

# Neutrons in soft matter

Lecture 2 – Reflectometry & Dynamics

João T. Cabral & Julia S Higgins  
Department of Chemical Engineering  
Imperial College London

David G Bucknall  
Heriot-Watt, UK

# Outline

## Lecture 1 – Structure & kinetics – SANS

### Introduction

soft matter & relevance of neutron scattering

Single objects: spheres, coils, rods...

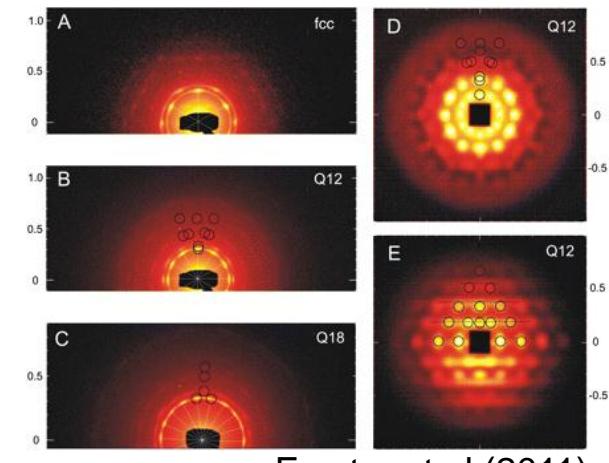
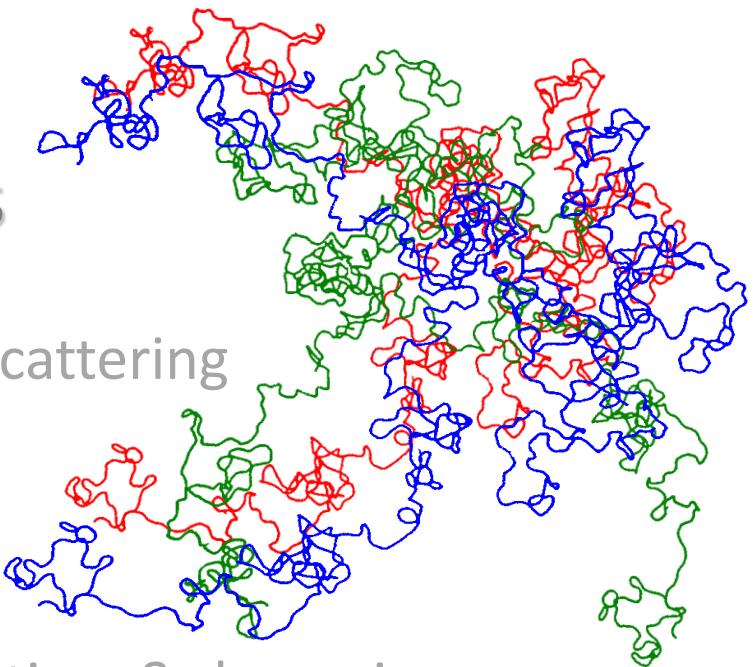
Single chain polymer conformation  
(solution and blends)

Polymer blends: interactions, conformation & dynamics  
(equilibrium and phase separation)

## Lecture 2 – Interfaces and dynamics

Reflectivity and diffusion

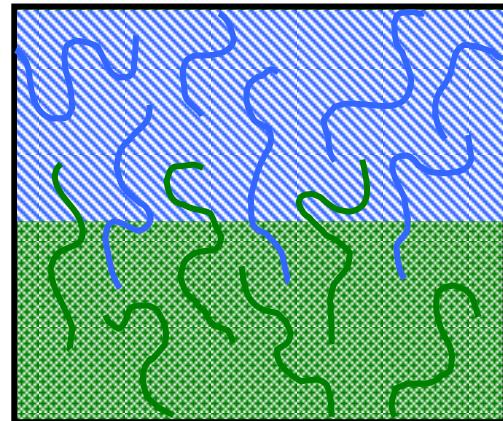
Dynamics in soft matter, QENS, BS, Spin-echo



# Reflectometry: study of interfaces

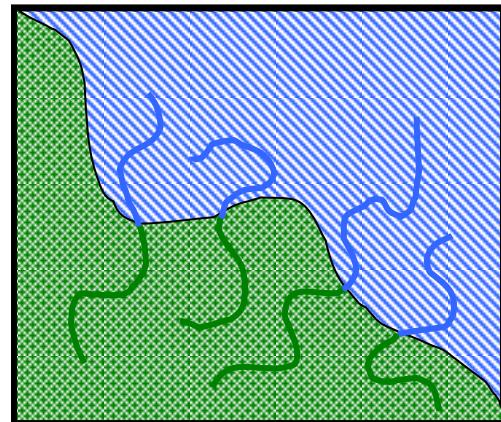
## Miscible systems

- Interdiffusion, e.g., welding

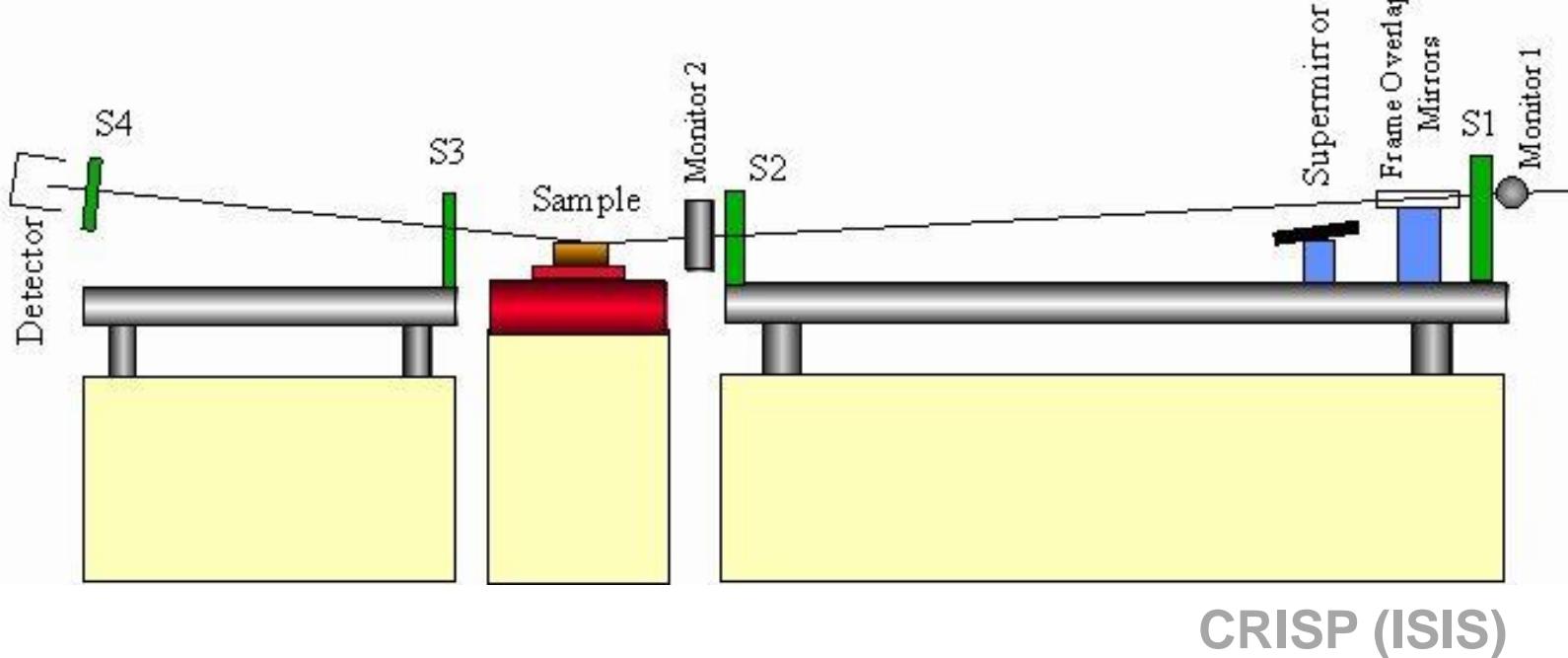


## Immiscible systems

- Copolymers, e.g., di-blocks
- Reduce interfacial tension  
→ smaller dispersed phase
- Entangle with homopolymers  
→ **increase strength**



# Reflectometry





# Significance of the interfacial width

Theoretical width

- Infinite molecular weight limit

$$w_t = \frac{2a}{(6\chi)^{0.5}}$$

E Helfand & AM Sapse  
*J Chem Phys* 62 (1975) 1327

where  $a$  (statistical segment length)

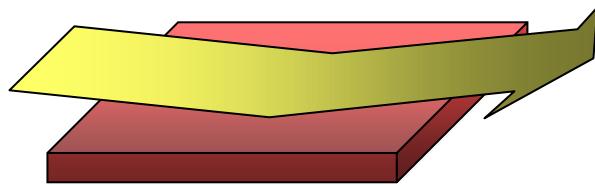
- Finite molecular weight limit

$$w_t = \frac{2a}{\sqrt{6}} \left( \chi - \frac{\pi^2}{6} (N_1^{-1} + N_2^{-1}) \right)^{-1/2}$$

M Stamm & DW Schubert  
*Ann Rev Mater Sci*  
25 (1995) 325

⇒ Measure interfacial width to find  $\chi$

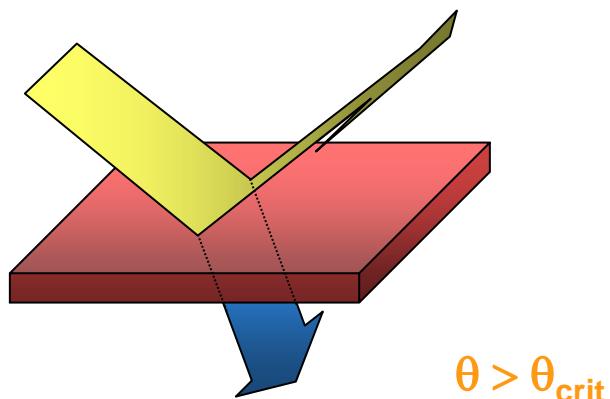
# Basics of Reflectivity



$\theta < \theta_{\text{crit}}$   
only reflection

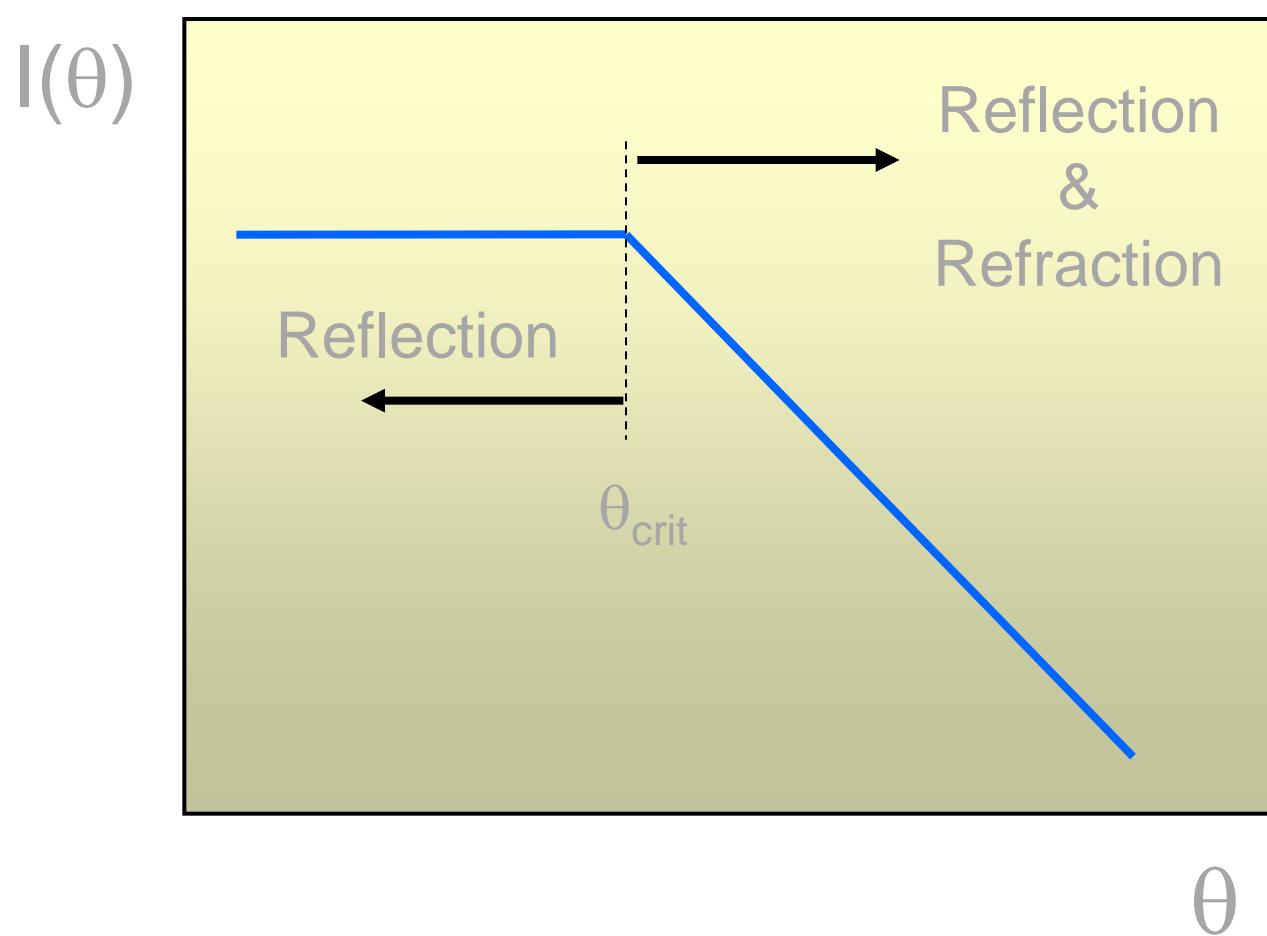


critical angle       $\theta = \theta_{\text{crit}}$

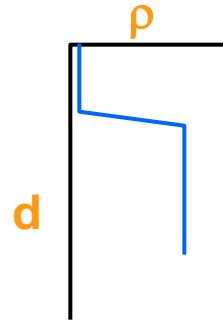
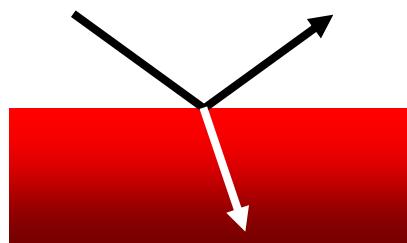


$\theta > \theta_{\text{crit}}$   
reflection and refraction

# The Reflectivity Profile



## Simplest Case

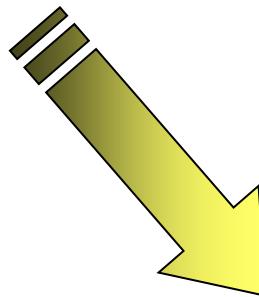


## Information Content

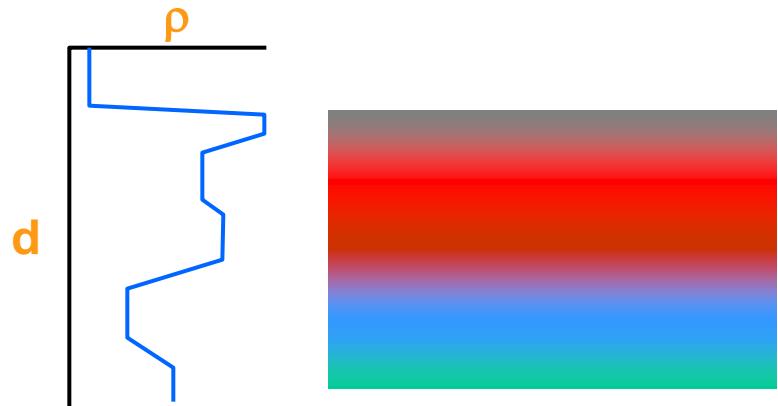
What about lateral  
information?



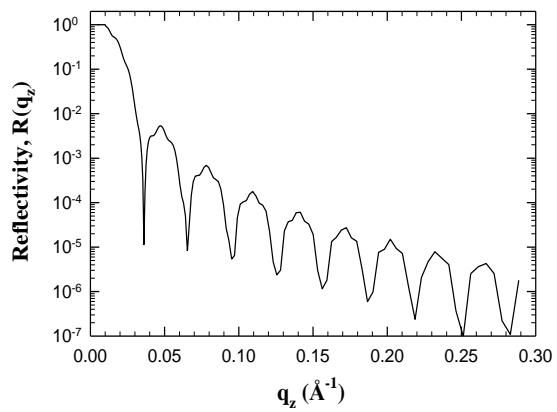
Off-specular !



## Complex Case



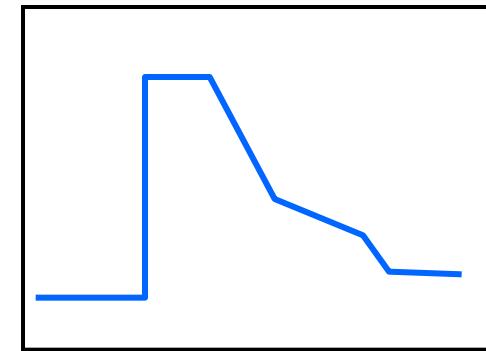
# Evaluating Reflectivity Data



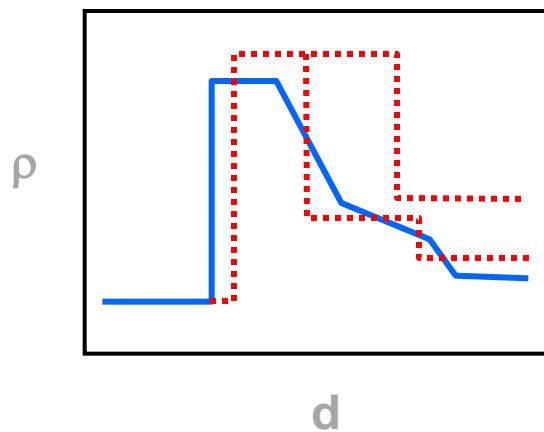
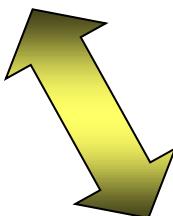
*fft*



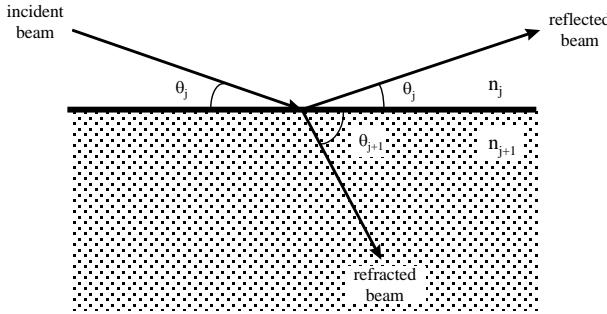
*Ideal*



*Real world*



# Single layers and bilayers



$$n_j = 1 - \frac{\lambda^2 N_d b}{2\pi} = 1 - \frac{\lambda^2 \rho_z}{2\pi}$$

$$n_j \cos \theta_j = n_{j+1} \cos \theta_{j+1}$$

**Snell's Law**

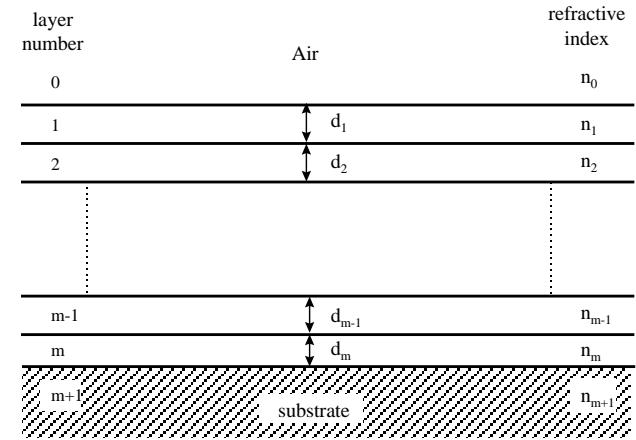
$$r_{j,j+1} = \frac{n_j \sin \theta_j - n_{j+1} \sin \theta_{j+1}}{n_j \sin \theta_j + n_{j+1} \sin \theta_{j+1}}$$

**Fresnel's law**

$$q = 2k = \frac{4\pi}{\lambda} \sin \theta$$

$$r_{j,j+1} = \left( \frac{q_{z,j} - q_{z,j+1}}{q_{z,j} + q_{z,j+1}} \right)$$

$$R = r_{j,j+1} r_{j,j+1}^*$$



$$r'_{m-1,m} = \frac{r_{m-1,m} - r_{m,m+1} \exp(2i\beta_m)}{1 + r_{m-1,m} r_{m,m+1} \exp(2i\beta_m)}$$

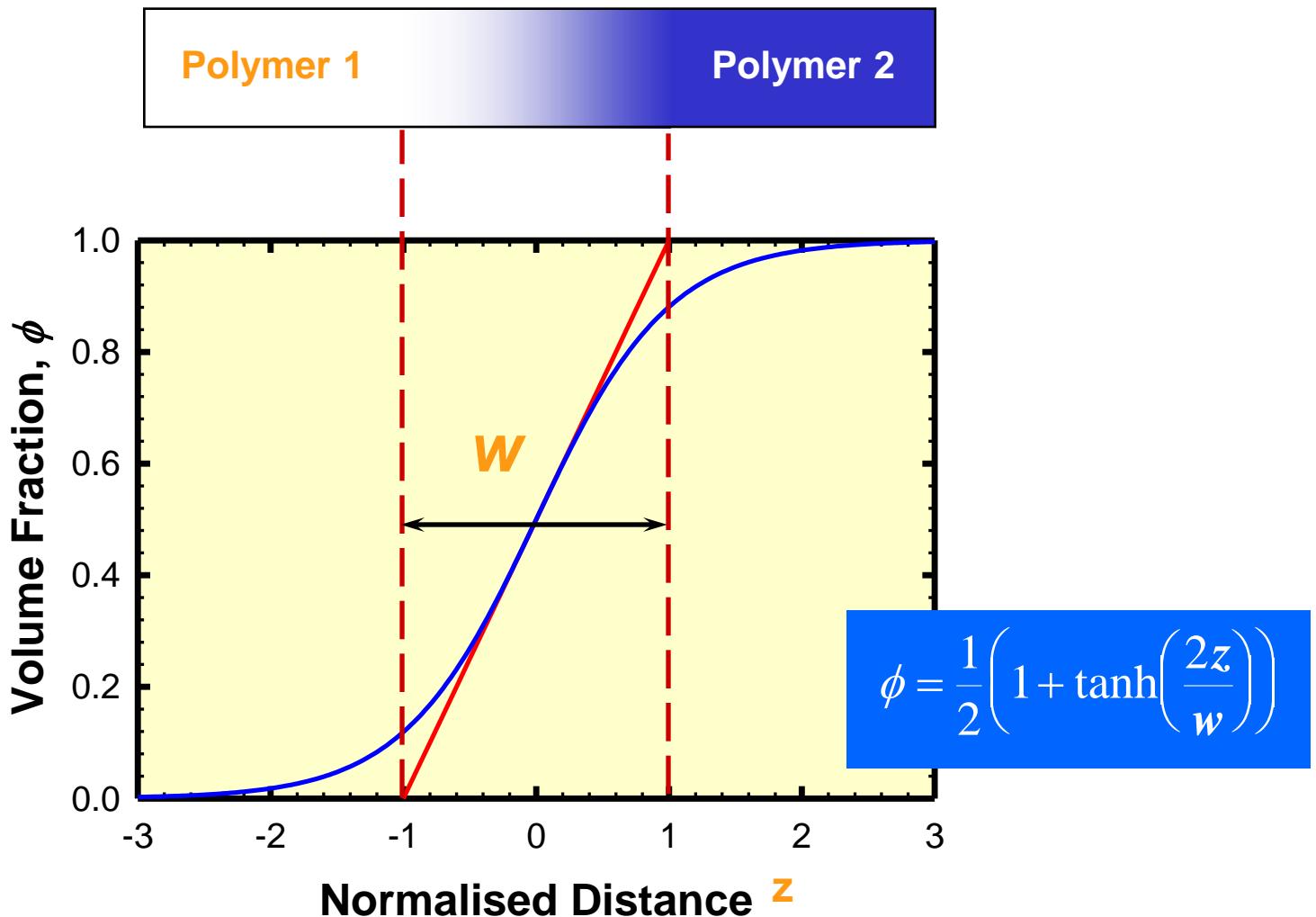
$$\beta_m = (2\pi/\lambda) n_m d_m \sin \theta$$

$$c_m = \begin{bmatrix} \cos \beta_m & -(i/\kappa_m) \sin \beta_m \\ -i\kappa_m \sin \beta_m & \cos \beta_m \end{bmatrix}$$

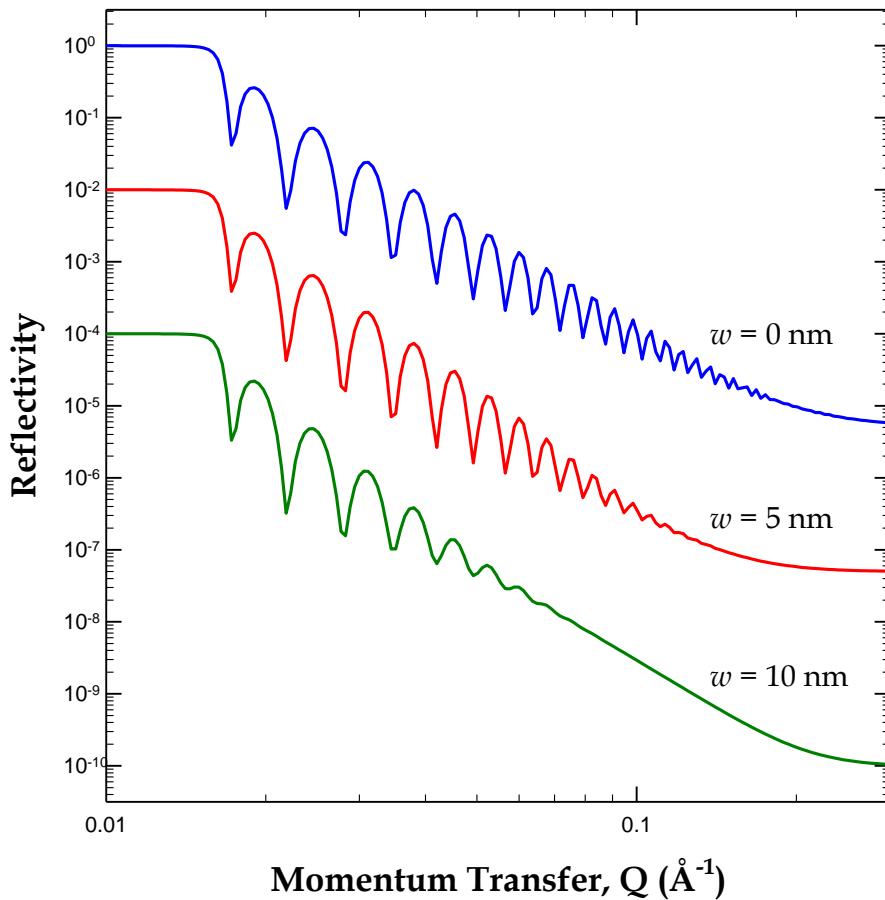
$$M = \prod_{m=0}^m c_m = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$$R = \left| \frac{(M_{11} + M_{12} \kappa_{m+1}) \kappa_0 - (M_{21} + M_{22}) \kappa_{m+1}}{(M_{11} + M_{12} \kappa_{m+1}) \kappa_0 + (M_{21} + M_{22}) \kappa_{m+1}} \right|^2$$

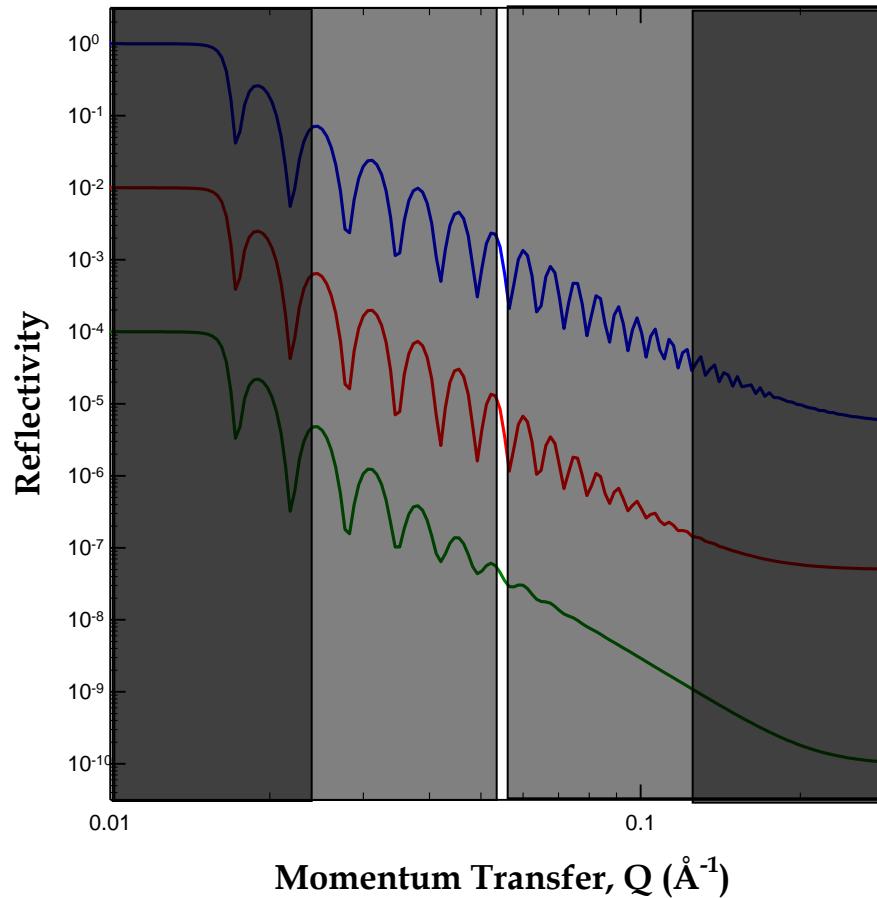
# Interfacial Width - Definition



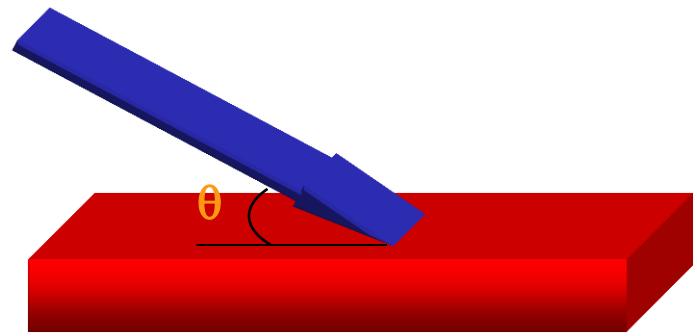
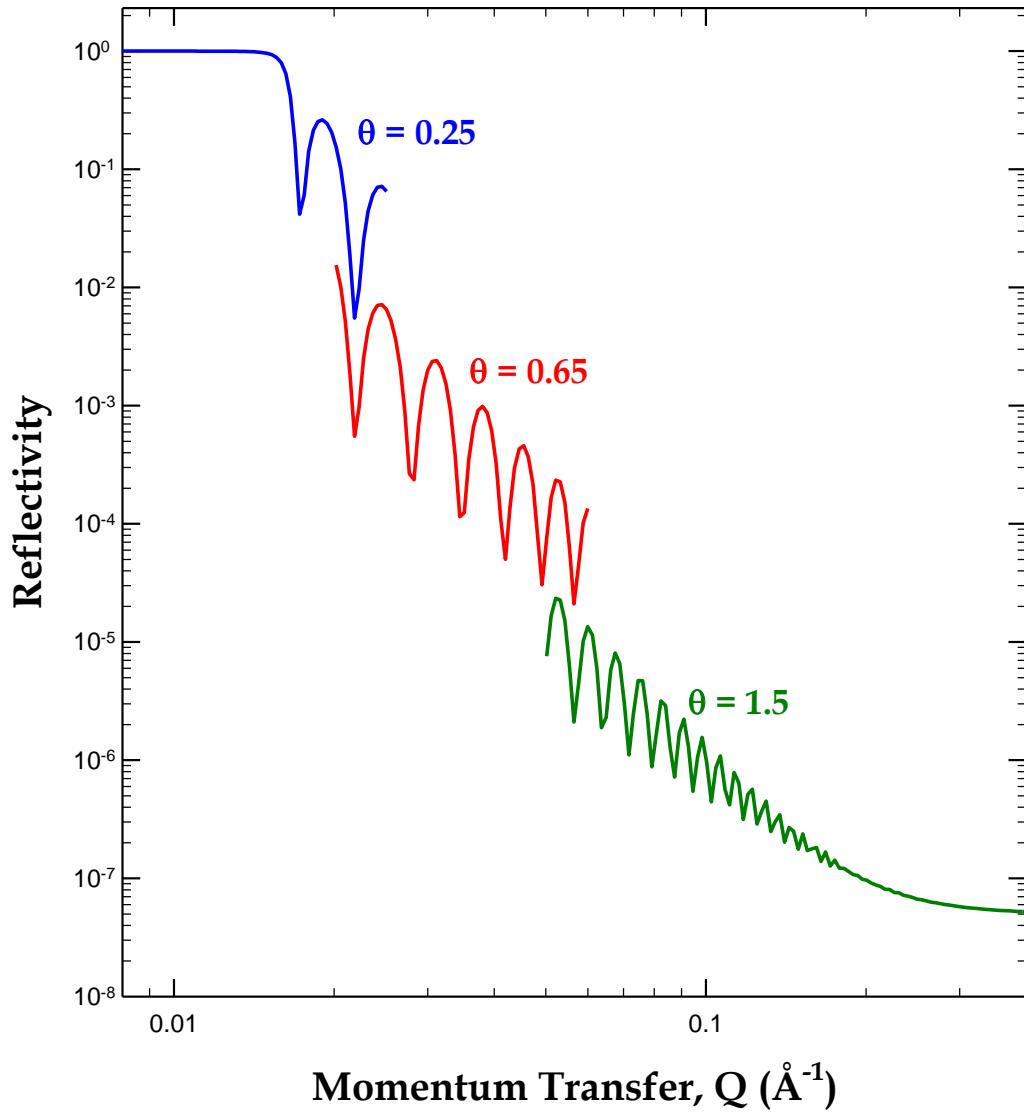
# Effect of Interdiffusion on Reflectivity Profiles



## Effect of Limiting Q range on Observation Window



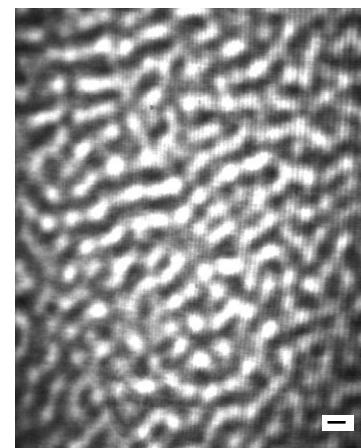
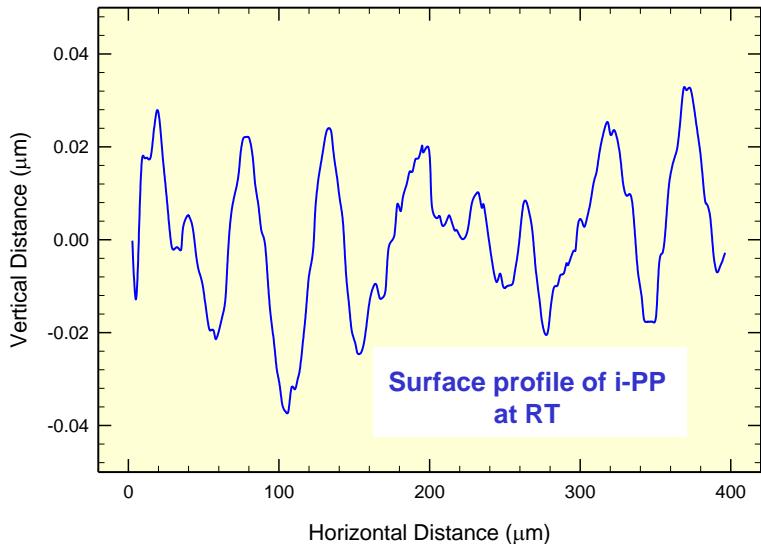
# Effect of Angle on the Q Range



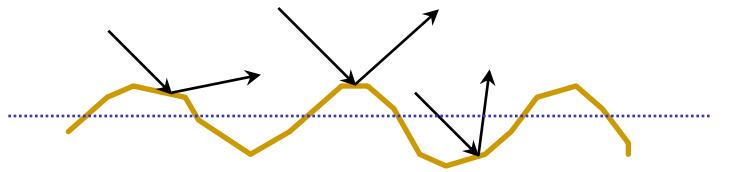
$$Q = \frac{4\pi}{\lambda} \sin \theta$$

$0.05 < \lambda \text{ (nm)} < 0.65$

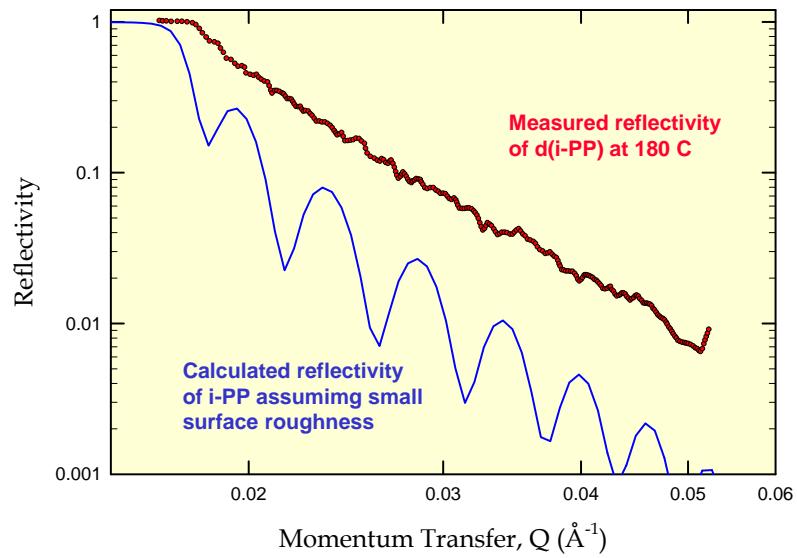
# Effect of Crystallinity on reflectivity



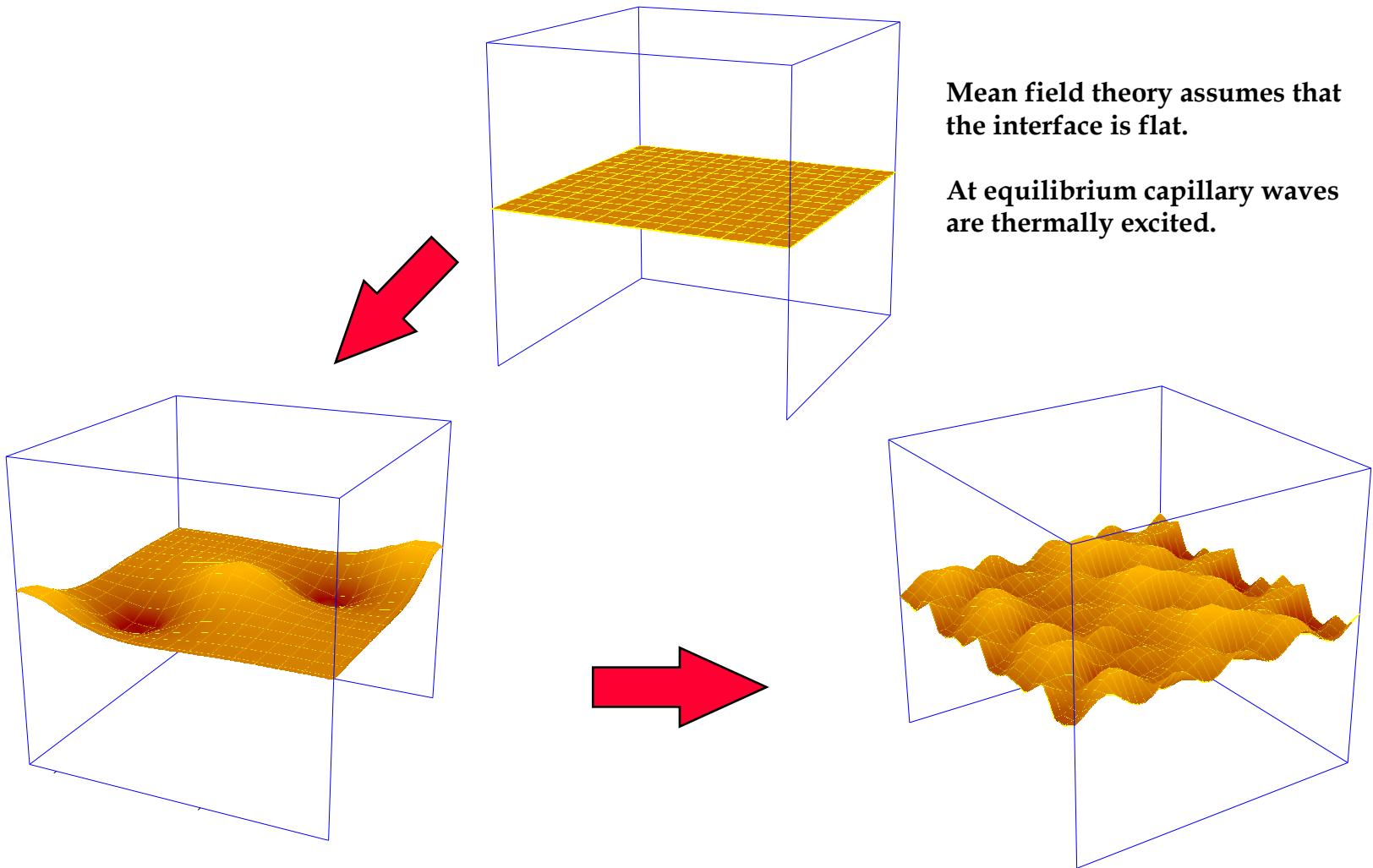
Brewster angle micrograph of surface of i-PP (bar 20 $\mu\text{m}$ )



Roughness causes off-specular scattering and increased resolution term.



# Thermally Excited Capillary Waves



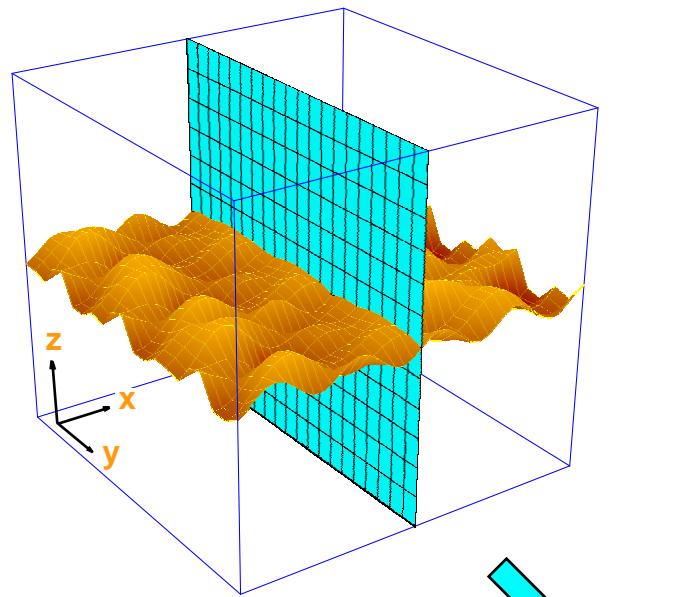
According to the equipartition theorem each mode increases the surface energy by  $0.5 kT$ .

The actual surface is roughened by a superposition of all possible capillary wave modes.

Mean field theory assumes that the interface is flat.

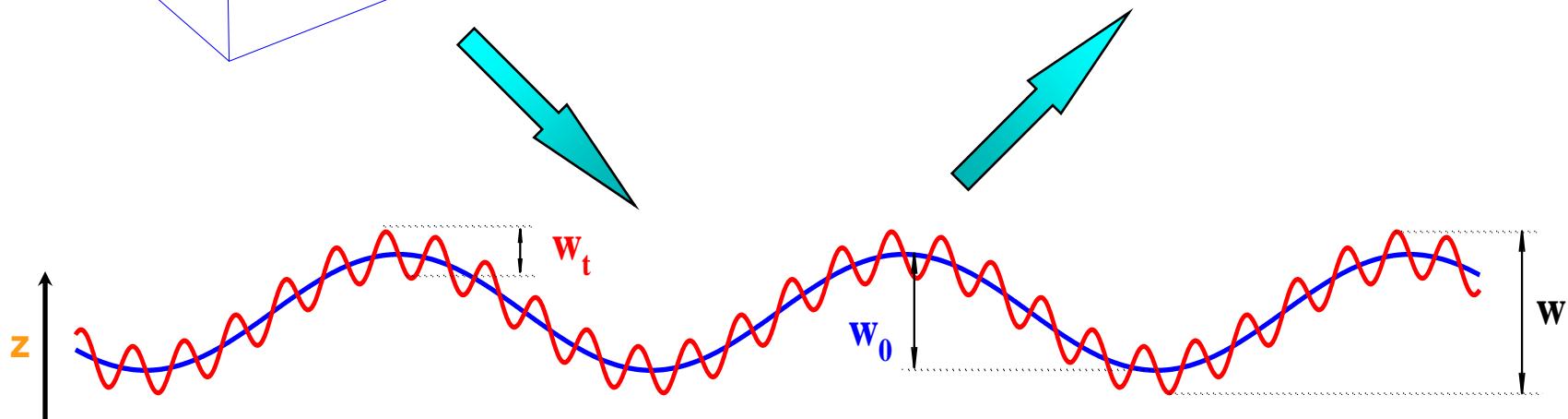
At equilibrium capillary waves are thermally excited.

# NR Measured Interfacial Width



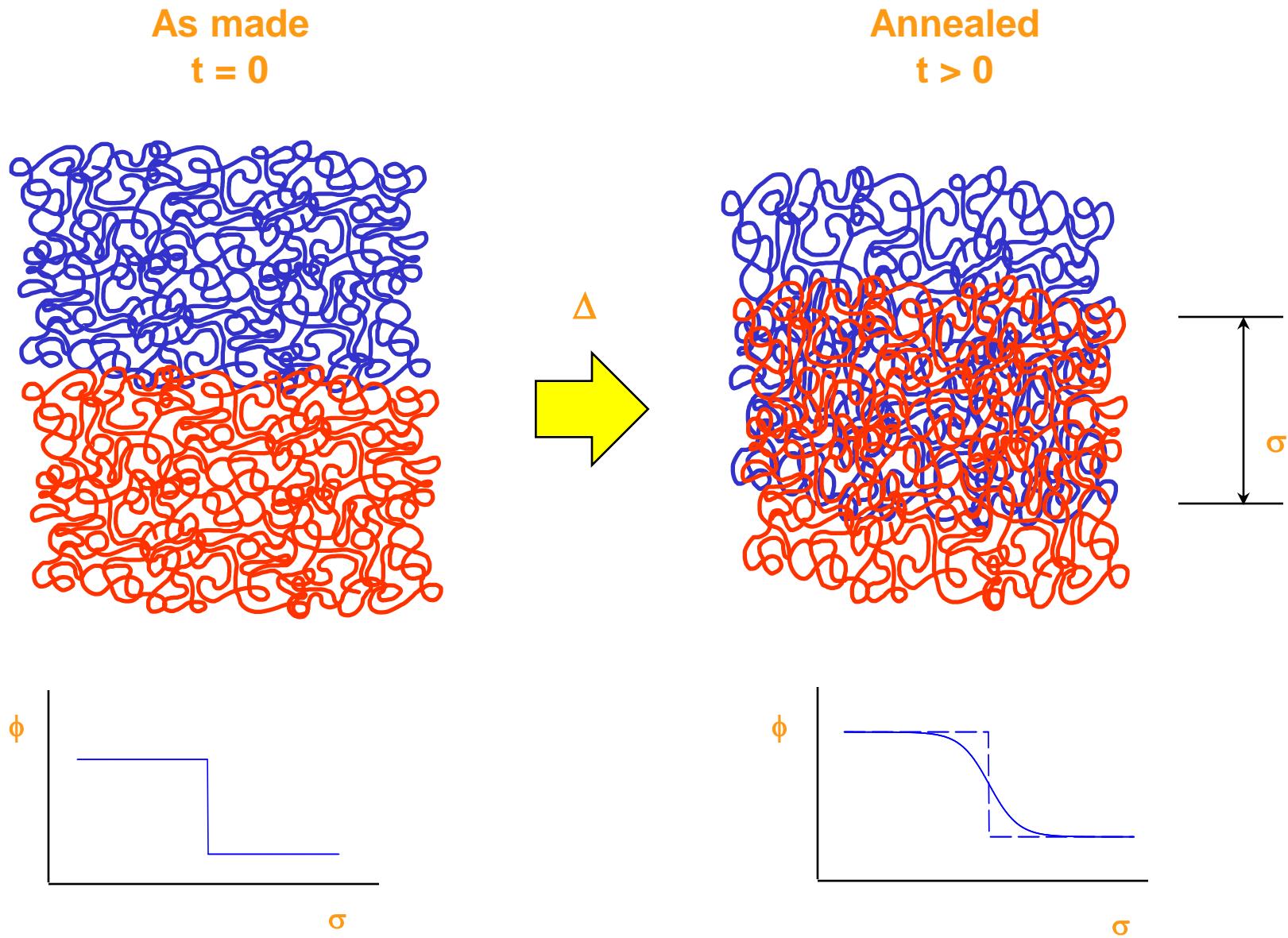
$$w = (w_t^2 + w_0^2)^{0.5}$$

Definition of  $w_0$  dominates derivation of  $w_t$



Projection onto z-y plan

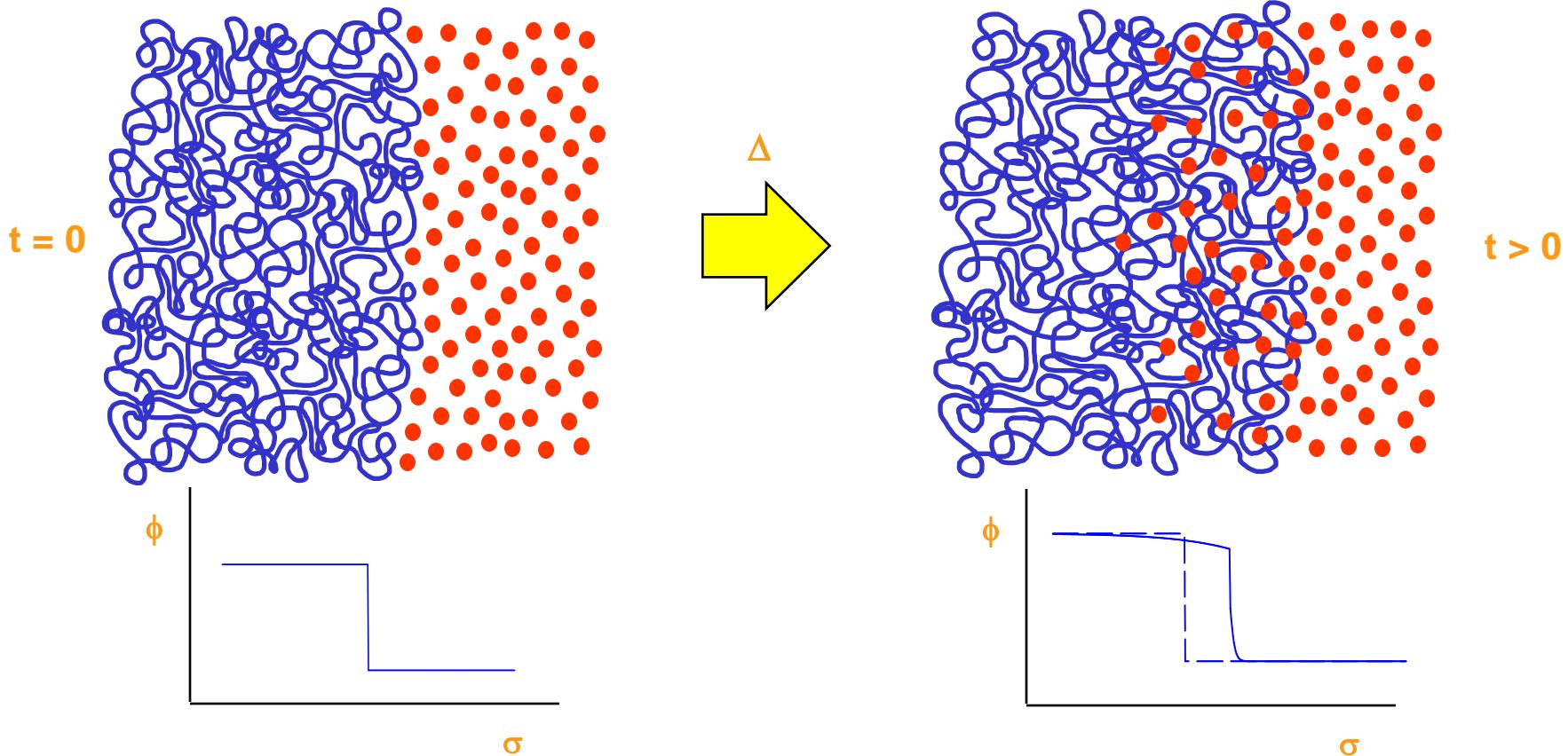
# Polymer Interdiffusion



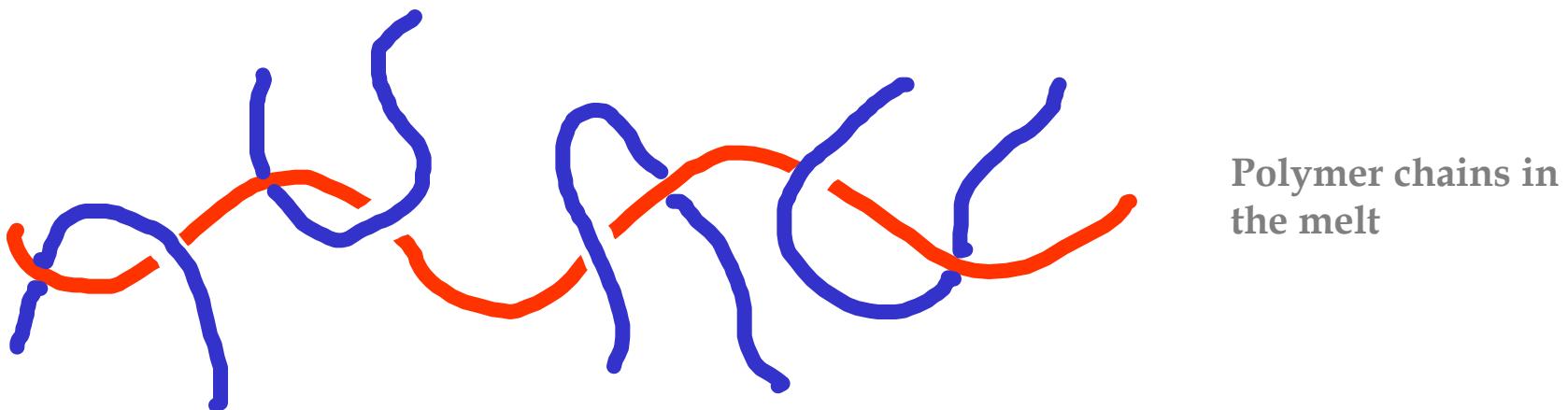
# Non-Fickian Diffusion - Case II Diffusion

$$\sigma \propto t^n \quad n = \frac{1}{2}$$

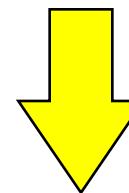
Non-Fickian Diffusion



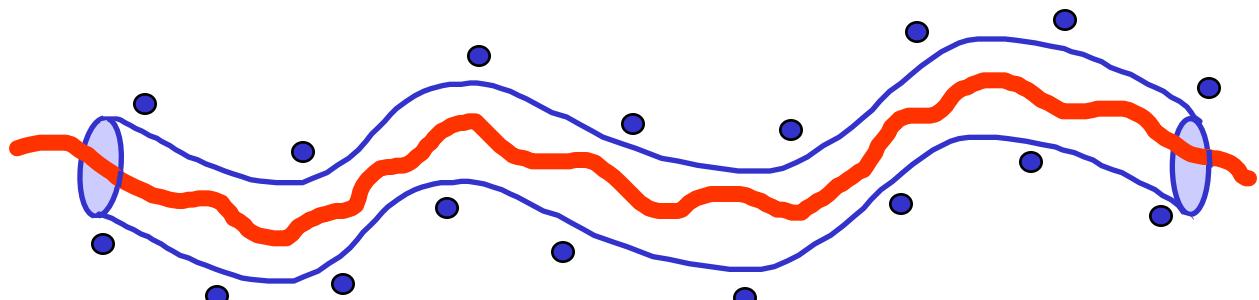
# The Tube Model



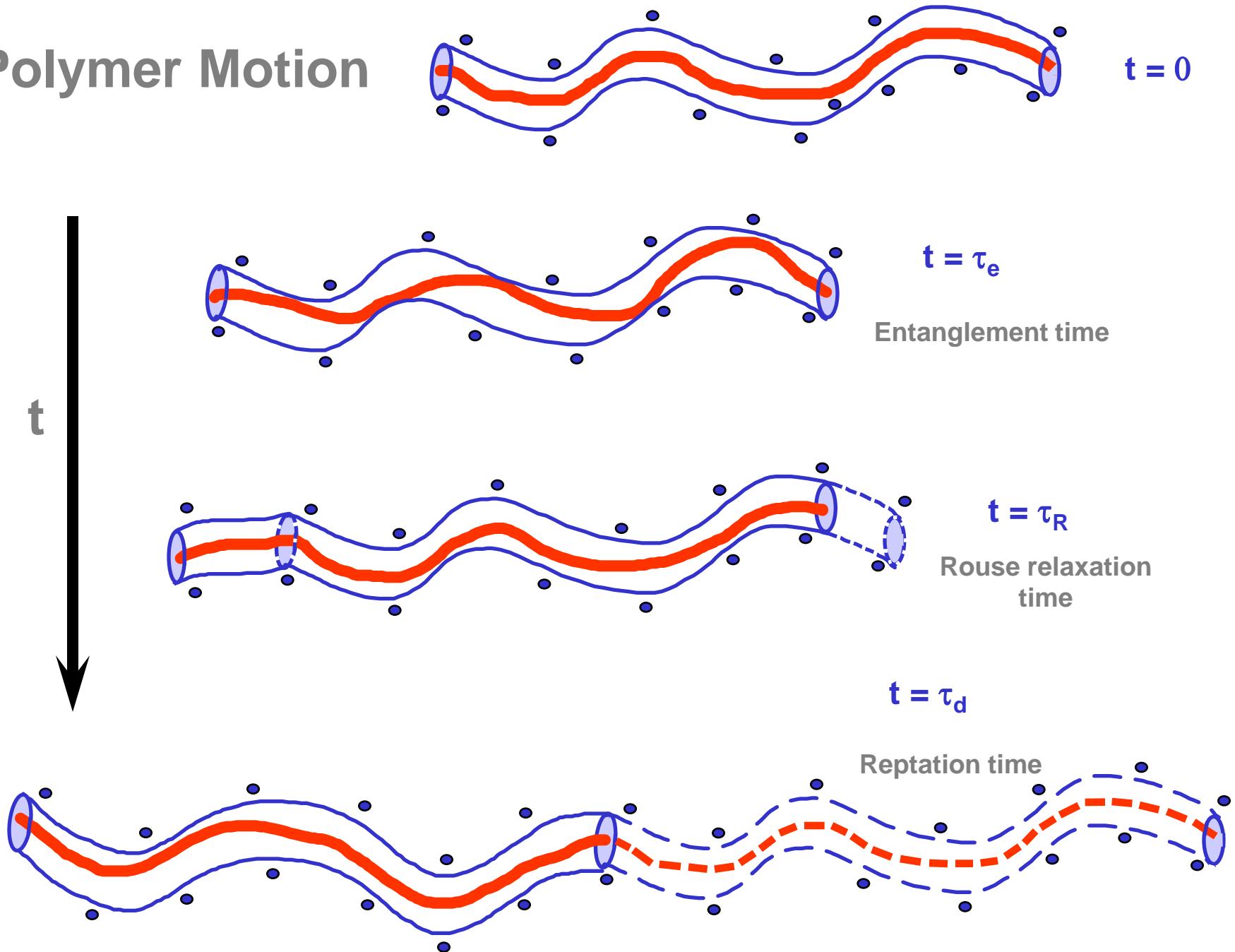
Polymer chains in  
the melt



Each chain can be  
considered to be  
constrained within  
a tube



# Polymer Motion



# Polymer Diffusion

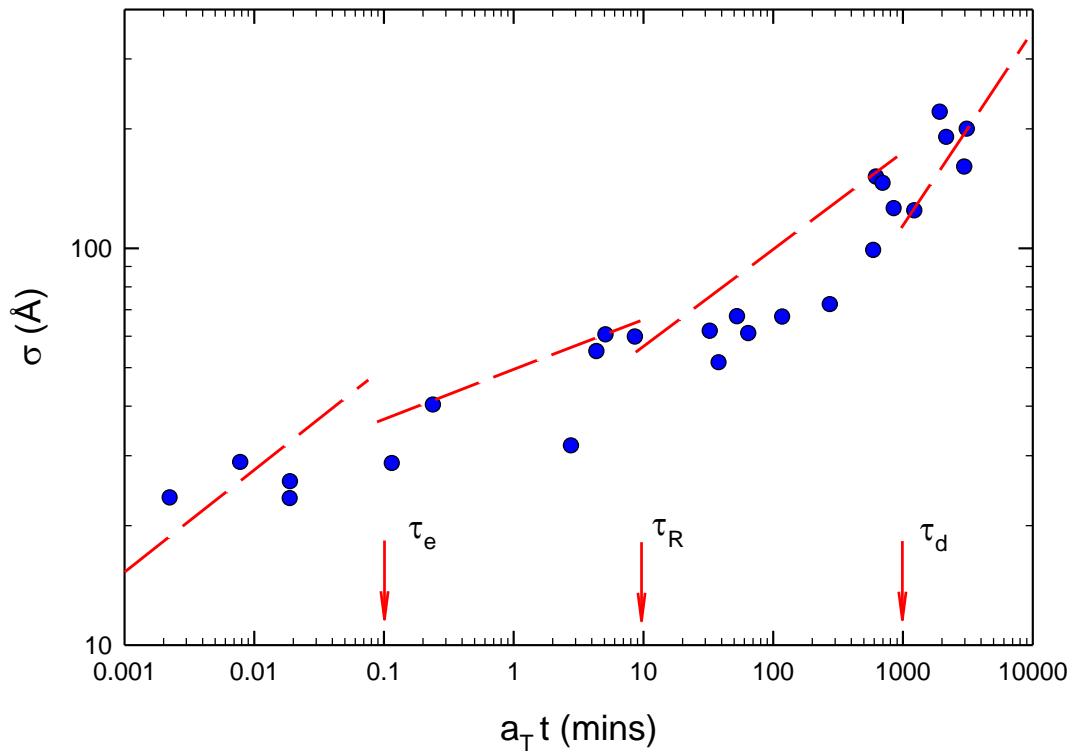
$$t < \tau_e \quad \Rightarrow \quad \sigma \propto t^{1/4}$$

$$\tau_e < t < \tau_R \quad \Rightarrow \quad \sigma \propto t^{1/8}$$

$$\tau_R < t < \tau_d \quad \Rightarrow \quad \sigma \propto t^{1/4}$$

$$t > \tau_d \quad \Rightarrow \quad \sigma \propto t^{1/2}$$

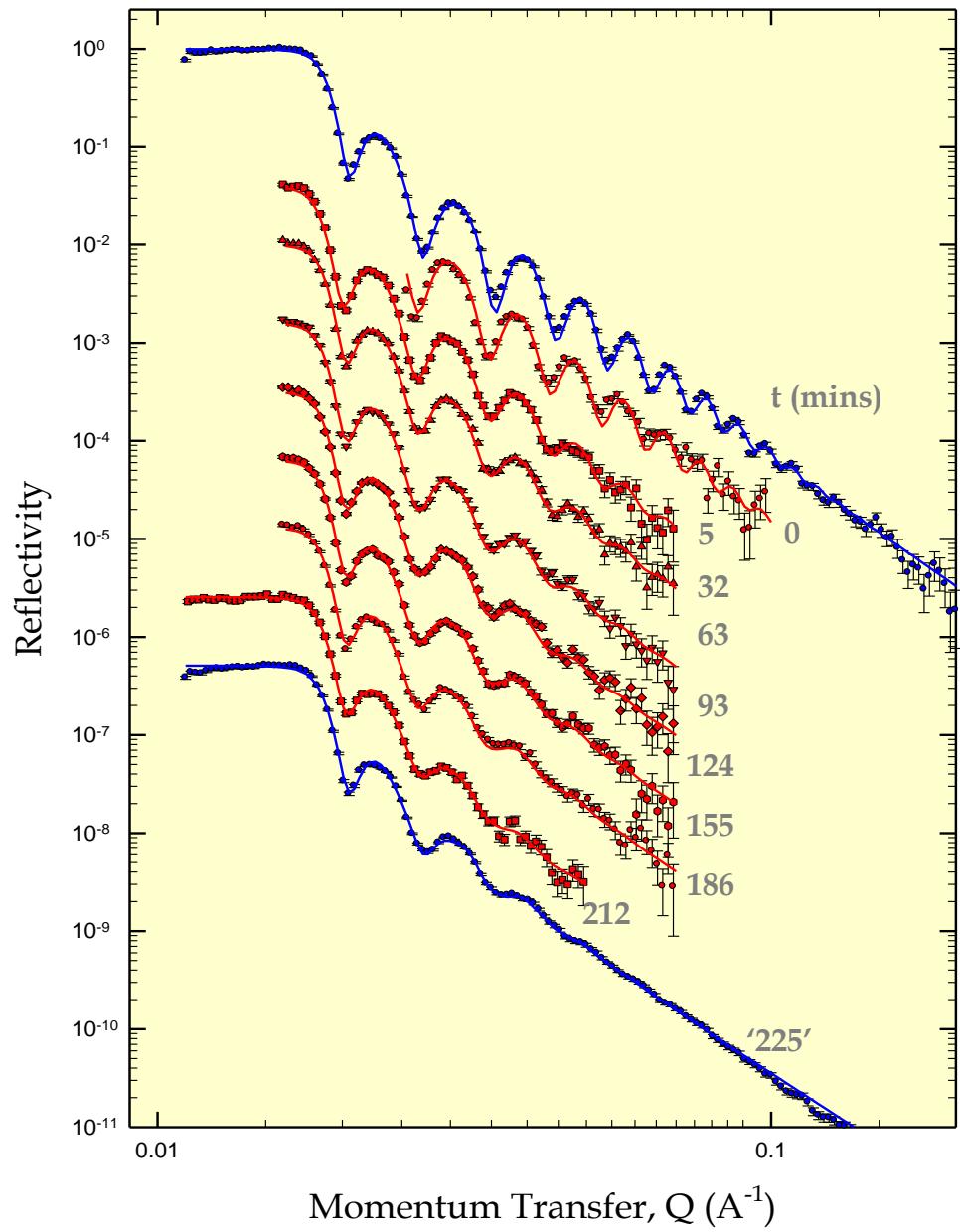
NR Results



A Karim et al, Phys Rev B 42 (1990) 6846

## Real Time Reflectivity Measurements

Si / PS (50k) / dPS (40k) @ 115 C



# Calculating a Diffusion Coefficient

$$w = \sqrt{4Dt}$$

For dPS-PS system:

$$D = (1.7 \pm 0.2) \times 10^{-17} \text{ cm}^2\text{s}^{-1}$$

$$D = \frac{k_B T d_T^2}{3N^2 \zeta b^2}$$

M Doi and SF Edwards  
*The Theory of Polymer Dynamics* (1986)

$$D = 2.81 \times 10^{-17} \text{ cm}^2\text{s}^{-1}$$

When  $\zeta$  (115C) = 0.199 dyne.s.cm<sup>-1</sup>  
and  $d_T$  = 5.7 nm

Reptation time:

$$\tau_r = \frac{Nb^2}{3\pi^2 D}$$

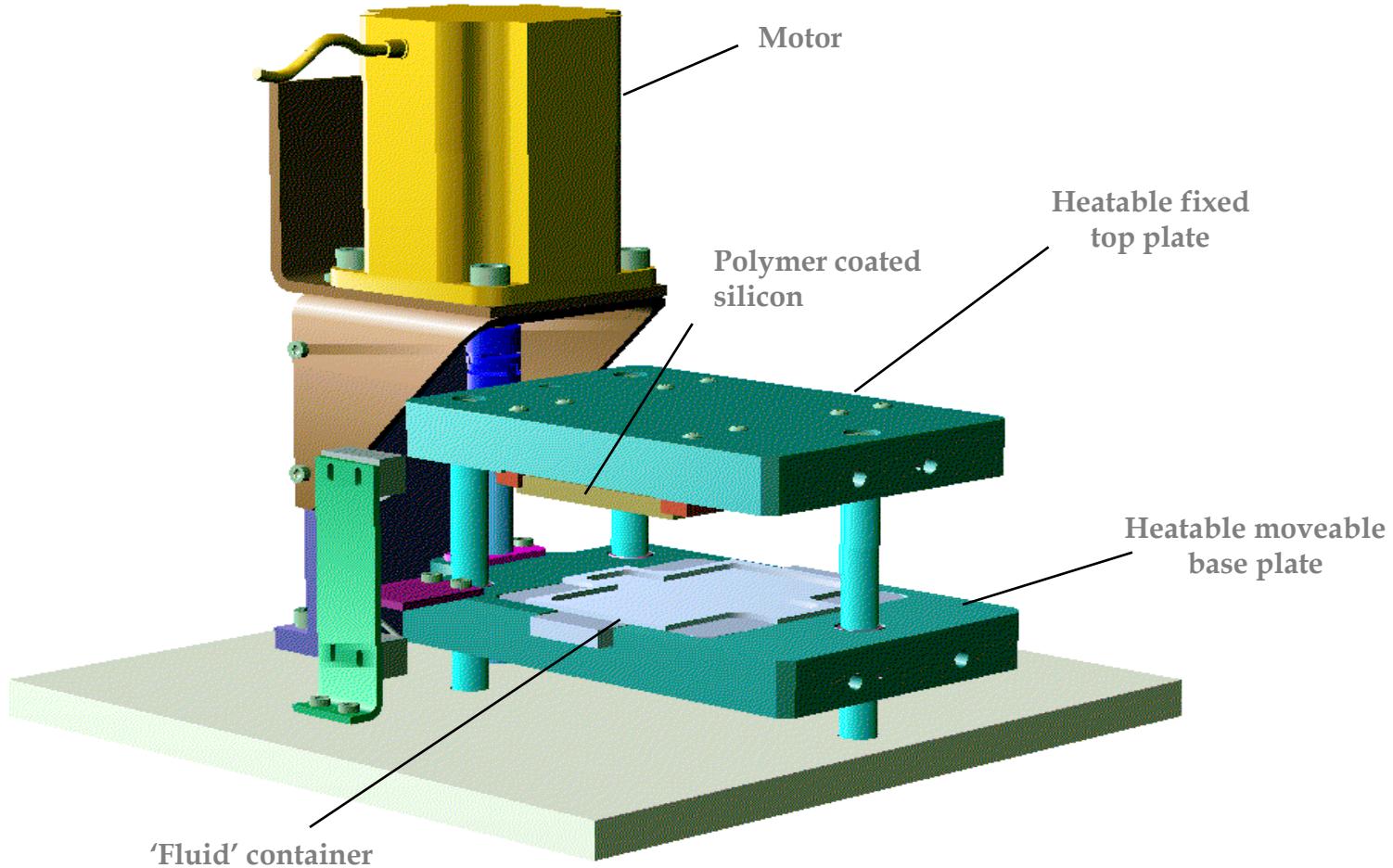
$$\begin{aligned}\tau_r &= 3223 \pm 363 \text{ s (dPS)} \\ &= 4333 \pm 489 \text{ s (hPS)}\end{aligned}$$

Rouse time:

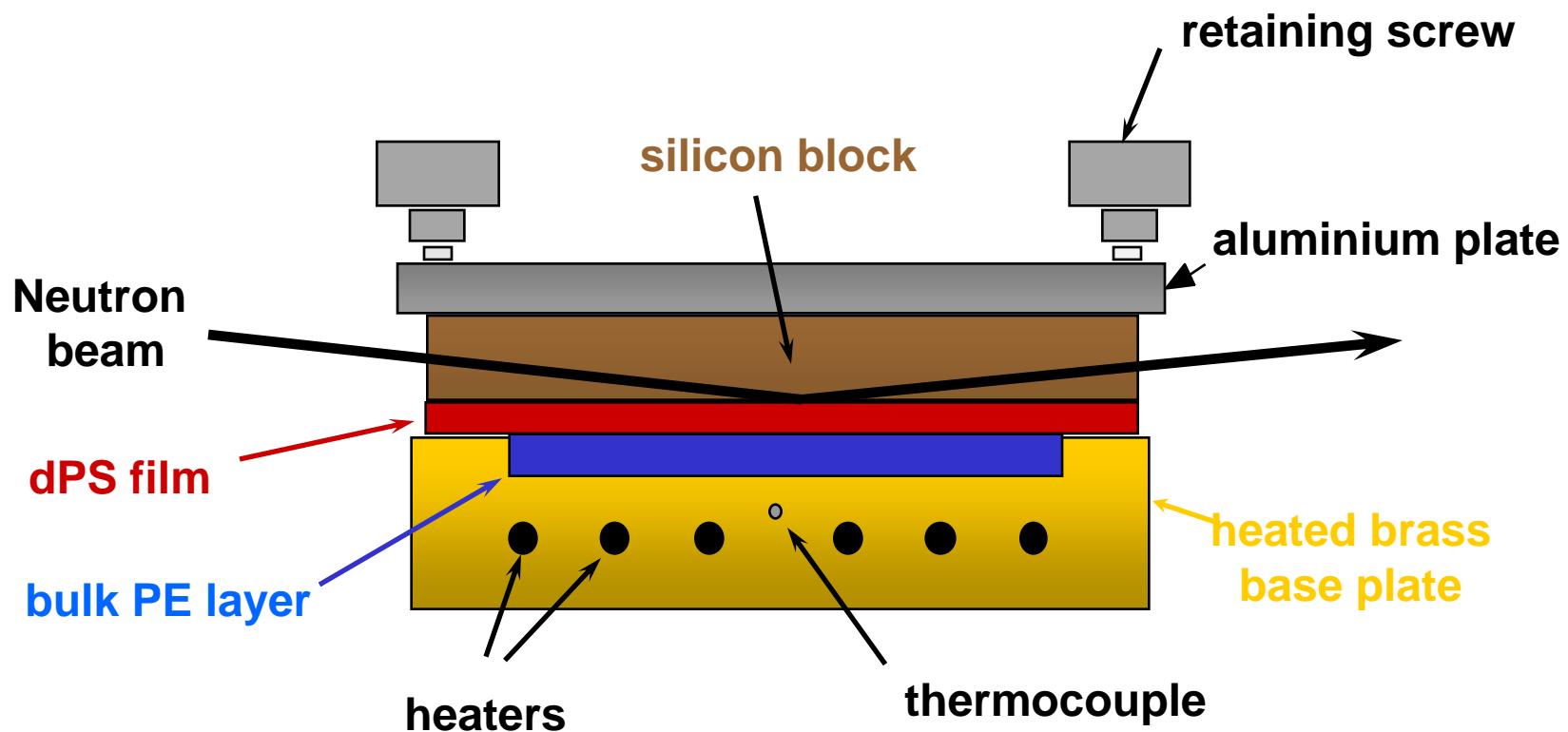
$$\tau_R = \frac{d_T^2}{9\pi^2 D}$$

$$\tau_R = 215 \pm 23 \text{ s}$$

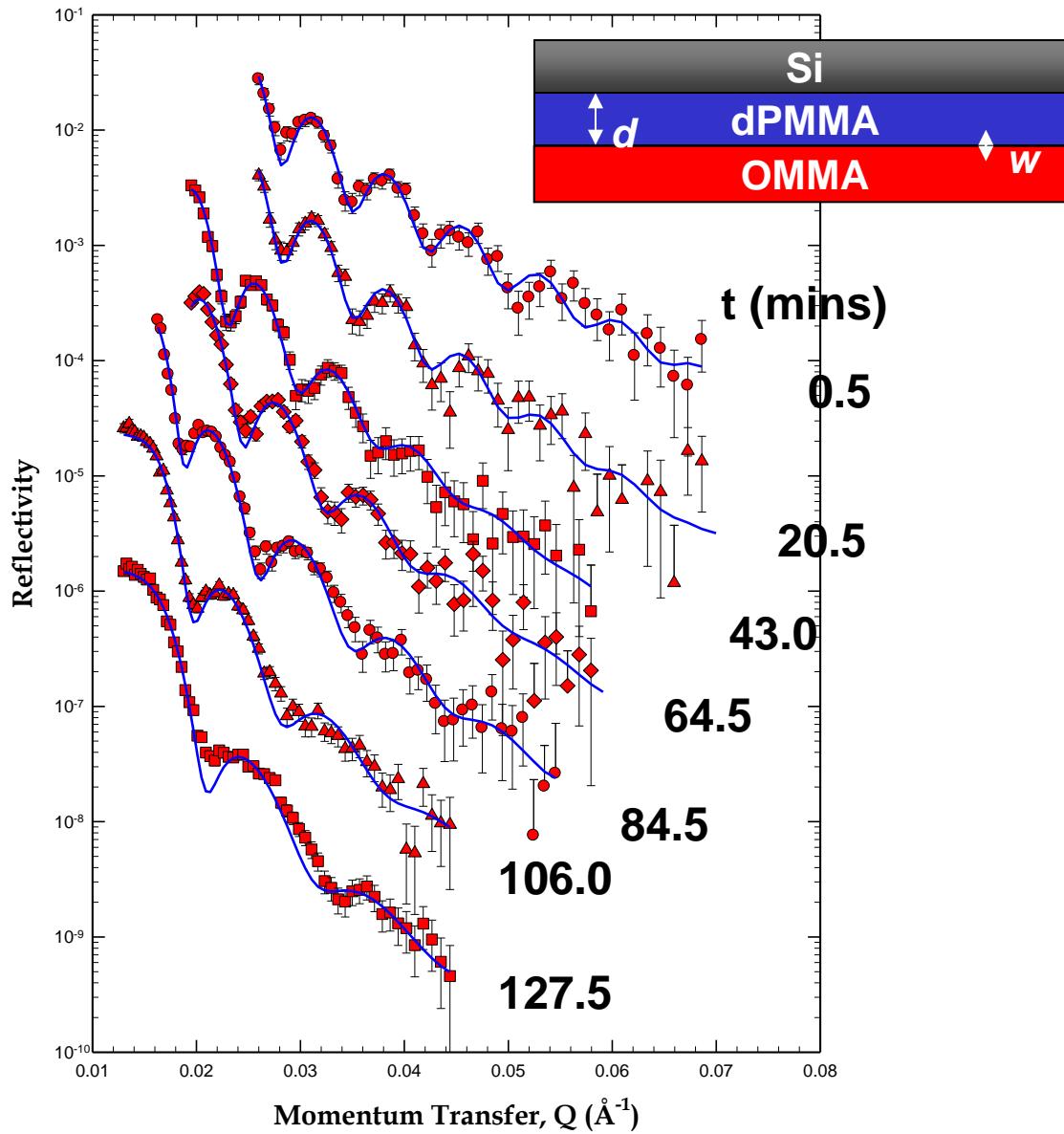
# Polymer-Oligomer Interdiffusion Reflectivity Cell



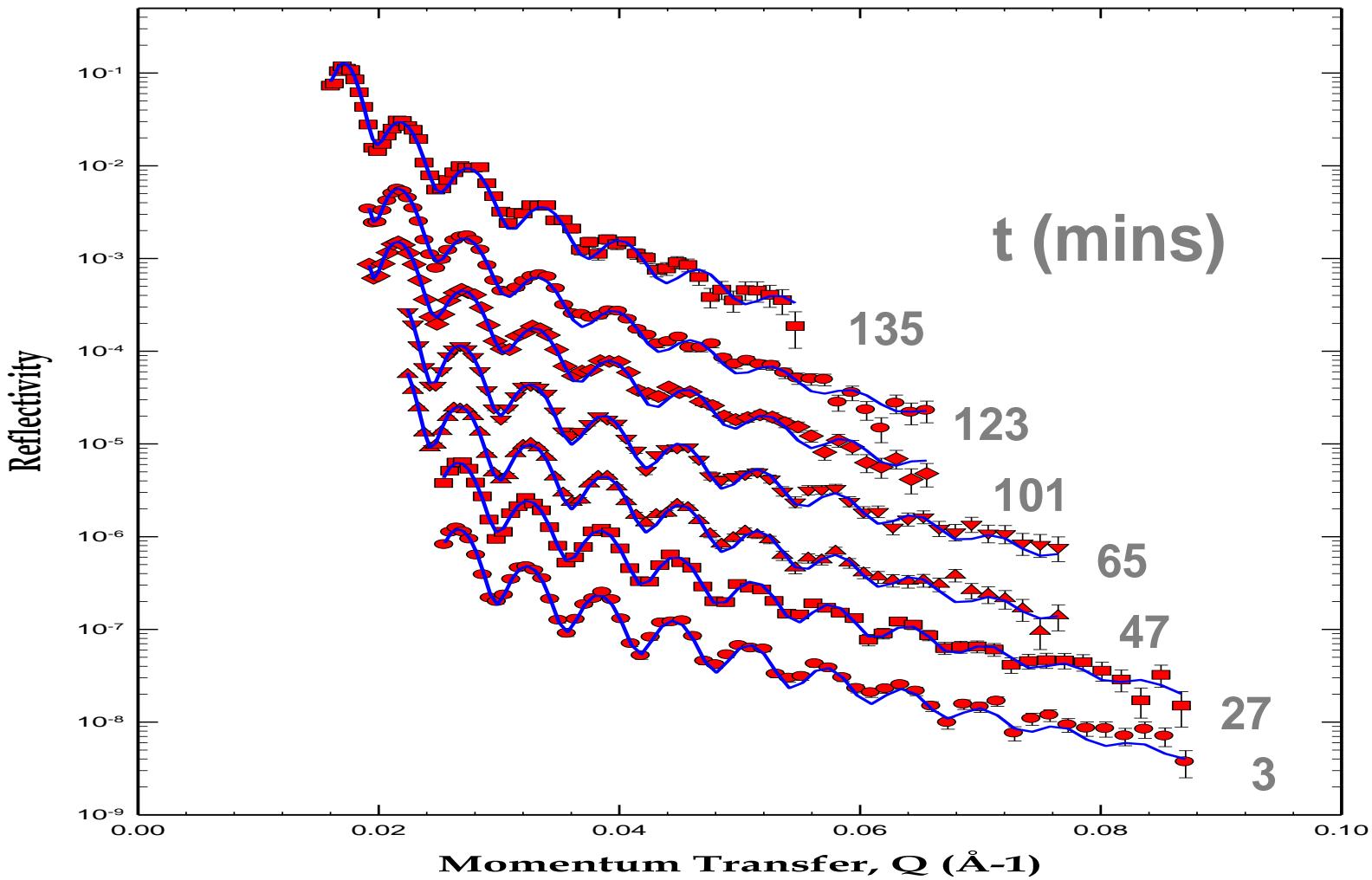
# Neutron Reflectivity Melt Cell



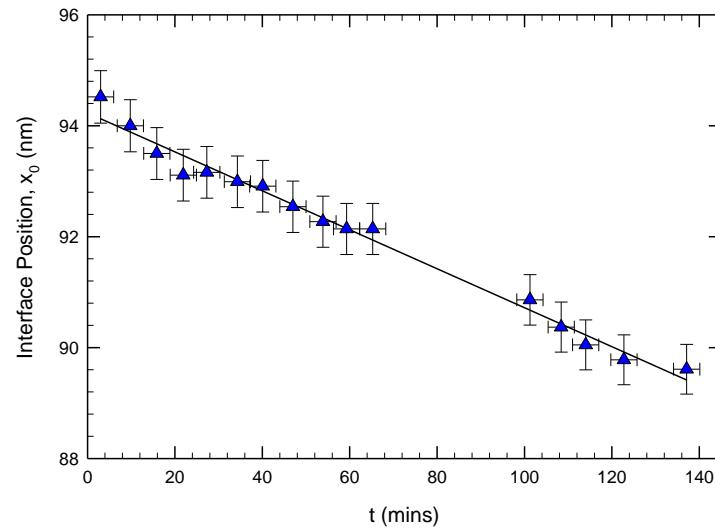
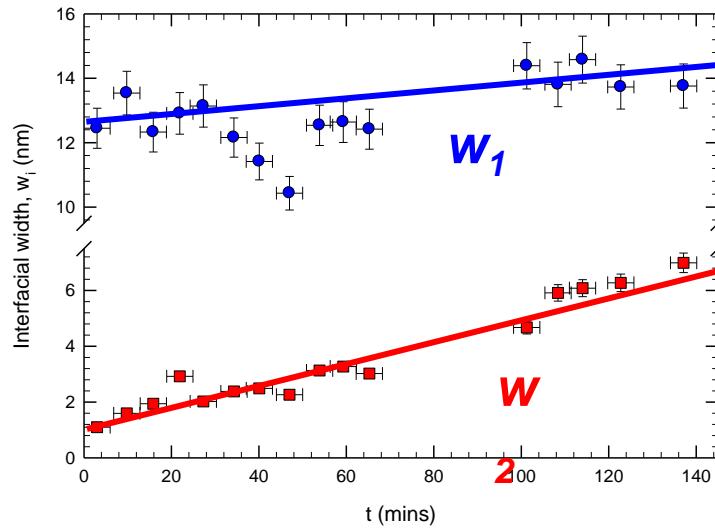
# dPMMA(100k) / OMMA(510) @ 45 C



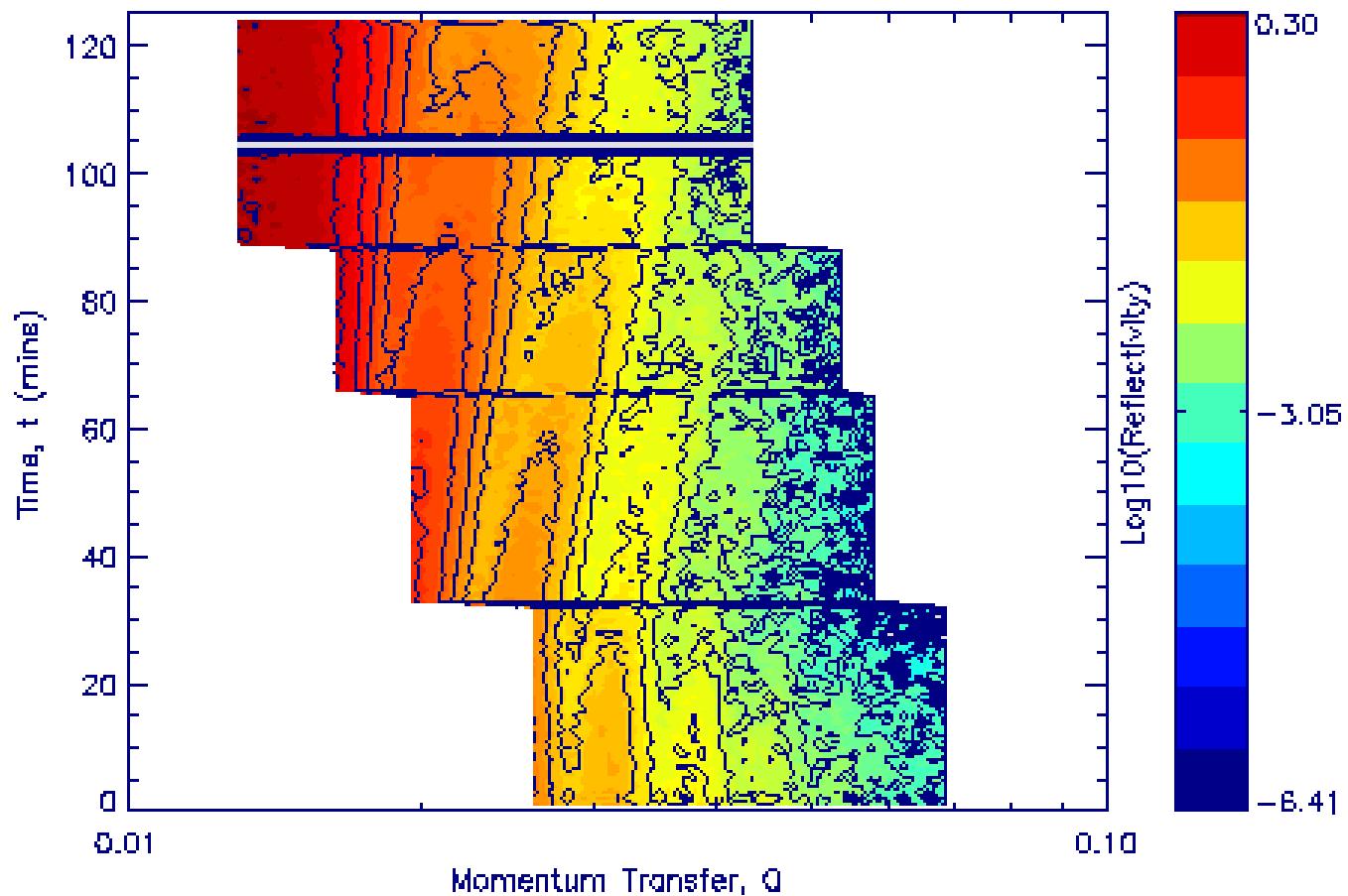
# dPS (101k) / OSt (1100) Interdiffusion @ 65C



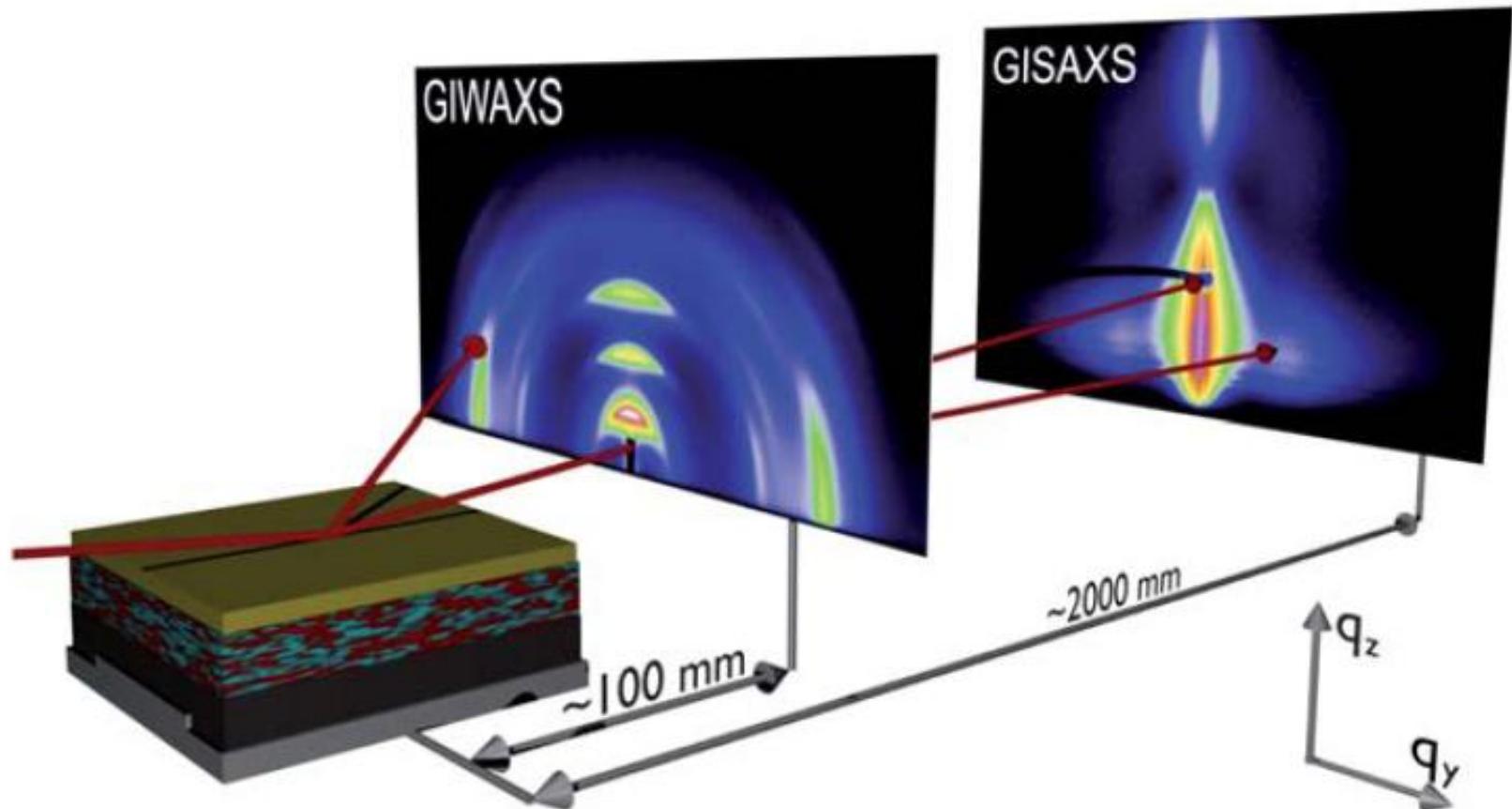
# dPS (101k) / OSt (1100) Interdiffusion @ 65C



# Off-specular reflection

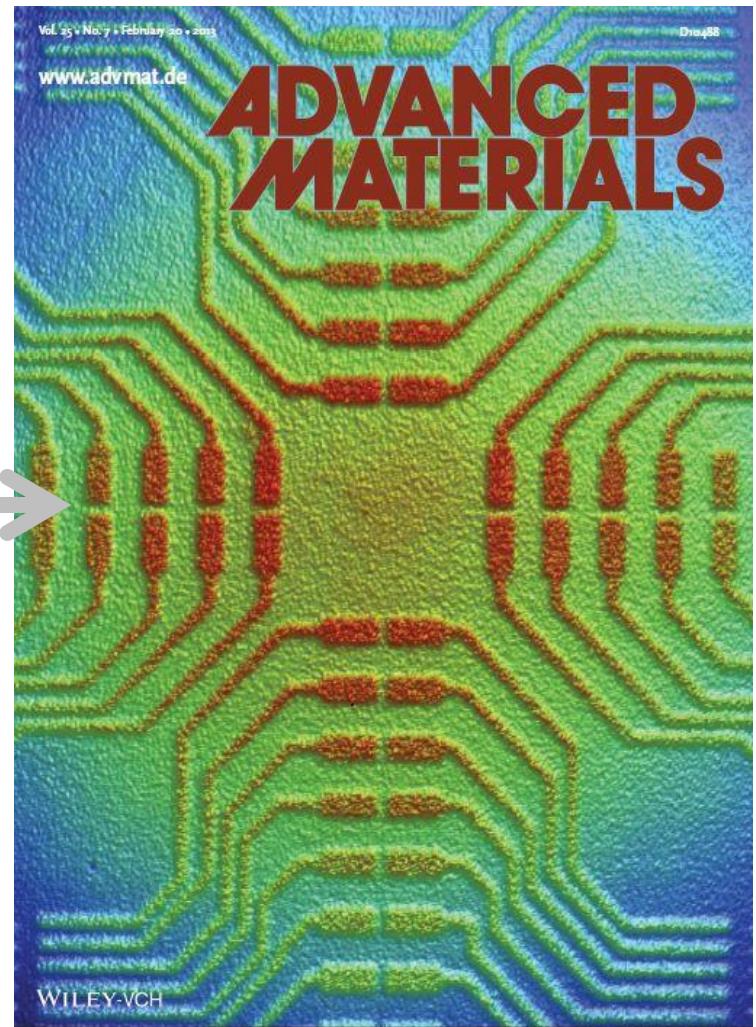
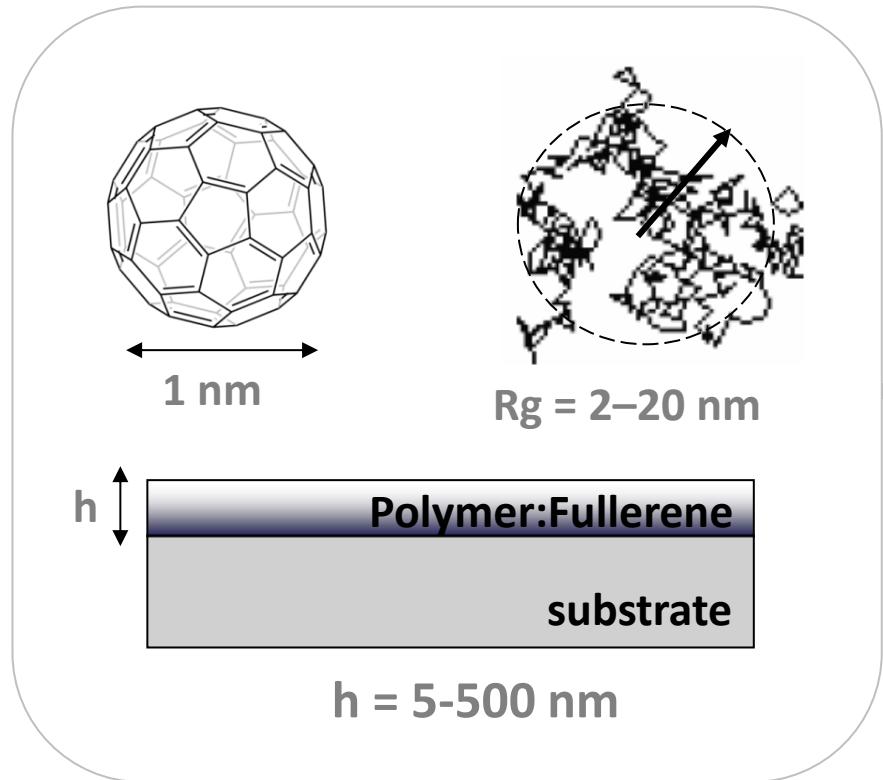


# Grazing incidence: wide and small angle



P Muller (2011)

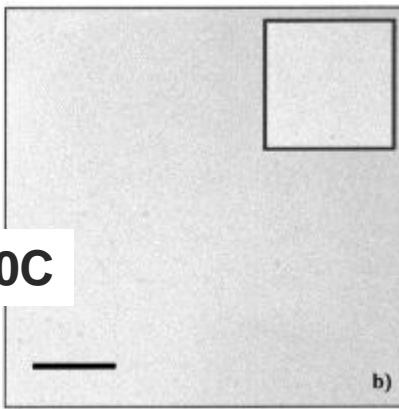
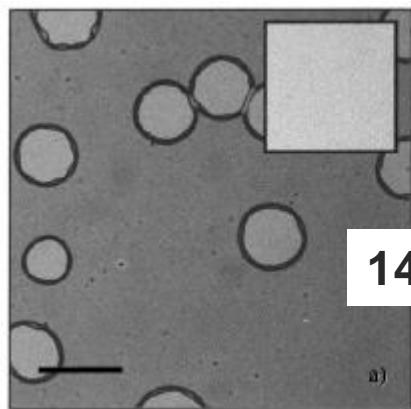
# Polymer-fullerene blends



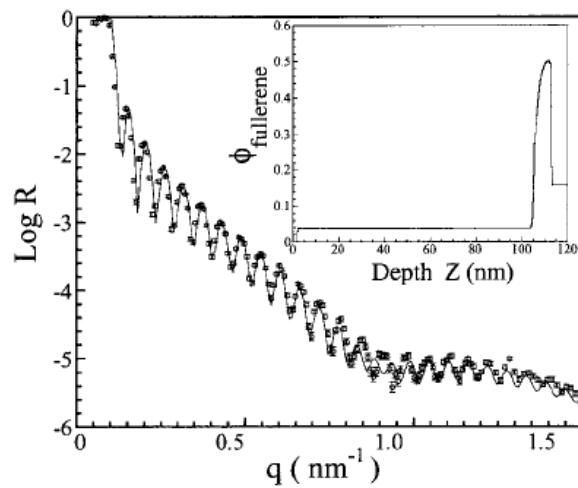
# Thin films

PS (2k), 30 nm

1% C<sub>60</sub>, 30 nm



140C

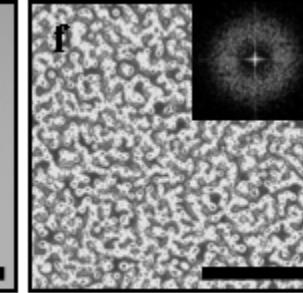
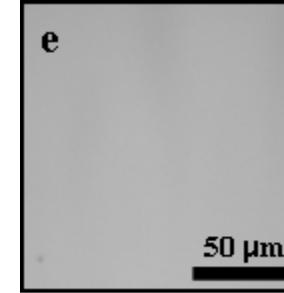
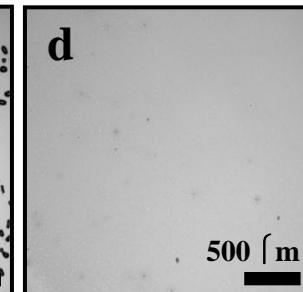
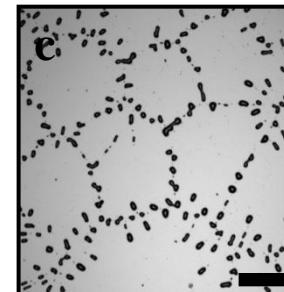
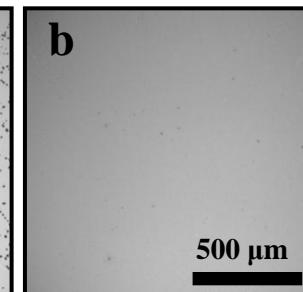
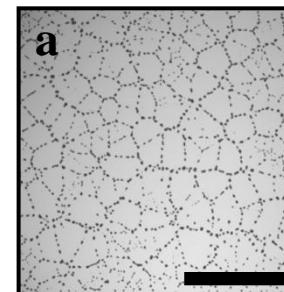


PS+5% C<sub>60</sub> h = 100nm

GISANS & reflectometry

PS

PS + 5% C<sub>60</sub>



140°C

180°C

2k, 30 nm

2k, 150 nm

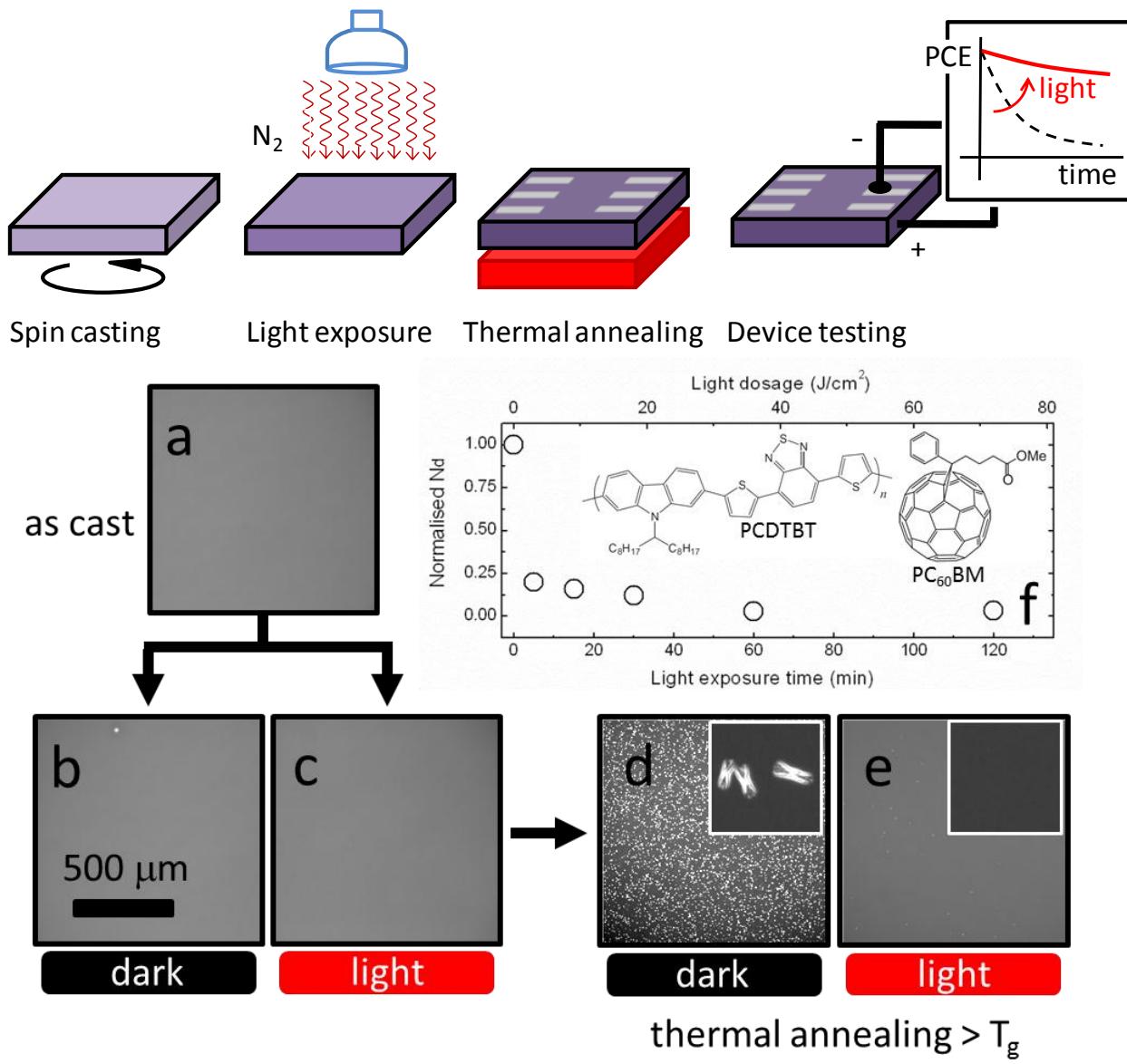
270K, 150 nm

180°C

'Spinodal nucleation'

Phys. Rev. Lett. **105**, 038301 (2010)  
Macromolecules **44**, 4530-4537 (2011)

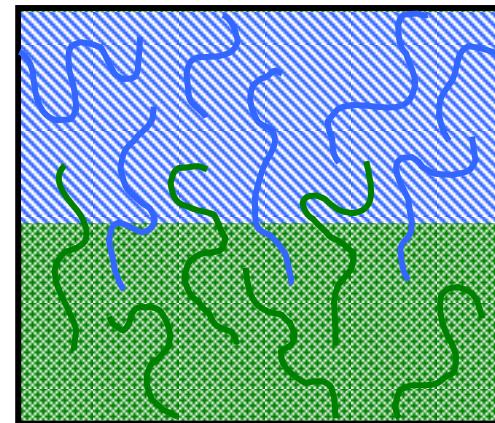
# Organic Solar Cell lifetime?



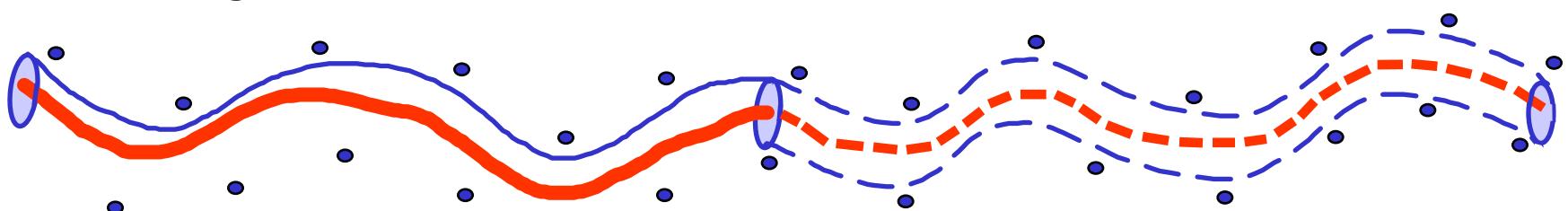
# Summary

## Reflectivity

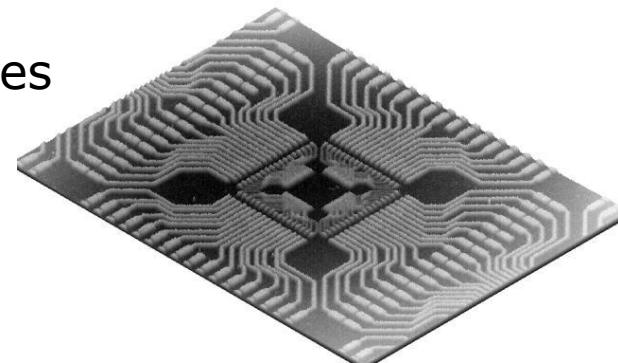
- study and design interfaces



- investigate diffusion mechanisms



- engineer 'functional' surfaces / devices

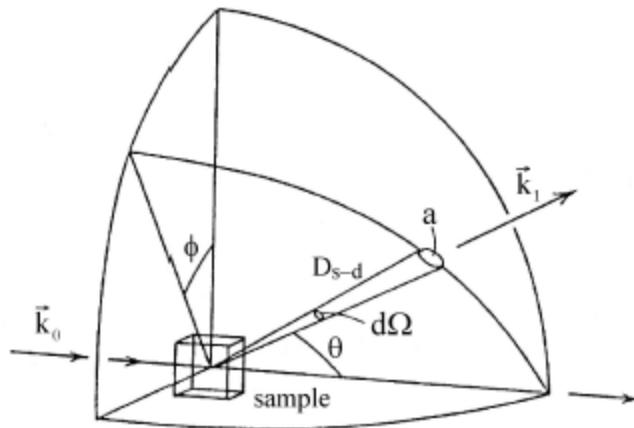


# Neutrons in soft matter

Lecture 2 (II) – Dynamics

João T. Cabral  
Department of Chemical Engineering  
Imperial College London

# Scattering theory reminder



## Scattering cross section

$$\frac{d^2\sigma}{d\Omega dE} = \left( \frac{d^2\sigma}{d\Omega dE} \right)_{coh} + \left( \frac{d^2\sigma}{d\Omega dE} \right)_{inc}$$

**coherent    incoherent**

$$\begin{aligned} \left( \frac{d^2\sigma}{d\Omega dE} \right)_{coh} &= \frac{1}{2\pi\hbar} \frac{k_1}{k_0} \frac{\sigma_{coh}}{4\pi} \int_{-\infty}^{+\infty} \sum_{i,j} \left\langle e^{-i\mathbf{q} \cdot \mathbf{R}_i(0)} e^{i\mathbf{q} \cdot \mathbf{R}_j(t)} \right\rangle e^{-i\omega t} dt \\ \left( \frac{d^2\sigma}{d\Omega dE} \right)_{inc} &= \frac{1}{2\pi\hbar} \frac{k_1}{k_0} \frac{\sigma_{inc}}{4\pi} \int_{-\infty}^{+\infty} \sum_i \left\langle e^{-i\mathbf{q} \cdot \mathbf{R}_i(0)} e^{i\mathbf{q} \cdot \mathbf{R}_i(t)} \right\rangle e^{-i\omega t} dt \end{aligned}$$

## Dynamic structure factor

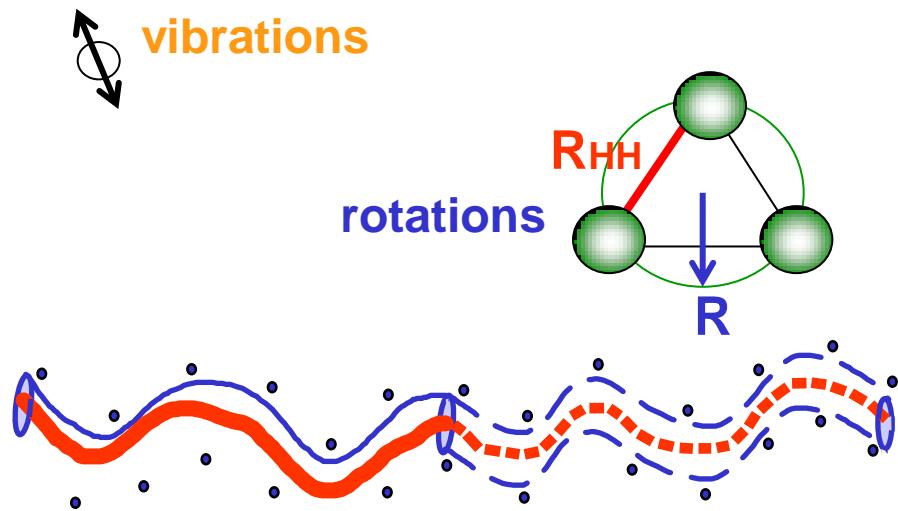
$$\text{FT } (t, \omega) \quad \uparrow \downarrow \quad S(\mathbf{q}, \omega) = \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} I(\mathbf{q}, t) e^{-i\omega t} dt.$$

## Intermediate scattering function

$$\text{FT } (r, q) \quad \uparrow \downarrow \quad I_s(\mathbf{q}, t) = \frac{1}{N} \sum_i \left\langle e^{-i\mathbf{q} \cdot \mathbf{R}_i(0)} e^{i\mathbf{q} \cdot \mathbf{R}_i(t)} \right\rangle e^{-i\omega t}.$$

## Pair correlation function

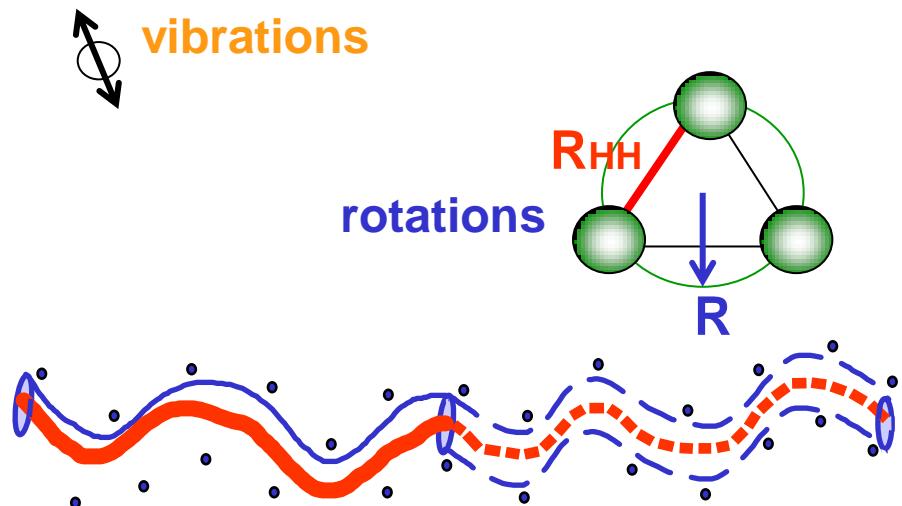
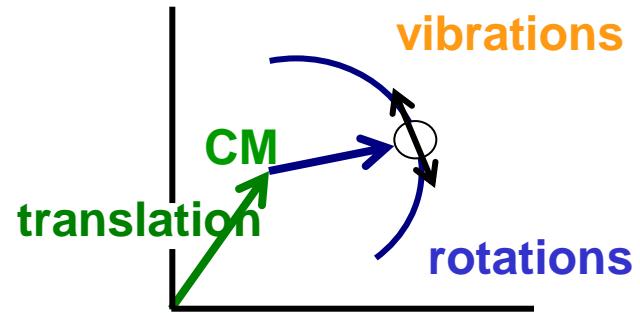
$$G(\mathbf{r}, t) = \frac{1}{(2\pi)^3} \int I(\mathbf{q}, t) e^{-i\mathbf{q} \cdot \mathbf{r}} d\mathbf{q}.$$



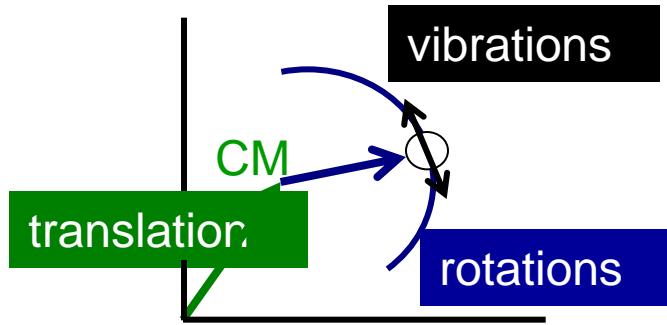
# single-particle dynamics

motion decomposition

$$I_{self}(Q, t) = \frac{1}{N} \sum_i \left\langle e^{iQ \cdot [V(t) - V(0)]} \right\rangle \left\langle e^{iQ \cdot [T(t) - T(0)]} \right\rangle \left\langle e^{iQ \cdot [R(t) - R(0)]} \right\rangle$$



# single-particle tools



motion decomposition

$$I_{self}(Q, t) = \frac{1}{N} \sum_i \left\langle e^{iQ \cdot [V(t) - V(0)]} \right\rangle \left\langle e^{iQ \cdot [T(t) - T(0)]} \right\rangle \left\langle e^{iQ \cdot [R(t) - R(0)]} \right\rangle$$

CM translation

frozen for polymers  $T \ll T_g$ .

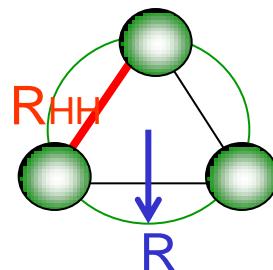
Proton delocalisation

DW factor:  $e^{-\frac{1}{3}Q^2 \langle u^2 \rangle}$

relevant proton reorientations: methyl and phenyl rotations about group's axis.

Methyl protons 3-fold jumps

$R \approx 1.032 \text{ \AA}$



$$S_{\text{rot}}(Q, \omega) = A_0(Q)\delta(\omega) + A_1(Q) \frac{1}{\pi} \frac{3/2\tau}{(3/2\tau)^2 + \omega^2}$$

with

$$A_0(Q) = \frac{1}{3} [1 + 2j_0(Qr\sqrt{3})]$$

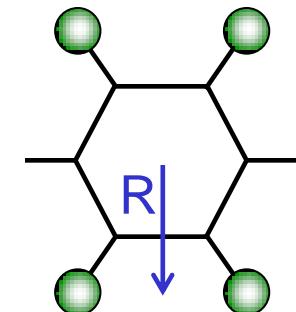
$$A_1(Q) = 1 - A_0(Q)$$

Phenyl proton 2-fold jumps

$R \approx 2.28 \text{ \AA}$

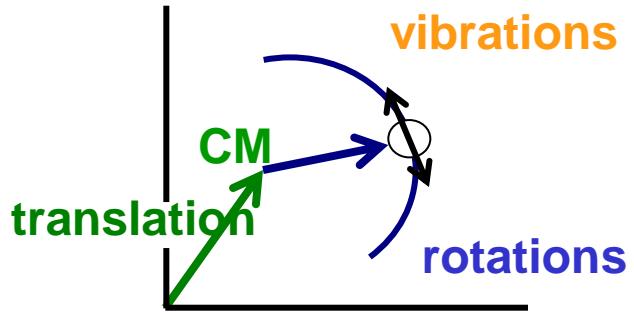
$$S_{\text{rot}}(Q, \omega) = A_0(Q)\delta(\omega) + A_1(Q) \frac{1}{\pi} \frac{2/\tau}{(2/\tau)^2 + \omega^2}$$

with  $A_0(Q) = \frac{1}{2} [1 + j_0(2Qr)]$



# single-particle dynamics

motion decomposition in the glass

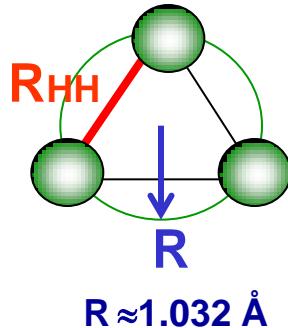


CM translation: frozen for polymers  $T \ll T_g$ .

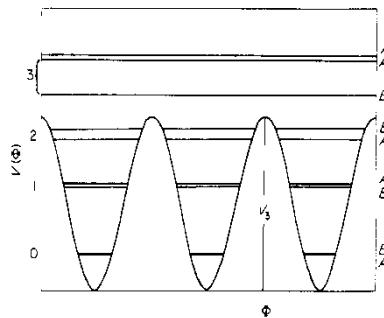
Proton delocalisation: DW factor:  $e^{-\frac{1}{3}Q^2\langle u^2 \rangle}$

example:

Side group rotations:



3-fold  $\text{CH}_3$  potential



Methyl protons 3-fold jumps

$$S_{\text{rot}}(Q, \omega) = A_0(Q)\delta(\omega) + A_1(Q) \frac{1}{\pi} \frac{3/2\tau}{(3/2\tau)^2 + \omega^2}$$

with

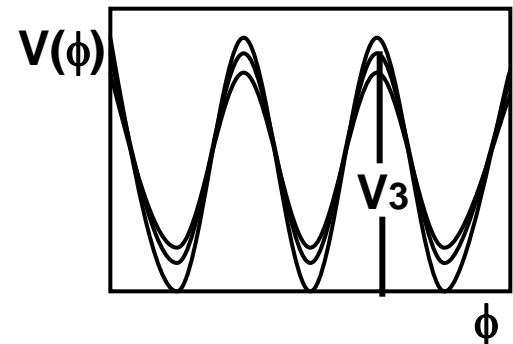
$$A_0(Q) = \frac{1}{3} [1 + 2j_0(Qr\sqrt{3})]$$
$$A_1(Q) = 1 - A_0(Q)$$

# distribution $\tau_{\text{correlation}}$

glassy polymers: no single relaxation time

variety local environments

intra- molecular  
inter-



(Gaussian) distribution of potential barriers:

$$g(E_i) = \frac{1}{\sigma_E \sqrt{2\pi}} e^{\frac{-(E_i - E_0)^2}{2\sigma_E^2}} \quad \text{if } \Gamma = \Gamma_0 e^{-\frac{E_A}{RT}}$$

(log-Gaussian) distribution of reorientation times:

$$g(\ln \Gamma_i) = \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{-\ln^2(\Gamma_i/\Gamma_0)}{2\sigma^2}}$$

Eo: average barrier height  
 $\sigma$ : distribution width

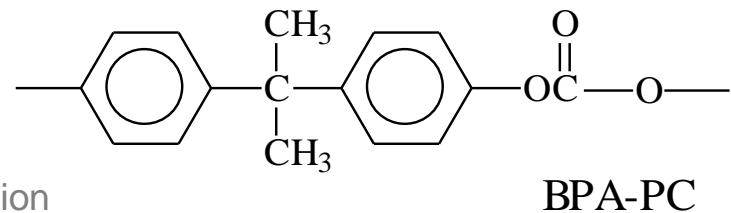
Dynamic structure factor:  $S_{\text{rot}}(Q, \omega) = A_0(Q)\delta(\omega) + A_1(Q) \sum_{i=1}^N g_i L_i(\omega)$

# Case study: Polycarbonates

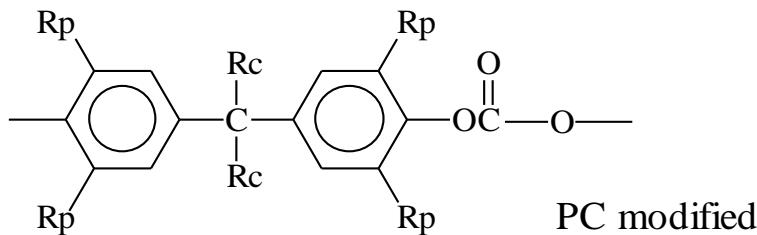
Bisphenol-A polycarbonate

thermoplastic polymer with remarkable

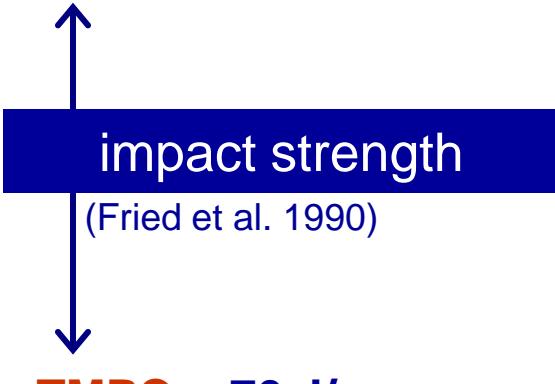
- optical clarity
- **mechanical properties**
  - high  $T_g$  glass transition
  - large impact strength
  - ductility.
- commercial applications



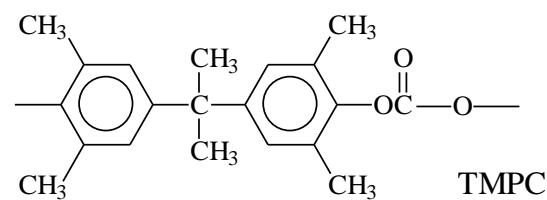
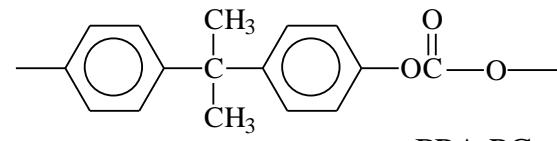
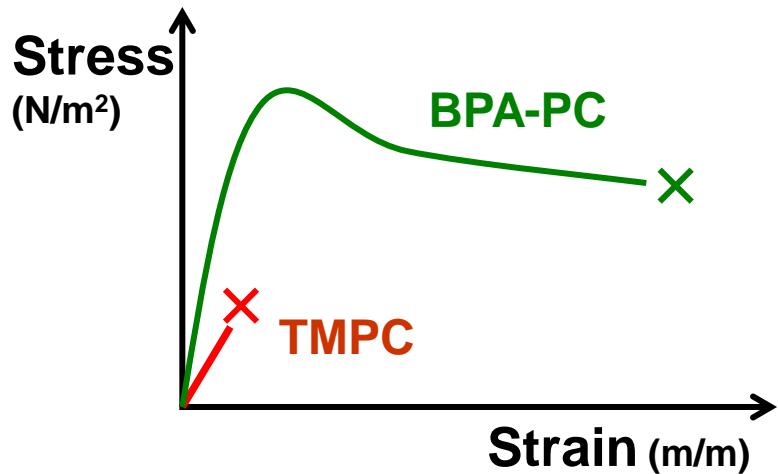
depend strongly on architecture



**BPA-PC: ~2400 J/m**



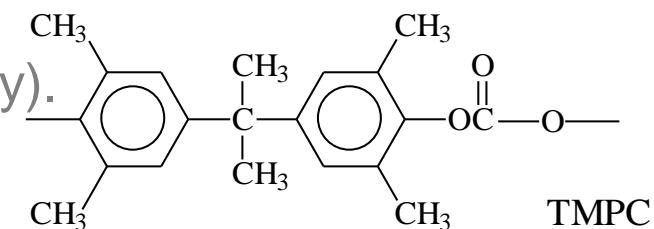
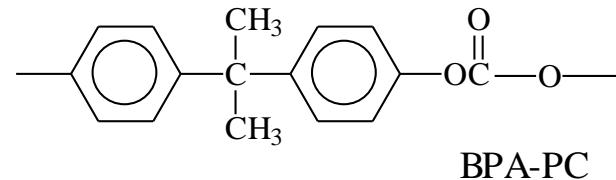
# Toughness



# Polycarbonates

Glassy BPAPC

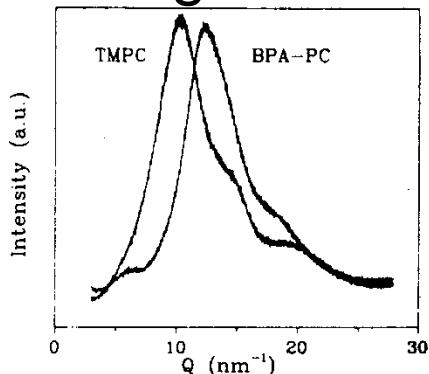
tough → co-operative phenyl motion,  
involve  $\geq 1$  monomer  
(account for dielectric/mechanical activity).



Glassy TMPC

most brittle PC → substituted CH<sub>3</sub> hinder backbone mobility;  
poor chain packing (large free volume).

Packing



$$\rho(\text{PC}) = 1.198 \text{ g/cm}^3$$
$$\rho(\text{TMPC}) = 1.084 \text{ g/cm}^3$$

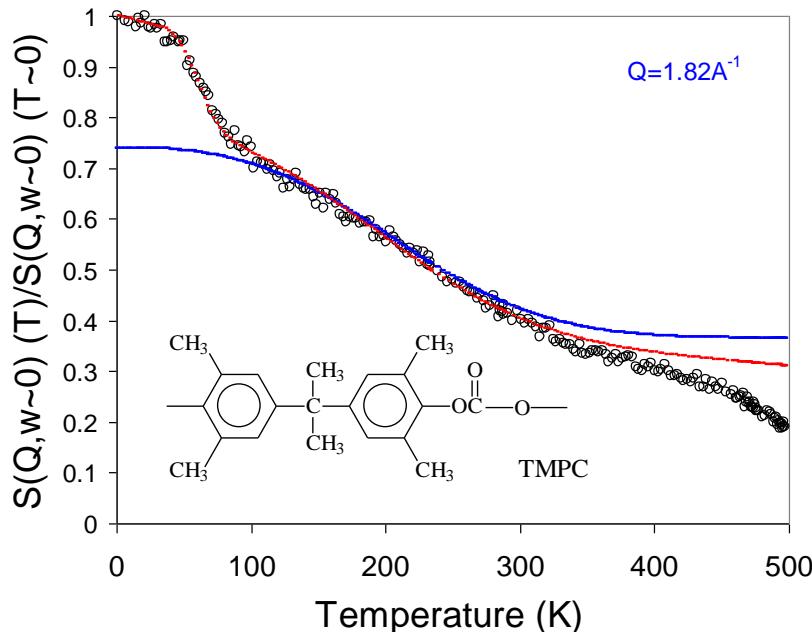
QENS:  
characterise dynamics of local reorientation.

# quantitative window scans

Elastic scans

$$S(Q, \omega \sim 0) = \int_{-\infty}^{+\infty} S(Q, \omega') R(\omega - \omega') d\omega' \Big|_{\omega=0}$$

for a Lorentzian resolution  $S(Q, \omega \sim 0) \approx A_0(Q) + \frac{2}{\pi} [1 - A_0(Q)] \arctan \left( \frac{\Gamma_{\text{res}}}{\Gamma} \right)$



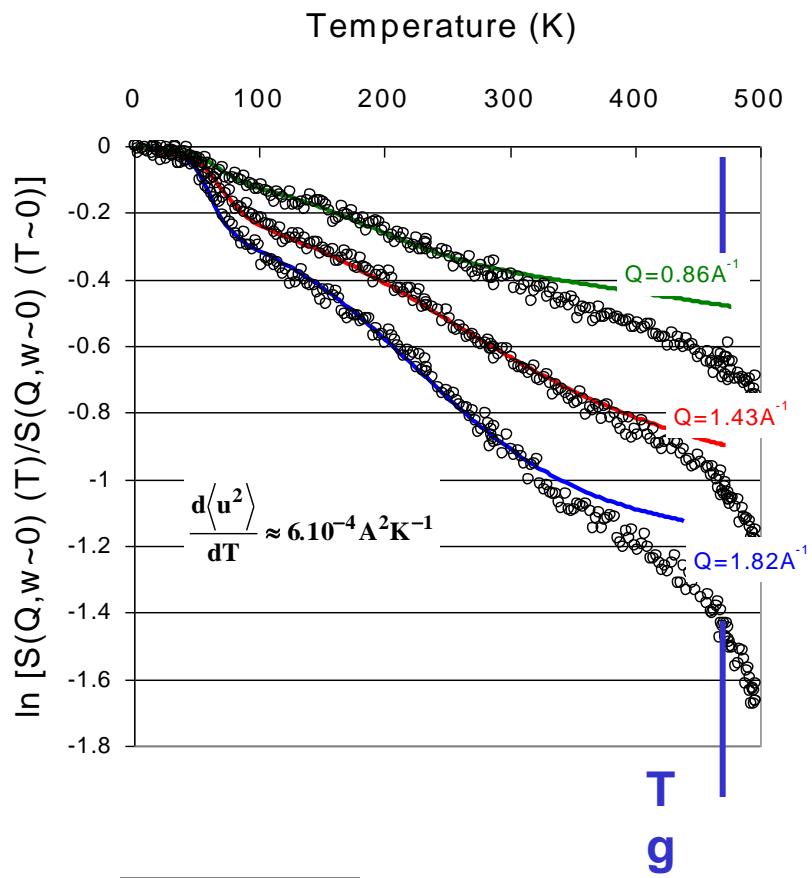
## PARAMETERS

- $\langle u^2 \rangle(T) \leftarrow$  initial slope
- distribution:  $E_A$  and  $\sigma$
- $\Gamma_0$

## ASSUMED

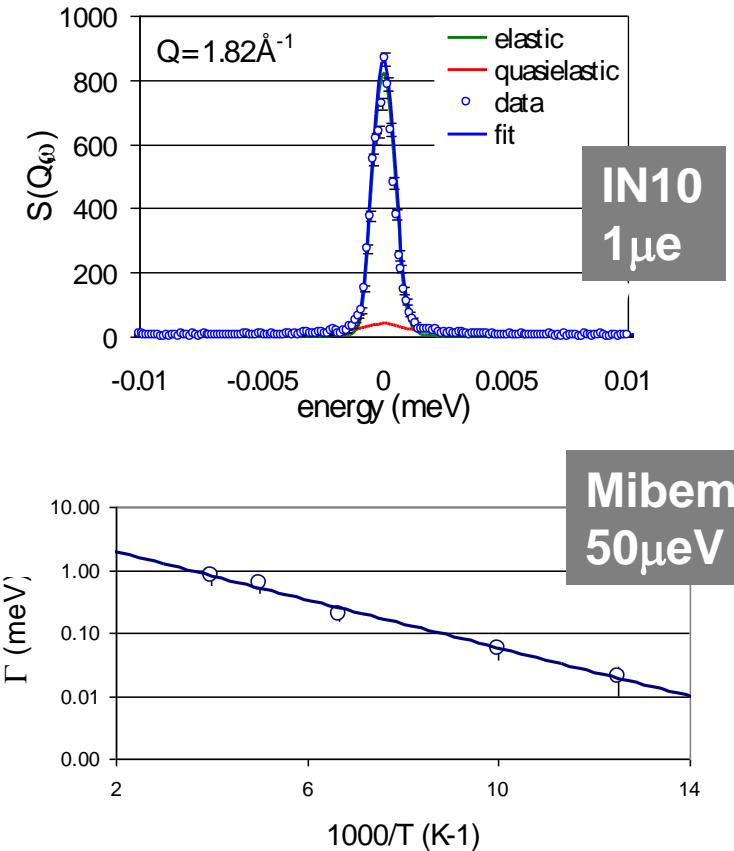
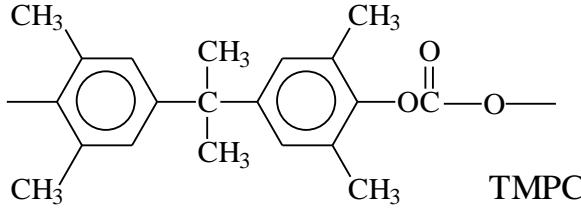
- geometry  $\leftarrow$  EISF
- activation ansatz:  $\Gamma = \Gamma_0 e^{-\frac{E_A}{RT}}$

# TMPC



Ea1~6  
kJ/mol  $\sigma 1 \sim 1$

Ea2=15  
kJ/mol  $\sigma 1 \sim 5$



# low temperature relaxation

TMPC first relaxation step:

- very low T → low  $E_0$
- rather sharp → narrow

→ candidate: rotational tunneling

Mathieu equation: inelastic lines

$$S_{\text{rot}}(Q, \omega) = \frac{5 + 4j\alpha(Qr)}{9} \delta(\omega) + \frac{2(1 - j\alpha(Qr))}{9} [\delta(\omega - \omega_t) + \delta(\omega + \omega_t)]$$

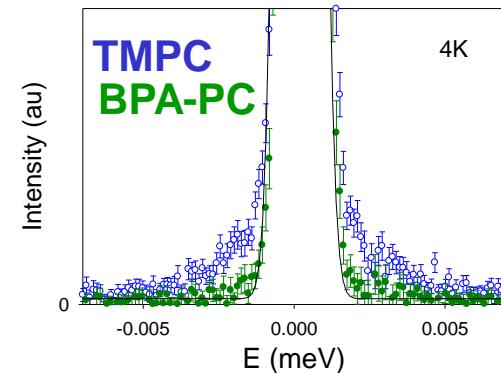
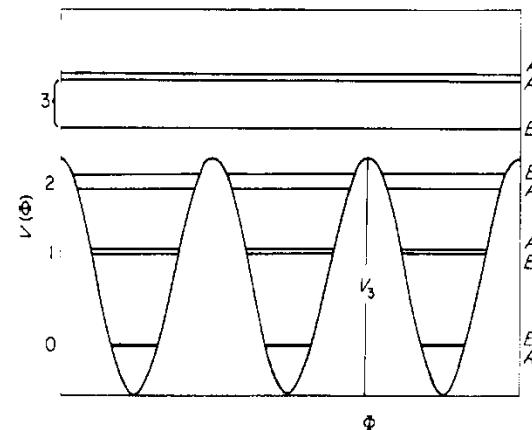
with  $\hbar\omega_t \propto E_A^{3/4} e^{-\sqrt{E_A}}$

Distribution of  $E_A \rightarrow$

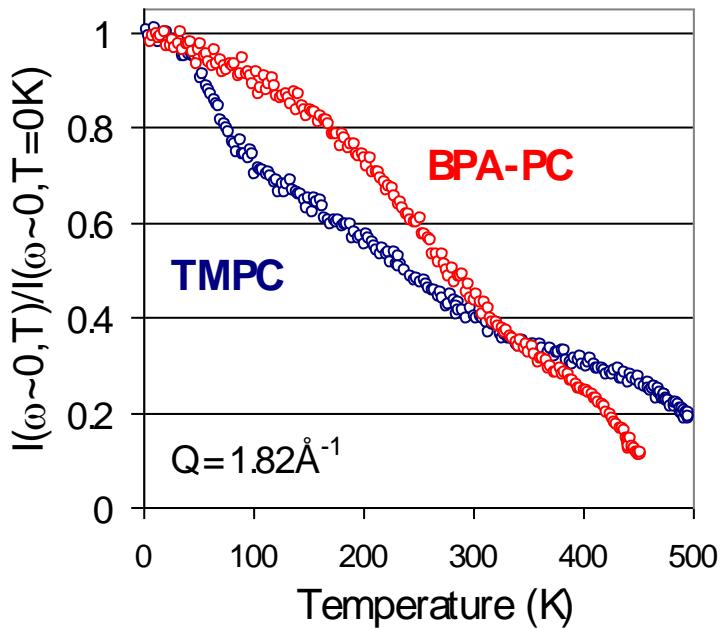
highly asymmetric distribution of  $\omega_t$

(Colmenero et al, PRL 1998)

3-fold  $\text{CH}_3$  potential



# BPA-PC

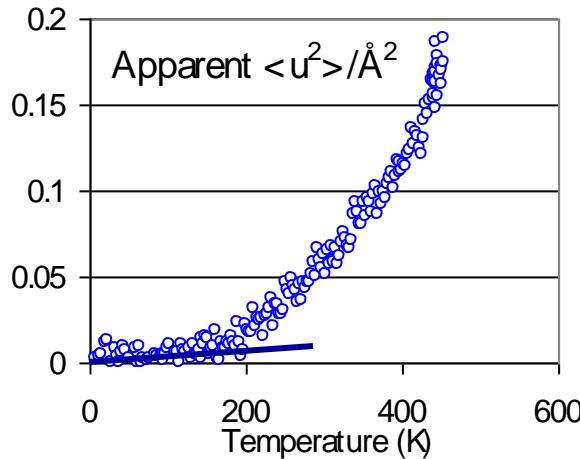
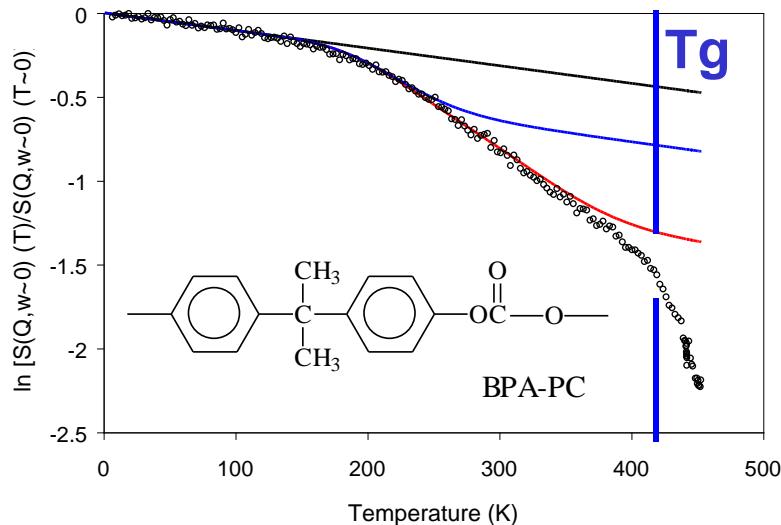


Ephenyl~37  
kJ/mol  $\sigma_1 \sim 6$

Ech3=15 kJ/mol  
 $\sigma_1 \sim 3$

compatible with TMPC

(after Spiess et al. 1987)



# Distribution?

Glassy polymers:

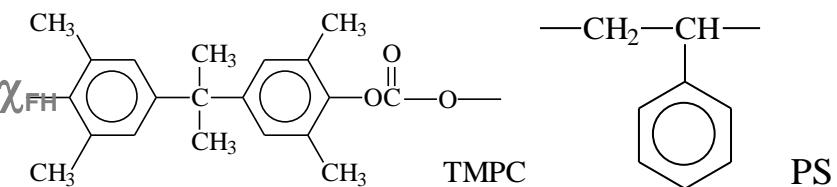
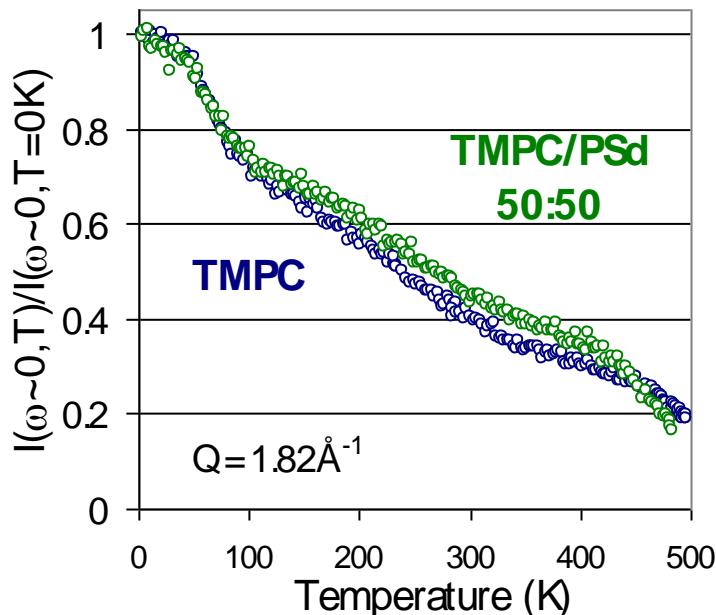
backbone chain conformation

Structural disorder

Blending

$\neq$  inter-molecular potential.  
 $\approx$  intra-

TMPC: only PC miscible with PS, large  $\chi_{FH}$



1st step: no resolvable perturbation

2nd step: broadened distribution

intramolecular environment

- average  $E_A$
- architectural considerations

intermolecular  $\rightarrow$  limited effect on  $\sigma$

# Conclusions: CASE STUDY

Characterisation local dynamics of PCs:

two architectures → toughest (BPA-PC) & most brittle (TMPC)

**Technique** combined backscattering window scans, inelastic BS & TOF

**TMPC**

exhibits two methyl relaxations of rather different distribution of potentials

**Blending**

affects  $\sigma(E_A)$

**BPA-PC**

Phenyl + methyl

