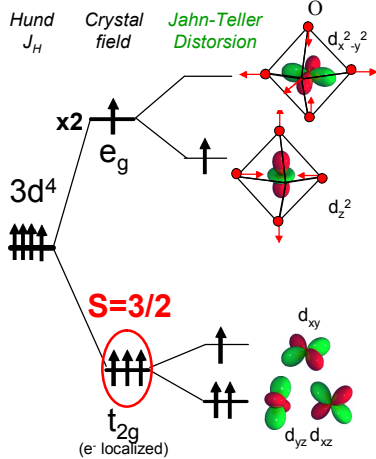


(PrCa)MnO₃ $x=0.50$ LLB-G4.2 (dec.99)

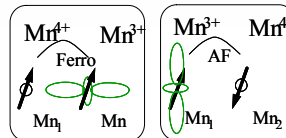


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(R^{3+} : La, Pr... D^{2+} : Ca, Sr...)

Electronic structure of Mn^{3+}

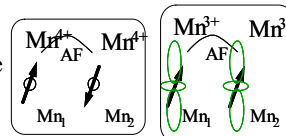


Couplings in the (a, b) plane



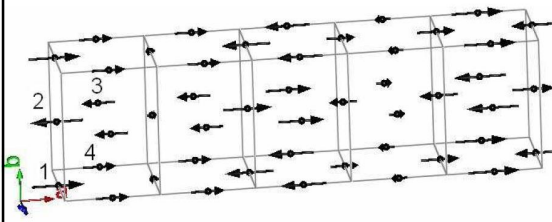
Goodenough-Kanamori-Anderson (GKA) rules for superexchange

Couplings along the c direction

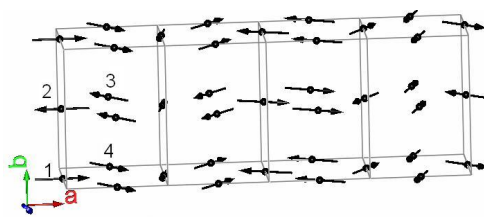


Example: RMnO_3 manganites

TbMnO_3 $T_{N2} < T < T_{N1} = 41\text{K}$ Sinusoidal

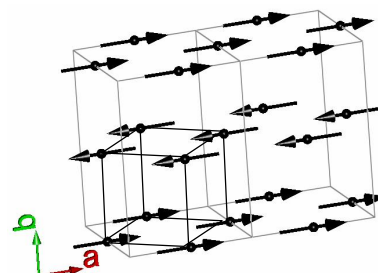


$T < T_{N2} = 23\text{K}$ Cycloid



Incommensurate structures

LaMnO_3 $T_N = 150\text{K}$



AF

Commensurate structure



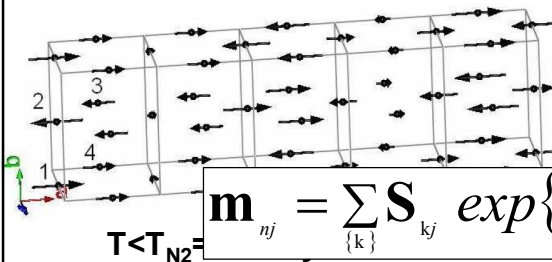
Why study magnetic order?

- **Fundamental properties of condensed matter. Exchange interactions related to the electronic structure.**
- **The first (necessary) step before determining the exchange interactions (generally, with inelastic neutron scattering)**
- **Permanent magnet industry. Chemical substitutions controlling single ion anisotropy, strength of effective interactions, canting angles, etc: NdFeB materials, SmCo_5 , hexaferrites, spinel ferrites.**
- **Spin electronics, thin films and multilayers**

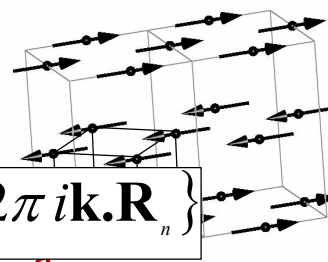


Example: RMnO₃ manganites

TbMnO₃ $T_{N2} < T < T_{N1} = 41\text{K}$ Sinusoidal

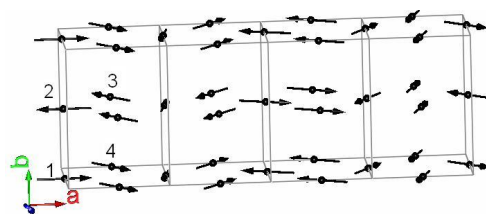


LaMnO₃ $T_N = 150\text{K}$



$$\mathbf{m}_{nj} = \sum_{\{\mathbf{k}\}} \mathbf{S}_{kj} \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_n\}$$

$T < T_{N2}$



AF

Commensurate structure

Incommensurate structures



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The magnetic structure factor

Practical aspects (Experimental choices / software, and methods for magnetic structure determination)

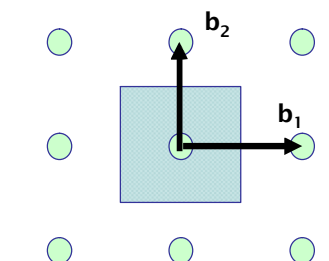
Note on available software:

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GSAS: Contains Shubnikov symmetry, but restricted to *commensurate* structures

General description (k-vector formalism)

Reciprocal k-space

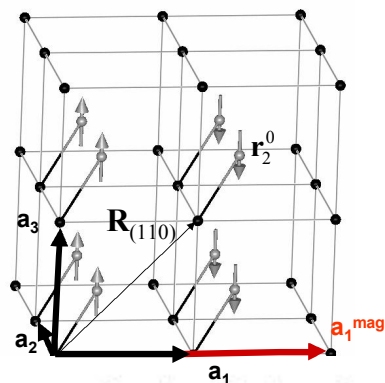


● Nuclear reflections

$$\mathbf{H} = h \cdot \mathbf{b}_1 + k \cdot \mathbf{b}_2 + l \cdot \mathbf{b}_3$$

$$\mathbf{a}_1^{\text{mag}} = 2\mathbf{a}_1 \quad \mathbf{a}_2^{\text{mag}} = \mathbf{a}_2 \quad \mathbf{a}_3^{\text{mag}} = \mathbf{a}_3$$

Crystal+AF mag structure



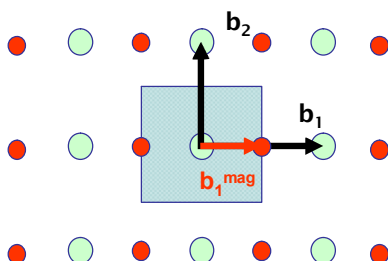
$$\mathbf{R}_n = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3$$

$$\mathbf{r}_j^0 = x_j \mathbf{a}_1 + y_j \mathbf{a}_2 + z_j \mathbf{a}_3$$

$$\mathbf{r}_j^n = \mathbf{R}_n + \mathbf{r}_j^0$$

General description (k-vector formalism)

Reciprocal space



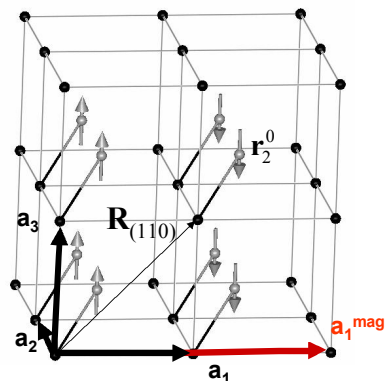
● Nuclear reflections

● Magnetic reflections

$$\mathbf{h} = h_{\text{mag}} \cdot \mathbf{b}_1^{\text{mag}} + k \cdot \mathbf{b}_2 + l \cdot \mathbf{b}_3$$

in fact the AF arrangement => ...
 h_{mag} necessarily odd (type, $2h+1$)

Crystal+AF mag structure



$$\mathbf{a}_1^{\text{mag}} = 2\mathbf{a}_1 \quad \mathbf{a}_2^{\text{mag}} = \mathbf{a}_2 \quad \mathbf{a}_3^{\text{mag}} = \mathbf{a}_3$$

$$\mathbf{b}_1^{\text{mag}} = \mathbf{b}_1/2$$

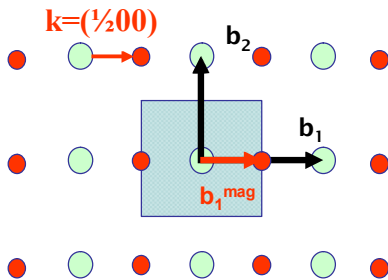


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General description (k-vector formalism)

Reciprocal space



● Nuclear reflections

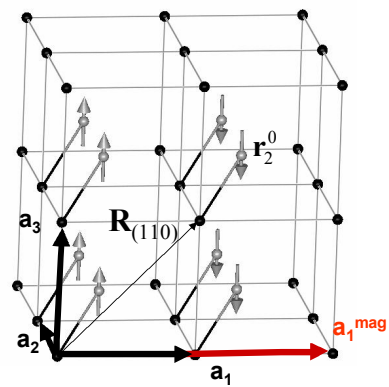
● Magnetic reflections

$$\mathbf{h} = (2h+1) \cdot \mathbf{b}_1/2 + k \cdot \mathbf{b}_2 + l \cdot \mathbf{b}_3$$

$$\mathbf{h} = h \cdot \mathbf{b}_1 + k \cdot \mathbf{b}_2 + l \cdot \mathbf{b}_3 + \mathbf{b}_1/2$$

$$\mathbf{h} = \mathbf{H} + \mathbf{k} \quad \text{« Propagation vector »}$$

Crystal+AF mag structure



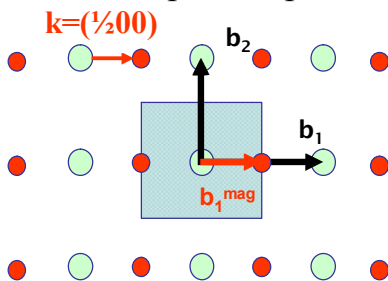
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$$\mathbf{b}_1^{\text{mag}} = \mathbf{b}_1/2$$



General description (k-vector formalism)

Reciprocal space



● Nuclear reflections

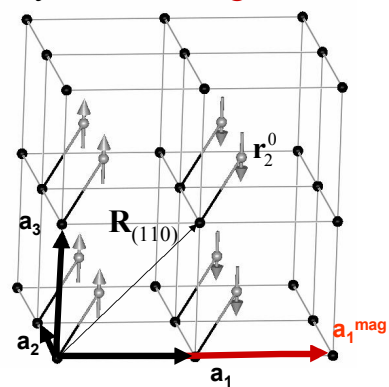
● Magnetic reflections

$$\mathbf{h} = (2h+1) \cdot \mathbf{b}_1/2 + k \cdot \mathbf{b}_2 + l \cdot \mathbf{b}_3$$

$$\mathbf{h} = h \cdot \mathbf{b}_1 + k \cdot \mathbf{b}_2 + l \cdot \mathbf{b}_3 + \mathbf{b}_1/2$$

$$\mathbf{h} = \mathbf{H} + \mathbf{k}$$

Crystal+AF mag structure



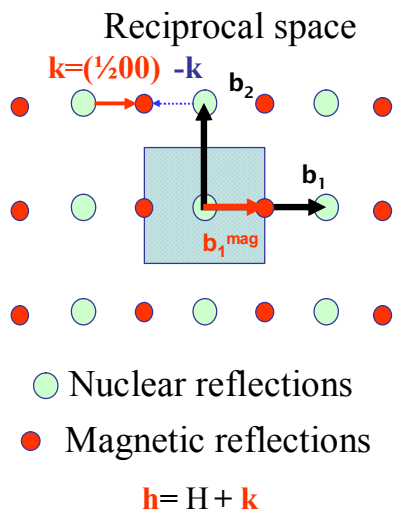
$$\mathbf{m}_{[n]i} = \mathbf{S}_i \cdot (-1)^{n_l}$$

$$\mathbf{R}_n = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3$$

$$\mathbf{m}_{nj} = \mathbf{S}_{kj} \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_n\}$$

$$\text{because } \mathbf{k} \cdot \mathbf{R}_n = n_l/2$$

General description (k-vector formalism)



Magnetic structure

$$\mathbf{m}_{nj} = \mathbf{S}_{kj} \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_n\}$$

In our example (AF):

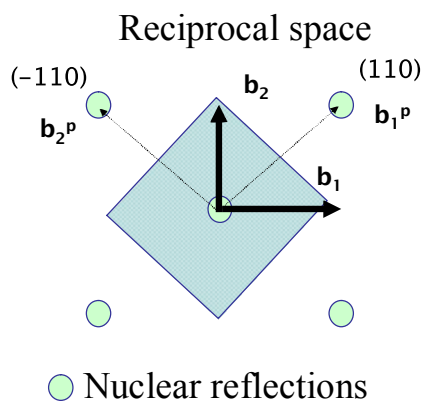
- \mathbf{k} **equivalent** to $-\mathbf{k}$
($2\mathbf{k}$ is a reciprocal lattice vector, or \mathbf{k} at the border of the Brillouin zone)
- Only \mathbf{k} sufficient to index all the magnetic reflections,
- And \mathbf{S}_{kj} must be real

In general :

- a set of $\{\mathbf{k}\}$ is needed
- And \mathbf{S}_{kj} are complex vectors

$$\mathbf{m}_{nj} = \sum_{\{\mathbf{k}\}} \mathbf{S}_{kj} \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_n\}$$

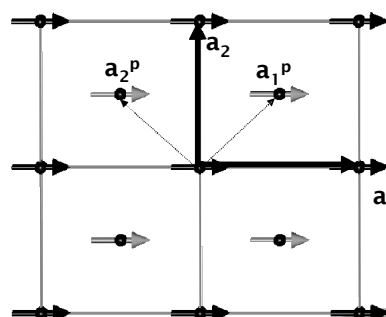
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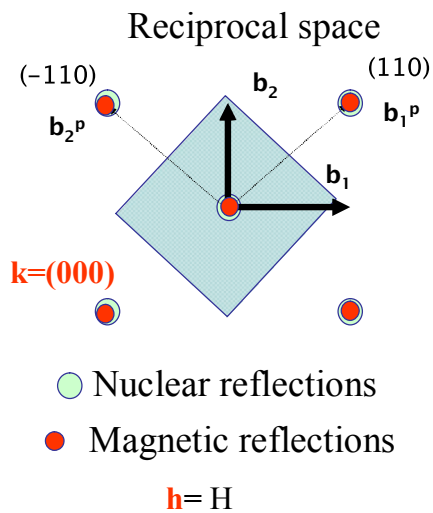
Magnetic structure

$$\mathbf{m}_{nj} = \mathbf{S}_{kj} \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_n\}$$

I/F-centred lattice (2D):



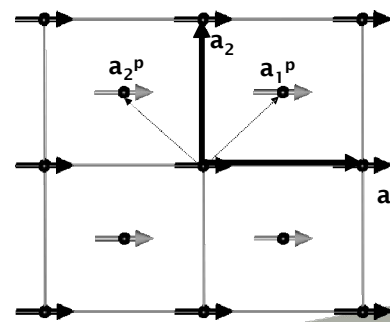
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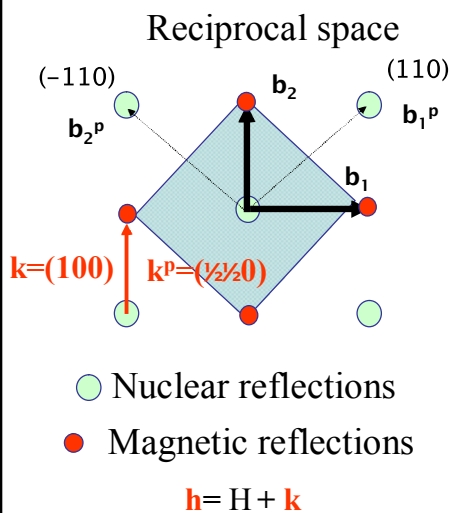
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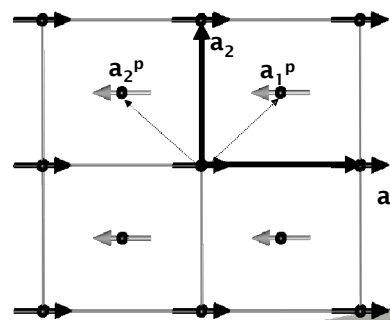
General description (k-vector formalism)



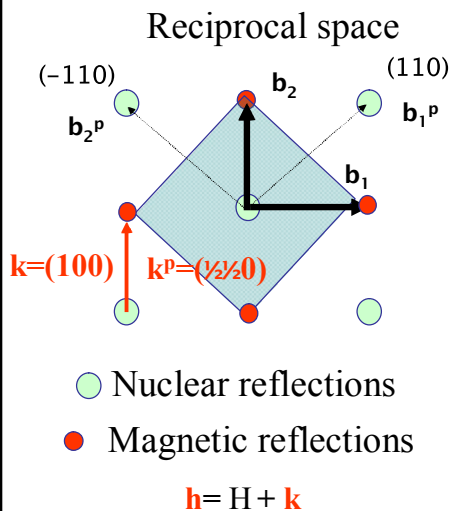
Magnetic structure

$$\mathbf{m}_{nj} = \mathbf{S}_{kj} \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_n\}$$

AF order on a centred lattice



General description (k-vector formalism)



Magnetic structure

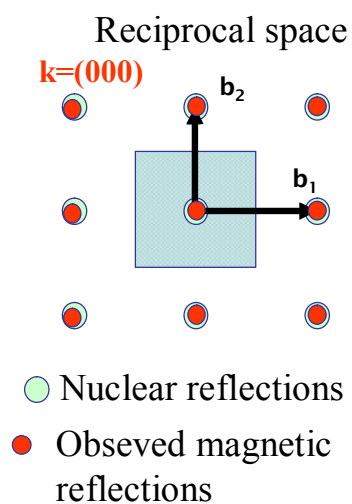
$$\mathbf{m}_{nj} = \mathbf{S}_{kj} \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_n\}$$

I-centred lattice :

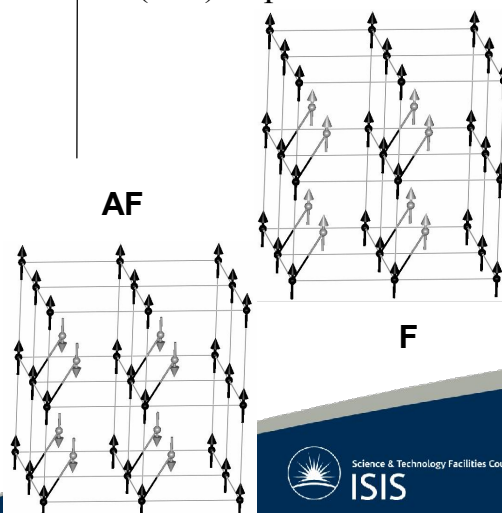
- \mathbf{k} **equivalent** to $-\mathbf{k}$
($2\mathbf{k}$ is a reciprocal lattice vector)
- Because \mathbf{R}_n contains type $(\frac{1}{2}\frac{1}{2}\frac{1}{2})\dots$ translations, \mathbf{k} vectors, which can have integer components, have in fact non-integer components in the *primitive* cell



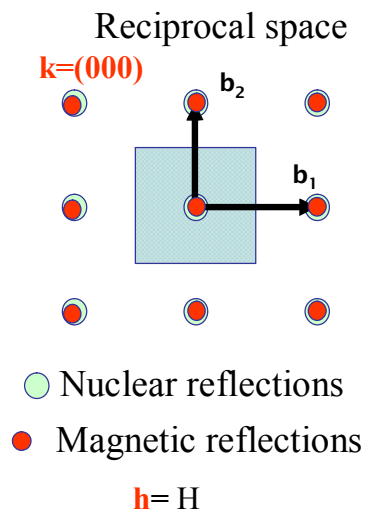
General description (k-vector formalism)



Magnetic structure
 $\mathbf{k} = (000)$ in primitive cell:



General description (k-vector formalism)



Magnetic structure

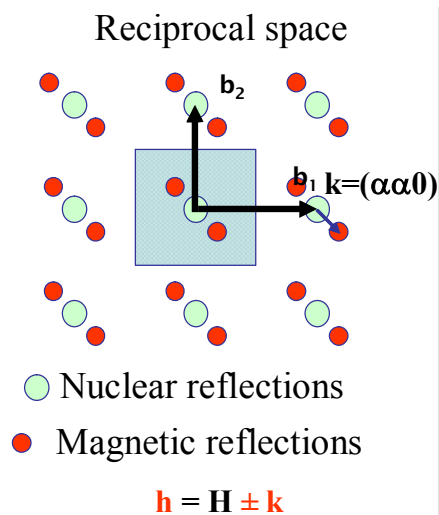
$$\mathbf{m}_{nj} = \sum_{\mathbf{k}_j} \mathbf{S}_{\mathbf{k}_j} \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_n\}$$

When $\mathbf{k}=(000)$ (primitive P):

- $\mathbf{m}_{li} = \mathbf{S}_{\mathbf{k}_j}$ whatever n :
this only means that the cell
of the the magnetic structure
is the same as that of the
nuclear structure (not
necessarily ferromagnetic)



General description (k-vector formalism)



Magnetic structure

$$\mathbf{m}_{nj} = \sum_{\{\mathbf{k}\}} \mathbf{S}_{\mathbf{k}_j} \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_n\}$$

Especially, when $\{\mathbf{k}\} = \mathbf{k}, -\mathbf{k}$:
and \mathbf{k} **NOT equivalent** to $-\mathbf{k}$

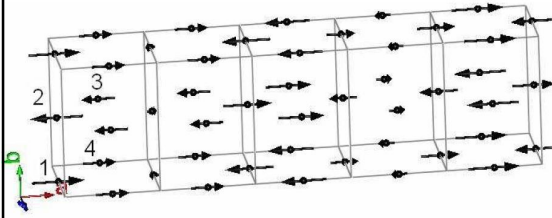
- we need $(\mathbf{k}, -\mathbf{k})$ to index
all the magnetic reflections,
- $(\mathbf{k}, -\mathbf{k})$ terms with complex
 $\mathbf{S}_{\mathbf{k}_j}$ vectors must be
summed to calculate \mathbf{m}_{ij} with
the necessary condition for
real \mathbf{m}_{nj} : $\mathbf{S}_{-\mathbf{k}_j} = \mathbf{S}_{\mathbf{k}_j}^*$

Incommensurate structures

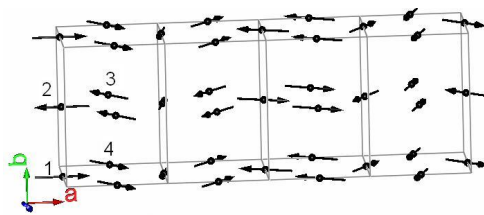


General description : Application

TbMnO_3 $T_{N2} < T < T_{N1} = 41\text{K}$ Sinusoidal

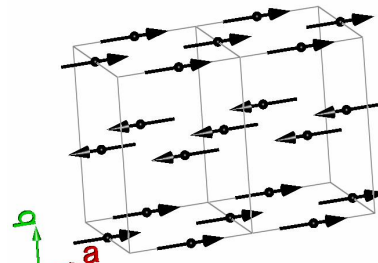


$T < T_{N2} = 23\text{K}$ Cycloid



Incommensurate structures

LaMnO_3 $T_N = 150\text{K}$



AF

Commensurate structure



General description : Application

$$\mathbf{m}_{nj} = \sum_{\{\mathbf{k}\}} \mathbf{S}_{kj} \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_n\}$$

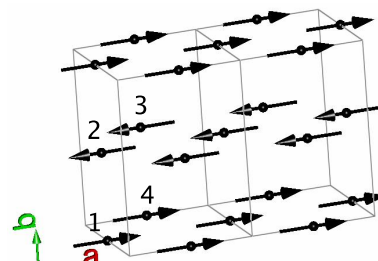
$\mathbf{k} = (000)$

$$\mathbf{m}_{li} = \mathbf{S}_{kj}$$

$$\mathbf{S}_{k1} = -\mathbf{S}_{k2} = -\mathbf{S}_{k3} = \mathbf{S}_{k4},$$

all reals vectors

LaMnO_3 $T_N = 150\text{K}$



AF

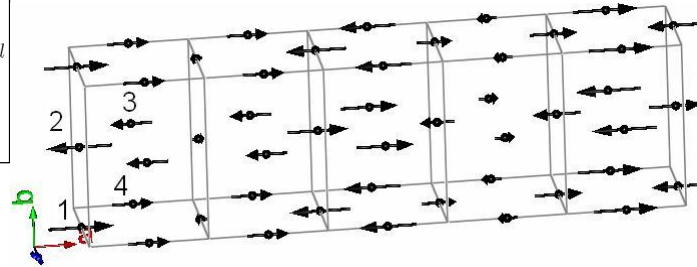
Commensurate AF structure



General description : Application

TbMnO₃ T_N<41K **k=(0.28 0 0)**

$$\begin{aligned} \mathbf{m}_{li} &= \mathbf{S}_i^{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{R}_l} \\ &+ \mathbf{S}_i^{-\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{R}_l} \\ \mathbf{S}_i^{-\mathbf{k}} &= \mathbf{S}_i^{*\mathbf{k}} \end{aligned}$$



$$\mathbf{S}_j^{\mathbf{k}} = 1/2(\mathbf{A}_j + i\mathbf{B}_j)e^{-i\phi}$$

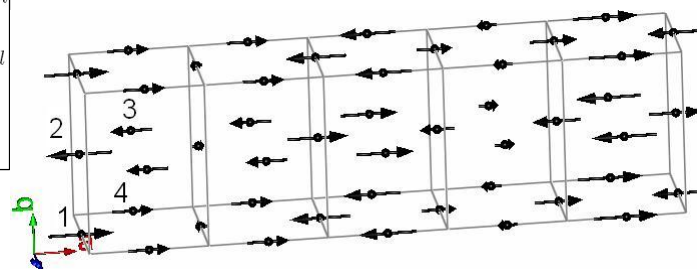
$$\mathbf{S}_{jl} = \mathbf{A}_j \cos(\mathbf{k} \cdot \mathbf{R}_l + \phi_j) + \mathbf{B}_j \sin(\mathbf{k} \cdot \mathbf{R}_l + \phi_j)$$



General description : Application

TbMnO₃ T_N<41K **k=(0.28 0 0)**

$$\begin{aligned} \mathbf{m}_{li} &= \mathbf{S}_i^{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{R}_l} \\ &+ \mathbf{S}_i^{-\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{R}_l} \\ \mathbf{S}_i^{-\mathbf{k}} &= \mathbf{S}_i^{*\mathbf{k}} \end{aligned}$$



$$\mathbf{S}_j^{\mathbf{k}} = 1/2(\mathbf{A}_j + i\mathbf{B}_j)e^{-i\phi}$$

$$\mathbf{B}_j = 0$$

$$\mathbf{S}_1^{\mathbf{k}} = (C_1, 0, 0)$$

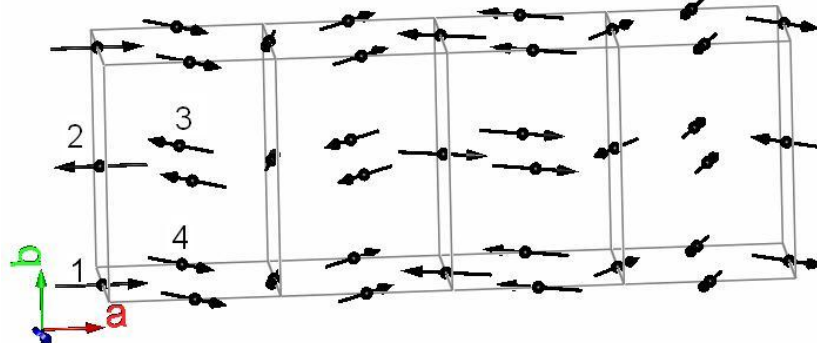
$$\mathbf{S}_2^{\mathbf{k}} = (-C_1, 0, 0) \quad \phi$$

$$\mathbf{S}_3^{\mathbf{k}} = (-C_1 e^{-2i\pi \cdot \mathbf{k}_x / 2}, 0, 0)$$

$$\mathbf{S}_4^{\mathbf{k}} = (C_1 e^{-2i\pi \cdot \mathbf{k}_x / 2}, 0, 0)$$

General description : Application

TbMnO₃ $\mathbf{k}=(0.28\ 0\ 0)$ $T < T_{N2}=23\text{K}$ Cycloid



$$S_j^{\mathbf{k}} = 1/2(\mathbf{A}_j + i.\mathbf{B}_j)e^{-i\phi}$$

$$\mathbf{B}_j \perp \mathbf{A}_j$$

$$S_1^{\mathbf{k}} = 1/2(u, 0, i.v)$$

$$S_2^{\mathbf{k}} = 1/2(-u, 0, -i.v)$$

$$S_3^{\mathbf{k}} = 1/2.e^{-2i\pi.\mathbf{k}_x/2}(-u, 0, i.v)$$

$$S_4^{\mathbf{k}} = 1/2.e^{-2i\pi.\mathbf{k}_x/2}(u, 0, -i.v)$$



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What's a magnetic structure?

Why study magnetic order?

Generic description (k-vector formalism)

- Related to representation/symmetry analysis (not covered)
- Alternative to Shubnikov symmetry, which are *TABULATED* crystallographic magnetic space groups : this is however restricted to *commensurate* structures

The magnetic structure factor

Practical aspects (Experimental choices / software, and methods for magnetic structure determination)

Note on available software:

Fullprof : General, k-vector formalism

GSAS: Contains Shubnikov symmetry, but restricted to *commensurate* structures

Magnetic scattering and structure factors

$\mu_N = -\gamma\mu_N\sigma$

Dipolar interaction term with one electron

$$\langle \mathbf{k}_F | V_M | \mathbf{k}_I \rangle = \int e^{i\mathbf{k}_I \cdot \mathbf{r}} \left[-\gamma\mu_N\sigma \cdot \frac{\mu_0}{4\pi} \nabla \left(\frac{2\mu_B S \times \mathbf{R}}{R^3} \right) \right] e^{-i\mathbf{k}_F \cdot \mathbf{r}} d\mathbf{r}$$

$\langle \mathbf{k}_F | V_M | \mathbf{k}_I \rangle = 4\pi \cdot \mathbf{Q} \times \left(\sum_i \exp(i\mathbf{Q} \cdot \mathbf{r}_i) \cdot \mathbf{s}_i \right) \times \mathbf{Q}$

Total S_i

\Rightarrow vector scattering amplitude for one atom

$$\langle \mathbf{k}_F | V_M | \mathbf{k}_I \rangle = 4\pi \cdot f(\mathbf{Q}) \cdot \mathbf{Q} \times \mathbf{S}_i \times \mathbf{Q}$$

Magnetic scattering and structure factors

Magnetisation density: $\mathbf{M}(\mathbf{r}) = \mathbf{M}_S(\mathbf{r}) + \mathbf{M}_L(\mathbf{r})$

$$\mathbf{M}(\mathbf{Q}) = \int \mathbf{M}(\mathbf{r}) \exp(i\mathbf{Q} \cdot \mathbf{r}) d\mathbf{r}$$
$$\langle \mathbf{k}_F | V_M | \mathbf{k}_I \rangle = 4\pi \cdot \mathbf{M}_\perp(\mathbf{Q}) \cdot \sum_{\mathbf{Q}=\mathbf{H}} \delta(\mathbf{Q} - \mathbf{H})$$

$\mathbf{M}_\perp(\mathbf{Q}) = \mathbf{Q} \times \mathbf{M}(\mathbf{Q}) \times \mathbf{Q}$ \Rightarrow Magnetic interaction vector for a crystal

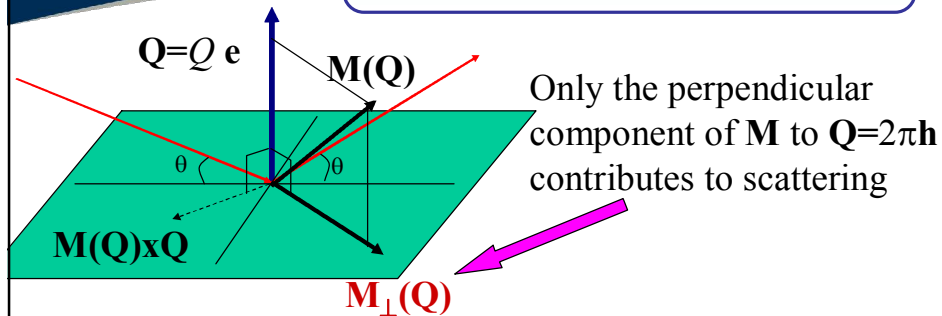
$\mathbf{M}(\mathbf{Q}) = \sum_i f_i(\mathbf{Q}) \cdot \mathbf{S}_i \exp(i\mathbf{Q} \cdot \mathbf{r}_i)$ \Rightarrow Magnetic structure factor

for a crystal \Rightarrow vector scattering amplitude for one atom

$$\langle \mathbf{k}_F | V_M | \mathbf{k}_I \rangle = 4\pi \cdot f(\mathbf{Q}) \cdot \mathbf{Q} \times \mathbf{S}_i \times \mathbf{Q}$$



Magnetic scattering and structure factors



$$\mathbf{M}_{\perp}(\mathbf{Q}) = \mathbf{Q} \times \mathbf{M}(\mathbf{Q}) \times \mathbf{Q}$$

\Rightarrow Magnetic interaction vector for a crystal

$$\mathbf{M}(\mathbf{Q}) = \sum_i f_i(\mathbf{Q}) \cdot \mathbf{S}_i \exp(i\mathbf{Q} \cdot \mathbf{r}_i)$$

\Rightarrow Magnetic structure factor

\Rightarrow vector scattering amplitude for one atom

$$\langle \mathbf{k}_F | V_M | \mathbf{k}_I \rangle = 4\pi \cdot f(\mathbf{Q}) \cdot \mathbf{Q} \times \mathbf{S}_i \times \mathbf{Q}$$

Magnetic scattering and structure factors

For non-polarised neutrons

$$I_h = \mathbf{M}_{\perp h} \cdot \mathbf{M}_{\perp h}^* + N_h N_h^*$$

Magnetic Phase:

$$\mathbf{h} = \mathbf{H} \pm \mathbf{k}$$

Nuclear Phase:

$$\mathbf{h} = \mathbf{H}$$

Scattering vector

Structural model

Arrangement of the moments

$$\mathbf{m}_{li} = \sum_{\{k\}} \mathbf{S}_k \exp\{-2\pi i \mathbf{k} \cdot \mathbf{R}_l\}$$

Atomic positions

$$\mathbf{r}_{li}^0 = \mathbf{R}_l + \mathbf{r}_i$$

Structure Factor/Intensity

$$\mathbf{M}(\mathbf{h}) = p \sum_{j=1}^n f_j(\mathbf{h}) \mathbf{S}_j \exp\{2\pi i \mathbf{h} \cdot \mathbf{r}_j\}$$

$$F(\mathbf{H}) = \sum_{j=1}^n b_j \exp\{2\pi i \mathbf{H} \cdot \mathbf{r}_j\}$$

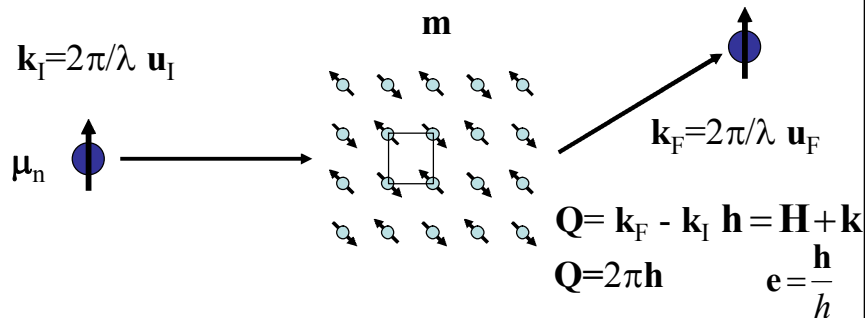
$$\mathbf{M}_{\perp h} = \mathbf{M}(\mathbf{h}) - \mathbf{e} (\mathbf{e} \cdot \mathbf{M}(\mathbf{h})) \text{ with } \mathbf{e} = \frac{\mathbf{h}}{h}$$

$$I_h^{\text{nuc}} = F(\mathbf{H}) \cdot F^*(\mathbf{H})$$

$$I_h = \mathbf{M}_{\perp h} \cdot \mathbf{M}_{\perp h}^*$$



Magnetic structure factor: Summary of working formulas



$$\mathbf{M}_{\perp h} = \mathbf{e} \times \mathbf{M}(\mathbf{h}) \times \mathbf{e} = \mathbf{M}(\mathbf{h}) - \mathbf{e}(\mathbf{e} \cdot \mathbf{M}(\mathbf{h}))$$

$$\mathbf{M}(\mathbf{h}) = p \sum_{j=1}^n f_j(\mathbf{h}) \mathbf{S}_k^j \exp \{2\pi i \mathbf{h} \cdot \mathbf{r}_j\}$$

$$I_h = \mathbf{M}_{\perp h} \cdot \mathbf{M}_{\perp h}^*$$



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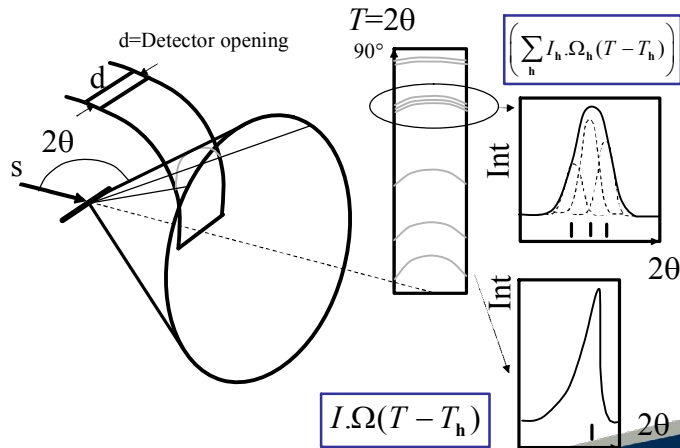
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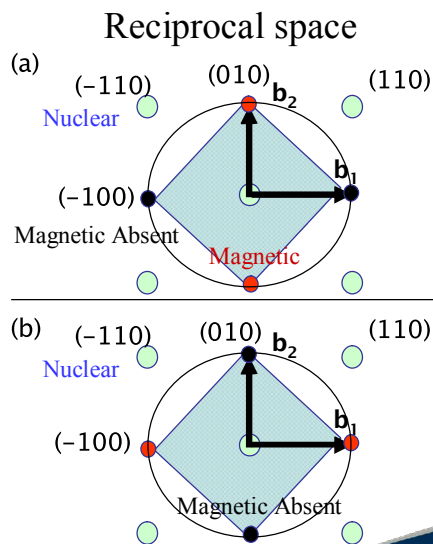
Practical aspects

Powder diffraction: benefit from the power of the rietveld technique, but problem of accidental overlap...

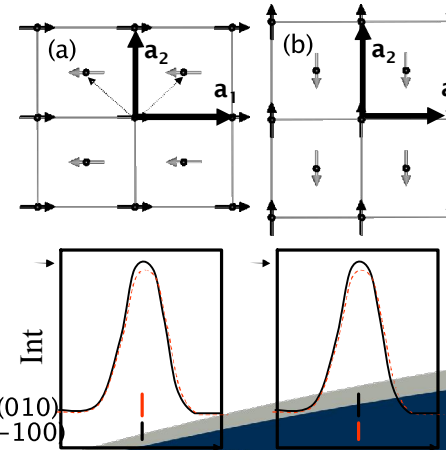


Practical aspects

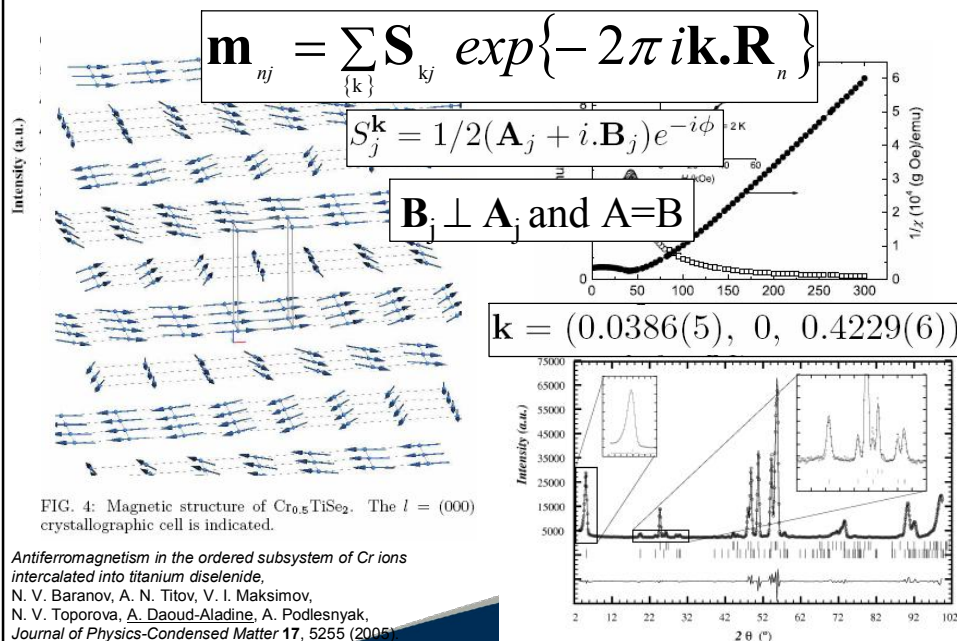
Powder diffraction: problem of accidental overlap...



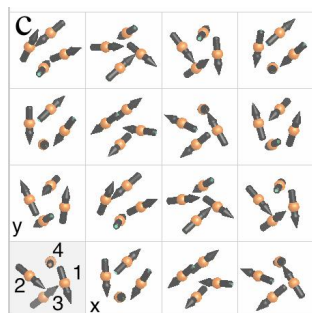
AF order on a centred lattice



From simple to complex cases...



From simple to complex cases...



Solution/refinement of Single XTAL data
 gives a magnetic structure
 that shows no AF plaquettes nor dimers,
 but canted pairs

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PHYSICAL REVIEW LETTERS

week ending
 19 NOVEMBER 2004

p. 217206-1

**Incommensurate Magnetic Ordering in $\text{Cu}_2\text{Te}_2\text{O}_5\text{X}_2$ ($\text{X} = \text{Cl}, \text{Br}$)
 Studied by Neutron Diffraction**

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 Laboratory for Neutron Scattering, ETHZ & PSI, CH-5232 Villigen, Switzerland

P.-J. Brown
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 Institut de Physique de la Matière Complexe, EPFL, CH-1015 Lausanne, Switzerland
 (Received 19 May 2004; revised manuscript received 14 September 2004; published 18 November 2004)

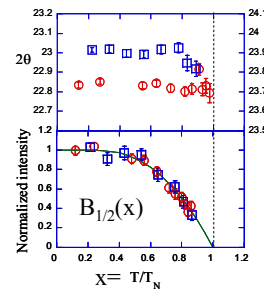
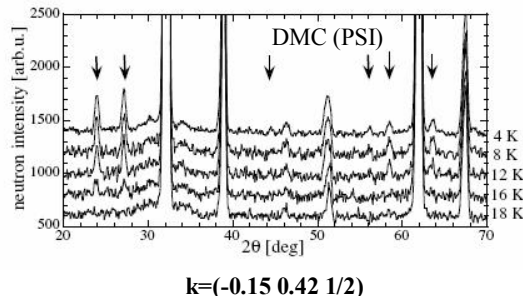
From simple to complex cases...

NDP

6-8
magnetic
peaks

4 Cu atoms
per cell
⇒

23 magnetic
params



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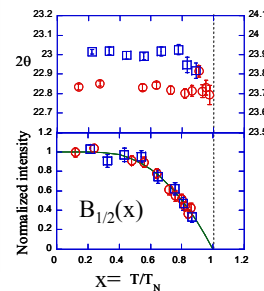
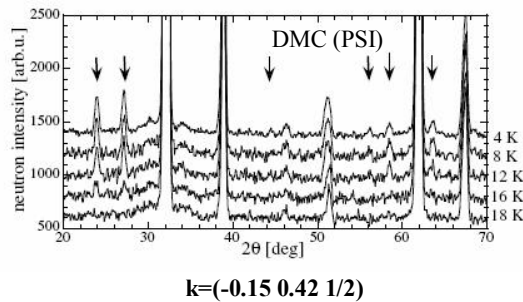
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Single XTAL

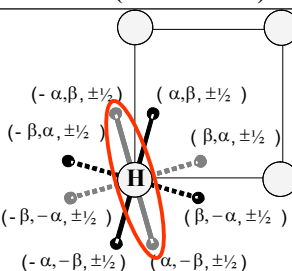
50
magnetic
reflections

Structure solution:

Simulated annealing

Structure refinement:

Least squares



Star of the wave vector

⇒ Complex domain structure

Even **focusing on one arm** of
the star (i.e on one domain)...

at TRICS (PSI) and D13 (ILL):



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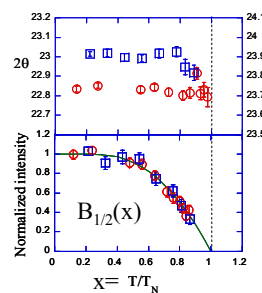
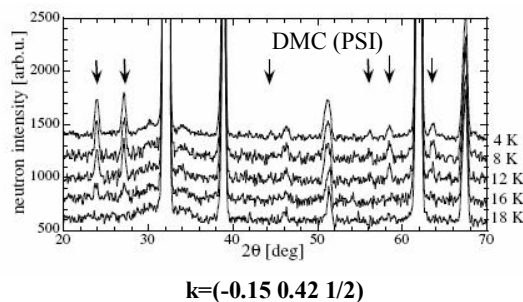
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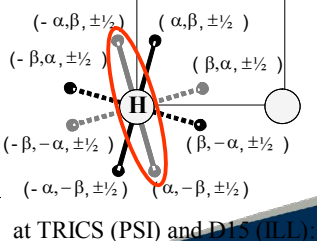
Single XTAL

50
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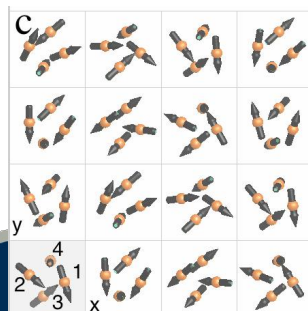
Structure solution:
Simulated annealing

Structure refinement:

Least squares



at TRICS (PSI) and D15 (ILL):

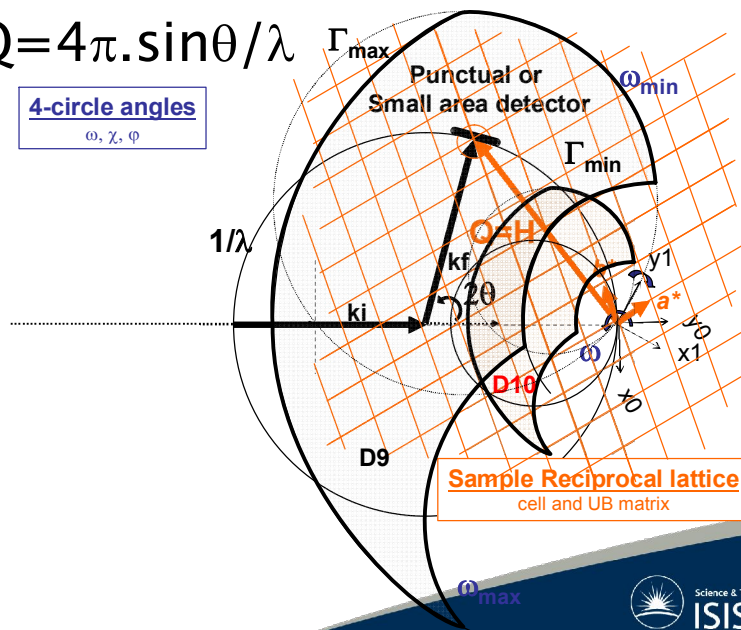


Single crystal diffraction: 4-circles

$$Q = 4\pi \cdot \sin\theta / \lambda$$

4-circle angles

ω, χ, ϕ

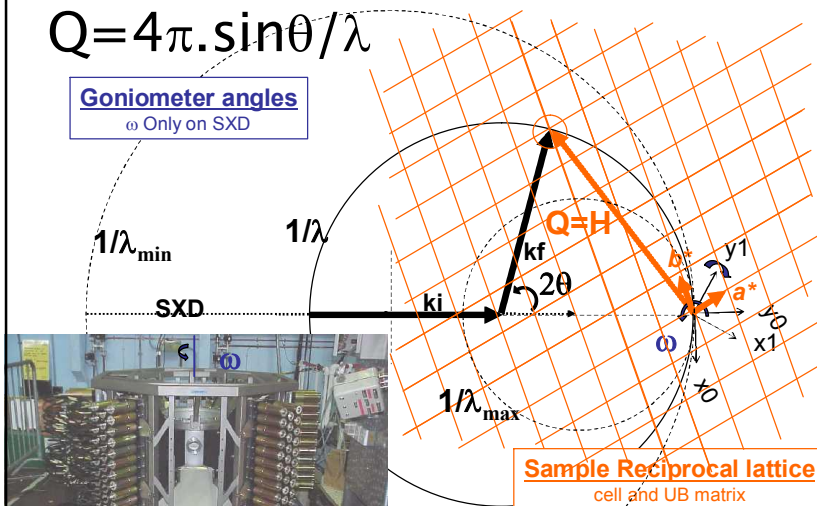


Single crystal diffraction: TOF-laue

$$Q = 4\pi \cdot \sin\theta / \lambda$$

Goniometer angles

ω Only on SXD

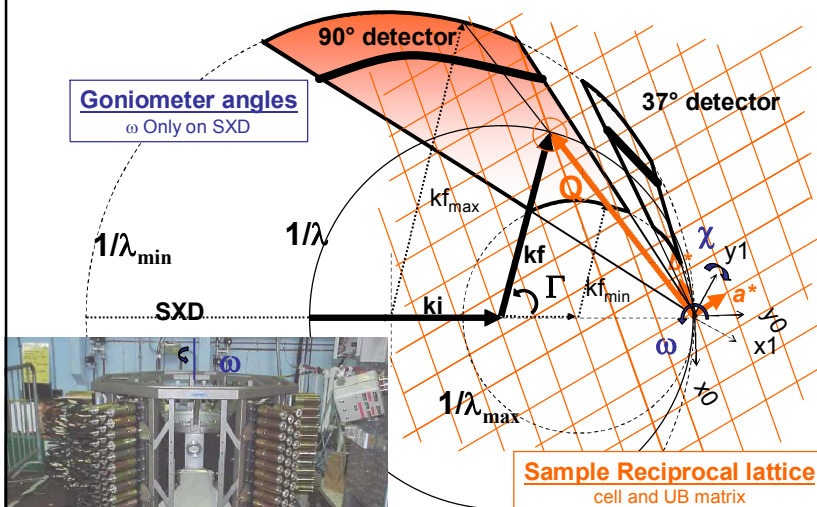


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Single crystal diffraction: TOF-laue

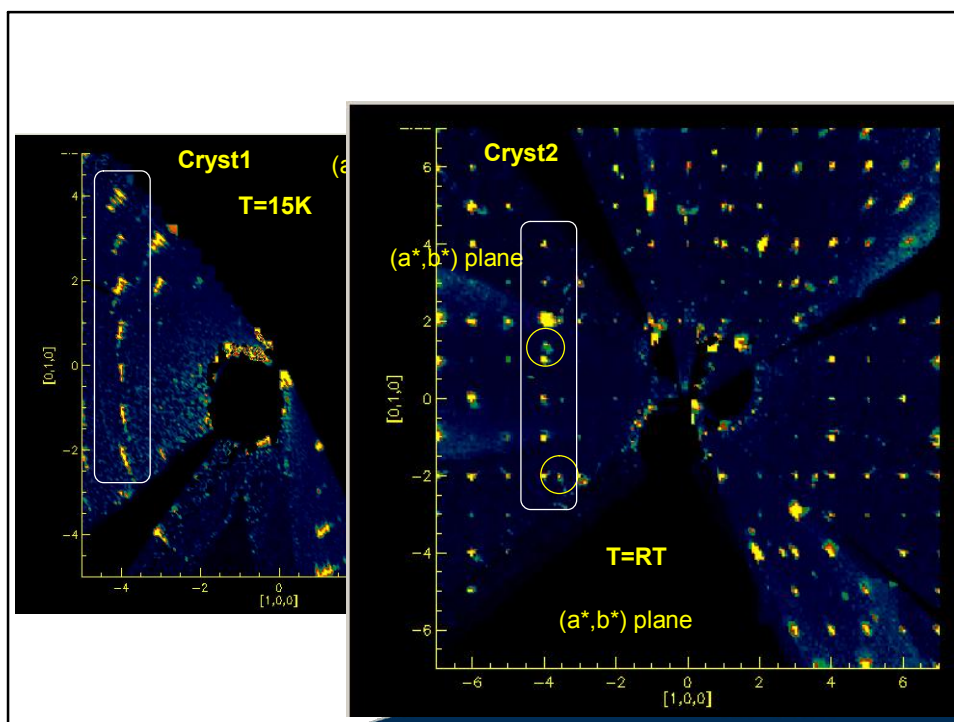
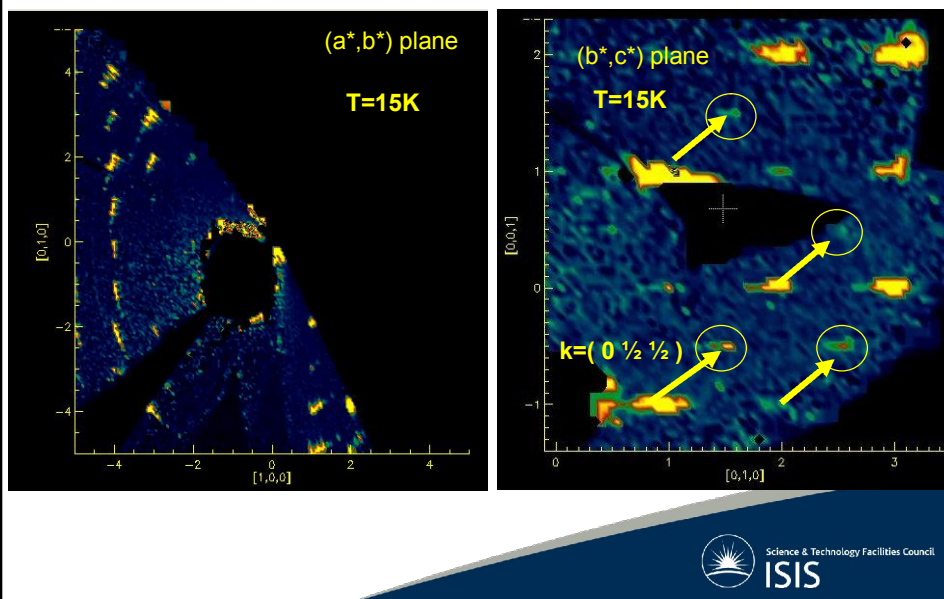
Goniometer angles

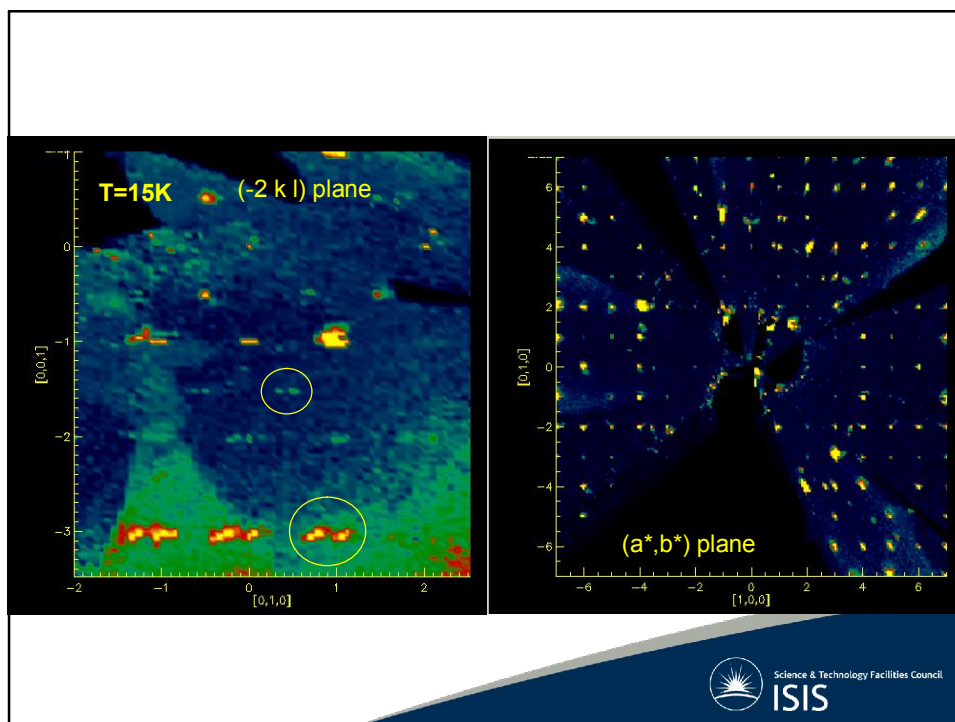
ω Only on SXD



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CaV₂O₄
Quasi one dimensional magnet V³⁺ S=1





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Structure determination methods

Except for simple cases, the Rietveld “refinement” can only be a final stage of a magnetic structure determination

Before using it, a maximum number of constraints on the magnetic model are desirable (ex: symmetry analysis), or starting models can be obtained using structure solution approaches

Single crystal data are always better, but can be tricky!

For advanced topics:

see the Fullprof Suite documentation and tutorials at:

<http://www.ill.fr/dif/Soft/fp/php/tutorials.html>



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